THE COMPETITIVE EFFECT OF MULTIMARKET CONTACT

Guy Arie[1], Sarit Markovich[2] and Mauricio Varela[3]

Changes in the extent of multi-market contact (MMC) between firms often affect market outcomes – quantities and prices. This paper challenges the standard economic interpretation of this phenomenon as an indication of tacit collusion. We show that a strategic but purely competitive effect of changes in MMC can change the quantity provided in a market by a firm by as much as 50%, and the prices a firm sets by as much as 20%. This may have important welfare implications, specifically with regards to horizontal mergers. Studying mergers that span several markets, we show that a myopic merger policy may thwart a surplus-increasing merger wave.

1. INTRODUCTION

Large firms are often active in more than one market and commonly compete with each other in many, but not necessarily all, markets.[1] There is by now a large empirical literature documenting the relation between the extent of firms' multi-market contact (MMC) and market outcomes.[2] While inconclusive, most empirical work finds a positive correlation between increases in firms' MMC and prices. The current microeconomics explanation for MMC effects on market outcome is that MMC facilitates mutual forbearance (i.e., tacit collusion).

This paper provides an alternative, purely competitive, microeconomics foundation for the relation between MMC and market outcomes. Whenever firms make investments that can be then used to serve multiple markets, changes in MMC will affect prices and outcomes by competitive (i.e., not collusive) responses of the competing firms. We call this the *competitive effect of MMC (*henceforth C-MMC effect*).*

May 2014. We thank seminar participants at Kellogg, Israel IO Day, UGA and Yale as well as Meghan Busse, David Dranove, Chaim Fershtman, Aaron Gellman, Gautam Gowrisankaran, Kei Hirano, Tom Hubbard, Mike Mazzeo, Michael Raith, William Rogerson, Mark Satterthwaite, Jeroen Swinkels and Michael Whinston for helpful comments. Arie gratefully acknowledges financial support by the Northwestern University Transportation Center Dissertation Year Fellowship and a Kellogg GM Center scholarship.

[1]University of Rochester, Simon School of Business (guy.arie@simon.rochester.edu).

[2]Northwestern University(s-markovich@kellogg.northwestern.edu).

[3]University of Arizona (mvarela@email.arizona.edu).

[1]In the airline industry, for example, each city-pair is typically considered a market and carriers overlap on some but not all of their markets. In other industries (e.g., retail, banking, restaurants) markets are typically defined by locality and product variety/quality. Large chains partially overlap on many but not all markets and also face competition from local establishments.

[2]Jayachandran et al. (1999) surveys the earlier studies. Earlier and recent examples include Heggestad and Rhoades (1978); Evans and Kessides (1994); Gimeno and Woo (1999); Parker and Roller (1997); Jans and Rosenbaum (1997); Fernandez and Marin (1998); Pilloff (1999); Young et al. (2000); Bilotkach (2011); Feinberg (2011); Ciliberto and Williams (2012) for MMC studies on the airline, banking, telecommunication, cement, software, hotel and the global cooking oil industries.

Focusing only on the case of completely identical markets, we show that changes in MMC can, through the C-MMC effect, change the quantity provided in a market by a firm by as much as 50%, the prices a firm sets by as much as 20% and the firm's profits by over 10%. In comparison, the mutual forbearance effect of MMC (henceforth MF-MMC effect) does not exist if the different markets are similar unless some additional, non-trivial conditions are met (see Bernheim and Whinston (1990) Proposition 1).

MMC is closely related to horizontal merger evaluation. Mergers often increase the number of markets a firm operates in, and by this drastically change the extent of MMC between the merged firm and its competitors. Section 4 shows that a merger can increase or decrease consumer surplus and total welfare without changing market power, production efficiency or facilitating collusion.

The main welfare implication of C-MMC is that asymmetry in scope between firms improves welfare. Welfare is typically maximized when a large multi-market firm competes with smaller local firms, subject to potential scope related cost savings. Section 1.1 provides an intuitive example for the C-MMC effect and the gains from asymmetry.

The competitive and the mutual forbearance effects may have qualitatively different welfare predictions and therefore understanding which effect applies has significant policy implications. The main implication pertains to dynamic merger policy. Current economic merger evaluation typically analyzes mergers on a market-by-market level. Under this setting, Nocke and Whinston (2010) show that when considering mergers within a specific market, a myopic merger policy is sufficient – a profitable merger increases consumer surplus whether or not other consumer-surplus-increasing mergers are approved. The C-MMC effect, however, implies that when considering mergers that span several markets, it may well be that one merger is surplus increasing while a second merger is not, even if both create the same cost efficiencies and have no implications on the within-market structure. We show in section 4 how this may cause a myopic regulator that is committed to reject surplus-decreasing mergers to thwart a surplus-increasing merger wave.

Our main comparative statics describing the C-MMC effect apply whether firms compete in prices or quantities and are independent of any specific demand form. We show that when firms compete in prices, any increase in MMC decreases prices for all firms in the market and in *all* other markets served by these firms. That is, an increase in MMC between firms A and B increases equilibrium prices also in markets served *only* by A or B. When firms compete in quantities, we show that an increase in a firm's MMC increases its own quantity and decreases its rival's quantity.

The model allows us to also characterize the implications of an increase in overlap between symmetric firms. This describes the changes in market outcomes as two firms gradually enter each-others markets, possibly to the point that all markets are served by both firms. Here, the C-MMC effect is non-monotonic for industries in which firms compete in quantities. If firms overlap in roughly less than half of the markets they serve, an increase in MMC increases quantities (and decreases prices). The effect is reversed if firms overlap in most markets. In industries characterized by price competition, an increase in overlap always decreases prices, leading to a similar prediction as the MF-MMC effect, though assuming identical markets.

While the comparative statics of the C-MMC and MF-MMC effects are often identical, there is an important qualitative difference between the two effects. Mutual forbearance implies that MMC causes multi-market firms to produce less than single-market firms. However, the C-MMC effect elicits *higher* production rates than in the no-MMC benchmark. Therefore, simple pre vs. post merger quantity or price evaluations can a mislead regulators (or policy makers) to the conclusion that MMC is enabling firms to produce at higher margins and lower surplus while in fact it is the absence of any MMC that strictly decreases surplus.

At the basis of the C-MMC effect is the assumption that firms make sunk investments that are transferable across markets. The formalization of 'invest then produce' dates back to at least Arrow (1968) and originally considered capital investment followed labor production costs. Such investments are common in multi-market industries. Airlines, for example, make fleet scheduling decisions many months before the first seat on the flight is offered for sale (see e.g. Barnhart et al. (2003) and Lohatepanont and Barnhart (2004)) . In many industries these capacity choices are reflected in plant size or number of plants[3]

Multi-market firms have the *flexibility* to reallocate a sunk investment across its different markets. If market conditions deteriorate for American Airlines on the route NYC-Chicago, for example, it can reallocate more of the seats on the flight from NYC to Chicago to the route NYC-Denver connecting in Chicago; the more routes American Airlines serves out of NYC with a connection at Chicago, the higher its flexibility in reallocating the seats on the flight NYC-Chicago.

The C-MMC effect arises when a *rival can take advantage of this flexibility.* If a rival is aggressive in an overlapping market, a multi-market firm can reallocate a larger share of the sunk investment

[3]See e.g. Friedman (1983) for an example in car manufacturing; Christensen and Caves (1997) for an analysis in the pulp and paper industry and Pesendorfer (2003) for a follow-up analysis considering merger effects; and Hortaçsu and Syverson (2007) for an example in the cement industry.

into markets in which the rival does not operate. Thus, flexibility increases the rival's profit from aggressive deviations.

This strategy requires the rival to *commit* to an aggressive behavior: increase the share of its sunk investments that will be used in the overlapping markets. The fewer markets the firm serves, the stronger its commitment power. We show that in industries with MMC, equilibrium outcome can be defined in terms of the firms' *flexibility* and *commitment power*.

This paper contributes to four strands of the literature: microeconomics analysis of multimarket contact, merger policy, preemption through investments, and competition in strategic substitutes vs. strategic complements.

The existing microeconomics analysis of MMC focuses on the possibility that MMC affects tacit cooperation. This *collusive effect of MMC* was suggested by Edwards (1955) and formalized and studied in detail by Bernheim and Whinston (1990). As firms interact over more markets, the long run returns from collusion are higher; increasing the likelihood of tacit collusion.[4] Our model complements the mutual forbearance literature and shows that MMC can alter firms' strategies and market outcomes absent any long term collusive strategies. Whether firms adopt collusive strategies when facing MMC or other competitive strategies is then an empirical question.

The different microeconomics foundations of C-MMC compared to the MF-MMC provide possible empirical approaches to distinguish between the two effects. In particular, the main empirical prediction of our paper is that the C-MMC effect is present *only* across markets that share a sunk transferable investment. In other markets, a change in MMC should not affect competitive behavior.

To the best of our knowledge, this empirical hypothesis was tested only by Gimeno and Woo (1999). The authors there find that in the airline industry, changes in MMC only affected markets that used the same hub, consistent with our model's prediction. Section 5 discusses this result.

Following Farrell and Shapiro (1990), horizontal merger analysis considers whether possible efficiency gains from a profitable merger compensate for potential surplus loss resulting from the increase in market power for the merging firm. However, mergers often occur between firms that operate in different markets.[5] Our framework extends the analysis to the evaluation of multi-market competitive effect of mergers.

[4]Bernheim and Whinston (1990) show, however, that the increase in the number of markets also implies an increase in the returns for deviating from any collusive agreement and so some additional conditions must be met (see Proposition 1 there). See also Feinberg (1984) for a formulation of mutual forbearance using conjectural variations.

[5]In two recent large horizontal mergers the merging firms had little overlap. Delta and Northwest argued that they do not compete directly in most markets. Comcast and Time Warner Cable, by law, did not compete directly in any market.

Spence (1977) and Dixit (1980) formalized the economic use of committed investments in capacity or in cost reduction to deter entry and help incumbents achieve a Stackelberg type leadership position in a market. By making committed investments in an early stage, incumbents can "push out" their reaction functions. Potential entrants that have not made such committed investments are thus deterred to enter or to obtain significant market share. In our framework, all firms make non-reversible investment decisions, but these are not fully committed to the markets in which firms overlap. The size of private markets limit how much of the initial investments are truly committed to the overlapping markets–the larger the size of the private markets, the weaker the commitment. As such, all firms' reaction curves are pushed out but the amount by which they are pushed out depends on the relative size of the non-overlapping markets.

The terms strategic complements and strategic substitutes were coined in the seminal work of Bulow et al. (1985) (hereafter BGK), which showed how the type of competition (complements or substitutes) and the cost structure of firms (economies of scale or diseconomies of scale) affected how firms competed when one firm had access to private markets. The basic model presented here extends the analysis in BGK in two important ways. First, we endogenize the investment decision and allow firms *to choose* investment decisions, which determine the extent of diseconomies across markets. In this sense we endogenize the cost structure of the firm. Second, we allow for both firms to have access to private markets. By adding these extensions, some of BGK's results regarding the effect of MMC no longer hold and the two forms of competition (substitutes and complements) no longer mirror each other.[6]

Specifically, our results depart from the results in BGK in three main areas: (i) in contrast to BGK, the effect of a change in the rival's MMC in our model is qualitatively the same whether firms compete in strategic substitutes or complements; (ii) we find a *non-monotonic effec*t for a joint change in MMC when firms compete in strategic substitutes; and (iii) when both firms have private markets, diseconomies across markets creates strategic links between otherwise unrelated markets.

Another implication of the analysis regards the interpretation of quantity differences as indicative of efficiency differences for international firms. There is now considerable evidence that exporting firms produce more and are more efficient and profitable than firms that sell domestically. See e.g., Aw and Hwang (1995); Clerides et al. (1998); Bernard and Bradford Jensen (1999); Pavcnik (2002). This

[6]Valletti et al. (2002) also allow firms to have overlapping and private markets. However, they force the firms to have uniform price in all markets and do not consider other multi-market effects.

is consistent with the theoretical results established in Melitz (2003), which shows that exporting firms tend to be more efficient than those limited to the domestic market. Our paper suggests caution, in particular when *empirically* inferring the *extent* of efficiency differences. The C-MMC effect implies that a firm that does not have private markets (a domestic firm) may strategically reduce output and increase price. A rival firm that does have private markets (exporter) would then follow with a reduction in output and a price increase, albeit of smaller magnitudes. As a result, even if the firms enjoy the same efficiency, the exporting firm would have a larger market share, higher profits, and lower prices than the domestic firm.

Section 2 presents and solves the model based on quantity competition. Section 3 analyzes price competition. Section 4 analyzes horizontal mergers and discusses the implications for merger policy, section 5 provides suggestive empirical evidence, and section 6 concludes.

1.1. *Illustrative Example*

We start with a simple example that illustrates the basic intuition behind the C-MMC effect.

Two firms – A and B – compete a-la Cournot in market 1. Demand is $P = 2 - Q$ and marginal cost is 1, so in equilibrium each firm produces $\frac{1}{3}$. Now suppose firm A also operates in an identical market, market 2 with rival C. In addition, assume a second stage after quantities are set in which multi-market firms, in our example firm A, can freely transfer quantities between markets.

If A sets the Cournot quantity of $\frac{1}{3}$ in each market, it is no longer optimal for B (or C) to respond by setting the Cournot quantity of $\frac{1}{3}$. To see why, suppose that B slightly increases its quantity. As a result, A's marginal revenue in market 1 is lower than A's marginal revenue in market 2. Because it can, A would use the second stage to transfer some units from market 1 to market 2. A's access to multiple markets allows it to react to B's deviation in a way it couldn't otherwise.

In the extreme case, if A services a large number of identical markets, each with a different competitor, it is costless for A to redistribute any marginal excess quantity from B's market to its other markets. The model, thus, converges to the Stackelberg model. The multi-market firm effectively decides on its market quantities after all the single market firms do. In other words, as the number of markets A services increases, its rivals become more and more aggressive.

To complete the example, suppose again that A only competes in two markets, with B in one and C in the other, and consider the effect of a merger between firms B and C. Clearly, the merger increases MMC between A and its rival–the now merged firm. However, the merger also forces the merged firm to become softer and return to the Cournot quantities in its markets. If the merged firm deviates in market 1, for example, from the Cournot quantity of $\frac{1}{3}$, A's reaction in market 2 decreases the merged firm's profits in market 2 by exactly the amount its profits increase in market 1.[7]

That is, an increase in MMC decreased quantities and increased prices. In the extreme case considered above, a merger of a large number of smaller firms would move the market from the Stackelberg equilibrium to the Cournot.

While the example assumed quantity competition in the second stage, the result is driven by the capacity *decision* that is made in the first stage. As a result, in contrast to the results in BGK, if firms first set capacities and then compete in prices, the same intuitive argument applies, rendering it profitable for the smaller firm (B) to deviate in the first stage and set excess capacity. This result formalized in section 3 and the corresponding appendix.

