VOLUME 18 | NUMBER 2 | WINTER 2006

Journal of APPLIED CORPORATE FINANCE

A MORGAN STANLEY PUBLICATION

In This Issue: Valuation, Capital Budgeting, and Value-Based Management

Morgan Stanley

The Role of Real Options in Capital Budgeting: Theory and Practice[1]

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The Black-Scholes-Merton approach to derivatives pricing has been an enormous intellectual and practical success, having transformed financial markets over the last 30 years. But the application of derivatives pricing technology to the valuation of capital projects—commonly referred to as "real options"—does not appear to have had the same effect on corporate capital budgeting practice. Surveys of corporate decision makers report that fewer than a quarter of companies say they use real options methods when making capital budgeting decisions. And the conventional wisdom is that most companies (with the possible exception of those in energy and pharmaceuticals) continue to rely mainly on standard discounted cash flow (DCF) calculations to make investment decisions. In this paper, I will argue that this perception is exaggerated in important ways, and that the differences between the DCF and real options valuation approaches are not as great as practitioners seem to believe.

In the first section I compare DCF and real options valuations and, in so doing, show that both methods provide the same answers when used correctly. With a series of examples, I show that it is helpful for managers to understand both techniques. In the second section, I address the complaint that the assumptions required to perform real options valuations are unrealistic by pointing out that the assumptions typically used to support DCF are at least as restrictive. In the third section, I argue that survey evidence that seems to show that corporate managers do not use real options techniques in capital budgeting almost certainly underestimates the extent to which managers take real options into account when making decisions, regardless of whether they use the formal apparatus of option pricing theory. Finally, I discuss empirical research on both corporate valuation practices and stock prices that is consistent with both managers and markets incorporating real options considerations into their decision making. The message here is that, regardless of what managers say in surveys, the outcomes we can observe appear consistent with the predictions of real options models.

Before we proceed, I will explain what I mean by "DCF" and "real options valuation." The standard DCF methodology calls for computing an expected future cash flow and then discounting its value at a discount rate that is equal to the expected return on an asset of comparable risk. At this level of generality, DCF is tautologically correct. Assumptions are required in order to implement any valuation model, and common practice when using DCF is to assume that the period-to-period discount rate is constant.[2] As I will show below, this assumption can lead to errors and is not required for real options models.

The real options methodology is a little harder to summarize briefly, but it typically begins by describing the future cash flows from a project as a function of some "state variable" (such as sales or revenues) that is assumed to evolve randomly over time. The next steps are to compute expected cash flows using a so-called "risk-neutral probability" and then discount those cash flows at the risk-free rate. This procedure is often called "risk-neutral valuation." In practice, it is common to simplify a real options valuation by assuming that characteristics governing the behavior of the state variable are constant over time. For example, we might assume that the degree of uncertainty about future sales remains constant from one period to the next. This is generally less restrictive than the constant discount rate assumption in a DCF valuation.

In comparing traditional DCF and real options valuations it helps to recognize three conceptually distinct issues that arise when valuing a project:

1. The mechanics of computing present value. Given assumptions about risk and future cash flows, the specific steps in computing a traditional discounted cash flow appear to be different when using DCF and when using real options methods. The expected cash flow calculation is different and different discount rates are used. (As noted before, however, the two methods yield the same answer when correctly applied.)

2. Constancy of inputs over time. Is it acceptable to use a constant discount rate in a multi-period DCF valuation? Is the risk-neutral probability constant over time in a real options valuation?

3. The underlying business model of the firm. Any valuation presumes a model of operating decisions: Can the

1. I thank Don Chew, Jan Eberly, Caroline Sasseville, Alex Triantis, and Lenos Trigeorgis for comments. Parts of Section 2 were inspired by an unpublished and stimulating talk given by Stewart Myers at the University of Maryland in 2003. Full citations of all articles referred to in the text or footnotes appear in the references at the end.

2. It is also common to assume that expected cash flows grow at a constant rate, but this assumption can be easily changed.

Table 1 **Key Characteristics of Projects 1, 2, and 3**

Characteristic	Variable	Project 1	Project 2	Project 3
Cash flow in up state	X_u	\$51.361	\$16.361	\$16.361
Cash flow in down state	X_d	\$31.152	-\$3.848	\$0
Probability of up state	p	0.571		
Beta for discounting cash flows	β	1.25		
Risk-free rate	r	6%		
Expected return on the market	r_M	10%		

Table 1. Assumptions used in valuation example. Project 2 cash flows are those of Project 1, less \$35. Project 3 cash flows are the greater of the Project 2 cash flows and zero.

company expand if profitable, contract if unprofitable, and so forth? Such assumptions are of first-order importance because they reflect the business model that implicitly underlies the valuation. The use of formal real option techniques may have the effect of encouraging managers to think more broadly about the dynamic nature of future business decisions, and thus choose from a different set of possible actions and investments. To the extent managers who use real options implicitly adopt a different *business model*, there is a real and important difference between the two valuation techniques.

The failure to separate these three issues undoubtedly contributes to the popular misconception that real options is fundamentally different from traditional DCF.

Since traditional DCF valuations often assume constant discount rates over time, it might be helpful to use the term "static DCF" to describe such valuations. I suggest the term "dynamic DCF" to describe valuations that take into account the possibility that risk changes with outcomes and over time.[3] Making this distinction is important if only because one major reason why the risks of projects change is that future operating decisions depend in large part on future outcomes—for example, if sales turn out higher than expected, then we invest more heavily in future growth. As I will argue below, much of the apparent difference between DCF and real options disappear once we admit the possibility of dynamic DCF valuations.

