

Modeling Joint Pricing and Product Assortment Choices

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Abstract

Research both in marketing and economics so far has treated product attributes as exogenous when estimating demand and supply models. While this assumption may be justified in the short run, in the long run managers make product-positioning decisions by taking into account the demand they face relative to the competitive landscape.

In this paper, we take a first step toward exploring the positioning strategies of oligopolistic firms by treating product choice as endogenous. Firms compete by choosing which products to offer and then setting prices. We use the framework developed in Seim (2004) to model firms' positioning decisions as a game of imperfect information where competitors have cost advantages to offering certain product types. We assume such cost advantages to be unobserved by the firm's rivals in the market at the time at which positioning decisions are made. In the resulting Bayesian Nash equilibrium, firms sort into product offerings that allow them to target demand optimally while limiting competitive intensity.

Keywords: endogenous product choice, multi-product firms

JEL Classification: L0, L1, L2, L8, M3

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1 Introduction

One of the fundamental decisions faced by a multi-product firm is which products to carry and how to price them, that is, the firm has to decide what product assortment to offer. There are multiple issues to be considered in determining whether and which additional product(s) to add to the firm's product assortment:

- cost of offering an additional product
- sources of potential demand: cannibalization, business stealing, or market expansion
- price premium that could be charged with a bigger assortment
- changes in competitive intensity

When selecting which products to include in its assortment, a firm in a competitive environment has to weigh the benefits of a “popular” location against the downsides of price competition. If it introduces close substitutes, then there is a risk of cannibalizing its own demand. This is similar to the monopoly case and can be described as an *expansion effect* (Shaked & Sutton 1990). On the other hand, if the firm decides to offer distant substitutes, then it leaves space open for a rival to offer the close substitutes which results in fiercer price competition (*competition effect*). Several researchers have examined this tradeoff and have tried to determine the conditions that lead to a segmentation equilibrium (one in which firms produce close substitutes) versus an interlacing equilibrium (one in which firms produce distant substitutes). Brander & Eaton (1984) and Bhatt (1987) assume a constellation of four products, two of which are produced by each firm. The firm thus only decides which pair of locations it wants to occupy. They show that both segmentation and interlacing are possible equilibria, depending on the type of game played. Martinez-Giralt & Neven (1988) find that when products' locations are endogenized by allowing for a continuum of possible locations, then in equilibrium, each firm will bunch its two products at the same point.

MD: We need to thin a bit about what the counterfactuals are or how we can estimate the above effects: (1) change in distribution of cost (2) cannibalization, business stealing, market expansion (?); (3) price premium

with a full line (?); (4) changes in competitive intensity: competitive and expansion effect (?)

In addition to the theoretical literature discussed above, there are two main streams of the empirical industrial organization literature that can be used to inform product assortment decisions: (1) structural demand models, and (2) the product-choice literature. We discuss them in turn.

To make product assortment decisions, firms need to understand the nature of demand, i.e., how consumers respond to the products offered and to the prices, at which they are offered. Structural demand models generate consistent estimates of demand elasticities given the characteristics of the products that firms have chosen to offer, but they assume that these characteristics or product type choices are fixed and thus exogenous. Therefore, counterfactual experiments that may entail a re-optimization of firms' product assortments yield potentially biased results if product choice is ignored. For example, Nevo (2000) uses structural demand estimates to simulate the effects of a merger between two competitors assuming that the characteristics of the products offered by the firms are the same pre- and post-merger. To the extent that the merged firm and its competitors adjust their offerings after the change in market structure, the true effect of the merger would be misstated if product choice is not considered as an endogenous variable.

In choosing which products to offer, firms need to consider the tradeoff between demand potential and intensity of competitive interaction, that is, the tradeoff between offering niche products with few immediate competitors for which the firm may charge a price premium, but have lower sales and offering products with higher demand that attracts a larger number of firms to offer competing products, putting downward pressure on prices. Recent advances in the empirical entry literature (Mazzeo 2002, Seim 2004, Einav 2004) have focused on competitive firms' choices of optimal product-space location. These papers, however, either model situations where prices are not a choice variable of the firm (Einav 2004) or use a reduced-form profit function that does not explicitly incorporate the prices and quantities of the products offered. Firms' product space locations and those of their competitors enter solely as arguments of the firms' objective function, thereby similarly limiting the scope of counterfactual exercises one can conduct using the model's estimated parameters.

Our goal in this paper is to develop an empirical model that incorporates both product-type selection by firms and the demand of consumers for those products. Because the available product space in most interesting empirical applications will invariably be quite large, our modeling framework will draw on Seim (2004). Competing firms are assumed to have incomplete information about each others' profitability of offering particular product assortments. This assumption allows us to generate an estimable empirical model with a wide variety of product-space locations available to firms. We augment this product-choice model with a structural model of demand for differentiated products.

To illustrate the potential of such an approach, we consider as an application the product assortment decisions by ice cream manufacturers. These firms choose which ice cream flavors to offer, and competing manufacturers have assortments that include unique flavors, flavors that are sold by a subset of competitors only, and staples that all ice cream producers offer. Manufacturers, industry analysts and regulators believe that flavor selection is an important concern for ice cream makers.¹ The data for the empirical analysis is provided by IRI's Infoscan service and has information on quarterly quantity sold, prices, promotions for all flavors/SKUs in 64 different regional markets in the U.S covering a representative sample of 3000 stores across the USA. In addition, we have advertising expenditure data from the AdSpender database and demographic information from the U.S. census.

