Party Labels and Information: The Implications of Contagion in Coelection Environments*

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Abstract

In related empirical work, we demonstrate that, relative to midterm elections, in presidential elections a more ideologically moderate electorate produces a more extreme and polarized senate. In particular, we find that senators first elected during presidential elections are relatively more extreme than their counterparts, first elected during midterm elections; conversely, we find that incumbents who leave office during presidential elections are relatively more moderate than those who leave during midterm elections. In this paper we propose a model in which the presence of party labels in an environment of incomplete information produces a contagion effect across contemporaneous races for office. In the aggregate, this contagion generates electoral outcomes that are consistent with our empirical findings and other well-known phenomena from the literature, such as midterm attrition.

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1 Introduction

The relationship between electoral representation and the characteristics of the citizenry is one of the fundamental questions in politics. While much research has demonstrated that electoral rules can affect the aggregation of preferences, less attention has been focused on the roll of concurrent races for office.[1] This is surprising as the number and importance of races being contested simultaneously varies systematically over a short period while the makeup of the citizenry is relatively unchanging. If the number of elections being simultaneously contested does have an effect on electoral outcomes, then it poses a variety of questions related to voter behavior and electoral competition.

In this paper, we present evidence that concurrent races for office have systematic and significant effects on voter behavior and election outcomes, and we propose a model that accounts for these differences. In particular, we compare voter behavior during midterm and presidential elections and the type of senator first elected during midterm elections to the type first elected during presidential elections. Although there was no explicit intent by the framers of the constitution to create electoral environments that would differ in any significant way, we find consistent and significant differences between these two cohorts of senators.[2] Surprisingly, we find that the cohort of senators first elected during presidential elections is ideologically more extreme than the cohort first elected during midterm elections.[3] Conversely, we find that senators who leave office during midterm elections

[1] There are, of course, important exceptions which we discuss further.

[2] Senators were originally chosen by state legislatures leaving the selection process explicitly separated from the electoral environment. Thus, the specific environment—presidential or midterm elections—in which senatorial candidates run for office is not intended to favor any one type of candidate over another.

[3] In the context of the United States, relative ideological extremism corresponds to more liberal positions for Democrats and more conservative positions for Republicans. As we will discuss, our left-right unidimensional ideology measure mirrors this liberal-conservative ideology framework.

are ideologically more moderate than those who leave during presidential elections. These results suggest that something beyond timing differentiates presidential-election from midterm-election environments.

We propose a model in which the presence of party labels in an environment of incomplete information produces a contagion effect across contemporaneous races for office. The presence of coattails alters the range of electorally viable positions in down-ticket races and, therefore, alters the expected type of winner and loser. Because coattails are only present in presidential elections, they constitute a significant structural difference between midterm and presidential elections. In the aggregate, this contagion generates electoral outcomes that are consistent with our empirical findings about both behavior and outcomes. In our model, candidates from the same party share a common general ideology, but differ in some very specific aspects of their respective ideologies due to idiosyncratic discrepancies and local electoral conditions.[4] Citizens have prior beliefs about party platforms, but do not know the exact party position. They may observe candidate positions in some races for office but not in all. Those who observe candidate positions in only a subset of races for office can update their beliefs about parties and candidates in other races. This phenomenon introduces a rational contagion effect, which alters the competitive landscape of races in which citizens do not observe candidate positions. A relatively more moderate candidate in the observed race creates a coattail effect for other members of her party running for office. This enables relatively more extreme and less electorally viable candidates to win. Conversely, a relatively more extreme candidate reduces the range of electorally attainable positions for his ticket. Our theoretical and empirical results suggest that previous attention to the seat-voteshare relationship and the effect of coattails on down-ticket performance alone may understate the impact that simultaneous elections have on

[4]We do not model parties or their candidate selection process directly. For examples of such models, see Snyder and Ting (2002) and Caillaud and Tirole (2002).

each other. A broader implication of our model is that the introduction of an unbiased public signal may have perverse effects on the aggregation of private information and preferences.

Before proceeding with a formal presentation of the model, we first describe the relevant institutional details and motivating empirical facts from Halberstam and Montagnes (2009a) and other sources. A brief discussion of the relevant literature follows. In section 3, we present our theory model in detail and in section 4 we derive the main results of our model as related to individual and aggregate voter behavior and electoral outcomes. Following a discussion of these results and other possible explanations for the empirical regularities in section 5, we conclude with a discussion of possible theoretical implications of our work on models of voting and party competition and the policy implications of our results.

2 Midterm and Presidential Elections

Federal elections are held on the first Tuesday following the first Monday in November during even-numbered years. When two or more elections for office are held concurrently, we term them coelections. Members of the House of Representatives, senators, and presidents serve terms of two, six and four years, respectively. Although all scheduled elections occur during even-numbered years, term-length variation produces disparate electoral cycles.

The staggered terms of the presidency and congressional offices in the United States create two electoral cycles: midterm elections and presidential elections. During midterm elections, the entire House of Representatives and one-third of the seats in the Senate are contested, whereas, during presidential-election years, in addition to congressional races, the presidency is contested. These two electoral environments generate two cohorts of senators: those who are first elected to the Senate concurrently with a president

and those who are first elected in midterm elections, between presidential-election years.[5] The existence of these two election cycles is an artifact of the variation in term length between senators and presidents.

Institutionally, midterm and presidential electoral environments differ only in the number of offices contested. The competitive landscape is similar in both elections: two major parties contest each race for office in which the winner is determined by a plurality rule. Nonetheless, turnout and voting decisions of citizens and the electoral outcomes they generate differ significantly between midterm and presidential elections. In this section, we review these differences with references to well-known phenomena as well as newly uncovered differences. We use this set of stylized facts to motivate our theoretical model, which follows.

2.1 Turnout

We begin by highlighting a well-known empirical regularity related to the cyclicality of midterm and presidential elections. Figure 1 illustrates the systematic oscillation in voter turnout between the low turnout rates in midterm elections and the significantly highers ones in presidential elections. We employ nationwide general-election turnout data from 1968 to 2008.[6] The bottom set of points in the figure is associated with turnout in midterm elections, and the top set, with presidential elections. The average turnout rate during this period is 58.1 percent and 42.4 percent in presidential and midterm elections, respectively. Over the 40-year period, the average increase in turnout for presidential elections was over 37 percent relative to midterm elections. The scatter plot demonstrates the magnitude of this variation by comparing the turnout in a given general election to the turnout in the preceding general election. The turnout ratio of presidential elections to

[5] We exclude from our analysis appointed senators.

[6] There are two common measures of voter turnout, Voter Age Population (VAP) and Voter Eligible Population (VEP). We use VAP rates; however, a similar pattern emerges for VEP rates. Source: McDonald and Popkin (2001) for years 1968-2000 and McDonald (2009) for 2002-2008.

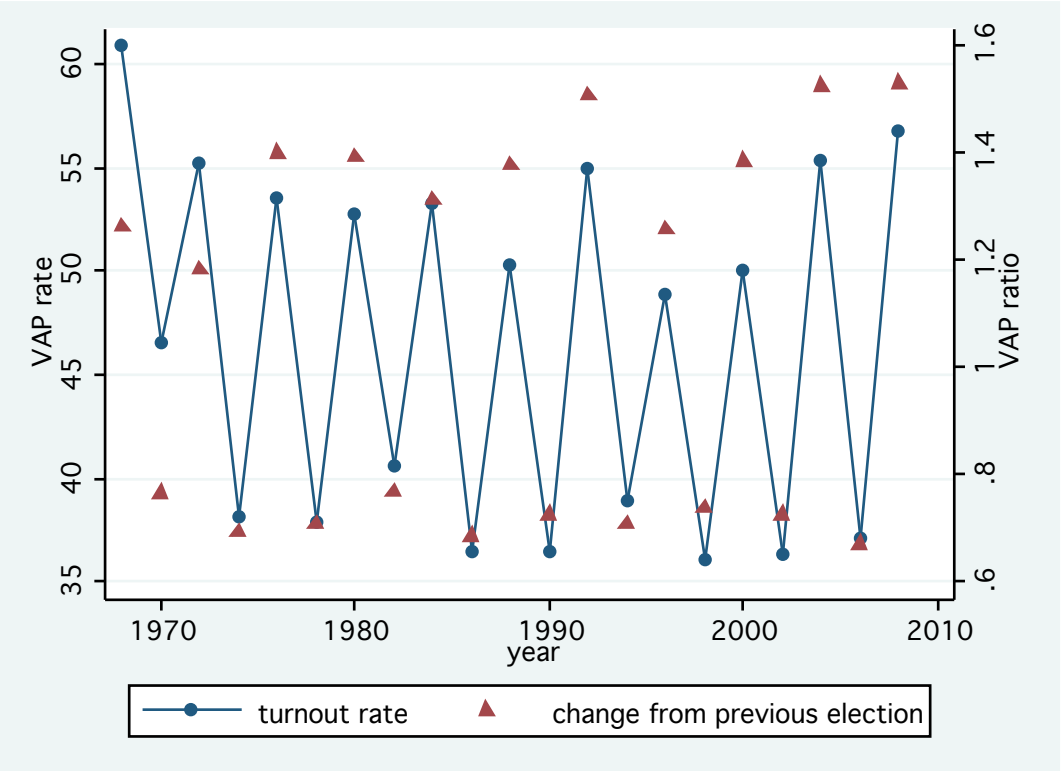


Figure 1: U.S. General-ElectionTurnout

midterm elections varies from a low of 125.6 percent in 1996 to a high of 153 percent in 2008. Similarly, the amount of turnout decline in midterm elections varies from a low of 67.1 percent in 2006 to a high of 77.7 percent in 1982.

In addition to the oscillation observed at the national level, the turnout cycle is also present and consistent at the state level. Looking at the number of votes cast at the state level, the empirical regularity is consistent across all fifty states. To account for the fact that statewide offices are not contested in every general election, we compare the number of votes cast in senatorial races during midterm elections to those cast during presidential elections. We compute the ratio of the average number of votes in presidential elections to that in miderm elections from 1916 to 2006.[7] In Figure 2 we plot the frequency histogram of these state-level ratios. In all states, on average, more votes are cast in senatorial races held during presidential-election years than during midterm elections. There is substantial variation in the magnitude of this ratio across states. The lowest variation is in Hawaii, where the average number of votes in senatorial races held during presidential-election years is only 102 percent of the midterm level. The greatest turnout swing is in Georgia, where the number of votes cast during presidential election years is 171 percent of the midterm level. Overall, the average number of votes for senators during presidential-election years is 24 percent greater than the midterm level.[8]

[7]Figure 2 does not account for growth in VEP, but since that growth does not vary systematically with electoral cycles, we need not control for it.

[8]The discrepancy between the rates reported here and the 137 percent turnout level increase arises for two reasons. First, our period of analysis for votes is much longer and includes periods in which the differences were less pronounced. And second, Figure 2 is an unweighted average of states; states with low midterm-presidential turnout variation tend to be less populous states. (The five states with the lowest variation are Hawaii, Wyoming, North Dakota, Montana and Minnesota; the five states with the greatest variation are Georgia, Mississippi, Kentucky, Texas and North Carolina).

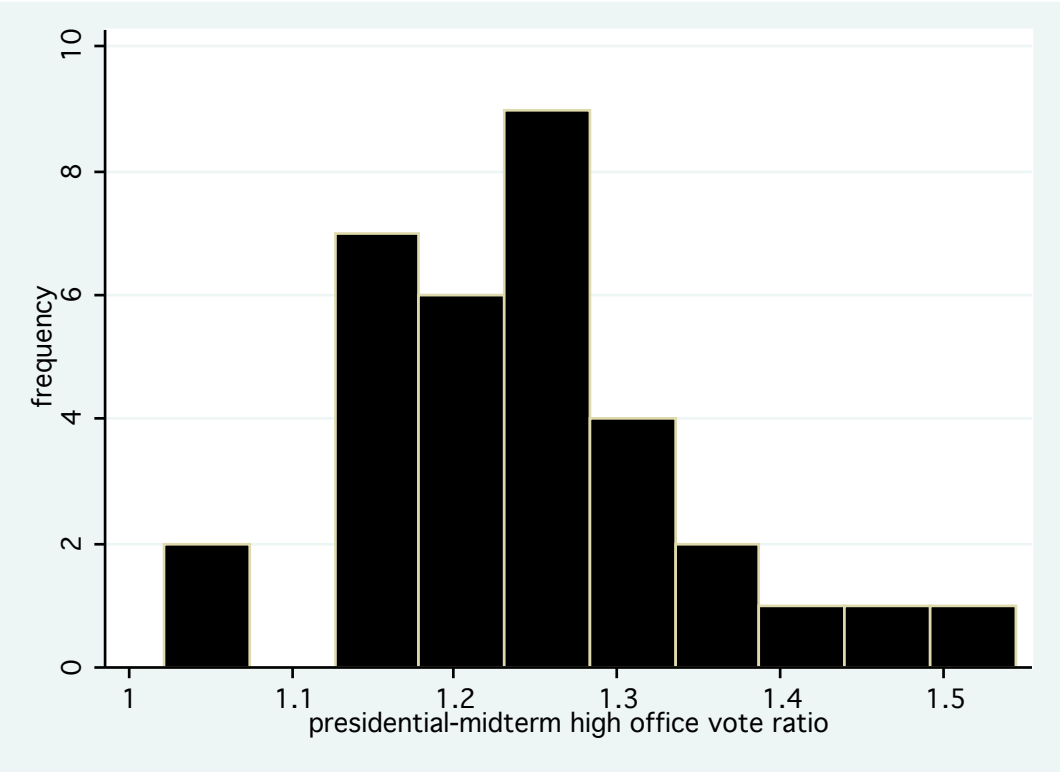


Figure 2: State-Level High-Office Turnout Ratio

2.2 Roll-Off

While turnout regularly increases during presidential-election years, not all voters cast their votes in all races for office in a given election. For example, many voters choose to vote for a presidential candidate (the up-ticket race), but abstain from voting for senatorial or house candidates (the down-ticket races). This phenomenon, termed roll-off, presents a puzzle for many models of voting; conditional on turning out and voting for a presidential candidate, the marginal incremental cost of casting a vote for a senatorial or house candidate is essentially zero.

Conventional wisdom holds that roll-off occurs when there are more votes for higher offices (such as that of the president) than for lower offices (such

as that of a senator.) Looking at the universe of 752 senatorial races held during presidential elections between 1916 and 2004, we find strong evidence of substantial roll-off.[9] Overall, when roll-on—a lower race for office receives more votes than a higher one—occurs, the difference in the number of votes between races is usually small, whereas with roll-off, the differences are of greater magnitude. In 680 observations with roll-off, the median number of votes in a presidential race had 3.4 percent more votes cast than in the corresponding senatorial race.[10]

While small magnitudes of roll-off and roll-on are the norm, there is considerable variation. In Figure 3 we plot a frequency histogram of the roll-off rate—the fraction of voters who cast their vote for president and abstain in the senatorial race. There are many elections with significant roll-off. In 29 percent of the cases, the amount of roll-off is greater than five percent (that is, the number of votes cast in the senatorial races is less than 95 percent of the number cast in the presidential race), and in 12 percent of the case, the amount of roll-off is greater than 10 percent.