Finally, observe that B's deviation from the Cournot equilibrium would have been qualitatively the same if A was a monopolist in some (or all) of its other markets, if these other markets would have been perfectly competitive, or anything in between. Consequently, the model assumes that any non-overlapping markets are monopolized. The implications are unaffected.

2. QUANTITY COMPETITION

2.1. *Setup*

Consider an industry with two firms, identified by $i \in \{A, B\}$ and three types of markets: *overlapping* markets in which both firms are active, and *private* markets for each firm – markets in which firm A operates but firm B does not, and markets in which firm B operates but firm A does not. All markets of each type are identical in terms of demand. However the number of markets of each type may vary. In particular, we denote by m_o, m_A, and m_B the measure, or number, of the overlapping markets, A's private markets, and B's private markets, correspondingly.

The model has two stages. In the first stage firms simultaneously make a sunk transferable investment, denoted k_A, k_B. Firms pay a constant marginal cost c per unit of investment.[8] To fix ideas, we refer to this investment as production capacity. However, it may be interpreted as any investment that is sunk, transferable across markets, and can be utilized to increase production.

[7] Intuitively, the two markets must have the same equilibrium as one market with double the size.

[8] The results are qualitatively unaffected when allowing for weakly convex costs and cost asymmetries across firms. See also the discussion following proposition 1.

After both capacities are fixed, in the second stage, firms *compete in quantities* subject to their installed capacity.[9] Each firm chooses how many units to provide to each of the overlapping markets and how many to provide to each of its private markets. For simplicity we assume there is no direct cost when choosing quantities. Nevertheless, a firm's total output across all markets cannot be larger than its installed capacity. Markets clear accordingly.

We denote by q_A and q_B the quantity sold by each firm in each of the overlapping markets.[10] That is, the total quantity offered in all overlapping markets is $m_o \cdot q_A + m_o \cdot q_B$. Similarly, denote by $\hat{q}_A$ and $\hat{q}_B$ the quantity sold in each private market so that the total quantity firm i offers in its private markets is $m_i \cdot \hat{q}_i$.

Firm i's inverse demand in each overlapping market is $P_i\left(q_i, q_{-i}\right)$. Similarly, firm's inverse demand in a private market is $\hat{P}_i\left(\hat{q}_i\right)$.

We assume the following standard regularity conditions on the inverse demand functions:

ASSUMPTION 1 *All inverse demand functions are twice differentiable, downward sloping, weakly concave everywhere, and satisfy:*

1. $\frac{\partial P_i}{\partial q_{-i}} \leq 0, \frac{\partial^2 P_i}{\partial q_i \partial q_{-i}} \leq 0$
2. $\hat{P}_i(0) > c$ *and* $P_i\left(0, q_{-i}^M\right) > c$, *where* q_j^M *is the monopoly quantity*[11]
3. $\frac{\partial^2 P_i}{\partial q_i^2} \leq \frac{\partial^2 P_i}{\partial q_i \partial q_{-i}} \leq \frac{\partial^2 P_i}{\partial q_{-i}^2}$

The first assumption implies that any sub-game equilibrium is unique and continuous in the first stage capacities k. The second assumption guarantees that no good dominates the overlapping markets such that if the firm acts as a monopoly in these markets, the other firm optimally stays out of the overlapping markets. The last assumption is convenient to guarantee that the game is always in strategic substitutes. Note that all assumptions on rivals' output hold trivially for any homogenous goods model.

An *equilibrium* $(\mathbf{k}^\star, \mathbf{q}_A^\star, \mathbf{q}_B^\star)$ is a pair of scalars $\mathbf{k}^\star = (k_A^\star, k_B^\star)$ indicating the capacity set by each firm, and two pairs $\mathbf{q}_A^\star = (q_A^\star, \hat{q}_A^\star)$ and $\mathbf{q}_B^\star = (q_B^\star, \hat{q}_B^\star)$ indicating the quantity allocation chosen by each firm in each market type.

The game is solved through backward induction. In the second stage, given capacities k_A and k_B,

[9] Competition in prices is solved in the next section.

[10] Since markets are identical in terms of demand, each firm will offer identical quantities in all markets of a certain type.

[11] That is q_j^M solves $q_j \cdot \frac{\partial P_j(q_j,0)}{\partial q_j} + P_j\left(q_j, 0\right) = c$

firms set quantities for each market type to maximize profit. As costs were already spent, this is simply revenue:

$$\Pi_i(k_A,k_B) = \max_{q_i,\hat{q}_i \geq 0} \quad m_o q_i \cdot P_i\left(q_i,q_{-i}\right) + m_i \cdot \hat{q}_i \cdot \hat{P}_i\left(\hat{q}_i\right) \quad \text{s.t.} \quad m_o q_i + m_i \hat{q}_i \leq k_i \tag{2.1}$$

A sub game equilibrium $(\mathbf{q}_A, \mathbf{q}_B; \mathbf{k})$ is the set of quantity allocations that form an equilibrium given the first stage capacities $\mathbf{k}$.[12] Throughout, we use η_i and $\hat{\eta}_i$ to denote firm i's marginal revenue curve in the overlapping and private markets, respectively:

$$\begin{aligned}\eta_i &= q_i \cdot \frac{\partial P_i\left(q_i,q_{-i}\right)}{\partial q_i} + P_i\left(q_i,q_{-i}\right)\\ \hat{\eta}_i &= \hat{q}_i \cdot \hat{P}_i'\left(\hat{q}_i\right) + \hat{P}_i\left(\hat{q}_i\right)\ .\end{aligned}$$

Assumption 1 allows us to describe the equilibrium strategies of the subgame as a function of the first stage capacities, which we define with a slight abuse of notation as: $q_i(k_A,k_B)$ and $\hat{q}_i(k_A,k_B)$.

The first stage problem for each firm is given by:

$$\max_{k_i \geq 0} \Pi_i(k_A,k_B) - ck_i \quad . \tag{2.2}$$

The first stage profit functions determine the equilibrium capacities (k_A,k_B), which in turn map into the equilibrium second stage quantities: $q_i = q_i(k_A,k_B)$.

The next lemma identifies a basic characteristic of the equilibrium that will be used throughout. All proofs are provided in the appendix.

LEMMA 1 *In equilibrium, in the second stage:*

1. *The capacity constraint binds:* $k_i = m_o q_i + m_i \hat{q}_i$.
2. *Firm i's marginal revenue is identical in both types of markets in which it is active:* $\eta_i = \hat{\eta}_i$.

The first claim is identical to those made in Spence (1977) and Dixit (1980).[13] The reasoning behind the second claim is standard. If the marginal revenue in one market type is larger than in the other type (e.g., $\eta_i > \hat{\eta}_i$), optimality requires diverting quantity from the markets with lower marginal revenue to the markets with the larger marginal revenue.

[12] As the sub-game profits for each player are not affected directly by the rival's first stage choice, it is irrelevant whether first stage capacities are observed.

[13] Firm B's best response in the second stage depends only on A's quantity in each overlapping market (q_A). Therefore, if for any first stage decisions k_A,k_B, firm A expects to have excess capacity ($k_A > m_0 q_A + m_A \hat{q}_A$), it should reduce k_A and set the same second stage quantities. Firm B's best response in the second stage is unaffected so firm A's revenues are unaffected, but A's costs are strictly lower.

2.2. *Measuring MMC*

Before characterizing the equilibrium we formalize three concepts: *MMC, flexibility,* and *commitment power.*

DEFINITION 1 The extent of MMC for firm i is $\lambda_i = \frac{m_o}{m_i}$

Firm i's MMC (with firm j) is the ratio between the number of overlapping markets for i and j, and i's private markets. When m_o is large relative to m_A and m_B, the degree of MMC is large. MMC can be asymmetric: for example, if $m_A \gg m_o \gg m_B$ then MMC is small for firm A and large for firm B.Our definition of MMC departs from most empirical studies that consider m_o as the extent of MMC.[14] As will become apparent in the model, ignoring differences between the firms in the non-overlapping markets can lead to misguided results.

DEFINITION 2 Firm i's *flexibility*, ϕ_i, is the firm's second stage reaction to a change in its competitor's quantity allocation: $\phi_i \equiv \frac{\partial q_i}{\partial q_{-i}}$

Flexibility measures A's best response to B's second stage deviation in the equilibrium neighborhood. Flexibility differs from the standard best response, which we denote $\overline{\phi}_i$,[15] because the economic cost of increasing q_i is varying. In particular, as all production costs have been paid for in the previous stage, the cost of an increase in quantity in the overlapping markets is the opportunity cost of the required decrease in the private markets quantity.

When A's extent of MMC (λ_A) is large, A has a relatively small number of private markets. In these settings, A's flexibility is very small ($\phi_A \to 0$) as given its small number of private markets its first stage capacity must primarily serve the joint markets. In contrast, when A's extent of MMC is small, its private markets can absorb any excess capacity without any effect.

LEMMA 2 *In equilibrium,*

$$\phi_i = -\frac{\frac{\partial \eta_i}{\partial q_{-i}}}{\frac{\partial \eta_i}{\partial q_i} + \lambda_i \frac{\partial \hat{\eta}_i}{\partial \hat{q}_i}} \in \left[\overline{\phi}_i, 0\right] \ .$$

[14] More recently, Bilotkach (2011) uses both our relative measure (λ_i) and the standard m_o measure.
[15] Using marginal revenues, the result for the standard Cournot is

$$\overline{\phi}_i = -\left(\frac{\partial \eta_i}{\partial q_{-i}} / \frac{\partial \eta_i}{\partial q_i}\right) \quad .$$

As expected, flexibility is negative as competition is in strategic substitutes, and as $\lambda_i \to 0$, flexibility approaches the standard Cournot.

While flexibility may be profitable when dealing with uncertainty, in our setting, flexibility in the second stage makes the firm vulnerable. If firm A is flexible (ϕ_A is large in absolute terms) while firm B is not ($\phi_B \to 0$), B can make its second stage decision in the first stage. The resulting dynamics are as if the flexible firm is a Stackelberg follower and its rival a Stackelberg leader. However, for this to work, firm B needs to be able to make a credible first stage commitment:

DEFINITION 3 Firm i's *commitment power*, σ_i, is the change in the firm's second stage quantity in the overlapping markets following an additional unit of capacity:

$$\sigma_i \equiv m_o \frac{\partial q_i}{\partial k_i}$$

Commitment power is simply the fraction of the additional unit of capacity that would be allocated to the overlapping markets. Ideally, the firm would like to set its capacity level equal to the sum of the optimal quantity in its private and overlapping markets. However, this is generally not possible. As lemma 1 indicates, the second stage decision equalizes marginal revenues across both types of markets for any first stage capacity. For example, suppose both firms set capacity so that each can set the monopoly quantity in its private markets and the duopoly quantity in the overlapping markets. With those allocations in the second stage, i's MR is the same in both market types. If i now tries to take advantage of its rival's flexibility and increase capacity so as to increase quantity in the overlapping markets, this will reduce i's MR in the overlapping markets. As a result, it is strictly optimal for i to use the capacity increase to also increase quantity in its *private* markets by a small amount.

A firm's commitment power therefore depends on the extent of the firm's MMC and the shape of the MR curves in all its markets. The MR curve in the overlapping markets, in turn, depends on how accommodating is the rival–i.e., on the rival's flexibility. Thus:

LEMMA 3 *In equilibrium,*

$$\sigma_i = \frac{\lambda_i \frac{\partial \hat{\eta}_i}{\partial \hat{q}_i}}{\frac{\partial \eta_i}{\partial q_i} + \frac{\partial \eta_i}{\partial q_{-i}}\phi_{-i} + \lambda_i \frac{\partial \hat{\eta}_i}{\partial \hat{q}_i}} \in [0, 1]$$

Since an increase in any quantity reduces the marginal revenue, σ_i is positive. If firm i has no private markets ($\lambda_i \to \infty$), any first stage investment will be used in the overlapping markets and $\sigma_i \to 1$. In contrast, if the number of its private markets is large relative to the number of overlapping markets ($\lambda_i \to 0$), the bulk of i's capacity-increase must be used in its private markets, and $\sigma_i \to 0$.

2.3. *Equilibrium*

The next proposition characterizes the equilibrium.

PROPOSITION 1 *The equilibrium of the game is given by the following equation*

$$c - \eta_i = c - \hat{\eta}_i = q_i \frac{\partial P_i\left(q_i, q_{-i}\right)}{\partial q_i} \cdot \phi_{-i} \cdot \sigma_i > 0 \qquad i \in \{A, B\}\ . \tag{2.3}$$

Firms behave more aggressively under MMC than they would have under standard Cournot.

- *If MMC is large for one firm, $\lambda_i \to \infty$, and small for the other, $\lambda_{-i} \to 0$, then the equilibrium in the overlapping markets is the Stackelberg equilibrium with firm i as the Stackelberg leader.*
- *If both firms' MMC is large, $\lambda_i \to \infty$, or small, $\lambda_i \to 0$, the equilibrium in the overlapping markets is the Cournot equilibrium.*

Because η, $\frac{\partial P}{\partial q}$, ϕ and σ depend only on the model primitives (and each other), a closed form solution should generally be obtained whenever it can be obtained in a model without MMC. This is illustrated in the appendix for the case where demand is linear. In addition, observe that if the firms have different marginal costs, the only adjustment required in the result is to replace c with c_i.

In the standard Cournot equilibrium, firms set marginal revenue at marginal cost ($\eta_i = c$). We refer to the difference between cost and marginal revenue in proposition 1 as firm *i's competitive effect of MMC*, or C-MMC effect. Given that σ_i and ϕ_i depend on the ratios λ_i and λ_{-i} rather than on the absolute values $m_o, m_i,$ and m_{-i}, so does the C-MMC effect. Underlying the C-MMC effect is the observation that the existence of a private market affects the firm's reaction in the second stage, and that the firm's rival may take advantage of this.

The ability to commit in the first stage is closely related to the Stackelberg analysis. Suppose firm A mainly serves the overlapping markets ($\lambda_A \to \infty$) while firm B mainly serves its private markets ($\lambda_B \to 0$). This would be the case, for example, if firm A is a local firm while firm B serves many localities. Assume that firms set quantities ignoring the C-MMC effect, so that in all markets marginal revenue equals marginal cost. Note that the marginal revenue in all markets for all firms is identical ($= c$). Now suppose that between the first and the second stage, demand in the overlapping markets m_o decreases. Both firms are "stuck" with over-capacity destined to the overlapping markets. Firm A does not have much alternative options for its excess capacity. However, firm B can divert the quantity destined for the overlapping markets to any of its other markets. As the marginal revenue in those other markets equals marginal cost, if there are enough of these other local markets, firm B's profits are almost unaffected. This is exactly the implication of B's low MMC: firm B is flexible and can absorb any excess capacity from the overlapping markets in its private markets. At the extreme ($\lambda_A \to \infty$ and $\lambda_B \to 0$), firm A effectively sets its quantity in the overlapping markets in the first stage while firm B sets it only in the second stage – the Stackelberg result is obtained.

MMC provides an extra incentive for firms to be aggressive: the rival has the flexibility to accommodate aggressive behavior by hurting other markets in which the aggressive firm is not active. However, as the rival's MMC (λ_{-i}) increases, the rival "runs out" of flexibility ($\phi_{-i} \to 0$), and the firms converge to the standard Cournot result.

