Real Options and Rules of Thumb in Capital Budgeting

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Abstract

Most firms do not make explicit use of real option techniques in evaluating investments. Nevertheless, real option considerations can be a significant component of value, and firms which approximately take them into account should outperform firms which do not. This paper asks whether the use of seemingly arbitrary investment criteria, such as hurdle rates and profitability indexes, can proxy for the use of more sophisticated real options valuation. We find that for a variety of parameters, particular hurdle-rate and profitability index rules can provide close-to-optimal investment decisions. Thus, it may be that firms using seemingly arbitrary "rules of thumb" are approximating optimal decisions.

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1.   Introduction

Suppose that a manager must decide whether to invest $500 million for a manufacturing facility which can be built today or at some later time. If the present value of cash flows from the facility is estimated at $500.001 million, NPV is $1000; hence by the NPV criterion the investment should be undertaken. Finance students often find the decision to invest $500 million in order to earn $1000 troubling, though they are often unable to articulate a reason.   This lack of comfort may extend to managers: it appears common for firms to use investment criteria which do not strictly implement the NPV criterion.

Anecdotal evidence suggests that firms making capital budgeting decisions routinely do a number of things that basic finance textbooks say they should not do:

- projects are taken based on whether or not internal rates of return exceed arbitrarily high discount rates (often called "hurdle rates"),

- hurdle rates are sometimes higher for projects with greater idiosyncratic risk,

- project selection is sometimes governed by a "profitability index", i.e., NPV/(Investment Cost) must be sufficiently great, and

- otherwise acceptable projects go untaken, i.e., firms engage in capital rationing.

Summers (1987) surveyed corporations on capital budgeting practices and found that 94% of reporting firms discounted all cash flows at the same rate, independently of risk; 23% used discount rates in excess of 19%. This behavior is suggestive of the use of hurdle-rate rules, and certainly at odds with textbook prescriptions for how to do capital budgeting.

This article asks whether these seemingly "incorrect" capital budgeting practices might serve as proxies for economic considerations not properly accounted for by the NPV rule. It is well-known by now that the NPV criterion has serious shortcomings. In particular, the project in the example above

could be delayed. Under uncertainty, the decision about when to invest is analogous to the decision about when to exercise an American call option, and the firm should generally invest only when the project NPV is sufficiently positive. Obviously, most managers do not formally perform this calculation as a routine part of capital budgeting.[1] Nevertheless, although managers may not use formal models to evaluate the options associated with an investment project, these options can be economically important and their effects grasped intuitively. Firms that make decisions ignoring these options should on average be less profitable than firms that somehow take them into account. This raises the question: is it possible that firms can make investment decisions that are *close* to optimal by following simple rules of thumb?[2]

We consider the extent to which observed investment decision-making behavior might be justified as an informal way to account for real options considerations, and in particular, investment timing. We take as a benchmark case the investment timing model of McDonald and Siegel (1986). In the context of that model, a firm should delay investing in a project until the NPV of the project is sufficiently positive, with the specific investment hurdle determined by inputs such as the volatility of the project, and the cash flows foregone by deferring investment. We focus on investment timing flexibility, since it is a simple option to evaluate and one that's likely to be important in a wide variety of real-world investment problems. We ask whether simple investment decision rules can approximate the optimal investment deferral implied by the investment-timing model.

It is obvious that in the simple case where the value of the project follows a time-homogeneous process, then for any particular set of project characteristics there is a corresponding hurdle-rate or profitability index rule which will give the correct decision about when to invest. For an investment project with a known and constant drift, variance, and required rate of return, investment in the project is

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[1]"To Wait or Not to Wait", *CFO* Magazine, Vol 13, No 5 (May 1997), pp. 91-94 reports on companies which have adopted explicit option valuation methods.

[2]While we show that some intuitively plausible rules of thumb can be reasonable decision rules, we do not try to explain how firms arrive at these particular rules.

optimal when the project value reaches a particular level. This project value in turn can be expressed in terms of an IRR, so there is always a correct investment rule of the form: "invest when the IRR reaches r*."[3] A more interesting question is whether simple rules are relatively robust to changes in project characteristics. For example, suppose that a firm has projects with a wide variety of characteristics, including discount rate and volatility. Can a single hurdle-rate rule yield approximately correct decisions for these projects?[4]

We perform experiments in which we fix the investment rule and vary project characteristics, such as the project discount rate and expected growth rate of cash flows. Our finding is that for a wide range of project characteristics, fixed hurdle-rate rules and profitability index rules can provide a good approximation to optimal investment timing decisions in the sense that the *ex ante* loss from following the suboptimal rule is small; it is possible to follow the wrong investment rule without losing much of the *ex ante* value of the investment timing option.[5] In fact, as the investment timing option becomes worth more and it becomes optimal to wait longer to invest, the option value becomes less sensitive to errors in investment rules.

We also consider the effect of permitting project abandonment, discussed by Brennan and Schwartz (1985) and Dixit (1989), and show that permitting non-trivial reversibility, for example being able to scrap the project for 50% of the investment cost, does not significantly alter the conclusions. There are, of course, other options besides the investment timing option which affect the value of projects and the optimal investment strategy: multi-stage investments, which allow the firm to abandon

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[3]Dixit (1992) shows how to compute the hurdle rate for a given real option, and Boyle and Guthrie (1997) show that there is always an equivalent payback rule.

[4]If a firm does not understand well the economics underlying investment decisions, there might be an advantage to specializing in projects of a particular type and applying an appropriate investment rule, compared to a conglomerate applying a "one-size-fits-all" rule to various projects.

[5]Cochrane (1989) poses a similar question in the context of consumption models, and finds that the loss from following non-optimal consumption rules are small.

the project before completion, options to shut-down production, strategic options, switching options, etc... In addition, while we focus on cash-flow uncertainty, a valuable investment timing option can also be generated by interest rate uncertainty (Ingersoll and Ross (1992)). Thus the findings here are suggestive, and not intended to suggest that particular rules of thumb should be universally adopted.

The results in this paper can help assess the relative value of knowing different characteristics of a project, and thus in principle help managers to allocate their time in investment decision-making. For example, knowledge of the project discount rate is extremely important for a standard NPV calculation. Nevertheless, it sometimes turns out to be unimportant for the investment timing decision, in the sense that a given rule of thumb might work well for projects with a variety of discount rates. Raising the project discount rate lowers the value of the project, but also lowers the value at which investment becomes optimal, so that a decision rule of the form "invest when the project has an internal rate of return of 20%" might in fact be appropriate for a wide variety of projects.

Section 2 presents the basic investment timing problem and explains the procedure we use for evaluating investment rules of thumb. A key result here is that as the investment option becomes more valuable, it also becomes less sensitive to errors in investment rules. Section 3 explores different investment rules in more detail and examines the loss associated with different rules under various parameter values in the basic investment timing model. Throughout the paper we use as benchmarks two somewhat arbitrary rules: a 20% hurdle-rate rule and a 1.5 profitability index. Section 4 examines robustness of the results to different assumptions about the evolution of project value, such as a negative growth rate and the possibility of a jump to zero in project value, and also considers the impact of adding a scrapping option. Section 5 concludes. The general conclusion is that the rules of thumb considered generally capture at least 50% of a project's option value, and often as much as 90%.