Finally, it is worth noting that the basic theory underlying this paper has appeared many times in textbooks and in the academic literature going back at least 30 years. My intent here is to present these ideas using simple examples that I hope will clarify the discussion about real options valuation *in practice*.

DCF and Real Options Valuation: a One-Period Discounting Problem

In this section I illustrate how DCF and real options pricing methods can be used to perform valuation in three simple cases. The goal is to understand how the two valuation methods can provide the same answer, and to show that traditional DCF is an essential component of risk-neutral valuations. I also show that, as projects become more complex, risk-neutral valuations can be much simpler than traditional DCF.

Project 1

Suppose an analyst is evaluating a project that will generate a single cash flow, X, at time T. As is common with investment projects, we will assume that there is no traded asset that is directly comparable to the project. Thus, there is no direct way to use stock returns to estimate the project's expected returns, the volatility of those returns, or the covariances of those returns with the stock market. Instead, the analyst must consider the economic fundamentals of the project and make educated inferences about them. The analyst might also look for public companies with a business that resembles the project, and then use information about such firms to infer the beta or other characteristics of the project.

Assume that, after examining all available data, the analyst estimates that the cash flow will be X_u if the economy is doing well—an event assumed to have probability p—and X_d if the economy is doing poorly. The analyst determines that projects with comparable risk have an effective annual expected rate of return of α.

1. DCF Valuation

Given these assumptions, the standard DCF method for finding the value of the project, V, is to compute the expected cash flow and discount it at a rate equal to the expected return on a project of comparable risk:[4]

$$V = \frac{pX_u + (1-p)X_d}{(1+\alpha)^T} \tag{1}$$

For example, suppose that the risk-free rate is r = 6%, the expected return on the market is r_M = 10%, the project

3. Here I define the term "dynamic DCF" differently than did Moel and Tufano (2002). They use the term to describe a firm repeating a static DCF calculation every period. I use the term to describe a firm that correctly takes into account changes in Beta over time.

4. By discounting at the expected return on a project of comparable risk, we are assuming that the returns of the project are spanned by existing traded assets; in other words, the addition of the project to the universe of assets does not materially change the opportunities available to investors. If this were not true, we would have to know more about the preferences of investors in order to evaluate the project.

beta is β = 1.25, p = 0.571, T = 1, X_U = \$51.361, and X_d = \$31.152. (For a summary of these assumptions, which will be used in a series of examples, see Table 1 under the heading Project 1. Projects 2 and 3 are discussed later.) With these assumptions, the expected return on an asset with the same risk as Project 1 is 11%, which is calculated as follows:

$$\alpha = r + \beta(r_M - r) = 0.06 + 1.25 \times (0.10 - 0.06) = 0.11$$

The expected cash flow is:

$$E(X) = 0.571 \times \$51.361 + 0.429 \times \$31.152 = \$42.692$$

Plugging this expected cash flow and an 11% discount rate into equation (1), we obtain the present value of the project cash flow:

$$V = \frac{\$42.692}{1+0.11} = \$38.461$$

The present value, V, is the time 0 market price for the Project 1 cash flow occurring in one year. There are two senses in which it is a time 0 price: First, it is quoted based on what is known at time 0; and second, *that market price is paid—and the investment is made*—at time 0. There is another common way to quote a market price for risky cash flows. The forward price for these cash flows, F_1, is the market price quoted at time 0 but paid at time 1. To understand the relationship between V and F_1, suppose that at time 0 you agree to buy the cash flows, but instead of paying for them at time 0 you agree to pay for them at time 1. You acquire the same risky cash flows in either case, but you have altered the timing of your payment. If V is the time 0 price, then the time 1 price will be the future value of V, or:

$$F_1 = (1 + r)V$$

The payment is risk-free (because you are *obligated* to pay V(1+r)), so we compute the future value at the risk-free rate. You might think that quoting a time 1 price at time 0 is odd, but deferring a payment in this fashion is so common that the price V(1+r) has a familiar name: the forward price.

In this example, we assumed that the analyst estimated future cash flows. For many commodities, however, there are forward markets (for example, for agricultural commodities such as corn and wheat, metals such as gold, aluminum, and copper, and energy products such as oil and natural gas) in which the forward price is a market-determined price for the future risky cash flow. In such cases, we can obtain the present value by discounting the forward price at the risk-free rate:

$$\frac{F_1}{1+r} = V$$

If there had been a forward market for Project 1 cash flows, the forward price would have been:

$$\text{Forward price} = F_1 = (1.06) \times \$38.461 = \$40.769 \qquad (2)$$

The forward price is the future value of the present value. *Thus, when you can observe a forward price, this means that the market has already performed the equivalent of a DCF valuation.*

Note that the forward price is lower than the expected cash flow of \$42.692. This difference between the forward price and the expected cash flow is a risk premium—a reward for bearing the risk of the cash flows. The dollar risk premium for the cash flows is computed as follows:

$$V\beta(r_M - r) = 38.461 \times 1.25 \times (0.04) = 1.923 \qquad (3)$$

We can express the forward price as the expected cash flow less the dollar risk premium, or:

$$42.692 - 1.923 = \$40.769$$

Another term for the forward price is the "certainty-equivalent cash flow." An investor is indifferent between accepting the risky cash flow or the certainty equivalent cash flow without risk.