We consider the introduction of new varieties, which depends on both demand characteristics and the competitive environment in each market. Table 1 presents an overview of the 64 IRI food markets. As we can see, there are two national brands, Breyers and Dreyer's/Edy's that are among the top three brands in most markets but there is considerable variation in the regional brands that compete with them. In addition, the store brands are major players: They are among the top three brands in 57 of the 64 markets. Note that although listed as one brand (private label), the identity of the store brands varies by region. The ice cream market therefore represents a promising empirical setting, in which to implement the endogenous product choice/structural demand estimation framework that we propose.

¹The U.S. Federal Trade Commission (FTC) has recently sought a preliminary injunction to block a proposed merger between two competing ice cream manufacturers on the grounds that it would "lead to anticompetitive effects . . . including less product variety and higher prices". Information from the FTC website at www.ftc.gov/opa/2003/03/dreyers.htm. Note that the FTC's concerns relate primarily to Dreyers' super-premium brands (Dreamery, Godiva and Starbucks).

In the model below, we treat each flavor as a separate product and model entry/exit for each flavor as well as all possible bundles of flavors that a firm can offer. For large flavor sets, this creates a dimensionality problem. To deal with this problem, the literature typically suggests projecting products onto a set of characteristics (Lancaster 1979). Treating a flavor as a bundle of discrete, observable attributes, however, entails similar dimensionality problems in a model of product assortment since the choice of an assortment would be based on all possible combinations of characteristics. Furthermore, in the ice cream setting, similar to many other consumer goods industries, each flavor’s observable characteristics (such as sugar or milk fat content) only incompletely describe differences to other available flavors. To uniquely identify flavors based on characteristics, one may need a level of detail that brings us back to the flavor level.

Alternatively, if we assumed that characteristics that drive similarity between products were primarily unmeasurable by observable flavor ingredients, we could infer the similarity between products by either using consumer switching data as in Goettler & Shachar (2001) or survey measures that allow for the construction of a perceptual map. Both of these approaches generally yield at least a two-dimensional, continuous attribute space in which products are placed, and the distance between products in this space measures their similarity. In our application, it is difficult to imagine which characteristics of a flavor such continuous attributes represent. Furthermore, the model would need to identify various points in this characteristic space as flavors a manufacturer wants to offer, which is difficult.

We reduce the dimensionality of the problem somewhat by conditioning on the staple flavors. **ADD EXAMPLES.** All firms are assumed to carry those in each market. The assortment decisions being made concern only what we call the “competitive” flavors. Our model does take into account the demand for staples in determining the price for the product lines but abstracts from the potential effect on the product offerings. Considering that all competitors offer the same staple flavors in all markets (this is how we define staple flavors), this assumption should not lead to loss of generality.

The remainder of this paper is organized as follows. Section 2 describes the ice-cream market and the data we use for the empirical analysis. In Section 3 we develop the modeling framework. We outline our estimation approach in Section 4 and then discuss a number of counterfactual analyses that the proposed modeling framework

would allow us to conduct in Section 5. Section 6 concludes with directions for future research.

2 The Ice Cream Market

Market definition. Ice cream is one of the most popular categories in supermarkets: 92.9% of households in the United States purchase in the category (Marketing Factbook, 1993). In the general category of ice cream, there is a distinction between regular ice cream, frozen yogurt and sorbets, and ice milk² with regular ice cream representing more than 60% of total sales. Approximately one-third of all ice cream sales is vanilla, but there are hundreds of other flavors that have been created over the years though the addition of various fruits, nuts, spices, and liquors.

Market structure. Ice cream is one of the few consumer product categories in the U.S. market not dominated by a single firm. The top national producer, Kraft (Breyers), had a 13.6% market share in 2003/2004, followed by Dreyers with about 10%, and Blue Bell and Häagen Dazs with about 5-6% each (Information Resources Inc., 2004). However, two major companies, Unilever and Nestle have been pushing for consolidation and fighting for dominance. Unilever bought Breyers from Kraft in 1993, and Ben & Jerry's in 2000. Nestle acquired Häagen Dazs in 1999, and in 2002 proposed to merge its ice cream operations with Dreyers.

3 Model

Consider a simple two-stage game. A total of $b = 1, \dots, B$ firms decide which subsets of flavors to offer in a given market and how to price them given their expectation of their competitors' offerings, demand, and a fixed cost of offering each subset of flavors. In the remainder of the section, we use firms and brands interchangeably.

In the first stage, the firms decide which flavors to offer. Each firm starts with a predetermined set of potential flavors to offer and picks the optimal subset of flavors among the potential set. The flavor choice model can be thought of applying to unique and competitive flavors that are offered by only some of the manufacturers

²Ice milk was renamed low-fat ice cream in 1994 due to new nutritional labeling requirements. After the name change, sales increased by 60% in the category.

and not in all of the markets, as opposed to the staple flavors that are offered by all manufacturers in all markets.

In the second stage, firms observe each others' flavor choices. Conditional on the firm's choice of flavors and its competitors' choice of offerings, firms choose prices. This separation is necessary to keep the model tractable and is well in line with our intuition that flavor choices are more rigid (more difficult to reverse) than pricing decisions.

Flavor Sets. We have $f = 1, 2, \dots, c, c + 1, \dots, F$ flavors, where $1, \dots, c$ are the competitive flavors and $c + 1, \dots, F$ are the staples. The option of not consuming ice cream in a market is indexed by $f = 0$.

Define the vector $d_{bt} \in \{0, 1\}^c$, where d_{bft} indicates whether the competitive flavor $f = 1, \dots, c$ has been chosen by competitor b in market t .

Let us index the set of all possible bundles by $r = \{0, 1, 2, \dots, 2^c\}$ where bundle 0 is the empty set (no competitive flavors offered, only staples); bundles $1, \dots, c$ are singleton sets representing offering only one competitive flavor etc. The flavor choice vector representing bundle r is denoted by d_{bt}^r .