The existence of MMC also affects the behavior in the private markets. In order to commit to being more aggressive in the overlapping markets, a firm must increase its private markets' quantity. As quantities never exceed the surplus maximizing level, the welfare effect is unambiguous. Total market quantity, total welfare and consumer surplus are always higher in all markets as a result of MMC, while total industry profit in equilibrium is always lower.

Proposition 2 *Consumer and total surplus is always at least as high in industries with MMC as without. Total industry profit is always at most as high in industries with MMC as without.*

2.4. *Comparative Statics*

The next two results summarize the comparative statics. We first focus on the case that MMC changes only for one firm. That is, λ_i changes holding λ_{-i} fixed, or vice versa. In the model, this is the case where the number of private markets of one firm changes. Overall, large MMC provides a first mover advantage. As A's number of private markets increases, its MMC (λ_A) and its first mover advantage decrease: A's quantity per market decreases and B's quantity per market increases.

The comparative statics require regularity assumptions on flexibility and commitment. These guarantee that our two stage game is still overall in strategic substitutes. All statements are on the third derivatives of the demand curve, and are trivially satisfied whenever these are all zero (e.g., linear demand at the equilibrium point). An alternative to assumption 2 is simply that the two-stage game is still in strategic substitutes.

Assumption 2 *In equilibrium:* $\frac{\partial \phi_i}{\partial k_i} \geq 0$, $\frac{\partial \sigma_i}{\partial k_{-i}} \leq 0$.

Proposition 3 *Holding one firm's MMC fixed, a firm's equilibrium per-market-quantity in both types of markets* ***increases*** *with the firm's MMC and* ***decreases*** *with the rival's MMC*

$$\frac{dq_i}{d\lambda_i} \geq 0, \quad \frac{d\hat{q}_i}{d\lambda_i} \geq 0, \quad \frac{dq_i}{d\lambda_{-i}} \leq 0, \quad \frac{d\hat{q}_i}{d\lambda_{-i}} \leq 0$$

The top left panel in figure 2.1 provides intuition. The figure shows firm A's and B's quantities in each overlapping market as a function of A's extent of MMC, while keeping B's MMC at a fixed high or low level. The figure uses an inverse linear demand model and the outcomes are presented relative to the standard Cournot outcomes for the same demand functions.

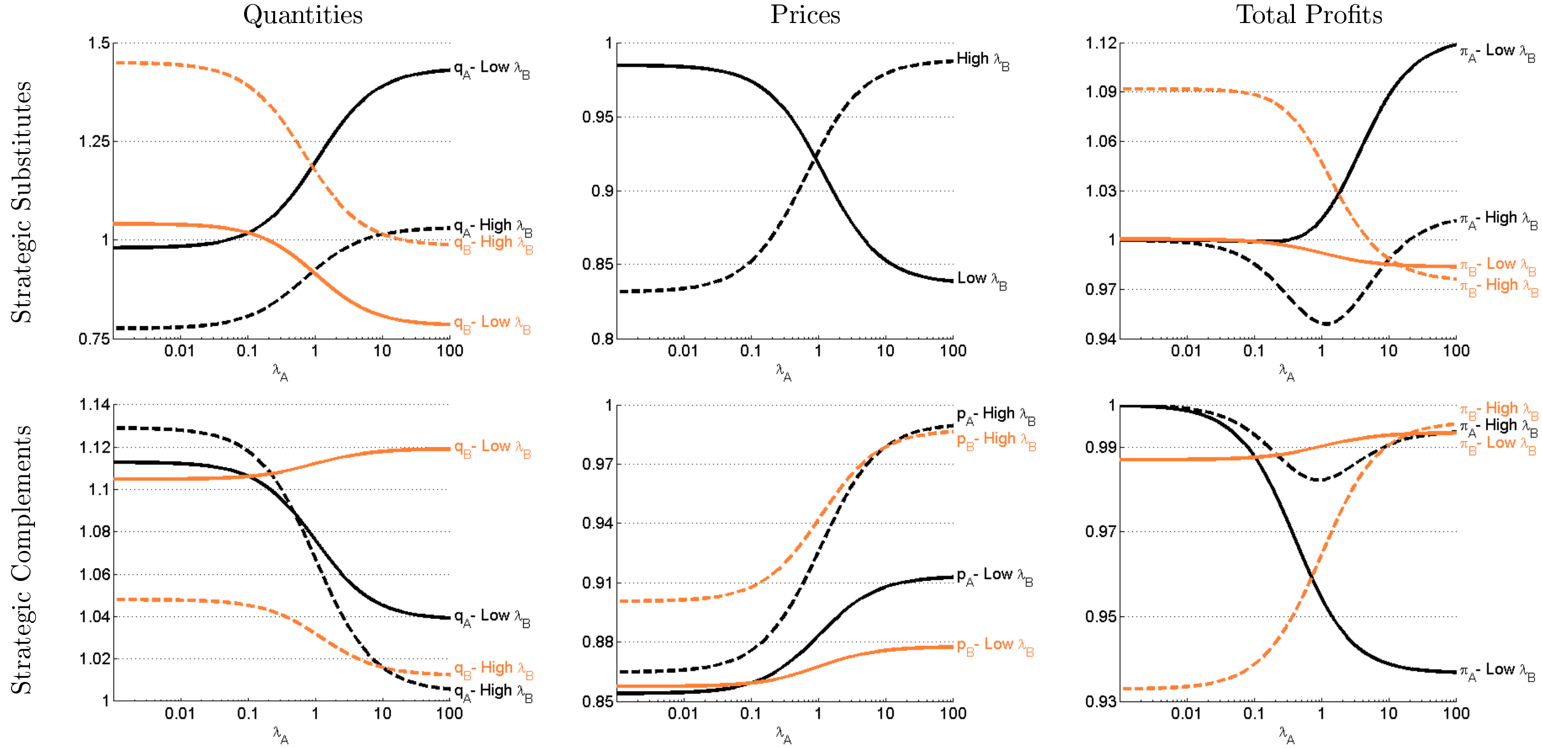
Proposition 3 identifies three implications of the model. First, a change in MMC has the same qualitative effect on the firm's *private and overlapping* markets. This is because MMC affects the firm's optimal marginal revenue in all markets in which it operates. Second, an increase in firm i's MMC makes firm i more aggressive in both the overlapping and private markets. This is an implication of the intuition developed above – an increase in MMC shifts the firm towards a 'first mover' strategy. Finally, an increase in firm i's MMC makes its rival less aggressive in both overlapping and private markets. This last result $\left(\frac{d\hat{q}_i}{d\lambda_{-i}} \geq 0\right)$ implies that a change in one of the firm's private markets conditions may affect outcomes in rivals' private markets–markets in which the firm does not operate at all. That is, firm A may decrease its quantities in markets it is serving as a monopolist as a result of a decrease in demand in markets monopolized by firm B.

Proposition 3 allows for a straightforward assessment of MMC on welfare in the private markets:

Corollary 1 *As a firm's MMC increases, total output and consumer surplus in its private markets increase. Total output and consumer surplus in the rival's private markets decrease.*

While the effect of MMC on total output and consumer surplus in the private markets is straightforward, the net effect in the overlapping markets is unclear. Specifically, proposition 3 states that

FIGURE 2.1.— Market Outcomes for Linear Demand Models



The figure shows firms' quantities and prices in the overlapping markets and firms' total profits across both market types. For strategic substitutes, inverse demand functions are linear: $P(q_i, q_{-i}) = a - bq_i - bq_{-i}$, $\hat{P}_i(\hat{q}) = P(\hat{q}, 0)$. For strategic complements, demand functions are also linear: $q_i(p_i, p_{-i}) = a - bp_i + \frac{b}{2}p_{-i}$, $\hat{q}_i(\hat{p}_i) = q_i(\hat{p}_i, 0)$. All variables are normalized by an appropriate benchmark (i.e. $q_i/q^\star$). For strategic substitutes, the benchmark is the standard Cournot, with $q^\star = \frac{a-c}{3b}$, $p^\star = \frac{a+2c}{3}$, $\pi^\star = m_o \frac{1}{b}\left(\frac{a-c}{3}\right)^2 + m_i \frac{1}{b}\left(\frac{a-c}{2}\right)^2$. For strategic complements, the benchmark is the two-shot game with no cross market spillovers, with $q^\star = \frac{3}{10}(2a - bc)$, $p^\star = \frac{1}{5b}(4a + 3c)$, $\pi^\star = m_o \frac{3}{b}\left(\frac{2a-bc}{5}\right)^2 + m_i \frac{1}{b}\left(\frac{a-bc}{2}\right)^2$. The x-axis is A's MMC ($\lambda_i = \frac{m_o}{m_i}$). Two lines are plotted for each firm: solid lines for $\lambda_B = 0.1$ (Low) and dotted lines for $\lambda_B = 10$ (High). The strategic substitutes plots are independent of the demand and cost constants. For the strategic complements plots, the constants used in the above illustrations are: $a = 10$, $b = 1$, and $c = 1$.

an increase in one of the firms' MMC has opposing effects on firms' equilibrium quantity in the overlapping markets. It is therefore not surprising that the effect on total output in the overlapping markets is unclear:

PROPOSITION 4 *When competition is in strategic substitutes there is a non-monotonic relationship between total output and the extent of MMC. In particular, for any* m_A, m_B*, there are* $\underline{m} \in (0, \infty)$ *and* $\overline{m} \in (\underline{m}, \infty)$ *such that for* $m_o \in [0, \underline{m}]$*, overlapping markets' quantity increases in* m_o *and for* $m_o \in (\overline{m}, \infty)$*, overlapping markets' quantity decreases in* m_o*.*

At lower levels of MMC an increase in MMC makes both firms more aggressive. While this result may be surprising, the intuition is quite simple. To see this, suppose that $\lambda_A = \lambda_B = \lambda > 0$ and that firm A sets its first stage capacity so that it can exactly provide the monopoly quantity in its private markets and the standard Cournot quantity in the overlapping markets. Firm B's best response in the first stage is to commit to a slightly larger capacity, knowing well that firm A will be able to accommodate B's additional quantity in the overlapping markets by shifting some of its quantity intended for the overlapping markets into its private markets. Since firms are symmetric, both firms deviate accordingly from the standard Cournot equilibrium.

Note that proposition 4 states the direction of the effect of MMC on total quantity in the overlapping markets only in the extremes. The proposition thus implicitly implies that in order to know the direction of the effect elsewhere, one must make strong assumptions about the shape of the demand function. In the appendix, we provide the explicit solution to the symmetric case where ($\lambda_A = \lambda_B = \lambda$) for linear demand. Figure 2.2 summarizes this case.

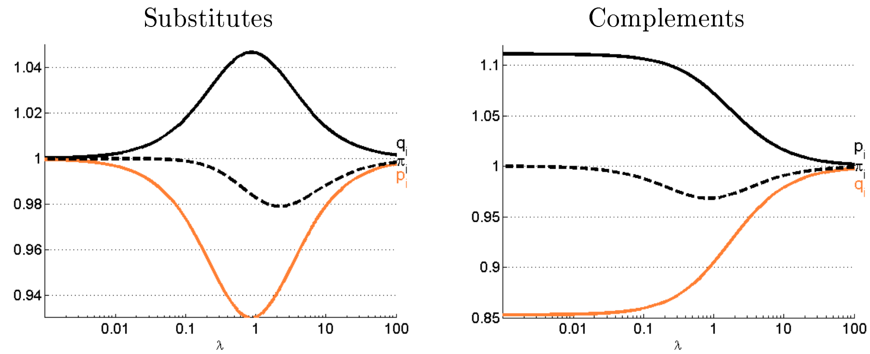
Our discussion so far has focused on the effect of MMC on firms' output. The top right panel of figure 2.1 and the dotted line in the left panel of figure 2.2 present the effect of MMC on each firm's total profits.[16] Since we are looking at firms' total profits across their overlapping and private markets, we take the benchmark profits to be the profits earned if the firm acts monopolistically in its private markets and according to the standard Cournot in the overlapping markets, denoted π^*.[17]

As expected, as the firm becomes more aggressive in the overlapping markets the rival responds softly and profits increase with MMC (see e.g., the case of low λ_B in the right panel of figure 2.1).

[16] Note that given our normalization, while the figure demonstrates total profits for each firm, the graph cannot be used to interpret total industry profits.

[17] The precise definition of $\pi^\star$ can be found in the caption of figure 2.1.

FIGURE 2.2.— Market Outcomes - Symmetric MMC



Graph is for symmetric MMC: $\lambda_A = \lambda_B = \lambda$. Plot shows firms' quantities (black) and prices (orange) in the overlapping markets and firms' total profits (dotted), all normalized (i.e. $q_i/q^\star$) by the appropriate benchmark. As in figure 2.1 the figure for strategic substitutes is independent of the non-MMC parameters (a, b, c). For the strategic complements figure, the same constants are used: $a = 10, b = 1, c = 1$. As firms are symmetric, only values of one firm are shown.

Interestingly, profits may be non-monotonic in MMC. Consider, for example, the case where $\lambda_B \gg \lambda_A$ (dotted line in the right panel of figure 2.1). If $\lambda_A \to 0$ then A's profits are driven predominantly by its monopolistic position in its private markets, and while A is soft in the overlapping markets, these markets correspond to a very little share of A's profits. Consequently, $\pi_A \to \pi^\star$. As A's MMC increases, the overlapping markets become a more important driver for A's profitability. Since A's MMC is still smaller than B's MMC, firm A remains the "Stackelberg follower" in the overlapping markets. That is, a larger share of A's profits comes now from markets where A's profitability is relatively low. As A's MMC increases, its flexibility decreases, and B's ability to behave aggressively in the overlapping markets diminishes. As a result, the price in the overlapping markets increases (see top middle panel of figure 2.1), and A's output in the overlapping markets increases as well (top left panel of figure 2.1). These effects combined improve A's profitability, and indeed as depicted in figure 2.1, A's profits start increasing. Once $\lambda_A \gg \lambda_B$, firm A becomes the "Stackelberg leader" in the overlapping markets. Since in this case A's profits are mainly driven from the overlapping markets, A's total profits are higher than the benchmark $\pi^\star$.