2.   The Investment Timing Problem

In this section we review the basic investment timing problem and explain how a given rule of thumb may be assessed in this framework.

2.1   The Basic Problem

Suppose that C_t, the instantaneous cash-flow rate from an irreversible investment project, follows the diffusion process:

$$\frac{dC_t}{C_t} = \alpha \, dt \; + \; \sigma \, dZ(t) \tag{1}$$

where α is the expected growth rate of cash flows and σ is the standard deviation of the cash-flow process.[6] Note that with α and σ constant, the project value is time-homogenous. If the project is infinitely-lived, the present value of the cash flows — conditional on the project being undertaken — is given by

$$V_t \; = \; \frac{C_t}{\rho \; - \; \alpha} \tag{2}$$

where ρ is the required rate of return on a project with the risk implied by (1). Note that since V is proportional to C, dV/V also follows a stochastic process of the same form as equation (1). The model can be expressed either in terms of C or V, but since we are interested in the effects of varying ρ and α, it is useful to specify the relation between C_t and V_t. We will be agnostic about the determination of ρ, although in practice it could be determined by the CAPM or some similar equilibrium model. Investment in the project costs I.

Let $\delta = \rho - \alpha$ be the difference between the required return on the project and the actual rate of appreciation in value, α. Note that $\delta \equiv C_t/V_t$, is a measure of the proportional cash flows foregone by not

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[6]This process assumes that net cash flows are always positive. Essentially the same model can be derived when C_t represents gross cash flows and there is a cost. See Dixit (1989).

investing in the project. For this reason we will refer to δ as the dividend yield on the project. The risk-free rate is given by r, so that the project's risk premium is ρ - r.

The basic investment timing problem with risky cash flows is analyzed in Brennan and Schwartz (1985), McDonald and Siegel (1986), and Dixit (1989).[7] The firm can acquire the project, worth V, by investing I. This raises two questions: when is it optimal to invest, and what is the value of following the optimal investment rule? The general solution is outlined in the Appendix. Consider the special case in which the project is completely irreversible (i.e., the scrap value is 0) and we follow the rule to invest when the value of the project is at an arbitrary threshold level, $V_A > I$.[8] Prior to undertaking investment, the value of the option to invest in the project, assuming that the firm invests when project value reaches a trigger value V_A, is

$$W(V,V_A) = (V_A - I)\left(\frac{V}{V_A}\right)^{b_1} \tag{3}$$

where

$$b_1 = \left(\frac{1}{2} - \frac{r-(\rho-\alpha)}{\sigma^2}\right) + \sqrt{\left(\frac{r-(\rho-\alpha)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{4}$$

The optimal policy is obtained by maximizing (3) with respect to V_A. This yields

$$V_H = \frac{b_1}{b_1 - 1} I \tag{5}$$

We shall refer to $W(V,V_A)$ as the value of the investment timing option and V_H as the *optimal* trigger value for investment. An important feature of the solution is that it is optimal to invest only when V is

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[7]Ingersoll and Ross (1992) analyze the case of risk-free cash flows and stochastic interest rates.

[8]If the scrap value of the project is positive, optimal scrapping is easily accommodated with a numerical solution.

Figure 1
Value of the investment timing option, $W(1,V_H)$, as a function of the project discount rate, ρ.
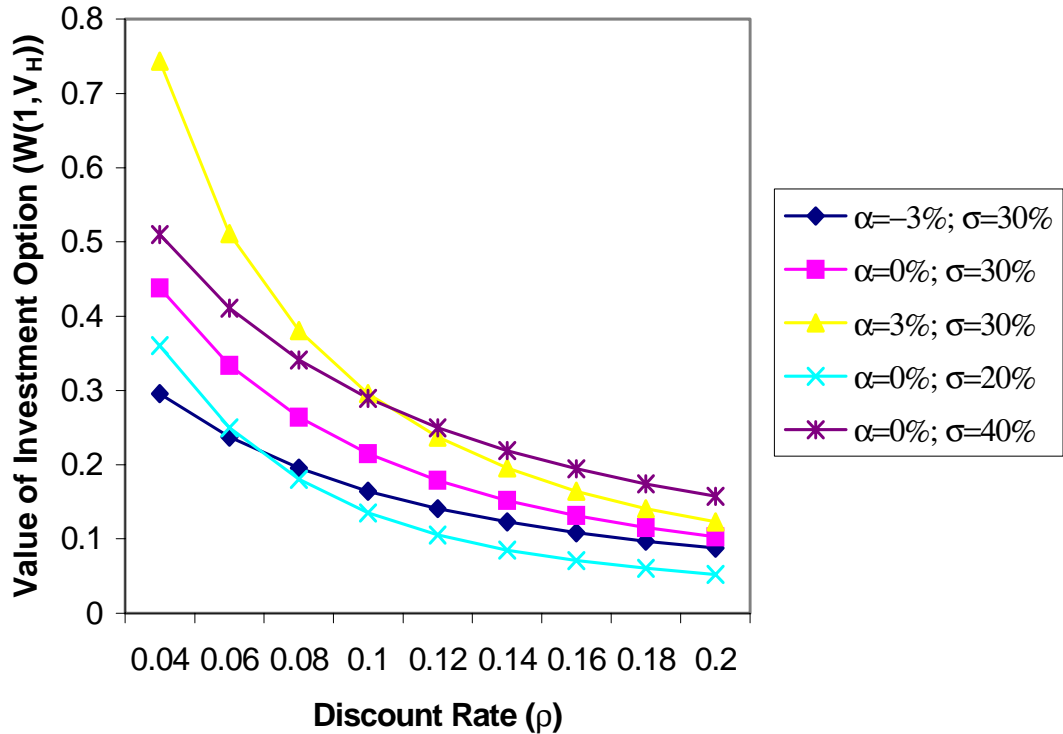

Note: Each point represents the value of the investment timing option for a different project with a zero NPV. Computed using equation (3) with $I = 1$: $W(1,V_H) = (V_H - 1)\left(\dfrac{1}{V_H}\right)^{b_1}$. Assumes the risk-free rate $r = 8\%$.

strictly greater than I.

