Modeling Risky Corporate Debt:
Pricing Default Risk Variance and Liquidity

Mara Baumgarten, Jason Brady, Sheila Hanley, Mark Rosen
May 29, 2001
Kellogg Graduate School of Management
Analytical Finance Practicum
Professor McDonald

Introduction

The increasing use of derivatives linked to credit events and the increase in availability of corporate bond data (including pricing, default, recovery, and ratings history) has led to a rise in the amount of research directed toward understanding credit risk pricing.[1] This paper discusses credit risk pricing generally, differentiates structural models from reduced form models, and briefly discusses other types of approaches. We pay special attention to comparing recent work by Elton et al. (2001) and Credit Suisse First Boston's CUSP Model (2001) with the earlier form of contingent claims models such as that set forth by Merton (1974). We conclude our discussion of credit risk pricing with a demonstration of our own model. Using a Value at Risk (VaR) methodology, we use credits that significantly underperformed in the latter half of 2000 to demonstrate both the usefulness and the limitations of our structural approach. Finally, we conclude our discussion of risky corporate debt pricing with an examination of liquidity concerns.

Issues in Credit Risk Pricing

Credit risk pricing seeks to define the likelihood of financial distress or default, identify the factors most likely to change the risk of those events, and model how participants in that market are likely to price that risk. Tony Kao (1999) identified four main "building blocks" for pricing credit risk: (1) a model of interest rates; (2) a determination of firm value; (3) a definition of default; and (4) an estimation of recovery value.

Each piece of this puzzle has spawned its own set of literature while at the same time relying on inputs from each other that highlight the difficulty in resolving the dynamic interplay of factors involved in credit risk. Interest rate modeling alone involves integrating a large and complex set of economic data and their interrelationships into a cohesive model of the term structure of interest rates. The health of the economy in turn incorporates some assessment of the state of affairs in the corporations that comprise it. Hence, both macroeconomic and microeconomic factors come into play.

Determining firm value is no less daunting. Each investor must have some model for ascertaining the value based on estimates of cash flows and growth as well as interest rates and firm-specific risk premia. Efficient market theory states that the market value of a firm's debt combined with the market value of its equity is the best indicator of this value. Nonetheless, research demonstrating the prevalence in various periods of momentum investing, January effects, and the pattern of "manics and panics"[2] give caution to the concept of complete faith in the market price as the true value in the short run.

Kao (1999) and Nandi (1998) discuss the difficulty in the definitions of default and estimates of recovery, each of which have several moving pieces. Credit risk is often defined in corporate spread models as the likelihood of default. However, financial distress short of default should also be considered. What is the likelihood that company management will need to engage creditors in a voluntary restructuring of debt or a series of covenant waivers that change the nature of their risk? What is the distribution of this risk over time? How does a particular model set a "trigger" for defining default or distress? Once investors consider the broader issue of default, varied estimates of recovery rates can vastly change one's valuation of the firm. Market participants need to consider lost coupons and principal, which are subject to the vagaries of bankruptcy negotiations that vary the time, amount, and adherence to "absolute priority." Finally, finding information and model recovery ratios for firms that undergo a financial restructuring without bankruptcy is difficult.

Forms of Credit Risk Pricing Models

The two dominant approaches to modeling credit risk pricing are referred to as "structural" and "reduced form." Structural models are centered on assessing the evolution of firm value. They take a contingent claims approach to valuing the debt and equity of the firm. Simply stated, contingent claims models see shareholders as owning a call option on the firm, with a strike price equal to the face value of the firm's debt. In other words, the shareholders can choose to exercise their option (fully pay off the debt) or not (default), in the latter case ceding ownership to the bondholders. (Thus the bondholders are effectively left the net position of being short a put on the assets of the firm.) Such models price the debt and equity of the firm using Black-Scholes option pricing theory (and are discussed in more detail in the "Archetypal Models" section).

Various structural approaches attempt to correlate the aforementioned value with an interest rate model. Others use statistical methods to incorporate jumps[3] in the price due to event risk. Some may or may not have separate models for the probability of default and the estimate of recovery value.

There are several practical challenges to the widespread use of structural models. As noted above, assessing firm value is a complex process and must account for the dynamic interplay with other models of interest rate and default. Valuing intangibles is notoriously difficult, as has been pointed out in the recent debate over accounting standards in this area. Structural models require extensive data and constantly revised computations to be timely and useful. Moreover, not all forms of debt are easily adaptable to contingent claims modeling. For example, sovereign debt issuers do not have an explicit equity component to their capital structures. Proxies for this equity component are likely to be highly imperfect, especially for state and municipal issuers. The strategic response of third-party actors is also difficult to capture in structural models. Examples might be the likelihood of an equity infusion or other indirect form of support from the government into strategically important firms, particularly if the authority holds a partial interest.

Reduced form models look at market prices and their implied credit spreads directly. Spreads or prices can be inferred from their relationship to risk-free rates by defining default and recovery parameters. Those parameters are externally defined, involving neither firm values nor credit fundamentals aside from the credit rating of the firm. Any rating changes are treated as random events. Some variations of this model incorporate overall default rates statically while others may attempt to incorporate the probabilities and implications of ratings transitions as well. J.P. Morgan's "CreditMetrics" is an example of such a model.[4]

One of the major practical challenges to reduced form models is that credit spreads and default probabilities can vary enormously within the same rating category. KMV, a private sector firm that advocates a structural model, points out the pitfalls of using bond ratings exclusively to ascertain credit risk.[5] They note that the wide dispersion of defaults can make the use of default rate averages for pricing guidelines highly misleading and may result in adverse selection problems.[6] Moreover, default rates and recovery estimates are based on stale historical data that may be irrelevant to current economic conditions. This issue is highlighted by the uncertainty over current recovery estimates in the defaulting high-yield bond sector today, where the nature of telecom and internet firms' assets may make for lower recovery rates than is the case with largely industrialized firms with readily saleable assets. Serial correlation of pricing changes may also complicate the use of static transition matrices.[7] Further, if ratings changes lag the market in pricing risk, the use of ratings-based methodologies will bias results. There is also some limitation on the use of a reduced form models for certain asset classes such as private debt or commercial and industrial loans.

Other credit approaches cited by Kao in his survey include risk-factor premium models, direct forward spread pricing, macroeconomic approaches, and credit fundamental approaches. Risk factor premium models are similar to macroeconomic approaches in that they seek to define a common set of factors against which to regress yield spreads. Risk factor premium models use credit-specific factors such as earnings and leverage. Macroeconomic models focus on general economic conditions with factors such as GDP and industrial production; they seek to isolate systematic risk factors from security-specific factors. Systematic risk factors can be either macroeconomic (as noted above) or based on equity market indicators such as those developed by Fama and French. Direct forward spread pricing uses two models, one for interest rates and another for credit spreads, then "stitches" them together, accounting for any correlation between the two. Credit fundamental approaches use a largely credit-specific model for estimated default risk and then price the bond based on a risk-neutral model.