2. Real Options Valuation

The present value calculation in the preceding section, like most present value calculations, is based on numerous assumptions. We specified the cash flows in two different states, the probabilities associated with those states, and the comparability of the project to a traded asset. Interestingly, it turns out that, in valuing the project, we have already made all the assumptions we need to make in order to value options or other derivatives related to the project. To see this, we will repeat the valuation exercise using the language and apparatus of option pricing.

The structure of the cash flows, with high and low outcomes for the project, may have reminded you of the binomial tree used in standard binomial option pricing.[5] In constructing such a binomial tree, we first measure the stock price volatility and then use this volatility to construct the up and down moves of the stock. Once we have the stock price tree, we have no further need to explicitly consider volatility, but it nevertheless affects valuation implicitly via the characteristics of the tree. In the case of Project 1, the analyst derived the up and down cash flows by considering the economics of the investment. The analyst *implicitly*

5. For an introduction to binomial option pricing, see McDonald (2006, Chapter 10).

specified the volatility of these cash flows, *but was never required to come up with an explicit specification of volatility*.

We now see how to use an option pricing approach to value the same cash flows. Option pricing appears to be quite different than traditional DCF. With option pricing, we compute expected cash flows using a probability—commonly called the *risk-neutral* probability, which we denote p^* —that is different from the true probability. We then discount these new expected cash flows at the risk-free rate:

$$V = \frac{p^* X_u + (1 - p^*) X_d}{(1+r)^T} \quad (4)$$

If we compare this to equation (1), this procedure appears odd, and perhaps incorrect, since there seems to be no consideration of the systematic risk of the cash flows. However, when you compare equations (1) and (4), it's apparent that there has to be *some* p^* such that equation (4) gives the same answer as (1).

To better understand the economic meaning of the risk-neutral probability, we can link the risk-neutral probability to the forward price: *The expected cash flow computed using the risk-neutral probability is the forward price*. That is,

$$p^* X_u + (1 - p^*) X_d = F_1 \quad (5)$$

Thus, if we observe a forward price and the possible cash flow realizations, we can compute the risk-neutral probability from equation (5) as follows:

$$p^* = \frac{F_1 - X_d}{X_u - X_d} \quad (6)$$

In our example, the risk-neutral probability is:

$$p^* = \frac{40.769 - 31.152}{51.361 - 31.152} = 0.476 \quad (7)$$

Given this p^*, we can then compute the value of the project using equation (4):

$$V = \frac{0.476 \times 51.361 + (1 - 0.476) \times 31.152}{1.06} = 38.461$$

Since the expected cash flow is the same as the forward price, this valuation method is the same as discounting the certainty equivalent at the risk-free rate.

Because we used the DCF value of \$38.461 to obtain the risk-neutral probability, this calculation seems circular—and it is. This implies that the DCF valuation is *a necessary input* into the risk-neutral valuation.[6] These examples illustrate the important point that, in order to do a real options valuation, *we need to know the forward price for the cash flows*. Sometimes the market will supply the DCF valuation, as when we observe a forward price or some kind of market comparable. In other cases, however, the analyst must perform the DCF valuation to infer the forward price. Thus, the mechanics of DCF—in particular, the estimation of risk premia—are in most cases an inevitable part of capital budgeting.

Project 2

Now let's consider a variant of the project in which the firm pays \$35 at time 1 to produce the cash flow, whether or not the resulting cash flow is positive. (That is, the firm has a binding commitment to produce.) This generates the cash flows shown in the "Project 2" column of Table 1. This example will illustrate one important difference between a DCF and a real options valuation.

We will demonstrate four ways to compute the present value of the Project 2 cash flows. First, we can recognize that the Project 2 cash flows are created by the difference between two cash flows of different risk, each of which should be discounted at a different rate. The present value of the difference between these cash flows is:

$$V_2 = \frac{\$42.692}{1.11} - \frac{\$35}{1.06} = \$5.443$$

A second way to compute the present value is to compute the forward price for the cash flows and discount it at the risk-free rate. To compute the forward price, we subtract the dollar risk premium from the expected cash flows. We already saw in equation (3) that the dollar risk premium for Project 1 is \$1.923. The dollar risk premium for Project 2 is the risk premium for Project 1, scaled to account for any difference in the relative dollar risk of the cash flows. The ratio of the dollar difference in cash flows for Project 2 vs. Project 1 is $(X_{2u} - X_{2d}) / (X_u - X_d)$, where X_2 denotes the cash flows for Project 2 and X the cash flows for Project 1. The dollar risk for Project 2 (\$16.361 – (–\$3.848)) is the same as the dollar risk for Project 1 (\$51.361 – \$31.152), so this scale factor is one. The dollar risk premium for Project 2 is therefore the same as the dollar risk premium for Project 1. The forward price for Project 2 is thus:

$$pX_{2u} + (1-p) X_{2d} - \frac{X_{2u} - X_{2d}}{X_u - X_d} V\beta (r_M - r)$$
$$= \$7.692 - \$1.923 = \$5.769$$

The present value of the forward price is:

$$\frac{\$5.769}{1.06} = \$5.443$$

A third way to compute the present value is to use DCF. Standard DCF applied to the net cash flow as an entity is