Cost. Firms incur a marginal cost of c_{bt} , $b = 1, \dots, B$, for each unit of a flavor offered in market t . The marginal costs of offering a flavor include costs for ingredients (such as milk, cream, sugar, and flavorings) and costs of packaging, labeling, and distributing the product. The flavor-specific cost component is relatively insignificant, therefore allowing us to assume that marginal costs are constant across flavors offered by a given firm. As is customary in the literature, we assume that firms observe each other's marginal costs when they choose prices (Berry, Levinsohn & Pakes 1995, Villas-Boas 2004).

In addition, firms incur a fixed cost per market t , ν_{bft} , to offer flavor f , where ν_{bft} is distributed according to probability distribution function G . The fixed costs of offering an additional flavor include the operating costs of producing the flavor (foregone economies of scale due to smaller batches, cost of cleaning machines, labeling, inventory cost), and stocking costs (time taken to stock shelves in the store increases in the number of flavors offered). For a discussion of the complexity of production with the addition of a new variety, see Eilon (1962) and Ravenscraft (1983).

We assume that this fixed cost varies by flavor and is only observed by the firm

itself, but not by its competitor. If a firm decides to offer more than one flavor, its total fixed costs are the sum of the individual fixed costs. Alternatively, fixed costs could be assumed to vary only by firm, but not by flavor, as a function of the total number of flavors a firm chooses to offer in that market. In that setting, the model below would uniquely predict the number of flavors to offer in a market, but not their identity.

The reason we assume that marginal costs are public information whereas fixed costs are private is that the cost of ingredients and other factors that are included in the marginal costs are typically observed by everybody, as opposed to fixed costs which may depend on how efficient the processes of each firm are.

3.1 Stage 2

In the second stage, we solve for optimal prices for every possible combination of flavor choices. These prices then flow into the first stage to determine profits for each of the flavors that a firm is considering.

Consumer Demand. Assume a discrete choice model of demand, in the simplest case a logit model. Given that we need to compute prices for all possible combinations of flavors that the B firms could offer, it is beneficial to have a demand model that yields easy-to-compute prices.

Let U_{bfmt} denote consumer k 's utility for brand b 's flavor f in market t , and let $b = f = 0$ denote the outside option of not purchasing ice cream in a given market.

$$\begin{aligned} U_{bfmt} &= X_{bfmt}\beta - \alpha p_{bt} + \epsilon_{bfmt} \\ &= \bar{u}_{bfmt} + \epsilon_{bfmt} \\ &\quad b = 0, \dots, B; f = 0, \dots, F \end{aligned} \tag{1}$$

In the above specification of utility, X_{bfmt} denotes observed characteristics of the flavor, such as firm and flavor fixed effects, whether the flavor is featured in the store ads or on display in the store in a given market. p_{bt} denotes the price charged by firm b in market t . The random component of utility, ϵ_{bfmt} , is assumed to be distributed according to an extreme value distribution. It is known to the consumer, but observed by the firms or the econometrician only in expectation. Our current modeling framework does not allow us to explicitly account for the presence of market-varying unobserved attributes

in our demand specification (or equivalently, necessitates setting the unobservables equal to zero). We explain this assumption in more detail in Section 4.

Normalizing utility from the outside good to zero results in logit market shares for the flavors that the firms offer:

$$\hat{s}_{bft} = \frac{\exp(\bar{u}_{bft})}{1 + \sum_b \sum_{f'=1}^c \exp(\bar{u}_{bf't}) d_{bf't} + \sum_b \sum_{f'=c+1}^F \exp(\bar{u}_{bf't})} \quad (2)$$

Firm Profits. For a given choice of flavors determined in the first stage, firm b chooses prices to maximize expected profit. Consistent with observed pricing practices in the industry, we constrain prices to be identical across flavors. Firms are assumed to compete in Bertrand-Nash fashion.

Firm b 's objective is to maximize profit of offering flavor f , given by

$$\max_{p_{bt}} (p_{bt} - c_{bt}) \left(\sum_{f=1}^c \hat{s}_{bft} d_{bft} + \sum_{f=c+1}^F \hat{s}_{bft} \right) - \sum_{f=1}^c \nu_{bft} d_{bft}. \quad (3)$$

Differentiating yields the competitors' first order conditions with respect to prices:

$$p_{bt} = c_{bt} - \frac{\sum_{f=1}^c \hat{s}_{bft} d_{bft} + \sum_{f=c+1}^F \hat{s}_{bft}}{\sum_{f=1}^c \frac{\partial \hat{s}_{bft}}{\partial p_{bt}} d_{bft} + \sum_{f=c+1}^F \frac{\partial \hat{s}_{bft}}{\partial p_{bt}}} \quad (4)$$

Solving the system of equations (4) yields equilibrium prices for the specific flavor offerings considered. We solve for equilibrium prices for the remaining possible flavor sets analogously. This gives us a vector of 2^c different prices for each firm, one for each possible bundle of flavors that could be offered. Note that there is no asymmetric information in the price-setting stage. Conditional on having made a flavor choice, prices are determined in a symmetric Nash equilibrium.

This is the main place where the proposed model differs from Einav's (2004) model of movie release date timing decisions. Since movie prices do not differ by movie, he does not need to solve for equilibrium prices. Instead, he estimates a seasonal demand function and uses the parameters to predict what demand (and profits) would be under each possible configuration of movie opening dates. The fact that pricing is not considered in Einav's (2004) model limits its applicability to industries such as movies where prices are uniform across competitors.

Marginal costs are specified as $c_{bt} = w_t \gamma + \eta_{bt}$, where w_t are cost shifters and η_{bt}

is a firm-specific component of marginal cost.