It is useful at this point to recall some basic intuition about the value of the investment timing option. The option value W and the optimal trigger value V_H depend on the parameters r, ρ, α, and σ. As we vary these parameters, the option value W and the optimal trigger value V_H change in the same

Figure 2
Optimal trigger value, V_H, as a function of the discount rate, ρ.
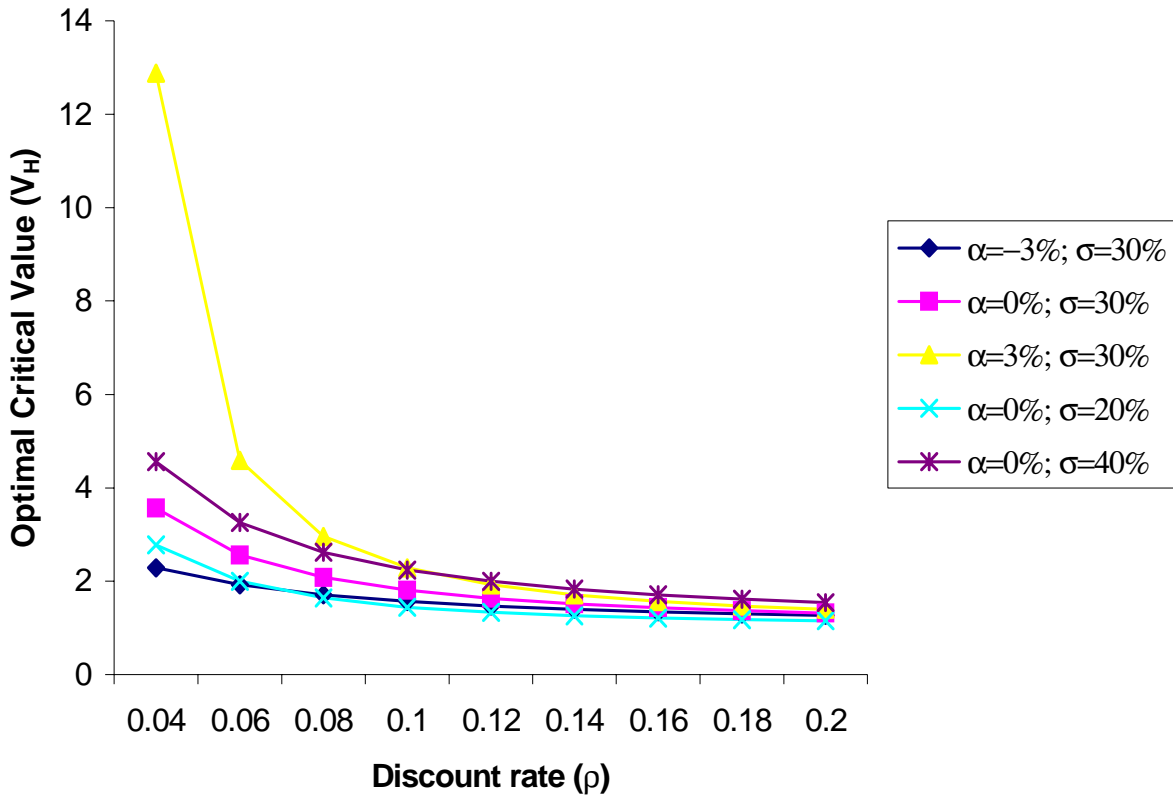
Note: Optimal critical value, V_H, computed as $b_1/(b_1 - 1)$. Assumes project value V, and investment cost, I = 1, and r = 8%.

direction: when V_H increases, $W(V,V_H)$ also increases.[9] The comparative statics of W are well-known.

First, deferring investment is valuable because the expenditure I is delayed and interest is earned, hence

as r increases, optimal deferral of investment increases. Second, deferring investment is more valuable

the greater the uncertainty, σ: the option to wait to invest implicitly provides insurance against declines

in the value of the investment project. Third, deferring investment is more costly when the cash flows

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[9]This is easily verified in general by noting that V_H is decreasing in b_1 (from equation (5)) and $W(V, V_H)$ is decreasing in b_1 (from equation (3)). By the envelope theorem, dV_H/db_1 can be ignored in evaluating $\partial W(V,V_H)/\partial b_1$. Hence since $V < V_H$, W is decreasing in b_1

lost by deferral, $\delta = \rho - \alpha$, are greater.

Figure 1 depicts the value of the investment timing option as a function of the project discount rate, ρ, and illustrates the effect of varying the cash-flow growth rate, α, and project volatility, σ.[10] Each point on the graph should be thought of as a separate project with a different discount rate, each of which currently has a zero NPV, i.e. $V = I = 1$. A firm which invests immediately at zero NPV would therefore lose the full value of the investment option depicted in the figure. Holding V fixed, the value of the option is an increasing function of the cash-flow growth rate, α, and the volatility, σ, and a decreasing function of the difference between the project discount rate, ρ, and the cash-flow growth rate, α.

Figure 2 shows how the optimal trigger value V_H varies with ρ, α, and σ. V_H declines as ρ rises, and increases with σ. In all cases, V_H asymptotes to infinity as ρ approaches α, i.e. as the project dividend yield, δ, approaches 0. This corresponds to the well-known result that an American call option on a non-dividend paying stock will never be exercised prior to expiration.

2.2 Measuring the Cost of Suboptimal Investment

In the typical real options problem we are interested in the optimal investment decision rule and the value of following that rule. However, equation (3) permits us to assess the option value associated with any arbitrary investment decision rule. For a given α, σ and ρ, the use of a particular investment decision rule — for example the hurdle-rate or profitability index — is equivalent to the choice of some investment trigger V_A. The central question in this paper is whether various approximations to the optimal rule given in equation (5) are "good enough" for practical purposes. In other words, if a manager does not explicitly calculate equation (5), is it possible that a seemingly arbitrary investment rule comes close, in the sense that the value lost from the approximation is small?

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[10]For a given V, the option value is a function only of $\delta = \rho - \alpha$. However, the capital budgeting rules we will later consider do sometimes depend separately on ρ and α, hence we consider them separately in the figures.

The value of following a non-optimal investment policy is depicted by Figure 3, which shows

how the value of the investment option, $W(1, V_A)$, varies as we vary the investment trigger, V_A, from 1 to

7, assuming that $r = 8\%$ and $\alpha = 0$. In each case, V_H is the level of V_A at which the option value W attains

a maximum. For example, at $\rho = 12\%$ and $\sigma = 30\%$, $V_H = 1.63$ is the level of V at which investment is

