THE COMPETITIVE EFFECT OF MULTIMARKET CONTACT

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AbstractChanges in the extent of multi-market contact (MMC) between firms often affect market outcomes – quantities and prices. This paper challenges the standard economic interpretation of this phenomena as an indication of tacit collusion. We show that a strategic but purely competitive effect of changes in MMC can change the quantity provided in a market by a firm by as much as 50%. Moreover, changes in demand for a firm one market may affect equilibrium quantities in markets where the firm is not active. The model is supported by novel empirical evidence from the domestic US airline industry.

1. INTRODUCTION

Large firms are often active in more than one market and commonly compete with each other in many, but not necessarily all, markets. For example, United Airlines and American Airlines compete on some but not all of their routes. There is a large empirical literature on the effect of changes in the extent of multi-market contact (MMC) between firms on market outcomes.[1] This relationship has been mostly interpreted as empirical evidence that MMC facilitates tacit collusion. This paper argues that MMC may also trigger a strategic but purely competitive effect. This effect is significant. Specifically, changes in the extent of MMC between firms serving a market may change the quantity provided by each firm in the market by as much as 50%. The effect is caused by simple best-response analysis, and does not rely on mutual forbearance or forward looking anti-competitive behavior by the firms. Thus, the paper implies that changes in market outcomes associated with changes in MMC should not be necessarily interpreted as evidence for (explicit or implicit) collusion.

We study a model with two firms serving two types of markets: (1) *overlapping markets* - markets

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[1]See e.g. Evans and Kessides (1994); Gimeno and Woo (1999); Parker and Roller (1997); Jans and Rosenbaum (1997); Fernandez and Marin (1998); Pilloff (1999)

served by both firms; and (2) *private markets* - markets served by one of the firms but not its rival.[2] Each firm first chooses a level of capacity (or any other common resource such as capital or shelf space). Capacities are then allocated across the different markets served by the firms. This setup creates joint diseconomies across markets in the sense that an increase in the utilization of the resource in one market requires a decrease in utilization in other markets. We examine the effect of changes in market structure, e.g. an increase in the number of overlapping markets relative to the number of private markets, on firms' behavior in the overlapping markets as well as in the private markets.

We complement the theoretic analysis with empirical evidence using data from the domestic US airline industry. We find that market shares of domestic US carriers react to changes in MMC as predicted in Lemma 5, and that the effect is economically significant. In particular, we find that a one standard deviation increase in a firm's demand from "private markets" increases its rival's market share in each overlapping market by 1.1 percentage points. Summing over all markets in which the firms overlap adds up to a large and economically significant efect. This finding supports the model and is not predicted by the traditional anti-competitive models of MMC.[3]

The basic model, presented in section 2, can be thought of as an extension of the analysis in Bulow, Geanakoplos, and Klemperer (1985) (BGK). There, one firm has a private market, and both firms make first stage decisions that affect second stage competition. In our model *both* firms may serve private markets. We find that allowing both firms to serve private markets has important implications for firms' behavior. In particular, while in BGK the comparative statics are all monotonic, in our settings some firms' behavior and market outcomes may follow a non-monotonic pattern. Most crucially, quantities in a market served by both firms may increase or decrease when the number of overlapping markets increases (see proposition 2).

The non-monotonic effect of MMC on market outcomes is a product of two related but sometimes contradicting forces we call *flexibility* and *commitment power.* The first step in understanding these forces is the realization that a firm that operates in multiple markets must equalize its marginal revenue across all its markets. Now consider, for example, that a rival chooses to deviate and increase quantity

[2]The qualitative results apply also when the private markets are served by other firms, but the analysis gets technically very tedious.

[3]The empirical analysis in this paper cannot be used to distinguish between the competitive and anti-competitive effects of MMC in the airline industry. We tackle this in follow-up work, currently in progress.

in any overlapping market. This reduces the firm's marginal revenue in that market. If the firm can reallocate supply in the second stage from low marginal revenue markets to higher marginal revenue markets, it will do so in response to the rival's deviation. Naturally, the more private markets a firm serves, the more *flexible* it is in reallocating its capacity. Rivals may take advantage of this *flexibility* and increase their supply to the overlapping market.

Increasing supply to overlapping markets implies a reduction in marginal revenue in these markets. Since firms equalize marginal revenue across private and overlapping markets, taking advantage of a rival's flexibility necessarily means that the firm also reduces marginal revenue in its own private markets. This implies that changes in a firm's private market (i.e., the firm's flexibility) will also affect prices and quantities in markets *served only by the firm's rivals*. As a firm serves more private markets and becomes more flexible, its rivals reduce marginal revenue in all the markets they serve, including markets not served by the firm.

If a firm wishes to take advantage of its rival's flexibility and increase capacity in the overlapping market, it must first set a higher first stage capacity. Indeed, when setting up its capacity, the firm may aim to allocate most of its capacity to the overlapping markets. However, depending on the rival's capacity in the overlapping market, profit maximization may dictate a lower than "wished for" capacity in the overlapping markets, and thus higher than "wished for" capacity in its private market.We define the extent to which the firm can commit to allocating any excess first stage capacity to the overlapping market as the firm's *commitment power*. Proposition 1 shows that the *flexibility* and *commitment power* together identify the competitive effect of MMC for each firm. Proposition 1 also shows that in any case, the competitive effect of MMC increases supply relative to the benchmark case of isolated markets. Interestingly, our results show that the overall level of competition may either increase or decrease, depending on the number of overlapping markets relative to the number of private markets. In particular, proposition 2 provides the market conditions under which an increase in MMC softens competition in the market. The proposition shows that if the firms serve a large number of private markets relative to the number of overlapping markets, an increase in MMC intensifies competition. However, if firms have more overlapping and less private demand, a further increase in overlapping demand makes competition less aggressive.

That firms' behavior is determined by the relative demand coming from the overlapping and non-overlapping markets is especially interesting when considering mergers that have no effect on productivity or on the actual number of firms in any market (for example, CVS's merger with People's Drug in 1990). Such mergers change the relative level of multimarket contact and would thus affect welfare in markets served by either chains.

An important extension of our model is when multiple resources are needed for production, as is often the case in real applications. A classic example of multiple resources across multiple markets is the allocation of labor and capital by multi-product firms. Another interesting example is seats allocation in the airline industry. For example, when flying a passenger from NYC to LA through Chicago, an airline carrier needs to employ an aircraft between NYC and Chicago and another aircraft between Chicago and LA in order to serve the route NYC-LA.[4] Seats on both flights can be, and are, sold as part of other markets (e.g., from NYC to Tuscon through Chicago and from LA to Boston through Chicago). Each carrier must first choose capacity levels for the NYC-Chicago and Chicago-LA flights. Then in the second stage, the carrier chooses how many seats to allocate to the NYC-LA, NYC-Chicago and Chicago-LA routes. Interestingly, carriers may overlap on the route NYC-LA and the route NYC-Chicago but not on the route Chicago-LA. Section 3 studies this extension and finds that while the effect in the overlapping markets is the same as before, the effect in a well defined subset of the private markets is reversed.

Traditionally, the literature has been concerned eith the notion that an increase in MMC facilitates anti-competitive behavior by firms. The possibility that MMC affects tacit cooperation was formalized and studied in detail by Bernheim and Whinston (1990).[5] The concern has been the subject of many empirical studies. For example, considering the US domestic airlines industry, Evans and Kessides (1994), Gimeno and Woo (1999) and more recently Ciliberto and Williams (2012) find that an increase in MMC between carriers reduces prices. This is considered by the authors as evidence of the mutual forbearance (anti-competitive) effect of MMC. Similar studies have been conducted in different industries. See e.g. Parker and Roller (1997), Jans and Rosenbaum (1997), Fernandez and Marin (1998) and Pilloff (1999). The implications of the competitive effect of MMC are closely related to the collusive effect of MMC. As large firms continue to grow, an increase in MMC decreases the firms' commitment power, increasing

[4]Investment in the context of labor and capital and multiple necessary resources has been widely studied. See, e.g. Wildasin (1984), Dixit (1997), and Eberly and Mieghem (1997).

[5]Bernheim and Whinston (1990) credit Edwards (1955) with the introduction of this idea to the academic economic discourse.

prices and reducing quantities in the overlapping markets. In this case, the competitive effect of MMC is difficult to distinguish from possible mutual forbearance, or a collusive effect. Critically, however, while the collusive effect suggests that the markets move from the standard competitive benchmark to a more collusive state, the competitive effect suggests that the markets move from a state of increased aggressiveness to the standard competitive benchmark.

Understanding the competitive MMC effect is therefore crucial to correctly evaluating both the efficiency advantages of larger firms entering (or merging into) new markets and any suggested evidence for anti-competitive actions. Section 5 illustrates the predictions of the competitive effect of MMC in various contexts – firms growth, mergers and international competition.

2. A COMPETITIVE MODEL OF MULTIMARKET CONTACT

2.1. *Model*

Consider an industry with two firms, identified by $j \in \{A, B\}$ and M identical markets.[6] Letting Q denote the total quantity supplied in a market, a market's inverse demand function is linear:

$$p(Q) = a - b \cdot Q$$

Of the M markets, m_o are *overlapping* markets – served by both firm A and firm B. The remaining markets are *private* –markets served by one of the firms, but not the other.[7] The number of private markets served by firm j is denoted m_j; so that $m_o + m_A + m_B = M$. If $m_A = m_B = m_o$, each firm serves exactly the same number of private and overlapping markets . When $m_j > m_o$ (resp. $m_j < m_o$), firm j serves more (resp. less) private markets than overlapping markets. The extreme case where the firms have no (resp. only) overlapping markets can be captured by setting $m_o = 0$ (or resp. $m_j = 0$ for both firms).

[6]The results are qualitatively the same if more firms are considered, but various technical complications arise. The discussion on mergers in section 5 provides an example.

[7]We will refer to the markets served by only one carrier as monopolistic markets. However, the analysis makes no assumption regarding the degree of competition in these markets relative to the degree of competition in the markets served by both carriers. Specifically, we do not rule out that each carrier faces competition from other carriers in the "monopolistic" markets. The only distinctive property of the monopolistic markets is that the "monopolist"'s rival does not internalize the effect of its actions on any change in profitability in these markets. Finally, adding markets that are not served by either firm has no effect.

The model has two stages. First, each firm j sets up capacity k_j at a linear cost of c per unit.[8] Second, after both capacities are fixed and known, each firm chooses quantity for each market, and markets clear. The quantity each firm sets in the second stage is limited by its first stage capacity. That is, the total quantity sold by firm j in *all* its markets cannot exceed k_j. A simple interpretation of the model is that firms first choose how many units of a good to produce, and then allocate the goods across various markets. Another, more nuanced, interpretation is similar to Olley and Pakes (1996) – firms first choose capital (k_j) that can be used in various markets, and then allocate labor across markets, which determines the quantity supplied in each market.

For simplicity, we assume that all costs are captured in the first stage capacity cost $k_j \cdot c$. To rule out pathological yet trivial cases, we assume that $a > c$.

An *equilibrium* $\{k_A, k_B, q_A, q_B\}$ is a pair of scalars (k_A, k_B) indicating the capacity set by each firm, and a pair of vectors (q_A, q_B) indicating the quantity sold by each firm in each market. We are interested in the effect of changes in the number of markets each firm serves–i.e., m_o and m_j–on outcomes. For example, an increase in m_o indicates an increase in multi-market contact between the firms.

Since markets are identical and the revenues in each market are concave in market quantity, in any equilibrium, each firm sets identical quantities q_j in all of the joint markets, and identical quantities $\hat{q}_j$ in all of its private markets. Thus, given our assumption of linear demand, there is no loss in analyzing all the joint markets as a single market of size m_o and all of firm j's private markets as a single private market of size m_j.

The analysis accordingly proceeds by considering a three-markets model – two private markets of sizes m_A and m_B and a single overlapping market of size m_o. Letting Q denote the total quantity in a market, the inverse demand functions are as follows:[9]

$$\begin{aligned} p_o(Q) &= a - \frac{b}{m_o} \cdot Q \\ p_{j\in\{A,B\}}(Q) &= a - \frac{b}{m_j} \cdot Q \end{aligned}$$

In this setting, equilibrium is defined by three values for each firm: firm j's capacity–k_j; quantity set

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[8] The results are qualitatively unaffected with any weakly convex cost function and cost asymmetries between the firms.

[9] Inverting a linear demand function $D(p) = \alpha - \beta p$ obtains the single market inverse demand function: $p(Q) = \frac{\alpha}{\beta} - \frac{1}{\beta} Q$. Thus, $a = \frac{\alpha}{\beta}$ and $b = \frac{1}{\beta}$

Inverting the demand function for m markets $D(p) = m \cdot \alpha - m \cdot \beta p$ obtains the multiple markets inverse demand function: $p(Q) = \frac{\alpha}{\beta} - \frac{1}{\beta m} Q$.

in the overlapping market–q_j; and quantity set in the private market–$\hat{q}_j$. Translating these back to the basic multiple markets model, $\frac{q_j}{m_o}$ denotes the quantity that j would set *in each* overlapping market, while $\frac{\hat{q}_j}{m_j}$ denotes the quantity that j would be set in each of its private markets. The total quantity supplied by firm j is k_j.

2.2. *The Cournot Benchmark*

To evaluate the effect of MMC, a benchmark must be set. The natural benchmark in our case is the standard Cournot equilibrium in which each firm sets marginal revenue in each market at marginal cost. We let an asterisk superscript denote the benchmark quantities. In the overlapping market, each firm's benchmark quantity is $q_j^* = m_o \frac{a-c}{3b}$. In the private market, each firm's benchmark quantity is $\hat{q}_j^* = m_j \frac{a-c}{2b}$. The overall firm benchmark capacity is $k_j^* = q_j^* + \hat{q}_j^*$ and the marginal revenue for each firm in each market is exactly the marginal cost, c. The benchmark results are obtained when assuming that each firm must set market level capacities in the first stage, or that firms only pay for quantity in the second stage.

2.3. *Equilibrium in the Three Markets Model*

This subsection identifies the economic forces that shape the solution. In particular, we isolate the deviation from the standard "marginal revenue equals marginal cost" result.