3.2 Stage 1

Each firm chooses the optimal set of flavors given its expectation of the other firms' choices and prices under each configuration. Consider for simplicity of notation firm 1's choice of its optimal bundle, $d_{1t}^{r'}$. The remaining firms' expected profits are determined similarly. Firm b chooses $d_{bt}^{r'}$ to maximize expected profits given by:

$$\begin{aligned}
& \mathbb{E} [\Pi_{bt}(d_{bt}^{r'}; d_{-bt}^r)] \tag{5} \\
&= \mathbb{E} \left[\left\{ \hat{p}_{bt}(d_{bt}^{r'}; d_{-bt}^r) - c_{bt} \right\} \left\{ \sum_{f=1}^c \hat{s}_{bft}(d_{bt}^{r'}; d_{-bt}^r) d_{bft}^{r'} + \sum_{f=c+1}^F \hat{s}_{bft}(d_{bt}^{r'}; d_{-bt}^r) \right\} - \sum_{f=1}^c \nu_{bft} d_{bft}^{r'} \right] \\
&= \sum_{b' \neq b} \sum_{r'_b=1}^{2^c} \left[\left\{ \hat{p}_{bt}(d_{bt}^{r'}; d_{b't}^{r'_b} \forall b' \neq b) - c_{bt} \right\} \left\{ \sum_{f=1}^c \hat{s}_{bft}(d_{bt}^{r'}; d_{b't}^{r'_b} \forall b' \neq b) d_{bft}^{r'} \right. \right. \\
&\quad \left. \left. + \sum_{f=c+1}^F \hat{s}_{bft}(d_{bt}^{r'}; d_{b't}^{r'_b} \forall b' \neq b) \right\} - \sum_{f=1}^c \nu_{bft} d_{bft}^{r'} \right] \Pr(d_{b't} = d_{b't}^{r'_b} \forall b' \neq b).
\end{aligned}$$

The marginal probability that firm b offers bundle r_b is:

$$\begin{aligned}
\Pr(d_{bt} = d_{bt}^{r_b}) &= \Pr \left\{ \mathbb{E} [\Pi_{bt}(d_{bt}^{r_b}; d_{b't}^{r'_b} \forall b' \neq b)] \geq \mathbb{E} [\Pi_{bt}(d_{bt}^{r'_b}; d_{b't}^{r'_b} \forall b' \neq b)] \forall r'_b \neq r_b \right\} \tag{6} \\
&= \int_{A_{bt}^{r_b}} dF \left(\sum_{f=1}^c \nu_{bft} d_{bft}^{r_b} \right),
\end{aligned}$$

where we let $A_{bt}^{r_b}$ denote the set of values for ν that induce the choice of flavor bundle $d_{bt}^{r_b}$:

$$A_{bt}^{r_b} = \left\{ \sum_{f=1}^c \nu_{bft} d_{bft}^{r_b} \mid \mathbb{E} [\Pi_{bt}(d_{bt}^{r_b}; d_{b't}^{r'_b} \forall b' \neq b)] \geq \mathbb{E} [\Pi_{bt}(d_{bt}^{r'_b}; d_{b't}^{r'_b} \forall b' \neq b)] \forall r'_b \neq r_b \right\}. \tag{7}$$

Assuming independence across firm cost shocks, ν_{bft} , entails that the joint probability of observing each of the $B - 1$'s product offerings $d_{bt}^{r_b}$ is simply the product of the joint marginal probabilities defined in equation (6). Substituting the flavor choice probabilities defined above into each firm's expected profit yields a measure of the attractiveness of each choice as a function of the competitors' probabilistic choice.

The probability that firm b chooses flavor offering r is then the probability that the expected profit of offering r exceeds expected profits of any other flavor offering, given its conjecture of competitor $-b$'s behavior.

The expressions defined in equations (5) and (6) characterize a system of $b \times 2^c$ equations in $b \times 2^c$ unknown flavor choice conjectures. We solve for each firm's probability of offering a given product assortment by numerically integrating over its unobserved cost shock ν , as a function of its competitors' assortment choice probabilities. The equilibrium probabilities of offering each flavor combination are found by searching for the fixed point of the system of equations for both competitors, the solution to which is the $b \times 2^c$ matrix of flavor offering probabilities. This fixed point is found numerically, by starting with an initial guess at the probabilities of flavor offerings. Using this value, we calculate expected profits for the two firms, which generates a new set of flavor offering probabilities based on equations (5) and (6). These probabilities serve as the new input for computing expected profits and we continue iterating in this way until there is convergence. The resulting fixed point in flavor offering probabilities is the Bayesian Nash equilibrium for the system of best response functions. For details of the solution algorithm, see Seim (2004).

Two-firm-two-flavor Example. As an illustration of the expected profit function and flavor choice conjectures, consider a two-firm problem where each firm has a choice of two competitive flavors to offer. To focus on the flavor choice stage, we restrict our attention to competitive flavors only. Each firm then chooses to offer that set of flavors that maximizes expected profit in a given market. With two flavors, there are four possible choices, offering either, both, or none of the flavors. The firms thus compare four expected profit levels and choose the flavor(s) that corresponds to the highest level of expected profit. We have $r = \{0, 1, 2, 3\}$, where $d_b^0 = \{0, 0\}$, $d_b^1 = \{0, 1\}$, $d_b^2 = \{1, 0\}$, and $d_b^3 = \{1, 1\}$. Figure 3.2 illustrates the example.