Figure 3
Value of the investment option, $W(1, V_A)$, as a function of the critical project value which triggers investment, V_A.
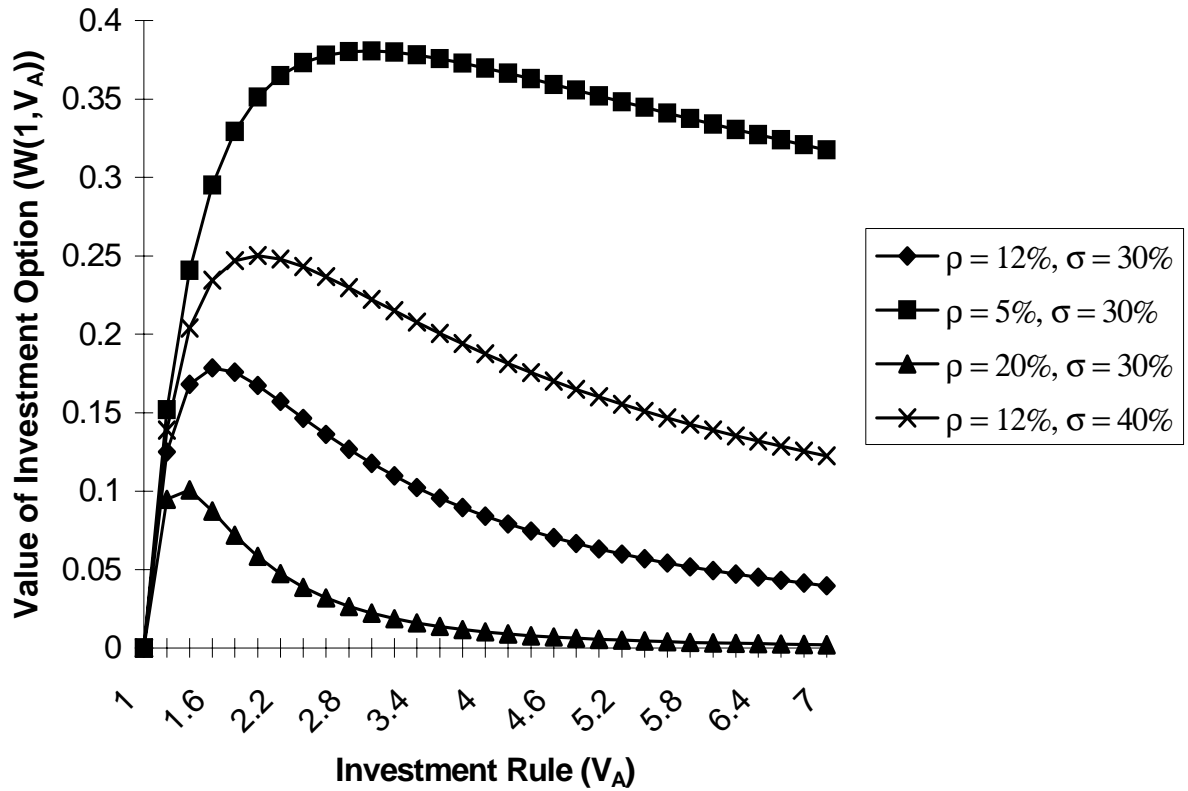
Note: $W(1,V_A)$ computed using equation (3) with $I = 1$: $W(1,V_A) = (V_A - 1)\left(\dfrac{1}{V_A}\right)^{b_1}$ *Assumes project*

value V and investment cost $I = 1$, and risk-free rate $r = 8\%$.

optimal. Also displayed are the results for discount rates of $\rho = 5\%$ and 20%, and a 40% volatility vs. 30% volatility. Several points are clear from the figure. First, the *worst* investment decision is to invest when $V = I = 1$, i.e. when NPV is zero. Second, the loss from selecting the wrong investment rule is asymmetric: it is usually better to wait too long to invest (i.e. to select too high a V_A) than to invest too soon. Finally, the conclusion is not that the investment rule does not matter (clearly it does) but rather that a broad range of rules gives roughly similar (albeit suboptimal) outcomes.

Table 1 is a counterpart to Figure 3, showing precisely what range of trigger values V_A will provide a specified minimum percentage of the optimal option value for a given set of parameters. For example, in the case where $\rho = .05$ and $\sigma = .3$, an investment trigger value range of 1.91 to 5.52 preserves 90% of the optimal option value, while a range of 1.28 to 23.71 preserves 50% of the optimal option value. In general, the greater the option value when the optimal critical project value is chosen, the wider the range of V_A at which a given percentage of the option value can be obtained. Put differently, for more valuable options, a given deviation from the optimal rule generates a smaller loss in project value.

Table 1
Ranges of critical project values, V_A, over which investment option attains at least a given percentage of its maximum possible value.

Percent of option value obtained when V_A is set equal to specified low and high values		$\rho=.05, \sigma=.3$	$\rho=.12, \sigma=.3$	$\rho=.20, \sigma=.3$	$\rho=.12, \sigma=.4$
	$W(1, V_H)$	0.38	.18	.10	.25
100%	V_H	2.96	1.63	1.32	2.00
90%	Low / High	1.91 / 5.52	1.35 / 2.12	1.19 / 1.53	1.52 / 2.92
75%	Low / High	1.56 / 9.29	1.23 / 2.63	1.12 / 1.75	1.33 / 4.00
50%	Low / High	1.28 / 23.71	1.12 / 3.81	1.07 / 2.09	1.17 / 6.83

Note: For example, if $\rho = .05$ and $\sigma = .3$, investment option attains at least 90% of its optimal value if $1.91 < V_A < 5.52$. Assumes investment cost $I = 1$, project value $V = 1$, risk-free rate $r = 8\%$, and cash-flow growth rate $\alpha = 0$. All examples are computed using equation (3):

$$W(1,V_A) = (V_A - 1)\left(\frac{1}{V_A}\right)^{b_1}.$$

This is evident in Figure 3, where $W(1, V_A)$ is flatter in the vicinity of V_H when the option value is greater. This property is true in general, as can be seen by differentiating equation (3):

$$\frac{\partial\left[\frac{\partial^2 W(1,V_A)}{\partial V_A^2}\Big|_{V_A=V_H}\right]}{\partial b_1} < 0 \tag{6}$$

Equation (6) is verified analytically in the Appendix. This characteristic of option value proves important when we later assess approximations to the optimal investment rule.

Of course, Table 1 depends on the assumption that cash flow follows geometric Brownian motion. If cash flows instead followed a mean-reverting process, the range over which a given fraction of the option value is preserved would be smaller, and the upper values of the range would not be as great.

3.   Assessing "Rules of Thumb"

In this section we examine the effect on investment value of using different *ad hoc* investment rules. Since we wish to investigate approximations to the optimal rule, we fix the investment rule and vary the assumptions to see how well a given investment rule performs in a wide range of situations. Here we consider variations in the cash-flow growth rate, α, the project volatility, σ, and the project discount rate, ρ. Varying the discount rate is a particularly interesting experiment since discount rates are hard to estimate in practice. Further, academics do not agree on an appropriate equilibrium model even for estimating firm-level discount rates, for which stock returns are observable; estimation of project-level discount rates is even more problematic. Thus it seems likely that there is a great deal of uncertainty associated with estimating project-specific discount rates. We next examine three rules: hurdle-rate, profitability index (V/I), and payback.

3.1   Hurdle Rates

As Dixit (1992) points out, for time-homogeneous cash flows, the optimal investment rule can be expressed as a constant hurdle-rate rule. For a given level of current project cash flow, C_t, the internal rate of return on the project, R, is

$$R = \frac{C}{I} + \alpha$$

A hurdle-rate rule calls for investing when the project's internal rate of return exceeds the hurdle rate, which we will denote as γ. A hurdle-rate rule, γ, is equivalent to a cash-flow rule in which the cash flow trigger, C_γ, is given by

$$C_\gamma = I(\gamma - \alpha). \tag{7}$$

or, in terms of project value, since from (2), $C = V(\rho - \alpha)$, we would invest when project value is

$$V_\gamma = \frac{\gamma - \alpha}{\rho - \alpha} I \tag{8}$$

For an arbitrary trigger value, V_A, the corresponding hurdle-rate rule would be to invest when the internal rate of return, R, equals γ_A, where

$$\gamma_A = \alpha + (\rho - \alpha) V_A / I$$

The zero-NPV rule is to invest when $V_A = I$, hence $\gamma = \rho$, i.e. the internal rate of return equals the cost of capital. Since the optimal investment trigger under uncertainty is $V = V_H$, the optimal hurdle-rate rule is

$$\gamma_H = \alpha + (\rho - \alpha) b_1/(b_1 - 1) \tag{9}$$

This is similar to Dixit's (1992) expression.