The results in this subsection are independent of the linear demand structure. Instead, it is sufficient that the inverse demand function in each of the three markets is strictly decreasing, twice continuously differentiable and concave whenever price is strictly positive. In addition we require that in all markets, price at zero supply is identical and strictly larger than marginal cost. This is a simple assumption to rule out pathological and trivial cases. The complete algebraic solution using the linear demand assumption is provided in subsection 2.4

It is intuitive, and proved formally in the appendix, that in equilibrium, firms equalize marginal revenue (MR) across their private and overlapping market, and set it to be at or below marginal cost (MC). If A's MR in higher in one market than in the other, A can increase its profits by shifting quantity from the low MR market to the high MR market. In addition, if A's MR in both of its markets is higher than MC, a small increase in first stage capacities should increase profits.

As MR in the benchmark case exactly equals MC, a natural way to measure the competitive effect of MMC is the difference between MR and MC:

Definition 1 Firm j's competitive effect of MMC, α_j, is defined by

$$\alpha_j \equiv c - MR_j$$

Where MR_j is firm j's marginal revenue in equilibrium.

If MMC has no effect on firm j's strategy then the competitive effect of MMC is zero, $\alpha_j = 0$. If MMC causes firm j to be more aggressive than the Cournot benchmark, $\alpha_j > 0$. In this case, for any quantity set by its rival, firm j's best response sets more quantity than specified in the Cournot benchmark. The opposite holds for $\alpha_j < 0$. Note that it is sufficient for one firm to have a non-zero MMC effect for the quantities supplied by both firms in the overlapping market to deviate from the Cournot benchmark.

The analysis proceeds using backward induction. Let $(q_{-j}, \hat{q}_{-j})$ denote firm j's rival's second stage quantities in each market. Firm j chooses the optimal quantities subject to its available capacity:

$$\max_{(q_j,\hat{q}_j)\geq 0} \; p_0\left(q_j + q_{-j}\right) q_j + p_j\left(\hat{q}_j\right) \hat{q}_j \tag{2.1}$$

$$\text{subject to} \qquad q_j + \hat{q}_j \leq k_j \tag{2.2}$$

For every (k_A, k_B), a second stage equilibrium is a set $\langle q_A, \hat{q}_A, q_B, \hat{q}_B \rangle$ such that $(q_j, \hat{q}_j)$ solves problem 2.1 given $(q_{-j}, \hat{q}_{-j})$. We prove in the appendix that there exists a unique second stage equilibrium for each pair of first stage capacities.

Consider any second stage equilibrium and suppose that firm B slightly deviates by increasing its quantity in the overlapping market, q_B. This reduces A's MR in the overlapping market. A's MR in its private market is not affected by B's deviation and is thus higher than A's MR in the overlapping market. Consequently, A can increase its profits by shifting some of its overlapping market quantity (q_A) to its private market $(\hat{q}_A)$. The extent to which A will react to B's deviation should depend on the equilibrium demand elasticities in each market. We call this measure *flexibility*, and formally define it as follows:

Definition 2 Firm j's *flexibility* measure, ϕ_j, is the firm's second stage reaction to a change in its competitor's quantity:

$$\phi_j \equiv \frac{\partial q_j}{\partial q_{-j}} \quad . \tag{2.3}$$

As an increase in the rival's quantity in the market reduces marginal revenue, A's reaction to an increased quantity by its rival will always be accommodating and thus $\phi_j \leq 0$. The two extreme cases of firm flexibility provide intuition. First, suppose A cannot reallocate any of the overlapping market capacity to its private markets. This would be the case if A has no or a very small private market. A can only react to B's deviation by removing quantity from the overlapping market. As the production costs were already paid for and marginal revenue is positive, this is a losing proposition. In this case, A is completely inflexible (cannot respond to B's deviation) and ϕ_A is zero.

In contrast, A's private market could be large enough to accommodate additional capacity. In this case, A will react to B's deviation by removing quantity from the overlapping market to A's private market. Thus, ϕ_A will be negative. A will reallocate exactly enough quantity so that, again, A's MR in the overlapping market equals A's MR in its private market. In the extreme case where A's MR in its private market is not affected by the reallocation, A's reaction will be identical to the Cournot reaction with the marginal cost replaced by the private market's MR.[10]

Let $\vartheta_j(k_A, k_B)$ denote the second stage equilibrium outcomes for firm j given the first stage capacity choices k_A and k_B. Suppose that firm A would like to deviate from a the first stage equilibrium (k_A, k_B) and increase its capacity, k_A. Suppose further that A would like to place the additional quantity *only in the overlapping market*. The second stage solution implies that this is never possible in equilibrium. Just like in the standard Cournot game, a marginal increase to q_A reduces A's marginal revenue in the overlapping market, even if firm B is completely flexible. As A optimally equalizes marginal revenues across its markets, in equilibrium at least some of the increase in k_A is allocated to A's private market. That is, a firm with a private market cannot commit to utilizing all of its increase in capacity only in the joint market. Some excess capacity will endogenously spill over to the private market. This leads to

[10]The reason is as follows: Since any quantity not allocated to the overlapping market is allocated to the private market,A's MR in its private market is essentially the true second stage "marginal cost" of allocating quantity to the overlapping market. As the private market's MR in equilibrium is at most the true marginal cost of capacity, c, it follows that A's flexibility cannot be lower than the Cournot equilibrium reaction. Accordingly, we can say that a firm is completely flexible if its response to a rival's deviation would be identical to the Cournot response.

our definition of commitment power:

Definition 3 The *commitment power* of firm j, σ_j, is

$$\sigma_j \equiv \frac{\partial \vartheta_j(\cdot)}{\partial k_j} .$$

σ_j measures the change in j's second stage equilibrium quantity in the overlapping market resulting from a marginal change in j's first stage capacity. If the overlapping market is much more elastic than j's private market, any change in j's capacity will be optimally absorbed in the private market, and thus $\sigma_j \to 0$. If j's private market is very elastic compared to the overlapping market, most of the change in j's capacity will be absorbed in the overlapping market, and $\sigma_j \to 1$. By optimality of any equilibrium, $0 \leq \sigma_j \leq 1$. The commitment power therefore measures the firm's ability to increase its overlapping quantity without increasing hurting its profitability in its private market. If a firm's rival is flexible, increasing the overlapping quantity above the Cournot benchmark level should be profitable for the firm. However, only a fraction σ_j of any increase in first stage capacity will be used in the overlapping market; profit maximization dictates that the rest will be allocated to the private market. This "undesired" increase in the private market's quantity reduces the firm's equilibrium profits. Therefore, all else being equal, a firm will set a higher capacity when its commitment power σ_j is higher.

Having defined the two economic forces at play – *flexibility* and *commitment power*, we characterize the equilibrium in terms of the firms' competitive effect of MMC, α, for a general demand function. The specific solution for linear demand is provided in the next subsection.

Proposition 1 *The game has a unique equilibrium, in which*

1. *For each firm* $q_j + \hat{q}_j = k_j$ *(full capacity utilization)*
2. $\min[q_j, \hat{q}_j] > 0$ *(each firm serves all possible markets), and*
3. *The competitive effect of MMC is weakly positive* ($\alpha_j \geq 0$) *and satisfies*

$$\alpha_j = p_o^{'}(q_A + q_B) \cdot q_j \cdot \phi_{-j} \cdot \sigma_j \tag{2.4}$$

Proof: All but the last statements are rather standard, and are proved in the appendix. The last statement is of interest and the proof also highlights the economic intuition of the model, and so is presented here. To simplify notation we solve for firm A and write $\vartheta_A(k_A, k_B)$ and $\vartheta_B(k_A, k_B)$ without the arguments (i.e. ϑ_A and ϑ_B). In the first stage, firm A's problem can be stated as

$$\max_{k_A} \quad p_o\left(\vartheta_A+\vartheta_B\right)\vartheta_A + p_A\left(k_A-\vartheta_A\right)\cdot\left(k_A-\vartheta_A\right) - c\cdot k_A$$

The first order condition is

$$\begin{aligned} & p_o^{'}\left(\vartheta_A+\vartheta_B\right)\vartheta_A\cdot\left(\tfrac{\partial\vartheta_A}{\partial k_A}+\tfrac{\partial\vartheta_B}{\partial k_A}\right)+p_o\left(\vartheta_A+\vartheta_B\right)\tfrac{\partial\vartheta_A}{\partial k_A} \\ & +p_A^{'}\left(k_A-\vartheta_A\right)\left(1-\tfrac{\partial\vartheta_A}{\partial k_A}\right)\left(k_A-\vartheta_A\right)+p_A\left(k_A-\vartheta_A\right)\left(1-\tfrac{\partial\vartheta_A}{\partial k_A}\right)=c \end{aligned}$$

Collecting terms, the first order condition is re-written as

$$\begin{aligned} & \tfrac{\partial\vartheta_A}{\partial k_A}\cdot\left(p_o^{'}\left(\vartheta_A+\vartheta_B\right)\vartheta_A+p_o\left(\vartheta_A+\vartheta_B\right)\right) \\ + & \left(1-\tfrac{\partial\vartheta_A}{\partial k_A}\right)\left(p_A^{'}\left(k_A-\vartheta_A\right)\left(k_A-\vartheta_A\right)+p_A\left(k_A-\vartheta_A\right)\right) \\ & +\tfrac{\partial\vartheta_B}{\partial k_A}p_o^{'}\left(\vartheta_A+\vartheta_B\right)\vartheta_A=c \end{aligned}$$

The terms in parenthesis in the first and second lines are, respectively, A's marginal revenue in the overlapping and private market. By lemma 8, the two are identical in equilibrium. Letting MR_A denote A's equilibrium marginal revenue, and replacing ϑ_j with the equilibrium values q_j yields

$$MR_A + p_o^{'}\left(q_A+q_B\right)\cdot q_A\frac{\partial\vartheta_B}{\partial k_A} = c\ .$$

Applying the envelope theorem to equation (2.1), in equilibrium

$$\frac{\partial\vartheta_B}{\partial k_A} = \frac{\partial q_B}{\partial q_A}\cdot\frac{\partial\vartheta_A}{\partial k_A}$$

Applying the definitions of ϕ_j and σ_j obtains the required equality.

The result that $\alpha_j \geq 0$ follows from $p\left(\cdot\right)$ weakly decreasing, $\phi_j \leq 0$ and $\sigma_j \geq 0$. *Q.E.D.*

Equation 2.4 shows that the competitive effect of MMC, α, is a result of changes in the profitability of all the infra-marginal consumers. The first term $p^{'}\left(\cdot\right)q_j$ is exactly the change in revenue from all the infra-marginal consumers in the overlapping market. This is multiplied by the *rival*'s flexibility ϕ_{-j} and the firm's own commitment power, σ_j . In other words, the firm increases its supply (decreases marginal revenue) to the extent that it can commit to place more supply in the overlapping market, and can expect its rival to accommodate the increase.

The result that $\alpha_j \geq 0$ implies that MMC results in overall market quantities that are at least as high as in the benchmark case with no MMC. MMC makes firm be weakly *more* aggressive – set a marginal revenue that is weakly *lower* than their marginal cost. That is, despite the fact that a private market makes each firm softer in the overlapping market, in equilibrium rivals take advantage of this soft behavior such that total quantity is higher than in the Cournot benchmark.

COROLLARY 1 *Each firm's capacity, the total quantity in the overlapping market and the quantity in the private markets are all at least as high with MMC as without any MMC.*

The extent to which total quantity is higher than in the Cournot benchmark depends on the degree of overlap relative to the size of the private market. Suppose, for example, that as in Bulow, Geanakoplos, and Klemperer (1985), firm A only serves markets that are also served by firm B, so that firm A has no private markets; $m_A = 0$. This implies that firm A can only allocate its capacity to the overlapping market and hence $\sigma_A = 1$ and $\phi_A = 0$.[11] Since A has no markets to divert its capacity to, firm B cannot gain by setting an aggressive first stage capacity. As a result, B sets its marginal revenue to marginal cost. In contrast, A's aggressiveness is affected only by B's private demand (m_B). As ϕ_B grows in absolute terms, A becomes more aggressive. In the extreme case where B is completely flexible ($m_B \to \infty$), the equilibrium becomes the Stackelberg equilibrium, with B as the Stackelberg follower.

COROLLARY 2 *If A is nested within B ($m_A = 0$), then $MR_B = c$ and $MR_A = c - P_o^{\prime}(q_A + q_B) \cdot q_A \cdot \phi_{-j}$*

In the next subsection we assume linear demand in order to derive some additional results on the effect of the degree of overlap relative to the size of the private market on quantities in the all markets.

2.4. *Multi Market Contact with Linear Demand*

This subsection uses the linear demand structure to further study the relationship between MMC and market outcomes. To isolate the competitive effect of MMC, the market quantities are shown as deviations from the benchmark quantities (q^* and $\hat{q}^*$) defined above. The equilibrium outcome is given in the next Lemma:

[11] Note that firm B's commitment power and flexibility (σ_B and ϕ_B) are not limited by the assumption that $m_A = 0$.

LEMMA 1 *In the linear model*

$$\phi_j = -\frac{1}{2}\frac{m_j}{m_j + m_o} \text{ and } \sigma_j = \frac{m_o + m_{-j}}{m_o + m_{-j} + m_j + \frac{3}{4}\frac{m_j m_{-j}}{m_o}}$$

and equilibrium quantities and capacities are functions only of the market sizes and the Cournot quantities absent any MMC:

$$\frac{q_A}{q_A^*} = \frac{3\kappa_A(\kappa_A - 2m_o m_A)}{\xi_A} \tag{2.5}$$

$$\frac{\hat{q}_A}{\hat{q}_A^*} = \frac{\kappa_A \rho_A}{\xi_A} \tag{2.6}$$

$$k_A = q_A + \hat{q}_A \tag{2.7}$$

With

$$\begin{aligned} \kappa_A &= 4m_o^2 + 4m_o m_B + 4m_o m_A + 3m_A m_B \\ \xi_A &= 16m_o^2\left(3m_o^2 + 5m_o m_B + 2m_B^2\right) + 20m_o m_A\left(4m_o^2 + 7m_o m_B + 3m_B^2\right) \\ &\quad + m_A^2\left(32m^2 + 60m_o m_B + 27m_B^2\right) \\ \rho_A &= 12m_o^2 + 10m_o m_B + 8m_o m_A + 9m_A m_B \end{aligned}$$

A closed form solution for $q_j, \hat{q}_j$ and k_j is provided in the appendix.