6. The same thing is true when pricing an option using the Black-Scholes formula. The formula requires knowing the current stock price. We observe the traded stock price; the market has already performed the necessary present value calculation.

problematic because it isn't obvious what the beta of the cash flow is. We can determine the risk, however, by computing the relative *percentage* variation of the cash flow between Project 2 and Project 1. Since we know the beta for Project 1, we can determine the beta for Project 2. The percentage variation in the Project 2 cash flow relative to that in Project 1 (also known as the project's *elasticity*) is:[7]

$$\frac{X_{2u}-X_{2d}}{X_u-X_d}\frac{V}{V_2} = \frac{16.361-(-3.848)}{51.361-31.152}\frac{38.461}{5.443} = 7.067 \tag{8}$$

This calculation tells us that the percentage risk of Project 2 is roughly seven times the percentage risk of Project 1. We obtain the Project 2 beta by multiplying the Project 1 beta by 7.067: 7.067 x 1.25 = 8.833. The DCF calculation is therefore as follows:[8]

$$\frac{0.571\times 16.361+(1-.0571)\times(-3.848)}{1+0.06+8.833\times(0.10-0.06)} = 5.443 \tag{9}$$

Here the calculation truly is circular in that we used the correct present value (5.443) to compute the elasticity, and thus as part of the DCF calculation. (As I show in the Appendix, when we perform the valuation without this circularity, we end up doing a certainty-equivalent valuation.) By using the incorrect 11% discount rate in a DCF valuation, we would have obtained a present value $6.930, a 27% error for a one-period calculation!

The final approach to valuing the Project 2 cash flows is to perform risk-neutral valuation. To do this, we need only recognize that the Project 2 cash flows are functions of the Project 1 cash flows; the only source of risk is variation in X. With this assumption, we can use the risk-neutral probability already computed for Project 1 and obtain the present value as follows:

$$\frac{0.476\times 16.361+(1-0.476)\times(-3.848)}{1.06} = 5.443$$

The risk-neutral probability depends on X, which is the underlying source of project risk, not on other characteristics of the project.

Project 3

Project 3 is the same as Project 2 except that the company chooses not to produce output if doing so results in a negative cash flow. In other words, the firm has an option to shut down production. Unlike Project 2, it is not possible to view the Project 3 cash flows as a simple difference of easy-to-value cash flows. Thus, to value the cash flows we must use either DCF or risk-neutral valuation. We will start by performing the risk-neutral valuation using the certainty equivalent cash flow, then use the risk-neutral probability, and finally value the project using DCF.

The dollar risk premium for Project 3 cash flows is computed by scaling the dollar risk premium for Project 1 cash flows by the relative variability of Project 3 and Project 1 cash flows. The dollar risk premium is calculated as follows:

$$\frac{X_{3u}-X_{3d}}{X_u-X_d}V\beta(r_M-r)$$
$$=\frac{16.361-0}{51.361-31.152}\times 1.923 = \$1.557$$

The forward price (or certainty equivalent cash flow) for Project 3 cash flows is computed by subtracting the dollar risk premium from the expected cash flow:

$$pX_{3u}+(1-p)X_{3d}-\frac{X_{3u}-X_{3d}}{X_u-X_d}V\beta(r_M-r)$$
$$=0.571\times 16.361+(1-0.571)\times 0-1.557 = \$7.786$$

The project present value is therefore:

$$\frac{\$7.786}{1.06} = \$7.345$$

Because we can shut down to avoid losses, the present value of cash flows is greater for Project 3 than for Project 2.

We can also value the project by computing the risk-neutral expected value of the cash flows as follows:

$$\frac{p^*X_{3u}+(1-p^*)X_{3d}}{1+r}$$
$$=\frac{0.476\times 16.361+(1-0.476)\times 0}{1.06}$$
$$=\$7.345$$

As in Project 2, a DCF valuation using an 11% discount rate is not appropriate. While the underlying cash flow has a beta of 1.25, the beta of the operating cash flow is greater. To calculate the beta of the project, we compare the percentage sensitivity of the project cash flow with the operating option to that without. The elasticity is:

$$\frac{16.361-0}{51.361-31.152}\times\frac{38.461}{7.345} = 4.239$$

7. The role of elasticity in computing discount rates has been discussed in this journal by Hodder, Mello, and Sick (2001). The observation that DCF and risk-neutral pricing give the same answer has appeared numerous times in the literature, including Black and Scholes (1973), Cox, Ingersoll, and Ross (1985), Cox and Rubinstein (1985), and McDonald (2006, Ch. 11). Full references for all citations can be found in the references section at the end of the article.

8. Had we simply discounted the expected cash flow at 11%, which was the appropriate rate of return for project 1, we would have obtained a present value (incorrect) of:

$$\frac{0.571\times 16.361+(1-.0571)\times(-3.848)}{1+0.06+1.25\times(0.10-0.06)} = 6.930$$

Table 2 **Cash flows, true probabilities, and risk-neutral probabilities from investments that pay risky cash flows for three years.**

Project 2 cash flows are Project 1 cash flows less $35. Project 3 cash flows are the greater of Project 2 cash flows and zero. The probabilities are the time 0 probabilities of reaching a given node, computing using either true or risk-neutral probabilities.