3. PRICE COMPETITION

We now turn to the case where firms compete in prices in the second stage. Using the same three market-types as before, the only change in our setting is that in the second stage, firms choose prices rather than quantities. The second stage problem for firm i is:

$$\Pi_i(k_A,k_B)=\max_{p_i,\hat{p}_i} \quad m_o q_i\left(p_i,p_{-i}\right)\cdot p_i + m_i\cdot \hat{q}_i\left(\hat{p}_i\right)\cdot \hat{p}_i \tag{3.1}$$
$$\text{s.t.} \qquad m_o q_i\left(p_i,p_{-i}\right)+m_i\hat{q}_i\left(\hat{p}_i\right)\le k_i$$
$$q_i(p_i,p_{-i})\ge 0 \quad ; \quad \hat{q}_i(\hat{p}_i)\ge 0$$

The second stage game is now in strategic complements. The economics of a capacity choice followed by a pricing game requires additional structure. Kreps and Scheinkman (1983) first solved this model for homogenous goods and showed that the game has the same solution as Cournot if the residual demand maintains the same distribution as the original demand. Davidson and Deneckere (1986), however, showed that the resulting equilibrium depends on the assumptions on residual demand. To avoid this sensitivity of the result, we assume that products are differentiated and can be described by a smooth demand function. The MMC setting and analysis are similar to the case where the second stage is in strategic substitutes. We therefore only present in this section the definitions, results, and intuition. Detailed analysis can be found in the appendix. The following assumption formalizes the necessary conditions which are common to models of price competition with smooth demand functions.[18]

Assumption 3 *At the equilibrium, all demand curves are twice differentiable, weakly concave and:*

1. *The goods are normal and substitutes:* $\frac{\partial q_i(\cdot)}{\partial p_i}\le -\frac{\partial q_i(\cdot)}{\partial p_{-i}}\le 0$
2. $\hat{q}_i(c)>0$, $q_i(c,p_j^M)>0$ *where* p_j^M *is the rival's monopoly price*[19]
3. *The second order derivatives satisfy* $\frac{\partial^2 q_i(\cdot)}{\partial p_i\partial p_{-i}}\ge 0$ and $\left|\frac{\partial^2 q_i(\cdot)}{\partial p_i^2}\right|\ge\left|\frac{\partial^2 q_i(\cdot)}{\partial p_i\partial p_{-i}}\right|\ge\left|\frac{\partial^2 q_i(\cdot)}{\partial p_{-i}^2}\right|$

The first stage problem is formally the same as in the quantity game, with the second stage profit Π_i referring to the pricing game 3.1 instead:

$$\max_{k_i\ge 0}\Pi_i(k_A,k_B)-ck_i \quad . \tag{3.2}$$

As in the quantity game, the marginal revenue[20] from a unit of capacity play a key role in determining the equilibrium. As in the quantity game, the capacity constraint binds and each firm's

[18] See Vives (2001), chapter 6, for a complete discussion. Assuming that $\frac{\partial^2 q_i(\cdot)}{\partial p_i\partial p_{-i}}\ge 0$ guarantees that the game is supermodular (Vives (2001) p. 151).

[19] That is p_j^M solves $\frac{\partial}{\partial p}\left(q(p,\infty)\cdot(p-c)\right)=0$

[20] Marginal revenue (with respect to quantity) in the pricing game accounts for the change in price. Smoothness of the demand functions implies that $\frac{dq_i}{dp_i}=\frac{1}{\frac{dp_i}{dq_i}}$ and therefore MR is given by $\hat{\eta}_i=\hat{p}_i+\hat{q}_i(p_i)/\hat{q}_i'(\hat{p}_i)$ and $\eta_i=p_i+q_i(p_i,p_{-i})/\frac{dq_i(p_i,p_{-i})}{dp_i}$.

marginal revenue is identical in all its markets (the proof is similar to that of lemma 1 and thus omitted).

We define *flexibility* and *commitment power* similarly to the quantity game.

DEFINITION 4 In the pricing game, firm i's *flexibility*, ϕ_i, is the firm's second stage reaction to a change in its competitor's overlapping markets' price and commitment power, σ_i is the change in the firm's second stage price in the overlapping market following an additional unit of capacity:

$$\phi_i \equiv \frac{\partial p_i}{\partial p_{-i}} \quad ; \quad \sigma_i \equiv m_o \frac{\partial p_i}{\partial k_i} \tag{3.3}$$

Consistent with the basic intuitions, flexibility is positive (prices are strategic complements) and commitment power is negative (selling more requires lower prices). The appendix provides ϕ_i and σ_i in terms of marginal revenue, μ, and MMC λ, leading to the equilibrium characterization:

PROPOSITION 5 *The equilibrium of the game is characterized by the following equations*

$$c - \eta_i = c - \hat{\eta}_i = -\left(\frac{\partial q_i}{\partial p_{-i}} / \frac{\partial q_i}{\partial p_i}\right) \cdot q_i \cdot \phi_{-i} \cdot \sigma_i < 0 \qquad i \in \{A, B\} \ . \tag{3.4}$$

Proposition 5 shows that the underlying fundamentals of the C-MMC effect (difference between cost and marginal revenue) are the same whether firms compete in prices or quantities. The difference between the two cases are the economic forces that drive firms' behavior, which change significantly as a result of the move from quantity to price competition. In particular, under price competition, *MMC makes all firms less aggressive*, as indicated by the sign of $c - \eta$ in 3.4.

The intuition is as follows: commitment power (from high MMC) allows the firm to credibly commit to increasing prices in the overlapping markets. To this end, MMC allows firms to be less aggressive in the first stage and choose a lower capacity level. This strategy is profitable as long as the rival is limited to reacting with prices rather than shifting capacity from its private markets, which happens when the rival also has high MMC.

Proposition 6 verifies the comparative statics for price competition:

PROPOSITION 6 *If prices in the overall game are strategic complements, then any increase in MMC* *increases prices for both firms*

$$\frac{dp_i}{dm_i} < 0 \quad \frac{dp_i}{dm_{-i}} \leq 0 \quad \frac{dp_i}{dm_o} \geq 0$$

$$\frac{d\hat{p}_i}{dm_i} < 0 \quad \frac{d\hat{p}_i}{dm_{-i}} \leq 0 \quad \frac{d\hat{p}_i}{dm_o} \geq 0$$

The results in proposition 6 are depicted in the middle panel of figure 2.1.[21] The main implication of proposition 6 is that when firms compete in prices, increases in MMC *always* reduce competition both in the overlapping and the private markets. It is therefore possible that the C-MMC effect would be confounded with mutual forbearance.

Increasing a firm's MMC increases its commitment power and reduces its flexibility. The increase in commitment power allows the firm to hold back capacity in the first stage and set a high price in the second. The decreased flexibility incentivizes the rival to also hold back additional capacity and increase price in the second stage, knowing that the other firm will follow. In contrast to the quantity game, both effects go in the same direction. Hence, *any* increase in MMC allows the firms to commit to competing softer.

Furthermore, while in the quantity game the C-MMC effect is minimal when MMC is large, in the case of strategic complements large MMC produces the strongest C-MMC effect. Commitment power increases in absolute terms with MMC because the firm must sell a larger share of its additional capacity in the overlapping markets, while flexibility increases with MMC as the game is in strategic complements. That is, in the case of strategic complements both ϕ_{-i} and σ_i increase in absolute terms with MMC, resulting in a stronger price response when MMC is large.

A final observation is that the effect of MMC on firm profits cannot be generally signed. Profits for a linear demand curve are provided in the bottom right panel of figure 2.1 and the dotted line in the right panel of figure 2.2. The case where λ_B is high demonstrates that as A's MMC increases, its profits increase as a result of both firms' soft behavior in the overlapping markets. When λ_B is low, however, the result may be reversed. If $\lambda_A \to 0$ then A's profits are driven predominantly by its monopolistic position in its private markets, and thus as in the quantity game $\pi_A \to \pi^\star$. As A's MMC increases, a larger portion of its profits comes from the overlapping markets. Since B's commitment power is low, the firms cannot commit to a large enough increase in price (relative to

[21] We take the benchmark prices to be the prices set if the firm acts monopolistically in its private markets and according to a two-stage game in the overlapping markets. Closed form solutions for prices and quantities are provided in the appendix.

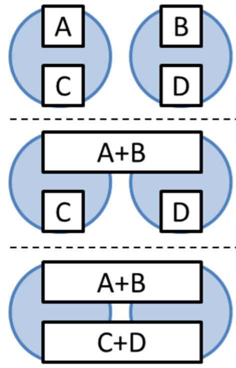


FIGURE 4.1.— Merger example: two markets (circles) and four firms (boxes). The first merger is between firms A and B. After the merger, firm C considers the second market as a private market of firm 'A+B'. The second merger is between firms C and D.

the benchmark)–negatively affecting A's profits.

4. MERGER ANALYSIS

The analysis above suggests that the C-MMC effect could potentially affect the profitability and welfare implications of a merger. Specifically, MMC considerations may discourage firms from otherwise profitable and consumer surplus increasing mergers as they try to avoid the "Stackelberg follower" outcome. In this case, mergers that do not provide additional benefits, such as cost-savings, would not be pursued.

In this section we explore how mergers and MMC interact. In particular, we extend the basic two firm model to allow for mergers by considering four firms (labeled A,B, C, and D) in which A only overlaps with C, and B only overlaps with D, as depicted in figure 4.1. We study two mergers: a merger between firms A and B, which forms firm AB, and a followup merger between C and D–forming firm CD. This last merger allows us to provide insight into merger waves.

In each case we study the effect of the merger on consumer surplus and profitability. If the merger is not profitable absent any cost efficiencies, we examine the size of cost efficiencies required to make such merger profitable. As both mergers keep the number of firms in each market constant, the analysis abstracts from reductions in the number of competitors. We also abstract from any effects due to mutual forbearance (which can arise in the second merger) as they are orthogonal to the C-MMC effect.

4.1. *Model*

We use the same setting as in section 3: firms simultaneously set capacities in the first stage and compete on prices in the second stage. Results for a parallel version in which second stage competition is in quantities are qualitatively similar to those provided here. We assume that all firms have a marginal cost of $c > 0$ and that demand is given by the linear form:[22]

$$q_i = a - b \cdot p_i + \frac{b}{2} p_{-i} \qquad (a, b) > 0 \tag{4.1}$$

Vives (2001) shows this would be the demand function for a representative consumer model with utility

$$U(q_1, q_2) = \frac{2}{b} \cdot \left[a \cdot (q_1 + q_2) - \frac{1}{3} \left(q_1^2 + q_2^2 + q_1 q_2 \right) \right]$$

As in figure 4.1, there are two separate markets: one served by firms A and C and one served by B and D.[23] We study the equilibrium of this game under three different ownership structures. First we analyze the case where all firms operate independently. We then study the case where firms A and B merge and operate jointly. Finally, in order to allow for the analysis of a merger wave, we study the structure where firms A and B are merged, as well as firms C and D, and contrast the outcomes to a setting where only A and B have merged . We allow mergers to increase production efficiency. In particular, a merger reduces the unit cost to $(1 - \rho) \cdot c$ for $\rho \in [0, 1)$.

4.2. *Benchmark Equilibrium*

Before the first merger, the extent of MMC of all firms is 1: A and C fully overlap as do B and D. Each firm's market quantity, price, and profits are:

$$k_i = q_i = \frac{3}{10}(2a - bc) \qquad p_i = \frac{4a + 3bc}{5b} \qquad \pi_i = \frac{3}{b}\left(\frac{2a - bc}{5}\right)^2$$

[22] Results for a rival price effect of $b_2 \neq \frac{b}{2}$ are qualitatively the same but require more notation without providing additional insights.

[23] The analysis requires only notational adjustment to allow each firm to serve any number of markets, as long as A only overlaps with C and B only overlaps with D before the merger. See appendix C for a discussion on more than two firms per market in the basic model.

Total welfare and consumer surplus per market are calculated using the representative consumer utility $U()$:

$$W = \frac{21}{50b}(2a-bc)^2 \qquad CS = \frac{9}{50b}(2a-bc)^2$$

As in section 3, the C-MMC effect induces quantities and welfare that are lower than in a game without MMC considerations.

To avoid corner-solutions, we assume that demand, costs and cost savings are such that profits (and quantities) are positive for all firms.

Assumption 4 *Benchmark profits are positive. In addition, cost efficiencies generated by a merger are small enough such that rivals would produce, in equilibrium, a positive amount absent any MMC considerations:*[24]

$$\rho < 3\left(\frac{2a-bc}{bc}\right)$$

To simplify notation, we denote $\underline{\pi} \equiv \frac{2a-bc}{bc}$.

4.3. *Evaluating the First Merger*

We first analyze the case where firms A and B merge to form AB. Post-merger, AB serves both markets, while the nonmerged firms serve a single market each.

Based on lemma 1, firms choose capacities in the first stage such that their installed capacity is fully utilized. Since the nonmerged firms serve a single market, they have no flexibility in the second stage and thus commit to a price-quantity pair when making their first stage capacity decision. The merged firm cannot make this same commitment. Serving two markets gives it the flexibility to reallocate quantity across its markets in the second stage. The nonmerged firms' strong commitment power combined with the merged firm's flexibility allows the nonmerging firms to be more aggressive in their capacity choice. This aggressiveness, however, may be offset by the cost efficiencies the merged firm may enjoy. Below we discuss the circumstances under which cost efficiencies compensate the merged firm for the soft behavior imposed by the C-MMC effect.

The following lemma provides the equilibrium quantities for the post-merger game. Interestingly, equilibrium quantities are such that the constraint required for the nonmerged firms to remain active

[24]This cutoff for cost efficiencies is driven from a single market duopoly, in which *wlog*, firm A's cost is $(1-\rho)c$ and firm C's cost is c. Equilibrium quantities and profits can be found in the appendix.

post-merger is the same as the upper bound on cost savings imposed by assumption 4–which is based on the benchmark model and does not incorporate the C-MMC effect. That is, MMC does not by itself make it easier for mergers to induce exit.

LEMMA 4 *If firms A and B merge, the resulting market quantities for the merged- (AB) and nonmerged (C, D) firms are*

$$\begin{aligned} q_{AB} &\equiv \tfrac{k_{AB}}{2} = \frac{3}{202} bc\,(20\underline{\pi} + 27\rho) \\ q_D = q_C &= \frac{3}{202} bc\,(21\underline{\pi} - 7\rho) \end{aligned}$$

The cost savings from the merger (ρ) and the C-MMC effect have two conflicting effects on the merged- and nonmerged firms' capacity decision. For the merged firm, the cost savings increase the merged firm's quantity. In contrast, the C-MMC effect makes the nonmerged firms more aggressive and thus has a negative effect on the merged firm's quantity (cf proposition 6). Nevertheless, the first order effect of both cost savings and the C-MMC effect is an increase in output. Any decrease in quantity is only a reaction to the rival's quantity increase. Therefore, the merger increases total quantity and consumer surplus. As pre-merger firms were producing less than socially efficient, total welfare increases as well.