PROOF: We use the Mathematica symbolic algebra solver to obtain the result. See attached annotated notebook printout. The notebook also proves proposition 1 for the linear case. *Q.E.D.*

The lemma shows that when demand is linear, the deviation from the Cournot benchmark depends *only* on the relative demands m_j and m_o. In particular, the deviations are independent from the specific demand intercept and slope as well as from the specific (constant) marginal cost.

Observe that as $m_j \to \infty$, $\phi_j \to -\frac{1}{2}$, and $\sigma_j \to 0$. The effect on ϕ_j implies that as $m_j \to \infty$, j's second stage response is similar to the standard Cournot response. The effect on σ_j implies that as $m_j \to \infty$, j cannot take any advantage of its rival's flexibility and sets MR=MC. Thus, in this case, j acts in the second stage as if it was the first stage. If j's rival's private market is not very large (m_{-j} is small), the outcome in the overlapping market should be very close to the Stackelberg outcome with j as the Stackelberg follower.In the Cournot benchmark ($m_A = m_B = 0$), all functions defined in Lemma

1 are the identity.

To evaluate the results in more detail, in the next subsections we first consider the symmetric case ($m_A = m_B$) and then provide comparative statics for the case where firms' private demand are asymmetric ($m_A \neq m_B$).

2.4.1. *Symmetry and Multi-Market Contact*

Let M denote a measure of demand, and suppose that $m_o = \lambda \cdot M$, and that the remaining markets are split evenly between the two firms: $m_A = m_B = M \cdot \frac{1-\lambda}{2}$. It is natural to interpret λ as the extent of multi-market contact (MMC). Lemma 2 shows that in the symmetric case the solution depends *only* on the level of MMC. Moreover, the effect on all markets, private and overlapping, is identical:

PROPOSITION 2 *If $m_0 = \lambda M$ and $m_A = m_B = M \cdot \frac{1-\lambda}{2}$, then each firm's market share is $\frac{1}{2}$ and*

$$\frac{q_j}{q_j^*} = \frac{\hat{q}_j}{\hat{q}_j^*} = \frac{3\left(3 + 10\lambda + 3\lambda^2\right)}{9 + 26 \cdot \lambda + 13\lambda^2} \quad ;$$

$$\frac{\partial\left(q_j/q_j^*\right)}{\partial\lambda} \gtreqless 0 \iff \frac{\partial\left(\hat{q}_j/\hat{q}_j^*\right)}{\partial\lambda} \gtreqless 0 \iff \lambda \lesseqgtr \frac{4\sqrt{3}-3}{13} \; .$$

PROOF: The result for market shares is trivial. The functions are obtained by plugging the values for m_0 and m_j in the functions obtained in lemma 1. This is done in the attached Mathematica notebook. *Q.E.D.*

Proposition 2 states that the deviation from the Cournot benchmark is non-monotonic in the degree of MMC, λ. If there is no MMC ($\lambda = 0$), the outcome is of course as in the benchmark and the ratios in the proposition are exactly one. As λ increases from zero (no MMC), the quantities in both the overlapping and private markets increase. The effect reverses at $\lambda \approx 0.3$, where the size of the overlapping market is almost the same as the size of each of the private markets (see figure 5.1). From that point on, an increase in MMC decrease quantities. Ultimately, when all markets are served by both firms ($\lambda = 1$), the outcome is again the same as in the benchmark.

The intuition behind the non-monotonicity is as follows: As the size of the overlapping market increases relative to the size of each firm's private market, the two determinants of firms' aggressiveness–flexibility and commitment power–are affected in opposite directions. An increase in overlapping demand enables firms to allocate more capacity to the overlapping market relative to the private markets, increasing each firm's commitment power (σ_j). In contrast, the relative decrease in the size of the private markets decreases both firms' flexibility (ϕ_j). The decrease in the rival's flexibility reduces the firm's ability to be aggressive in the overlapping market. When the overlapping market is much smaller than each of the private markets, both firms' flexibility is relatively high. In this case, firms flexibility is not affected much by an increase in MMC. Consequently, the commitment power effect outweighs the flexibility effect, and the deviation from the Cournot quantity increases with MMC. In contrast, when the size of the overlapping market is larger than the size of each of the private markets, both firms' ability to divert capacity to their private market is relatively weak–flexibility is low. In this case, the increase in the firms' commitment power as a result of a further increase in MMC is not large enough to offset the negative effect decreased flexibility has on quantities. Consequently, the deviation from the Cournot quantity decreases with MMC, and $\frac{\partial q_j}{\partial \lambda} < 0$.

Interestingly, the effect of MMC on quantities in the private market and on overall capacity is identical to the effect of MMC on quantities in the overlapping market; both in terms of the direction and magnitude. It is easy to understand this result when thinking about the change in firms' marginal revenue as quantity in the overlapping market changes. When both firms increase their quantity in the overlapping market, marginal revenue in this market goes down. Since the firms can still reallocate quantities in the second stage, they will both shift capacity away from the overlapping and onto their private market until marginal revenues in all three markets are equal. Thus, firm j's aggressiveness in the overlapping market is matched with an increase in quantity in its private market–$\hat{q}_j$ goes up. As a result, both firms' overall capacity is higher than in the Cournot outcome by the same magnitude as well.[12]

2.4.2. *Private Demand and Firm Behavior*

An important implication of lemma 1is that changes to a firm's private demand affect *both* firms' behavior. In particular, for A, an increase in m_A reduces its commitment power (σ_A). From B's point of view, an increase in m_A increases its rival's flexibility (ϕ_A) and its own commitment power (σ_B). The

[12]This is proved in the appendix.

effects enhance each other – A accommodates B's increased aggressiveness and changes its quantities in both its markets, and therefore the total effect of a change in a single firm's private demand is always monotonic:

LEMMA 2 *The following decrease in the size of the firm's private market and increase in the size of the rival's private market:*

- *The firm's competitive effect of MMC* (α_j)
- *Firm quantity and market share in the overlapping market* (q_j *and* s_j)
- *Quantity per market in the private market* ($\frac{\hat{q}_j}{m_j}$)

PROOF: The detailed algebraic terms are derived by taking the derivatives of the the terms obtained in the proof of Lemma 1. For the first statement, we show that $\frac{\partial \alpha_j}{\partial m_{-j}} > 0$ and $\frac{\partial \alpha_j}{\partial m_j} \leq 0$, with the inequality strict whenever $m_{-j} > 0$. For the second statement we show that $\frac{\partial q_j}{\partial m_j} < 0$ and $\frac{\partial q_j}{\partial m_{-j}} > 0$. By symmetry, this also implies the result for shares. For the third statement, first normalize the quantity to the market size ($\frac{\hat{q}_j}{m_j}$ rather than simply $\hat{q}_j$) as otherwise the change in market size dominates the effect. The result then follows from $\frac{\partial \frac{\hat{q}_j}{m_j}}{\partial m_j} \leq 0$ and $\frac{\partial \hat{q}_j}{\partial m_{-j}} > 0$, with that the first inequality strict when $m_{-j} > 0$.

All partial derivatives are provided in the attached Mathematica notebook. The notebook also shows that similar comparative statics cannot be obtained with respect to the overlapping market size (m_0). *Q.E.D.*

Lemma 2 states that the quantity in the *private* market deviates from the standard monopoly solution, and is affected by the entire industry structure. In particular, firm A's quantity and profits in markets not served by B are affected by B's demand in markets not served by A $\left(\frac{\partial \hat{q}_A}{\partial m_B} > 0\right)$. This may be surprising at first, but is reminiscent of the result in Bulow, Geanakoplos, and Klemperer (1985). Whenever a firm has some flexibility, its rival will take advantage of that by allocating more than the standard Cournot equilibrium capacity to the overlapping market; reducing marginal revenue in this market. Since firms optimally equalize marginal revenues across markets, the rival will reduce its marginal revenue in the private market as well. It does so by increasing its quantity there. The oversupply in the private market, relative to the monopolistic case, is therefore generated by the same forces that affect the marginal revenue in the overlapping market.

Figure 2.1 illustrates the comparative statics and the extent to which the competitive effect of MMC affects market outcomes. All values are shown based on the functions defined in lemma 1.[13] The top left panel of figure 2.1 shows the change in A's quantities in the overlapping market as a function of A's private demand (m_A) when B has a very small private demand ($m_B = 1$). Since m_B is very small, firm B can almost fully commit to use any first stage capacity in the overlapping market ($\sigma_B \to 1$) and has almost no second stage flexibility ($\phi_B \to 0$). As m_A increases, A becomes more flexible and is forced to accommodate B's aggressiveness. Taking advantage of this, B increases its quantity in the overlapping market (q_B). A's accommodation is reflected in the reduction of its quantity in the joint market (q_A). As m_A becomes very large, q_B and q_A converge, respectively, to the Stackelberg leader and follower quantities.

As noted above, the increase in B's quantity in the overlapping market reduces its marginal revenue in that market. While B's private market in this case is relatively small, it can still reallocate some quantity in the second stage across its private and overlapping market to the point where marginal revenues in both markets are equal. Thus, B's aggressiveness in the overlapping market is accompanied by an increase in quantity in its private market, $\hat{q}_B$. This is shown in the bottom left panel of figure 2.1. Finally, because B has almost no flexibility, firm A cannot gain from setting an aggressive capacity – firm B cannot accommodate firm A. As a result, A is as aggressive as in the standalone Cournot case ($\alpha_A \to 0$) and sets capacities so that its equilibrium marginal revenue in all markets is always c. Therefore, $\hat{q}_A$ in the bottom left panel is almost fixed at one.

The middle and right panels of figure 2.1 present the more complicated case in which both firms have meaningful private demand. When A's private demand is very small, the intuitions from the left panel still apply, with A taking the place of B as the aggressive firm. As A's private demand grows, firm B becomes more aggressive – q_B increases. However, in contrast to the left panel, as B has some private demand, A can be aggressive as well and q_A is, at least initially, larger than in the Cournot quantity. The differences between the middle and right panel are illustrative. If B's private demand is about as large as the overlapping market, it has significant commitment power. This allows B to take advantage of A's flexibility as A's private demand increases. As a result, quantities in the overlapping market are always significantly higher than in the Cournot benchmark.

[13]That is, as a multiplier on the standard Cournot solution.

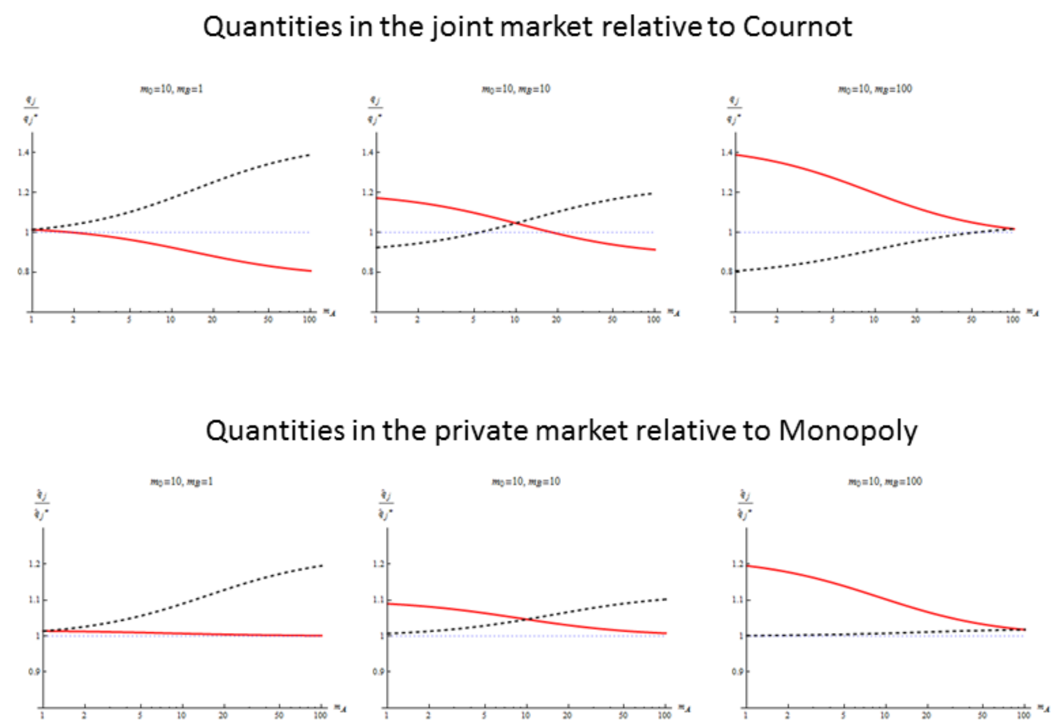

FIGURE 2.1.— Equilibrium quantities and shares. The top panel shows the quantities in the joint market for each firm as multiples of the Cournot quantity $\frac{a-c}{3b}$. The bottom panel shows quantities for each firm in its private markets as multiples of the monopoly quantity $\frac{a-c}{2b}$. Firm A's quantities are the thick solid line. Firm B quantities are the dashed line. The x-axis in all plots is firm A's private demand, m_A, using a logarithmic scale. The left plots ($m_B = 1$,$m_o = 10$) are for the case that firm B has very small private demand. The middle plots are for the case that firm B's private demand is about the same as the overlapping demand. The right plots are for the case that firm B's private demand is very large compared to the overlapping demand. The figures show the extent to which an increase in A's private demand size reduces A's quantities in all markets it serves, and increases B's quantities.

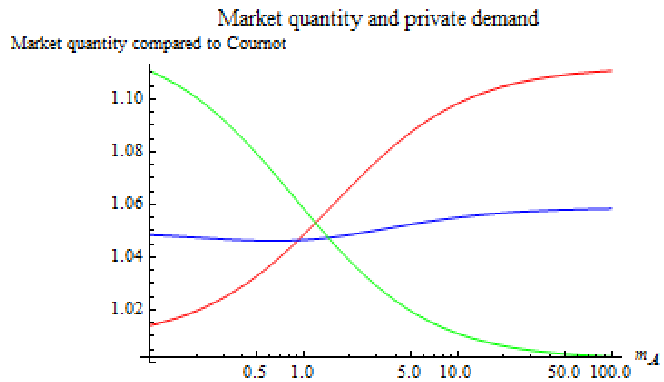

FIGURE 2.2.— The total quantity in the overlapping market relative to the Cournot benchmark $\left(\frac{q_A+q_B}{\frac{2}{3}\frac{a-c}{b}}\right)$, as a function of firm A's private demand (x-axis in log scale). The line starting closest to the origin corresponds to the case where B has no private demand ($m_B = 0$). The middle line corresponds to the case where B's private demand is the same as the overlapping demand ($m_B = 1$). The line starting at the top corresponds to the where B has significant private demand ($m_B = 10$).