Although roll-off is widespeard, the drop in votes due to roll-off in particular is dwarfed by the drop in turnout in midterm elections relative to presidential elections. Combining these two empirical facts, the relative size of the electorate can be ranked as follows: on average, the number of people in a given state voting for president is greater than the number casting their vote in the senatorial race for office, which in turn is greater than the number voting for senator during midterm elections.

[9] There are a non-trivial number of exceptions (i.e., roll-on). In 9.6 percent of cases, the number of votes cast in a senatorial race is higher than the number of statewide votes cast in the presidential race.

[10] In the 72 races with roll-on, the median number of votes cast in a senatorial race was less than one percent greater than the number cast in the presidential race.

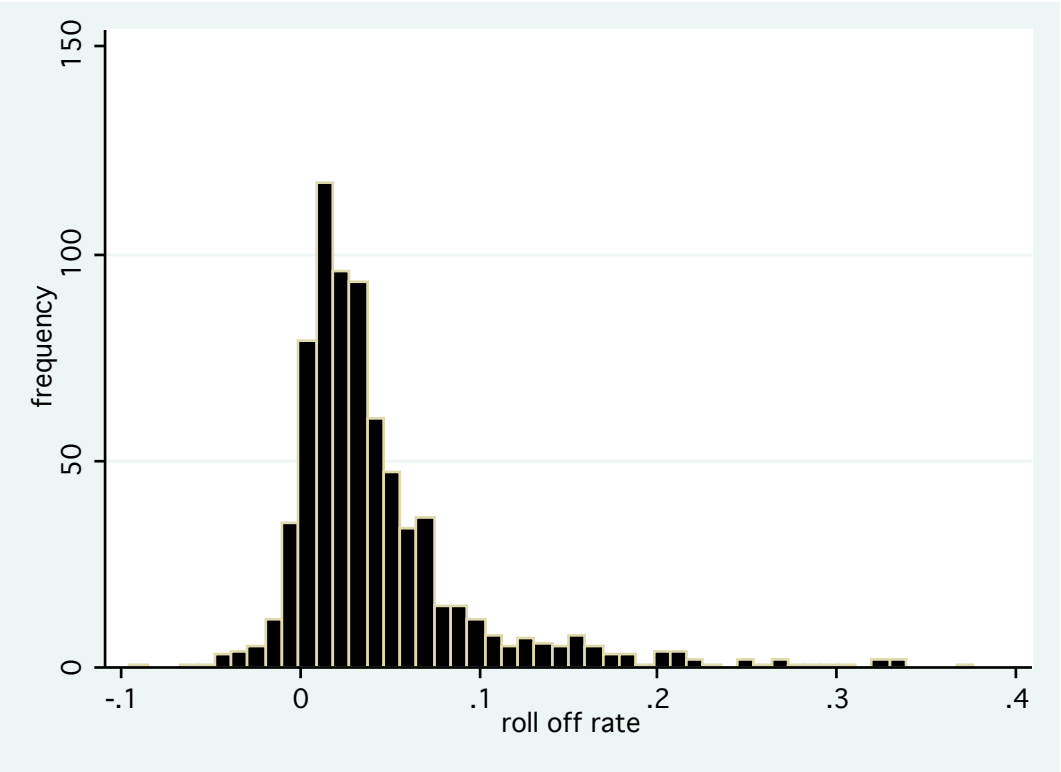


Figure 3: Presidential to Senatorial Race Roll-Off

2.3 Presidential Coattails and Midterm Attrition

The discussion thus far has focused on voting behavior, we now begin to present facts regarding outcome difference and regularities between the two environments. Perhaps the most widely known facts about outcomes in presidential and midterm elections are the associated surge and decline in support of the president's party in congress. Many scholars have studied the relationship between presidential voteshares and the the change in the president's party's share of congressional seats following a presidential election, commonly referred to as presidential coattails. In all but a handful of cases, the president's party experiences a surge in support of other candidates from his party resulting in an increase of its share in congress. Conversely, during

Table 1: President's Party Gain/Loss, 1932-2008

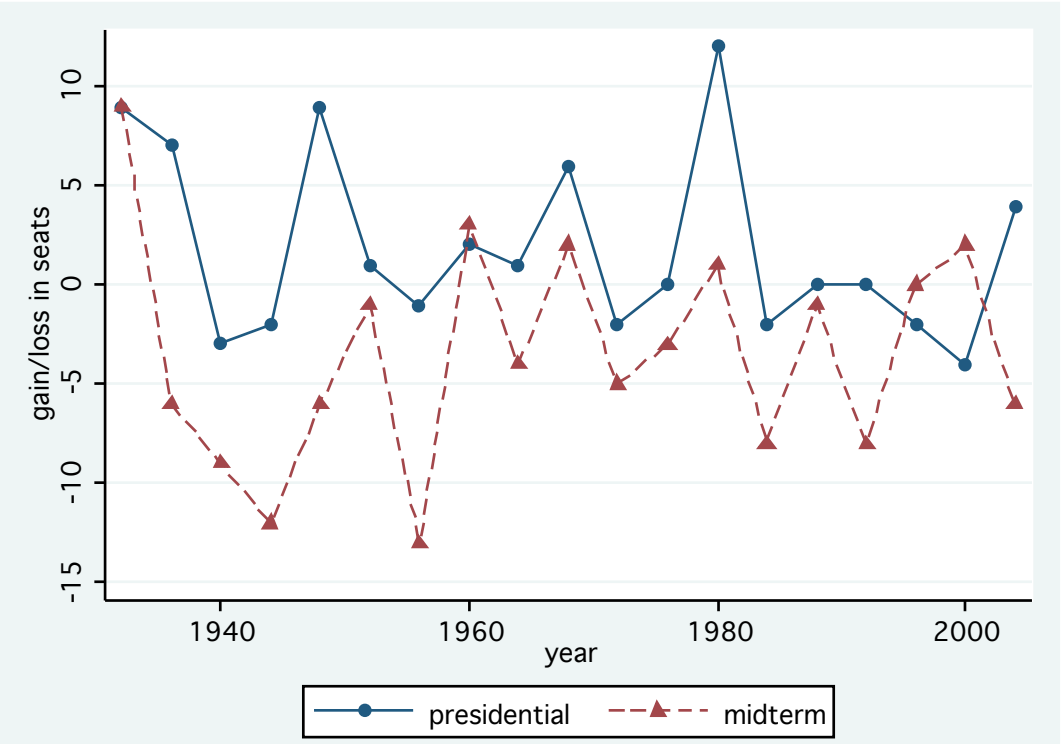
Independent Variable: Seat Change		
	(1)	(2)
	Senate	House
Presidential dummy	5.571**	41.39**
	(1.66)	(8.23)
Constant	-3.421*	-25.84**
	(1.28)	(5.49)
Observations	39	39

Robust standard errors in parentheses
** p<0.01, * p<0.05

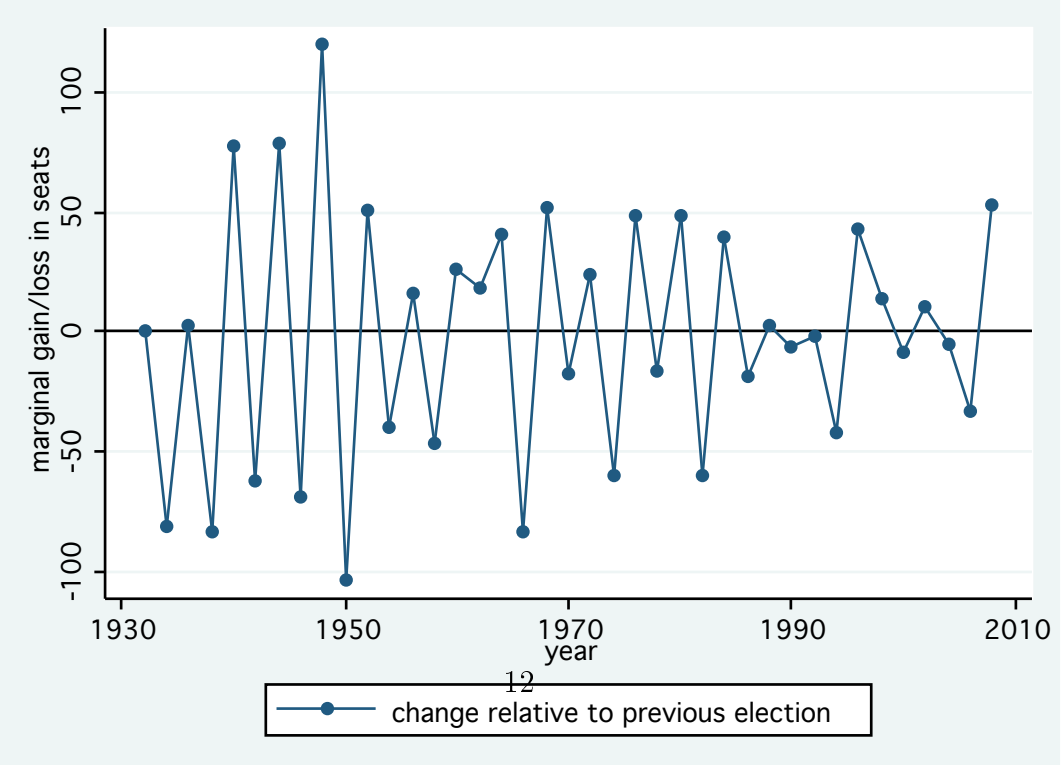
midterm elections, the president's party experiences a decline. In all but four elections since the Civil War, the party of the president has lost seats in the House of Representatives.[11] The regular surge and decline has motivated a variety of theories that explore the differences between midterm and presidential elections (see Campbell (1960), Campbell (1991), Campbell (1997), Tufte (1975), Kernell (1977), Erikson (1988), Alesina and Rosenthal (1989), Alesina and Rosenthal (1995) and Alesina and Rosenthal (1996).

In both the House and the Senate, the presidential surge and midterm decline is significant. In Figure4a , we compare the change in the president's party's share in the Senate following presidential elections to the change following midterm elections two years later. The difference between the presidential and midterm plots amounts to the likelihood that a senatorial candidate from the president's party will win office during presidential elections

[11]See Patty (2005). Even the four exceptions are somewhat instructive. The most recent exception occurred in 2002 after a presidential race in which the winner failed to win the popular vote. The most recent exception prior to that occurred in 1998, following a presidential race in which a third-party candidate received approximately eight percent of the vote. A third exception followed the extraordinarily close 1960 race, and the very first exception followed Roosevelt's first election and occurred in the midst of the depression.



(a) Senate Seats



(b) House Seats

Figure 4: Surge and Decline of President's Party, 1932-2008

relative to the likelihood of winning during midterm elections. In nearly all general elections over the past 80 years, the gain following a presidential election was higher than that after a midterm election. Another way to demonstate the regularity is to compare the marginal change in the president's party's share following a given general election to the change following the previous election. In Figure 4b we plot the number of seats in the House gained (or lost) following one election subtracted by the change in the number of seats following the previous election. In the vast majority of cases, the net difference in presidential elections is positive, and in midterm elections, negative. The handful of outliers here coincide with those in Figure 4a for the Senate. In Table 1, we display the average gain (or loss) for the president's party in the House and the Senate. Overall, midterm elections are associated with significant losses in both chambers of congress, whereas presidential elections are associated with significant gains.

The surge and decline phenomena coupled with the differences in voting behavior suggest that there may be outcome differences between the presidential and midterm environments beyond the change in congresisonal seats. In the next section, we explore these differences.

2.4 Electoral Bias During Entry and Exit

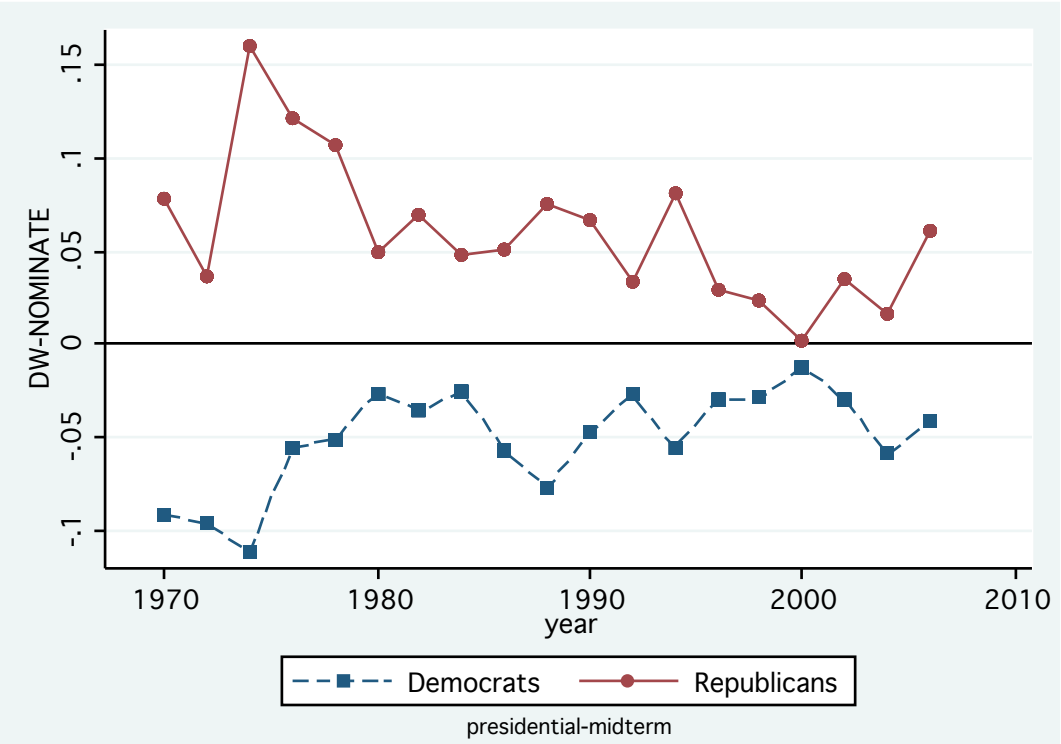
While midterm decline suggests that outcomes may differ between presidential and midterm elections, it leaves open the question of whether this is merely a partisan effect or if outcomes are different in a more substantial manner. For example, do representatives elected during midterm and presidential elections differ in their voting records or their degree of party loyalty? In Halberstam and Montagnes (2009a), we suggest that they do in fact differ significantly and in a surprising way–senators first elected during presidential elections are more ideology extreme and party loyal than those first elected during midterm elections.[12]

[12]See Halberstam and Montagnes (2009a) for amore detailed discussion of the methodology and robustness of the results.

