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An options-based approach to evaluating the risk of Fannie Mae and Freddie Mac ☆

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Abstract

Fannie Mae and Freddie Mac assume a significant amount of interest and prepayment risk and all of the credit risk for about half of the $8 trillion U.S. residential mortgage market. Their hybrid government–private status, and the perception that they are too big to fail, make them a potentially large, but largely unaccounted for, risk to the federal government. Measuring the size and risk of this liability is technically difficult, but important for the debate over the appropriate regulation of these institutions. Here we take an options pricing approach to evaluating these costs and risks. Under the base case assumptions, the estimated value of the guarantees is $7.9 billion over 10 years, with a combined .5 percent value at risk of $122 billion. We evaluate the sensitivity of these estimates to various modeling assumptions, and also to the regulatory regime, including forbearance policies and capital requirements. The analysis highlights the benefits, but also the challenges, of taking an options-based approach to evaluating the value of federal credit guarantees.

Keywords: Financial institutions; Options pricing; Value at risk

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1. Introduction

Fannie Mae and Freddie Mac (F&F) assume a significant amount of interest and prepayment risk, and all of the credit risk, for about half of the roughly $8 trillion U.S. residential mortgage market. Their hybrid government–private status, and the likelihood that they would not be allowed to fail because of the disruption it would cause to housing and financial markets, make them a potentially costly risk to the federal government, and ultimately to taxpayers.

How large is this risk? The answer, although much debated in the literature (e.g., CBO, 2001; Jaffee, 2003; Naranjo and Toevs, 2002; Hubbard, 2004; Passmore, 2005; Stiglitz et al., 2002),[1] remains poorly understood for a variety of reasons including limited regulatory oversight and financial disclosure; and the inherent difficulties in modeling F&F's risk exposure. Compared to commercial banks and thrifts, F&F are lightly regulated. Their regulator, the Office of Federal Housing Enterprise Oversight (OFHEO),[2] cannot force prompt closure, has limited modeling capabilities, and has very little latitude in adjusting capital requirements (Falcon, 2004). The absence of an explicit federal guarantee means that no cost for F&F is recorded in the federal budget, diluting the incentive for lawmakers to strengthen regulatory oversight or increase legislative restrictions on risk-taking. F&F's charter act exempts them from SEC disclosure requirements, although they voluntarily comply with a number of its provisions for equity. Modeling F&F's risk exposure is complicated by its sensitivity to the poorly measured tails of the distributions of interest rates and housing prices. Even if those risks could be accurately assessed, F&F's extensive and non-transparent use of derivatives and their complicated liability structures make it impractical to model their risk exposure using reported on- and off-balance-sheet quantities.

This paper takes an options-based approach to evaluating two related questions: what is the insurance value of the implicit government guarantee to F&F, and how does the distribution of possible losses change under alternative policy regimes? An advantage of this approach is that it requires making assumptions about a relatively limited set of variables—firm asset variability, debt policy, and the trigger point for bankruptcy. It avoids having to explicitly model F&F's use of derivatives or the many sources of risk affecting their solvency, since these factors are implicitly reflected in the distribution of firm asset and liability values. Further, the option approach properly incorporates the price of market risk into the estimate of the guarantee value. This is important because credit guarantees are equivalent to highly levered positions in the assets of the guaranteed firms, and hence their value reflects their considerable market risk (CBO, 2004b).

Despite the relative simplicity of an options-based approach, serious difficulties remain. These include: whether the relatively short historical time series observed provides information about the tails of the relevant statistical distributions; the extent to which F&F could and would adjust their investment policy and debt in response to a rapid

[1]Few disinterested parties participate in this debate. Many of the studies that found costs to be low were commissioned by the GSEs (e.g., Hubbard, 2004; Naranjo and Toevs, 2002, Stiglitz et al., 2002) whereas the studies that provided evidence for higher costs were under the auspices of federal government entities (e.g., CBO, 2001 and Passmore, 2005, who is at the Federal Reserve Board of Governors.) Feldman (1999) surveys the results of older studies.

[2]OFHEO is part of the Department of Housing and Urban Development (HUD), an agency whose primary mission is to promote affordable housing, not financial market stability.

change in circumstances; and how quickly F&F would be forced to cease operating in the event of serious financial distress. Because of these uncertainties, rather than emphasizing a set of fixed assumptions, we focus on the sensitivity of cost estimates and value at risk (VaR) to a variety of assumptions. The options-based estimates also are compared to earlier estimates of guarantee value based on credit spreads (Passmore, 2005, and CBO, 2001), and also other volatility-based approaches (Stiglitz et al., 2002; Hubbard, 2004).

To briefly summarize the main findings, under the base case assumptions the combined value of the implicit guarantee to F&F over a 10-year horizon is \$7.9 billion. The value at risk at the 1 percent level is \$74 billion for Fannie and \$48 for Freddie. Extending the horizon to 25 years increases the estimated guarantee value to \$28 billion, and increases the value at risk to \$89 billion and \$60 billion, respectively. A novel feature of the analysis is to increase the conditional volatility of assets in periods of distress. We show that this significantly increases guarantee value and value at risk, although it generates an undetectable increase in measured asset volatility in time series data. Different policy options for limiting risk have very different effects. Increasing capital requirements significantly lowers guarantee value, but has a relatively small effect on catastrophic outcomes, as measured by the value at risk. Conversely, a reduction in regulatory forbearance—either in the form of more frequent monitoring or a more stringent bankruptcy trigger—has a minimal effect on guarantee value, but significantly reduces the probability of a catastrophic outcome.

The remainder of the paper is organized as follows: In Section 2 we provide some background on Fannie and Freddie. In Section 3 we introduce the basic model, discuss some of the critical sensitivities, and lay out the alternative assumptions that are considered. Quantitative results are presented in Section 4. Section 5 concludes with a discussion of policy options to reduce the risk exposure from these entities.

2. Background

F&F were created by acts of Congress to provide liquidity and stability in the home mortgage market. Fannie Mae was originally created as a wholly owned government corporation in 1938, for the purpose of buying mortgages from originators and holding them in its portfolio. It was converted into a Government Sponsored Enterprise (GSE) in 1968. Freddie Mac was created in 1970 as part of the Federal Home Loan Bank System to purchase mortgages from thrifts. Rather than holding mortgages in its portfolio, Freddie Mac pooled the mortgages and attached a guarantee for credit risk. Claims to the cash flows from the insured mortgage pools were sold to investors, creating mortgage-backed securities (MBS).