Suppressing market subscripts for ease of readability, firm 1's expected profit if it chooses flavor 1, or $d_1^r = d_1^1$, is given by:

$$E[\Pi_1^1] = E[(\hat{p}_1 - c_1)\hat{s}_{11}(p_1, p_2) \cdot M] - \nu_{11} \quad (8)$$

Since firm 1 does not observe firm 2's fixed cost, it has to form an expectation of firm 2's optimal flavor choice, that is, a probability assessment of how likely it is that firm

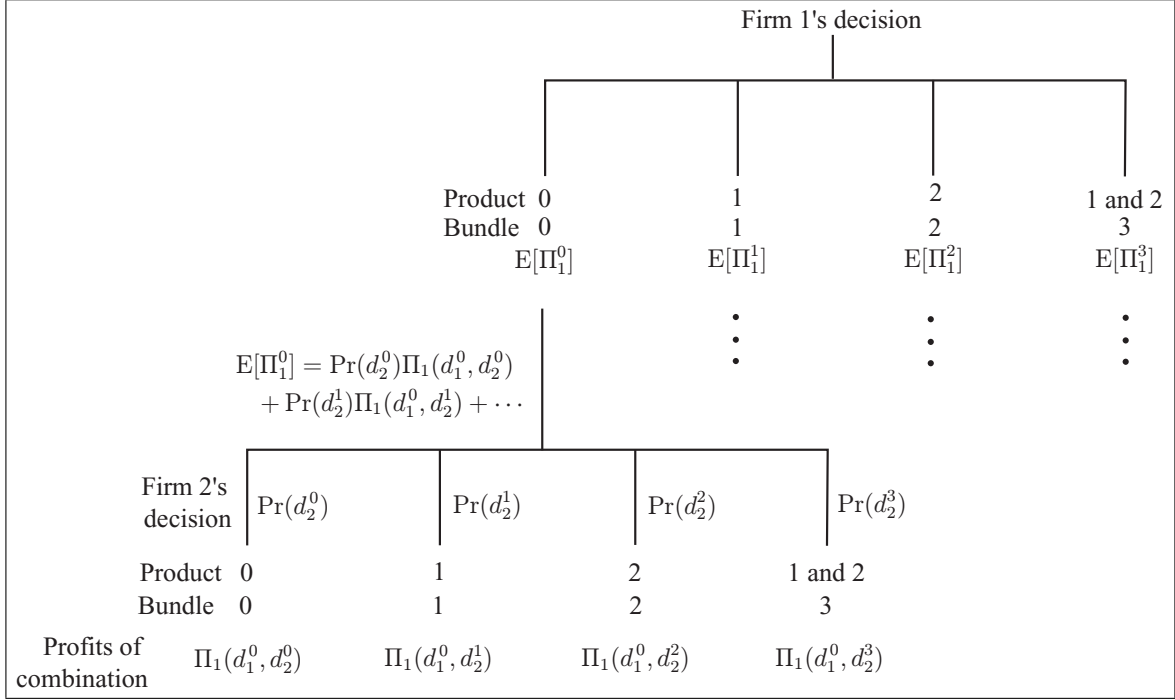


Figure 1: Expected profits.

2 will choose any one of its four possible flavor sets. Integrating over firm 2's cost type yields expected profit of the form:

$$\begin{aligned}
 E[\Pi_1^1] &= \sum_r (\hat{p}_1(d_1^1, d_2^r) - c_1) \hat{s}_{11}(p_1, p_2) M \cdot \Pr(d_2 = d_2^r) - \nu_{11} \\
 &= E[\bar{\Pi}_1^1] - \nu_{11}
 \end{aligned} \tag{9}$$

where $\hat{p}_1(d_1^1, d_2^r)$ denotes firm 1's optimal price as determined in stage 2 if it offered flavor 1 and firm 2 offers the flavor set d_2^r , while $\Pr(d_2 = d_2^r)$ denotes the probability that firm 2 will offer flavor set d_2^r . The flavor offering considered by firm 1 and the possible flavors offered by firm 2 are thus reflected in both the price firm 1 charges and its expected market share. Firm 1's expected profit for flavor 2 is computed similarly, while firm 1's expected profit if it does not offer any flavor is normalized to zero.

The expected profit if firm 1 offers both flavors, i.e., chooses flavor set d_1^3 , is given

by:

$$\begin{aligned}
\mathbb{E}[\Pi_1^3] &= \sum_r [(\hat{p}_1(d_1^3, d_2^r) - c_1)(\hat{s}_{11}(p_1, p_2) + \hat{s}_{12}(p_1, p_2))M] \\
&\quad \cdot \Pr(d_2 = d_2^r) - (\nu_{11} + \nu_{12}) \\
&= \mathbb{E}[\bar{\Pi}_1^3] - (\nu_{11} + \nu_{12})
\end{aligned} \tag{10}$$

Firm 2's expected profits are derived analogously.

Each firm's expected profit depends on its assessment of how likely it is that its competitor will offer each of its possible flavors and flavor combinations. Four flavor choice conjectures need to be formed: firm 1's assessment of firm's 2 probability of not offering any flavor, offering flavor 1, offering flavor 2, and offering both flavors.

Firm 1's assessment of firm 2's probability of offering flavor 1 is given by

$$\begin{aligned}
\Pr(d_2 = d_2^1) &= \Pr(\mathbb{E}[\Pi_2^1] > \mathbb{E}[\Pi_2^3] \wedge \mathbb{E}[\Pi_2^1] > \mathbb{E}[\Pi_2^0] \wedge \mathbb{E}[\Pi_2^1] > \mathbb{E}[\Pi_2^2]) \\
&= \Pr \left[-\nu_{22} < \mathbb{E}[\bar{\Pi}_2^1] - \mathbb{E}[\bar{\Pi}_2^3] \wedge \nu_{21} < \mathbb{E}[\bar{\Pi}_2^1] \right. \\
&\quad \left. \wedge \nu_{21} - \nu_{22} < \mathbb{E}[\bar{\Pi}_2^1] - \mathbb{E}[\bar{\Pi}_2^2] \right].
\end{aligned} \tag{11}$$