The analysis above demonstrates that as a firm's private demand increases, its quantity in the overlapping market decreases, while its rival's quantity in the overlapping market increases. Moreover, the effect of changes in m_A on quantity in the overlapping market depends on the size of B's private demand. This, then, raises the question: what is the overall effect of changes in m_j on total quantity in the overlapping market? Figure 2.2 plots the total quantity in the overlapping market, relative to the Cournot outcome, as a function of firm A's private demand for $m_B = 0$, 1 and 100.

PROPOSITION 3 *Given m_{-j}, total quantity and welfare in the overlapping market:*[14]

- *Increases in m_j for any $m_j \geq 0$ iff $m_{-j} < .7$*
- *Is non-monotonic in m_j iff $m_{-j} \in (0.7, 1.9)$*
- *Decreases in m_j for any $m_j \geq 0$ iff $m_{-j} > 1.9$*

PROOF: See appendix B. *Q.E.D.*

When B's private market is very small, an increase in A's private demand allows B to be more aggressive in the overlapping market. The increase in B's quantity dominates A's soft response, and total quantity

[14] The result is provided for the normalization $m_o = 1$.

in the overlapping market increases. As the proposition indicates, this is the case when B's size is no bigger than 70% of the overlapping market size.

In contrast, if m_B is large, A is the more aggressive firm. In this case, as A's private demand increases, its commitment power (σ_A) decreases, and A becomes less aggressive. As figure 2.2 shows, in this case the decrease in A's quantity dominates and total quantity in the overlapping market decreases with A's private demand. In particular, as m_A increases from zero to 100, total quantity in the overlapping market decreases by 12%; essentially reflecting a move from the standard Stackelberg outcome, with A as the Stackelberg leader, to the benchmark Cournot outcome. As shown in the proposition, this is the case when B's private market size is more than twice the size of the overlapping market size

Finally, when the sizes of the overlapping market and B's private market are similar ($m_B = 1$), the effect of A's private demand on overall quantity is small and non-monotonic. When m_A is very small, the main effect of an increase in m_A is similar to the case where m_B is large and total market quantity decreases. When m_A is large, the effect of an increase in m_A is similar to the case that m_B is small and total market quantity increases. Note that since total quantity is always lower than efficient,the change in total quantity also reflects a change in welfare.

2.5. *Total Welfare*

The previous results show that quantities and welfare in market l may be significantly affected by changes in the size of other markets, even if those are served by firms that do not serve market l (e.g., $\hat{q}_A$ is affected by m_B). Clearly, if some private markets become overlapping, the increase in competition should dominate all other effects and thus improve welfare. It is less clear, however, how different distributions of private demand affect welfare. That is, setting $m_A + m_B = \overline{m}$, is there a distribution of private demand that maximizes welfare?

Proposition 1 together with Lemma 2 show that a firm's aggressiveness increases with its rival's private market size and decreases in its own market size. Symmetry should, therefore, be expected to maximize surplus, which is indeed proved in the following proposition:

PROPOSITION 4 *(a) For any $m_A + m_B + m_o = M$, total welfare increases with m_o .*

(b) Fix m_0 and set $m_A + m_B = \overline{m}$. Total welfare over all markets is maximized when the firms are symmetric: $m_A = m_B$. Total welfare increases in m_A iff $m_A < m_B$.

PROOF: See appendix B. *Q.E.D.*

3. EXTENSION – MULTI-RESOURCE FIRMS WITH APPLICATION TO THE AIRLINE INDUSTRY

In the baseline model presented in the previous section, each firm chose one capacity level. However, firms often make more than one capacity choice, and the choices complement and substitute each other in the different production processes.[15] For example, car manufacturers set an overall wage bill as well as a total procurement bill, but these can be used for several different models that potentially compete with different manufacturers. This section extends the model to allow for multi-resource firms. To make matters concrete, we focus on a specific setting – the airline industry. This industry is an important and ideal application for a multi-resource and multiple markets model. The airline industry also provides the added feature of widely available data that can be used to evaluate the empirical implications of the model, as we do in section 4. A similar analysis can, of course, be applied to other industries, replacing the routes in the model below with different products and the legs with different inputs or resources used for production.

The industry is characterized by a large set of markets – common practice is to consider each city pair as a market (often also called a route). Among the markets a firm serves, some are also served by other firms while others are not: paralleling the private and overlapping markets characterized in our model.

The operational structure and timeline of the industry also fits the assumptions of the model. Most routes are served in one of two ways: non-stop service, in which the route is served with a single leg; and connecting service, where the route is served with two legs connecting in a midpoint. Legs (flights) typically serve multiple routes and their capacity and frequency is determined much earlier than the market for tickets is active. As such, determining the number of seats on a given leg is similar to investing in a capacity that will later be used to serve multiple markets, some of which will also be served by a rival carrier.

Constructing a flight schedule is a very complex process. Barnhart, Cohn, Johnson, Klabjan, Nemhauser, and Vance (2003) and Lohatepanont and Barnhart (2004) document the operational timeline of the airline industry as being determined in four stages: seats per leg, aircraft type, frequencies and finally crews. Tickets on routes are sold when this process is completed. Changes to this flight schedule are

[15] See e.g. Wildasin (1984) for a detailed discussion.

very expensive and avoided as much as possible.[16] Moreover, schedules are well known to all carriers before most tickets are sold. Thus, the carriers competing on a specific market first determine overall capacity that is available for this and other routes and then compete in each market subject to the set and commonly known capacity limits.

As most routes use two legs, most products depend on *two* decisions made by each firm – the capacity on each leg. These investment decisions will in general be used in different sets of overlapping and private markets. For example, the route SFO-LAX-PHX by United Airlines will have two legs: SFO-LAX and LAX-PHX. The first leg, SFO-LAX will be serving many routes that are also served by American Airlines (e.g., SFO-TUS and SFO-SDO).[17] In contrast, while the second leg, LAX-PHX, might also be used by United Airlines for other routes, American Airlines's network structure prevents it from serving most of these routes. When United and American compete, the capacity chosen by United to the SFO-LAX can be used for many overlapping (with American) markets, while the capacity chosen by United to the LAX-PHX route is mostly used for markets in which American does not compete. The model below captures the implications of this structure.

3.1. *Extending the Baseline Model for the Airline Industry*

To highlight the difference from the baseline model, we refer to the firms as 'carriers'. As before, we assume a duopoly with two airline carriers (denoted by j) and I cities (with a city denoted i). Each pair (i_1, i_2) of cities is a market (a route). Both carriers serve the cities via a single hub.[18] Carrier j only serves a subset of the cities, denoted $\overline{I}^j \subset I$. In the first stage, each carrier j sets a capacity for each hub-spoke leg i, denoted k_i^j. In the second stage, each carrier sets a quantity for each market q_{i_1,i_2}^j, subject to the constraint imposed by the first stage capacity:

$$\forall i \in \overline{I}^j : \sum_{i_2 \in I^j} q_{i,i_2}^j \leq k_i^j$$

This constraint requires that the total number of seats sold by carrier j on all routes using the leg

[16] The only common exception is switching planes of the same type and capacity due to maintenance requirements.

[17] Some carriers may also offer direct flights, e.g., SFO-PHX, a feature we use in the empirical section below.

[18] The analysis is technically simplified by allowing also for the market (i, i). This has no qualitative effect on the results as long as the number of cities is large, but allows algebraic simplifications. It is possible to consider this as the market from the spoke to the hub.

between spoke i and the hub do not exceed the number of seats scheduled on this leg by the carrier.

If all markets are identical, the set of all cities can be separated into three subsets: cities served *only* by carrier A (denoted I^A), cities served *only* by carrier B (denoted I^B) and those in which both carriers compete (denoted I^o).[19] This separation defines the following sets of markets:

1. Overlapping markets – served by both carriers. These are markets between two cities in I^o. As each city pair is a market, there are $m_o \equiv (I^o)^2$ overlapping markets.
2. Private markets – between two cities that only carrier j serves ($i_1, i_2 \in I^j$). The number of private markets for carrier j is $m_j \equiv \left(I^j\right)^2$.
3. Mixed markets – between a city that only carrier j serves and a city that both carriers serve ($i_1 \in I^j$, $i_2 \in I^o$). The number of mixed markets for carrier j is $\tilde{m}_j \equiv I^j \cdot I^o$.
4. Unserved markets – city pairs in which one city is served only by carrier A and the other only by carrier B. This set of markets can be ignored in the remainder of the analysis.

Figure 3.1 presents an example. The overlapping markets are $(1,1)$,$(1,2)$, $(2,1)$ and $(2,2)$. A's private markets are $(3,3)$, $(3,5)$, $(5,3)$ and $(5,5)$. A's mixed markets are $(1,3)$,$(1,5)$,$(2,3)$ and $(2,5)$.

As in the three markets model, the symmetric markets assumption and concavity of revenues implies that in equilibrium each carrier's second stage quantities are the same for all the markets in each set and that the equilibrium is unique. Since second stage quantities are symmetric, first stage capacities must also be. Therefore, as before, we consider each market set as a single market with size determined by the number of markets.

The symmetric equilibrium is then characterized by five quantities for each carrier:

- k^j – the first stage allocation by each carrier for the overlapping legs. These can be used for the overlapping or mixed markets.
- $\hat{k}^j$ – the first stage allocation by each carrier for its monopoly legs. These can be used for the private or mixed markets.
- q^j – the second stage allocation by each carrier for the overlapping market.
- $\hat{q}^j$ – the second stage allocation by each carrier for its private market.
- $\tilde{q}^j$ – the second stage allocation by each carrier for its mixed market.

Solving backward, carrier A's problem in the second stage is to maximize revenue in all of its markets,

[19] We will refer to the markets served by only one carrier as monopolistic markets. The effect of adding other rival carriers in the private markets is the same as adding other rivals in the private markets in the baseline model.

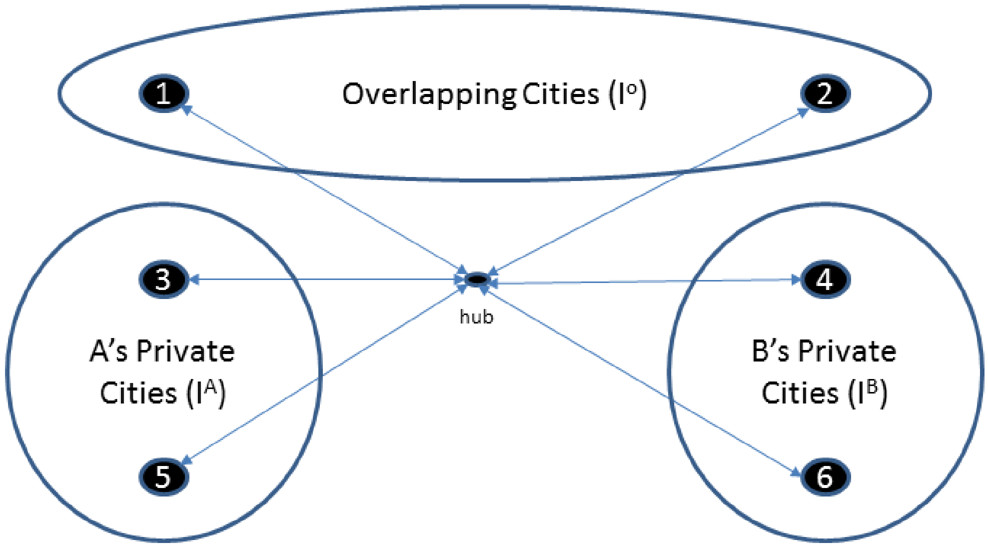

FIGURE 3.1.— Airline Setting Example. The overlapping markets are $(1,1)$ (i.e. from 1 to the hub and back) ,$(1,2)$, $(2,1)$ and $(2,2)$. A's private markets are $(3,3)$, $(3,5)$, $(5,3)$ and $(5,5)$. A's mixed markets are $(1,3)$,$(1,5)$,$(2,3)$ and $(2,5)$.

subject to the capacity constraints k^A and $\hat{k}^A$ and given its rival allocation in the overlapping market q^B. Letting MR denote the marginal revenue, the second stage solution is characterized by the following equality:[20]

$$\hat{MR}^j + MR^j = 2\tilde{MR}^j \tag{3.1}$$

Condition 3.1 captures the second stage allocation tradeoff by each carrier. A seat on an overlapping route $\left(q^A\right)$ requires two seats on overlapping legs. A seat on a private route $\left(\hat{q}^A\right)$ requires two seats on private legs. Finally, a seat on a mixed route $\left(\tilde{q}^A\right)$ requires a seat on an overlapping leg and a seat on a private leg. Thus, two 'mixed'-route seats can alternatively be used to add a seat on an overlapping route and a seat on a private route.

As in the baseline model, each carrier would like to commit to a higher supply in the overlapping market, pushing its rival to divert some capacity to a non-overlapping market. In the baseline model, a firm could divert its capacity between the overlapping and private market at will. In the multi-resource case, carriers divert overlapping capacity to their mixed market. However, as serving a mixed market

[20]The derivations and proofs in this section are provided in appendix C.

requires also a seat on a private leg, the carrier's ability to divert overlapping capacity depends on the carrier's private capacity $\left(\hat{k}^j\right)$. If a carrier has no available private capacity (i.e., no available seats on private legs), it can only use its overlapping capacity $\left(k^j\right)$ in the overlapping markets. Thus, the carrier's *flexibility* in the airline setting is endogenous. In particular, the carrier can reduce its flexibility by setting a lower private capacity compared to the standalone Cournot benchmark.