Given the opposing effects costs saving and the C-MMC effect have on firms' output decision, whether post-merger firms increase or decrease output depends on which effect dominates. Proposition 7 identifies the relevant conditions:

PROPOSITION 7 *If firms A and B merge, total market quantity, total welfare and consumer surplus increases. However, per market:*

- *The merged firm produces more post-merger iff cost savings (ρ) satisfy $\rho > \frac{1}{135}\underline{\pi}$*
- *The merged firm produces more than the nonmerged firm iff cost savings (ρ) satisfy $\rho > \frac{1}{34}\underline{\pi}$*
- *Each nonmerged firm produces **less** post-merger iff cost savings (ρ) satisfy $\rho > \frac{4}{35}\underline{\pi}$*

The first result in proposition 7 identifies the cost savings needed for the merged firm to increase output; i.e., the conditions under which the cost-savings effect outweighs the C-MMC effect. Larger cost-savings, identified in the second condition, are required for the merged firm to offset the C-MMC effect and dominate the market in shares. The final condition identifies the significant cost-savings required to completely overturn the C-MMC effect and decrease the rival's quantity to below the pre-merger levels. Note that the final condition requires cost savings roughly 16 times larger than the first. As ρ is bounded above by one, there may be circumstances in which cost savings may never be large enough to decrease rivals' quantities.[25]

All conditions require a non-trivial cost saving. For example, if we set $a = 80, \quad b = 2$ and $c = 10$, (so $\underline{\pi} = 7/2$) the required cost savings are roughly 5%, 20% and 80% for the merged firm to increase production , increase market share, and for the nonmerged firm to decrease production, respectively. Note that ignoring any MMC considerations, the merger would increase the merged firm's (and overall market) quantity for any cost saving.

The same forces that affect quantities also affect profits.[26] Post merger, the merged firm is flexible and thus more accommodating. Consequently, absent any cost savings, the merged firm's joint profits are lower than pre-merger. However, this negative effect may be offset by cost savings. That is the merger is profitable only under large enough cost savings:

Proposition 8 *If firms A and B merge:*

- *The merger increases the merged firm's profits (relative to the pre-merger joint profits) iff cost savings satisfy* $\rho > \frac{1}{135}\underline{\pi}$
- *The non merged firms' profits increase iff* $\rho < .00535\underline{\pi}$

Corollary 2 *Any profitable merger* ***reduces*** *profits for the nonmerged firms.*

Taking propositions 7 and 8 together we see that the merger is profitable if, and only if, the merged firm produces more after the merger. Nonetheless, there is a large range of cost savings for which the C-MMC effect is strong enough to increase the nonmerged firms' output beyond the merged firm's increase and consequently dominate the market in shares.

Corollary 3 *For moderate cost savings:* $\rho \in \left[\frac{1}{135}\underline{\pi}, \frac{1}{34}\underline{\pi}\right]$*, the merger is profitable, both firms produce more* ***and*** *the nonmerged firms increase their market share relative to the merged firm.*

Since costs cannot be lower than zero, in industries with low production costs, cost efficiencies from mergers will be such that either corollary 3 applies or the merger isn't profitable: :

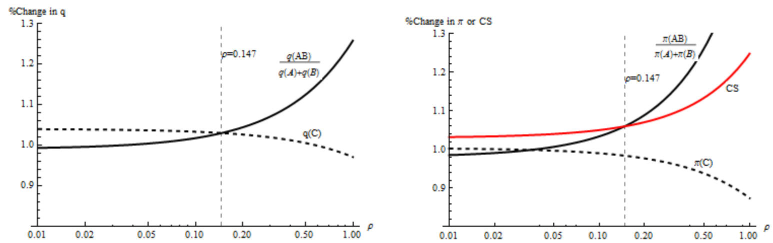
[25] This occurs when $a - 5bc > 0$.

[26] Firm profits per market post-merger are:

$$\begin{aligned} \pi_m^M &= 3bc^2\left(\frac{20\underline{\pi} + 27\rho}{101}\right)^2 \\ \pi_m^N &= 3bc^2 \cdot \frac{91}{2}\left(\frac{3\underline{\pi} - \rho}{101}\right)^2 \end{aligned}$$

FIGURE 4.2.— Market Outcomes - Merger with MMC

Quantities Profits and Welfare



The change in firms' market quantities (left) and profits (right) when firms A and B merge as a function of the cost-savings generated by the merger (ρ). The dark solid line and dashed line represent the merged and nonmerged firms, respectively. The CS line in the right plot identifies the change in consumer surplus. The parameters for the plots are $a = 300$, $b = 10$, $c = 10$. The merger is profitable whenever $\rho > .037$. The merged firm produces more than its rivals only when $\rho > .147$, which is the exact point in which the profit increase (in %) is greater than the increase in consumer surplus. CS increases by about 3% without any cost savings ($\rho = 0$) and about 25% if cost savings are almost full ($\rho \to 1$).

COROLLARY 4 *If costs are extremely low relative to demand* $\left(c \leq \frac{2}{136}\frac{a}{b}\right)$, *the merger cannot be profitable. If costs are sufficiently low relative to demand* $\left(c \leq \frac{2}{35}\frac{a}{b}\right)$, ***any** profitable merger provides the outcome described in corollary 3.*

We have just established that there is a non-trivial range of cost-saving, profitable, surplus-increasing mergers in which *the merged firm's market share decreases post-merger.* This implies that one should be cautious when evaluating mergers' welfare-gains based on firms' market share (see for example, Jeziorski (2013) and Pesendorfer (2003)). Again, to illustrate, if $a = 80, \quad b = 2$ and $c = 10$, a merger that generates cost savings between 5% to 20% could be mistaken for decreasing consumer surplus if the estimation assumes that the rival's quantities reflect market fundamentals without accounting for MMC effects.

Figure 4.2 illustrates the effects of the merger on quantities, profits and welfare. Absent cost reductions, the merger reduces profits for the merged firm. A 3.7% reduction in marginal cost makes the merger profitable, and a 14.7% reduction in marginal cost is required for the merged firm to increase market share. In other words, for modest cost savings, the main determinant of the market outcomes of a merger is the C-MMC effect.

In conclusion, this section finds that MMC considerations reduce the profitability of a merger by increasing the aggressiveness of the nonmerged firms. The main implication is that under MMC, some cost-saving mergers may not be pursued. Furthermore, using market outcomes to estimate the cost-savings implied by a merger may provide qualitatively wrong conclusions if the C-MMC effect is not accounted for. Specifically, cost-*decreasing* mergers that meet the criteria in corollary 3 may be estimated as cost-*increasing*. Finally, note that the predicted effects above are consistent with the predictions generated by the mutual forbearance theories of MMC. However, no firm in our models implemented any cooperative practices. This suggests caution when analyzing mergers so as to not confound mutual forbearance with the purely competitive effects of MMC

4.4. *Dynamic Merger Analysis*

Having established the effect of MMC on a single merger, we now consider the policy implications for merger evaluation of a followup merger, and the resulting overall dynamic merger policy. In particular, suppose that the nonmerged firms (C and D) now consider merging too; expecting the same cost savings (ρ) as in the first merger. If the second merger is carried out, the degree of MMC for both merged firms will again be 1. The equilibrium would be the same as the benchmark described in section 4.2 with costs $c \cdot (1-\rho)$ for each firm.

The second merger has two effects on quantities and profits. First, consistent with proposition 6, the increase in MMC decreases quantities and increases prices for both firms. Second, the cost-savings for C and D decrease overall prices, increase CD's profits and decrease AB's profits. The next proposition summarizes these results:

PROPOSITION 9 *The second merger is profitable iff $\rho \geq 0.00134\underline{\pi}$. In particular, if cost efficiencies are large enough such that the first merger is profitable, then so is the second merger. Moreover, the firm that merged first benefits from the second merger iff the first merger induced a decrease in the merged firm's market share; i.e.:$\rho < \frac{1}{34}\underline{\pi}$.*

As in the first merger, the second merger has two opposing effects on each firms' profits. The second merged firm (CD) loses the "Stackelberg-leader" advantage it enjoyed due to the C-MMC effect, but gains from the cost savings. The first merged firm (AB) gains as the increase in its rival's MMC decreases the rival's aggressiveness, but loses from having a more efficient rival.

If cost savings were sufficient to make the first merger profitable, they are sufficient to make the second one profitable. As shown in corollary 3, there is a range of cost savings for which the merger is profitable yet the merged firm's market share is smaller than the nonmerged firms'. Proposition 9 shows that it is for this range of cost savings that the second merger increases firm AB's profits, such that both firms gain from the second merger. If, however, cost savings are large enough such that AB dominates the market when competing against firms C and D, then the second merger hurts AB's profitability.

Finally, proposition 10 considers the welfare effects of a merger wave.

PROPOSITION 10 *The second merger increases consumer surplus iff it decreases profits for the first merged firm ($\rho > \frac{1}{34}\underline{\pi}$). If $\rho < 0.0082\underline{\pi}$ the second merger decreases total welfare as well as consumer surplus.*

The second merger brings the degree of MMC back to the pre-merger MMC levels yet with lower costs, suggesting that consumer surplus and total firm profits should increase. However, the decrease in MMC following the second merger makes firm CD a softer competitor and consequently decreases welfare. The second merger thus increases consumer (or total) surplus, in addition to the increase delivered by the first merger, iff the cost-savings effect is large enough to outweigh the C-MMC effect. Note that the cutoff for consumer surplus was already identified in proposition 9 as the cutoff that determines the effect of the second merger on AB's profitability.

Proposition 10 implies that a regulator that is committed to approve only CS-increasing mergers would approve the first merger but reject the second merger whenever $\rho < \frac{\underline{\pi}}{34}$. This dynamic dependency can potentially distort merger decisions. For example, suppose that $\rho = \frac{\underline{\pi}}{200}$. A single merger is not profitable (proposition 8). Nevertheless, if AB merges, the second merger is profitable for CD (proposition 9) and results in higher profits for AB relative to the pre-merger equilibrium. Thus, if firms are forward looking, the industry will consolidate; increasing consumer surplus by the cost reduction.

Note, however, that the second merger is surplus decreasing compared to the first (proposition 10) and would thus be blocked by a regulator committed to blocking CS-decreasing mergers.[27] If the firms expect the second merger to be blocked, no merger would take place. The regulator's myopic policy would thus result in lower consumer surplus than otherwise. The potential for such distortions increases if mergers require a fixed cost, as this increases the profit requirement in propositions 8 and 9 without affecting the consumer surplus cutoff in proposition 10.

Proposition 10 suggests that the C-MMC effect provides an alternative explanation for one merger to be welfare improving while a second, identical, merger may not. However, all the effects considered are pro-competitive. The market is as competitive as can be expected even after both mergers. There-

[27]The regulator objection could be expressed, for example, as a limit on the number of "mega firms" in the industry.

fore, rather than suggesting a potential reason to reject the second merger, we interpret proposition 10 to suggest caution when using post-merger quantities, prices or even welfare levels to evaluate the merits of a regulator's decision to approve a sequence of mergers.

5. EMPIRICAL RELEVANCE – THE US AIRLINE INDUSTRY

Based on Evans and Kessides (1994) and Gimeno and Woo (1999), this section considers an example from the US domestic airline industry. These two studies use route pricing data for the years 1984 to 1988 and find that MMC is positively correlated with firm prices.[28] The airline industry example illustrates the applicability of the underlying sequence of events in our model: capacity choices followed by competition. In addition, the discussion below demonstrates that, at least in the airline industry, it is possible to empirically distinguish between the MF-MMC and C-MMC effects. Finally, the example maintains that it is plausible that the empirical findings reflect C-MMC rather than MF-MMC effects.

The airline industry is an ideal industry to apply our model of competitive MMC. Airlines schedule flights from spokes to their hub (or between spokes) well in advance of actually selling tickets on the routes using these flights.[29] Moreover, the prevalence of the Hub and Spoke system implies that virtually all domestic flights serve many routes (markets).[30] For example, a flight between New-York City (NYC) to Chicago can be used to fly passengers from NYC to Chicago as well passengers from NYC to Denver, Austin, and dozens of other destinations; all connecting in Chicago. However, a flight between NYC to Chicago cannot be used to fly passengers from San Diego to Seattle. Thus, for example, United Airline's NYC-Chicago flights provide it with capacity that is sunk (the specific plane will takeoff, regardless of any marginal sale) and *transferable* across the different routes that utilize the NYC-Chicago flight leg. When United makes market level decisions (i.e., prices per market), the total number of seats available for all markets that use the same flight is fixed, and the majority of the cost is already committed.

The main empirical prediction of our model is that the C-MMC effect is present *only* in markets

[28] Ciliberto and Williams (2011) provide a more current and expanded analysis of MMC in the airline industry. Their analysis finds the same price effects. However, they do not perform the specific test we are interested in. In an ongoing work, we construct an empirical model that captures the hub-and-spoke structure and the implied capacity problem. This allows us to directly distinguish between MF-MMC and C-MMC effects and evaluate the effects of the recent major mergers in the airline industry.

[29] See e.g. Barnhart et al. (2003). Roughly speaking, an airline scheduling process includes four steps. The airline first allocates to each city-pair planes by type (e.g., two 747 flights a day between Chicago and Cleveland). Then allocates specific planes (i.e., tail numbers), then determines a maintenance schedule, and finally assigns crews. The procedure is done using demand forecasts and flights are typically scheduled more than six months before takeoff.

[30] The hub and spoke structure dominated the US airline market since deregulation. Southwest, the only major airline to challenge the hub and spoke structure, was a very small player at the time.

that share a sunk transferable investment. This allows us to separate between MMC effects that could be competitive or collusive and MMC effects that could only be collusive. For example, suppose that two carriers have three hubs each. One for travel on the east coast, one for travel on the west coast and one for all other travel.[31] The C-MMC effect applies only for sunk, transferable investment. Since it is impossible to transfer any sunk investment between the hubs (e.g., an East-coast flight cannot be used to serve West-coast routes), following changes in MMC on the East coast, the C-MMC effect would only affect East coast prices. In contrast, for MF-MMC analysis, the location of the MMC change makes no difference. Therefore, an empirical approach that distinguishes between MMC changes on the same hub and MMC changes on different hubs can differentiate between the two effects. In particular, finding that that MMC changes in a remote hub affect prices in *all* shared routes supports the existence of MF-MMC effects. Finding that prices changed only on routes that connect at the remote hub supports the C-MMC effect.

The study by Gimeno and Woo (1999) (hereafter, GW) performs exactly this empirical test. For every given route, GW define two distinct measures of MMC between airline i and j on that market (route): (i) the “standard” MMC measure–counting **all** routes served by both airlines; and (ii) an MMC measure that counts all routes served by both airlines **and** share a common endpoint with the current market. This second measure of MMC corresponds to our definition of MMC. That is, if United enters one of American’s markets in the East coast (e.g., Portland,Maine – Durham,N.C.) the standard MMC measure for United and American increases. However, the second MMC measure for these carriers increases on East coast routes from either Portland or Durham (e.g., Boston – Durham), but not on the West coast routes (e.g., San Diego – Seattle).

Running a regression with airlines’ yield (price per mile) as the dependent variable and the two measures of MMC as the explaining variables, GW find that an increase in MMC with routes that share a common endpoint was correlated with higher yield but an increase in MMC on routes that do not share an endpoint was not.

GW interpret their results as suggesting that common endpoints are required for MMC to facilitate collusion. In contrast, our model posits that the positive relationship between MMC and prices may in fact be the outcome of a competitive, rather than a collusive, behavior driven by shared sunk, transferable investment. That is, GW’s results provide compelling evidence that the possible relationship between MMC and price in the US airline industry may be due to the C-MMC effect

[31] United, for example, has hubs in Newark, Chicago, and San-Francisco

rather than MF-MMC.[32]

The distinction between empirical support for C-MMC and MF-MMC effects has significant policy implications when considering mergers in the industry. The three recent mergers (Delta-Northwest, United-Continental and American-US Airways) all significantly affect MMC. If MMC in the industry facilitates tacit collusion, the mergers may well decrease welfare. However, if the main effect of MMC is through the C-MMC effect, regulatory intervention is not required.