Archetypal Models: Merton, Elton, and CUSP

Merton's 1974 papers on valuing corporate debt were the first to apply option pricing to default risky debt valuation. This is the model that we have worked to modify and empirically evaluate.

Merton's concept of firm value is a structural model that adapts the then year-old Black-Scholes model for valuing European options. Merton values debt exposed to default risk by assuming a capital structure consisting only of equity and one zero-coupon bond. This capital structure implies that default can only occur at maturity; if the firm's market value is greater than the face value of its debt, the bondholder gets the face value. If, however, the market value of the firm is less than the face value of the debt, the firm defaults and the bondholder gets only the market value of the firm. The debt holders' payoff resembles the payoff an investor receives for having written a put option, and the equity holders' payoff resembles that of a purchased call option. The inputs into the model are the value and volatility of the firm rather than the value and volatility of equity used by Black and Scholes to value equity options. These inputs are the core of the model that we are seeking to create, with some additional refinements to compensate for Merton's simplifying assumptions.

Merton's model makes several important assumptions that he outlines in his seminal article in the *Journal of Finance*. Many are market-based assumptions — for example no transaction costs, the ability to sell short securities, and trading taking place in continuous time — that are the familiar assumptions of most financial models. Later authors have subsequently shown that some of Merton's assumptions are problematic, including the assumption that an investor can buy or sell as much of an asset as he or she wants at the market price and that the only opportunity for firm default is at maturity.

Still, what Merton creates is a term structure of credit. "For a given maturity, the risk premium is a function of only two variables: (1) the variance or volatility of the firm's operations, σ^2, and (2) the ratio of the present value (at the riskless rate) of the promised payment to the current value of the firm, d. Because d is the debt-to-firm value ratio where debt is valued at the riskless rate, it is a biased upward estimate of the actual (market-value) debt-to-firm value ratio."[8] In our model, we too build upon the assumption that increased leverage causes increased risk of default. Merton's model also demonstrates differing shapes of credit curves for different rating classes of issuers. Nandi summarizes this quality of the credit structure nicely:

> The term structure of credit spreads is upward sloping for high-credit-quality issuers, downward-sloping for low-credit-quality issuers, and humped-shaped for intermediate-quality issuers. Why do these papers hold true? The value of default risky bonds depends upon their risk of default, which in turn depends upon the value of the firm. If a firm is currently enjoying high credit quality, the impact on the bond value of further improvement in the credit quality through further increases in the value of the firm is limited because the payoff from the bond at maturity is capped at its face value. On the other hand, the firm's credit may deteriorate with the passage of time, thus increasing the risk of default…. On the other hand, for a firm that currently has low credit quality, the downside risk is limited and the upside potential is substantial as time elapses. Therefore, one would expect credit spreads to decrease with maturity, yielding a downward-sloping structure of credit spreads.[9]

The recently published piece by Elton, Gruber, Agrawal, and Mann (2001) set out to show that corporate bonds carry a risk premium independent of default risk and similar to systematic risk premia on common stocks. Their study combines elements of the reduced form approach with a macroeconomic approach. First, they determine the portion of credit spread that can be explained by default; to accomplish this, they develop "corporate spot rates" for various rating classes of corporate bonds. Several bonds in the data are excluded from the study as outliers, including bonds they assume to be mispriced, those that may be undergoing restructuring, or those that may be in the midst of a ratings transition.

Elton et al. then use a ratings-based default probability matrix to estimate the portion of the spread attributable to default rates under a risk-neutral assumption by applying the marginal probability of default estimated by the matrix to the cash flows in the numerator. For example, the probability of default applied to year-one cash flows of a given bond is simply the probability of default for a bond in that rating class. The default probability for year two first estimates the probability of transitioning to a new rating class and then takes a weighted average of the default probabilities of each rating class. This process is continued over a 10-year period. Recovery upon default is estimated from historical data relating recovery rates to original ratings on the debt.[10] The assumed coupon used for each bond that is "stripped" to create the spot rate is one that would exist such that a 10-year bond trading with that coupon would be selling close to par in all periods. The cash flows are then discounted by the risk-free Treasury rate. The authors apply this methodology to all the bonds in that universe to determine what the spread should be in a risk-neutral world, assuming the only factor differentiating the bond's return from government bonds is this adjustment for default and recovery.

Elton et al. then address the fact that corporate bonds are subject to state taxes while government bonds are not. The effect of taxes is incorporated into the analysis both through its impact on the coupon as well as on the recovery value insofar as there will be a tax-deductible capital loss in the event of default. The tax rate used was one that resulted in the smallest mean-squared error between actual versus calculated prices.

After accounting for both state taxes and default rates, the Elton study concludes that a surprisingly small portion of the premium on corporate bonds can be explained by default probabilities. In fact, state taxes can account for a larger portion of the spread premium than that accounted for by default.[11] The study then takes the remaining "unexplained" spread and regresses this against systematic factors — specifically, the Fama-French factors frequently used to assess systematic risk for common stocks: excess returns on the market, differentials in returns of high versus low book-to-market stocks, and differentials in small stock versus big stock returns. The authors find that up to 85% of the unexplained spread on industrial bonds can be explained by sensitivity to these three risk factors.[12]

In terms of financial economics, the Elton study offers support for the argument that investors will demand a premium for bearing marked-to-market volatility in their bond returns and that these returns will be related to the same risk factors that affect common stocks. The methodology used in the study also takes into account two oft-ignored problems in comparability: taxes and duration/convexity differentials (eliminated via the corporate "spot" rates). This methodology can provide insights to be incorporated into future studies. The Elton model can also be viewed in the context of evaluating an asset-allocation model that is based on an investor's view of systematic factors.

That said, the model has several drawbacks. First, while it provides valuable insight into the issue of systematic risk, the study is not easily translated into a portfolio strategy that can be used by practitioners. A corporate bond manager cannot buy and sell the Fama-French risk factors themselves. At the aggregate level, he/she may choose to weight various rating classes of bonds according to their expected behavior given the portfolio manager's view of the underlying factors. In practice, however, instruments that allow one to buy the "A-index" or the "BBB-index" are not highly developed in terms of size and have various contractual limitations and transactions costs.[13] Liquidity and pricing data can be poor, and borrowing corporate bonds to short is difficult.