GSEs are hybrids of private corporations and federal entities. Although they are required to issue their debt securities with the explicit statement that they do not bear a government guarantee, their many federal ties convince investors otherwise, as evidenced by the relatively narrow spread between GSE and Treasury debt issues compared to other financial institutions. According to the Congressional Budget Office (2001), these federal connections include that they are chartered by federal statute, exempt from state and local income taxes, exempt from the Securities and Exchange Commission's (SEC's) registration requirements and fees, and may use the Federal Reserve as their fiscal agent. The U.S. Treasury is authorized to lend \$2.25 billion to each of them. GSE debt is eligible for use as collateral for public deposits, for unlimited investment by federally chartered banks and thrifts, and for purchase by the Federal Reserve in open-market operations. GSE securities are explicitly government securities under the Securities Exchange Act of 1934 and are exempt from the provisions of many state investor protection laws.

Concern about systemic risk has escalated along with the rapid growth of these enterprises (see Frame and White, 2005, for a recent survey). Their assets on-balance sheet grew rapidly through 2003, but fell slightly in 2004. Combined liabilities were $1.677 trillion in 2004 (Table 1). The risk represented by on-balance-sheet holdings is considerably greater than for the MBS they guarantee. Their mortgage holdings are financed largely with fixed rate debt of various maturities, creating exposure to interest rate and prepayment risk that is partially hedged with derivatives. The exposure on MBS is limited to credit risk, which causes little concern because of the collateral value of houses and historically low credit losses. Nevertheless, a large negative shock to house prices could lead to significant losses on the $2.255 trillion of MBSs guaranteed by the two firms, as well as on their balance-sheet holdings. A further concern is that the implicit government subsidy accrues to F&F's shareholders rather than to the housing market. Historically these firms have been extremely profitable for their investors (see Fig. 1). The returns on

Table 1
Outstanding MBS and debt, year-end 1985–2004 (in billions of dollars)

	Fannie Mae		Freddie Mac	
	MBS[a]	Debt[b]	MBS[a]	Debt[b]
1985	55	94	100	13
1986	96	94	169	15
1987	136	97	213	20
1988	170	105	226	27
1989	217	116	273	26
1990	288	123	316	31
1991	355	134	359	30
1992	424	166	408	30
1993	471	201	439	50
1994	486	257	461	93
1995	513	299	459	120
1996	548	331	473	157
1997	579	370	476	173
1998	637	460	478	287
1999	679	548	538	361
2000	707	643	576	427
2001	859	763	653	578
2002	1029	851	749	666
2003	1300	962	773	740
2004	1403	945	852	732

Source: Congressional Budget Office based on data from the Department of Housing and Urban Development's Office of Federal Housing Enterprise Oversight, and the Federal Housing Finance Board. The 2004 numbers are based on data from Fannie Mae and Freddie Mac.
As of June 30, 2004, data from FHLB Office of Finance.

[a]MBSs = mortgage-backed securities issued by the enterprise; excludes holdings of the enterprise's own MBSs held in its portfolio. These are off-balance sheet, with exposure only to credit risk.

[b]On-balance sheet debt, financing primarily mortgages with exposure to interest rate, pre-payment, and credit risk.

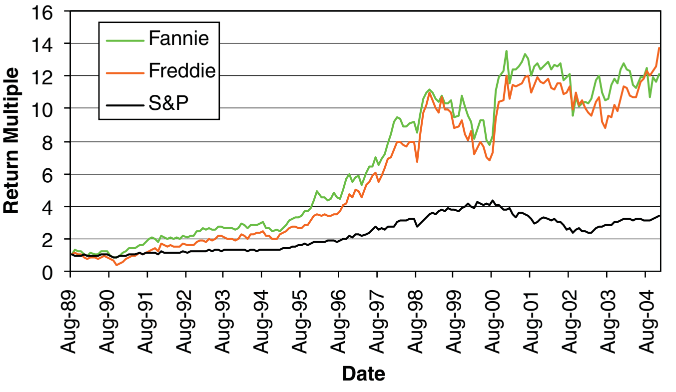

Fig. 1. Cumulative monthly returns September 1989–December 2004.

the two firms are highly correlated, confirming that they are affected by common risk factors. The correlation in monthly data from October 1989 to December 2004 is 79.5 percent.

2.1. *Risk experience and disclosure*

The potential susceptibility of the GSEs to economic shocks, particularly when mortgages are held on-balance sheet rather than securitized, was demonstrated during the late 1970s and early 1980s. As for the savings and loan industry, high and volatile interest rates weakened Fannie Mae as the value of 30-year mortgages fell and the cost of financing increased. Because interest rate and prepayment risk are transferred to the buyers of MBS, Freddie Mac was much less exposed to interest rate risk during this earlier time period, and hence was less adversely affected than was Fannie Mae.

This experience caused Fannie Mae to take measures to reduce balance-sheet risk exposure, and beginning in the early 1980s, it rapidly increased its reliance on MBS. For both companies, however, the ratio of mortgages held in portfolio to MBSs guaranteed only against credit losses grew rapidly in the 1990s, and has flattened out since 2000 (see Fig. 2, which is based on Table 1).

Although the growth in portfolio holdings appears to represent a considerable increase in risk-taking over the last decade, F&F take measures to hedge their exposure using derivatives such as swaps, swaptions, and callable debt, as well as dynamic hedging strategies (see Jaffee, 2003 for a more detailed discussion). The effectiveness of these hedges is difficult to evaluate because of the very limited risk disclosure by both firms, and because there has not been a shock large enough to test the functioning of the derivatives markets upon which the hedges rely.

One measure of risk exposure that both firms report is their duration gap, which measures the difference in the sensitivity of portfolio assets and liabilities to changes in interest rates. For instance, a 6-month duration gap means that a 1 percent increase in interest rates will cause the value of assets to fall by about $\frac{1}{2}$ percent more than the value of liabilities, thereby eroding capital by an amount equal to $\frac{1}{2}$ percent of assets. The measure proved informative in September of 2002, when Fannie reported that its duration gap had

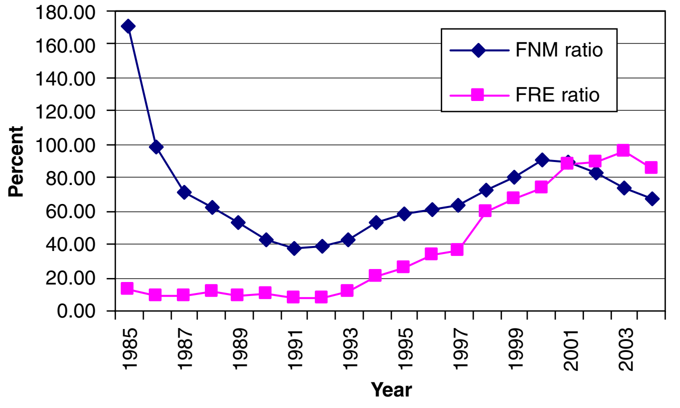

Fig. 2. Ratio of outstanding debt to MBS.