6. CONCLUSION

With globalization, the analysis of MMC is more relevant now than ever. While there is rich empirical research, the theoretical foundation is focused on collusive considerations. Our analysis shows that the basic microeconomics forces in our model are just as important as the collusive ones. We, therefore, believe that empirical research on MMC would largely benefit from accounting for these effects. Our analysis complements the mutual forbearance literature;[33] focusing on markets with sunk, transferable investments..

Our framework is especially important for the evaluation of horizontal mergers. The current horizontal mergers literature assumes away any MMC considerations. Our analysis shows that while for a single merger the C-MMC effect reduces the merged firm's profitability, a followup merger may increase the first merged firm's profits to a point above the initial profitability level. More importantly, such a followup merger will also increase welfare above the initial level–before any merger took place. This suggests that when considering multi-market mergers, a policy that rejects all surplus-decreasing mergers may in fact harm consumers. Multi-market mergers should not be considered at the merger-by-merger level. Rather, the regulator should account for the dynamic effect of each merger on subsequent mergers, and thereby on the ultimate market structure.

Our formal analysis considered only the case of two firms in a specific market. As exemplified in section 4, the qualitative results easily extend to more complicated models. Appendix C extends the model to the case where the overlapping markets are served by more than two firms and shows that

[32] Our conclusions are reserved mainly because of the significant changes in empirical methods, in particular for establishing causality, in the last two decades. In addition, our followup work constructs independent variables that are more closely related to our definition of MMC.

[33] In fact, extending our model to an infinitely repeated game generates results that are similar to the mutual forbearance results in the literature. Specifically, BW show that a simple increase in the number of overlapping markets is not enough for MMC to exhibit collusion when firms have linear marginal costs. The same holds in our model. Note that if firms can observe the rival's capacity before making the second stage decision, or observe the rival's private market outcome, the existence of private market might change the BW rationale and MMC may indeed affect the incentive to collude. As this is orthogonal to the scope of this paper, we leave this for future research.

results stay qualitatively unchanged.

Previous literature on MMC focused on perceived differences between large and small firms in terms of productivity or mutual forbearance. The C-MMC effect can be confounded with either of the two. Thus, an important empirical challenge is to distinguish between the competitive effect and the mutual forbearance effect of MMC. A key difference between the two effects is that while the former relies on specific physical diseconomies across markets (i.e., sunk, transferable investments), the latter does not. This opens possibilities for future research to empirically distinguish between the two hypotheses by separating the effect of changes in MMC that use the same capacity (i.e., use of a common distribution center or airline hub) and changes in MMC that do not.

REFERENCES

Arrow, Kenneth J., "Optimal capital policy with irreversible investment," *Value, Capital, and Growth*, 1968, p. 1–20.

Aw, B. Y. and A. R. Hwang, "Productivity and the export market: A firm-level analysis," *Journal of Development Economics*, August 1995, *47* (2), 313–332.

Barnhart, Cynthia, Amy Cohn, Ellis Johnson, Diego Klabjan, George Nemhauser, and Pamela Vance, "Airline Crew Scheduling," in Randolph W. Hall, ed., *Randolph W. Hall, ed.*, Springer US, 2003, pp. 517–560.

Bernard, Andrew B. and J. Bradford Jensen, "Exceptional exporter performance: cause, effect, or both?," *Journal of International Economics*, February 1999, *47* (1), 1–25.

Bernheim, B. Douglas and Michael D. Whinston, "Multimarket Contact and Collusive Behavior," *The RAND Journal of Economics*, 1990, *21* (1), 1–26.

Bilotkach, Volodymyr, "Multimarket Contact and Intensity of Competition: Evidence from an Airline Merger," *Review of Industrial Organization*, January 2011, *38* (1), 95–115.

Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer, "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, January 1985, *93* (3), 488.

Christensen, Laurits Rolf and Richard E. Caves, "Cheap Talk and Investment Rivalry in the Pulp and Paper Industry," *The Journal of Industrial Economics*, March 1997, *45* (1), 47–73.

Ciliberto, F and J Williams, "Does Multimarket Contact Facilitate Tacit Collusion? Inference on Conjectural Parameters in the Airline Industry," 2011, (434).

Ciliberto, Federico and Jonathan W. Williams, "Does Multimarket Contact Facilitate Tacit Collusion? Inference on Conduct Parameters in the Airline Industry," 2012.

Clerides, Sofronis K., Saul Lach, and James R. Tybout, "Is Learning by Exporting Important? Micro-Dynamic Evidence from Colombia, Mexico, and Morocco," *The Quarterly Journal of Economics*, August 1998, *113* (3), 903–947.

Davidson, Carl and Raymond Deneckere, "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model," *The RAND Journal of Economics*, October 1986, *17* (3), 404–415. ArticleType: research-article / Full publication date: Autumn, 1986 / Copyright © 1986 RAND Corporation.

Dixit, Avinash, "The Role of Investment in Entry-Deterrence," *The Economic Journal*, March 1980, *90* (357), 95–106. ArticleType: research-article / Full publication date: Mar., 1980 / Copyright © 1980 Royal Economic Society.

Edwards, Corwin D., "Conglomerate Bigness as a Source of Power," in George J. Stigler, ed., *Business Concentration and Price Policy*, Princeton University Press, 1955, p. 329–358.

Evans, William N. and Ioannis N. Kessides, "Living by the "Golden Rule": Multimarket Contact in the U.S. Airline Industry," *The Quarterly Journal of Economics*, May 1994, *109* (2), 341–366.

Farrell, Joseph and Carl Shapiro, "Horizontal Mergers: An Equilibrium Analysis," *The American Economic Review*, March 1990, *80* (1), 107–126.

Feinberg, Robert M., "Mutual forbearance as an extension of oligopoly theory," *Journal of Economics and Business*, May 1984, *36* (2), 243–249.

———, ""Exports-at-Risk": the Effect of Multi-Market Contact in International Trade," 2011.

Fernandez, Nerea and Pedro L. Marin, "Market Power and Multimarket Contact: Some Evidence from the Spanish Hotel Industry," *The Journal of Industrial Economics*, 1998, *46* (3), 301–315.

Friedman, James, *Oligopoly Theory*, CUP Archive, September 1983.

Gimeno, Javier and Carolyn Y. Woo, "Multimarket Contact, Economies of Scope, and Firm Performance," *The Academy of Management Journal*, June 1999, *42* (3), 239–259.

Heggestad, Arnold A. and Stephen A. Rhoades, "Multi-Market Interdependence and Local Market Competition in Banking," *The Review of Economics and Statistics*, November 1978, *60* (4), 523–532.

Hortaçsu, Ali and Chad Syverson, "Cementing Relationships: Vertical Integration, Foreclosure, Productivity, and Prices," *Journal of Political Economy*, April 2007, *115* (2), 250–301.

Jans, Ivette and David I. Rosenbaum, "Multimarket contact and pricing: Evidence from the U.S. cement industry," *International Journal of Industrial Organization*, 1997, *15* (3), 391 – 412.

Jayachandran, Satish, Javier Gimeno, and P. Rajan Varadarajan, "The Theory of Multimarket Competition: A Synthesis and Implications for Marketing Strategy," *Journal of Marketing*, July 1999, *63* (3), 49–66.

Jeziorski, Przemyslaw, "Estimation of Cost Efficiencies from Mergers: Application to U.S. Radio," SSRN Scholarly Paper ID 1700762, Social Science Research Network February 2013.

Kreps, David M. and Jose A. Scheinkman, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *The Bell Journal of Economics*, 1983, *14* (2), 326–337.

Lohatepanont, Manoj and Cynthia Barnhart, "Airline Schedule Planning: Integrated Models and Algorithms for Schedule Design and Fleet Assignment," *TRANSPORTATION SCIENCE*, February 2004, *38* (1), 19–32.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, November 2003, *71* (6), 1695–1725.

Nocke, Volker and Michael D. Whinston, "Dynamic Merger Review," *The Journal of Political Economy*, December 2010, *118* (6), 1201–1251.

Parker, Philip M. and Lars-Hendrik Roller, "Collusive Conduct in Duopolies: Multimarket Contact and Cross-Ownership in the Mobile Telephone Industry," *The RAND Journal of Economics*, 1997, *28* (2), pp. 304–322.

Pavcnik, Nina, "Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants," *The Review of Economic Studies*, January 2002, *69* (1), 245–276.

Pesendorfer, Martin, "Horizontal Mergers in the Paper Industry," *The RAND Journal of Economics*, October 2003, *34* (3), 495–515.

Pilloff, Steven J., “Multimarket Contact in Banking,” *Review of Industrial Organization*, 1999, *14* (2), 163–182. 10.1023/A:1007779814751.

Spence, A. Michael, “Entry, Capacity, Investment and Oligopolistic Pricing,” *The Bell Journal of Economics*, October 1977, *8* (2), 534–544. ArticleType: research-article / Full publication date: Autumn, 1977 / Copyright © 1977 RAND Corporation.

Valletti, Tommaso M., Steffen Hoernig, and Pedro P. Barros, “Universal Service and Entry: The Role of Uniform Pricing and Coverage Constraints,” *Journal of Regulatory Economics*, March 2002, *21* (2), 169–190.

Vives, Xavier, *Oligopoly Pricing: Old Ideas and New Tools*, MIT Press, January 2001.

Young, Greg, Ken G. Smith, Curtis M. Grimm, and Daniel Simon, “Multimarket Contact and Resource Dissimilarity: A Competitive Dynamics Perspective,” *Journal of Management*, December 2000, *26* (6), 1217–1236.

APPENDIX A: PROOFS OF PROPOSITIONS AND LEMMAS

We present the formal proofs to all Lemmas, Propositions, and claims in the text.

A.1. *Quantity Competition*

Throughout this section, we denote with subscripts the following partial derivatives: $\eta_{i,1} \equiv \frac{\partial \eta_i}{\partial q_i}$, $\eta_{i,2} \equiv \frac{\partial \eta_i}{\partial q_{-i}}$, $\eta_{i,12} \equiv \frac{\partial^2 \eta_i}{\partial q_i \partial q_{-i}}$, $p_{i,1} \equiv \frac{\partial p_i}{\partial q_i}$, $p_{i,2} \equiv \frac{\partial p_i}{\partial q_{-i}}$, $p_{i,22} \equiv \frac{\partial^2 p_i}{\partial q_{-i}^2}$, $p_{i,12} \equiv \frac{\partial^2 p_i}{\partial q_i \partial q_{-i}}$. Recall that η_i is marginal revenue: $\eta_i \equiv \frac{\partial}{\partial q_i} q_i p_i = p_i + q_i p_{i,1}$ and $\hat{\eta}_i = \frac{\partial}{\partial \hat{q}_i} \hat{q}_i \hat{p}_i$.

Lemma 1 *In equilibrium, in the second stage:*

1. *The capacity constraint binds:*$k_i = m_o q_i + m_i \hat{q}_i$.
2. *Firm i's marginal revenue is identical in both types of markets:*$\eta_i = \hat{\eta}_i$.

Proof: For the first point, assume it does not bind for firm A. Firm A can reduce k_A so that the constraint now binds. This saved A a strictly positive cost. A now assigns the same quantities in the second stage as before. B cannot have any profitable deviation, because that deviation was also possible in the original game. The second stage equilibrium is unaffected and A's total profit increased, contradicting the original equilibrium.

For the second point, the subgame problem is (eq. 2.1):

$$\max_{q_i, \hat{q}_i \geq 0} m_o q_i p_i + m_i \hat{q}_i \hat{p}_i \quad .s.t.\ m_o q_i + m_i \hat{q}_i \leq k_i$$

so the FOC are:

$$m_o \eta_i = m_o \mu_{k_i} - \mu_i \qquad \text{and} \qquad m_i \hat{\eta}_i = m_i \mu_{k_i} - \hat{\mu}_i$$

where μ_{k_i} is the Lagrangian multiplier on the capacity constraint for firm i and $\mu_i, \hat{\mu}_i$ are the Lagrangian multipliers on the non-negativity constraints. If $q_i, \hat{q}_i > 0$, these last multipliers are zero. As the partial derivatives are simply the marginal revenue (MR) curves, the FOCs are $m_o \eta_i = m_o \mu_{k_i}$ and $m_i \hat{\eta}_i = m_i \mu_{k_i}$, giving the desired result.

It remains to show $q_i, \hat{q}_i > 0$: *Q.E.D.*

- Observe that for any equilibrium $k_A, k_B, q_A, \hat{q}_A, q_B, \hat{q}_B$ and $\varepsilon \in [-\hat{q}_A, \infty)$, if A deviates to $k_A^{'} = k_A + \varepsilon$, $\hat{q}_A^{'} = \hat{q}_A + \varepsilon$, then q_B and $\hat{q}_B$ are still the equilibrium sub-game quantities by B.
- If $\hat{q}_i = 0$ then firm i can strictly increase profits by setting $k_i = k_i + \hat{q}_i^M$ and $\hat{q}_i = \hat{q}_i^M$ where $\hat{q}_i^M > 0$ is the monopoly quantity for the private market. Hence it must be that $\hat{q}_i > 0$
- If $q_i = 0$, it must be that $\hat{q}_i = \hat{q}_i^M$ and so $\hat{\eta}_i = \mu_{k_i} = c$. Placing in the first FOC obtains $\eta_i < c$. As $q_i = 0$, this implies that $P_i(0, q_{-i}) < c$. By assumption, 1, $q_{-i} > q_{-i}^M$. From the previous bullet, $\hat{q}_{-i} > 0$. Thus, for the rival, $\eta_{-i} = \hat{\eta}_{-i} < c$. Then the rival can increase profits by slightly decreasing k_{-i} and $\hat{q}_{-i}$.

Lemma 2 *In equilibrium,*

$$\phi_i = -\frac{\frac{\partial \eta_i}{\partial q_{-i}}}{\frac{\partial \eta_i}{\partial q_i} + \lambda_i \frac{\partial \hat{\eta}_i}{\partial \hat{q}_i}} \in \left[\overline{\phi}_i, 0\right] .$$

Proof: By lemma 1, the second stage FOCs for firm i are characterized by

$$\eta_i = \hat{\eta}_i \quad \text{and} \quad m_o q_i + m_i \hat{q}_i = k_i$$

Taking full derivatives of these two equations with respect to rival's quantity yields:

$$\eta_{i,1}\phi_i + \eta_{i,2} - \hat{\eta}_{i,1}\frac{\partial \hat{q}_i}{\partial q_{-i}} = 0 \qquad \text{and} \qquad m_o\phi_i + m_i\frac{\partial \hat{q}_i}{\partial q_{-i}} = 0$$

Isolating $\frac{\partial \hat{q}_i}{\partial q_{-i}}$ in the second equation and placing in the first yields:

$$\begin{aligned} 0 &= \eta_{i,1}\phi_i + \eta_{i,2} - \hat{\eta}_{i,1}\left(-\frac{m_o}{m_i}\phi_i\right) \\ \phi_i &= \frac{-\eta_{i,2}}{\eta_{i,1} + \frac{m_o}{m_i}\hat{\eta}_{i,1}} \end{aligned}$$

Isolating ϕ_i and replacing $\lambda_i = \frac{m_o}{m_i}$:

$$\phi_i = \frac{-\eta_{i,2}}{\eta_{i,1} + \lambda_i \hat{\eta}_{i,1}}$$

To determine the bounds, assumption 1 (in equilibrium, $p_{i,1} \leq 0$, $p_{i,2} \leq 0$, $p_{i,11} \leq 0$ and $p_{i,12} \leq 0$) implies that:

$$\eta_{i,1} = 2p_{i,1} + q_i \cdot p_{i,11} \leq 0$$

$$\eta_{i,2} = p_{i,2} + q_i \cdot p_{i,12} \leq 0$$

and so

$$0 \geq \phi_i \geq -\frac{\eta_{i,2}}{\eta_{i,1}} = \overline{\phi}_i$$

Q.E.D.