Perhaps more importantly, the Elton study seems to have given short attention to the notion that the systematic factors may be dynamically affecting default probabilities. Their adjustments for potential default changes used perfect forecasting and lagged/lead models. The exact format for these tests is not cited specifically in the study and appears to be based on realized historical default rates versus more dynamic measures to incorporate a form of "distance to default" changes for various credit classes.

For active managers, out-performance from asset-allocation decisions may be quickly wiped out by outliers. This is particularly true for those who cannot "buy the index" for reasons noted above. For these managers, they must attempt to replicate the index, necessarily resulting in "overweights" and "underweights" resulting from asset selection. Elton's study systematically eliminated most of the "outliers" (credits likely to be in the midst of restructuring or rating change), yet it can be argued that these are the most important credits to identify in terms of asset selection. As noted earlier, the wide level of dispersion of default risk within rating classes may promote adverse selection if pricing guidelines are based on aggregate, ratings-based risk categories. Finally, the Elton paper did not incorporate any discussion of liquidity issues, a bond market reality that in our opinion must be addressed.

Backward-looking default prediction models are increasingly seen as obsolete — even rating agencies are struggling to produce more accurate forecasting. New models, such as KMV, Moody's Public Firm Risk model, and CSFB's CUSP model are incorporating equity market indicators to capture the dynamic dimension of default risk[14]. This evolving breed of contingent claims models attempt to account for leverage and equity price volatility in assessing default risk, thus making an explicit attempt to ascertain the interplay between systematic risk and default. KMV, a proprietary contingent claims model, claims to produce an "estimated default frequency" based on this notion by incorporating leading signals from the stock markets. This appears to provide better default forecasting than ratings — which is most useful for high-yield and distressed credit investors. This argument also implies that equity are more efficient and have information which can be exploited by debt investors. The KMV contingent claims model does not, however, address the issue of risk aversion and market spread changes for high-grade, marked-to-market fund managers.

CSFB's recently introduced CUSP model, which we discuss in greater quantitative detail below, seeks to correct this shortcoming. Based on Merton's scholarship, CUSP's contingent claims model attempts to estimate changes in bond spread by evaluating the heightened default risk due to changes in a firm's capital structure. The model incorporates equity volatility and leverage indicators to estimate the change in the value of the equity default option. Thus, CUSP incorporates both issues of fluctuating default risk and risk aversion. Spreads are measured relative to LIBOR, attempting to isolate relative credit changes from more systematic factors that may affect credit spreads overall. This approach also seeks to eliminate the issue of taxes and alleged "technical noise" from the Treasury curve.

The CUSP model provides visibility into how systematic risk affects default probability implied by the debt percentage of the firm's capital structure. It can be used as a filter for identifying particular bonds whose default risk may be rising. If the model were used on a widespread basis, there would be no market advantage to be gained from it in terms of trading rules. However, there are eventual applications for risk management of bond portfolios based on mean-variance analysis derived both from equity relationships and observed responses in the mean-variance equation on the debt side.

A major drawback of the CUSP model is that is does not provide valuable feedback for most higher-rated credits (e.g. AA). Also, in its current form it applies best to industrial credits; highly levered industries such as banking and finance may require a separate iteration of the model. While attempting to isolate relative risk of spread widening, CUSP requires a separate swaps curve model for the "term structure of credit." This is controversial and also inconsistent with the approach to asset allocation for investors with a Treasury-based benchmark.

The following is a discussion of a structural model based loosely on Merton's theories applied by the CUSP model to value risky debt from the perspective of portfolio managers with frequent marking-to-market. We gathered daily bond data, swap data, and historical implied call volatility from Bloomberg, total debt information from Compustat, stock data from CRSP, and yield-adjusted spread information from the Salomon Smith Barney Yield Book.

Basic Objective

We have attempted to forecast the market price for corporate debt under certain Value at Risk scenarios. It is our belief that, in accordance with much of the academic work discussed above, bond investors require a return above that which represents mere expected loss according to historical default and recovery rates. It is a commonly held belief that there exists a market risk premium for equities to compensate investors for their risk aversion; because equity returns can be below the risk-free rate in any particular year, on average the market requires that the equity returns be significantly larger than the risk-free rate. In other words, there is a risk associated with mean return as well as a risk associated with variance to those returns. The market requires compensation for holding both of these risks. Similarly, historical default rates represent the average outcome for a given class of bonds. However, there exists significant variance from these rates. We believe that there is a compensation for the risk that in any particular year the probability of default may be higher than average. Because the price of investment-grade bonds is closely tied to the outlook the market has for their potential for default (hence the high reliance on ratings), this might be called “price risk.” In practice it is clear that bond managers do not necessarily hold their purchases to maturity. Furthermore, even if these managers bought a bond planning to hold it for five or ten years, their compensation (or opportunity to continue to be employed) might be directly dependent not on the performance of that bond over the entire holding period, but rather on its performance over one or two years.

Because of our belief that investors’ compensation must be described in both mean and variance terms, we sought to build a model that takes mainly market-based inputs. These inputs would necessarily incorporate the risk aversion of the market.

Theoretical Underpinnings

Using the Black-Scholes formulation for options pricing, a delta-hedging market maker will require a certain premium for a written call. Similarly, a debt holder of a firm (with a superior claim on that firm’s assets) will require a premium, or payment, for the call bought by the equity investors. This premium is not fixed at the IPO price, but rather varies with the value of the underlying assets. Put simply, the equity’s value depends on the total value of the firm and varies with that value.

Black and Scholes showed that the value of a European call can be found with the following inputs: underlying asset price, strike price of the call, volatility of the underlying asset, risk-free rate, time to expiration of the call, and dividend yield of the underlying asset. Merton's formulation of a firm's capital structure requires a different understanding of the roles of these variables in options pricing. Whereas typically investors think of equity being the underlying asset, in this case, equity represents the option premium. The table below summarizes the role of each variable in both typical equity derivative pricing and in a Merton-based model.

Option Pricing Variable	**Equity Derivative Pricing**	**Merton Firm Value**
Underlying Asset Value (S)	Stock Price	Total Enterprise Value of Firm
Strike Price (K)	Strike Price of Option Contract	Firm Default Point
Asset Volatility (v)	Stock Volatility (forward-looking over life of contract)	Volatility of Firm's total assets
Risk-Free Rate (r)	Risk-Free Rate	Risk-Free Rate
Time to Expiration (t)	Time to Expiration of Option	Total Time relevant to equity holders
Dividend Yield (d)	Dividend Yield on Underlying Stock	Dividend Yield on total assets of firm
Call Premium (C)	Call Premium	Equity Price

As the table above shows, the variables, while having a similar relationship to one another, are less clearly defined in a Merton model, because the contract between the equity holders and the debt holders is less defined than a typical equity derivatives option contract.