Let $\nu_{21} \sim G_1$ and $\nu_{22} \sim G_2$ independent of each other and denote $\mathbb{E}[\bar{\Pi}_2^1] - \mathbb{E}[\bar{\Pi}_2^2]$ as a , $\mathbb{E}[\bar{\Pi}_2^1]$ as b , and $\mathbb{E}[\bar{\Pi}_2^1] - \mathbb{E}[\bar{\Pi}_2^3]$ as c . The probability of offering flavor 1 is thus

$$\Pr(d_2^r = d_2^1) = \Pr(-\nu_{22} < c, \nu_{21} < b, \nu_{21} - \nu_{22} < a),$$

which in $\nu_{21} \times \nu_{22}$ space is the area left of b and above $-c$ minus the triangle spanned by $(b, -c)$, $(a - c, -c)$, and $(b, b - a)$. Hence,

$$\begin{aligned}
\Pr(d_2^r = d_2^1) &= (1 - G_1(b))G_2(-c) - \int_{\nu_{21}=a-c}^b \int_{\nu_{22}=-c}^{\nu_{21}-a} g_2(\nu_{22})d\nu_{22}g_1(\nu_{21})d\nu_{21} \\
&= (1 - G_1(b))G_2(-c) - \int_{\nu_{21}=a-c}^b (G_2(\nu_{21} - a) - G_2(-c))g_1(\nu_{21})d\nu_{21} \\
&= (1 - G_1(b))G_2(-c) + G_2(-c)(G_1(b) - G_1(a - c)) \\
&\quad + \int_{\nu_{21}=a-c}^b G_2(\nu_{21} - a)g_1(\nu_{21})d\nu_{21}.
\end{aligned}$$

The above presumes $b \geq a - c$. If $b < a - c$, then the probability simplifies to:

$$\Pr(-\nu_{22} < c, \nu_{21} < b, \nu_{21} - \nu_{22} < a) = (1 - G_1(b))G_2(-c). \quad (12)$$

With normal distributions for G_1 and G_2 , a closed form solution for these probability expressions does not exist. However, since we only need to compute a one-dimensional integral, we find the single-product probabilities using numerical integration techniques. If we assumed that G_1 and G_2 are distributed extreme value, then closed-form solutions exist for both probabilities.

The probability that flavor 2 is chosen over flavor 1 or flavors 1 and 2 together is obtained analogously as

$$\begin{aligned} \Pr(d_2 = d_2^2) &= \Pr(\mathbb{E}[\Pi_2^2] > \mathbb{E}[\Pi_2^3] \wedge \mathbb{E}[\Pi_2^2] > \mathbb{E}[\Pi_2^0] \wedge \mathbb{E}[\Pi_2^2] > \mathbb{E}[\Pi_2^1]) \\ &= \Pr \left[-\nu_{21} < \mathbb{E}[\bar{\Pi}_2^2] - \mathbb{E}[\bar{\Pi}_2^3] \wedge \nu_{22} < \mathbb{E}[\bar{\Pi}_2^2] \right. \\ &\quad \left. \wedge \nu_{22} - \nu_{21} < \mathbb{E}[\bar{\Pi}_2^2] - \mathbb{E}[\bar{\Pi}_2^1] \right]. \end{aligned} \quad (13)$$

The probability that firm 2 will offer both flavors, flavors 1 and 2, is given by

$$\begin{aligned} \Pr(d_2^r = d_2^3) &= \Pr(\mathbb{E}[\Pi_2^3] > \mathbb{E}[\Pi_2^1] \wedge \mathbb{E}[\Pi_2^3] > \mathbb{E}[\Pi_2^2] \wedge \mathbb{E}[\Pi_2^3] > 0) \\ &= \Pr(\nu_{22} < \mathbb{E}[\bar{\Pi}_2^3] - \mathbb{E}[\bar{\Pi}_2^1] \wedge \nu_{21} < \mathbb{E}[\bar{\Pi}_2^3] - \mathbb{E}[\bar{\Pi}_2^2] \wedge \\ &\quad \nu_{21} + \nu_{22} < \mathbb{E}[\bar{\Pi}_2^3]) \end{aligned} \quad (14)$$

while the probability that firm 2 chooses not to offer any flavors, or flavor 0, equals

$$\begin{aligned} \Pr(d_2^r = d_2^0) &= \Pr(\mathbb{E}[\Pi_2^1] < 0 \wedge \mathbb{E}[\Pi_2^2] < 0 \wedge \mathbb{E}[\Pi_2^3] < 0) \\ &= \Pr(\nu_{21} < \mathbb{E}[\bar{\Pi}_2^1] \wedge \nu_{22} < \mathbb{E}[\bar{\Pi}_2^2] \wedge \nu_{21} + \nu_{22} < \mathbb{E}[\bar{\Pi}_2^3]), \end{aligned} \quad (15)$$

which can be found similarly to the single-flavor probabilities.

4 Estimation

Identification. Before presenting the estimation approach, a brief discussion of potential identification strategies for flavor portfolios – and the data that would be

needed for identification – is in order. From a practical point of view, variation in the observed portfolios offered by firms is a prerequisite for any empirical analysis as discussed above. To proceed further, there must be something (preferably exogenous) in the economic environment that helps to distinguish among observations with different portfolios. Simply put, if we observe a particular firm offering vanilla and chocolate in one market and vanilla and strawberry in the other, all else cannot be equal. Finding a source of independent variation that is correlated with such changes is therefore essential for any modeling approach.