Although it is not obvious by inspection of equation (9), comparative statics for the optimal hurdle-rate, γ_H, mimic those for the optimal trigger value, V_H. First, an increase in the project discount rate, ρ, increases the optimal hurdle rate, γ_H. Second, an increase in the project cash-flow growth rate, α, decreases γ_H. Third, an increase in σ raises the optimal hurdle rate. These results can be verified by

Table 2
Optimal hurdle rate, γ_H, for representative parameters.

Cash flow volatility, σ	Cash flow growth rate, α	Project Discount Rate, ρ		
		8%	12%	16%
	-3%	12.2	15.5	19.1
20%	0	13.1	16	19.4
	3%	14.5	16.7	19.8
	-3%	15.8	19.1	22.5
30%	0	16.7	19.6	22.9
	3%	17.8	20.3	23.4
	-3%	20.2	23.5	26.9
40%	0	20.9	24	27.3
	3%	21.8	24.7	27.8

Note: computed using equation (9): $\gamma_H = \alpha + (\rho - \alpha) b_1/(b_1 - 1)$. Assumes risk-free rate r = 8%.

differentiating equation (9).

To get a sense of magnitudes, Table 2 reports γ_H for representative parameters. The optimal hurdle rate, γ_H, is sensitive to changes in ρ and σ, and relatively less sensitive to changes in α. Obviously if one were to adopt a fixed hurdle rate for all projects, say 20%, there would generally be errors in the timing of investment. For example if $\rho = 8\%$, $\sigma = 30\%$, and $\alpha = 3\%$, the correct decision is to invest when project cash flow, C, satisfies

$$C/(.178 - .03) = 1,$$

or C = .148. A 20% hurdle-rate rule would entail investing when

$$C/(.2 - .03) = 1$$

or C = .17. While investment generally occurs at the wrong time using a 20% hurdle rate, the question

Table 2 does not address is whether the use of the wrong hurdle rate creates a significant loss of value.

Our earlier analysis suggests that the loss in *value* is not necessarily large, even if the investment decision

is made at the wrong time.

Figure 4 depicts the fraction of project value, $W(1,V_\gamma)/W(1,V_H)$, obtained by using a 20% hurdle-

rate rule for cases depicted in Figures 1 and 2 (also considered in 5 of the 9 rows in Table 2). To see how

Figure 4
Fraction of maximum option value obtained by basing investment decisions on a 20% hurdle-rate rule.
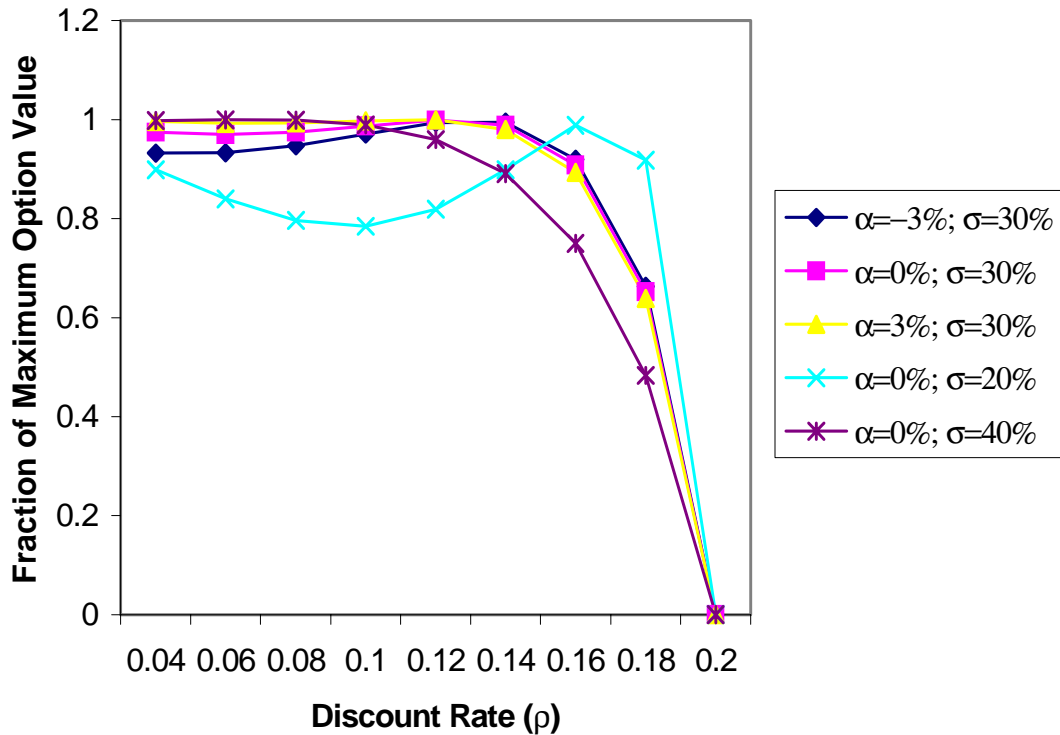
Note: Computed as W(1,V$_\gamma$)/W(1,V$_H$), the ratio of option value when investment trigger value is computed using equation (8), i.e. $V_\gamma = I\, \dfrac{\gamma - \alpha}{\rho - \alpha}$, *with hurdle rate $\gamma = 20\%$, to the option value when the investment trigger is optimal. Assumes risk-free rate r = 8%.*

the figure is constructed, consider the case where $\rho = 12\%$, $\alpha = 0$, and $\sigma = 40\%$. The optimal hurdle rate from Table 2 is 24%. From Table 1, following this rule gives an option value of .25, and we should invest when V = 2. By following a 20% hurdle-rate rule, we invest when C is such that C/.2 = 1, or C = .2, which implies a trigger value of V_A = .2/.12 = 1.67. This in turn yields an option value of .24, 96% of the optimal option value of .25. For those cases with optimal hurdle rates γ_H below 20%, the effect of the 20% hurdle-rate rule is to delay investment beyond the optimal point, while it accelerates investment when γ_H exceeds 20% The 20% hurdle-rate rule works quite well as long as the true project discount rate is 16% or below: in almost all cases, the project is worth at least 80% of its maximal value.

The 20% hurdle-rate rule works least well in the low volatility case, i.e. when $\sigma = 20\%$. When project volatility is low, the investment option is worth the least, and it is optimal to invest at relatively low project values. In this case the 20% hurdle-rate rule leads to excessive delay. Since the optimal trigger, V_H, is declining in volatility, performance of the rule would be even worse for lower volatilities.

Figure 4 is intentionally constructed to show that the hurdle-rate rule provides zero value if the true project discount rate is 20%. This occurs for the following reason: if we adopt a hurdle rate equal to the true project discount rate, then we are back to following the NPV rule. In that case we lose all value from the investment timing option. The potential gain with a hurdle-rate rule comes from selecting a hurdle rate which is higher than the true discount rate, in order to delay investment under uncertainty.

3.2   Profitability index

The profitability index criterion entails investing when the ratio of the project value per unit cost, V/I, reaches some pre-selected level, which we will denote by Π. The profitability index is usually presented in textbooks as a criterion used to rank projects when investment funds are limited. It may also be, however, that the profitability index has survived as a capital budgeting practice because in some situations it produces better results than the NPV rule, even though the textbook rationale for its use is spurious. The operational difference between the hurdle-rate rule and the profitability index is that with

the hurdle-rate rule, the trigger value V_A implicitly changes as the true underlying parameters change.