Model Methodology

One important upshot of a Merton-based approach to looking at the relationship between corporate debt and equity is that changes in capital structure must be the main determinant in changes in corporate bond yield spreads to a default-risk-free instrument. In order to determine the validity of the relationship between capital structure changes and spread changes on corporate debt issues, we ran multiple regressions on individual firms and particular bond issues. Some of the more illustrative results are found in the "Results" section below.

After determining the empirical validity of the relationship between changes in capital structure and corporate bond yield spread changes in individual debt issues, we set out to calculate the relationships between the Black-Scholes variables above. We actually run these variables through the Black-Scholes formula two separate times. The first time we attempt to calculate the firm's asset volatility. Our inputs include the following: the total market value of the firm (the firm's market value of debt plus the market value of equity), a default point for the firm related to its face value of the debt, a risk-free rate based on swap rates, a holding period defined by the time to maturity of the bond issue, a dividend yield on the assets (related to the effective interest rate on the firm's debt and the dividend yield on the equity), and the current equity price. The output is the volatility on the assets implied by the above variables.

In our model the second iteration of the Black-Scholes formula takes a look at the capital structure implied by possible movements of the stock. The implied volatility of the stock (calculated by the market value of traded equity derivatives) gives a look at the market's expectations of possible forward prices. Using Value at Risk methodology, we calculate the 5% downside equity level. By putting this value into our Merton-based model, we can solve for the firm's total enterprise value rather than the asset volatility. By subtracting the 5% equity downside level from the total enterprise value implied by this equity level, we calculate the market value of debt implied by the 5% downside equity level.

The final step in our analysis involved finding a hypothetical debt value given equally weighted probability buckets. We calculated a hypothetical debt price for each percentile (1%, 2%, 3%,

…, 98%, 99%) for the entire lognormal stock price distribution. We then averaged these debt prices and found what price we would expect to see for the debt given the relationship between the market-based inputs described by the Black-Scholes equation. As a final note, the price that results from this process does not equal the 50% output for the lognormal stock price distribution. This is due to two factors. First, Jensen's inequality makes the mean and the median of the lognormal distribution unequal. Second, the Black-Scholes-based function that uses the predicted possible equity values is not linear.

Some of the more illustrative results are found in the "Results" section below.

Discussion of Input Variables

Total Enterprise Value of Firm

This variable is defined in our model as the sum of the firm's market value of debt and the firm's market value of equity. In the first iteration of the Black-Scholes formulation, we assume that the bond being examined has a price which is consistent across the stated book value of the firm's debt. For example, if a particular bond is priced at 90, we assume that the total market value of the firm's debt is 90% of the face value of the firm's debt. Equity market value is more transparent: we simply multiply current price by shares outstanding. In the second iteration of the Black-Scholes formulation, we solve for the Total Enterprise Value of the Firm and subtract the hypothetical equity value.

Default Point

This variable assumes that the firm will default at a point related to the face value of the firm's debt. Equity holders will not be subject to this downside risk and the option will expire. As a result, the use of a down-and- out barrier option is appropriate. This type of option expires when a certain barrier is reached. In this case, the barrier value and the strike price are identical.

One difficulty that arose upon implementation was a violation of put-call parity if the strike price used was simply the face value of the firm's debt. In this case, the total equity value was less than the difference between the total enterprise value of the firm and the present value of the face value of the firm's debt. Put in simple option pricing terms, $S - Ke^{rt} > C$. The default point for most firms comes when they violate one of their debt covenants. These covenants require not only debt service in the form of interest payments, but also typically require a certain amount of principal repayment and minimum financial ratios. As a result, the effective default point would be higher than the face value of the firm's long-term debt. This point would differ from firm to firm given different covenants and from industry to industry given different characteristics of underlying assets. Furthermore, there is a significant portion of a firm's liabilities that are not subject to coupon payments (e.g. trade payables). As a result, we have built a multiplier into our model that adjusts the market value of debt upwards to compensate for these factors.

Asset Volatility

This variable is the output of our first iteration of the Black-Scholes formula. It describes the volatility of the value of the assets of the firm as a whole. Because we obtain this value based on equity prices, we are forced to assume that the risk aversion of equity investors is similar to that of bond holders. Because their risk aversions are analogous, the corollary to this assumption is that their requirement of compensation for variance will be similar. Thus we further have assumed that the asset volatility, though based on equity price, can be used to calculate the inherent value of the firm's assets.

Risk-Free Rate

Though many traditional models use a U.S. T-Bill rate, we believe that this would introduce significant idiosyncrasies to the model. The U.S. Treasury market, especially in the current era of diminishing supply, reflects not only the return investors require for default-risk-free bonds, but also constraints certain classes of investors have regarding in which types of securities they are allowed to invest. (For example, pension funds often have written into their charters that they must hold a certain percentage of their assets in U.S. Treasuries.) Because of the idiosyncratic pricing pressures inherent in Treasuries, we decided to use quoted swap rates of different maturities based on LIBOR. While these rates contain some default risk (in the form of counterparty risk), they are nevertheless a good proxy for extremely high-grade bond yields. Calculating spreads to these rates would therefore circumvent any problems that might occur when using Treasuries as a default-free proxy.

Time to Expiration

Estimating this variable caused significant difficulty as an input for our model. In the case of run-of-the-mill equity derivatives contracts, the time to expiration is merely the time remaining until the option contract expires. With a Merton-based approach to firm value and capital structure, however, there is no clear definition of "expiration." While the debt holders have a certain period of time before the firm's obligation to them expires, the equity is, in theory, perpetual. The contract should then be priced as a perpetual option. On the other hand, if we were privy to the investment thesis of every investor, we could calculate their particular holding period. This would be when they would exercise their option, in the event that the firm does not default first, and would then be the price they would be willing to pay for an option on the firm's assets as a whole for that particular period of time. Again, as a practical matter, we chose the maturity on the debt to be a reasonable time for the time to expiration. Other models, including the CUSP model, use an arbitrary fixed period of time.