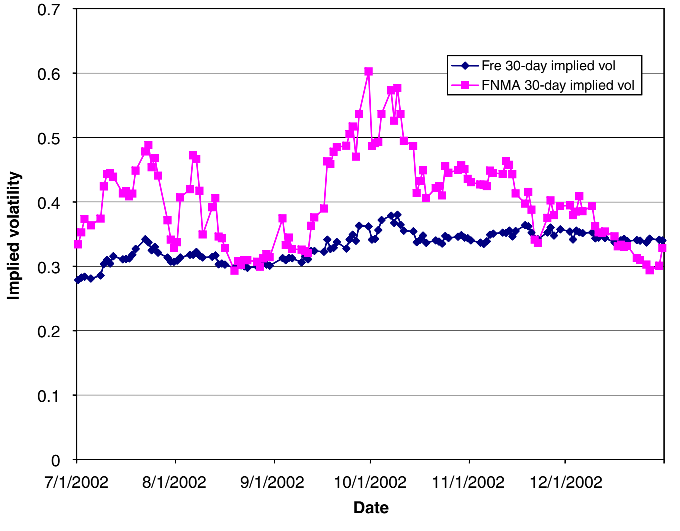

Fig. 3. The effect of the 9/02 spike in Fannie's duration gap on implied volatility differentials.

ballooned to 14 months, well outside its target range of less than 6 months. Capital markets responded to this information, as reflected in the increase in the implied volatility of Fannie's equity value at that time (see Fig. 3). Implied volatilities are derived from options price data using an options pricing model. The return of implied volatilities to normal levels suggests that the concerns raised by the event were short-lived.

F&F also present a "fair value balance sheet" designed to give a more accurate picture of their economic situation than their official balance sheet that reflects accounting conventions (such as mixing market and book values) that can be misleading. While Freddie reports its fair value quarterly, Fannie reports it only annually.

Although F&F are exempt from SEC registration requirements and fees, recently Fannie opted to register their common stock with the SEC, and Freddie has expressed the intention to follow. In doing so, information from Form 10k and other mandatory disclosures for publicly traded firms will become available, but these reports have only limited quantitative information on derivatives, and will add little to disclosures already made by the enterprises. Neither registers their debt or MBS securities with the SEC.

Credit ratings provide an additional external evaluation of default risk, and have been used in some past studies as the basis for estimating the government subsidy (CBO, 2001; Passmore, 2005). The ratings agencies have refined their analysis in recent years, and now make a clear distinction between a rating that reflects the risk to investors in agency debt securities and the companies' stand-alone risk. Moody's rating, taking into account the regulatory regime and implicit guarantee, is AAA for the unsecured debt of both firms, and P-1 for short-term issues. However the ''bank financial strength rating,'' which corresponds more closely to a measure of the risk to the government, was recently lowered to B+ from and A− for Fannie. For Freddie the bank financial strength rating is A−. The credit default swap (CDS) market offers another piece of data on how the risk of F&F compare to other financial institutions. CDSs are derivatives that make a payment to the protection buyer if there is a default event on an underlying security, in exchange for a fixed premium payment to the protection seller. Recent data from the CDS market suggest that F&F are perceived to have default risk similar to other large financial institutions.

2.2. Regulatory controls on risk

Regulation limits risk-taking through two main mechanisms: restrictions on investments and minimum capital requirements. F&F's charters restrict their investments primarily to conventional mortgages. They predominantly hold and securitize conforming mortgages. The definition of a conforming mortgage is based on loan size, the loan to value ratio, and property type. The conforming mortgage ceiling is indexed to house prices and hence has been increasing rapidly. The conforming limit rose from $252,700 in 2000 to $359,650 in 2005. That limitation excludes F&F from only about 10–20 percent of the residential mortgage market. In fact the fraction of the market they intermediate has grown more rapidly than the available conforming mortgages, as the share of non-GSE competitors for conforming mortgages has decreased over time. The loan-to-value ratio on a conforming mortgage cannot exceed 80 percent, unless supplemental insurance or some other permissible guarantee mechanism is obtained. The vast majority of conforming mortgages are for single-family homes, but dwellings with up to four units also can qualify as conforming.

F&F are required to hold a minimum amount of capital as a buffer against adverse shocks. The fixed part of the requirement stipulates capital equal to 2.5 percent of on-balance-sheet assets, and .45 percent of off-balance-sheet obligations and assets. Additional capital is required if they fail to pass a stress test that subjects them to several hypothetical large and sustained interest rate changes over ten years, and also a severe credit event. These capital requirements have recently become a binding constraint on Fannie Mae, although both firms generally have maintained slightly more than the regulatory minimum capital. As for commercial banks with deposit insurance, economic theory predicts that to maximize the value of the implicit guarantee F&F would manage liabilities so that capital remains close to the regulatory minimum.

3. The model

The options-based approach to modeling credit guarantees is based on the insights of Sharpe (1976) and Merton (1977), that such insurance can be valued as a put option on the assets of the underlying firm's assets. A put option gives the holder the right but not the obligation to sell an asset at a pre-specified strike price, and will be exercised if the market value of the underlying asset falls below the strike price. In the case of a firm with just one maturity of zero coupon-guaranteed debt outstanding, the strike price of the option is the face value of the debt, and the maturity of the option equals the maturity of the debt.

This approach, modified to take into account complications such as the term structure of debt obligations and state-contingent triggers for bankruptcy, is an alternative to traditional predictors of bankruptcy risk that rely on financial ratio analysis. KMV, a subsidiary of Moody's, has invested considerable resources in developing it as a commercial tool, and provides a clear description of the basic method (Crosbie and Bohn, 2003). Moody's now uses this information as an input in some of their credit ratings.

For pricing F&F's implicit guarantee, our analysis uses Monte Carlo simulation with risk neutral probabilities, which accommodates a variety of assumptions about debt policy, time variation in risk, and regulatory regime. Monte Carlo valuation uses the risk-neutral distribution of outcomes, but we also estimate the true distributions in order to estimate the distribution of possible observed losses.