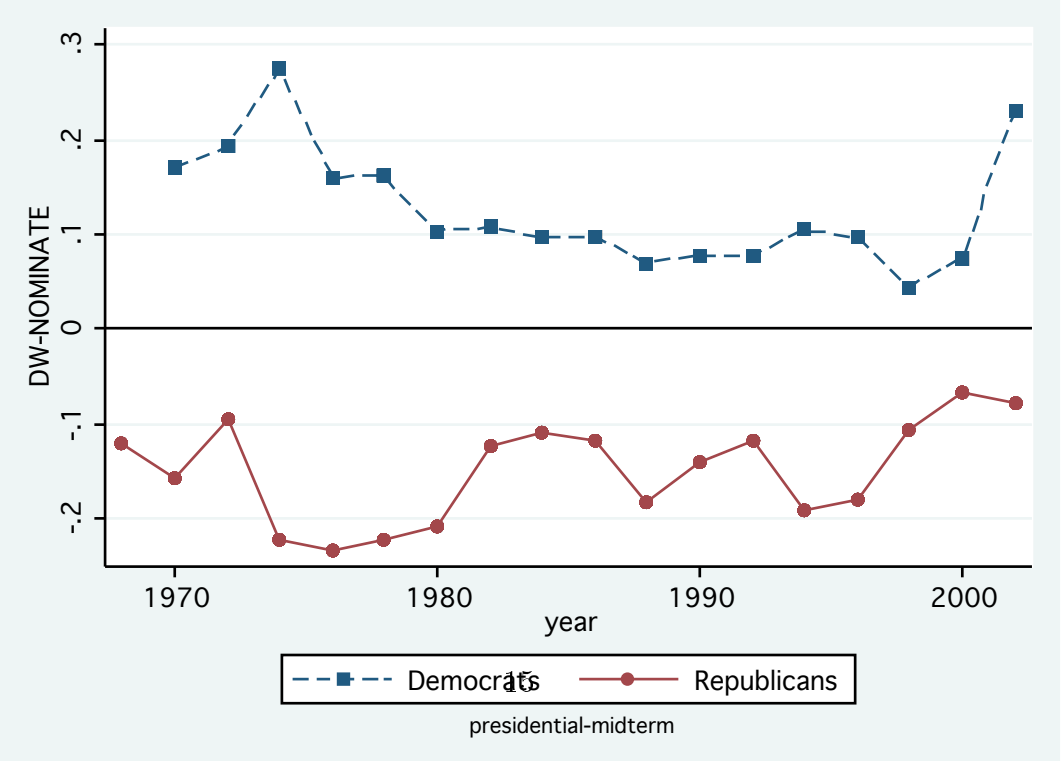
If we segment the Senate by party, and then further divide senators by the electoral environment in which they won office, we arrive at four distinct groups. One is the cohort of Democratic senators first elected to the Senate during midterm elections, a group we term the Democratic midterm cohort. Another is the cohort of Democratic senators first elected to the Senate during presidential elections, the Democratic presidential cohort. The final two, similarly obtained cohorts are the Republican midterm cohort and the Republican presidential cohort.

For each party, we can compare the ideology of senator cohorts by computing the average DW-NOMINATE scores of the entry and exit cohorts. In Figure 5a we demonstrate the relative extremism of presidential-entry cohorts by comparing them to midterm-entry cohorts. We normalize the scores by subtracting the average midterm ideology scores from the presidential ones; thus, the baseline horizontal line at zero represents the normalized average DW-NOMINATE scores for senators first elected during midterm elections. The solid and dashed curves correspond to the normalized average scores for Republican and Democrat presidential cohorts, respectively. In this figure, we note that for both parties in each congress in our dataset, when splitting the Senate into midterm-election entrants versus presidential-election entrants, we find that presidential-election entrants are consistently more ideologically extreme than their counterparts, i.e., those first elected during midterm elections.

In addition to observing the differences between senator entry during midterm and presidential electios, we establish a similar, but distinct, effect related to exit. As in senator entry, we divide the Senate into cohorts by party and exit environment into midterm and presidential exit cohorts. We note that incumbents who leave or are defeated during presidential elections tend to be significantly more moderate than those who exit during midterm elections. We display this regularity in Figure 5b. In Table 2 we report the regression results for entry and exit.



(a) Entry



(b) Exit

Figure 5: Ideology Differences Between Midterm and Presidential Cohorts

Table 2: Weighted Least Squares

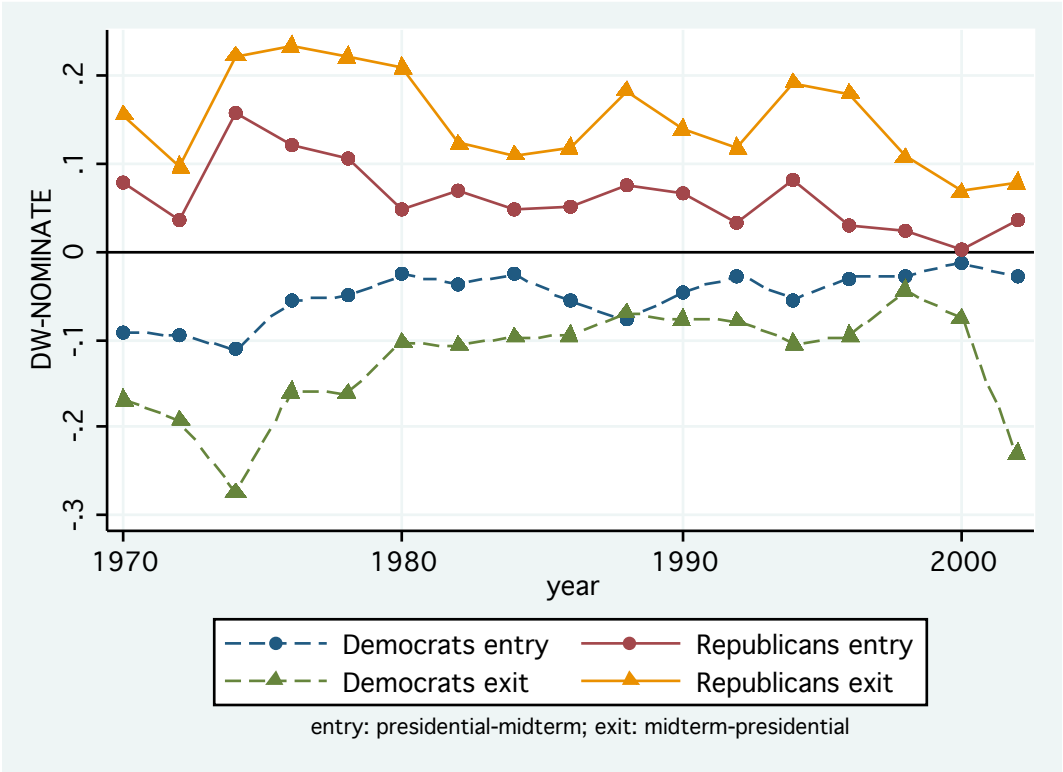
Independent Variable: DW-NOMINATE				
	Entry		**Exit**	
	Democrat	Republican	Democrat	Republican
Presidential dummy	-0.0547*	0.0890**	0.136**	-0.171**
	(0.022)	(0.001)	(0.000)	(0.000)
Constant	-0.257**	0.319**	-0.356**	0.439**
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	697	742	379	504

Robust p values in parentheses

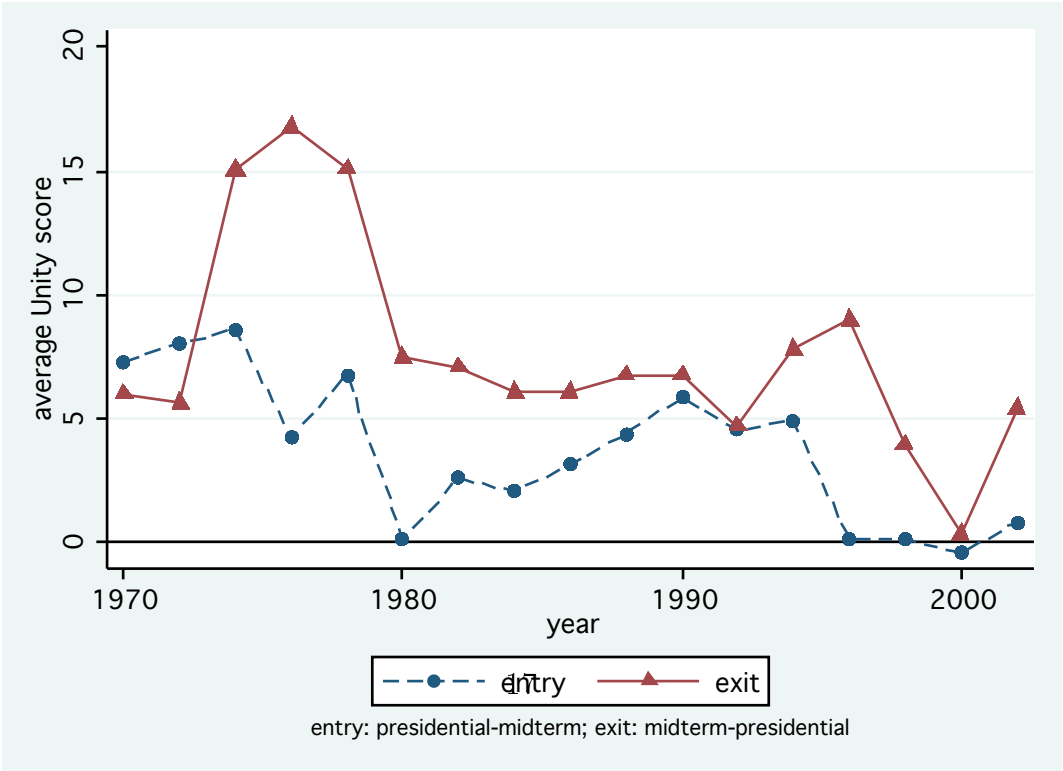
** p<0.01, * p<0.05

Looking at these results and comparing them to the previous results on entry, we find that the ideological differences during exit are substantially greater than the ones during entry. This allows us to nest the differences for entry within the differences for exit. In Figure 6a we nest figure 5a within the inverse of figure 5b. This amounts to comparing the ideological differences between entry and exit for each party. The plots suggests that the overall differences are not mitigated by the turnover of senators. On the contrary, Figure 6a suggests that during presidential elections, while moderates are leaving, they are not being replaced with senators of a similar ideology; rather, relative extremists are replacing relative moderates. Whatever the ideological differences are between midterm-entry and -exit cohorts, the ideological differences that is generated by senator turnover during presidential elections is significantly amplified.

Finally, we find an ancillary result related to the party loyalty of senators. Using Unity scores, we run an analogous analysis to the one we executed



(a) Nesting Ideology



(b) Nesting Party Discipline

Figure 6: Nesting Entry in Exit

using ideologies.[13] Unlike ideology, however, we do not need to segment senators by party. We display our results in Figure 6b. In this figure, we observe a similar pattern to the results on ideology. The presidential-entry cohort is persistently more party disciplined than the midterm one; conversely, the presidential-exit cohort departs from party line voting significantly more often than the midterm-exit cohort.

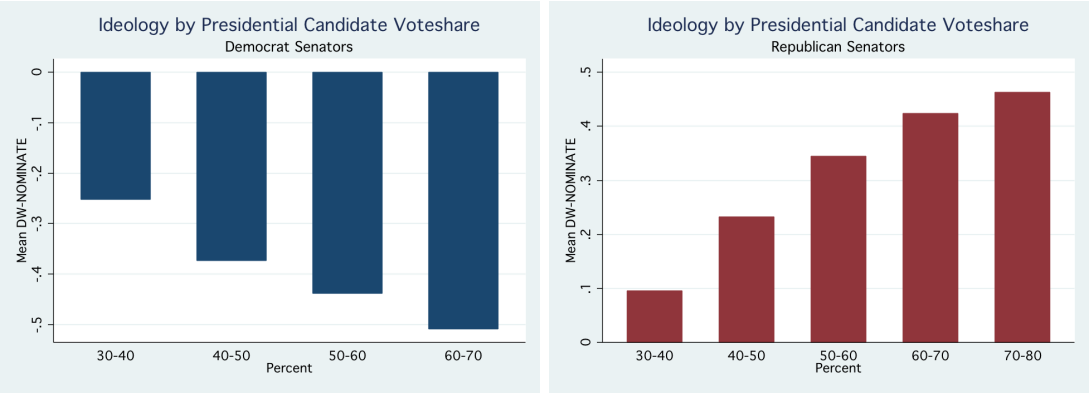
These patterns in electoral outcomes are surprising given that the ideological composition of the midterm electorate is more ideologically extreme than the one found in presidential elections.

2.5 Party Matching and Ideology

Our previous findings suggest that the presidential electoral environment returns a more extreme and polarized Senate than the midterm electoral environment. Before proceeding with a detailed description of the model and our results, we examine more closely the underlying structure of these different returns. Figure 7 demonstrates that increased support for a president is associated with increasing ideological extremism for senators from the same party. Restricting our attention to senators who enter during a presidential-election cycle, we classify each senator in our dataset by her party identification and the voteshare decile of her party's presidential candidate who ran for office at the time of her entry. Following this classification, we calculate the mean DW-NOMINATE score for all our observations that fall into each of these decile groups. Note that not all decile groups contain observations. The notable observation is that for each party the mean DW-NOMINATE score becomes (strictly) more extreme as we move up the presidential voteshare decile group.

Next, we demonstrate across time the relationship between the ideolo-

[13] A Unity vote is defined as a roll call in which at least fifty percent of Republicans vote against at least fifty percent of Democrats. A Unity score is the percent of votes one voted with her party from the total Unity roll calls on which she cast her vote; thus, a higher score implies greater party loyalty (e.g., a score of 100 implies that on all Unity roll calls in which a senator participated he voted party line).