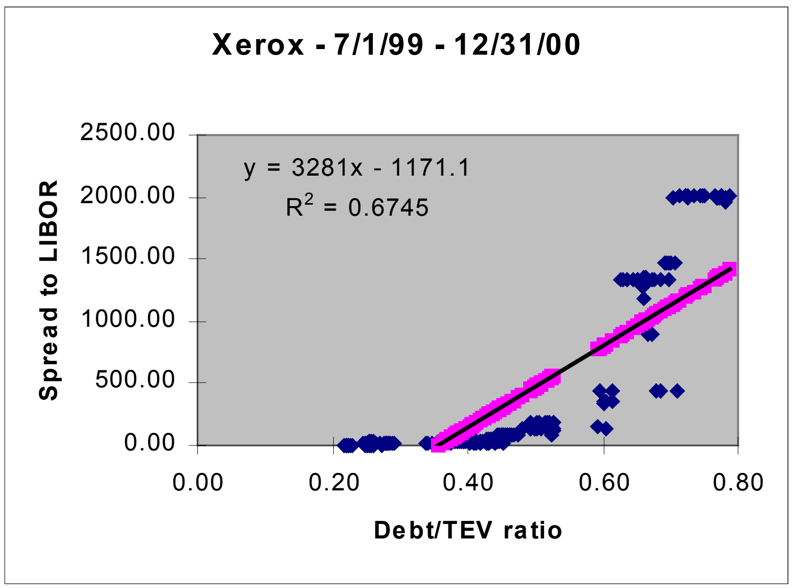
Dividend Yield

The dividend yield caused some difficulty in that different estimates of dividend yield often led to dramatically different outputs in the model. On a theoretical level, the dividend yield represents a payment to the asset holders that the call holders do not receive. In typical equity derivatives, this is generally a dividend paid by the firm to its equity holders to which option holders are not entitled but which affects the price of the underlying stock. In real options, however, the dividend yield is often negative, representing a value to being able to put off a project until a later date. In this case, the actual dividend that the firm pays to its stockholders would certainly represent a negative dividend yield in the Merton conception of capital structure; the dividend is received by the option holders rather than the holders of the assets. When the option holders sell their claim (i.e. the stock) they no longer are entitled to this dividend. On the other hand, the coupon payments received by the debt holders would represent a positive dividend yield; the coupon payments affect the value of the underlying assets (cash is paid out which would otherwise go to the equity holders). As a result, the correct dividend yield assumption for many of these firms would be their effective interest rate minus their actual dividend yield. As a practical matter, for our calculations we used one percent. This is consistent with an effective interest rate of approximately half of a firm's par coupon yield (given a percentage of the firm's liabilities are non-coupon debt such as trade payables) and a dividend yield between one and three percent.

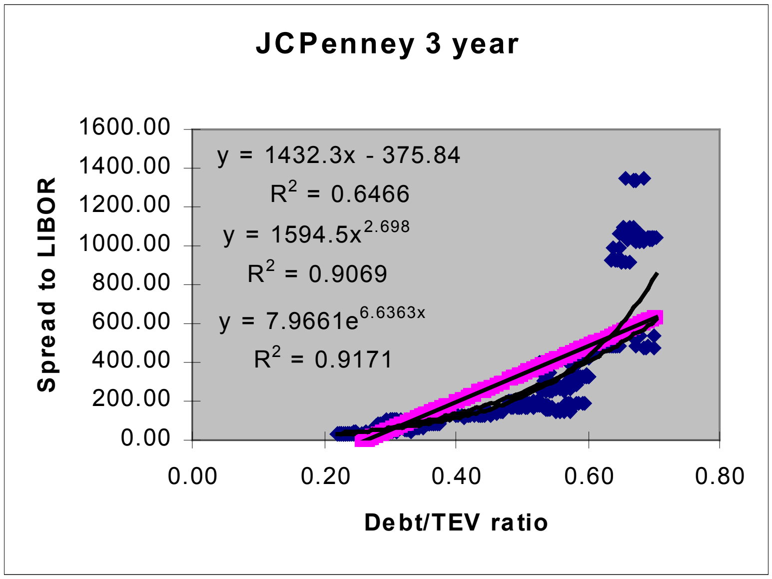
Results

We regressed a debt/total enterprise value ratio to bond yield spreads to comparable swap rates for multiple issues. The results are summarized in several figures below.

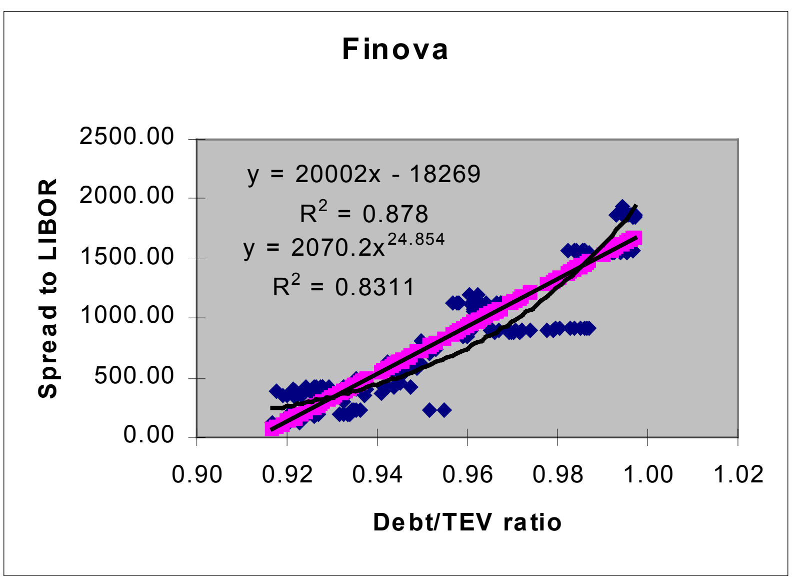
Xerox – Though the R-squared value of a linear regression between the two axis variables shown below is relatively high, the graphical representation clearly shows that the relationship is non-linear. As the debt/TEV ratio becomes larger, the spread increases at a rate at first lower, and then much greater, than the slope of a linear regression would indicate.



J.C. Penney – Further illustrating the non-linearity of this relationship, data for J.C. Penney over three years can be seen below. Regression equations are shown for linear, exponential, and power relationships. R-squared values for non-linear equations are noticeably higher.