3.1. Evolution of assets

The standard abstraction for the evolution of asset value is that the expected return on existing assets reflects the market risk of those assets (e.g., the asset beta), and that the returns are log-normally distributed.[3] In addition, firm assets may increase due to purchases financed with additional debt and equity issues, or retained earnings. In a risk-neutral representation, returns are log-normally distributed but on average equal the risk-free rate. The risk-neutral discrete time representation of the evolution of assets in the Monte Carlo can be represented as

$$A_{t+h} = A_t \,\mathrm{Exp}\left[(r_{\mathrm{f}} + g_t - \delta \frac{E_0}{A_0} - .5\sigma_A^2)h + \sigma_A \varepsilon \sqrt{h} \right], \tag{1}$$

where h is the time step, subscripts represent time, E is equity, A is assets, r_{f} is the risk-free rate, g_t is externally financed firm asset growth, δ is the dividend yield on equity (hence $\delta E_0/A_0$ is the dividend yield on assets), σ_A is the volatility of firm assets, and ε is a draw from a standard normal distribution under the risk neutral probability measure. The same evolution in terms of the true probability distribution can be written as

$$A_{t+h} = A_t \,\mathrm{Exp}\left[(r_A + g_t - \delta \frac{E_0}{A_0} - .5\sigma_A^2)h + \sigma_A \hat{\varepsilon} \sqrt{h} \right]. \tag{1a}$$

[3]The log-normal representation is standard, and allows a simple transformation between the actual and the risk-neutral measure. Some have suggested that financial institutions bear fatter-tailed risks. Although we do not consider this possibility directly, we do consider the possibility of higher asset volatility in the sensitivity analysis in part as a proxy for fatter tails.

In Eq. (1a) $\hat{\varepsilon}$ is a draw from a standard normal distribution under the true probability measure, and r_A is the risk-adjusted required return on assets. These representations for firm assets should be interpreted as the present value of all expected future cash flows, including those from on- and off-balance-sheet activities. The drift is equal to required returns (equal to the risk-free rate under the risk-neutral measure, and the risk-adjusted return under the true probability measure) net of payouts.

Time variation in the assumed volatility of assets can significantly affect cost estimates, with periods of high volatility having a large influence on guarantee value. Consideration of the possibility of time variation is important also because of the relatively short historical time series of available data, and the limited number of large shocks observed. The effect of time variation in volatility, particularly increased volatility during times of financial distress, is considered in the sensitivity analysis.

The initial market value and volatility of firm assets must be estimated since it is not directly observable. The market value of firm assets is the sum of the market value of liabilities and owners equity. Although many of the liabilities of Fannie and Freddie are liquid and publicly traded, the complexity of the liability structure makes it difficult to directly estimate their outstanding market value. Further, the traded debt prices reflect the value of the implicit guarantee, whereas we also are interested in the market value of the liabilities in the absence of insurance. Using Merton's framework, we can use options pricing theory to impute the value and volatility of firm assets from the value and volatility of firm equity. The idea is that equity can be valued as a call option on the firm's assets, with a strike price equal to the future book value of liabilities, according to

$$E = A\mathrm{e}^{-qT}N(d_1) - X\mathrm{e}^{-r_f T}N(d_2) + A(1 - \mathrm{e}^{-qT}), \tag{2}$$

$$\sigma_A = \sigma_E \frac{E}{A}[N(d_1)\mathrm{e}^{-qT} + (1 - \mathrm{e}^{-qT})]^{-1}, \tag{3}$$

$$d_1 = [\ln(A/X) + (r - q + .5\sigma_A^2)T]/(\sigma_A T^{.5}),$$
$$d_2 = d_1 - \sigma_A T^{.5}.$$

where T is the maturity of liabilities, and X is the strike price, $q = \delta E_0/A_0$ is the payout rate of assets, and all values are as of time 0. Eq. (3) comes from the fact that in the Merton model, $\sigma_E = \partial E/\partial A(A/E)\sigma_A$. Given the other parameters, Eqs. (2) and (3) are solved simultaneously for A and σ_A.

Although we model asset risk abstractly through asset volatility imputed from equity value and volatility, it is worth enumerating the types of risk F&F are exposed to. These risks can be broken into three broad categories: interest rate risk, credit risk, and other risks. An advantage of the options pricing approach is that stock price volatility is arguably the most comprehensive measure of risk, and perhaps the only available measure that includes all three sources.

Interest rate risk, and the accompanying prepayment and extension risk that arise due to the prepayment option on residential mortgages,[4] has received the most attention. As

[4]Both prepayment risk and extension risk arise from the option to prepay mortgages. Prepayment risk is due to rapid prepayments in a falling interest rate environment, whereas extension risk results from mortgages remaining outstanding longer than predicted at the time of pricing in a rising interest rate environment. The two risks together give mortgages the property of negative convexity, whereby small changes in interest rates cause a reduction in value relative to an option-free bond with comparable duration.

discussed by Jaffee (2003), interest rate related risk is absent for the MBS they guarantee, since it is borne by the holders of those securities. The mortgages held on balance sheet, to the extent they are not hedged through the debt structure and the use of derivatives, are the source of interest rate risk. Hedges protect F&F from small to moderate rate changes, but the results of stress tests suggest that they are exposed to large and rapid movements in rates.

Credit risk is present both from mortgages held on balance sheet, and from the MBS they guarantee. Credit risk on conforming mortgages is mitigated by the restriction that the loan to value ratio not exceed 80 percent. On MBS, F&F charge approximately 20 bps for this insurance. Historically credit losses have been modest, and more than covered by these fees. Nevertheless, recent concern about regional housing bubbles and the possibility that higher interest rates could trigger a crash suggest that credit risk could become more significant. It is notable that although F&F actively manage interest rate risk with derivatives, it does not appear they have tried to directly hedge this risk, for example by creating and selling derivatives on mortgage credit risk. Thus, their levered position (via the MBSs they guarantee) in credit risk appears to be entirely retained.

The "other risks" category is a catch-all for political risk, accounting risk, fraud, liquidity risk, model risk, counterparty risk in the derivatives market, etc. These other risks are potentially quite important, although they are emphasized less often in the academic literature and are more difficult to quantify. Political risks include a weakened public perception of the guarantee value, which could increase the cost of funds and lower market capital. It also includes the possibility of legislation that restricts growth or increases competition, reducing franchise value. Seiler (2003) stresses the importance of political risk, and uses an event study methodology to show its effect on stock price. Liquidity risk is related to political risk, as the market for Fannie and Freddie issues might lose liquidity in the event of an unanticipated government action. Accounting misrepresentations or fraud not only may diminish equity capital, but also can prolong the time between when a problem arises and is recognized, increasing the severity of losses. The risk associated with accounting irregularities is evidenced by the recent restatement of $9 billion of earnings by Fannie Mae, and the subsequent precipitous drop in their stock price.

3.2. Evolution of liabilities

In the simplest form of the Merton model, liabilities have a single fixed maturity and are constant over the life of the option. This assumption is unrealistic for most firms, and especially so for highly leveraged financial institutions. Theoretically, debt should be managed to maximize the value of equity, and closed form solutions for optimal or stationary debt policy have been derived for a few special cases (e.g., Leland, 1994; Collin-Dufresne and Goldstein, 2001). In the case of insured financial institutions, the standard argument is that holding equity at the regulatory minimum maximizes the value of the government guarantee, particularly as firms become distressed. Little is known, however, about how Fannie and Freddie would manage their liabilities and hedging strategies in the event of sudden financial distress, or the speed with which they could adjust their liability positions.