Project 1			Project 2			Project 3		
1	2	3	1	2	3	1	2	3
51.361	65.949	84.680	16.361	30.949	49.680	16.361	30.949	49.680
31.152	40.000	51.361	-3.848	5.000	16.361	0.000	5.000	16.361
	24.261	31.152		-10.739	-3.848		0.000	0.000
		18.895			-16.105			0.000
Expected cash flows (true probabilities)								
42.692	45.566	48.633	7.692	10.566	13.633	9.343	12.542	16.117
Expected cash flows (risk-neutral probabilities)								
40.769	41.553	42.352	5.769	6.553	7.352	7.786	9.503	11.180

True Probabilities			Risk-neutral Probabilities		
1	2	3	1	2	3
0.571	0.326	0.186	0.476	0.226	0.108
0.429	0.490	0.420	0.524	0.499	0.356
	0.184	0.315		0.275	0.392
		0.079			0.144

The discount rate for the project is therefore:

$$4.239 \times 1.25 \times (0.10 - 0.06) = 0.272$$

And the DCF valuation is:

$$\frac{0.571 \times 16.361 + (1 - 0.571) \times 0}{1 + 0.272} = 7.345$$

Using the (incorrect) 11% discount rate, we would have computed a DCF value of $8.417, about a 15% error.

Summary

This series of simple valuation examples demonstrates two important points: First, even if we perform a real options valuation, a DCF valuation is necessary unless there is a forward price or present value that we can observe in the market. Second, the DCF discount rate was different in all three projects, whereas the risk-neutral probability remained the same. Using the same DCF discount rate in all three projects produces materially incorrect values for Projects 2 and 3, while using the same risk-neutral probability always produces the correct valuation. Although DCF valuation is essential as a first step, risk-neutral valuation can greatly simplify the calculation in a dynamic DCF problem where risk and beta change with different assumptions about how the project is operated.

Multi-Period Valuations

Most projects span multiple periods. A common assumption when using DCF is that expected cash flows are discounted at a constant rate. However, this is correct only when cash flow uncertainty is expected to resolve over time at a constant rate.[9] Similar assumptions are typically

9. This point has long been discussed in the discounting literature. See Fama (1977), and Myers and Turnbull (1977) and Brennan (2003) for a modern treatment. A standard textbook treatment is Brealey, Myers, and Allen (2006, Section 9.4). See also Trigeorgis (1996) for a discussion in the context of real options valuation.

made when performing a real options valuation. Table 2 summarizes the three examples we will consider in this section; all are multi-period versions of the projects in Table 1 Project 1 is a static DCF calculation. Project 2 is a dynamic DCF problem since (as we will see) the project beta changes over time and across the cash flow tree. It is dynamic even though operations are fixed in advance (the project always operates). Project 3, where we have the option to shut down, is also clearly a dynamic problem. We will see that traditional static DCF, using a constant discount rate, works well only in Project 1.

The purpose of these examples is to highlight the assumptions commonly made in performing multi-period DCF valuations, and to compare DCF and real options valuations given these assumptions. The examples will illustrate the comparative simplicity of risk-neutral valuation when making plausible assumptions about a project.

Project 1

As in the previous section, we assume that the cash flow forecasts are based on projections of how the project will perform conditional on the economy's performance. If the project does well in a given period, it may do better or worse the following period. For example, if the cash flow in year one is $51.361, the following year it may be $65.949 (with probability 57.1%) or $40 (with probability 42.9%). The magnitude of the up and down moves is a measure of the analyst's uncertainty about subsequent cash flows. The cash flows in the table assume that both the amount of uncertainty, and the rate at which uncertainty resolves is constant over time, in that the change in cash flow in each period is assumed to be a constant percentage of the previous period's cash flow. Specifically, all cash flow increases are 128.4% of the previous period cash flows, and cash flow decreases are 77.9% of the previous period cash flows.[10] Also, from the perspective of time 0, the risk of the project between period 1 and period 2 is assumed to be the same as the risk from any node between period 2 and period 3. When the cash flow is greater in one period, all subsequent cash flows are assumed to be higher as well. This is the sense in which there is a constant resolution of uncertainty.

The panel labeled "true probabilities" reports the time 0 probabilities of reaching any given node.[11] You can multiply these probabilities by the corresponding cash flow to calculate the expected cash flow in each period. Risk-neutral probabilities are also constant, and the time 0 risk-neutral probabilities of reaching any node are computed in the same way.

1. DCF Valuation

How do we discount the multi-period Project 1 cash flows in Table 2? The key is to recognize that the proportional risk going forth from each node is the same. It is then plausible to assume that the systematic risk (β) is the same at each node. This same level of systematic risk is projected forward. So the appropriate one-period discount rate at each node is 11%, and the appropriate discount rate for cash flows two periods away is 11% each period, and so forth. We can therefore employ traditional DCF, discounting each year's expected cash flows at 11%, and obtain the following:

$$\frac{\$42.692}{1.11}+\frac{\$45.566}{1.11^2}+\frac{\$48.633}{1.11^3}=\$111 \tag{10}$$

The cash flow assumptions for Project 1 may seem highly artificial and unrealistic, but such assumptions implicitly underlie the typical traditional DCF analysis.