This is the main additional economic insight from the multi-resource model compared to the baseline model: firms set a higher overlapping capacity $\left(k^j\right)$ and a lower private capacity $\left(\hat{k}^j\right)$. The additional overlapping capacity exploits the rival's flexibility and causes both the overlapping and mixed quantities to be higher than the Cournot benchmark. The lower private capacity results in lower than the benchmark quantity in the private markets, and in mixed routes quantity that is lower than the overlapping quantity.

In equilibrium, an increase in the private capacity makes the firm, in the second stage, divert quantity from the overlapping market to the mixed market. In particular, if carrier A adds one unit of private capacity and uses only ε of it in its fully private markets, it will use the remainder in its mixed markets. This will require removing $(1-\varepsilon)$ units of capacity from the overlapping market to the mixed market. Thus, the extent to which the firm can commit to allocating additional private capacity to its fully private markets (m_j) has an important role in determining outcomes. We call $(1-\varepsilon)$ the firm's *private commitment power,* $\hat{\sigma}_j$, defined by*:*

$$\hat{\sigma}_j = 1 - \frac{\partial \hat{\vartheta}^j}{\partial \hat{k}^j} \quad .$$

Flexibility and commitment power are defined as in the baseline model:

$$\phi_j = \frac{\partial q^j}{\partial q^{-j}} \quad ; \quad \sigma_j = \frac{\partial \vartheta^j}{\partial k^j}$$

To recreate the first result of the baseline model, we first define the firm's aggressiveness:

Definition 4 In the multiple resource model, the firm's competitive effect of MMC is the difference between the sum of marginal revenue in the private and overlapping markets and their marginal costs

$$\alpha_j \equiv 2c - MR_j - \hat{MR}_j$$

The next Lemma parallels proposition 1. The proof follows the same lines and is provided in the appendix.

LEMMA 3 *In the multi resource model,* $\alpha_j = P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \phi_B\left(\sigma_A - \hat{\sigma}_A\right)$

We derive a closed form solution using a similar (but more notationally intensive) procedure as in the baseline model.[21] We provide below the parallels for proposition 2 (the symmetric case) and Lemma 2 (the asymmetric comparative static).

There are two main differences in the findings, compared to the baseline model. Both result from the fact that a firm's private market capacity increases its flexibility, as discussed above. In the symmetric case, this implies that a firm's per market supply to its private market initially decreases in the overlap between firms, in order to constrain flexibility.

In the asymmetric case, carrier A's per-market quantity *decreases* with its rival's private size $\left(I^B\right)$. As B becomes more flexible, A can gain more by being aggressive. For this, A reduces its alternative use for overlapping capacity in the mixed markets. This is done by reducing the private capacity $\hat{k}^A$, which in turn requires lower quantities in A's private markets. In contrast, if A's private market increases $\left(I^A\right)$, the number of mixed market for A increases. As a result, A has less to gain by constraining its flexibility through reducing $\hat{k}^A$ and A's quantity per private market increases.

To state the symmetric result, as in the baseline model, an asterisk denotes the standard Cournot (or monopoly) quantities: $q^{j^*} = m_o\frac{a-c}{3b}$, $\hat{q}^{j^*} = \hat{m}_j\frac{a-c}{2b}$, $\tilde{q}^{j^*} = \tilde{m}_j\frac{a-c}{2b}$.

LEMMA 4 *Suppose* $I^A = I^B$, *fix* $I^o + I^A + I^B = I$ *and let* $I^0 = \lambda \cdot I$ *for* $\lambda \in [0,1]$. *Then in equilibrium:*

$$\frac{q^j}{q^{j^*}} = \frac{9 + 156\lambda + 342\lambda^2 - 420\lambda^3 + 105\lambda^4}{9 + 140\lambda + 382\lambda^2 - 452\lambda^3 + 113\lambda^4}$$

$$\frac{\tilde{q}^j}{\tilde{q}^{j^*}} = \frac{9 + 148\lambda + 358\lambda^2 - 428\lambda^3 + 105\lambda^4}{9 + 140\lambda + 382\lambda^2 - 452\lambda^3 + 113\lambda^4}$$

$$\frac{\hat{q}^j}{\hat{q}^{j^*}} = \frac{9 + 140\lambda + 374\lambda^2 - 436\lambda^3 + 105\lambda^4}{9 + 140\lambda + 382\lambda^2 - 452\lambda^3 + 113\lambda^4}$$

[21] The detailed solution is available from the authors upon request.

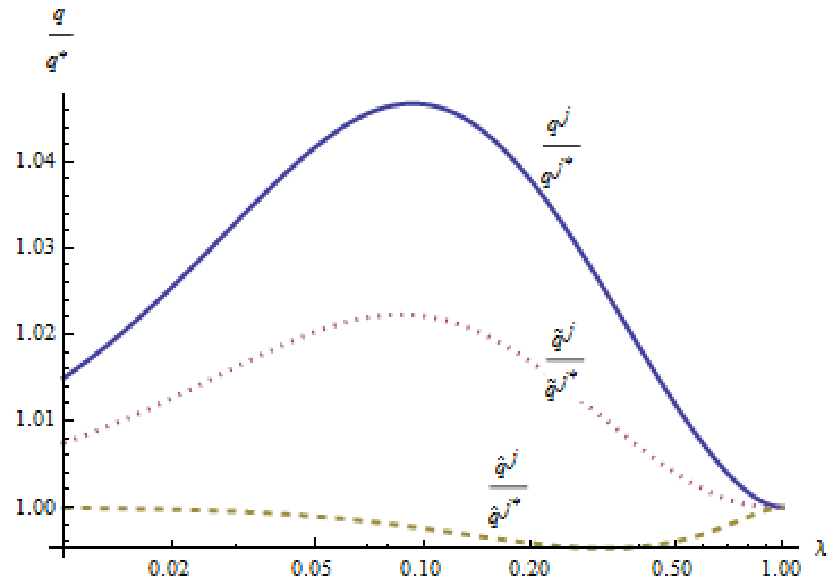

FIGURE 3.2.— Quantities deviation from Cournot in the symmetric airlines setting. $I^o = \lambda \cdot I$, $I^A = I^B = \frac{I}{2}(1-\lambda)$.

The next plot illustrates the three quantities and the non-monotonicity for each. Observe that, as explained above, the private market quantity behavior is opposite of the overlapping market quantity behavior. Initially both carriers constrain the private capacity to limit their flexibility.

The next lemma states the comparative statics with respect to one carrier's private size $\left(I^j\right)$:

LEMMA 5 *For any I^o:*

- *A firm's overlapping market quantity $\left(q^j\right)$ increases with its rival private size $\left(I^{-j}\right)$ and decreases in its own private size $\left(I^j\right)$*
- *A firm's per-market private quantity $\left(\frac{\hat{q}^j}{\hat{m}_j}\right)$ decreases with its rival private size $\left(I^{-j}\right)$ and increases in its own private size $\left(I^j\right)$*
- *A firm's mixed market quantity $\left(\tilde{q}^j\right)$ increases with its rival private size$\left(I^{-j}\right)$*

The Lemma shows that the asymmetric results from the single resource case extend as expected. An increase in the rival's private market size makes the rival more flexible. As a result, the firm increases its market quantity (reduces its marginal revenue). In addition, an increase in the rival's private market size reduces the rival's commitment power. This makes it harder for the rival to take advantage of the firm's flexibility, reducing the rival's quantity even more, and in turn increasing the firm's quantity.

Observe that the lemma does not specify the relation between the quantity in the mixed market, $\tilde{q}^j$ (or $\frac{\tilde{q}^j}{\tilde{m}_j}$), and the firm's own private size $\left(I^j\right)$. It can be shown that this relation is non-monotonic.

4. AN EMPIRICAL TEST USING THE US DOMESTIC AIRLINE INDUSTRY

One clear implication of the model (Lemma 5) is that a firm's market share increases as the size of a rival's private market increases. The predicted effect is most prevalent when the focal firm (the firm under observation) has small private markets.This section tests this prediction. We find that carriers may gain up to 7.5% market share when rival carriers' private demand increases significantly. We also show that the effect is not caused by a rival's limited capacity – that is, the increase in market share is not a result of the rival moving resources to service the increase demand. Note that the standard, anti-competitive effect of MMC is not affected by changes to the rival's private market and thus the anti-competitive model does not provide a prediction similar to the one tested here.

We use passenger data from the BTS's DB1B Origin and Destination database for the years 1993 to 2010. The database reports quarterly a ten percent sample of all itineraries sold for domestic flights. We define a route as a non-directional city pair, and aggregate itineraries to a carrier-route-quarter level. We obtain information on the number of passengers, the mode city used for connecting service, the percentage of passengers flying non-stop, and other covariates. For example, an observation will be United Airlines, servicing San Francisco to Phoenix, on the fourth quarter of 2010. For that observation, we will record that 36,700 passengers flew on that quarter, of which 93% flew non-stop and those that had connecting service connected mostly through Los Angeles.

Given this data structure, a route will be characterized by three legs: a leg from the origin to the 'hub' city, a leg from the 'hub' city to the destination, and, alternatively a 'direct' leg from the origin to the destination.[22] If no passengers fly non-stop, then this last 'direct' leg is nonexistent. If all passengers fly non-stop, then the first two legs are nonexistent. For each one of these legs, when they exist, we calculate the total number of passengers flying that leg. More technical details are given in the appendix.

[22]The use of the terms 'origin' and 'destination' is arbitrary, as routes are defined as non-directional.

4.1. *Regressions Details and Control*

Ideally, the key independent variable for each carrier-route-quarter would be the size of a rival's private demand. As this cannot be observed, we proxy for the rival's flexibility by taking the ratio between the quantity sold by the rival in its mixed markets that use one of the legs of the focal route and the total quantity sold by the rival in all the markets that use one of the legs of the focal route. In the notation of the model, this is equivalent to measuring $\frac{\tilde{q}^B}{\tilde{q}^B+q^B}$.[23] Note that the range of the this measure is 0 to 1 and that a higher number indicates a relatively larger private market. We denote this measure of carrier j's rival's private market size when considering route r in period t as $\Gamma_{j,r,t}$. Letting $s_{j,r,t}$ denote j's market share on route r in period t, our specification is:

$$s_{j,r,t} = \beta\Gamma_{j,r,t} + \gamma X_{j,r,t}$$

The vector $X_{j,r,t}$ are controls, including fixed effects, discussed below. A positive β supports our model. Furthermore, since both market-share and Γ are between 0 and 1, β represents the percentage points gain in market-share from moving from no private markets for the rival to the case where all of the rivals' other markets that use a common leg are private. In a symmetric duopoly, this is equivalent to going from the Cournot outcome to the Stackelberg outcome, in which case β should be 0.16.[24]

Controls for the regression include the market-share of the focal carrier at the endpoint cities, and the size of the route. We also include route-carrier fixed effects, carrier-year-quarter fixed effects, rival carrier-year-quarter fixed effects, and market structure dummies of the number and type of carriers that vary by year-quarter.

The route-carrier fixed effects control for long-standing differences across routes and carriers, such as American Airlines' strong position on the Chicago-San Francisco route, or that Delta tends to have high market-shares on routes to and from Atlanta. These fixed effects should control for significant differences in persistent market power across routes.[25]

[23]In our setting, the measure is constructed separately for each leg and rival for each carrier-route-period observation and then aggregated to form one measure per observation. See appendix for details.

[24]In a symmetric duopoly, the share for each firm is $\frac{1}{2}$. In the linear Stackelberg game, the leader's share is $\frac{2}{3}$ (a 0.16 increase from $\frac{1}{2}$) and the follower's share is $\frac{1}{3}$ (a 0.16 decrease).

[25]Market power could arise both on the demand side: brand effects, consumer switching costs, etc.; and on the supply side: landing slots, gate access, etc.

We use endpoints market share covariate to control for changes in market power on a specific route. Nationwide changes in market power of a given carrier, arising from debt restructuring, national policy changes, or cost shifts (fuel contracts, labor unions, etc.) are controlled for with the carrier-year-quarter fixed effects and the rival carrier-year quarter fixed effects. The carrier-year-quarter fixed effects control the changes in market power of the focal carrier, while the rival-year-quarter fixed effects controls for changes in market power of a rival of the focal carrier. For example, an American Airlines-2004-Q1 fixed effect allows AA to have a higher market share on all AA routes that year-quarter relative to other years and quarters, while a Rival-American Airlines-2004-Q1 fixed effect allows for all carriers on all routes in which AA competes to have a lower market share in the first quarter of 2004 relative to other years and quarters. Adding rival-year-quarter fixed affects allows to control for changes in market shares that are correlated with changes in rivals' network structures that are caused by nationwide shifts in market power and not by regional shifts in demand.

Finally, the market structure dummies control for entry and exit, and changes in type of service (connecting vs non-stop). These are dummies for the type of service of the focal carrier: non-stop[26] or connecting, and the number and type of service of rival carriers: one non-stop, two non-stop, one connecting, one non-stop and one-connecting, etc. As entry and exit can cause both market shares and rival carrier's flexibility to change, we allow these dummies to vary from quarter to quarter. This large set of dummies, along with the carrier-year-quarter and the rival carrier-year-quarter fixed effects, should control for most of the variation that is driven by entry, exit, or changes in service type.

We run the regression on observations in which the focal carrier j has a private market ratio $\left(\frac{\tilde{q}_j}{\tilde{q}_j+q_j}\right)$ lower than 0.1 and services at least 55 daily passengers (approx. three non-stop flights a week). That is, we focus on routes that are not too small and for which the carrier is relatively inflexible. Typically this would mean the carrier serves most passengers on the route with direct flights.

When calculating the carrier's flexibility, we drop all routes with less than 5 weekly passengers, and all carriers with less than 1% national market share. A carrier is considered a competitor on a route if it has at least 5% market share on the route.[27]

[26] A carrier is considered non-stop if at least 50% of its passengers on the route fly non-stop and the carrier services more than 110 daily passengers.

[27] This last restriction is not applied when counting the number of passengers on a leg, only when considering a given carrier as a rival to another carrier and when running the regressions.