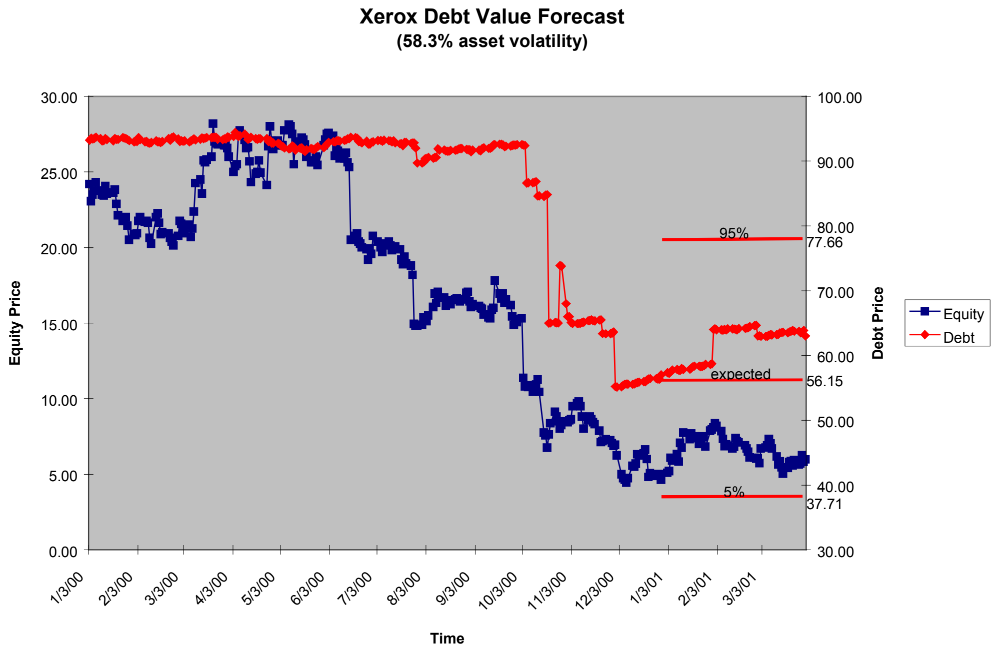


Finova – Though the same relationship exists between capital structure and required spread for financial services firms, their requirements and use of leverage are very different from a typical industrial firm. While the regression still has good explanatory power given its high R-squared value, the debt/TEV ratios are far higher for a given spread. As a result of such high leverages, we have excluded pure financial services firms from our model. We believe, that merely modifying assumptions, rather than creating an entirely different model, could yield good results.

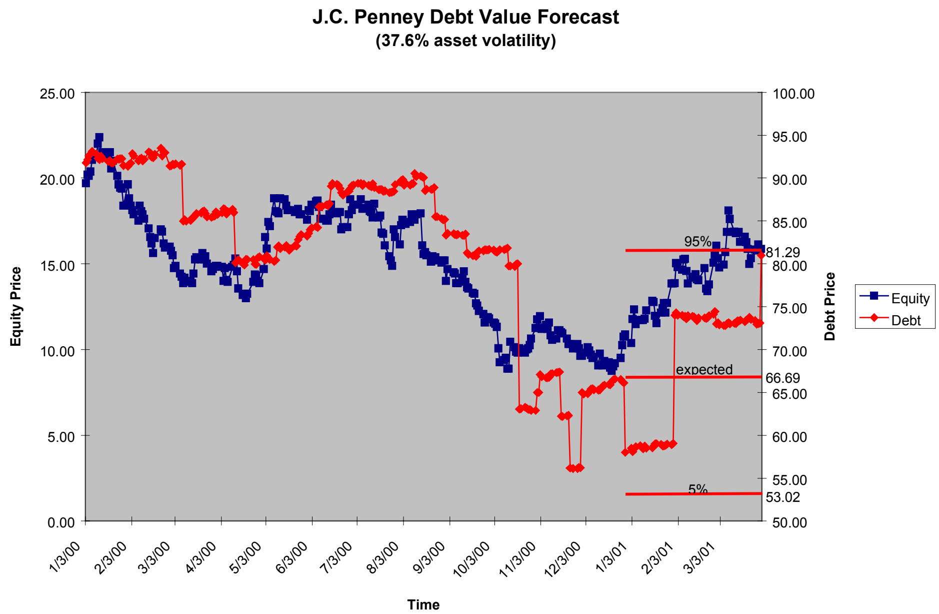


After empirically investigating the relationship between debt/TEV ratios and corporate bond spreads to LIBOR, we set out to run the iterations of the adapted Black-Scholes formula (methodology described above).

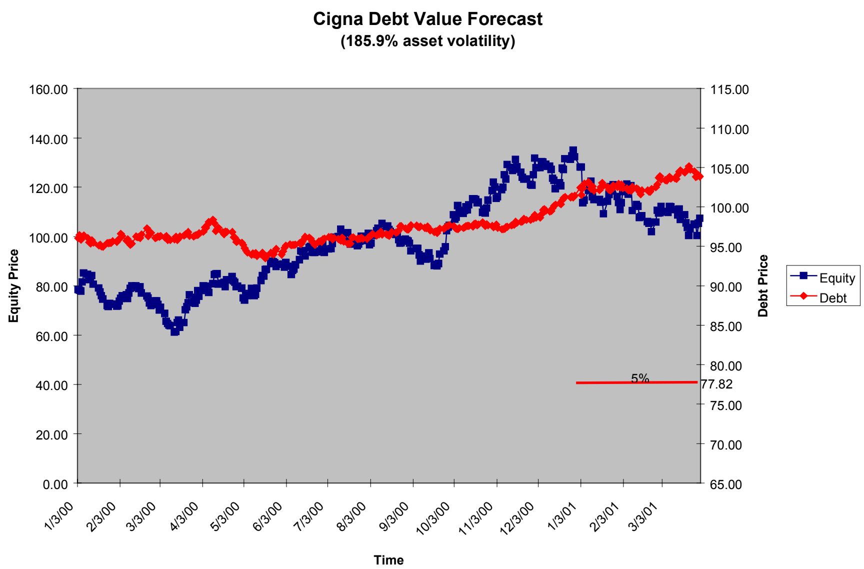
Xerox – For December 28, 2000, we calculated (through our first iteration) that Xerox had a 58.3% asset volatility. At the same time, its equity volatility was quoted at 140.12%. These high implied volatilities described a firm with a very uncertain future. Equity prices had fallen 90% in the past year and a half and from the high $20s earlier that year. As a result, our second iteration calculations produced a wide possible spread and a high VaR. In the worst 5% of cases, we calculated Xerox bonds to reach or fall below $37.71 in the next three months. At the same time, our equal probability bucket calculations showed that the bond was fairly priced. (Our calculation implied a current market price of $56.15 with the actual market price at $56.34.) This means that the corresponding upside potential was relatively large. In the best 5% of cases, we calculated that the Xerox bonds would reach or exceed $77.66 before the end of March, 2001. Though many high-grade bond managers would not want to hold this bond, if it were in their portfolio, this model shows several scenarios which would help these managers appreciate the risks inherent in these bonds. The chart below show that the debt price over the next three months recovered slightly but remained well within the predicted range.



J. C. Penney – On the same date, December 28, 2000, J. C. Penney was also going through uncertain times. We calculated its asset volatility to be 37.6%. At the same time the quoted three-month equity volatility implied by the firm's call options was 95.98%. The current market price of the bonds was $66.12. Because of these high volatilities, we again calculated a relatively large VaR spread. With 5% probability, we calculated that these bonds would fall to or below $53.02. Similarly, 5% of the scenarios showed the debt would rise to or above $81.29. Our equal-weighted probability bucket average predicted a market price of $66.69, again relatively close to the actual market price.