2. Real Options Valuation

Just as the true probabilities of cash flow moves are constant in Table 2, so are the risk-neutral probabilities. The risk-neutral present value is computed by discounting expected cash flows at the risk-free rate. We obtain:

$$\frac{\$40.769}{1.06}+\frac{\$41.553}{1.06^2}+\frac{\$42.352}{1.06^3}=\$111 \tag{11}$$

Project 2

As in Table 1, Project 2 cash flows are the result of subtracting $35 from each of the Project 1 cash flows. Earlier we saw that the discount rate for the project was increased by the leverage (relative to Project 1) implicit in the cash flows. The multi-period version of Project 2 is even more complicated, because implicit leverage changes from period to period and node to node.

The easiest way to value Project 2 is to compute the difference of the present value of the cash flows as follows:

$$\$111-\$35\times\left[\frac{1}{1.06}+\frac{1}{1.06^2}+\frac{1}{1.06^3}\right]=\$17.448$$

10. These cash flow moves imply a continuously compounded cash flow volatility of $0.5\times\ln(X_u/X_d) = 0.5\times\ln(51.361/31.152)=0.25$.

11. This calculation takes into account all possible ways of reaching a given cash flow node. For example, there are two ways to reach the $40 node in period 2: by reaching 51.361 and then 40, or by reaching 31.152 and then 40. Either has the probability of an up move and a down move in the cash flows, or $0.571\times(1-0.571)=0.245$. Since there are two ways to get to 40, each with the same probability, the probability of reaching 40 is $2\times 0.245=0.490$.

We can obtain the same answer using risk-neutral valuation and the expected cash flows in Table 2:

$$\frac{\$5.769}{1.06}+\frac{\$6.553}{1.06^2}+\frac{\$7.352}{1.06^3}=\$17.448 \qquad (12)$$

Even though firm behavior is static, a correct DCF is considerably more complicated than the real options valuation. We have already seen that the cash flow beta for the first period is approximately 7.0. The beta going forward varies from node to node. For example, in period 1, when the cash flow is $51.36, the beta going forward is less than 7 because the implicit leverage is lower: The present value of the $35 fixed cost is low relative to the present value of risky cash flows. When cash flow is $31.15, the beta is greater than 7 because the implicit leverage is greater. It is possible to perform traditional DCF calculations in this case by accounting for the different discount rates on a path-by-path basis, but it is a complicated procedure. Even in this relatively simple case where operating decisions are static, the valuation is dynamic because proportional risk changes from node to node on the cash flow tree.[12]

To see how the DCF discount rates change, we can compute the internal rate of return for the individual period cash flows. We have already seen that a 41.3% discount rate is appropriate for period 1. For periods 2 and 3, a single discount rate of 34.6% and 30.2% would give the correct answers. If we were to discount the Project 2 cash flows at 11%, we would obtain an incorrect DCF value of $25.474, as opposed to the correct value of $17.448. This is about a 45% error.

Project 3

From the perspective of valuation, Project 3 is like Project 2 except that it cannot be valued as the difference of two simple-to-value cash flows. The cash flow beta changes from node to node. A valuation using the risk-neutral probability (or certainty equivalent) is straightforward. Using the expected cash flows in Table 2, we would calculate the present value as follows:

$$\frac{\$7.786}{1.06}+\frac{\$9.503}{1.06^2}+\frac{\$11.180}{1.06^3}=\$25.190 \qquad (13)$$

By contrast, the DCF calculation is complicated just as in Project 2, because systematic risk changes node-to-node. The single discount rates that would give the correct answer are 27.2% for period 1, 21.8% for period 2, and 19.7% for period 3. Using an 11% discount rate, we obtain a present value of $30.381, a 20% error.

Summary

The examples in this section have illustrated the well-established point that traditional static DCF, using a constant discount rate, is a correct discounting method only under the special assumption that risk resolves at a constant rate over time. A real options valuation, by contrast, automatically addresses the problem of discounting cash flows even when risk premiums change dynamically. A real options valution also requires special assumptions; but in the examples we have examined, stricter assumptions are required for a constant risk premium than are required for constant risk-neutral probabilities. In any case, however, the concepts of a DCF valuation are still required to characterize the risk of the project.

Real Options and Capital Budgeting in Practice

In this section I will discuss what we know about capital budgeting practices and what academic research tells us about the way businesses seem to behave. Although surveys of corporate decision makers tell us that real option methods are not much used by managers, the academic evidence suggests that prices and behavior are consistent with real options effects.

How do Firms Make Capital Budgeting Decisions?

Most academic knowledge about capital budgeting practice is based on surveys and anecdotes. Surveys report that DCF is used very widely in practice. For example, in a much cited survey by John Graham and Campbell Harvey (2001), 75% of the 398 responding CFOs said they "always or almost always" use net present value (NPV)—which relies on DCF—as a capital budgeting technique.[13] But only 25% of those CFOs claimed to use real option methods. At the same time, many CFOs in the survey reported always or almost always using many other methods, including IRR, payback, and P/E multiples. A similar picture emerges from a survey by Patricia Ryan and Glenn Ryan (2002), which reports that 85% of companies use NPV "always or often" while at most 15% use methods such as real options, option pricing, and simulation.

One way to interpret this survey evidence is that most companies routinely use multiple capital budgeting methods in making decisions. This suggests that, rather than basing decisions on a single metric, managers perform a variety of formal calculations and then make

12. For a discussion of DCF when risk varies in this fashion, see Hodder, Mello, and Sick (2001) and McDonald (2006, Sections 11.2 and 19.1).