TABLE I
MARKET-SHARES REGRESSION RESULTS

	A	B	C
Rivals' Private Demand(Γ)	0.276*	0.075*	0.075*
	(0.007)	(0.005)	(0.006)
Avg Mkt Share at Endpoints		0.937*	0.789*
		(0.019)	(0.022)
Ln(route passengers)		-0.039*	-0.032*
		(0.002)	(0.003)
Market Structure Dummies		YES	YES
Route-Carrier FE		YES	YES
Carrier-Year-Quarter FE		YES	YES
Rival Carrier-Year-Quarter FE		YES	YES
# of Observations	58,361	58,361	37,903
Adjusted R-Sq	0.028	0.936	0.938

The unit of observation is a route-carrier-year-quarter. The dependent variable in all regressions is market share (between 0 and 1). Rivals' Private Demand is a measure between 0 and 1 and is described in the text. Market Structure Dummies are dummies for the number and type (non-stop or 1-stop flights) of carriers on the route and vary by quarter (one set for each year-quarter). Heteroskedastic robust standard errors in parenthesis. (*) statistically different than 0 at a 5% p-value. Column C excludes NYC routes, excludes Q3 and Q4 of 2001, all 2002, 2008 and 2009. It also increases the carrier size restriction to be above 110 daily passengers.

4.2. *Regression Results*

Table I presents the main results. The first column shows the results without any controls. Column B, our preferred specification, includes all of the controls discussed above. Column C restricts the sample in many ways, so as to avoid estimating the effects from potentially non-representative sectors of the industry. It excludes the six quarters following the 9/11 attacks and the four quarters of the 2008 financial crisis. It also excludes all routes to and from New York City and restricts carriers to operate at least 110 daily passengers (approximately at least one daily flight).

The coefficient on rival carriers' private demand, Γ, is significant across all specifications. Column B shows an effect of 0.075. This represents a full 7.5 point gain in market share when private demand of rival carriers increases significantly. Given that the average market share of the observations used in the regression is of 43 points, a 7.5 point gain is equivalent to a 17% increase in revenue. This effect could be large enough to discourage merger activity, as the merging parties could see aggregate market shares decrease by up to 7.5 points in most markets of the merged carrier's network.[28]

The 7.5 point change in market share assumes private demand changes from 0 to 1, which is effectively saying demand in overlapping markets is miniscule compared to demand in private markets. This shift is rarely observed. Nevertheless, rival carriers' private demand does shift significantly across the data. The standard deviation on this measure is 0.15 (see appendix). Thus, a one standard deviation change in the rival's private demand can increase a focal carrier's market shares by 1.1 points. To the extent that this increase is across all markets in which the carriers overlap and use the same resources (flights), a 1.1 percentage point gain can be very significant. The standard deviation on market shares is 25 percentage points, thus the competitive effect of MMC accounts for more than 4% of the differences in market shares across observations.

The coefficients of the controls in table I are as expected. Having a strong presence at the endpoint airports translates into a strong presence on the route. A coefficient of 0.9 implies that a 1 percentage point gain at the airport translates into a gain of 0.9 percentage on the route. Finally, larger routes result in lower market shares, surely from the entry of more carriers.[29]

As robustness, reported in table II, we re-estimate the specification in column B in table I including only legacy carriers in the regression.[30] These are the carriers that mostly operate under a Hub & Spoke system and that are the most able to cancel/reschedule flights. In addition, low-cost carriers typically have the highest ability to gain market share in response to increases in rival carriers' private demand, since they can credibly commit to a market. Legacy carriers have a lesser ability to do so, but still can commit somewhat. Table II presents several variations of this robustness test. The regression for column A takes the specification in column B in table I and limits the focal carriers to Legacy carriers. The coefficient on Rival's Private Demand is 0.047, which represents an almost three point gain in market share as a response to a significant increase in rivals' private demand.

Columns B and C consider only changes in private demand by specific types of carriers. For column B, the rivals' private demand, Γ , is averaged only across legacy carriers, as opposed to all carriers. The

[28] Mergers may increase carriers' flexibility tremendously. We discuss the effect of mergers in more detail below.

[29] The market structure dummies bucket routes with 3+ carriers into a limited number of buckets: more than two direct carriers, one direct carrier and more than two hub carriers, and more than two hub carriers with no direct carrier. For the extremely large routes, route size can be proxying for the number of carriers that can compete on the route.

[30] American Airlines, Continental, Delta, Northwest Airlines, TWA, United Airlines, and US Airways.

TABLE II

Legacy Carriers' Market Share Regression

	A	**B**	C
Rivals' Private Demand(Γ)	0.047*		
	(0.009)		
Rival's Private Demand (Legacy Carriers only)		0.035*	
		(0.009)	
Southwest's Private Demand($\tau^{WN,i}$)			0.023*
			(0.006)
# of Observations	25,010	24,085	21,873
Adjusted R-Sq	0.941	0.942	0.931

Sample includes only Legacy Carriers: AA, CO, DL, NW, TW, UA, and US. The dependent variable is market share. Controls are the same as in Table I, Column B. Heteroskedastic robust standard errors in parenthesis. (*) and (†) statistically different than 0 at a 5% and 10% p-value. See table I for more details.

effect is still significant, albeit smaller: 3.5 point gain in market share for a significant increase in legacy carriers' private demand.

Column C considers only the private demand of Southwest as a rival. This is our most stringent regression. We additionally restrict the sample to routes in which Southwest has a positive private demand. As such, the results cannot arise from the effect of entry of Southwest into a focal route. Furthermore, as before, the sample is restricted to legacy carriers only. Thus, we are measuring the legacy carrier's ability to gain market share on routes in which they compete with Southwest in response to an increase in Southwest's private demand (demand Southwest has on other routes that use one of the legs Southwest is using on the focal route). [31] The effect we find is smaller but still significant: Routes on which Southwest has private demand, legacy carrier's are able to gain up to 2.3 points in market share when Southwest's private demand increases significantly.[32]

In summary, a carrier's market share increases by up to 7.5 points when rival carriers' private demand

[31]Since Southwest does not operate many connecting service routes, there is only a relatively small number of routes on which legacy carriers' private demand is smaller than 10% and that Southwest services with connecting service -an example of such route is Las Vegas to San Francisco, on which 16% of Southwest's passengers have connected in Los Angeles over the years-. In order to a have a large enough sample to identify the effect we are after, we increase in this specification the cutoff of own private demand from 0.1 to 0.25.

[32]Southwest's standard deviation on their private demand used in the regression is 0.064, but varies from 0 to 0.73. Thus, the largest effect actually observed in the data is one of a 1.7 point difference in market share.

for connected routes increases significantly. This supports the theoretical model in which a carrier's ability to commit to a market can be used to gain market share on such market, at the expense of a rival firm that cannot commit to such market because they have the flexibility to utilize investments in alternative markets.

5. DISCUSSION AND APPLICATIONS

The level of MMC can be affected by organic firm growth, mergers, acquisitions, alliances, international trade and many other forces. In this section we discuss how such applications relate to our model and examine their welfare effects and policy implications. As before, our analysis is based on comparative statics and, in particular, on firms' capacity and quantity choices under different market structures.

5.1. *Firm Growth and Multi-Market Competition*

We start with analyzing the relationship between firm growth and MMC. An interesting example of a market that saw a variety of growth patterns is the discount department stores industry, and in particular Target's and Wal-Mart's growth during the 60s through the 90s. Roughly, Target's and Wal-Mart's growth during that time can be described by the following stages: (1) The two chains do not overlap at all; (2) Target enters some of Wal-Mart's markets;[33] (3) the chains mostly enter new private markets: Wal-Mart grows West and Target grows East; (4) the chains enter each others' market and eventually completely overlap as they both serve the entire US market. Figure 5.1 presents a simplified demonstration of the Target/Wal-Mart growth.[34] We think of the movement from each market structure to the other as different growth phases and analyze below each of the three growth phases in terms of welfare and policy implications.

In the first phase, MMC increased as Target encroached on Wal-Mart's market. During this entire growth phase Target's private market remained larger than the overlapping market. Proposition 2 implies that during this first growth phase, welfare in each overlapping market *increased* with Target's growth.[35]

[33] While Wal-Mart has opened new stores during the 60s and 70s they were mostly within the same geographical areas. Therefore, our analysis of the first growth phase focuses on Target's entry into Wal-Mart's private market–i.e., an increase in the size of the overlapping market while the size of Target's private market remains unchanged.

[34] A detailed representation of the chains' growth can be found in http://projects.flowingdata.com

[35] That is, welfare in the m markets that were served by both companies before the entry into a new market. Obviously, Target's entry likely increased welfare in the market it entered. The discussion assumes for expositional simplicity that

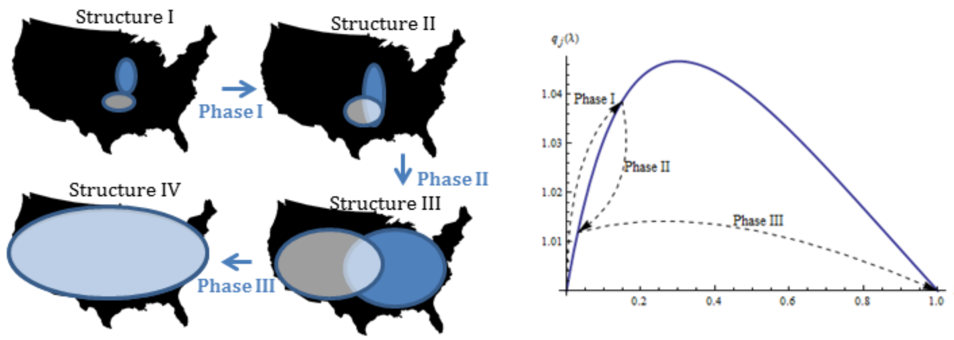

FIGURE 5.1.— Target's and Wal-Mart's growth example. In the first phase, the firms move from no overlap ($\lambda = 0$) to Target entering about a third of the markets served by Wal-Mart ($\lambda = 0.2$). As a result, quantity per market increases by three percentage points from the Cournot benchmark. In the second phase, the firms expand their private markets and the overlapping market share drops to about a tenth of the private market for each firm ($\lambda = .05$). This reduces firms' quantities to only one percentage point over the Cournot benchmark. In the third and final stage, both firms serve almost all markets, and overlap significantly increases ($\lambda \approx 1$), resulting in all markets converging to the Cournot benchmark.

During the second growth phase, Wal-Mart and Target entered into new private markets such that the *relative* size of the overlapping market decreased over time. We, therefore, think about this phase as a period during which the level of MMC declined. Based on proposition 2, this type of growth could have *decreased* each chain's quantity in each overlapping market, and thus negatively affected welfare in markets where both chains were already active.[36] As a result, welfare in the overlapping markets could have decreased, while the growth of Target and Wal-Mart during this phase has likely increased total welfare when considering the new markets..

During the final growth phase, MMC grew tremendously to the point where the chains overlap in all their markets. Proposition 2 then implies that, during this last growth phase, per-market welfare in the overlapping markets perhaps temporarily increased but then decreased to the Cournot benchmark level (see figure 5.1). Such dynamic can be easily misinterpreted as evidence of implicit collusion.

Target did not enter any new private markets. Such additional entry would slightly weaken the increase in multi-market contact.

[36]For simplification, as well as in order to focus on changes in MMC, we can assume that both firm's grew at about the same rate throughout this phase and that the total overlapping demand was smaller than the private demand for each firm.

5.2. *International Trade*

With slight modification, the first growth phase in the example above can be extended to consider international trade. Specifically, one can think of structure I in figure 5.1 as an example of a German and French small-appliance manufacturers that choose to enter a duopolistic international market–e.g., Denmark–replacing the current local competitors.[37] That is, overall market structure in all three markets–Germany, France, and Denmark–did not change. The only change is in the identity of the firms that serve the Danish market, and in particular in their access to private markets. Since pre-entry the two Danish firms had no private markets, entry by the German and French competitors will result in a higher quantity in the Danish market, relative to the levels set by the Danish local competitors. Moreover, as shown in figure 2.1, following the international growth, the German and French manufacturers will be more aggressive with their quantity also in their private markets. As a result, total quantity and thus welfare in all three markets increases following this international growth. Note that this dynamic can be confounded with the perceived higher efficiency of the international (German and French) firms.

A more complicated case is one in which, before entry into the Danish market, the German and French manufacturers overlap in some of the markets they serve–e.g., Belgium. That is, entry into the Danish market has increased MMC between the German and French manufacturers such that they now overlap both in Denmark and Belgium. While the effect on competition in the Danish market is still positive, its magnitude now depends on the size of the Danish and Belgian (i.e., overlapping markets) relative to the size of the German and French (i.e., private) markets. Moreover, depending on the relative sizes of these markets, the effect on welfare in the Belgian market may be negative.

5.3. *Mergers*

An analysis of the effect of horizontal mergers on welfare typically focuses on cost reduction and market power. Mergers, however, often affect the level of MMC in the market–either increasing or decreasing it. General results require a model with more than two firms, and is thus beyond the scope of this paper. Our discussion below provides several examples to demonstrate the importance of incorporating the competitive effect of MMC when evaluating the effect of mergers on welfare. As our analysis emphasizes

[37] In order to stay within the diseconomies across markets assumption, we assume that both firms choose to export their products to the new market, rather than produce locally. Alternatively, one can think multinational firms that manufacture its products somewhere in Asia and ships around the world.

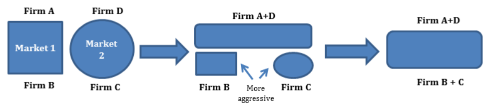

FIGURE 5.2.— Merger example. The markets start with the structure depicted left with A and B competing in market 1 and C and D competing in market 2. The first merger, between A and D changes the market structure so that A competes with firm B in market 1 and firm C in market 2. The second merger, between B and C changes the market structure so that A competes with B in markets 1 and 2.

the competitive effect of MMC, we assume away any cost reductions mergers may generate. Furthermore, given that market structure (i.e., number of firms) always dominates the competitive effect of MMC in terms of welfare, our example focuses on a case where there is no change in the number of firms serving each market.

Assume two markets, 1 and 2, and four firms: A, B, C and D, such that firms A and B both serve market 1; and firms C and D serve market 2. Furthermore, assume that none of the firms serve any additional markets. That is, A and B completely overlap and so do C and D. Thus equilibrium quantities in both markets equal the Cournot benchmark quantities. Assume now that firms A and D merge. The industry now has two markets and three firms: A, B and C, where market 1 is served by firms A and B, and market 2 is served by firms A and C. Firm A serves now markets that are not served by its competitors, and can thus be thought of as having private markets. More precisely, firm B can think of market 2 as firm A's private market, while firm C can consider market 1 to be firm A's private market. The top panel in figure 5.2 presents the market structure before and after the merger.