To examine the sensitivity of cost estimates to different dynamic liability management strategies, we assume that the book value of liabilities adjusts towards a target liability to asset ratio at several different adjustment rates. Liabilities, L, evolve according to

$$L_{t+h} = L_t e^{(r_d+\gamma g_t)h} + I_t \alpha_t h[\lambda^* - L_t e^{r_d h}/A_t]A_t, \tag{4}$$

where α_t is the annual rate of adjustment, which may be state dependent, λ^* is the target liability to asset ratio, and I_t is an indicator variable that equals one in a period where liabilities are adjusted, and 0 otherwise. Liabilities grow at a rate r_d to cover promised coupons. In addition, a fraction γ of externally financed growth is supported by debt. This representation applies to both the actual and risk-neutral measure, but the realized paths differ because the return on debt and externally financed growth take on different values in each instance.

Computationally it would be straightforward to add volatility to liabilities in this model. We maintain the assumption of non-stochastic liabilities, however, because the estimated volatility of assets implicitly captures volatility arising from all sources, including liabilities.

3.3. Modeling the trigger for insolvency

In the basic, continuous time Merton model, bankruptcy is triggered by the market value of assets falling below some trigger value. If the trigger value were equal to the book value of liabilities, the cost of a credit guarantee would always be zero, since debt holders take over the firm at the point where it equals the value of their claim. In reality managers tend to postpone bankruptcy, and creditors recover far less than the promised value of their claims. The conditions under which managers seek the protection of bankruptcy courts varies considerably, leading to wide variations in recovery rates, even within a single seniority class or industry.

Modeling the bankruptcy trigger for Fannie and Freddie is further complicated by the regulatory situation. They do not operate under the U.S. bankruptcy code. Their regulator, OFHEO, only has the powers of a conservator, not a receiver (Carnell, 2005). This means that although OFHEO has some control, it does not have the authority to close F&F down, nor to pay off creditors.

Several insolvency triggers are commonly used in models that take into account more complicated liability structures and trigger strategies (e.g., see Crosbie and Bohn, 2003). One is that the firm is liquidated when the market value of assets falls below the level of current liabilities plus half of the book value of long-term liabilities. Another is that the market value of assets falls below a fraction, sometimes taken to be 70 percent, of the total book value of liabilities. We consider a variety of trigger levels based on the value of assets relative to book liabilities. The distinction between long- and short-term liabilities is less meaningful for F&F than for many other firms because of the constant maturity conversion taking place through the derivatives market, so no distinction is made on that basis.

We assess the cost of a drawn-out reorganization or closure process—which might also be described as regulatory forbearance—in two ways. First, we use the ratio of assets to liabilities that triggers bankruptcy to represent forbearance, with a higher trigger level representing a more stringent regulatory policy. Second, following Merton (1978) on deposit insurance, we assume that the trigger for bankruptcy is only checked periodically. The longer the time between "audits," the greater the probability that the financial condition of a distressed firm will have deteriorated, and the greater the cost to the government contingent upon bankruptcy. The probability of bankruptcy, however, may be lower under the assumption of less frequent auditing when asset returns have a positive drift. This offsetting effect makes the net cost of forbearance in the form of lower frequency audits indeterminate. We consider a variety of audit frequencies in the sensitivity analysis.

An element of the cost associated with a delayed closure procedure is that asset volatility could increase significantly, for reasons such as unusually volatile markets, reduced availability of derivatives for hedging, or a purposeful increase in risk-taking to gamble to regain solvency. This increase in volatility is unlikely to be observed in historical data, both because its occurrence is a low probability event, and because it is likely to persist for relatively short periods of time when it does occur. The quantitative analysis below suggests that this may be the most important cost of forbearance.

4. Results

4.1. Initial values

Inputs into Eqs. (2) and (3) to estimate the initial value of firm assets and asset volatility are summarized in Table 2. All values are based on year-end 2004 data, except the equity volatility, which is based on its historical average. The Table also shows the resulting estimates of asset value and volatility. Since the liabilities have multiple maturity dates, the maturity of the option is based on the average maturity of reported liabilities, and the strike price is the future value of total reported book liabilities. The volatility of equity is a critical input, serving as a sufficient statistic for the net effect on risk of all on- and off-balance-sheet activities including interest rate risk, derivatives used in hedging strategies, and MBS guarantees. Estimated volatility varies considerably over time. For instance, for Fannie Mae it averages 31 percent from January 4, 1996 to December 31, 2004, but ranges from 16.7 to 60 percent (see Fig. 4).

The model generates an estimated asset volatility of only 2.06 percent for Fannie Mae, and 1.92 percent for Freddie Mac. The sensitivity analysis reveals that the cost estimates and distribution of losses are very sensitive to this parameter.

Table 2
Market value and volatility of assets—estimates and inputs (dollar values in millions)

	Fannie Mae	Freddie Mac
Market value assets	$1,026,194	$793,670
Volatility assets	0.0206	0.0192
Market value equity	$68,924	$50,740
Volatility equity	0.3	0.3
Book value liabilities	$945,000	$731,697
Wtd. maturity liabilities	2.65	3.05
Dividend yield on equity	0.0225	0.0178

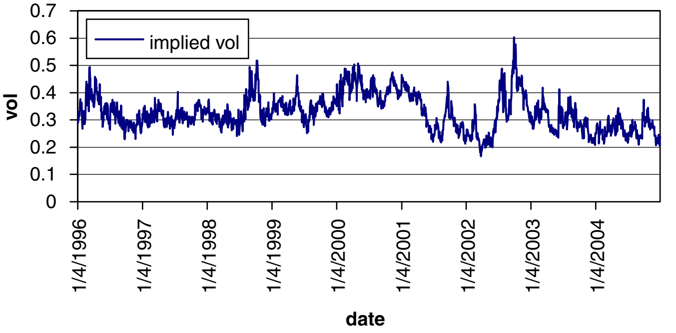

Fig. 4. Fannie Mae implied volatility. *Source*: Optionmetrics.

4.2. Base case

The parameters for the base case simulations are reported in Table 3, with a brief explanation of what they represent. This case is not necessarily the most likely, but seems a reasonable starting point. The estimate is based on a 10-year time horizon, mitigating the greater estimation error and model uncertainty in longer-term projections. Asset volatility is assumed to increase to four times its normal level when assets fall to 101 percent of liabilities, representing increased volatility in periods of financial distress. Management and regulatory decisions (debt adjustment and solvency determination) are evaluated at a quarterly frequency, while assets returns are calculated at a monthly frequency. Liabilities adjust gradually and asymmetrically towards their target level—80 percent per year adjustment up versus 40 percent per year adjustment down. The asymmetry reflects that it is more difficult to rapidly reduce liabilities when asset values are declining than it is to increase them when times are good. To reflect the historically high rate of asset growth, assets are assumed to grow at a rate equal to their realized return net of dividends, plus an additional 6 percent annually when the target capital ratio is met or exceeded. The external growth is financed entirely with an increase in debt. When capital is below the target level there is no additional debt-financed growth.