Cigna – Our final illustrative example shows the need for any model to incorporate boundary conditions. Cigna, on December 28, 2000, was a strong firm with relatively a relatively low debt/TEV ratio. We calculated an asset volatility of 185.93% and an equity volatility of 41.87%. The then-current market price of Cigna debt was $101.415. Though Cigna still had a significant equity cushion, our calculations show that these volatilities imply a movement of the debt price to $77.82 or below with 5% probability. While this calculation is useful from a Value at Risk perspective, our model needs to further incorporate the boundary conditions associated with Cigna's debt to accurately gauge a predicted market price. Because bonds are often callable at high values, and because the maximum value of a particular bond is the coupon and principal payments discounted at a predicted risk-free rate, the value of these bonds cannot grow, equity-like, much above a quoted price of $105. In addition, this example shows the importance of placing a term structure of interest rates within any debt value forecast. Cigna debt value is being partially driven by the increasing volatility of its stock and the underlying assets. However, the drift upwards in debt value is directly attributable to a sinking interest rate environment. Because our model uses current swap rate quotes, it does not perfectly capture the effect of interest rate changes on the firm's debt. By adding these boundary conditions and additional dimensions into our model, we would be able to more usefully predict both the price reached on the upside in 5% of scenarios, but could also model a fair market price based on the Merton contingent claims ideology.



The bonds and firms presented above illustrate the potential of a structural approach to Value at Risk in corporate bond valuation. Bond managers, like their equity counterparts, are concerned with the potential for underperforming firms to ruin their overall results, but because of the lack of volatility information inherent in less-transparent bond markets were until now unable to quantify possible loss. The model we have presented above should help to ameliorate this difficulty from a credit perspective. Another concern that bond managers must deal with, however, is liquidity risk. This additional variable crucial to understanding corporate bond spreads is discussed below.

Liquidity Risk

After quantifying the risk associated with the value of the security, there is the risk that the holder of the security will not be able to sell at the mid-price. Corporate bonds have large and volatile bid/ask spreads sensitive to macroeconomic and issuer-specific factors. Prices are not as transparent as equity markets as all trading is done "over-the-counter" without a central quote or clearing system. Analysis of liquidity risk can be exceedingly difficult for exactly the same reasons that it is so important to quantify — there is so little visibility, volume, and liquidity. And yet with the constraints placed on managers regarding the types and ratings of securities they may hold in their portfolios, it may be necessary to liquidate positions at the most inopportune times. Because the information is so difficult to attain, we have attempted two types of liquidity analyses. The first analysis (using Dodge and Cox practitioner Jim Dignan's research) values the approximate return demanded to compensate the holder of the bond for the liquidity risk they bear relative to a benchmark. The second approach, adapted from Anil Bangia's liquidity risk model for currencies, attempts more on a Value at Risk basis, so that the holder of the security can predict his/her worst possible loss due to lack of liquidity.

Types of Liquidity Risk

We focus, as did Bangia et al. (1998), on the exogenous liquidity risk as opposed to the endogenous liquidity risk that comes from the risk that a specific trade might be large enough to move the market. Still, we would like to acknowledge that in the corporate debt market, with low average daily volumes and heavily concentrated ownership, endogenous risk poses large problems and presents areas for further study once more complete volume data is available.

Initial Analysis

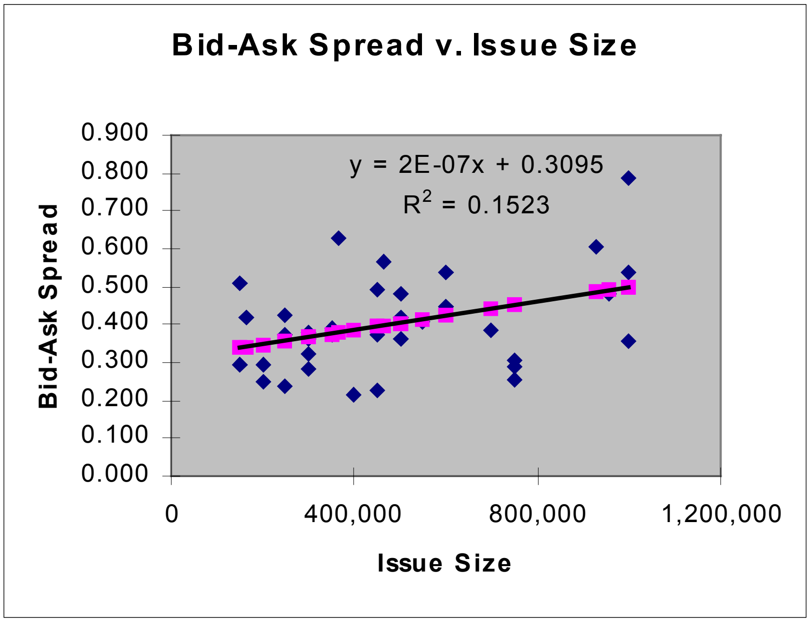
Seeking to understand the drivers of liquidity risk in the bond market, we performed multiple regressions on the data we were able to gather in an attempt to uncover relationships. We found few significant relationships, leading us to believe that each bond is different in terms of liquidity risk. This is clearly in part due to the fact that our sample size is limited (though this in turn is related to the scarcity of bid/ask data and the thinly-traded nature of corporate debt). As you can see from this table, our data set was limited both in size and in issuer diversity.

CUSIP	Amount Outstanding	Equity Ticker	Issue Date	Maturity Date	Bid/Ask Spread	
					Average	Standard Deviation
549463ae75	$ 1,360,000,000.00	LU	3/15/99	3/15/29	44.63%	20.40%
549463ad9	$ 50,000,000.00	LU	11/24/98	11/15/08	29.10%	18.36%
984121aw	$ 600,000,000.00	XRX	11/16/98	11/15/03	31.08%	19.41%
694308EW31	$ 300,000,000.00	PCG	9/27/93	10/1/05	35.64%	13.60%
694308EU74	$ 400,000,000.00	PCG	8/2/93	8/1/03	18.99%	8.23%
694308fa02	$ 350,000,000.00	PCG	11/18/93	3/1/04	20.29%	12.47%
694308fb84	$ 350,000,000.00	PCG	11/18/93	3/1/24	53.60%	4.39%
694308EA11	$ 400,000,000.00	PCG	3/19/92	3/1/02	22.40%	11.98%
694308dv88	$ 200,000,000.00	PCG	4/25/91	5/1/24	118.77%	0.42%
694308fu6	$ 680,000,000.00	PCG	11/1/00	11/1/05	22.13%	14.25%
620076an9	$ 325,000,000.00	MOT	10/20/98	10/15/08	30.62%	19.36%
620076aro	$ 1,200,000,000.00	MOT	11/13/00	11/15/10	41.77%	22.72%
620076ah2	$ 400,000,000.00	MOT	5/15/95	5/15/25	49.51%	15.94%
620076aG4	$ 200,000,000.00	MOT	3/8/93	3/1/08	35.77%	10.94%