Post-merger, while firm A does not have a monopolistic private market, from B's and C's perspective, firm A has flexibility in the second stage of the game in the allocation of its capacity, and thus $\phi_A > 0$ while $\phi_B = \phi_C = 0$. Consequently, B and C can be aggressive in their capacity choice and equilibrium quantities in markets 1 and 2 will be above the Cournot benchmark. The exact quantity in each market depends on the size of market 1 relative to the size of market 2. Figure 5.3 presents average per-market quantities in the industry, per-market quantity in market 1, firm A's and B's per market quantity in market 1, and firm A's total capacity as a function of the relative size of markets 1 and 2. All values are relative to the Cournot benchmark. In order to understand the effect of changes in the relative sizes of markets 1 and 2, we assume in figure 5.3 that $m_1 + m_2 = \bar{m}$.

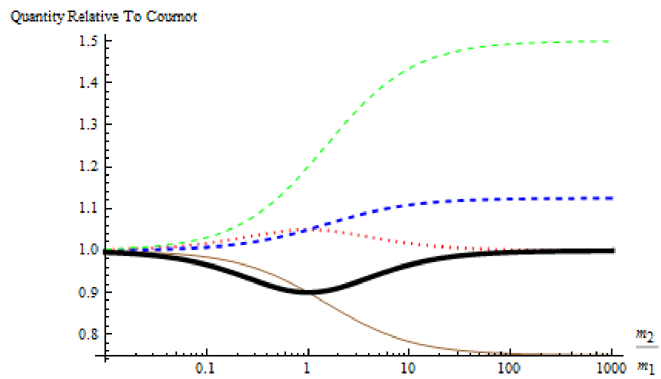

FIGURE 5.3.— Quantities in markets 1 and 2 after the merger between A and D as a function of the relative market sizes $\left(\frac{m_2}{m_1}\right)$.

The dotted line is the average quantity per market, $\frac{k_A+k_B+k_C}{m_1+m_2}$, which is maximized when $m_2 = m_1$. The thick dashed line is the quantity per market size in market 1 ($\frac{q_A+q_B}{m_1}$), increasing as m_1 reduces in size relative to m_2. The thin dashed line is B's quantity per the market size it serves ($\frac{q_B}{m_1}$), which reaches the Stackelberg leader level as m_2 becomes much larger than m_1. The thin solid line is A's quantity in the market served by B ($\frac{q_A}{m_1}$), which reaches the Stackelberg follower level as m_2 becomes much larger than m_1. The thick solid line is A's total quantity ($\frac{k_A}{m_1+m_2}$), minimized when $m_2 = m_1$.

As figure 5.3 shows, total capacity in the market is always at least as high as in the Cournot game, and is maximized when markets 1 and 2 are symmetric: $m_1 = m_2 = \bar{m}/2$. That is, in this example, a merger *increases* welfare even absent any cost efficiencies. Since firms B and C perceive firm A to be flexible, they behave more aggressively and total quantity increases. When markets are symmetric, firm A's capacity (and profits) are the lowest, due to its rivals' high aggressiveness. Firm B (resp., C) increases its quantity by as much as 50% as the size of market 1 (resp. 2) decreases relative to the size of market 2 (resp., 1), as it is easier for firm A to allocate a larger share of its capacity to market 2 (resp., 1). Indeed, as the graph shows, firm A responds by an up to 22% reduction in its quantity. Nevertheless, overall quantity in the market increases as its relative size decreases, such that in the extreme case, where market 2 is much larger than market 1, competition in market 1 resembles a Stackelberg game, with firm A as the Stackelberg follower.

Horizontal mergers are often followed by subsequent mergers by the remaining firms. Assume, for example, that as a response to A's and D's merger, firms B and C choose to merge as well (see bottom structure in figure 5.2). That is, post merger, markets 1 and 2 are both served by the same firms–firm

A and the newly merged firm, and thus none of the firms has a private market. Post-merger equilibrium quantities are therefore exactly at the Cournot level. Comparing this to the quantities after the first merger, we see that overall aggregated quantities in the industry decrease; and therefore so does welfare. Interestingly, at the outset, subsequent mergers typically appear to better balance competition and are thus expected to be "welfare-enhancing" mergers. Nevertheless, as our example demonstrates, when considering the competitive effect of MMC, a subsequent merger may in fact have a negative effect on welfare.[38]

5.4. *International Airline Alliances and Competition*

In the international market, antitrust immunity is common for international airline alliances. International alliances that are given antitrust immunity are essentially given permission to act as a merged firm for the short term analysis. These alliances are typically between very large carriers that have high flexibility in their existing routes. In this case, the merger does not reduce the level of competition in any market. However, the merged carriers serve many more routes post merger and thus make the merged carriers more flexible. As a result, our model predicts that the merged carrier's rivals become more aggressive and overall quantity and welfare increase.

Park and Zhang (2000) study the main North Atlantic alliances in the 1990's (British Airways/USAir, Delta/Sabena/Swissair, KLM/Northwest, and Lufthansa/United Airlines).[39] As predicted by our model, their analysis concludes that a complementary alliance between carriers with low network overlap (e.g., BA/USAir, KLM/NW, LH/UA) is likely to increase total seat miles sold and consumer surplus.

6. CONCLUSION

The paper studies the competitive effect of multi-market contact (MMC) in oligopoly settings. We find that changes in the extent of multi-market contact can have a significant effect on quantities and welfare. In our setting, firms that operate in multiple markets generate more welfare on average than single-market firms. However, *increases* in the extent of MMC may decrease or increase welfare in markets. In particular, proposition 2 shows that whether an increase in MMC strengthens or softens

[38]Recall that our discussion assumes that the merger did not affect costs. Naturally, a merger between firms B and C may create some efficiencies which in turn may decrease costs and thus overall increase welfare.

[39]Not all alliances were granted complete "antitrust immunity". See the discussion in Park and Zhang (2000) for details.

competition depends on the number of markets in which the firms overlap relative to the number of markets in which the firms do not interact.

The analysis highlighted two related forces through which MMC affects competition. The size of a firm's private markets determines the firm's *flexibility* – its ability to shift resources across its different markets. Firms in oligopoly respond to a rival's flexibility by increasing their own aggressiveness, knowing that their rival will have the flexibility to accommodate. The second force – the firm's *commitment power* measures the extent to which a firm can commit to allocating productive capacity to be used in the overlapping markets.

While the detailed results rely on a linear demand function and a second stage Cournot setting, the underlying economics are general. In particular, while it is beyond the scope of our discussion, analysis similar to the Kreps and Scheinkman (1983) argument relating a Cournot to a two stage Bertrand game should be applicable to our model: The second stage results in Kreps and Scheinkman (1983) directly apply to our model[40], and the first stage equilibrium therefore follows.

Previous literature has focused on perceived differences between large and small firms in terms of productivity or anti-competitive behaviors. The competitive MMC effect can be confounded with either of the two. Thus, an important empirical question is whether it is possible to distinguish between each. A key difference between the competitive and anti-competitive effect of MMC is that the former relies on specific physical diseconomies across the markets (i.e., the first stage capacities in our model) while the latter does not. This opens possibilities for future research to distinguish between the competitive and anti-competitive effects of MMC by separating the effect of changes in MMC that use the same capacity (i.e., flights through the same hub) and changes in MMC that do not (i.e., flights through different hubs). We intend to further explore this point in our future research.

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[40]Treating the marginal revenue in the private market as marginal cost of the second stage obtains the exact same setting as Kreps and Scheinkman (1983) with increasing and convex second stage cost.

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APPENDIX A: ALGEBRAIC CALCULATIONS

See attached Mathematica notebook.

APPENDIX B: PROOFS FOR THE MAIN MODEL

LEMMA 6 For any $(k_A, k_B) > 0$, there is a unique tuple $\langle q_A, \hat{q}_A, q_B, \hat{q}_B \rangle$ that is a second stage equilibrium, such that:

1. *If j's capacity constraint does not bind* $(q_j + \hat{q}_j < k_j)$*, then the marginal revenue for j is zero in both of its markets.*
2. *If j serves the overlapping market* $(q_j > 0)$ *, then j serves its private market* ($\hat{q}_j > 0$) *and the marginal revenue for j is identical in both markets*
3. *If* $q_j = 0$*, then* $\min[q_{-j}, \hat{q}_{-j}] > 0$.

PROOF: For #1, if firm j's capacity constraint does not bind $(q_j + \hat{q}_j < k_j)$, then j's cost for increasing quantity in either of its markets is zero. It must therefore be that j's MR in all of its markets is zero.

For #2, it is sufficient to consider the case that $q_j > 0$ and $q_j + \hat{q}_j = k_j$. In this case, an increase in the quantity firm j supplies to the overlapping market, q_j, requires a decrease in the quantity supplied to the private market, $\hat{q}_j$. The marginal *opportunity cost* of quantity in the second stage in one market is the marginal revenue in the other market. Therefore, in equilibrium, marginal revenue (MR) in both of j's markets must be equal.

For #3, if firm j serves only its private market, (so that $q_j = 0$), then it must be that firm j's marginal revenue in its private market is strictly higher than its marginal revenue in the overlapping market. Since price at zero output is identical in both markets and decreasing in output, the rival's output in the overlapping market must be strictly positive. The previous argument now requires that the rival's marginal revenue in its private market is lower than the price at zero, and thus $\hat{q}_{-j} > 0$.

Finally, in all cases firm j's second stage profit is strictly concave in q_j whenever the capacity constraint binds, and in both q_j and $\hat{q}_j$ whenever the constraint does not bind. Thus the second stage equilibrium is unique (see e.g. Kreps and Scheinkman (1983), Lemma 1) *Q.E.D.*

LEMMA 7 *Full capacity utilization: In any equilibrium* $q_j + \hat{q}_j = k_j$

PROOF: Suppose $k_A > q_A + \hat{q}_A$. The second stage equilibrium remains the unique equilibrium if firm A slightly decreases k_A. Therefore, reducing k_A decreased A's costs without affecting its revenues and $k_A, q_A, \hat{q}_A$ cannot be part of an equilibrium. *Q.E.D.*

Next, we show that first stage allocations are set so that q_j is strictly positive in equilibrium.

LEMMA 8 *All equilibrium quantities are strictly positive. Firm j's equilibrium marginal revenue is equal in the joint and private market.*

PROOF: Given lemma 6, the second statement follows directly from the first statement. We prove the first statement. Suppose without loss of generality that $q_A = 0$. By lemma 7 $\hat{q}_A = k_A$ and by lemma 6 $\min[q_B, \hat{q}_B] > 0$ and B's marginal revenue in both markets is identical. We now show that this cannot be an equilibrium.

1. First we show that, in this equilibrium, marginal revenue in A's private market is c:

(a) If A's marginal revenue in its private market is strictly larger than c, A can slightly increase k_A. Regardless of B's reaction, A can allocate the additional capacity to its private market (i.e. increase $\hat{q}_A$). This strictly increases A's profits. Thus, A's marginal revenue in its private market is at most c.

(b) If A's marginal revenue in its private market is strictly smaller than c, A can slightly decrease k_A. Regardless of B's reaction, by decreasing $\hat{q}_A$ A's profits strictly increase. Thus, A's marginal revenue in its private market is at least c.

2. Firm A's marginal revenue in the overlapping market must be at most c. If it is higher than c, A can strictly increase profits by reducing $\hat{q}_A$ by some positive ε and increasing q_A by the same amount.

3. B's marginal revenue in the overlapping market is strictly lower than A's , as B's quantity in the overlapping market is non-zero.

4. Given that $q_B > 0$, lemma 6 implies that B's marginal revenue in its private market equals its marginal revenue in the overlapping market.

5. B's profits strictly increase by slightly decreasing k_B – the loss in marginal revenue is strictly lower than the costs saving.

Q.E.D.

LEMMA 9 *The game has a unique equilibrium, in which* $q_j + \hat{q}_j = k_j$ *and* $\min[q_j, \hat{q}_j] > 0$.

PROOF: Immediate from combining the preceding three lemata. *Q.E.D.*

LEMMA 10 *Given* m_{-j}*, total quantity and welfare in the overlapping market:*[41]

- Increases in m_j for any $m_j \geq 0$ iff $m_{-j} < .7$
  - Is non-monotonic in m_j iff $m_{-j} \in (0.7, 1.9)$
  - Decreases in m_j for any $m_j \geq 0$ iff $m_{-j} > 1.9$

PROOF: The proof relies on properties on the aggregate quantity as a function of the firms' flexibilities. Setting $m_o = 1$, each firm's flexibility ϕ_j is a monotone transformation of the private market size $\left(\phi_j \equiv -\frac{1}{2}\frac{m_j}{1+m_j}\right)$. The properties we wish to exploit exist only when considering the aggregate quantity as a function of flexibilities. To simplify the analysis we define $\tilde{\phi}_j \equiv \frac{m_j}{1+m_j}$ so the valid range of $\tilde{\phi}_j$ is $[0,1]$ rather than $\left[-\frac{1}{2}, 0\right]$. In this case $m_j = \frac{\tilde{\phi}_j}{1-\tilde{\phi}_j}$.

The proof proceeds as follows:

1. Define $Q\left(\tilde{\phi}_A, \tilde{\phi}_B\right)$ as the total quantity in the overlapping market as a function of the firms' flexibilities
2. Show that $Q(\cdot)$ is convex in each $\tilde{\phi}_j$ and has decreasing differences in $\tilde{\phi}_A, \tilde{\phi}_B$.

[41] The result is provided for the normalization $m_o = 1$.

3. Given the previous steps, it is sufficient to show that :

   (a) $Q(0,0)$ increases in either argument and $Q(1,1)$ decreases in either argument.

   (b) For some $\tilde{\phi}_B^1$, $\frac{\partial Q(\tilde{\phi}_A,\tilde{\phi}_B^1)}{\partial \tilde{\phi}_A}=0$ at $\tilde{\phi}_A=0$. Then total quantity in the overlapping market increases for any m_j iff $m_{-j}\leq\frac{\tilde{\phi}_B^1}{1-\tilde{\phi}_B^1}$.