(a) Democrat Senators (b) Republican Senators

Figure 7: Presidential Voteshares and Senator Ideology

gies of senators at their time of entry and presidential voteshares. For each congress in our dataset we first segment the Senate by party identification. Next, we segment each party into three groups of senators. In the first group are those first elected during midterm elections. The next group, includes those elected during presidential elections in states where their party's presidential candidate garnered the plurality vote. The final group contains senators elected during presidential elections in states where their party's presidential candidate was defeated. For each of the twenty congresses in our dataset, we compute the respective average DW-NOMINATE scores for each of these groups. For expositional reasons, we then normalize these averages by using the midterm averages computed for each congress; thus, the zero horizontal reference line is associated with the midterm cohort of senators in each given congress. Senators who ran with defeated presidential candidates ("Unmatched") are, in expectation, more moderate than those elected during midterm elections. In turn, midterm-entry senators are more moderate than senators who entered during presidential elections in which the presidential race in their state was won by their party's candidate ("Matched"). Figure

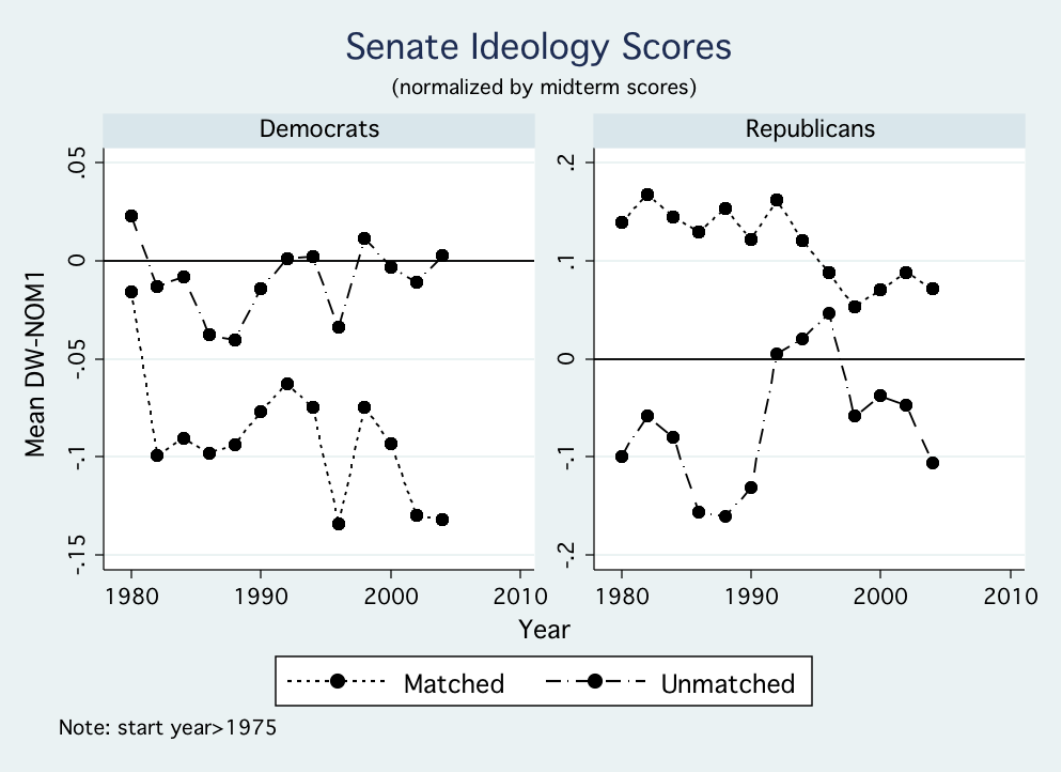


Figure 8: Matched and Unmatched Senators

8 corresponds with these statements. We observe that the average ideology scores associated with Matched senators are consistently more extreme than those of midterm senators. In turn, the ideologies of midterm-entry cohorts are on average more extreme than those of Unmatched senators. Less distinct is the ranking between Unmatched and midterm senators. Overall, 68 percent of entering senators since 1975 are Matched senators. Equivalently, 32 percent of exiting senators are Matched with the presidential candidate who received the plurality vote.

3 Model

In the previous section, we argued that midterm and presidential electoral environments are distinguished by significant, systematic differences in both

voting behavior and electoral outcomes. In this section, we present a model that explains these variations in a systematic and parsimonious manner. The key assumption of our model is that voter utility depends on information, and that party labels have informational value.[14] The presence of multiple races at a single time generates more information for voters, but also introduces information contagion between the races. As we will demonstrate, this contagion generates significant, but predictable, differences in voting behavior, which in turn generate the observed differences in aggregate behavior and outcomes.

In order to focus on elections with systematic variation in the number of races being contested, we model voting behavior for statewide races for Senate in both electoral environments. Thus, we model senate and presidential races at the state level and consider in our analysis the results of the state-level senate races.[15]

3.1 Policy Space and Parties

Let the policy space be $P \subseteq \mathbb{R}$ and let $s \in \{1, 2, 3, ..., 50\}$ denote one of the 50 states. In each race there are candidates from two parties, L and R, running for office. As discussed, a major difference between presidential and midterm elections for senators is the presence of concurrent presidential elections. Depending on the election cycle, there are one or two races for office.[16] We denote presidential and senatorial candidates with superscripts p and s, respectively.

[14] For recent work documenting the interaction of information and spatial voting, see Jesse (2009 and forthcoming).

[15] An analysis of the strategic interaction of party-platform choice across multiple heterogeneous districts (as in our model) is beyond the scope of this paper. For analysis of such issues, see Austen-Smith (1981) and Callander (2005).

[16] Some states may not include a race for Senate during a given presidential election cycle. Because we are interested in model results of Senate races, we will not consider such cases in our analysis. To clarify, we categorize a senate race as occurring during presidential elections if it is held concurrently with the presidential race, and deem it a midterm race otherwise.

Let $C_q^r \in P$ denote the position of a candidate from party $q \in \{L, R\}$ running for office $r \in \{p, s\}$. Candidates from the same party are ideologically similar, but differ due to individual idiosyncratic reasons, local state conditions, or other factors; however, in a given race for office, the candidate from party L will always be to the left of the candidate from party R in any given race.[17] That is,

$$C_L^r \leq C_R^r \quad \forall r \in \{p, s\}.$$

As we will see shortly, the underlying preferences of voters in our model are a form of proximity preferences. Thus, given our assumption about candidate ordering in the policy space, a sufficient statistic of any given race is the midpoint of the candidate position. It will often be clearer both notationally and conceptually to discuss the midpoint of a race, so let $M^r \equiv \frac{C_L^r + C_R^r}{2}$ be the ideological midpoint of candidate positions running in the race for office r.

In our model, senatorial and presidential candidates from the same party share a degree of ideological similarity.[18] There are variety of mechanisms that may account for such congruence of candidate positions. From a candidate choice perspective, Snyder and Ting (2002) make the argument that candidates with similar ideological perspectives may join parties in order to reduce information costs for voters. Joining a party is costly if the party presents a different ideological position than the candidate. This cost imposes ideological consistency across party members and allows the party label to convey information to voters.[19] Alternatively, the candidate-selection

[17] Empirically, we observe that Democrat senators are almost always to the left of Republican senators; however, our assumption is made for state-level races, and in our data the assumption is always true when we observe senators from both parties in one state.

[18] While we model this congruence in a particular parametric manner for tractability, the essential assumption we need to make is that there is some commonality in ideological positioning.

[19] This concept of party labels as informational signals is very much in the spirit of our paper. For other work that makes this argument, see Cox and McCubbins (1993) and

process, fundraising, and the behavior of party elites may serve to generate common ideological positions among candidates from the same party.[20]

While senatorial and presidential candidates may share some common ideological positioning, they are not perfectly aligned with respect to the underlying platforms of their parties. Some state-level Senate races occur under different political environments than the presidential race or Senate races in other states. The electoral environment and citizenry attributes of states vary, with some states being more conservative and others more liberal. These differences in state characteristics are also reflected in the positions of senatorial candidates across parties.

As citizens' conditional voting decisions depend on proximity preferences, given our ordering of candidates by party, the essential element of party competition for citizen decision making is the distribution of the midpoint of candidate positions in races.[21] Modeling only the midpoints of party competition has several advantages. The first is tractability; by not modeling the underlying party competition process, it is much easier to aggregate the underlying decision process of citizens. Additionally, the updating process between races that citizens employ and comparative statics on the relevant model primals will be transparent. Second, this approach highlights the robustness of our results to a variety of models of party competition. Many spatial models of party competition will result in some distribution of candidate positions (including degenerate distributions). These distributions of candidate positions will in turn generate a distribution of midpoints, as in our model. While we employ the Normal distribution for tractability and transparency purposes, our results are robust to any underlying midpoint-generating process. [22]

Aldrich (1995).

[20]See Gerber and Morton (1998) and Besley and Case (2003) for a discussion of the candidate-selection process.

[21]Since the particulars of party strategy are in and of themselves not of interest to voters, we focus on the decision-relevant consequences of party competition.

[22]For empirical evidence that corresponds to our model of candidate selection, see Ansolabehere, Snyder and Steward (2001).

Let M^p be the midpoint of candidates in the presidential race, and M^s be the midpoint in senatorial races. Equations (1) and (2) describe the model of midpoints in presidential and senatorial races, respectively.

$$\underbrace{M^p}_{\substack{\text{Presidential Race}\\ \text{Midpoint}}} = \underbrace{\Omega}_{\substack{\text{Party}\\ \text{Midpoint}}} + \underbrace{\epsilon^p}_{\substack{\text{Presidential Race}\\ \text{Idiosyncratic Effect}}} \tag{1}$$

$$\underbrace{M^s}_{\substack{\text{Senatorial Race}\\ \text{Midpoint}}} = \underbrace{\Theta^s}_{\substack{\text{State}\\ \text{Fixed Effect}}} + \underbrace{\Omega}_{\substack{\text{Party}\\ \text{Midpoint}}} + \underbrace{\epsilon^s}_{\substack{\text{Senatorial Race}\\ \text{Idiosyncratic Effect}}} \tag{2}$$

Let the overall midpoint of party competition, Ω, be fixed but otherwise unknown to citizens. Let citizens' prior beliefs over the unknown party midpoint be represented by a Normal distribution with mean μ_ω and variance σ^2_ω. Additionally, let the idiosyncratic, race-specific effects be independently distributed Normal with mean zero and variance $\sigma^2_{\epsilon,r}$ for $r \in \{p, s\}$. The state fixed effect is assumed to be non-stochastic. The overall priors over midpoints in presidential and senatorial races are

$$M^r \sim N\left(\mu_r, \sigma^2_r\right) \quad \forall r \in \{p, s\}, \tag{3}$$

where $\mu_s = \mu_\omega + \Theta^S$, $\mu_p = \mu_\omega$, $\sigma^2_p = \sigma^2_\omega + \sigma^2_{\epsilon,p}$ and $\sigma^2_s = \sigma^2_\omega + \sigma^2_{\epsilon,s}$.

Since the senatorial and presidential candidates share a common party element, citizens can update their priors about the senatorial race using information from the presidential race; thus, it follows that the updated prior over Ω conditional upon observing the presidential-race midpoint m^p, is[23]

[23] See Degroot page 147.

$$(\Omega|M^p = m^p) \sim N\left(\mu_\omega + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right), \sigma_\omega^2\left(1 - \rho_{\omega,p}^2\right)\right), \qquad (4)$$

where $\rho_{\omega,p}$ is the correlation coefficient between presidential-candidate midpoints and the prior distribution over Ω.

Two comments about our model of midpoints and party competition are warranted, the first being incidental, and the second, substantial. We have chosen to have the midpoints of candidate position be Normally distributed around a competition mean. While the choice of a Normally distributed idiosyncratic shock is innocuous, the choice of a Normally distributed prior over midpoints has no direct basis in theory per say. The choice is made in order to make the updating process for voters clear and tractable. Our results do not depend substantially on these assumptions and are robust to a variety of underlying models of party competition. The second, more substantial assumption in our model of parties is that the choices of candidates are not conditional on the selections in other races.[24]

3.2 Citizens

3.2.1 Citizen Preferences

The main objective of our model is to explain voting behavior and regular differences in outcomes between midterm and presidential elections; thus, we are less concerned with the particular details of the micro foundations of preferences and party competition. Nonetheless, we face a trade off when studying large elections: the tractability and clarity characteristic of modeling agents in a continuum comes at the cost of providing little motivation for strategic rational citizens to turn out and vote in any race for office since

[24] We implicitly assume that parties are playing a simultaneous move game. This assumption depends on the sequential nature of candidate selection, entry and primaries at the state level. In our view, this process is best viewed as a simultaneous game.

each citizen's likelihood of being pivotal is, essentially, zero. Since we focus on two-candidate races for office, however, we note that an equilibrium and its outcomes when voters vote sincerely is equivalent to one of the equilibria that exist when they vote strategically.[25] As we are modeling the two-party competition of the United States, our results are not compromised by focusing on underlying proximity preferences and voting.[26]

Our model of citizen micro foundations is one in which uncertainty and ideological proximity preferences interact to drive both turnout and spatial voting decisions. Our model is inspired by Degan and Merlo's (forthcoming) framework, which the authors test using individual-level voting data.[27] They estimate a structural model of voter choice employing a version of the voter preferences used here and find that their estimated model is able to replicate the observed levels of abstention, selective abstention, split-ticket voting, and straight-ticket voting. This provides strong empirical support for the value of our preferences in a model of uncertainty and spatial preferences.

The general utility framework is one in which a citizen benefits from voting in a given race but is subjected to an ex-ante cost of voting for the wrong candidate, that is voting for the candidate whose position is less like her own.[28] Obviously, in an environment in which there is no uncertainty about candidate positions, all citizens vote for the "right" candidate and obtain the benefit of voting. However, in our setting these preferences have great appeal

[25]Sincere voting in a two-party election is equivalent to the unique Nash equilibrium in weakly undominated strategies.

[26]For a recent example of a model where voters have preferences over actions, see Feddersen, Gailmard and Sandroni (2009).

[27]We consider aggregate voting behavior and crucially expand the analysis to outcomes.

[28]See Degan and Merlo (2007) for a discussion of the psychological motivations behind these preferences.

as they directly incorporate the role of different levels of information.[29] [30]

Let $I_s \subseteq \mathbb{R}$ be the set of citizens in state s. We specify for each citizen $i \in I_s$ a corresponding ideal point $y^i \in P_s \subseteq P$, which is citizen i's most preferred policy position. At the state level, the preferences of voters are distributed symmetrically and unimodally with full support over P_s around the state preference mean, μ_s, with the corresponding cumulative distribution function, F_s. [31]

We assume that citizens have proximity preferences and incur symmetric disutility from voting for candidates with policy positions equally diverging from their own.[32] Formally, we assume that $u_i(.)$ achieves a global maximum at y^i is strictly decreasing away from y^i and that $u_i(w) = u_i(-w)$.

We next follow a model specification similar to that of Degan and Merlo (forthcoming) and assume that each citizen i can receive a benefit $b \in \left(0, \frac{1}{2}\right)$ for voting in a particular race for office.[33] This can be thought of as the

[29] These preferences also have an interesting interpretation in light of the swing voter's curse (Feddersen and Pesendorfer (1996)). The cost of a voter's uncertainty as to which candidate's position is closest to her can be thought of as a psychological proxy for the voter's strategic concerns. Higher uncertainty corresponds with a greater probability of making a mistake. As a voter's preferences become more extreme, the voter is less likely to make a mistake voting for one party or the other and, therefore, becomes a partisan voter.

[30] While we present out model in terms of the preferences presented here, we are able to generate the same variation in individual behavior and, hence, aggregate results using a different utility specification. In particular, we are able to generate the same set of results using the ambiguity-aversion preference framework as applied to voting as developed by Ghirardato and Katz (2006). At the moment, we are also attempting to implement a version of preferences that rely on voter preferences over outcomes.

[31] Our assumptions about the distribution of citizens is similar to those made elsewhere in the literature. See, for example, Callander (2005).

[32] The actual functional form of these preferences is not essential to our model. Our result will hold if we employ any form of symmetric-loss preferences. More generally, our results hold for any single peaked preference but at the expense of clarity and tractability.

[33] See Riker and Ordeshook (1968) for a conical model in which voters derive an intrinsic benefit from voting. A large literature has also model the benefit associated with voting, see Tullock (1971), Brennan and Buchanan (1984), Brennan and Lomasky (1993), Schuessler (2000), Fedderson and Sandroni (2006a,b) and Fedderson, Gailmard and Sandroni (2009).

utility of doing one's civic duty or the right to brag about one's participation in the Democratic process. In a case in which there is no uncertainty, there is no associated cost of voting and each citizen votes for the candidate who shares a policy position that is closest to her own.