The results of the base case Monte Carlo simulations are summarized in Table 4, where we report the guarantee value, the risk neutral and actual default probabilities, and the value at risk (VaR) at the .5 percent level under the actual probability measure. The risk neutral default probability is always greater than the actual default probability due to the effect of market risk; the risk neutral result is presented to illustrate the magnitude of this difference. The true default probabilities are consistent with Moody's historical 10-year cumulative default probabilities for corporate bonds with comparable ratings—for instance bonds rated Baa2 have a cumulative default probability over 10 years of 5.5 percent. Value at risk measures the worst outcome in terms of present value of the cost of the government payoff at the one percent level over the 10 years of the simulation period. The VaR is more relevant to the question of systemic risk than the guarantee value, since it gives a sense of the size of rare but catastrophic outcomes. Figs. 5 and 6 show the distribution of the present value of costs conditional upon default for each firm in the base case.

4.3. Sensitivity and policy analyses

4.3.1. Asset volatility

The experiments reported in Table 5 show that assumed asset volatility is a critical sensitivity. The guarantee cost, the probability of default, and the VaR all increase rapidly

Table 3
Base case parameter values

Short name	Value	Description
Fannie Mae		
FAinit	$1,026,194	Initial market value of assets ($ millions)
FLinit	$ 945,000	Initial book value of liabilities ($ millions)
MVEquity	$68,924	Initial market value of equity ($ millions)
Equity dividend yield	0.0225	
Freddie Mac		
FAinit	$793,670	Initial market value of assets ($ millions)
FLinit	$731,697	Initial book value of liabilities ($ millions)
MVEquity	$50,740	Initial market value of equity ($ millions)
Equity dividend yield	0.0178	
Common values		
Rf	0.025	Risk free rate
Rd	.0275	Promised return on debt
FAer_a	0.03307	Firm asset expected return (actual)
FAer	0.025	Firm asset expected return (risk-neutral)
FAvol	0.0206	Firm asset volatility during normal conditions
FAvol_h	0.0824	firm asset volatility in high volatility state
FLrate_d	0.4/4	Quarterly adjustment of liabilities to lower target
FLrate_u	0.8/4	Quarterly adjustment of liabilities to higher target
Growth	0.06	Externally financed growth if enough capital
Growth_trig	.93	If target liability to asset ratio met, then growth
Growth_debt	1	Proportion of external financing that is debt
Trigger	0.98	Bankruptcy trigger assets/liabilities
Trig_volh	1.01	Trigger of assets/liabilities for higher volatility
Look	4	Frequency of checking bankruptcy trigger per year
Look_l	4	Frequency of updating debt
FLFAtarget	.93	Target liability to asset ratio
New FLFA	1	Proportion of debt financed exogenous asset growth
nmonte	20,000	Number of Monte Carlo simulations
nyear	10	Number of years in each simulation run
nfreq	12	Time steps per year

Table 4
Base case results

	Guarantee value ($ billions)	10-year default probability (actual)	10-year default probability (risk neutral)
Fannie Mae	5.14	.064	.152
Freddie Mac	2.78	.042	.115

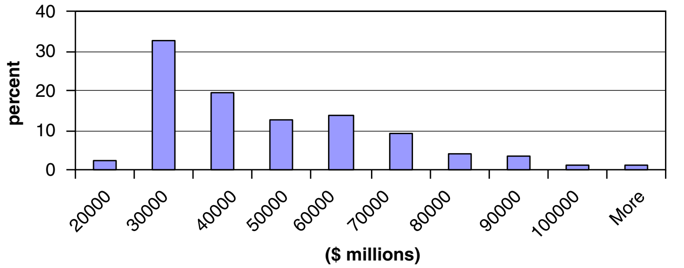

Fig. 5. Present value cost of Fannie Mae conditional on default.

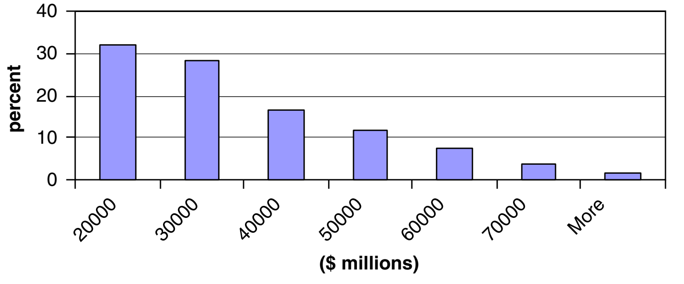

Fig. 6. Present value cost of Freddie Mac conditional on default.

Table 5
The effect of increased asset volatility

	Guarantee value ($ billions)	10-year default probability (actual)	VaR at .5 percent ($ billions)
Fannie Mae (.025 vol)	9.60	.130	100
Freddie Mac (.025 vol)	7.24	.126	80
Fannie Mae (.03 vol)	15.43	.215	126
Freddie Mac (.03 vol)	12.03	.216	97

in volatility over the range considered. Results for both firms are reported for a fixed 2.5 and 3 percent asset volatility. All other parameters are as in the base case.

An alternative to higher steady-state volatility that increases guarantee value without significantly increasing the probability of default is to increase the conditional volatility when the enterprises become distressed. In general, higher conditional volatility might be attributable to the same factors that caused financial distress, or it could result from deliberate risk-taking in an effort to avoid insolvency. To examine this effect, the non-distress volatilities and all other parameters are set to base case values. In the first variation conditional volatility is held constant at the base case non-distress level. In the second variation, for an asset to liability ratio between the default trigger of 98 and 101 percent, volatility is assumed to increase six-fold for each firm. These values bracket the base case,

Table 6
High volatility conditional on high leverage

	Guarantee value ($ billions)	10-year default probability (actual)	VaR at .5 percent ($ billions)	Observed average vol
Fannie (0 × high vol)	4.32	.020	31	.0206
Freddie (0 × high vol)	2.40	.009	19	.0192
Fannie (6 × high vol)	5.39	.065	96	.0210
Freddie (6 × high vol)	2.84	.043	63	.0195

Table 7
Alternative debt adjustment rules

	Guarantee value ($ billions)	10-year default probability (actual)	VaR at .5 percent ($ billions)
Fannie (symmetric, +.25/−.25)	7.20	.075	71
Freddie (symmetric, +.25/−.25)	4.50	.056	50
Fannie (asymmetric, +.85/−.15)	9.96	.10	79
Freddie (asymmetric, +.85/−.15)	6.26	.075	54

in which volatility increases fourfold for the same range of liability ratios. The results are reported in Table 6.