   (c) For some $\tilde{\phi}_B^2$, $\frac{\partial Q(\tilde{\phi}_A,\tilde{\phi}_B^2)}{\partial \tilde{\phi}_A}=0$ at $\tilde{\phi}_A=1$. Then total quantity in the overlapping market decreases for any m_j iff $m_{-j}\geq\frac{\tilde{\phi}_B^2}{1-\tilde{\phi}_B^2}$.

   (d) For any $m_{-j}\in\left(\frac{\tilde{\phi}_B^1}{1-\tilde{\phi}_B^1},\frac{\tilde{\phi}_B^2}{1-\tilde{\phi}_B^2}\right)$ total quantity in the overlapping market decreases at $m_j=0$ and increases at $m_j\to\infty$ and is therefore non-monotonic.

The steps are proved algebraically in the attached Mathematica notebook. *Q.E.D.*

PROPOSITION 5 *(a) For any $m_A+m_B+m_o=M$, total welfare increases with m_o .*

(b) Fix m_0 and set $m_A+m_B=\overline{m}$. Total welfare over all markets is maximized when the firms are symmetric: $m_A=m_B$. Total welfare increases in m_A iff $m_A<m_B$.

PROOF: The welfare maximizing quantity in each market is $m_j\frac{a-c}{b}$. The dead weight loss in each private market j is thus given by

$$DWL_j=\frac{1}{2}\left(a-\frac{b}{m_j}\hat{q}_j-c\right)\left(\frac{a-c}{b}m_j-\hat{q}_j\right)$$

The DWL in the overlapping market is

$$DWL_o=\frac{1}{2}\left(a-\frac{b}{m_o}\left(q_A+q_B\right)-c\right)\left(\frac{a-c}{b}m_o-q_A-q_B\right)$$

Define

$$DWL\equiv DWL_A+DWL_B+DWL_o\ .$$

1. For (a):

   We need to consider a change in m_o while maintaining $m_A+m_B+m_o=M$. Let $\xi\in[0,1]$ be the share of the change in m_o that is compensated by a change in m_A, and $(1-\varepsilon)$ be the share of the change that is compensated by a change in m_B. Define

$$\Delta\left(\xi,m_o,m_A,m_B\right)\equiv\frac{\partial DWL}{\partial m_o}-\xi\frac{\partial DWL}{\partial m_A}-\left(1-\xi\right)\frac{\partial DWL}{\partial m_B}$$

   To prove the result it is required that for any valid parameter set:

$$\Delta\leq 0$$

- First, it is clear from the formulation that, while $\frac{\partial DWL}{\partial m_j}$ may depend on the exact distribution of market sizes, both directly and through the market quantities (the q's), $\frac{\partial \Delta}{\partial \xi}$ is independent of ξ.
- Next, some algebra (see notebook appendix) shows that $\frac{\partial \Delta}{\partial \xi}$ is continuous and equals zero only when $m_A = m_B$
- By the previous step, it is sufficient to consider only the case $\xi = 0$. We show (in the notebook appendix) that $\Delta \leq 0$ for all possible m_A and m_B when $\xi = 0$.

2. For (b), it is sufficient to show that

$$\frac{\partial DWL}{\partial m_A} - \frac{\partial DWL}{\partial m_B} \gtreqless 0 \iff m_A \gtreqless m_B \ .$$

This is done in the Mathematica notebook appendix.

Q.E.D.

APPENDIX C: PROOFS AND DERIVATIONS FOR THE AIRLINE MODEL EXTENSION

The second stage problem is given by

$$\max_{q^A, \hat{q}^A, \tilde{q}^A \geq 0} \quad P\left(q^A, q^B; m_o\right) q^A + P\left(\tilde{q}^A; \tilde{m}_A\right) \tilde{q}^A + P\left(\hat{q}^A; m_A\right) \hat{q}^A$$

$$\text{(C.1)} \qquad \text{subject to} \qquad q^A + \tfrac{1}{2}\tilde{q}^A \leq k^A$$

$$\text{(C.2)} \qquad \hat{q}^A + \tfrac{1}{2}\tilde{q}^A \leq \hat{k}^A$$

As in the main text, all quantities are strictly positive and the constraints bind.

A standard transformation of the first order conditions implies the second stage optimality condition 3.1.

Letting $\vartheta^j\left(k^A, \hat{k}^A, k^B, \hat{k}^B\right)$ denote the second stage solution for any first stage allocation, the first stage problem for carrier A, given the rival first stage choices of k^B and $\hat{k}^B$ is

$$\max_{k^A, \hat{k}^A} \quad P\left(\vartheta^A, \vartheta^B; m_o\right) \vartheta^A + P\left(\tilde{\vartheta}^A; \tilde{m}_j\right) \tilde{\vartheta}^A + P\left(\hat{\vartheta}^A; m_j\right) \hat{\vartheta}^A - c \cdot \left(k^A + \hat{k}^A\right)$$

C.1. *Proof for Lemma 3*

PROOF: In equilibrium, any first stage allocation $k^j, \hat{k}^j$ defines three equations per carrier: equation 3.1 and inequalities C.1 and C.2 as equalities. The simultaneous solution of these six equations determines the second stage sub-game equilibrium.

Letting $\vartheta^j\left(k^A, \hat{k}^A, k^B, \hat{k}^B\right)$ denote the second stage solution for any first stage allocation, the first stage problem for carrier A, given the rival first stage choices of k^B and $\hat{k}^B$ is

$$\max_{k^A, \hat{k}^A} \quad P\left(\vartheta^A, \vartheta^B; m_o\right) \vartheta^A + P\left(\tilde{\vartheta}^A; \tilde{m}_j\right) \tilde{\vartheta}^A + P\left(\hat{\vartheta}^A; m_j\right) \hat{\vartheta}^A - c \cdot \left(k^A + \hat{k}^A\right)$$

An interior solution is again guaranteed, as in the main text.

The first order conditions are

$$\frac{\partial \vartheta^A}{\partial k^A} \cdot MR^A + P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \frac{\partial \vartheta^B}{\partial k^A} + \frac{\partial \tilde{\vartheta}^A}{\partial k^A} \cdot \tilde{MR}^A + \frac{\partial \hat{\vartheta}^A}{\partial k^A} \cdot \hat{MR}^A = c$$

$$\frac{\partial \vartheta^A}{\partial \hat{k}^A} \cdot MR^A + P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \frac{\partial \vartheta^B}{\partial \hat{k}^A} + \frac{\partial \tilde{\vartheta}^A}{\partial \hat{k}^A} \cdot \tilde{MR}^A + \frac{\partial \hat{\vartheta}^A}{\partial \hat{k}^A} \cdot \hat{MR}^A = c$$

The second stage capacity constraints imply:

$$\begin{aligned} \frac{\partial \vartheta^A}{\partial k^A} + \frac{1}{2}\frac{\partial \tilde{\vartheta}^A}{\partial k^A} &= 1 \\ \frac{\partial \hat{\vartheta}^A}{\partial k^A} + \frac{1}{2}\frac{\partial \tilde{\vartheta}^A}{\partial k^A} &= 0 \end{aligned}$$

Simple algebra yields:

$$\frac{\partial \tilde{\vartheta}^A}{\partial k^A} = 2 - 2\sigma_A \quad ; \quad \frac{\partial \hat{\vartheta}^A}{\partial k^A} = \sigma_A - 1$$

And similarly

$$\frac{\partial \tilde{\vartheta}^A}{\partial \hat{k}^A} = 2\hat{\sigma}_A \quad ; \quad \frac{\partial \vartheta^A}{\partial \hat{k}^A} = -\hat{\sigma}_A$$

Applying the envelope theorem to the second stage response, in equilibrium

$$\begin{aligned} \frac{\partial \vartheta_B}{\partial k_A} &= \frac{\partial q_B}{\partial q_A} \cdot \frac{\partial \vartheta_A}{\partial k_A} = \phi_B \sigma_A \text{ , and} \\ \frac{\partial \hat{\vartheta}_B}{\partial k_A} &= \frac{\partial q_B}{\partial q_A} \cdot \frac{\partial \vartheta_A}{\partial \hat{k}_A} = -\phi_B \hat{\sigma}_A \end{aligned}$$

Using these, the first order conditions are:

$$\sigma_A \cdot \left(MR^A + P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \phi_B\right) + (2 - 2\sigma_A) \cdot \tilde{MR}^A + (\sigma_A - 1) \cdot \hat{MR}^A = c$$
$$(-\hat{\sigma}_A) \cdot \left(MR^A + P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \phi_B\right) + 2\hat{\sigma}_A \tilde{MR}^A - (1 - \hat{\sigma}_A) \cdot \hat{MR}^A = c$$

Collecting terms and using equation 3.1 :

$$\begin{aligned} \sigma_A \cdot P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \phi_B + 2\tilde{MR}^A - \hat{MR}^A &= c \\ (-\hat{\sigma}_A) \cdot P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \phi_B - MR^A + 2\tilde{MR}^A &= c \end{aligned}$$

Adding the two equalities yields

$$P'\left(\vartheta^A, \vartheta^B; m_o\right)\vartheta^A \cdot \phi_B\left(\sigma_A - \hat{\sigma}_A\right) + 4\tilde{MR}^A - \hat{MR}^A - MR^A = 2c\,.$$

Using equation 3.1 again obtains

$$P'\left(\vartheta^A,\vartheta^B;m_o\right)\vartheta^A\cdot\phi_B\left(\sigma_A-\hat{\sigma}_A\right)+\hat{MR}^A+MR^A=2c\,.$$

Rearranging yields the desired result. *Q.E.D.*

APPENDIX D: EMPIRICAL CONSTRUCTS AND DATA SUMMARY

This section shows how the measures of "a carrier's rival carriers' private demand" ,$\Gamma_{i,m,t}$, and "a carrier's own private demand", (denoted below $\Lambda_{i,m,t}$) are built. As a reminder, a unit of observation in the database is a route-carrier-year-quarter.

A route will be characterized by three legs: a leg from city n[42] to the 'hub' city, a leg from the 'hub' city to city m, and a leg from city n to city m. If no passengers fly non-stop, then this last leg is nonexistent for this route. If all passengers fly non-stop, then the first two legs are nonexistent for this route.[43] For each one of these legs we calculate the total number of passengers using that leg. Formally, suppressing the year-quarter indices and allowing i to index a focal carrier (j will be used for rival carriers), let $x^i_{m,n}$ be the number of passengers serviced by carrier i with non-stop service between cities m and n, and let $z^i_{m,h,n}$ be the number of passengers serviced with connecting service between cities m and n, where the connection is in city h. Then, we can define the number of passengers on a leg as:

$$l^i_{m,n}=x^i_{m,n}+\sum_s z^i_{m,n,s}+\sum_s z^i_{s,m,n}$$

Let $\rho^i_{m,n}$ be an indicator equal to 1 if carrier i *does not* service route (m,n), irrespective of the type of service (non-stop or connecting). That is, if carrier i has less than 5% market share on the route. Then, for leg (m,n), for carrier i, and for rival carrier j, we can define a carrier's own private demand as:

$$\text{(D.1)}\qquad \lambda^{i,j}_{m,n}=\frac{x^i_{m,n}\rho^j_{m,n}+\sum_s z^i_{m,n,s}\rho^j_{m,s}+\sum_s z^i_{s,m,n}\rho^j_{s,n}}{l^i_{m,n}}$$

To aggregate across the three legs on a route, let $\delta^i_{m,n}$ be the percentage of passengers flying non-stop: $\delta^i_{m,n}=x^i_{m,n}/(x^i_{m,n}+z^i_{m,h,n})$. Then , assuming a carrier's private demand on a *route* is constrained by the weakest private demand on a *leg*, carrier i's private demand on route (m,n) with respect to carrier j is:

$$\text{(D.2)}\qquad \tau^{i,j}_{m,h,n}=\delta^i_{m,n}\lambda^{i,j}_{m,n}+\left(1-\delta^i_{m,n}\right)\min\left\{\lambda^{i,j}_{m,h},\lambda^{i,j}_{h,n}\right\}$$

We can then aggregate across rival carriers to form a single measure of a carrier's own private demand and a carrier's rival

[42] We use the terminology of city n and city m instead of the more commonly understood *origin* and *destination* since routes are defined as non-directional. Moreover, it is understood that if route (m,n) exists, then route (n,m) does not exist.

[43] For the specific algebraic calculations below we assign 0 passengers from that route to that leg, but do allow all three legs to exist (as a mathematical construct).

TABLE III

SUMMARY STATISTICS

	Mean	**SD**	**Min**	**Max**
Market Share	0.43	0.25	0.05	0.95
Rival's Private Demand (Γ)	0.21	0.15	0	0.98
Price ($)	139	66	11	2,650
Endpoint Market Shares	0.20	0.12	0.00	0.85
Southwest's Private Demand ($\tau_{m,n}^{WN,i}$)	0.02	0.05	0	0.83
# of Non-Stop Carriers*	1.33	1.21	0	7
# of Carriers*	3.32	1.28	2	9
Daily Passengers on Route*	941	1,548	59	27,370

Unit of observation is a route-carrier-year-quarter. The sample includes only observations used in the regression: carriers with Private Demand ≤ 0.1 , that service more than 55 daily passengers, and that have more than 5% market share on the route. 58,361 obs. (*) Unit of observation is a route-year-quarter; 44,554 observations.

TABLE IV

PRIVATE DEMAND MEASURES

	Mean	**SD**	**Min**	**Max**
Rival's Private Demand (Γ)	0.192	0.125	0	1
Own's Private Demand (Λ)	0.197	0.132	0	1

Unit of observation is a route-carrier-year-quarter. 2,347,470 observations.

carriers' private demand:

$$\Lambda_{m,n}^{i} = \sum_{j \neq i} \frac{s_{m,n}^{j}}{\sum_{j \neq i} s_{m,n}^{j}} \tau_{m,h,n}^{i,j}$$

$$\Gamma_{m,n}^{i} = \sum_{j \neq i} \frac{s_{m,n}^{j}}{\sum_{j \neq i} s_{m,n}^{j}} \tau_{m,h,n}^{j,i}$$

These are the variables used in the regression and in the sample selection. Summary statistics of these variables and of the main controls are presented in tables III and IV.