Lemma 3 *In equilibrium,*

$$\sigma_i = \frac{\lambda_i \frac{\partial \hat{\eta}_i}{\partial \hat{q}_i}}{\frac{\partial \eta_i}{\partial q_i} + \frac{\partial \eta_i}{\partial q_{-i}}\phi_{-i} + \lambda_i \frac{\partial \hat{\eta}_i}{\partial \hat{q}_i}} \in [0, 1]$$

PROOF: From the equations characterizing the firm's 2nd stage equilibrium:

$$\eta_i = \hat{\eta}_i \quad \text{and} \quad m_o q_i + m_A \hat{q}_i = k_i$$

Take full derivatives with respect to capacity (k_i):

$$\eta_{i,1}\frac{\sigma_i}{m_o} + \eta_{i,2}\frac{\sigma_i}{m_o}\phi_{-i} - \hat{\eta}_{i,1}\frac{d\hat{q}_i}{dk_i} = 0 \qquad \text{and} \qquad \sigma_i + m_i\frac{d\hat{q}_i}{dk_i} = 1$$

Use the second equality to replace $\frac{d\hat{q}_i}{dk_i}$ in the first:

$$\eta_{i,1}\frac{\sigma_i}{m_o} + \eta_{i,2}\frac{\sigma_i}{m_o}\phi_{-i} - \hat{\eta}_{i,1}\left(\frac{1-\sigma_i}{m_i}\right) = 0$$

Solve for σ_i :

$$\sigma_i = \frac{\hat{\eta}_{i,1}}{\frac{m_i}{m_o}\left(\eta_{i,1} + \phi_{-i}\eta_{i,2}\right) + \hat{\eta}_{i,1}}$$

To show that $\sigma_i \in [0,1]$:

- $|\phi_{-i}| \leq \left|\overline{\phi}_{-i}\right| \leq 1$ by assumption 1 and lemma 2.
- By assumption 1 $|\eta_{i,2}| \leq |\eta_{i,1}|$:

$$2p_{i,1} + q_i p_{i,11} \leq p_{i,2} + q_i p_{i,12}$$

- Combining the last two bullets, as $\eta_{i,1} \leq 0$, we have that $\eta_{i,1} + \phi_{-i}\eta_{i,2} \leq 0$.
- As $\hat{\eta}_{i,1} \leq 0$, $\sigma_i \in [0,1]$

Q.E.D.

PROPOSITION 1 *The equilibrium of the game is given by the following equation*

$$c - \eta_i = c - \hat{\eta}_i = q_i\frac{\partial P_i\left(q_i, q_{-i}\right)}{\partial q_i} \cdot \phi_{-i} \cdot \sigma_i > 0 \qquad i \in \{A, B\} \; .$$

Firms behave more aggressively under MMC than they would have under standard oligopoly.

- *If MMC is large for one firm $\lambda_i \to \infty$ and small for the other $\lambda_{-i} \to 0$, then the equilibrium in the overlapping market is the Stackelberg equilibrium with firm i the Stackelberg leader.*
- *If MMC is large $\lambda_i \to \infty$ or small $\lambda_i \to 0$ for both firms, the equilibrium in the overlapping market is the Cournot equilibrium.*

PROOF: The first stage problem is:

$$\max_{k_i} \pi_i(k_i, k_{-i}) - ck_i$$

where $\pi_i(k_i, k_{-i}) = m_o q_i p_i + m_i \hat{q}_i \hat{p}_i$ and $(q_i, \hat{q}_i, q_{-i}, \hat{q}_{-i})$ are functions of k_i and k_{-i} given by the second stage equilibrium. The first stage FOC is then:

$$\frac{\partial \pi_i(k_i, k_{-i})}{\partial k_i} - c = 0$$

$$m_o \eta_i \frac{dq_i}{dk_i} + m_o q_i p_{i,2} \frac{dq_{-i}}{dk_i} + m_i \hat{\eta}_i \frac{d\hat{q}_i}{dk_i} \quad = \quad c$$

Using the definitions of σ_i and ϕ_{-i} and the condition:

$$\sigma_i + m_i \frac{d\hat{q}_i}{dk_i} = 1$$

The first order condition is:

$$\eta_i \sigma_i + q_i p_{i,2} \sigma_i \phi_{-i} + \hat{\eta}_i (1 - \sigma_i) \quad = \quad c$$

The subgame equilibrium implies $\eta_i = \hat{\eta}_i$. So the FOC can be written as

$$c = \eta_i + q_i p_{i,2} \sigma_i \phi_{-i}$$

The last term is positive as $p_{i,2} \leq 0$, $\phi_{-i} \leq 0$, and $\sigma_i \geq 0$.

For the second part, when $\lambda_i \to \infty$, then $\phi_i \to \overline{\phi}_i$ and $\sigma_i \to 0$. When $\lambda_{-i} \to 0$, then $\phi_{-i} \to 0$ and $\sigma_{-i} \to 1$. The FOCs of the two players are then

$$c = \eta_i \qquad \text{and} \qquad c = \eta_{-i} + q_{-i} p_{-i,2} \overline{\phi}_i$$

which are exactly the same FOC as the Stackelberg game with firm i as the follower. *Q.E.D.*

Proposition 2 *Consumer and total surplus is always at least as high in industries with MMC as without. Total industry profit is always at most as high in industries with MMC as without.*

Proof: Consumer and total surplus are a monotonic function of total market output as long as price is above cost. Total industry profits decrease in total market output if quantity is at least the monopoly level.

That quantity is at least the monopoly level is obvious. If price is below cost, each firm gains by producing less. As marginal revenue for all firms in all markets is lower with MMC than without it, quantities in all markets are higher. *Q.E.D.*

Proposition 3 *Holding one firm's MMC fixed, a firm's equilibrium per-market quantity in both types of markets* ***increases*** *with the firm's MMC and* ***decreases*** *with the rival's MMC*

$$\frac{dq_i}{d\lambda_i} \geq 0 \quad \frac{d\hat{q}_i}{d\lambda_i} \geq 0 \qquad\qquad \frac{dq_i}{d\lambda_{-i}} \leq 0 \quad \frac{d\hat{q}_i}{d\lambda_{-i}} \leq 0$$

Proof: The proof is given using comparative statics on m_i which is equivalent to changing only one firm's MMC (λ_i or λ_{-i}). We show that $\frac{dq_i}{dm_i} < 0$ and that $\frac{dq_i}{dm_{-i}} > 0$. The proof proceeds in 4 steps:

1. Change the variable of the first stage optimization from k_i to q_i and show that $\frac{d\hat{q}_i}{dm_i} \leq 0 \iff \frac{d\hat{q}_i}{dm_{-i}} \geq 0$.

2. Show the FOC w.r.t. q_i is decreasing in m_i, holding rival's output fixed.

3. Show the FOC w.r.t. q_i is increasing in m_{-i}, holding rival's output fixed.

4. Show that strategic substitution is preserved. Thus, if a change causes the firm to increase it's quantity holding it's rival quantity fixed and causes the rival to decrease quantity hold the firm's quantity fixed, the two effects reinforce each other.

Step 1. Change of variables.

The four equations that characterize the second stage equilibrium provide a mapping between $(q_i, \hat{q}_i)$ and (k_i, k_{-i}). These four equations are the two equations equating marginal revenue (one for each firm) and the two constraints (one for each firm). Apply the implicit function theorem on these four equations to express k_i, $\hat{q}_i$, and q_{-i} as a function of (q_i, k_{-i}). Denote these functions by κ_i ,$\hat{\vartheta}_i$, and ϑ_{-i} so that $\kappa_i(q_i, k_{-i}) = k_i$, $\tilde{\vartheta}_i(q_i, k_{-i}) = \hat{q}_i$, and $\vartheta_{-i}(q_i, k_{-i}) = q_{-i}$.

The definition of flexibility (ϕ) implies that $\frac{d\vartheta_{-i}}{dq_i} = \phi_{-i}$.

The definition of commitment (σ) implies that $\frac{d\kappa_i}{dq_i} = \frac{m_o}{\sigma_i}$.

The second stage capacity constraint implies that $\frac{d\hat{\vartheta}_i}{dq_i} = \frac{m_o}{m_i}\frac{1-\sigma_i}{\sigma_i}$[34]

The first stage objective function can now be expressed in terms of q_i:

$$\pi_i(q_i) = m_o q_i p_i(q_i, \vartheta_{-i}) + m_i \hat{\vartheta}_i \hat{p}_i(\hat{\vartheta}_i) - c\kappa_i$$

The derivative with respect to q_i is :

$$\begin{aligned} \frac{\partial \pi_i}{\partial q_i} = \pi_{q_i} &= m_o \eta_i + m_o q_i p_{i,2} \frac{d\vartheta_{-i}}{dq_i} + m_i \hat{\eta}_i \frac{d\hat{\vartheta}_i}{dq_i} - c\frac{d\kappa_i}{dq_i} \\ &= m_o \eta_i + m_o q_i p_{i,2} \phi_{-i} + m_o \hat{\eta}_i \frac{1-\sigma_i}{\sigma_i} - \frac{cm_o}{\sigma_i} \\ &= \frac{m_o}{\sigma_i}\left(\sigma_i \eta_i + (1-\sigma_i)\hat{\eta}_i - c + q_i p_{i,2}\sigma_i \phi_{-i}\right) \end{aligned}$$

where the last equality used the fact that in all subgame equilibria $\eta_i = \hat{\eta}_i$.

To simplify notation, let $\tilde{\pi}_{q_i} \equiv \sigma_i \eta_i + (1-\sigma_i)\hat{\eta}_i - c + q_i p_{i,2}\sigma_i \phi_{-i}$.

In equilibrium, $\tilde{\pi}_{q_i} = 0$. Moreover, $\eta_i = \hat{\eta}_i$ which simplifies the first order condition to the result of proposition 1:

$$c - \eta_i = q_i p_{i,2} \sigma_i \phi_{-i}$$

Step 2. The FOC is decreasing in m_i when holding q_{-i} fixed.

As $\frac{\partial \eta_i}{\partial m_i} = \frac{\partial \hat{\eta}_i}{\partial m_i} = \tilde{\pi}_{q_i} = 0$, the sign of the derivative of the FOC when holding q_{-i} fixed is given by the sign of $\frac{\partial}{\partial m_i}\tilde{\pi}_{q_i}$.

$$\frac{\partial}{\partial m_i}\tilde{\pi}_{q_i} = \frac{\partial \sigma_i}{\partial m_i}(\eta_i - \hat{\eta}_i) + q_i p_{i,2}\left(\phi_{-i}\frac{\partial \sigma_i}{\partial m_i}\right)$$

[34] Differentiating the capacity constraint with respect to q_i yields:

$$m_i \frac{d\hat{\vartheta}_i}{dq_i} + m_o = \frac{d\kappa_i}{dq_i} = \frac{m_o}{\sigma_i} \quad .$$

As $\eta_i = \hat{\eta}_i$ the first term cancels. Both ϕ_{-i} and $p_{i,2}$, are negative. Thus, the sign is determined by the sign of $\frac{\partial \sigma_i}{\partial m_i}$. From lemma 3$\sigma_i$ can be written as

$$\sigma_i = \frac{-x}{-y - \frac{m_o}{m_i}x}$$

with a, x, y all positive and do not depend on m_i or m_o. Thus, $\frac{\partial \sigma_i}{\partial m_i} < 0$ and so $\frac{\partial}{\partial m_i}\tilde{\pi}_{q_i} < 0$

Step 3. The FOC is increasing in m_{-i} when holding q_{-i} fixed.

As in the previous step, the sign of the derivative of the FOC when holding q_{-i} fixed is given by the sign of $\frac{\partial}{\partial m_{-i}}\tilde{\pi}_{q_i}$.

$$\frac{\partial}{\partial m_{-i}}\tilde{\pi}_{q_i} = q_i p_{i,2}\left(\phi_{-i}\left(\frac{\partial \sigma_i}{\partial \phi_{-i}}\frac{\partial \phi_{-i}}{\partial m_{-i}} + \frac{\partial \sigma_i}{\partial m_{-i}}\right) + \sigma_i \frac{\partial \phi_{-i}}{\partial m_{-i}}\right)$$

To show that $\frac{\partial}{\partial m_{-i}}\tilde{\pi}_{q_i} > 0$:

- $\frac{\partial \sigma_i}{\partial m_{-i}} = 0$ is immediate from lemma 3.
- From lemma 2, $\frac{\partial \phi_{-i}}{\partial m_{-i}}$ has the form

  $$\phi_{-i} = -\frac{-x}{-y - \frac{z}{m_{-i}}} = -\frac{x}{y + \frac{z}{m_{-i}}}$$

  with x, y, z positive and do not depend on m_i or m_o. Thus, $\frac{\partial \phi_{-i}}{\partial m_{-i}} < 0$.
- From lemma 3, $\frac{\partial \sigma_i}{\partial \phi_{-i}}$ has the form

  $$\sigma_i = \frac{-x}{-x - y - \phi_i z}$$

  with x, y, z positive and do not depend on m_i or m_o. Thus, $\frac{\partial \sigma_i}{\partial \phi_{-i}} < 0$.
- As $\phi_{-i} \leq 0$, $\sigma_i > 0$ the term inside the parenthesis is negative.
- As $p_{i,2}$ is negative, $\frac{\partial}{\partial m_{-i}}\tilde{\pi}_{q_i} > 0$

Step 4. Strategic substitution is preserved: $\frac{\partial^2 \pi_i}{\partial q_i \partial q_{-i}} \leq 0$

Here, note the a change of q_{-i} affects the first order condition through a change of k_{-i} that implies the change in q_{-i}. As $\sigma_{-i} \geq 0$, formally, we need to show that $\frac{\partial^2 \pi_i}{\partial q_i \partial k_{-i}} \leq 0$.

