

CORRELATION PROBLEMS

1. Suppose that we are planning to make investments in three mutual funds.

Suppose that the annual growth ratios of the three funds are Multivariate Normal random variables with the following means, standard deviations, and correlations.

The annual growth ratio of Fund 1 has expected value 1.12 and standard deviation 0.15.

The annual growth ratio of Fund 2 has expected value 1.14 and standard deviation 0.18.

The annual growth ratio of Fund 3 has expected value 1.10 and standard deviation 0.19.

The annual growth ratios of Funds 1 and 2 have correlation 0.8.

The annual growth ratios of Funds 1 and 3 have correlation 0.5.

The annual growth ratios of Funds 2 and 3 have correlation 0.3.

We have a \$50,000 in cash that we can invest now. Any cash that we do not invest in these mutual funds will be put into risk-free short-term bonds that pay 5% annual interest (not compounded).

(a) Set up a spreadsheet to compute, for any level of investments in these three funds:

(i) the expectation of our portfolio's value one year from now,

(ii) the standard deviation of our portfolio's value one year from now,

(iii) the probability that our portfolio's value one year from now will be less than \$50,000,

(iv) the certainty equivalent of the portfolio one year from now, if

RiskTolerance=\$10,000.

Compute these quantities when for the plan of investing \$20,000 in Fund 1, \$15,000 in Fund 2, \$10,000 in Fund 3, and \$5000 in the risk-free bonds.

(b) Find the portfolio that maximizes the certainty equivalent of our portfolio's value next year if we evaluate such gambles with a constant risk tolerance of \$10,000.

(c) How does your answer to part (b) change if we add the constraint that all \$50,000 must be invested now in the mutual funds? (That is, nothing can be in bonds.)

2. An investor is considering two mutual funds. In each year, the logarithmic rates of return for these two funds are Multivariate Normal random variables. The logarithmic rate of return for Fund 1 each year has expected value 0.08 and standard deviation 0.20. The logarithmic rate of return for Fund 2 each year has expected value 0.09 and standard deviation 0.18. The correlation coefficient between the two funds' logarithmic rates of return each year is 0.6.

Suppose that the rates of return in any one year are independent of all past and future rates of return of these funds.

(a) Make a simulation model of the values that will be returned 5 years from now per dollar invested today in each of these two funds.

(b) The investor has \$20,000 to invest. Suppose that she puts \$5000 in Fund 1 and \$15,000 in Fund 2 today, and she plans to hold these investments for 5 years. Using at least 400 simulations of your model, show the cumulative risk profile for the total value of her portfolio in 5 years. Also, estimate the expected value and the standard deviation of her portfolio's value in 5 years.

(c) Now consider alternative investment plans putting \$X into Fund 1 and \$20,000-X into Fund 2, for X=0, X=1000, X=2000, ..., up to X=20,000. Make a table and a chart showing, for each of these plans, the expected value and the standard deviation of the total value of her portfolio after 5 years.

(d) For each of the investment plans in (c), estimate her certainty equivalent of her portfolio 5 years from now, assuming that she evaluates such gambles with constant risk tolerance \$25,000.