

Problems for Introduction to Probability and Simulation in Spreadsheets

1. The Connecticut Electronics company produces sophisticated electronic modules in production runs of several thousand at a time.. It has been found that the fraction of defective modules can be very different in different production runs. These differences are caused by micro-irregularities that sometimes occur in the electrical current. For a simple first model, we may assume first that there are just two possible values of the defective rate.

In about 70% of the production runs, the electric current is regular, in which case every module that is produced has an independent 10% chance of being defective.

In the other 30% of production runs, when current is irregular, every module that is produced has an independent 40% chance of being defective.

Testing these modules is quite expensive, so it is valuable to make inferences about the overall rate of defective output based on a small sample of tested modules from each production run.

(a) Make a spreadsheet model to study the conditional probability of irregular current given the results of testing 10 modules from a production run. Make a simulation table with data from at least 1000 simulations of your model (where each simulation includes the results of testing 10 modules), and use this table to answer the following questions.

(b) Before testing any modules, what is the probability of finding exactly 2 defective modules among the 10 tested?

(c) What is the conditional probability that the current is regular given that 2 defective modules are found among the 10 tested?

(d) Make a table showing the estimates of $P(\text{irregular current} | k \text{ defectives among 10 tested})$, for $k = 0, 1, 2, \dots$. (You may have trouble with some k greater than 7, but the answer in those cases should be clear.)

(e) Let \mathbf{D} denote the number of defective modules found among 10 tested, and let I denote the event that the current is irregular. Make a table showing $P(\mathbf{D} \leq k)$, $P(I \ \& \ \mathbf{D} \leq k)$, and $P(I | \mathbf{D} \leq k)$ for $k = 0, 1, 2, \dots$. (Here "&" denotes intersection, which is also often denoted by " \cap ".)