Formally, citizen i votes for C_L^r in the race for office r if and only if

$$u_i\left(C_L^r\right) \quad > \quad u_i\left(C_R^r\right) \tag{5}$$

and obtains a benefit of voting, b. Given our assumption that $C_L^r \leq C_R^r$, condition (2) simplifies to

$$y^i < \frac{C_L^r + C_R^r}{2} = M^r. \tag{6}$$

Note that the only information a citizen considers when deciding for whom to vote is the midpoint of candidate positions. Essentially, a citizen is concerned with the *relative* ideological position of a candidate rather than her absolute position. Given this insight, our approach henceforth will focus on the candidates' ideological midpoint in a race for office rather than on their particular positions.

3.2.2 Incomplete Information

We now introduce an environment in which, a priori, not all citizens are perfectly informed about the policy positions of candidates. Let Δ_i^r denote the information set ("beliefs") that citizen i has about the ideological midpoint of candidate positions in race r and let $G_i^r\left(M^r\right)$ denote the subjective distribution that represents citizen i's beliefs in race r. Citizens are aware of the underlying party conditions and labels; thus, the subjective beliefs about candidate positions correspond with their actual distribution. If citizen i has no uncertainty about the ideological midpoint in race r, then $\Delta_i^r = M^r$; otherwise, $\Delta_i^r = G_i^r\left(M^r\right)$. In this case, uncertainty can produce a non-zero ex-ante psychological cost of voting.

Let $c_i\left(q^r; y^i, \Delta_i^r\right)$ denote citizen i's psychological cost associated with voting for the candidate from party q in race r with information Δ_i^r. Specifically, citizen i's cost of voting for a candidate from party L in the race for office r is

$$c_i\left(L^r; y^i, \Delta_i^r\right) = \int\limits_{\{M^r : M^r \leq y^i\}} dG_i^r\left(M^r\right) = \mathbb{P}\left(M^r < y^i\right). \tag{7}$$

This expression is a sum of all states in which voting for the candidate from party R is the better choice weighted by the subjective probability of the state. Overall, we arrive at a closed-form solution that bears some appeal: citizen i's cost of voting for the candidate from party L is the probability of "making a mistake"–the candidate from party R shares a closer position with citizen i than the candidate from party L. Since this probability ranges from zero to one and the benefit from voting is less than half, there are now two decisions each citizen must face: whether to vote in a given race and for whom.[34]

3.2.3 Citizen Classification

We now segment citizens into two classes differentiated by the degree to which they are informed about races for offices p and s. Although we now assume that the degree to which a citizen is informed is uncorrelated with her preferences, this assumption can be relaxed and is not essential for generating our results.[35]

Let $\Delta_i = \{\Delta_i^p, \Delta_i^s\}$ denote citizen i's information about the ideological

[34] A strategic interpretation of these preferences is that as the possibility of making a mistake decreases, then regardless of pivot probabilities, a voter is more likely to vote for her preferred party.

[35] To the degree that preferences are correlated with how well-informed a citizen is about candidate positions, empirical work indicates that ideological extremism is associated with more-informed citizens, in which case our primary aggregate results would be even more profound. See Palfrey and Poole (1987).

midpoints in both races. Accordingly, each citizen in a given state, s, is classified in one of the following two ways[36]

1. Fully Informed Citizen (FIC): a citizen who observes the ideological midpoints of candidate positions in both the presidential and senatorial races for office. For citizen i in this group, $\Delta_i = \{M^p, M^s\}$

2. Partially Informed Citizen (PIC): a citizen who observes the ideological midpoint of candidate positions in the presidential race only. For citizen i in this group, $\Delta_i = \{M^p, G_i^s(M^s)\}$

Under our reduced-form model of party competition, candidates from the same party in races p and s are linked by their party labels. As shown earlier, citizens observing a candidate position in one race will use Bayes' rule to update their priors upon observing presidential candidate positions; thus, they condition their subjective distribution over senatorial candidates using information about the positions that presidential-candidates are observed to have. Consequently, in each race for office, citizens use either party labels or their observations of races to make turnout and voting decisions.

Given our specification of preferences, a few results are immediate. First, FICs always observe candidate positions, turn out in both races and vote for the candidates whose positions are closest their own. Second, PICs always turn out in the presidential race, use their proximity preferences to vote, then update their beliefs about senatorial-candidate positions and accordingly decide whether and for whom to vote using party labels.[37]

[36]Two classes of voters that we do not model are those who are uninformed about both races, $\Delta_i = \{G_i^p(M^p), G_i^s(M^s)\}$, and those who observe ideological positions in the senatorial race but not in the presidential race, $\Delta_i = \{G_i^p(M^p), M^s\}$. The first type of voter does not alter her behavior in the two electoral environments and is not sensitive to the realized positions of candidates. The second type of voter is empirically less relevant, and including this type of voter increases the complexity of the model without contributing anything to our understanding. If we were to include voters who are more informed about the senatorial race, then for our results to hold, we would need to assume there are fewer of them than our PICs.

[37]Although, theoretically, there may exists other groups of voters–namely, those who

3.2.4 Citizen Choices and Objective

Denote citizen i's turnout decision in race r by $t_i^r \in \{0,1\}$, where if she decides to vote in race r, $(t_i^r = 1)$, and if she abstains, $(t_i^r = 0)$, and let the ballot she casts be $v_i^r \in \{L^r, R^r\}$. Given these specifications, citizen i maximizes the following objective function:

$$\underset{t_i^r \in \{0,1\}, v_i^r \in \{L^r, R^r\}}{Max} t_i^r \left[b - c_i \left(v_i^r ; y^i, \Delta_i^r \right) \right]. \tag{8}$$

3.2.5 Citizen Behavior

Given the previous objective function, each citizen's voting and turnout decisions can be solved using backward induction. A citizen first chooses her preferred candidate, and then decides whether the benefit of voting outweighs the cost of voting for her preferred candidate. Solving for the program, we derive for each citizen i a conditional voting rule and a turnout rule as follows.

Lemma 1*: *Voting Rule
Conditional on voting, vote for party L's candidate in race r if and only if:

$$\mathbb{P}\left(M^r < y^i \right) \quad < \quad \frac{1}{2} \tag{9}$$

and for party R's candidate otherwise.

Proof:
($\Rightarrow$)If $\mathbb{P}\left(M^r < y^i\right) < \frac{1}{2}$, then $\mathbb{P}\left(M^r > y^i\right) < \frac{1}{2}$. Therefore, $c_i\left(L^r; y^i, \Delta_i^r\right) < \frac{1}{2}$ and $c_i\left(R^r; y^i, \Delta_i^r\right) > \frac{1}{2}$, and, thus, $c_i\left(L^r; y^i, \Delta_i^r\right) < c_i\left(R^r; y^i, \Delta_i^r\right)$. This implies that condition on voting $b - c_i\left(v_i^r; y^i, \Delta_i^r\right)$ is maximized at $v_i^r = L^r$.
($\Leftarrow$)If $b - c_i\left(v_i^r; y^i, \Delta_i^r\right)$ is maximized at $v_i^r = L^r$, then $c_i\left(L^r; y^i, \Delta_i^r\right) < c_i\left(R^r; y^i, \Delta_i^r\right)$. Since $c_i\left(L^r; y^i, \Delta_i^r\right) = 1 - c_i\left(R^r; y^i, \Delta_i^r\right)$ and $c_i\left(v_i^r; y^i, \Delta_i^r\right) \in$

observe senatorial candidates but not presidential ones and who would employ up-ticket inferences–we believe that, given the evidence about cyclical turnout in the U.S. in conjunction with the fact that substantially more information is conveyed about the presidential race than a senatorial one, for simplicity, we can assume this fraction and other marginal types of voters away.

$[0,1]$, we know that $c_i(L^r; y^i, \Delta_i^r) < \frac{1}{2}$ and $c_i(R^r; y^i, \Delta_i^r) > \frac{1}{2}$, which implies that $\mathbb{P}(M^r < y^i) < \frac{1}{2}$. Q.E.D.

Lemma 2: Turnout Rule Turn out to vote in race r if and only if

$$Min\left\{\mathbb{P}\left(M^r < y^i\right), \mathbb{P}\left(M^r > y^i\right)\right\} \;<\; b \tag{10}$$

and abstain otherwise.

Proof:
($\Rightarrow$)If $Min\{\mathbb{P}(M^r < y^i), \mathbb{P}(M^r > y^i)\} < b$, then either $\mathbb{P}(M^r > y^i) < b$ (and $\mathbb{P}(M^r < y^i) > b$) or $\mathbb{P}(M^r < y^i) < b$(and $\mathbb{P}(M^r > y^i) > b$). WLOG assume $\mathbb{P}(M^r < y^i) < b$. Therefore, $c_i(L^r; y^i, \Delta_i^r) < b$ and $b - c_i(L^r; y^i, \Delta_i^r) > 0$. This implies that $t_i^r[b - c_i(L^r; y^i, \Delta_i^r)]$ is maximized at $t_i^r = 1$.
($\Leftarrow$)WLOG we consider the case where $v_i^r = L^r$. If $t_i^r[b - c_i(L^r; y^i, \Delta_i^r)]$ is maximized at $t_i^r = 1$, then $b - c_i(L^r; y^i, \Delta_i^r) > 0$. That implies that $c_i(L^r; y^i, \Delta_i^r) < b$, which in turn implies that $\mathbb{P}(M^r < y^i) < b$. If $\mathbb{P}(M^r < y^i) < b$, then clearly $Min\{\mathbb{P}(M^r < y^i), \mathbb{P}(M^r > y^i)\} < b$. Q.E.D.

Note that the voting rule is independent of whether a citizen decides to actually turn out and vote; it just specifies that, conditional on voting in race r, a citizen should cast her vote for the candidate from the party whose associated cost incurred by voting is less than one-half. It follows that since the sum of the costs of voting for the candidates from parties L and R in race r is equal to one if the cost associated with voting for one candidate is less than one-half then the cost of voting for the other is greater than one-half. Thus, conditional on voting in the race for office r, a citizen votes for the party whose candidate's position, she expects, is likeliest to be closet to her own.

Since the cost of voting is less than half for only one candidate and the benefit of voting is no greater than half, the *Turnout Rule* implies a cutoff position at which a citizen is indifferent about whether to obtain the benefit of voting and incur the cost associated with voting for her preferred party's candidate or not turn out to vote in race r at all.

3.3 Electoral Rule

In each senatorial race, the electoral winner is decided by a plurality rule. In the context of our model, this implies that the candidate (L or R) who garners the highest share of the combined PIC and FIC votes will be the winner. [38] In particular, let δ_s be the proportion of PIC in a given state, s. Let $\pi_{L,s}^{PIC}$ and $\pi_{L,s}^{FIC}$ be party L's voteshare in senate race s among the PICs and FICs, respectively. Under the plurality electoral rule, party L's senatorial candidate then wins if and only if

$$\delta_s \pi_{L,s}^{PIC} + (1 - \delta_s)\, \pi_{L,s}^{FIC} \geq \frac{1}{2}$$

.

4 Results

With the basics of the model and institutional details described, we now proceed to deriving the results related to differences in observed aggregate behavior and outcomes between midterm and presidential elections. To do so, we first examine the difference in individual turnout and voting behavior of citizens between the two electoral environments. As described, our model has two different classes of citizens, distinguished by their level of information about various races. In Senate races, the two types of citizens will be distinguished by behavior, particularly between the two different electoral environments. As FICs observe candidate positions in all races, they will impart centripetal discipline on outcomes. In an electorate of only FICs, the candidate with a position closest to that of the median voter will win. How-

[38] We resolve ties in favor of party L; Our results are not sensitive to this assumption. For simplicity, we model only two-party competition, so the plurality rule will simply be the majority rule. In fact, two states, Georgia and Louisiana, employ an absolute majority rule for Senatorial races, and employ runoffs if no candidate garners more than half the votes. The empirical regularities presented earlier are strengthened by excluding these two states from the analysis, but they are included for completeness.

ever, the presence of PICs and their reliance on prior beliefs and partially informative signals will introduce centrifugal forces and produce outcomes skewed away from the median. Importantly, the effect of a partially informed voter will be systematically related to the presidential race, thus demonstrating the role of party labels and information contagion on voter behavior and electoral outcomes.

4.1 Individual Citizen Behavior

Citizens' decisions whether to turn out and, conditional on turning out, for whom to vote in Senate elections depend on their expectations and certainty about candidate positions. Citizens differ not only in their preferences, but also with respect to their information about candidates. Electoral environments also present citizens with differing amounts of information. In midterm elections, citizens can only observe candidates in the race of interest, but in presidential races an additional signal in the position of presidential candidates is available to citizens. In order to highlight the role of information and electoral environments, we present the behavior of citizens by information levels according to electoral environments.

4.1.1 Fully Informed Citizens

As FICs are not uncertain about candidate positions and do not rely on priors and signals to shape their voting decisions, their behavior is not systematically biased in any election. Nonetheless, FICs play an important role in deciding outcomes. Before turning to a presentation of PIC-voter behavior in each environment, we describe FIC voting behavior, which does not vary by environment.

Proposition 1 : Turnout and Conditional Voting of Fully Informed Citizens

Turnout

All FICs turn out to vote in every Senate election.

Conditional Voting

An FIC, i, votes for party L's senatorial candidate if and only if

$$y^i \;\; < \;\; m^s.$$

Proof: Follows directly from $\Delta_i = \{M^p, M^s\}$ and *Lemmas 1 and 2.*

The behavior rules governing conditional voting and turnout for FICs do not vary by electoral environment. In both electoral environments, the cost of voting for FICs is zero and, thus, given any positive benefit to voting, FICs will turn out. Thus, the set of FICs turning out to vote is constant. This suggests that the variation in turnout will come from the set of PICs. The voting decisions of FICs are likewise simple to characterize: FICs vote for the party whose candidate's realized (actual) position is closest to their own. Their voting decision depends solely on the realized position of candidates and does not incorporate their prior (*ex-ante*) beliefs. While FICs might exhibit an *ex-post* bias in voting for one party over another, they will always vote for the candidate that is closest to them in ideology regardless of party labels. It should be noted, that because senate candidates differ from presidential candidates for both local and idiosyncratic reasons, FICs may exhibit both ticket-splitting and straight-ticket voting.[39]

[39]The voting behavior of FICs may be described as that of swing voters, as they rely upon the observed candidate position to make their voting decision and not on party labels. This behavior constrasts that of a completely uninformed voter who relies solely on party labels, a behavior is, in a sense, partisan: the actual positions of candidates do not matter and decisions are made solely on the basis of party labels.

4.1.2 Partially Informed Citizens

Partially Informed Citizens

Unlike that of FICs, the turnout and voting behavior of PICs varies according to electoral environment. For this reason, we consider the turnout and conditional voting behavior of PICs separately, but compare them across electoral environments.

Proposition 2 : Turnout of Partially Informed Citizens
Midterm Electoral Environment
A PIC, i, turns out to vote in midterm senatorial elections if and only if

$$y^i \notin \left[\Phi^{-1}(b)\,\sigma_s + \mu_s, -\Phi^{-1}(b)\,\sigma_s + \mu_s\right]$$

and abstains otherwise.