One source of such independent variation are differences across observations in flavor-specific preferences. We anticipate that there would be substantial regional variation on that dimension driven by the different demographic composition in each area. Another source of variation is the set of competitors that each brand encounters in a given market. As evident from Table 1, there are only a few markets that have the exact same brands competing. In addition to these cross-sectional sources of variation, we observe markets on a quarterly basis and can therefore use the time-series variation in the flavors offered by a brand to identify the parameters of our model.

One could also imagine that the set of available flavors from which each firm chooses its optimal portfolio is itself prone to exogenous shocks rather than endogenously determined through some sort of flavor R & D process. If a new product appears in the available set, and the profits from offering this new product are sufficiently high, the entire optimal product portfolio (for all firms) could potentially change. Again, there may need to be an embellishment to the model to accommodate the arrival process for the available new flavors. Since we observe examples of new flavors being introduced within our sample and because no additional data are required, it may be fruitful to explore this last identification strategy as well.

Estimation. For a given set of parameters for the demand and pricing equations, the second stage of the model yields predicted market shares for the flavors offered in a given market. Furthermore, the pricing stage yields marginal costs implied by the observed prices and an assumption on market conduct. These marginal costs flow into the first-stage profit function to determine profits of all potential assortment choices. The first stage in turn yields an equilibrium probability for each flavor in the firm’s potential flavor set that the flavor is offered in that given market.

Since we need to predict profit for all potential flavors, including those that are ultimately not chosen, in the first stage, we cannot use observed market shares to infer flavor-specific marginal costs or unobserved product attributes as is common in the demand literature. If we did so, we would not be able to predict profits for flavors that are not in the household's choice set. By omitting flavor-specific unobservable preference shifters, we for now ignore any potential endogeneity problems of firms' pricing decisions. One way around this is perhaps to infer unobserved product characteristics from another market in which the flavor was offered, assuming that the unobserved preference shifters remain constant.

For each brand, we observe its assortment decisions, denoted by indicator variable \mathcal{I}_{bt}^r , where $\mathcal{I}_{bt}^r = 1$ if bundle r is offered in market t , 0 otherwise, the flavors that form the assortment, indicated by $\mathcal{I}_{bft} = 1$ if flavor f is part of the assortment, the actual market share, s_{bft} , for all flavors f that are part of the assortment chosen in the first stage, and the price, p_{bt} charged by the brand for all flavors in the chosen assortment (recall that the price for a given brand is uniform across flavors).

To estimate the parameters of the model, we match firms' behavior to the three sets of predictions generated by the model using a method-of-moments estimator. The first set of moment conditions results from matching the firms' actual assortment choices to the ones predicted by the model. The remaining moment conditions then match the predicted market shares for the chosen assortment to the ones observed in the data and the predicted prices to the actual ones. We combine the moments using a minimum distance estimator to minimize the sum of least square errors. The errors that are minimized in the estimation routine consist of assortment prediction errors (the difference between the predicted choice probability and the actual assortment choice), market share prediction errors and pricing prediction errors:

$$\begin{aligned}
e_{bt}^s &= \sum_f \{s_{bft} - \hat{s}_{bft}\} \mathcal{I}_{bft}, \\
e_{bt}^p &= \sum_r \{p_{bt} - \hat{p}_{bt}\} \mathcal{I}_{bt}^r = \sum_r \hat{\eta}_{bt} \mathcal{I}_{bt}^r, \\
e_{bt}^r &= \sum_r \mathcal{I}_{bt}^r - \Pr(d_{bt} = d_{bt}^r),
\end{aligned} \tag{16}$$

where \hat{p}_{bt} is given by the RHS of equation (4). The assortment, market share, and

price prediction errors are then interacted with instruments to generate the empirical moment conditions that we minimize:

$$\bar{m}^s(\Theta) = \frac{1}{BT} \sum_{t=1}^T \sum_{b=1}^B e_{bt}^s(\Theta) l'_{bt} \quad (17)$$

$$\bar{m}^p(\Theta) = \frac{1}{BT} \sum_{t=1}^T \sum_{b=1}^B e_{bt}^p(\Theta) X_{bt}, \quad (18)$$

$$\bar{m}^r(\Theta) = \frac{1}{BT} \sum_{t=1}^T \sum_{b=1}^B e_{bt}^r(\Theta) l'_{bt},$$

l'_{bt} denotes the score, or the derivative of the log of the likelihood of the observation with respect to the parameter. X_{bt} denotes the vector of instruments for the pricing equation. The instruments include all exogenous variables in the model and possibly instruments for price. We combine the two sets of moments using a minimum distance estimator. The optimal parameters, which are obtained using a numerical minimization routine, minimize the objective function:

$$Q(\Theta) = \bar{m}(\Theta)' W \bar{m}(\Theta), \quad (19)$$

where $\bar{m}(\Theta)$ is the column vector of moment conditions and W is the optimal weighting matrix.³

5 Results

[TO BE COMPLETED]

Successful estimation of a model like the one proposed above will yield parameters describing consumer preferences for brands and flavors, as well as tradeoffs with price that can be used to compute demand elasticities. Such parameter estimates are the output of a typical structural demand model in the literature, and are often used to simulate changes in market structure or product availability, as described below. In addition, by modeling the product choice stage of the game we will obtain estimates of the firms' fixed cost distribution. Combining these with the structural demand

³Another possibility is to write down the errors from the first stage as the usual discrete choice problem and then take the first order condition from that as the moment condition, instead of matching the indicators to predicted shares.

estimates allows for an assessment of the importance of cannibalization within the product line in product assortment decisions, the magnitude of business stealing effects from offering an additional flavor, and the size of the price premium a firm is able to charge due to the introduction of an additional flavor. The explicit modeling of the first-stage assortment decision may also allow for more complete simulation exercises that allow for equilibrium responses to unilateral market structure or product portfolio changes. Below, we consider what may potentially be added to some existing empirical examples with the availability of the first stage parameters.