The profitability index, on the other hand, explicitly specifies a fixed V_A / I. Obviously, setting $\Pi = V_H / I$

yields optimal investment decisions; here we are interested in how a given profitability index performs

across different projects.

Suppose we set $\Pi = 1.5$. Figure 5 depicts the fraction of the maximum option value obtained by

following this investment rule for projects with different characteristics. By definition the rule performs

Figure 5
Fraction of maximum option value obtained by investing when the project is worth 1.5 times investment cost, i.e. $\Pi = 1.5$.
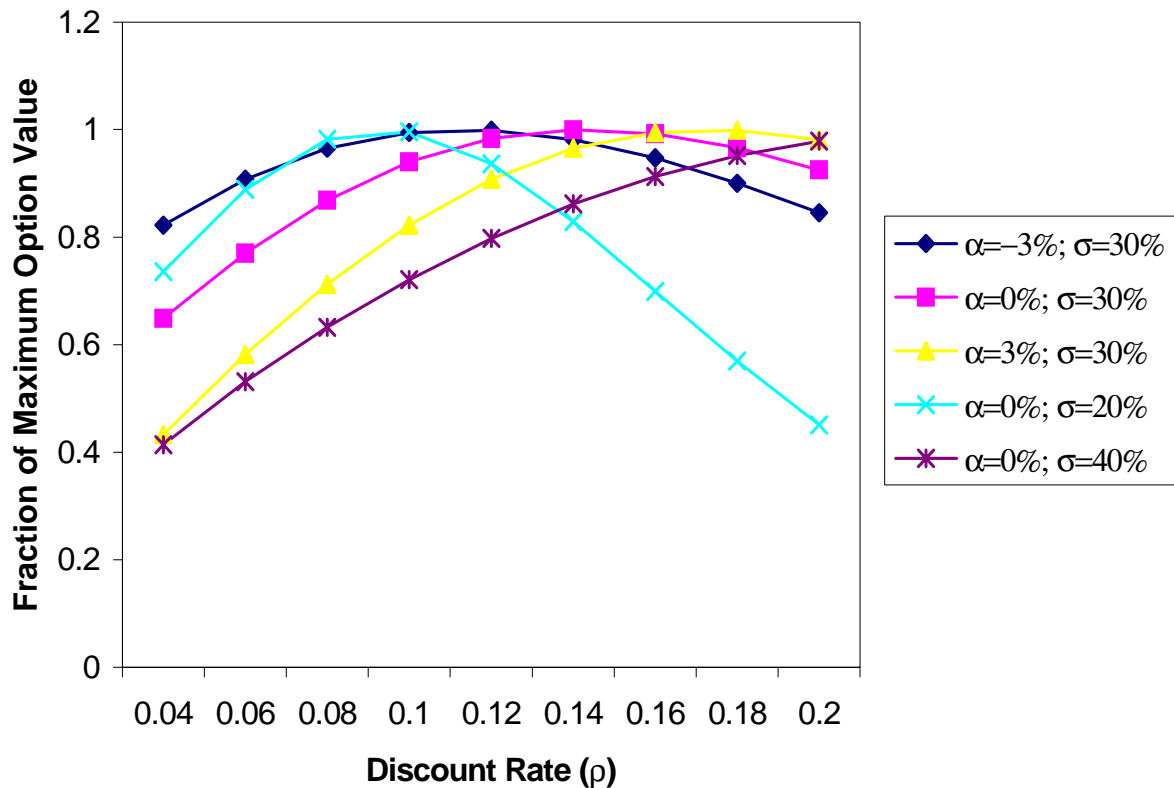
Note: Computed as $W(1,1.5)/W(1,V_H)$, the ratio of option value when investment trigger value is 1.5 to the option value when the investment trigger is optimal. Assumes risk-free rate $r = 8\%$.

best in those cases where V_H/I is close to 1.5. For the project with $\sigma = 20\%$, the profitability index rule

works best for lower discount rates. As σ increases, the rule works better for progressively higher

discount rates, reflecting the movement in V_H as the project parameters change. The profitability index

rule works best, relative to the hurdle-rate rule, when discount rates are very high. Where the hurdle-rate

rule provides insufficient project delay (because the hurdle rate is close to the true project discount rate),

the profitability index rule provides at least some delay.

3.3   Payback Rules

Payback rules are also useful to examine in this framework.[11] Payback is defined as the time

until the sum of expected future cash flows equal the investment cost. Cash flows in this calculation can

be either discounted or undiscounted; we focus on the latter. The payback period is the horizon T such

that

$$
\begin{aligned}
I &= C_t \int_t^{t+T} e^{\alpha(s-t)}ds \\[2mm]
&= \frac{C_t}{\alpha}(e^{\alpha T} - 1)
\end{aligned}
\tag{10}
$$

If we set an arbitrary payback period, then the payback criterion is satisfied when $V = V_P$, where V_P is

given by[12]

$$
V_P = \begin{cases} \left(\dfrac{\alpha}{\rho-\alpha}\right)\dfrac{I}{e^{\alpha T}-1} & \alpha \neq 0 \\[4mm] \dfrac{I}{T\rho} & \alpha = 0 \end{cases}
\tag{11}
$$

Note first that when $\alpha = 0$, from equations (8) and (11), the payback rule with payback period T is

equivalent to a hurdle-rate rule with hurdle rate $\gamma = 1/T$. Thus, the behavior of the two rules differs only

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[11]Boyle and Guthrie (1997) examine payback in a similar context.

[12]With discounted payback, the critical project value is given by $I/(1-e^{(\alpha-\rho)T})$.

when $\alpha \neq 0$. For the parameters reported in Figure 4, the differences between the hurdle rate and payback calculations are mostly slight and hence are not reported here. The material differences occur for high discount rates: when α is positive, the payback rule generates a critical project value which is greater than 1, and hence this rule avoids the sharp drop-off in value generated by the hurdle-rate rule Figure 4 as ρ approaches 20%.

3.4   Comparison of Profitability Index and Hurdle-rate Rules

In comparing Figures 4 and 5, the hurdle-rate and profitability index rules tend to be most inaccurate for different sets of parameter values. The profitability index rule $\Pi = 1.5$ works least well for low discount rate projects, when it is optimal to wait until V reaches a substantially higher critical value. The 20% hurdle-rate rule works least well when project discount rates are close to the hurdle rate. This suggests constructing a third rule as a hybrid of the two rules, for example selecting the maximum critical project value implied by the two rules. By implementing this rule, we generate high threshold Vs from the hurdle rate rule when discount rates are low, and threshold Vs significantly above 1 when discount rates equal or exceed the hurdle rate.

Such a hybrid rule can prevent the large errors at extreme discount rates generated by either rule alone. For the cases we have examined, in all but the $\sigma = 20\%$ case, the hybrid rule captures at least 85% of the value of the optimal rule in all cases, and captures 95% or greater in the vast majority of cases. Although this example stretches a bit the concept of a "rule of thumb", it demonstrates that firms in practice might find it useful to consider multiple rules at once, perhaps using a rule which best justifies making intuitively-plausible investment decisions.