Presidential Electoral Environment
A PIC, i, turns out to vote in presidential-year senatorial elections if and only if

$$y^i \notin \left[\mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) \pm \Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2}\right]$$

and abstains otherwise.
Proof: See Appendix.

PICs choose to abstain when the uncertainty over which party's candidate is closer to them in ideological position makes voting too costly.[40] This cost

[40]Note, that this is different from risk consideration, where for almost all voters one of the choices is in expectation closer and, thus, less risky.

of voting results in a zone of abstention centered around the expected midpoint of candidate positions. PICs find it too costly to vote when their ideal points lie near the expected midpoint of the candidates. The size of this zone of abstention depends on the level of uncertainty about party midpoints as measured by σ_s and the amount of benefit that PICs derive from voting.[41] As the uncertainty increases or the benefit of voting decreases, the range of abstention increases on both sides. Likewise, in presidential elections, the senatorial turnout decision of a PIC depends on her observation of presidential candidates. The effect of observing presidential candidates is two-fold. First, the overall uncertainty about the positions of senatorial candidates is reduced, and the zone of abstention shrinks. Second, the center of the zone of abstention can move depending on the realization of presidential-candidate positions. As will be discussed later, the overall effect is that the turnout of PICs increases during presidential elections relative to midterm elections.[42]

Having described the turnout decisions of PICs, we now characterize their conditional voting decisions.

[41] Even if voters have a strict preference in expectation for one of the two choices, they still may not turn out to vote.

[42] A general point about roll-off can be made at this point. For our population of PICs, the position of candidates in the presidential race is certain and, thus, all PICs are expected to turn out for the presidential race. Note, however, that even during presidential elections, the zone of abstention for the senatorial race has a positive measure. This implies that there will be a measurable amount of roll-off. However, we also show that in some instances there is a degree of roll-on, or higher turnout in down-ticket races than in up-ticket races. To generate this effect in our model, we would need to assume that there are voters who observe candidate positions in senate races, but not in presidential races. While we have assumed these voters away for simplicity, it might be reasonable to assume a positive measure of these voters in practice. Note that, in order to generate a positive roll-on, these voters do not need to be of greater measure than our set of PICs.

Proposition 3 : Conditional Voting of Partially Informed Citizens
Midterm Electoral Environment
During midterm elections, conditional on turning out, a PIC votes for party L's senatorial candidate if and only if

$$y^i \quad \leq \quad E\left[M^s\right] = \mu_s$$

and votes for party $R's$ candidate otherwise.

Presidential Electoral Environment
During presidential elections, conditional on turning out, a PIC votes for party L's senatorial candidate if and only if

$$y^i \quad < \quad E\left[M^s | M^p = m^p\right] = \mu_s + \sigma_\omega \rho_{\omega,p} \left(\frac{m^p - \mu_p}{\sigma_p} \right)$$

and votes for party R's candidate otherwise.

Proof: See Appendix .

The conditional voting behavior of PICs is simply to vote for the candidate whose position is in expectation closest to their ideal position.[43] In midterm elections, this means that the conditional voting rule for PICs incorporates only their priors about party competition. During presidential elections, conditional voting for PICs incorporates both their prior beliefs about the positions of Senate candidates and their observations of realized presidential-candidate positions, with the degree of updating depending on the amount of correlation and the degree of variance.[44] This updating can introduce bias in the conditional voting decisions of PICs, something that we

[43]Since our distribution is continuous, indifference occurs with a probability of zero. Nonetheless, our voting rule assigns indifferent voters to party L. For voters with incomplete information, indifference will also imply that they abstain. Thus, indifference in senatorial races is only relevant for the behavior of FICs.

[44]As candidates for the Presidency and the Senate share a common party element, an observation of the position of presidential candidates confers information about the party's overall position.

explore in more detail in the following section. Note that much like FICs, PICs can exhibit both ticket-splitting and straight-ticket voting.

4.2 Aggregation of Citizen Behavior

In the following sections, we discuss and present formal results about differences in aggregate behavior by electoral environment. These results form the connection between the micro foundations of voter behavior and information in our model and the differences in outcomes that we have presented. We first discuss the overall differences in turnout by electoral environment and then turn to the question of a coattail effect generated by rational informational contagion. In the following section, we focus on the differences in electoral outcomes in the senatorial race generated by midterm and presidential elections.

Turnout decisions of FICs are the same in both electoral environments. Any differences in turnout between midterm and presidential elections is, thus, generated by the behavior of PICs, and we can focus our attention on comparisons of the range of abstention between the two environments. We present our analysis for a given realization of midpoints, $M^p = m^p$ and $M^s = m^s$, where we assume that the senatorial midpoint is identical in both electoral environments.

Recall, that in midterm elections the range of PICs who choose to abstain is a connected set of length–

$$-2\Phi^{-1}(b)\,\sigma_s$$

–centered around μ_s, while in the presidential elections, the range of abstention is of length

$$-2\Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2\rho_{\omega,p}^2} \qquad Equation\ *,$$

centered around $\mu_s + \sigma_\omega\rho_{\omega,p}\left(\frac{m^p-\mu_p}{\sigma_p}\right)$; thus, for any non-zero correlation, the

zone of abstention will be narrower in presidential elections than in midterm elections, implying a higher turnout for any given midpoint.[45] Additionally, since μ_s is the median of the unimodal symmetric distribution of preferences for PICs, shifting the range of abstention (even if it is of the same size) decreases the mass of citizens abstaining. This result, combined with a strictly smaller zone of abstention, implies increased turnout. The necessary condition for increased turnout is the presence of contagion across races for office. Once this condition is satisfied, the correlation and the difference between m^p and μ_p have a complementary effect on turnout. This result is summarized in *proposition* 4.

Proposition 4: Turnout

Citizen turnout in presidential elections is strictly greater than turnout in midterm elections if and only if $\rho_{\omega,p} \neq 0$.

Proof: See Appendix.

Proposition 4 demonstrates the connection between electoral environments, and the aggregate variation in voter turnout between these two environments. Additionally, as a long as (*) is not of zero measure, our model is also consistent with the observation of positive levels of roll-off.

We now turn to an examination of the difference in voting decisions in Senate elections between electoral environments. The conditional voting decisions of FICs in the senatorial races for office are independent of the realization of the presidential-candidate idiosyncratic effect. Focusing on PICs, we establish a relationship between the voteshare of party L's presidential candidate and party L's senatorial candidate conditional on a realization of party midpoints.

[45] Recall that $\sigma_s^2 = \sigma_\omega^2 + \sigma_{\epsilon,s}^2$, and $\rho_{\omega,p} \in [0,1]$ implies that $\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2} \leq \sigma_s$, with strict inequality for $\rho_{\omega,p} \neq 0$.

Consider a particular realization of $M^p = m^p$. We can rewrite the realized presidential effect, $\epsilon^p = e$, in terms of the fixed party effect and the realized midpoint of presidential candidates, such that $e = m^p - \Omega$. The presidential voteshare among PICs for party L is now a function of the realization of the idiosyncratic presidential effect of the two candidates. Let $\pi_{L,p}^{PIC}(e)$ be the presidential voteshare for party L among PICs conditional the realization of m^p.[46]

$$\pi_{L,p}(e) = F_s(m^p) = F_s(\Omega + e) \tag{11}$$

Similarly let $\pi_{L,s}^{PIC}(e)$ be the senatorial voteshare for party L among PICs conditional the realization of party positions.[47] Note that this it is a function of the presidential idiosyncratic realization and not of the senatorial one. Let $\zeta_s(e)$ be the fraction among PICs who choose to turn out and vote in race for office r as a function of the presidential idiosyncratic effect, conditional on the realization of $M^p = m^p$ (in the case of the presidential race, $\zeta_p(e) = \zeta_p = 1$). We then have

$$\pi_{L,s}^{PIC}(e) = \frac{F_s\left(\Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2\rho_{\omega,p}^2} + \mu_s + \sigma_\omega\rho_{\omega,p}\left(\frac{\Omega+e-\mu_p}{\sigma_p}\right)\right)}{\zeta_s(e)}. \tag{12}$$

Note that since $\pi_{L,s}(e)$ and $\pi_{L,p}(e)$ are both strictly increaing in e, among PICs the relationship between the presidential voteshare and the senatorial voteshare is positive.[48] We term this relationship the coattail effect as it is

[46] Note, presidential voteshares at the state level are monotonically related across states. In an abuse of notation, we sometimes refer to state-level presidential voteshare as $\pi_{L,p}^{PIC}$, when the interpretation is clear.

[47] Unless mentioned otherwise, we will refer to $\pi_{L,r}$ as the voteshare for party L in race r among PICs associated with a presidential idiosyncratic draw, e.

[48] Let $A(e) \equiv F_s\left(\Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2\rho_{\omega,p}^2} + \mu_s + \sigma_\omega\rho_{\omega,p}\left(\frac{\Omega+e-\mu_p}{\sigma_p}\right)\right)$ and $B(e) \equiv 1 - F_s\left(-\Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2\rho_{\omega,p}^2} + \mu_s + \sigma_\omega\rho_{\omega,p}\left(\frac{\Omega+e-\mu_p}{\sigma_p}\right)\right)$. We can now rewrite $\pi_{L,s}$ in terms of A and B. That is, $\pi_{L,s} = \frac{A}{A+B}$. Since F_s has full support we can devide through by A

independent of the actual realizations of the party platform and is generated solely by good or bad (for party L) draws of the idiosyncratic characteristics of presidential candidates.

Proposition 5: Contagion
Among PICs, favorable idiosyncratic draws in the presidential race are associated with greater support for presidential *and* senatorial candidates. Formally, for any two distinct realizations of ϵ^p,e' and e^*, if $e' > e^*$ then $\pi_{L,p}(e') > \pi_{L,p}(e^*)$ and $\pi_{L,s}(e') > \pi_{L,s}(e^*)$
Proof: Follows directly from derivations of $\pi_{L,p}()$ and $\pi_{L,s}()$.

Instead of relating idiosyncratic realizations of presidential positions to the senatorial race, we can employ presidential-race voteshares. Since proposition 5 establishes the strict monotonic relationship between ϵ^p and party L's presidential voteshare, $\pi_{L,p}$, we can instead condition $\pi_{L,s}$ on presidential voteshare, which is an observable quantity. In particular, since F_s has full support, we can rewrite $\pi_{L,s}$ as a function of $\pi_{L,p}$ as follows:

$$\pi_{L,s}(\pi_{L,p}) = \frac{F_s\left(\Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2\rho_{\omega,p}^2} + \mu_s + \sigma_\omega\rho_{\omega,p}\left(\frac{F_s^{-1}(\pi_{L,p})-\mu_p}{\sigma_p}\right)\right)}{\zeta_s(\pi_{L,p})}.$$

Corollary 5:
For any pair $\pi'_{L,p}, \pi_{L,p} \in [0,1]$, such that $\pi'_{L,p} > \pi_{L,p}$, we have that $\pi_{L,s}(\pi'_{L,p}) > \pi_{L,s}(\pi_{L,p})$
Proof: Follows directly from Proposition 5 and equation for $\pi_{L,s}(\pi_{L,p})$ above.

Proposition 5 and its corollary is the key result that links observed differences in aggregate voting behavior to consistent and systematic difference in outcomes bias. The presence of informational coattail voting will create

and get that $\pi_{L,s} = \frac{1}{1+\frac{B}{A}}$. Since A is strictly increasing in e and B is strictly decreasing in e, then $\frac{B}{A}$ is strictly decreasing in e and, thus, $\pi_{L,s}$ is strictly increasing in e.

a systematically biased electoral landscape, which affects the type and party of candidates elected to office. We explore these outcome effects in the next section.

4.3 Electoral Outcomes in the Senatorial Race for Office

In this section, we focus on the differences in electoral outcomes in the senatorial race generated by midterm and presidential elections. Without loss of generality, we focus on party L to demonstrate the results for aggregate electoral outcomes of candidates in the senatorial race for office in state s. The following results will symmetrically hold for party R. Our model of candidate selection contains an idiosyncratic stochastic component. Thus, for a fixed population of citizens, the electoral outcomes of senate races will not be deterministic, and our results will be presented as expectations. Before presenting our main propositions, we present a set of general results for the midterm and presidential electoral environments.

Midterm Elections

In a midterm election, a senatorial candidate from party L in state s with senatorial race midpoint $M^s = m^s$ wins office if and only if

$$(1 - \delta_s) F_S(m^s) + \delta_s \pi_{L,s}^{PIC} \geq \tfrac{1}{2} \tag{13}$$

The first term on the LHS is associated with the measure of FICs whose ideal point is to the left of the candidate-position midpoint (and who, thus, will vote for the candidate from party L) weighted by their proportion in the overall population in state s. The second term relates to the measure of PICs in the population who vote for the senatorial candidate from party L. Since the proportion of PICs' votes is split equally between the candidates from both parties independent of the realized ideological midpoint in the

senatorial race, that measure is equal to one-half.[49] Overall, this condition simplifies to

$$m^s > F_s^{-1}\left(\frac{1}{2}\right) = \mu_s,$$

which reads that party L's candidate wins if the realized senatorial-race midpoint is to the right of the median citizen's ideal point in state s. Thus, the probability that a candidate from party L wins when $M^s = m^s$ is

$$\mathbb{P}\left(Win_L^s | M^s = m^s\right) = \begin{cases} 1 & m^s \geq \mu_s \\ 0 & otherwise. \end{cases} \tag{14}$$

Note that the FICs determine which candidate will be the winner in this case. Given their proximity preferences, if the senatorial-race midpoint is to the right of the FICs' ideal point, then the candidate from party L wins office. Formally, the range of midpoints that result in a victory for the senatorial candidate from party L is (μ_s, ∞).