13. In addition, 75% of the respondents report always or almost always use the internal rate of return (IRR) method. Although IRR has well-known flaws (e.g., see Brealey, Myers, and Allen 2006), it is a DCF-like method. In personal communication, John Graham reports that only 10 out of 398 firms in the Graham and Harvey (2001) sample use neither NPV nor IRR, suggesting that almost all firms perform DCF-like calculations.

decisions by weighing the results and relying on subjective judgment. (It would be surprising if this were not the case.) It is possible that part of this subjective judgment represents managers' "adjustments" of DCF methods in ways that, however informal, take account of real option value. For example, two important options that companies should consider in making investments are the ability to delay a project to gain more information, and the ability to make strategic "toehold" investments that pave the way for possible expansions. In a study published a few years ago (McDonald (2000)), I showed that the common corporate practice of using artificially high hurdle rates in project evaluation causes managers to delay some major investments in a way that may be optimal, given present uncertainty about their payoffs. In similar fashion, the use of low hurdle rates when evaluating "strategic" investments may also have a justification in terms of real options. Consistent with this possibility, a 1996 survey of CEOs by Jim Poterba and Larry Summers reported that companies often use very different hurdle rates when valuing different kinds of projects. The average spread between high and low hurdle rates in their sample was 11%. While this could be due to differences in systematic risk, Poterba and Summers said that those companies that offered an explanation of such differences suggested that their "strategic projects" were assigned a low hurdle rate.

If corporate managers do not strictly use traditional DCF methods to make investment decisions, is there evidence that DCF either does or does not describe market prices? In a much cited 1995 article, Steve Kaplan and Rick Ruback examined highly-leveraged transactions (management buyouts and leveraged recapitalizations) in which the transaction documentation contained forecasts of cash flows. Using standard assumptions for DCF valuation, Kaplan and Ruback found that the DCF estimates were relatively accurate, on average, but with a large standard deviation. (The median log ratio of estimated value to transaction price was about 1.05; but 40-50% of the estimates had errors greater than 15% and the standard deviation of the valuation errors was around 25%.) By focusing on the average error, one would conclude that DCF accurately values the transaction prices. But one could reach the opposite conclusion by focusing on the standard errors.

In many respects, the Kaplan and Ruback study seems to provide as good an assessment of DCF (in a corporate context) as one could hope for. The forecasts are constructed by knowledgeable participants for cash flows of existing companies that have been publicly traded (which means that historical data for the firms is available). Moreover, the cash flow forecasts are reported in legal documents supported by fairness opinions and constructed with the recognition that transactions would occur based in part on those estimates.[14] Given these favorable conditions, the large errors suggest that one should view DCF calculations as inherently noisy estimates of value.[15]

If traditional DCF is a noisy estimate of value, it would make sense for managers to use multiple valuation methods. What, then, are managers doing? An alternative to surveys is to look at studies that examine market outcomes.

Academic Research Related to Real Options

While most companies profess not to use real options, academics routinely use such models to describe how managers behave and prices are determined. Many studies find evidence that is consistent with real options playing a role in the thinking of managers and markets.[16] This should not be surprising since, however managers describe their behavior, ignoring an option assigns it a value of zero. Powerful market forces work against companies and managers that systematically make such mistakes. Here I will discuss a sampling of the many academic studies that bear on this question.

Some studies have viewed real options models as a way of explaining observed prices. For example, in a 1998 study of oil tract leasing by Jim Paddock, Dan Siegel, and Jim Smith—which was the first study to use real options to study behavior and pricing—the authors found that the market prices for oil lease tracts exceeded both the DCF and real options values that they computed.

In another study of real options and natural resource extraction, Alberto Moel and Peter Tufano examined managerial decisions to open and close gold mines in the context of real options models. Consistent with the models' predictions, they found that decisions to open closed mines are delayed when the volatility of gold prices increases.

Eduardo Schwartz and Mark Moon have used real options methods to perform valuations of Amazon (Schwartz and Moon, 2000) and e-Bay (Schwartz and Moon, 2001). In the Schwartz-Moon model, the main source of option value is the right to walk away from an unprofitable operation. In both cases, Schwartz and Moon concluded that the prices of the companies observed at the time of their studies

14. Of course, the firms rendering fairness opinions may be subject to pressure from transaction participants, who may have reasons to bias the forecasts either up or down. Kaplan and Ruback discuss such possible biases.

15. Along these lines, Fama (1996) offers this assessment of DCF: "One can argue that a discounting rule is a way to get on with the task of project valuation that at least has some economic justification." However, "Given the massive uncertainties inherent in all aspects of project valuation …the conclusion [that DCF is superior to methods such as payback] is based more on faith than evidence."

16. Of course there is selection bias (a paper finding a negative result is less likely to be published) but it is plausible that real options effects should exist because they reflect optimizing behavior.

far exceeded the estimated real options values of the companies, which in turn significantly exceeded DCF values.[17] Thus, although real option models have not necessarily been accurate in valuing companies, in these cases they produced prices closer to those observed than DCF valuations. One can imagine other sources of value stemming from real options, such as the right to expand profitable operations, that would close the gap between real options model prices and observed prices.

Another study that compared traditional DCF values to real option values is Berger, Ofek, and Swary (1996), who attempted to determine whether abandonment values are reflected in corporate stock prices. The idea is that the value of a firm should be the DCF of existing operations plus, at a minimum, the right to abandon the firm. After controlling for cash flows from ongoing operations, they found that companies with more abandonment value (which the authors estimated using a sample of firms that sold or liquidated operations) and more fungible assets had higher market values.