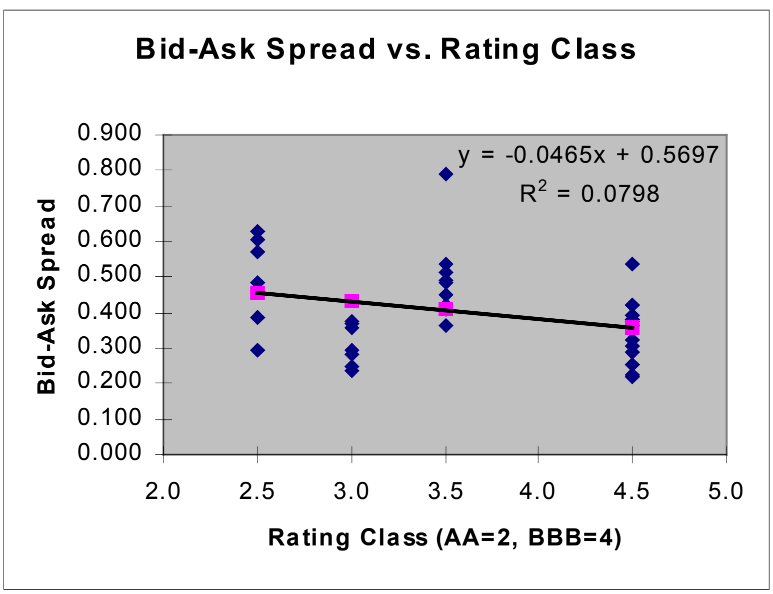
Bond sample obtained from James Dignan, 2001.

Despite these limitations, we regressed three variables versus bid/ask spreads: issue size, rating class, and issue age.

Issue Size – While seemingly the most fruitful variable (the regression was significant with an R-squared value of 0.1523), we expected the slope of the regression equation to be negative. In other words, it is a widely held belief among practitioners that larger issue sizes lead to lower bid/ask spreads and less liquidity risk. Our sample size is such that we are in no position to dispute this.



Rating Class – Because more highly rated debt has a wider investor base (due to restrictive charters which constrain many mangers), and because the most highly rated debt (U.S. Treasuries) has the greatest liquidity, we expected that more highly rated debt would have better liquidity and a lower bid/ask spread. Though this is, in fact, what we discovered in our sample, the R-squared value shows the difficulty of ascribing liquidity concerns to one factor.



Liquidity Risk Measure

The next step was finding and incorporating a dynamic liquidity risk measurement tool since our spread valuation model has produced is a dynamic range of spread values. Our model's output is a mid-price; what is the risk that the bid/ask around that midpoint will widen? To this end, we have adapted a method proposed by Bangia for currencies to bonds. This method seems to work even better for corporate bond spreads than it did for currencies, because our data on bid/ask spreads appears more normally distributed than the emerging market currencies Bangia studied.

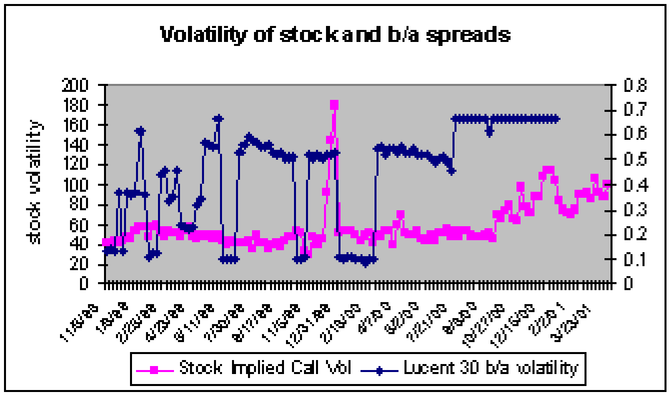
This allows us to layer on a Value at Risk analysis of the bid/ask spread if we make the same intellectual leap that Bangia makes — the "simplifying, but reasonable assumption that in adverse market environments extreme events in returns and extreme events in spreads happen concurrently. Loosening this assumption merely complicates the algebra without bringing anything conceptually new. The correlation between mid-price movements and spread is not perfect, but it is nevertheless strong enough during extreme market conditions." To illustrate this, see the below graph of the equity volatility of Lucent and the bid/ask spread of its 30-year bond.

We simply apply the 99% VaR from the spread to the VaR of our credit risk analysis.

Example Using Lucent Bonds:

Using Bangia's VaR formula $\frac{1}{2}[P_t(s+\alpha\sigma)]$ for 99% confidence (as our distribution is more normal we are not adjusting our α and we are not adjusting for kurtosis because the kurtosis of our distributions were relatively low 2 to 3, whereas the kurtosis for the currencies used by

Bangia et al. ranged from 4.9 to 18.1) where P_t is the bond price at time t, s is the relative spread (ask-bid)/mid, and σ is the standard deviation of the relative spread:



For the Lucent bonds our value at risk was calculated as follows:

	Mid-Price (P)	Relative Spread	Standard deviation of relative spread	VaR for 99% confidence
Lucent 10 Year	100.17	0.0037936	0.002067	0.4311856
Lucent 30 Year	99.775	0.001317	0.002740553	0.3801398

These numbers are calculated based on price and would be incorporated into our spread calculations.

Conclusion

A bewildering array of factors must be considered when pricing risky corporate debt. Past studies have examined historical default probabilities as a major factor (outside of interest rate risk). However, Elton et al. (2001) clearly demonstrate that this sole variable is woefully insufficient. Though that study leans toward a reduced form solution to explaining corporate bond spreads, we believe that structural models hold greater explanatory power and are more useful for bond managers in practice.

Following in the steps of Merton (1974) and using some of the methodologies laid out by CSFB's CUSP model (2001), we created a model to apply the Black-Scholes equation to capital structure and debt and equity values. We found excellent relationships for individual issues between capital structure and the spreads over swaps that investors required. Furthermore, by using equity prices and volatilities, our model yield results for declining credits that show that this model is ripe for use in a Value at Risk setting.

In a separate analysis, we examined the effect of liquidity in corporate bond markets and found that investors must adjust their expectations of returns significantly to account for exogenous liquidity concerns.

Modeling debt exposed to default and liquidity risk is problematic, but though a structural approach to modeling credit risk has some difficulties, we believe the measures presented here are a useful toolkit for quantitative credit risk evaluation.

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