Merger Simulation. In recent years a number of papers have used structural demand estimation combined with a model of competition among manufacturers on the supply side to evaluate the effects of potential mergers (Nevo 2000, Pinkse & Slade 2004, Dubé 2005). These models have an advantage over traditional analyses based on the Herfindahl index in determining the extent of market power in markets with differentiated products because they take into account the level of substitution between the products offered and not only their market shares.

For example, Nevo (2000) uses structural demand parameter estimates to simulate the price and welfare effects of a merger between competing brands of ready-to-eat breakfast cereal. These effects are computed by assuming jointly optimal price and quantity choices post-merger, using the demand parameters to calculate the new market shares for all products once prices have changed. Nevo (2000) predicts significant price increases post-merger due to the increased concentration. In his analysis, he holds the portfolio of competing products after the merger constant.

There are several effects that a merger may have on firms' assortment choices, however, which may alter these conclusions regarding post-merger pricing and consumer surplus. In addition to charging higher markups, the merging parties might remove close substitutes from their joint assortment, leading to even greater loss in consumers surplus due to the decreased variety. On the other hand, it is conceivable that because of the higher markups, firms may be incentivized to offer more variety, which would have a positive effect on consumer surplus. What happens is difficult to predict ex ante.

Product Portfolio Changes. Other studies, especially those using data from the auto industry, have used structural demand estimates to determine the market-wide

responses to one firm's change in the set of products offered. For example, Berry, Levinsohn & Pakes (2004) use price and quantity data to simulate the effects of General Motors dropping the Oldsmobile brand. They predict that demand would shift to cars that are close substitutes to the Oldsmobile models. Other GM divisions offer most of these substitutes, such that the predicted impact on GM profits is quite large. However, the authors admit that, "we have done very little which examines the longer term responses of the other characteristics (other than price) of the vehicles marketed to changes in the environment." To the extent that they do not already do so, one might expect competing firms to introduce products that would be close substitutes to the discontinued Oldsmobiles. Therefore, the profit gains from dropping the Oldsmobile brand may be overstated without incorporating the potential for such a response.

6 Conclusions

The contribution of this paper can be seen along two dimensions. Methodologically, we develop an approach for dealing with endogenous product choices. Substantively, this paper allows for an in-depth analysis of all aspects of firms' product assortment decisions by providing estimates of the fixed cost of offering additional flavors as well as the price premium that can be charged, the cannibalization effect, and the extent of primary versus secondary demand creation.

The model in this paper does not focus on the initial decision to develop a flavor of certain characteristics. Instead, it assumes that firms have a set of flavors at their disposal and decide which of these flavors to offer in any given market. While the initial product development decision is potentially more interesting, determining which characteristics to choose for such a flavor is difficult since the number of different combinations of possible ingredients that could form flavors is extremely large.

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Table 1: Brand presence among the top three brands in the 64 IRI food markets.

State / City	Private Label	Breyers	Dreyers	Blue Bell	Wells Blue Bunny	Haagen Dasz	Friendly
AL, Birmingham/Montgom	x	x		x			
AR, Little Rock	x			x			
AR, Phoenix/Tucson	x	x	x				
CA, Los Angeles	x	x	x				
CA, Sacramento	x	x	x				
CA, San Diego	x	x	x				
CA, San Fran/Oakland	x		x			x	
CO, Denver	x	x	x				
CT, Hartford/Springfield		x	x				x
FL, Jacksonville	x	x		x			
FL, Miami/Ft. Lauderdale	x	x				x	
FL, Orlando	x	x	x				
FL, Tampa/ St.Petersburg	x	x	x				
GA, Atlanta	x	x					
IA, Des Moines	x	x			x		
ID, Boise	x	x	x				
IL, Chicago	x	x	x				
IL, Peoria/Springfield	x				x		
IN, Indianapolis	x	x	x				
KS, Kansas City	x		x		x		
KS, Wichita	x	x			x		
KY, Louisville	x	x	x				
LA, New Orleans/Mobile, AL	x	x		x			
MA, Boston							x
MI, Detroit	x	x	x				
MI, Grand Rapids		x					
Mississippi	x	x		x			
MN, Minneapolis/St Paul	x	x					
MO, St. Louis	x		x				
NC, Charlotte	x	x	x				
NC, Raleigh/Greesboro	x	x	x				
Ne, Omaha	x				x		
New England	x						
NY, Albany	x	x					x
NY, Buffalo/Rochester	x	x					
NY, New York		x	x			x	
NY, Syracuse	x	x					
OH, Cincinnati/Dayton	x	x					
OH, Cleveland	x	x	x				
OH, Columbus	x	x	x				
OH, Toledo	x	x	x				
OK, Oklahoma city	x	x		x			
OK, Tulsa	x			x	x		
OR, Portland	x		x				
PA, Harrisburg/Scranton	x	x					
PA, Pittsburgh	x	x				x	
PA,Philadelphia		x	x				
RI, Providence		x					x
SC, South Carolina	x	x					
TN, Knoxville	x	x					
TN, Memhis	x			x			
TN,Nashville	x	x					
TX, Dallas/FT. Worth	x		x	x			
TX, Houston	x			x			
TX, San Ant/Corpus Chr	x			x			
TX, West Tex / New Mexico	x			x	x		
UT, Salt Lake City	x		x				
VA, Roanoke	x	x					
VA, Richmond/Norfolk	x	x	x				
WA, Baltimore	x	x	x				
WA, Seattle/Tacoma	x	x	x				
WA, Spokane	x		x		x		
WI, Green Bay	x				x		
WI, Milwaukee			x		x		
Total Markets	57	45	31	12	10	4	4

Note: To conserve on space, only brands present in 4 or more markets are listed.