One potentially important application of real options is real estate development.[18] Commercial real estate is a risky and irreversible investment. Theory suggests that a developer should not develop a marginally profitable building: If the market turns down the developer loses; while if the market later turns up, the developer could have built anyway. Thus, developers will delay investing until the market is strong. For example, the existence of undeveloped plots of land in a large city is consistent with real options. Two separate studies of real estate prices in Seattle—Quigg (1993) and Cunningham (2006)—found that real options considerations help explain the variations in such prices. Cunningham reported that greater uncertainty about prices reduced development and raised prices, both of which effects are consistent with the predictions of real options models.

Finally, the Black-Scholes formula—as described in Black and Scholes (1973)—provides a famous example of practice predating a formula. In a 1989 article, Fischer Black described how he along with colleagues Robert Merton and Myron Scholes tried to profit from use of the formula. They found that, although the degree of apparent mispricings were within the bid-ask spread, there appeared to be some arbitrage opportunities. They invested in one seemingly underpriced warrant and ended up losing money when the market had information they did not (the company turned out to be a takeover target). While the Black-Scholes formula unquestionably permitted more accurate pricing and transformed financial markets, option prices were reflecting the economics of options before the formula existed.

Conclusion

In this paper I argue that DCF and risk-neutral valuation both play an essential role in capital budgeting calculations. Traditional DCF is a necessary input for a real option valuation, and real option methods can greatly simplify otherwise daunting valuation problems. Even with valuation problems that seem relatively straightforward, option valuation methods are helpful. But rather than discuss valuation methods using the term "real options," it may be more productive to concentrate on the details of the economic and valuation issues that arise in specific contexts and then decide the best way to obtain accurate prices.

Despite survey evidence reporting that most managers do not claim to use real options methods when making capital budgeting decisions, academic studies generally find both managerial behavior and market pricing to be consistent with the predictions of real option models. Managers can be expected to find rules of thumb and *ad hoc* modifications of DCF and other traditional valuation approaches that lead to better evaluation of the economics of a given project.

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Appendix

DCF calculations use true expected cash flows and adjust for risk in the discount rate, while risk-neutral calculations adjust for risk when computing risk-neutral expected cash flows, and discount at the risk-free rate. This appendix shows that performing a correct DCF valuation is equivalent to performing a risk-neutral valuation. In demonstrating this equivalence, I show that the "circularity" described in the text is resolved by computing a certainty-equivalent (i.e., risk-neutral) cash flow. We will denote the original (Project 1) cash flows as X_u and X_d, and the Project 2 cash flows as X_{2u} and X_{2d}. The present values of these cash flows are V and V_2 respectively. We wish to compute V_2.

17. Pástor and Veronesi (forthcoming) find that uncertainty about future earnings is a possible explanation for the high NASDAQ stock prices prior to the NASDAQ crash in 2000. This explanation has a real options flavor in that volatility is positively linked to value. Of course, traditional DCF done correctly would reach the same conclusion.

18. See Titman, 1985.

As discussed in the text, the beta for Project 2 is related to that for Project 1 by the elasticity of Project 2 cash flows with respect to Project 1 cash flows:

$$\Omega = \frac{X_{2u} - X_{2d}}{X_u - X_d} \times \frac{V}{V_2}$$

The DCF equation can be written in terms of the elasticity:

$$\frac{pX_{2u} + (1-p)X_{2d}}{1 + r + \Omega\beta(r_M - r)} = V_2$$

Notice that V_2 appears both on the left (as part of the elasticity, Ω) and right sides of this equation. We can algebraically manipulate the equation, however, to obtain:

$$\frac{pX_{2u} + (1-p)X_{2d} - \{[(X_{2u} - X_{2d})/(X_u - X_d)]V\beta(r_M - r)\}}{1 + r} = V_2 \tag{14}$$

The term in curly braces in the numerator of this equation has a simple interpretation. The dollar value of the systematic risk in Project 1 is $V\beta(r_M\text{-}r)$. The fraction of this systematic risk borne by Project 2 is $(X_{2u} - X_{2d}) / (X_u - X_d)$. Thus, the numerator is the expected cash flow for Project 2, less the dollar value of risk in this project. The numerator is the *certainty equivalent cash flow* for Project 2, and it is discounted at the risk-free rate.

Finally, we can also see how risk-neutral probabilities can be obtained from the certainty equivalent. Rewrite equation (14) to obtain:

$$\frac{p^*X_{2u} + (1-p^*)X_{2d}}{1 + r} = V_2 \tag{15}$$

Where

$$p^* = p - \frac{V\beta(r_M - r)}{X_u - X_d}$$

That is, the risk-neutral probability p^* is the true probability less the dollar risk premium $(V\beta(r_M - r))$ as a percentage of the dollar risk of the underlying asset is $(X_u - X_d)$.

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Journal of Applied Corporate Finance (ISSN 1078-1196 [print], ISSN 1745-6622 [online]) is published quarterly, on behalf of Morgan Stanley by Blackwell Publishing, with offices at 350 Main Street, Malden, MA 02148, USA, and PO Box 1354, 9600 Garsington Road, Oxford OX4 2XG, UK. Call US: (800) 835-6770, UK: +44 1865 778315; fax US: (781) 388-8232, UK: +44 1865 471775.

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