Presidential Elections

Recall that when $\rho_{\omega,p} = 0$, the behavior of PICs during presidential elections is equivalent to their behavior during midterm elections and there is no contagion between races for office during presidential elections. Now, instead, suppose that $\rho_{\omega,p} > 0$, but that $\epsilon^p = 0$. While there is no bias in vote choices, PICs are more informed about candidate positions and turnout increases. As $\rho_{\omega,p}$ increases and ϵ^p remains zero, the number of citizens induced to vote for each party increases evenly and the proportion of PICs voting for each party remains the same (i.e., $\pi_{L,s}^{PIC}(0) = \pi_{R,s}^{PIC}(0) = \frac{1}{2}$). Thus, if the realized pres-

[49] Uniformed voters provide no advantage for any given candidate since they vote with equal proportions for both candidates.

idential idiosyncratic error is identical to its expectation, then the votes of PICs in the senatorial-race are split equally; however, more PICs turn out to vote relative to midterm elections. Essentially, an increase in $\rho_{\omega,p}$ signifies that PICs have more information when facing the senatorial-race for office, which in turn reduces their ex-ante probability of voting for the wrong candidate *ceteris paribus*. Now, suppose that $\rho_{\omega,p} > 0$ and that $\epsilon^p \neq 0$; the sign of the realized error will determine which senatorial candidate will benefit from a built-in advantage passed down from the presidential race for office. We will focus the following comparative statics on the presidential idiosyncratic error while conditioning on a fixed level of correlation, $\rho_{\omega,p} > 0$.[50]

In a presidential election with $\epsilon^p = e$, a senatorial candidate from party L in state s with senatorial-race midpoint $M^s = m^s$ wins office if and only if

$$(1-\delta)F_s\left(m^s\right) + \delta\pi_{L,s}\left(\pi_{L,p}\right) \quad \geq \quad \frac{1}{2}. \tag{15}$$

As before, the first term on the LHS of equation (1) is the measure of FICs who vote for the candidate from party L weighted by their measure in the population, while the second term is the weighted measure of PICs who vote for party L, respectively. The inequality simplifies to[51]

[50] Note that $\rho_{\omega,p} = \frac{cov(\Omega,M^p)}{\sigma_p\sigma_\omega} = \frac{\sigma_\omega^2}{\sigma_p\sigma_\omega} > 0$

[51] Note that since $\pi_{L,s}(e) \in [0,1]$,

$$h_s(e) \quad \in \quad \left(F_S^{-1}\left(\frac{1}{2} - \frac{\delta}{2(1-\delta)}\right), F_S^{-1}\left(\frac{1}{2} + \frac{\delta}{2(1-\delta)}\right)\right).$$

Also, note that for $\rho > 0$,

$$\frac{\partial h_s(e)}{\partial e} \quad < \quad 0$$

and that

$$h_s(e) \quad = \quad \mu_s \iff e = 0.$$

in which case this result boils down to the baseline midterm-election win condition, though

$$m_s \geq F_S^{-1}\left[\frac{1}{2} + \frac{\delta}{2\left(1-\delta\right)}\left(1 - 2\pi_{L,s}\left(\pi_{L,p}\right)\right)\right] \equiv \Lambda\left(\pi_{L,p}\right). \tag{16}$$

Thus, the probability that a candidate from party L wins when $M^s = m^s$ is:

$$\mathbb{P}\left(Win_L^s | M^s = m^s\right) = \begin{cases} 1 & \text{if } m_s > \Lambda\left(\pi_{L,p}\right) \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

Suppose there is a positive draw, such that $e > 0$; then $\mu_s > \Lambda\left(\pi_{L,p}\right)$. Consequently, given a positive draw of ϵ^p, the range of ideological midpoints that result in a win by party L's candidate in the senatorial-race contains the corresponding range derived for midterm elections. As before, the range of midpoints that result in a win by party L is (μ_s, ∞), whereas in presidential elections the range is $\left(\Lambda\left(\pi_{L,p}\right), \infty\right)$ .We now arrive at the following proposition.

The basic result is that in presidential elections as the support for the presidential candidate of a party increases, the range of electorally viable positions for that party's senatorial candidate increases. This increased range of electoral viability occurs as a larger set of PICs are induced to vote for a senatorial candidate independent of the realized idiosyncratic ideology of th candiatdate.

Proposition 6: Increased Range of Electoral Support
A positive draw of the idiosyncratic error in the presidential race increases the range of winnable electoral positions for the senate candidate from party L. Formally,

$$\left(\mu_s, \infty\right) \subsetneqq \left(\Lambda\left(\pi_{L,p}\right), \infty\right) \iff e > 0 \tag{18}$$

Proof: Immediately follows from the derivations above.

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turnout increases.

Corollary 6: Decreased Range of Electoral Support

$$(\mu_s, \infty) \supsetneqq (\Lambda(\pi_{L,p}), \infty) \iff e < 0 \tag{19}$$

Proposition 6 and its corollary establishes the main mechanism by which electoral bias is introduced into observed outcomes. The observation of presidential-candidate positions is informative and it is rational for PICs to condition their turnout and voting decisions upon it. However, any generic realization of the idiosyncratic error in the presidential race introduces bias into the behavior of PICs, which in turn alters the electoral landscape as presented in proposition 6.

We now present propositions on the aggregate effects of our model of voting, information and party competition. We will assume a fixed underlying structure, holding constant the relative population of citizens by type, and the underlying party competition structure. Of interest is describing the relationship between presidential success and the positions of entering and exiting senators.

We will now consider three possible draws of presidential support for party L: $\pi'_{L,p}$, $\bar{\pi}_{L,p}$ and $\pi_{L,p}$ where $\pi'_{L,p} > \bar{\pi}_{L,p} > \pi_{L,p}$.[52] As the results are analogous for party R, we present proofs for party L.

Proposition 7 : Entry and Coattails
The expected ideological position of winning senatorial candidates for party L (R) is decreasing (increasing) in support for the presidential candidate from party L (R). Formally

$$E\left[C_L^s | Win_L^s, \pi'_{L,p}\right] \leq E\left[C_L^s | Win_L^s, \pi_{L,p})\right].$$

[52] Each of these presidential voteshares is the result of a particular realization of the idiosyncratic realizations in the presidential race. As we will want to compare presidential elections to midterm elections, we let $\pi'_{L,p} = \pi_{L,p}(e')$,$\pi_{L,p} = \pi_{L,p}(e)$ and $\bar{\pi}_{L,p} = \pi_{L,p}(0)$, where $e' > 0 > e$. $\bar{\pi}_{L,p}$ is the presidential voteshare that corresponds to a state-level split among PICs, or equivalently, the midterm voting proportions for PICs.

Proof: See Appendix.

Sketch of Proof: By previous result, the range of winning midpoints for a party L Senatorial candidate is $[h_s(e), \infty)$. As there is a one-to-one relationship between the realization of e and realized presidential voteshare $\pi_{L,p}(e)$, we can write this range in terms of presidential voteshare:

$$[\Lambda(\pi_{L,p}), \infty).$$

If we consider a fixed party R senatorial candidate with position $C_R^s = c$, we can rewrite this range in terms of party L's senatorial-candidate positions:

$$[2\Lambda(\pi_{L,p}) - c, c].$$

A higher presidential voteshare $\pi'_{L,p}$ implies a greater coattail effect and an increased range of viable midpoints; thus,

$$\Lambda\left(\pi'_{L,p}\right) < \Lambda\left(\pi_{L,p}\right),$$

and the overall range of winning positions for senatorial candidates from party L increases strictly on the leftward boundary and remains the same on the right boundary:

$$[2\Lambda(\pi_{L,p}) - c, c] \subsetneq \left[2\Lambda(\pi'_{L,p}) - c, c\right].$$

As the positions of party L's possible senatorial candidates range over this entire support, the expected position of those who win will be strictly lower.

The next proposition establishes a similar result for exit. For a senator from party L, less support for party L's presidential candidate implies a greater range of losing positions. The increase in losing positions occurs at the middle of the distribution, implying that more-moderate candidates are

expected to lose when presidential support is low.

Proposition 8: Exit and Coattails
The expected ideological position of an exiting Senator for party L (R) is increasing (decreasing) in support for the presidential candidate from party R (L). Formally,

$$E\left[C_L^s | Lose_L^s, \pi_{L,p}\right] > E\left[C_L^s | Lose_L^s, \pi'_{L,p}\right].$$

Proof: See Appendix

Sketch of Proof*:* The logic of the proof is similar to that of the previous proposition. Decreased support for a party L presidential candidate implies an anti-coattail effect. This increases the range of losing positions for a party L's senatorial candidates on the right boundary. As the position of candidates ranges over this entire range, increasing losers on the right raises the expected position of losing candidates for party L.

The following proposition establishes that increased support for a party's presidential candidate is associated with a greater probability of same party senatorial candidates winning during presidential elections.

Proposition 9: Matched Winners
During presidential elections, a senatorial candidate is more likely to win when his party's presidential candidate has stronger support. Formally,

$$P\left(Win_L^s | \pi'_{L,p}\right) > P\left(Win_L^s | \pi_{L,p}\right).$$

Proof: See Appendix.

Respectively, Proposition 10 establishes that decreased support for a party's presidential candidate is associated with a greater probability of losing for senatorial candidates from the same party.

Proposition 10: Unmatched Losers
During presidential elections, a senatorial candidate is more likely to lose when his party's presidential candidate has weaker support. Formally,

$$P\left(Lose_L^s|\pi'_{L,p}\right) < P\left(Lose_L^s|\pi_{L,p}\right).$$

Proof: See Appendix.

Propositions seven to ten connect our observations about the altered presidential electoral landscape to differences in expected outcomes. For entry, candidates from a party benefiting from a positive coattail effect are presented with an advantageous electoral environment. As a result, more of these candidates, particularly the more-extreme candidates, are able to win elections. This increases the expected ideological extremism relative to an environment without coattails—such as during midterm elections. Conversely, negative coattails disadvantage candidates and increase the importance of ideological moderation and FIC votes to win office. Our results for exit are analogous. Positive coattails allow relatively more extreme candidates to stay in office, but negative coattails disadvantage moderate candidates relative to midterm elections. Overall, the presidential electoral environment with coattails will return a more polarized Senate.

These results also connect our model to previous accounts of coattails (see the next section for a discussion of these models). While previous models focus on the relationship between presidential support to party success downticket, we suggest that such studies may overlook the broader implications of such support. Not only is there a party effect, but the type of candidates that prevail from both parties is affected by coattails.

We also provide microfoundations for the coattail effect that can easily be related to characteristics of political competition and the electorate. Coattails arise in our model due to informational cues and proximity preferences. If party discipline and, hence, the correlation among candidates from the same party increase, then the degree to which voters rely on party labels should increase and ticket-splitting should become less common. Similarly as the proporation of PICs in the electorate increase, we would expect the effect of informational contagion on outcomes to increase. In our model, a

larger proportion of voters who rely on party lables results in greater coattail swings and more extreme outcomes.

5 Conclusion

In the United States, elections for office are rarely held in isolation. Instead, many offices are contested simultaneously, and candidates across the ticket are linked through party identification. At the same time, because acquiring information is costly, voters possess limited information about candidates. The degree to which voters are informed about candidates is expected to vary by office. For example, greater media coverage of tickets for higher-level office may reduce the cost of information acquisition for that particular race, but may do so to a lesser degree for other races.

There are two main reasons for holding concurrent races for office. First, since the variable costs of running an election—hiring monitors and polling staff, printing ballots, delivering equipment, securing polling locations—do not vary much with the number of offices being contested, the marginal cost of holding an additional race for office is relatively low. The fixed costs, however, can be substantial. A recent special election for a single congressional seat in Illinois was estimated to cost over $3.5 million or $33 per vote.[53] With all 435 seats in the House and one-third of the 100 seats in the Senate contested every general-election year, the possible savings are considerable. A second reason is the cost reduction for prospective voters. Holding races for different offices simultaneously decreases the time and effort required for citizens to turn out and vote, and mitigates the cost of acquiring information about policy issues, parties and candidates.

While the direct effects of holding concurrent races for office are unambiguously positive, there are a variety of indirect effects that make the overall

[53] F.N. D'Alessio, *The Associated Press State & Local Wire*, March 4, 2009, http://www.huffingtonpost.com/2009/03/05/board-wants-mail-in-speci_n_172157.html.

net benefit of such elections less obvious. The behavior of voters, candidates, parties, the media and other players may differ systematically between presidential and midterm elections. Citizens may have strategic and informational concerns that alter their voting behavior in each environment. In response, parties may implement their own set of changes. In addition, holding many races for office simultaneously can alter the parties' strategic landscape. For example, differences in fundraising between the two environments can affect the candidate selection process. Thus, if changes in voter or party behavior are widespread, they can lead to persistent differences in electoral outcomes.[54]

Formal literature on the interaction between contemporaneous elections is somewhat lacking. With the important exception of Alesina and Rosenthal (1989), (1995) and (1996), most of the theoretical modeling has been informal. This literature has tended to view down-ticket-voting in light of presidential politics, and has primarily been preoccupied with midterm decline. There are two broad themes in this literature. The first views midterm elections as a reversion to the mean in terms of presidential support. For example, in the surge and decline models (see Angus Campbell (1960) and James Campbell(1991)), the major difference between the two electorates is the presence of presidential partisan voters. Thus, midterm elections are distinguished from presidential elections by their lack of voters in support of the president's party. Conversely, another strand of the literature characterizes midterm elections by the presence of voters who vote against the president's party (see Erikson (1988), Kernell (1977) and Tufte (1975)). Both approaches, however, fail to provide an account of why we might observe consistent differences in ideology by electoral environment across parties and over time.

[54] An evaluation of the normative implications of such differences in the aggregation of preferences depends on the underlying mechanism that generates them and the measure of the degree to and manner in which they differ. In the concluding section, we suggest a possible mechanism, and provide an example of the direct positive effects of these differences, but leave out a discussion of the normative concerns.

There are three main reasons why these models are ill-suited for explaining the facts we uncover in this paper. First, they do not directly consider down-ticket races, but instead focus on the effect of presidential politics on voter decision making. Second, the models are purely partisan and do not consider races in light of a spatial setting. And third, it is difficult to incorporate spatial competition into models that posit that voting decisions in congress are based on the perception of the presidency.

A model that accounts for the effects of multiple simultaneous elections in a spatial setting is Alesina and Rosenthal's balanced government model. In their model, voters attempt to balance the policy produced by congress and the president by electing a divided government. This produces a more moderate policy outcome, which better reflects the preferences of voters. Our results on electoral entry and exit environments are difficult to reconcile with such a model, unless voters are systematically electing senators who are extreme in the opposite direction of the presidential preference. Furthermore, presidential candidates' preferred by such split-ticket voters would have to be sufficiently ideologically extreme such that the balancing senators from the opposing party would necessarily be even more extreme than their midterm counterparts. In fact, our results on party matching and ideology suggests that the opposite is true: states that vote for Democratic presidential candidates elect more-liberal candidates during presidential elections then during midterm elections.

Previous research has suggested that an important role of parties in elections is to serve as a cue or brand that conveys information about candidate positions. In a recent series of papers, Stephen Jessee has found strong support for both spatial voting and the importance of information in making correct spatial choices. In this paper, we have examined the aggregate effect of party labels in the context of incomplete information and citizens' distaste for making mistakes. Our first substantive result establishes that the presence of these labels creates informational contagion between races for office.

Since candidates from the same party share ideological characteristics, citizens make rational inferences about candidate positions in one race based upon their observations in another. This informational contagion creates a coattail effect between races, due to which an electorally advantageous draw of presidential candidates from one party creates an electoral advantage for the same party on the down-ticket senatorial race.

Our second theoretical contribution is to examining the role of this coattail effect in the context of a spatial model of electoral competition. We demonstrate that beyond the effect on a senatorial candidate's prospect of winning, positive presidential coattails affect the expected positions of winners and losers. By swaying a portion of the electorate towards one party, the coattail effect alters the range of electorally viable positions for parties. This result suggests that previous related work may understate the importance of coattails on electoral outcomes.

Finally, if party labels introduce informational contagion between races, then policy-makers may want to consider the role and implications of party labels in other contexts, such as judicial and local elections. In addition to the partisan effects of party labels, our model implies that their presence in presidential elections generates a less informed electorate relative to midterm elections. This finding warrants further research on the availability of such cues, their effects on the type of information being processed by the electorate, and the resulting consequences.