4.   Robustness to Alternative Project Assumptions

We have so far assumed that projects are irreversible and that cash flows, and hence project values, follow geometric Brownian motion. In this section we briefly consider the effect of alternative

assumptions about the project. One objection to the prior analysis is that it does not accommodate cases where there is an intuitive sense that the project will be lost if it is not taken quickly. We consider two ways to model this: negative expected cash-flow growth rates, and the possibility that project can take a Poisson jump to zero. Finally, we also examine the effect of scrapping, which amounts to permitting costly reversibility of the investment.

4.1 Negative Cash-flow Growth

Suppose project cash flows are high but are expected to decline quickly. This is expected in an industry in which competitive entry is anticipated. A *ceteris paribus* reduction in the growth rate reduces both the value of the investment timing option, W, and the optimal trigger value, V_H. Figures 1 and 2 confirm that even a small negative cash flow growth rate, α, noticeably lowers the value of the timing option and V_H.

With a cash-flow growth rate, α, of -20%, the value of the timing option falls below .15 per dollar of investment cost and V_H is below 1.45 for all cases we have previously considered. The 20% hurdle rate rule performs better than the 1.5 profitability index rule in this case because the trigger value implied by the hurdle rate, V_γ, declines as the growth rate α declines. The profitability index, by contrast specifies a fixed trigger value which is too high. For example, if the hurdle rate, γ, is 20% and the cash flow growth rate $\alpha = -20\%$, then from equation (8), the trigger value for the 20% hurdle rate is $V_\gamma = (.2-(-.2))/(\rho-(-.2))$. For ρ between 8% and 16%, V_γ ranges from 1.42 (when $\rho = 8\%$) to 1.11 (when $\rho = 16\%$), while V_H ranges from 1.36 to 1.06. Although the 20% hurdle rate induces excessive delay, nonetheless, for volatilities of 30% and 40%, the loss from following the 20% hurdle rate rule with $\alpha = -20\%$ is similar to the $\alpha = -3\%$ case depicted in Figure 4. When volatility is 20%, however, the 20% hurdle rate rule performs significantly worse, particularly at low discount rates. The 1.5 profitability index rule, on the other hand, induces even greater delay, causing a greater percentage loss in the value of the investment option than with the 20% hurdle rate.

In the cases considered here, the percent loss in value due to following rules of thumb is greater as the growth rate becomes more negative. However, the timing option is worth less to begin with in these cases.[13] The net effect is that the rules of thumb we consider, especially the 20% hurdle rate, preserve much of the value of the investment timing option, even for growth rates as large in absolute value as -20%.

4.2   Probability of Project Becoming Worthless (Jumps)

In practice there may be first-mover advantages from early investing, which the standard investment-timing model ignores. One way to incorporate this effect is to model a random chance that the investment option will become worthless, for example due to preemption by a rival. Following Merton (1971), this is modeled as a Poisson process in which the option value can jump to zero with instantaneous probability λdt. Permitting the option value to jump to zero reduces the value of the option and accelerates investment. McDonald and Siegel (1986) show that the investment-timing model in this case is modified by replacing r with $r+\lambda$ and δ with $\delta+\lambda$.[14] An increase in λ lowers the optimal trigger value V_H and the option value W, but has less of an effect than an identical change in α.[15]

Introducing a moderate probability of a jump to zero, for example, 20% per year (i.e. an average of one jump every five years) improves the performance of the 1.5 profitability index rule at low discount rates relative to the accuracy shown in Figure 5. The reason is that the possibility that the project will become worthless eliminates those cases where it is optimal to wait until V_H is very high, so setting $\Pi = 1.5$ provides a reasonable approximation to the optimal trigger values. In cases where the investment

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[13]This is partly a consequence of equation (6); as the option has less value, the loss from following a suboptimal investment policy is greater.

[14]The intuition is that if there is an instantaneous probability λ that the project value can jump to zero, this increases the instantaneous discount rate by λ. Since the discount rate is increased by λ but the expected cash-flow and α are unchanged, the dividend yield, $\delta = \rho - \alpha$, is also increased by λ. This differs from the result for an option on a stock, in which case only the risk-free rate is increased by λ (see Merton (1976)).

[15]The difference in derivatives for the two parameters is proportional to $b_1 - 1$, which is positive.

option has little value, i.e., for high discount rates and low volatility, even 1.5 is too high a trigger value,
and the performance of the profitability index is poor, capturing as little as 20% of option value in the
worst case. Otherwise, the profitability index rule performs at least as well as in the no-jump case.

Regarding the hurdle rate, the main issue is how managers evaluate the hurdle rate when there is
a possibility of a jump. If managers incorporate the jump probability by increasing the discount and
hurdle rates, from equation (8) the effect on the hurdle rate is algebraically equivalent to a decrease in the
cash-flow growth rate α, the case analyzed in the previous section. If, on the other hand the jump
probability is ignored in computing the hurdle rate, then a jump to zero can lead to a substantial error
when using the hurdle rate. The reason is that in many cases the hurdle rate induces waiting when it is no
longer optimal to do so. This is particularly a problem with very low discount rates and positive growth
rates.

4.3   Impact of Scrapping (Abandonment) Option

The analysis thus far has assumed that the scrap value of a project is 0, i.e., the investment is
completely irreversible. A positive scrap value has two effects: the value of the investment timing option
increases, but the firm should invest at a lower project value. The option to scrap raises the value of the
investment option because it increases the value of insurance against a decline in the value of the asset —
investing creates both a project and a put option to scrap the project. At the same time, the existence of
the scrapping option creates partial reversibility that makes the firm less willing to lose cash flows by
deferring the project.[16]

In practice, scrapping does not significantly alter the optimal trigger value V_H as long as the scrap
value is not close to $I = 1$.[17] This can be understood by considering the solution of the investment

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[16]See Trigeorgis (1996), Chapter 7, for a discussion of option interactions along these lines.

[17]A similar finding is in Abel and Eberly (1995). They study the effects of a difference in the purchase and sale price of
capital, and show that even a small difference in the purchase and sale price of capital leads to an investment rule which is close to
that with full irreversibility.

problem with scrapping, presented in the Appendix. If V_H is large and the scrap value as a fraction of the investment cost is significantly less than 1, then the value of the option to scrap, which is acquired along with the project when $V = V_H$, will be the value of a deep out-of-the-money put option. The value of the option will be small, hence the optimal critical investment level V_H will not be affected very much.

Table 3 illustrates the effect on the profitability index and hurdle rate rules of different project scrap values. To provide a benchmark, entries in the column with scrap value equal to zero (full irreversibility) correspond to points depicted in Figures 4 and 5. Consider first the profitability index rule. When the project discount rate ρ is 8%, the optimal trigger value V_H is generally above 1.5. In that

Table 3
Fraction of investment option obtained by using 1.5 profitability index and 20% hurdle rate, under different assumptions about scrap value of project.