What is most striking about results in the high conditional volatility case in Table 6, as compared to the results with the smaller but constant increases in volatility considered in Table 5, are similarly high VaRs but much lower 10-year default probabilities. The difference is explained by the fact that higher conditional volatility tends to make losses more severe when bankruptcy occurs, but distress events remain rare. Also noteworthy is that the observed average volatility (calculated across simulations and time periods) is only slightly higher than the volatility during normal periods (last column in Table 6). The low observed volatility occurs because periods of heightened volatility are rare and short-lived. This suggests that time variation in volatility may be an important, but intrinsically unobservable, factor driving guarantee costs and systemic risk.

4.3.2. Liability rules

How the fixed portion of liabilities (equated initially to the book value of on-balance-sheet debt) should vary over time with firm asset value is an open but important question. Mathematically it affects the results because it is the reference point for the bankruptcy trigger. In the base case, the difference in difficulty of adjusting debt up in good times versus down in bad times is reflected in differential adjustment speeds of 80 percent per year towards the target versus 40 percent per year towards the target. The sensitivity analysis reported in Table 7 shows the effect of two alternatives: a symmetric rate of adjustment up and down of 25 percent, and a severe asymmetry of 85 percent up and 15 percent down. As expected, greater asymmetry, and reducing the rate at which liabilities can be reduced with or without asymmetry, increases the guarantee cost by as much as a factor of two relative to the base case.

4.3.3. Changing capital requirements

The effect of altering capital requirements can be represented by a change in the target ratio of liabilities to assets. In the base case the target ratio is set to .93, slightly higher than the initial estimated ratio. Actual capital requirements are more complicated than a simple ratio of the market value of equity to the market value of assets, as they depend on book values and stress test results. How tightly management can and does maintain a target ratio also is not observable. Table 8 shows the effect of increasing the assumed target ratio to .95 and decreasing it to .91, with all other parameters as in the base case. As would be expected of such highly levered institutions, relatively small changes in the target leverage ratio have an enormous impact on the guarantee value and risk exposure, suggesting that this is a powerful policy lever for controlling government risk exposure.

4.3.4. Growth and horizon

In the past decade F&F's portfolio growth rate exceeded that of the conforming mortgage market as they captured an increasing market share. Growth rates going forward likely will be lower, both because market share is bounded and because of regulatory pressures to restrain growth. In the base case we assume that both companies grow according to the rate of return on their assets net of dividends, which averages about 3 percent, and that if the capital ratio is at or above its target, assets increase an additional 6 percent, funded entirely by new debt issues. A natural question is what happens to estimated costs if debt-funded growth is higher or lower than initially assumed? Table 9 reports what happens when externally financed growth is assumed to be 4 percent or 8 percent, with all other parameters as in the base case. While guarantee value and VaR both increase with the growth rate, the increases are modest. The small effect can be attributed

Table 8
Changing capital requirements

	Guarantee value ($ billions)	10-year default probability (actual)	VaR at .5 percent ($ billions)
Fannie (target L/A = .91)	1.93	.023	54
Freddie (target L/A = .91)	1.01	.014	33
Fannie (target L/A = .95)	11.14	.149	97
Freddie (target L/A = .95)	6.70	.108	70

Table 9
Varying exogenous growth

	Guarantee value ($ billions)	10-year default probability (actual)	VaR at .5 percent ($ billions)
Fannie (4% growth)	4.58	.057	68
Freddie (4% growth)	2.56	.039	48
Fannie (8% growth)	5.38	.068	79
Freddie (8% growth)	2.88	.043	52

Table 10
Costs over 25 year horizon

	Guarantee value ($ billions)	25-year default probability (actual)	VaR at .5 percent ($ billions)
Fannie Mae	17.41	.199	89
Freddie Mac	10.67	.136	60

Table 11
Varying monitoring frequency

	25-year guarantee value ($ billions)	25-year default probability (actual)	VaR at .5 percent ($ billions)
Fannie (monthly)	16.72	.230	54
Freddie (monthly)	10.77	.160	37
Fannie (semi-annually)	17.83	.172	121
Freddie (semi-annually)	10.96	.122	80

to the assumption that externally financed growth only occurs if the target capital ratio is satisfied.

The analysis focuses on a 10-year horizon to avoid the greater approximation errors at longer horizons. For comparison to other studies, and to illustrate the effect of horizon on cost, Table 10 reports results under the base case assumptions, but with a 25-year horizon. In comparison to the 10-year horizon, the guarantee value increases by a much larger percentage than the VaR because while the number of bankruptcy events increases with horizon, the severity of the events is limited by the assumption that liabilities revert toward a target level and by the closure rule.

4.3.5. Forbearance

In the base case, regulators are assumed to monitor the enterprises quarterly, and to shut them down if the ratio of assets to liabilities falls below the bankruptcy trigger of .98. One way to model greater forbearance is to decrease the frequency at which the bankruptcy trigger is monitored. Another approach is to lower the liability to asset ratio that serves as the bankruptcy trigger.

The effects, shown in Table 11, of varying the monitoring frequency are based on a 25-year horizon. Table 10 serves as the base case for comparison, as it is also for a 25-year estimation period. The longer horizon permits a more accurate assessment of the effect of changing the monitoring frequency. The results indicate that increasing the frequency of monitoring dramatically reduces the VaR—going from semi-annual to monthly monitoring more than cuts it in half. With more frequent oversight, regulators are able to cut off rapidly mounting losses, significantly reducing the likelihood of a catastrophic event. Increasing monitoring frequency, however, has a relatively small effect on the estimated guarantee value. In fact, for some parameters, increasing monitoring frequency has the effect of slightly increasing guarantee value. The reason is that while more frequent

Table 12
Varying bankruptcy trigger

	Guarantee value ($ billions)	10-year default probability (actual)	VaR at .5 percent ($ billions)
Fannie (asset/liability .97)	4.63	.051	85
Freddie (asset/liability .97)	2.52	.035	62
Fannie (asset/liability .99)	4.43	.072	52
Freddie (asset/liability .99)	2.61	.049	33

monitoring tends to reduce costs conditional on hitting the trigger, an offsetting effect is that it increases the frequency with which the trigger is hit.

For the bankruptcy trigger, we maintain all base case assumptions (returning to a 10-year horizon and quarterly monitoring), but bracket the base case assumption that an asset to liability ratio of .98 triggers bankruptcy, setting the trigger to .97 and .99. As with the frequency of monitoring, and for the same reasons, changing the bankruptcy trigger has an insignificant effect on guarantee value, but a very large impact on the VaR and the probability of catastrophic losses (Table 12).