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Appendix

Appendix

Proofs

Proposition 2

Midterm Elections: ($\Rightarrow$) It follows from Lemma 2 and equations (2) and (3) that a PIC, i, turns out if (a) $\mathbb{P}\left(M^s < y^i\right) < b \iff \Phi\left(\frac{y^i-\mu_s}{\sigma_s}\right) < b \iff y^i < \Phi^{-1}\left(b\right)\sigma_s + \mu_s$ (since $\Phi\left(\right)$ is continuous and strictly increasing) or if (b) $\mathbb{P}\left(M^s > y^i\right) < b \iff 1 - \Phi\left(\frac{y^i-\mu_s}{\sigma_s}\right) < b \iff \Phi\left(\frac{\mu_s-y^i}{\sigma_s}\right) < b$ (since $\Phi\left(x\right) = 1 - \Phi\left(-x\right)$) $\iff y^i > -\Phi^{-1}\left(b\right)\sigma_s + \mu_s$. Q.E.D.

($\Leftarrow$) Suppose PIC i does not turn out and

$$y^i \notin \left[\Phi^{-1}\left(b\right)\sigma_s + \mu_s, -\Phi^{-1}\left(b\right)\sigma_s + \mu_s\right].$$

By Lemma 2 it must be that if citizen i does not turn out that $\mathbb{P}\left(M^s < y^i\right) > b$ and $\mathbb{P}\left(M^s > y^i\right) > b$. But then according to the derivation above $y^i > \Phi^{-1}\left(b\right)\sigma_s + \mu_s$ and $y^i < -\Phi^{-1}\left(b\right)\sigma_s + \mu_s$. Contradiction. Q.E.D.

Presidential Elections: ($\Rightarrow$) For a given realization $M^p = m^p$, It follows from Lemma 2 and equations (2), (3) and (4) that a PIC, i, turns out if (a) $\mathbb{P}\left(M^s < y^i | M^p = m^p\right) < b \iff \Phi\left(\frac{y^i-\mu_s-\sigma_\omega\rho_{\omega,p}\left(\frac{m^p-\mu_p}{\sigma_p}\right)}{\sqrt{\sigma_s^2-\sigma_\omega^2\rho_{\omega,p}^2}}\right) < b \iff y^i < \Phi^{-1}\left(b\right)\sqrt{\sigma_s^2 - \sigma_\omega^2\rho_{\omega,p}^2} + \mu_s + \sigma_\omega\rho_{\omega,p}\left(\frac{m^p-\mu_p}{\sigma_p}\right)$ or if (b) $\mathbb{P}\left(M^s > y^i | M^p = m^p\right) < b \iff 1 - \Phi\left(\frac{y^i-\mu_s-\sigma_\omega\rho_{\omega,p}\left(\frac{m^p-\mu_p}{\sigma_p}\right)}{\sqrt{\sigma_s^2-\sigma_\omega^2\rho_{\omega,p}^2}}\right) < b \iff \Phi\left(\frac{\mu_s+\sigma_\omega\rho_{\omega,p}\left(\frac{m^p-\mu_p}{\sigma_p}\right)-y^i}{\sqrt{\sigma_s^2-\sigma_\omega^2\rho_{\omega,p}^2}}\right) < b \iff y^i > -\Phi^{-1}\left(b\right)\sqrt{\sigma_s^2 - \sigma_\omega^2\rho_{\omega,p}^2} + \mu_s + \sigma_\omega\rho_{\omega,p}\left(\frac{m^p-\mu_p}{\sigma_p}\right)$. Q.E.D.

($\Leftarrow$) Suppose PIC i does not turn out and

$$y^i \notin \left[\mu_s + \sigma_\omega \rho_{\omega,p} \left(\frac{m^p - \mu_p}{\sigma_p}\right) \pm \Phi^{-1}(b) \sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2}\right]$$

By Lemma 2 it must be that if citizen i does not turn out that $\mathbb{P}(M^s < y^i) > b$ and $\mathbb{P}(M^s > y^i) > b$. But then according to the derivation above $y^i > \Phi^{-1}(b) \sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2} + \mu_s + \sigma_\omega \rho_{\omega,p} \left(\frac{m^p - \mu_p}{\sigma_p}\right)$ and $y^i < -\Phi^{-1}(b) \sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2} + \mu_s + \sigma_\omega \rho_{\omega,p} \left(\frac{m^p - \mu_p}{\sigma_p}\right)$. Contradiction. Q.E.D.

Proposition 3

Midterm Elections: ($\Rightarrow$) It follows from Lemma 1 and equations (2) and (3) that a PIC, i, votes for party L if and only if $\mathbb{P}(M^s < y^i) < \frac{1}{2} \iff \Phi\left(\frac{y^i - \mu_s}{\sigma_s}\right) < \frac{1}{2} \iff y^i < \Phi^{-1}\left(\frac{1}{2}\right)\sigma_s + \mu_s = \mu_s = E[M^s]$, since $\Phi^{-1}\left(\frac{1}{2}\right) = 0$. Q.E.D.

($\Leftarrow$) Suppose PIC i votes for party R and $y^i < \mu_s$. By Lemma 2 it must be that $\mathbb{P}(M^s > y^i) < \frac{1}{2}$. But then by the derivation above $y^i > \mu_s$. Contradiction. Q.E.D.

Presidential Elections: ($\Rightarrow$) It follows from Lemma 1 and equations (2), (3) and (4) that a PIC, i, votes for party L if and only if $\mathbb{P}(M^s < y^i | M^p = m^p) < \frac{1}{2} \iff \Phi\left(\frac{y^i - \mu_s - \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)}{\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2}}\right) < \frac{1}{2} \iff y^i < \Phi^{-1}\left(\frac{1}{2}\right)\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2} + \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) = \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) = E[M^s | M^p = m^p]$. Q.E.D.

($\Leftarrow$) Suppose PIC i votes for party R and $y^i < \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$. By Lemma 2 it must be that $\mathbb{P}(M^s > y^i) < \frac{1}{2}$. But then by the derivation above $y^i > \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$. Contradiction. Q.E.D.

Proposition 4

By Proposition 1 all FICs turn out to vote in every senatorial race; thus, changes in turnout are generated by PICs only.

($\Rightarrow$) Suppose then that turn out strictly increases in presidential elections but $\rho_{\omega,p} = 0$. Let $T_m = [\Phi^{-1}(b)\,\sigma_s + \mu_s, -\Phi^{-1}(b)\,\sigma_s + \mu_s]$ and $T_p = \left[\mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) \pm \Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2}\right]$ be the zones of abstention in midterm and presidential elections, respectively. Then turnout increases implies that $\delta \int\limits_{I \setminus T_p} f_s(e)\,de > \delta \int\limits_{I \setminus T_m} f_s(e)\,de$. But $T_p = T_m$ since $\rho_{\omega,p} = 0$. Contradiction. Q.E.D.

($\Leftarrow$) Suppose $\rho_{\omega,p} \neq 0$ and turnout decreases in presidential elections. Since by Proposition 1 all FICs turn out to vote in every senatorial race, it must be that

$$\delta \int\limits_{I \setminus T_p} f_s(e)\,de < \delta \int\limits_{I \setminus T_m} f_s(e)\,de \tag{20}$$

Lemma: For some $d>0$, let $A \equiv \{[a,b] \in \mathbb{R} :\ b > a,\ b - a = d\}$ and $x = \left[\mu_s - \frac{d}{2}, \mu_s + \frac{d}{2}\right]$. Then given our assumptions about F_s, for any $x' \in A$: $x' \neq x$, $\int\limits_x f_s(e)\,de > \int\limits_{x'} f_s(e)\,de$.

Given the Lemma above, for a given $\rho_{\omega,p}$ and for any $m^p \neq \mu_p$, $\int\limits_{I \setminus T_p(\mu_p)} f_s(e)\,de < \int\limits_{I \setminus T_p(m^p)} f_s(e)\,de$; thus, it suffices to show that inequality 20 is violated for $T_p = T_p(\mu_p)$. Note that $T_p(\mu_p) = \left[\mu_s + \Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2}, \mu_s - \Phi^{-1}(b)\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2}\right]$ and recall that $\sigma_s^2 = \sigma_\omega^2 + \sigma_{\epsilon,s}^2$. Then for any $\rho_{\omega,p} \neq 0$, $\sqrt{\sigma_s^2 - \sigma_\omega^2 \rho_{\omega,p}^2} = \sqrt{\sigma_\omega^2\left(1 - \rho_{\omega,p}^2\right) + \sigma_{\epsilon,s}^2} \leq \sigma_s \Longrightarrow T_p(\mu_p) \subseteq T_m \Longrightarrow \int\limits_{I \setminus T_p(\mu_p)} f_s(e)\,de > \int\limits_{I \setminus T_m} f_s(e)\,de$. Contradiction. Q.E.D.

Prosposition 7: $E\left[C_L^s | Win_L^s, \pi'_{L,p}\right] \leq E\left[C_L^s | Win_L^s, \pi_{L,p}\right]$

Notice that from equation (17) we have that $E\left[C_L^s | Win_L^s, \pi_{L,p}\right] = E\left[C_L^s | M^s \in (\Lambda(\pi_{L,p}), \infty)\right]$. We can rewrite the interval of winnable midpoints in terms of the function Λ and consider an arbitrary draw of $C_R^s = c$. Then we have that

$E\left[C_L^s | M^s \in \left(\Lambda\left(\pi_{L,p}\right), \infty\right), C_R^s = c\right] = E\left[C_L^s | C_L^s \in \left(2\Lambda\left(\pi_{L,p}\right) - c, c\right)\right]$. Note that since Λ is strictly decreasing in $\pi_{L,p}$, $\Lambda\left(\pi_{L,p}\right) > \Lambda\left(\pi'_{L,p}\right)$. We need to show $E\left[C_L^s | C_L^s \in \left(2\Lambda\left(\pi'_{L,p}\right) - c, c\right)\right] \leq E\left[C_L^s | C_L^s \in \left(2\Lambda\left(\pi_{L,p}\right) - c, c\right)\right]$. By law of iterated expectations, we have

$$E\left[C_L^s | C_L^s \in \left(2\Lambda\left(\pi'_{L,p}\right) - c, c\right)\right] =$$

$$E\left[C_L^s | C_L^s \in \left[2\Lambda\left(\pi'_{L,p}\right) - c, 2\Lambda\left(\pi_{L,p}\right) - c\right]\right] P\left[C_L^s \in \left[2\Lambda\left(\pi'_{L,p}\right) - c, 2\Lambda\left(\pi_{L,p}\right) - c\right] | C_L^s \in \left(2\Lambda\left(\pi'_{L,p}\right) - c\right.\right.$$

$$E\left[C_L^s | C_L^s \in \left[2\Lambda\left(\pi_{L,p}\right) - c, c\right)\right] P\left[C_L^s \in \left[2\Lambda\left(\pi_{L,p}\right) - c, c\right) | C_L^s \in \left[2\Lambda\left(\pi'_{L,p}\right) - c, c\right)\right]$$
$$\leq E\left[C_L^s | C_L^s \in \left[2\Lambda\left(\pi_{L,p}\right) - c, c\right)\right]$$

Since $E\left[C_L^s | C_L^s \in \left[2\Lambda\left(\pi_{L,p}\right) - c, c\right)\right] \geq E\left[C_L^s | C_L^s \in \left[2\Lambda\left(\pi'_{L,p}\right) - c, 2\Lambda\left(\pi_{L,p}\right) - c\right]\right]$. Q.E.D.

Proposition 8: $E\left[C_L^s | Lose_L^s, \pi_{L,p}\right] \geq E\left[C_L^s | Lose_L^s, \pi'_{L,p}\right]$
Notice that from equation (17) we have that $E\left[C_L^s | Lose_L^s, \pi_{L,p}\right] = E\left[C_L^s | M^s \in \left(-\infty, \Lambda\left(\pi_{L,p}\right)\right)\right]$. We can rewrite the interval of winnable midpoints in terms of the function Λ and consider an arbitrary draw of $C_R^s = c$. Then we have that $E\left[C_L^s | M^s \in \left(-\infty, \Lambda\left(\pi_{L,p}\right)\right), C_R^s = c\right] = E\left[C_L^s | C_L^s \in \left(-\infty, 2\Lambda\left(\pi_{L,p}\right) - c\right)\right]$. We need to show $E\left[C_L^s | C_L^s \in \left(-\infty, 2\Lambda\left(\pi'_{L,p}\right) - c\right)\right] \leq E\left[C_L^s | C_L^s \in \left(-\infty, 2\Lambda\left(\pi_{L,p}\right) - c\right)\right]$. By law of iterated expectations, we have

$$E\left[C_L^s | C_L^s \in \left(-\infty, 2\Lambda\left(\pi_{L,p}\right) - c\right)\right] =$$

$$E\left[C_L^s | C_L^s \in \left[2\Lambda\left(\pi'_{L,p}\right) - c, 2\Lambda\left(\pi_{L,p}\right) - c\right]\right] P\left[C_L^s \in \left[2\Lambda\left(\pi'_{L,p}\right) - c, 2\Lambda\left(\pi_{L,p}\right) - c\right] | C_L^s \in \left(-\infty, 2\Lambda\left(\pi_{L,p}\right.\right.\right.$$

$$E\left[C_L^s | C_L^s \in \left(-\infty, 2\Lambda\left(\pi'_{L,p}\right) - c\right]\right] P\left[C_L^s \in \left(-\infty, 2\Lambda\left(\pi'_{L,p}\right) - c\right] | C_L^s \in \left(-\infty, 2\Lambda\left(\pi_{L,p}\right) - c\right)\right]$$
$$\geq E\left[C_L^s | C_L^s \in \left(-\infty, 2\Lambda\left(\pi'_{L,p}\right) - c\right)\right] \quad .$$

Q.E.D.

Proposition 9: $P\left(Win_L^s | \pi'_{L,p}\right) > P\left(Win_L^s | \pi_{L,p}\right)$

Note that $P\left(Win_L^s | \pi'_{L,p}\right) = P\left(M^s \in \left(\Lambda\left(\pi'_{L,p}\right), \infty\right)\right)$. Since Λ is strictly decreasing in $\pi_{L,p}$, $\left(\Lambda\left(\pi_{L,p}\right), \infty\right) \subsetneq \left(\Lambda\left(\pi'_{L,p}\right), \infty\right)$. This implies that $P\left(M^s \in \left(\Lambda\left(\pi'_{L,p}\right), \infty\right)\right) > P\left(M^s \in \left(\Lambda\left(\pi_{L,p}\right), \infty\right)\right)$. Q.E.D

Proposition 10: $P\left(Lose_L^s | \pi'_{L,p}\right) < P\left(Lose_L^s | \pi_{L,p}\right)$

Note that $P\left(Lose_L^s | \pi'_{L,p}\right) = P\left(M^s \in \left(-\infty, \Lambda\left(\pi'_{L,p}\right)\right)\right)$. Since Λ is strictly decreasing in $\pi_{L,p}$, $\left(-\infty, \Lambda\left(\pi'_{L,p}\right)\right) \subsetneq \left(-\infty, \Lambda\left(\pi_{L,p}\right)\right)$. This implies that $P\left(M^s \in \left(-\infty, \Lambda\left(\pi_{L,p}\right)\right)\right) > P\left(M^s \in \left(-\infty, \Lambda\left(\pi'_{L,p}\right)\right)\right)$. Q.E.D