	Project discount rate, ρ	Cash Flow Growth Rate, α	Scrap Value as Fraction of Investment Cost			
			0	**0.25**	**0.5**	**0.75**
1.5 Profitability Index	0.08	-0.03	0.965	0.978	0.997	0.991
		0	0.868	0.883	0.919	0.966
		0.03	0.712	0.721	0.750	0.803
	0.12	-0.03	0.999	0.989	0.943	0.818
		0	0.983	0.993	1.000	0.967
		0.03	0.908	0.923	0.956	0.992
	0.16	-0.03	0.947	0.894	0.765	0.545
		0	0.992	0.974	0.907	0.752
		0.03	0.995	1.000	0.991	0.928
20% Hurdle Rate	0.08	-0.03	0.947	0.936	0.903	0.838
		0	0.975	0.972	0.960	0.937
		0.03	0.993	0.993	0.991	0.987
	0.12	-0.03	0.995	0.981	0.928	0.799
		0	0.999	0.996	0.976	0.914
		0.03	1.000	1.000	0.998	0.982
	0.16	-0.03	0.920	0.984	0.981	0.792
		0	0.908	0.962	1.000	0.918
		0.03	0.893	0.932	0.987	0.990

Note: Profitability index panel computed as $W(1, 1.5)/W(1,V_H)$. 20% hurdle rate panel computed as $W(1,V_\gamma)/W(1,V_H)$, where $V_\gamma = (\gamma-\alpha)/(\rho-\alpha)$ and $\gamma = 20\%$ is the hurdle rate. W is computed as described in the Appendix. Assumes risk-free rate, $r = 8\%$, and cash flow volatility $\sigma = 30\%$.

case raising the scrap value reduces the optimal trigger value and hence improves the performance of the profitability index. By comparison when the discount rate is 16%, V_H is generally below 1.5. The profitability index then performs more poorly with a higher scrap value.

In the case of the hurdle-rate rule, the hurdle rate declines with the project discount rate, ρ, tracking the similar decline in the optimal trigger value V_H. Except when scrap value reaches 75% of investment cost, the use of the 20% hurdle rate captures at least 90% of the value of the optimal investment timing option. In general, the introduction of scrapping does not significantly alter conclusions about the performance of either rule of thumb.

5.   Conclusion

We have suggested that seemingly arbitrary investment rules of thumb can proxy for optimal investment timing behavior   This is so because when the timing option is most valuable, it is also least sensitive to deviations from the optimal investment rule. Thus, managers may use approximately correct investment timing rules without losing much value. We do not argue that managers *should* use these rules of thumb, but rather that their use in practice might stem from the success of apparently arbitrary rules which are revealed over time to be close to optimal. Managers likely observe the capital budgeting practices in their own and other companies, and in most cases probably mimic what seems to work.

One problem with analyzing firm investment decisions is that we do not know very much about how managers actually behave. We know that hurdle-rate and payback rules are used in practice, but it must be the case that managers adjust these rules in extreme situations, for example when an investment is strategic and expiring. One might guess that managers also think differently about projects with different volatilities, even though textbook finance is of little guidance in this regard. A project with no volatility is intuitively like a bond, and standard NPV analysis would be appropriate. In fact the rules of thumb we consider here work least well for low volatility projects. A project with high volatility, on the other hand, may intuitively seem to call for a higher discount rate, which is how the use of hurdle rates

might have arisen. The interesting point is that this intuition is in fact justified, since projects with high volatility have higher optimal trigger values, justifying investment only at a higher hurdle rate.

There are certainly several caveats attached to the specific examples in the paper. For many kinds of projects, for example natural resources, it is plausible that output prices and hence NPVs might evolve as mean-reverting processes. This would lower long-run volatility and the value of the timing option (Schwartz, 1997) and, if mean reversion is ignored, can lead to excessive delay. Investment decisions may also involve strategic options which can alter these results. It is also unclear how managers evaluate "platform investments", i.e., investments which generate the possibility of profitable investments or other options in the future. Excessive waiting (induced by a hurdle-rate rule) could be detrimental for such investments. Of course, if the investment decision is based on cash-flow projections which presume the future option is profitably exercised, then the value of the future option may in practice be overstated. The critical issue is how firms actually make these decisions, which is not yet well understood.

It would be interesting to see if industry characteristics could be correlated with capital budgeting practice. For example, are industries with strong mean reversion in project values likely to display less use of rules of thumb, since there would be a smaller gain to deviating from standard NPV calculations? One might also expect to see hurdle-rate rules used for low cash flow, long-lived, non-strategic projects (for which significant delay is optimal), and profitability-index type rules used in other cases. It is a challenge to find data which could be used to test these kinds of predictions.

Appendix

The Investment-Timing Option

Let W_0 denote the value of the initial investment option, and W_1 the value of the scrapping option which may exist once the investment is undertaken. Following standard arguments (see e.g. Dixit and Pindyck, 1994), the value of these two options are described by the partial differential equations

$$\frac{1}{2}\sigma^2 V^2 W_{i,VV} + (r - \delta)V W_{i,V} = r W_i \qquad i = 0,1 \qquad \text{(A1)}$$

where r is the risk-free rate. A general solution to this equation is

$$W_i(V,\sigma,r,\delta) = A_1 V^{b_1} + A_2 V^{b_2} \qquad \text{(A2)}$$

where

$$b_1 = \left(\frac{1}{2} - \frac{r-(\rho-\alpha)}{\sigma^2} \right) + \sqrt{\left(\frac{r-(\rho-\alpha)}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}}$$

$$b_2 = \left(\frac{1}{2} - \frac{r-(\rho-\alpha)}{\sigma^2} \right) - \sqrt{\left(\frac{r-(\rho-\alpha)}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}}$$

with A_1 and A_2 determined by appropriate boundary conditions.

The investment problem is further characterized by boundary conditions. In each case we need to solve for V_L and V_H, the trigger values at which scrapping and investment are optimal. There are two boundary conditions each for the options to invest or scrap, high-contact and value-matching conditions:

Value-Matching

$$\text{(A3)}$$

$$W_0(V_H) = W_1(V_H) - I$$

$$W_1(V_L) = K - V_L$$

High contact

$$W_0{}'(V_H) = W_1{}'(V_H)$$

$$W_1{}'(V_L) = -1$$

(A4)

In order to compute the value of the option for an arbitrary investment boundary, we simply omit the first high-contact condition and set $A_1 = V_A - I$, where V_A is the arbitrary investment level.

Verification of Equation (6)

By differentiating equation (3) we obtain

$$\frac{\partial^2 W}{\partial V_A^2} = -b_1 V^{b_1} V_A^{-(b_1+2)}$$

(A5)

We want to show that the absolute value of this expression is increasing in b, i.e. that W is more concave as the option becomes less valuable. Without loss of generality set $V = 1$. Taking the log of the absolute value of the right hand side and differentiating with respect to b we get

$$\frac{1}{b_1} - \ln(\frac{b_1}{b_1-1}) - (b_1+2)\left[\frac{1}{b_1} - \frac{1}{b_1-1}\right]$$

The behavior of this expression as b goes to 1 depends on the behavior of $1/(b - 1) + \ln(b - 1)$.

$$\frac{1}{b_1-1} + \ln(b_1-1) = \ln\left(\frac{\exp^{\frac{1}{b_1-1}}}{\frac{1}{b_1-1}}\right) = \ln(\frac{e^x}{x})$$

(A6)

which has a limit of infinity as x goes to infinity. Thus the degree of concavity of W increases with b.

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