4.4. *Comparisons to previous studies*

Many of the results in the literature cannot be directly compared because they vary with regard to the time period covered, whether the estimate is a present value or an annual flow number, and exactly what the cost represents. Nevertheless, it is useful to make some comparisons of these estimates with those of other studies.

Several authors have estimated the value of the implicit government guarantee by comparison of yields on F&F debt with issues from similarly rated financial institutions of equal maturities.[5] The subsidy on MBSs guaranteed was similarly found by looking at comparable guarantees made by private entities. For instance, CBO (2004a) reports an estimated present value subsidy of $12.6 billion for Fannie Mae on debt and MBS, and $5.9 billion for Freddie Mac. The CBO numbers represent the incremental value of a year of new business, rather than the value of a 10-year guarantee on the enterprises. Passmore (2005) reports a present value over 25 years in the range of $122 to $182 billion as the subsidy to Fannie and Freddie. His estimates are conceptually similar to the ones reported here, but considerably larger. The closest comparison is with the results in Table 10, which shows a combined guarantee value of $28 billion over 25 years, or approximately 23 percent of F&F's combined market capitalization.

Several Fannie Mae papers have examined risk using volatility-based methods. Stiglitz et al. (2002) conclude that the cost of an implicit guarantee to the government would not exceed $200 million, which is more than an order of magnitude smaller than our estimates. One reason for the discrepancy is that they rely on the relatively benign historical time series on interest rates and credit risk of the post-WWII period, rather than using forward-looking stock market prices as the basis for calibrating volatility. They do not describe the details of their valuation model, but it appears that they do not consider the possibility of

[5] Ambrose and Warga (2002) and Nothaft et al. (2002) estimate the funding advantage of the GSEs.

time variation in volatility, nor the possibility of changing behavior in periods of distress. Hubbard's (2004) approach has similarities with the one taken here, although it relies on a different underlying model of assets and liabilities. He focuses on default probabilities and losses at a one-year horizon, so the analysis cannot be used to assess the present value of the guarantee.

A possible explanation for the higher subsidy values that emerge from spread-based analyses versus options-based methods is that the benefit of the implicit government guarantee to the GSEs is greater than the cost of expected defaults to the government. The difference arises, for instance, because the agency classification of the debt allows regulated financial institutions to hold less capital against these investments, and because liquidity, also a valuable if hard to quantify commodity, is enhanced. The fact that the market places value on other attributes that accompany the implicit government guarantee means that from an opportunity cost perspective, the cost to the government includes these other benefits created, and also is greater than the cost of expected defaults. The estimates here, then, should be considered a conservative estimate of the cost to the government of the implicit guarantee.

5. Summary and further discussion

The analysis suggests several conclusions about the cost and risk of the implicit government guarantee to F&F. As expected, the present value of the guarantees are far smaller than the potential losses from a catastrophic outcome, as represented here as by the .5 percent value at risk of the present value of 10-year costs. The base case estimates show guarantee values over 10 years that total 7.9 billion. The VaR is $74 billion for Fannie, and $48 billion for Freddie. A novel feature of this analysis is to quantify the effect of correlation between volatility and financial distress, a feature that significantly increases estimated costs and risk exposure. The analysis also demonstrates that such a correlation may be present but unidentifiable in historical data because periods of distress leading to high volatility are rare and relatively short-lived. The policy options considered—strengthening capital requirement or increasing oversight—are shown to have very different effects. Tighter capital requirements could significantly reduce guarantee values, but have a smaller effect on catastrophic outcomes. Increased oversight in the form of more frequent monitoring or a tighter shutdown trigger has a minimal effect on guarantee value, but could significantly reduce the probability of catastrophic losses.

It is important to emphasize what these estimates do, and do not, measure. The estimates represent the market value of providing credit insurance to F&F's debt-holders, but not the economic efficiency loss or benefit resulting from the subsidy. As a first approximation, the subsidy is a pure transfer to the stakeholders[6] in F&F from the government, and hence from taxpayers at large. The size of any social efficiency losses or gains, such as whether the subsidy causes overinvestment or reduces underinvestment in housing, cannot be inferred from these numbers; a fundamentally different type of analysis would be required to assess such questions. It also should be emphasized that the options-pricing approach takes a narrow view of the value of the guarantee, abstracting from

[6] Many of the spread-based analyses cited earlier attempt to tease out the incidence of the subsidy between shareholders and homeowners, usually using information on the spread between conforming and jumbo mortgages. The analysis here does not address this issue.

benefits such as added liquidity or lower associated capital requirements for the regulated institutions that invest in their securities that increase the market value of GSE securities. Hence, these estimates are lower than the cost to the government in terms of opportunity cost, which is a conceptually broader measure of the benefit conferred.

The absolute size of the reported numbers suggests that while financial distress at F&F could be costly, it is unlikely to be large enough to severely disrupt financial markets. However, poor management of an evolving crisis could lead to much higher costs and greater market disruption than is captured in this exercise. As a point of reference, the combined value at risk in most of the scenarios considered is less in real terms than the cost of the S&L crisis in the 1980s.

To construct a less stylized model, or to better calibrate and evaluate this one, would require information not currently reported by Freddie or Fannie. Providing more information about duration and convexity would significantly improve transparency, and imposes a relatively small additional reporting requirement. Currently, neither Fannie nor Freddie report the separate duration of assets and liabilities, only the gap between them. Neither reports convexity, either in public disclosures or privately to OFHEO. The negative convexity associated with mortgages implies that the duration gap is only informative about risk for very small changes in interest rate. It also implies that losses will increase at a faster than linear rate for large interest rate changes. The convexity gap is a measure of this divergence that, together with the duration gap, would provide a more reliable estimate of interest rate and prepayment risk.

Although mortgages are highly collateralized and historically credit losses have been small, F&F have a highly leveraged exposure to credit risk that is not hedged. Other financial institutions, including insurers, use reinsurance markets, or increasingly, CDSs, to better spread such risks. An option that has not received much attention is to encourage credit hedging by F&F.

In considering how the regulation of the GSEs should be changed to limit potential systemic risk, a number of fundamental considerations that are beyond the scope of this study must also be taken into account. One is whether government support for Fannie and Freddie on net stabilizes the housing market. While the increased liquidity of the mortgage market increases the supply of capital to housing in normal times, it may also encourage excessive investment in housing or inflation in housing prices that exacerbates problems during periods of heightened risk. Another open question is whether there are mechanisms that would allow the government to credibly avoid or lower the expected cost of interventions, for instance by offering a more limited but explicit guarantee financed by premiums charged to the enterprises.

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