$$\begin{aligned}
\frac{\partial}{\partial k_{-i}}\tilde{\pi}_{q_i} &= \frac{\partial}{\partial k_{-i}}\sigma_i \eta_i + \frac{\partial}{\partial k_{-i}}(1-\sigma_i)\hat{\eta}_i + \frac{\partial}{\partial k_{-i}} q_i p_{i,2}\sigma_i \phi_{-i} \\
&= \frac{\partial \eta_i}{\partial k_{-i}} \cdot \sigma_i + \frac{\partial \hat{\eta}_i}{\partial k_{-i}}(1-\sigma_{-i}) + \frac{\partial \sigma_i}{\partial k_{-i}}(\eta_i - \hat{\eta}_{-i}) + q_i \frac{\partial}{\partial k_{-i}}(\sigma_i \phi_{-i} p_{i,2})
\end{aligned}$$

By the change of variables done above, $\hat{\eta}_i$ is a function of q_i and k_{-i} that is determined by $\hat{\eta}_i = \eta_i$ and so

$\frac{d\eta_i}{dk_{-i}} = \frac{d\hat{\eta}_i}{dk_{-i}}$ independent of the reaction in q_i.[35] The second is zero as $\eta_i = \hat{\eta}_{-i}$. Thus we have:

$$\frac{\partial}{\partial k_{-i}}\tilde{\pi}_{q_i} = \frac{\partial \eta_i}{\partial k_{-i}} + q_i \frac{\partial}{\partial k_{-i}}\left(\sigma_i \phi_{-i} p_{i,2}\right)$$

The first element is simply $\sigma_{-i}\eta_{i,2}$. Thus the effect simplifies to

$$\begin{aligned} \frac{\partial}{\partial k_{-i}}\tilde{\pi}_{q_i} &= \sigma_{-i}\eta_{i,2} + \sigma_{-i}\sigma_i\phi_{-i}q_i p_{i,22} + q_i p_{i,2}\left(\sigma_i \frac{\partial \phi_{-i}}{\partial k_{-i}} + \phi_{-i}\frac{\partial \sigma_i}{\partial k_{-i}}\right) \\ &\leq \sigma_{-i}\cdot\left(\eta_{i,2} + \sigma_i\phi_{-i}q_i p_{i,22}\right) \end{aligned}$$

The last line followed from the fact that the term in parenthesis is positive by assumptions 2 and 2. Expanding the $\eta_{i,2}$ term and collecting terms gives

$$\frac{\partial}{\partial k_{-i}}\tilde{\pi}_{q_i} \leq \sigma_{-i} \cdot \left(p_{i,1} + q_i\left(p_{i,12} + \sigma_i\phi_{-i}p_{i,22}\right)\right)$$

- q_i ,σ_i and σ_{-i} are positive and $\phi_{-i} \leq 0$
- By assumption, 1 $p_{i,1} \leq 0$,$p_{i,12} \leq 0$ and $p_{i,12} \leq p_{i,22}$
- If $p_{i,22} \geq 0$ all terms in the RHS are negative and we're done.
- If $p_{i,22} \leq 0$, the largest value is obtained for the RHS at the most negative $\phi_{-i} = -1$ and the most positive $\sigma_i = 1$:

  $$\frac{\partial}{\partial k_{-i}}\tilde{\pi}_{q_i} \leq p_{i,1} + q_i\left(p_{1,12} - p_{i,22}\right)$$

  As $p_{i,12} \leq p_{i,22}$ so the term in parentheses is still negative.

This concludes the proof. *Q.E.D.*

Proposition 4 *When competition is in strategic substitutes there is a non-monotonic relationship between total output and the extent of MMC. In particular, for any m_A, m_B, there are $\underline{m} \in (0,\infty)$ and $\overline{m} \in (\underline{m},\infty)$ such that for $m_o \in [0,\underline{m}]$, overlapping markets' quantity increases in m_o and for $m_o \in (\overline{m},\infty)$, overlapping markets' quantity decreases in m_o.*

Proof: It is sufficient to show that an increase in m_o decreases (for the first case) or increases (for the second case) the marginal revenue for both firms.

Starting from the FOC

$$c - \eta_i = S_i \equiv q_i \frac{\partial P_i}{\partial q_{-i}}\sigma_i\phi_{-i}$$

As c is fixed, it is equivalent to show that S_i increases (for the first case) and decreases (for the second). For any $m_A, m_B < \infty$, the C-MMC effect $S_i = 0$ if $m_o = 0$ (by $\sigma_i = 0$) and if $m_o \to \infty$ (as $\phi_{-i} \to 0$). However, outside the limits, for any m_o, $\sigma_i > 0$, $\phi_{-i} < 0$ and $S_i > 0$. As S_i is continuous, there is some $\underline{m}(m_A, m_B)$ and $\overline{m}(m_A, m_B)$ such that for any $m_o < \underline{m}$, S_i must increase for both firms and for any $m_i > \overline{m}$, S_i must decrease for both firms. *Q.E.D.*

[35] For example, if q_i does not change, the reduction in MR in the overlapping market generates an increase in $\hat{q}_i$ and k_i so that the MR in both markets stays equal.

A.2. *Price Competition*

First, observe that, in contrast to the well known models of capacity followed by price competition (Kreps and Scheinkman (1983); Davidson and Deneckere (1986)), the products in our model are differentiated. This removes the need to determine a rationing rule (which Davidson and Deneckere (1986) show is critical in the homogenous good case). Vives (2001) provides various general conditions and specific demand models for which the resulting demand function is smooth and meets our assumption 3.

The smooth demand functions imply that the need for rationing no longer exists. Neither firm ever gains by specifying a price for which demand exceeds supply – the firm can simply increase it's private market price,

Throughout this section, we denote with subscripts the following partial derivatives: $\eta_{i,1} \equiv \frac{\partial \eta_i}{\partial p_i}$, $\eta_{i,2} \equiv \frac{\partial \eta_i}{\partial p_{-i}}$, $\eta_{i,12} \equiv \frac{\partial^2 \eta_i}{\partial p_i \partial p_{-i}}$, $q_{i,1} \equiv \frac{dq_i}{dp_i}$, $q_{i,2} \equiv \frac{dq_i}{dp_{-i}}$. It is useful to recall that $q_{i,1} \leq 0$ and $q_{i,2} \geq 0$. Also recall that η_i is marginal revenue: $\eta_i \equiv p_i + \frac{q_i}{q_{i,1}}$ and $\hat{\eta}_i \equiv \hat{p}_i + \frac{\hat{q}_i}{\hat{q}_{i,1}}$.

Lemma *In the equilibrium of the pricing game:*

1. *The capacity constraint binds:*$k_i = m_o q_i + m_i \hat{q}_i$.
2. *Firm i's marginal revenue is identical in both of it's markets:*$\eta_i = \hat{\eta}_i$.

Proof: The proof is identical to that of lemma 1 and so omitted. *Q.E.D.*

The following claim will be used throughout:

Claim 1 $\eta_{i,1} \geq 0$, $\hat{\eta}_{i,1} \geq 0$,$\eta_{i,2} \leq 0$ and $\eta_{i,1} \geq -\eta_{i,2}$

Proof: $\eta_{i,1} = 2 - \frac{q_i}{q_{i,1}^2} q_{i,11}$ and $\eta_{i,2} = \frac{q_{i,2}}{q_{i,1}} - q_i \frac{q_{i,12}}{q_{i,1}^2}$. Since $q_{i,1}$ is negative and $q_{i,2}$ is positive (demand is upward sloping in rival's price) and $q_{i,11} \leq 0$ (demand is concave) and $q_{i,12} \geq 0$ (assumption 3) the claim holds true. The proof for $\hat{\eta}_{i,1} \geq 0$ is identical to $\eta_{i,1} \geq 0$.

For the last claim:

$$\eta_{i,1} - (-\eta_{i,2}) = 2 - q_i \frac{q_{i,11}}{q_{i,1}^2} + \frac{q_{i,2}}{q_{i,1}} - q_i \frac{q_{i,12}}{q_{i,1}^2} = \frac{1}{q_{i,1}} \left[2q_{i,1} + q_{i,2} - \frac{q_i}{q_{i,1}} (q_{i,11} + q_{i,12}) \right]$$

As $q_{i,1} < 0$, the claim requires

$$2q_{i,1} + q_{i,2} - \frac{q_i}{q_{i,1}} (q_{i,11} + q_{i,12}) \leq 0$$

Using $q_{i,1} + q_{i,2} \leq 0$ (assumption 3) and again $q_{i,1} \leq 0$, a sufficient condition is $q_{i,11} + q_{i,12} \leq 0$, which follows from assumption 3. *Q.E.D.*

Lemma *In equilibrium of the price setting game,*

$$\phi_i = -\frac{\lambda_i \frac{\partial \hat{q}_i}{\partial \hat{p}_i} \cdot \frac{\partial \eta_i}{\partial p_{-i}} + \frac{\partial q_i}{\partial p_{-i}} \cdot \frac{\partial \hat{\eta}_i}{\partial \hat{p}_i}}{\lambda_i \frac{\partial \hat{q}_i}{\partial \hat{p}_i} \cdot \frac{\partial \eta_i}{\partial p_i} + \frac{\partial q_i}{\partial p_i} \cdot \frac{\partial \hat{\eta}_i}{\partial \hat{p}_i}} \in [0,1] \ .$$

Proof: By lemma A.2, the second stage FOCs for firm i are characterized by

$$\eta_i = \hat{\eta}_i \quad \text{and} \quad m_o q_i + m_i \hat{q}_i = k_i$$

Taking full derivatives of these two equations with respect to rival's prices gives the desired result:

$$\eta_{i,1}\phi_i + \eta_{i,2} - \hat{\eta}_{i,1}\hat{\phi}_i = 0 \qquad \text{and} \qquad m_o\left(q_{i,1}\phi_i + q_{i,2}\right) + m_i \hat{q}_{i,1}\hat{\phi}_i = 0$$

$$0 \quad = \quad \eta_{i,1}\phi_i + \eta_{i,2} + \frac{m_o}{m_i}\hat{\eta}_{i,1}\left(\phi_i q_{i,1} + q_{i,2}\right)/\hat{q}_{i,1}$$

Isolating ϕ_i and replacing $\lambda_i = \frac{m_o}{m_i}$:

$$\phi_i \quad = \quad -\frac{\hat{q}_{i,1}\eta_{i,2} + \lambda_i \hat{\eta}_{i,1} q_{i,2}}{\hat{q}_{i,1}\eta_{i,1} + \lambda_i \hat{\eta}_{i,1} q_{i,1}}$$

When $\lambda_i \to 0$, $\phi_i \to -\frac{\eta_{i,2}}{\eta_{i,1}} = \overline{\phi}_i^1$. When $\lambda_i \to \infty$, $\phi_i \to -\frac{q_{i,2}}{q_{i,1}} = \overline{\phi}_i^2$.

- For $\phi_i \geq 0$: By claim 1, $\eta_{i,2} \leq 0$ and all other partials of the MR are positive. By assumption 3 $q_{i,2} \geq 0$ and all other partials of the quantities sold are negative. Thus, as $\lambda_i \geq 0$, the numerator is always positive and the denominator is always negative, so $\phi_i \geq 0$.
- For $\phi_i \leq 1$:
  - By assumption $|\eta_{i,2}| \leq |\eta_{i,1}|$ and $|q_{i,2}| \leq |q_{i,1}|$. Therefore, $|\hat{q}_{i,1}\eta_{i,1}| \geq |\hat{q}_{i,1}\eta_{i,2}|$ and $|\lambda_i\hat{\eta}_{i,1}q_{i,1}| \geq |\lambda_i\hat{\eta}_{i,1}q_{i,2}|$.
  - Both summands in the denominator have the same sign ($\hat{q}_{i,1}\eta_{i,1} \leq 0$ and $\lambda_i\hat{\eta}_{i,1}q_{i,1} \leq 0$), and thus

$$|\hat{q}_{i,1}\eta_{i,1} + \lambda_i\hat{\eta}_{i,1}q_{i,1}| \geq |\hat{q}_{i,1}\eta_{i,2} + \lambda_i\hat{\eta}_{i,1}q_{i,2}|$$

Note that holding all else fixed,

$$\frac{\partial \phi_i}{\partial \lambda_i} = -\frac{\hat{\eta}_{i,1}q_{i,2}}{\hat{q}_{i,1}\eta_{i,1} + \lambda_i\hat{\eta}_{i,1}q_{i,1}} + \frac{\hat{\eta}_{i,1}q_{i,1}\phi_i}{\hat{q}_{i,1}\eta_{i,1} + \lambda_i\hat{\eta}_{i,1}q_{i,1}} \geq 0$$

The denominator is the same as above so always negative. The first numerator is positive so the first element is positive. The second element is always negative ($\phi_i > 0$). *Q.E.D.*

LEMMA *In the equilibrium of the pricing game:*

$$\sigma_i = \frac{\frac{\partial \hat{\eta}_i}{\partial \hat{p}_i}}{\frac{m_i}{m_o}\left(\frac{\partial \eta_i}{\partial p_i} + \phi_{-i}\frac{\partial \eta_i}{\partial p_{-i}}\right)\frac{\partial \hat{q}_i}{\partial \hat{p}_i} + \frac{\partial \hat{\eta}_i}{\partial \hat{p}_i}\left(\frac{\partial q_i}{\partial p_i} + \phi_{-i}\frac{\partial q_i}{\partial p_{-i}}\right)} < 0$$

PROOF: The second stage FOCs can be characterized by:

$$\eta_i = \hat{\eta}_i \quad \text{and} \quad m_o q_i + m_i \hat{q}_i = k_i$$

Take the full derivative of these with respect to k_i:

(A.1) $$\eta_{i,1}\frac{\sigma_i}{m_o} + \eta_{i,2}\frac{dp_{-i}}{dk_i} - \hat{\eta}_{i,1}\frac{d\hat{p}_i}{dk_i} \quad = \quad 0$$

(A.2) $$\sigma_i q_{i,1} + m_o\frac{dp_{-i}}{dk_i}q_{i,2} + m_i\frac{d\hat{p}_i}{dk_i}\hat{q}_{i,1} \quad = \quad 1$$

Holding k_{-i} fixed, by definition :

$$\frac{dp_{-i}}{dk_i} = \frac{dp_i}{dk_i} \cdot \frac{dp_{-i}}{dp_i} = \frac{\sigma_i}{m_o} \cdot \phi_{-i}$$

Substituting for $\frac{dp_{-i}}{dk_i}$ and isolating $\frac{d\hat{p}_i}{dk_i}$ in A.2 yields

$$\frac{d\hat{p}_i}{dk_i} = \frac{1 - \sigma_i q_{i,1} - \sigma_i \phi_{-i} q_{i,2}}{m_i \hat{q}_{i,1}}$$

Placing both $\frac{dp_{-i}}{dk_i}$ and $\frac{d\hat{p}_i}{dk_i}$ in A.1 yields

$$0 = \frac{\eta_{i,1}}{m_o}\sigma_i + \eta_{i,2}\frac{\sigma_i}{m_o} \cdot \phi_{-i} - \hat{\eta}_{i,1}\frac{1 - \sigma_i q_{i,1} - \sigma_i \phi_{-i} q_{i,2}}{m_i \hat{q}_{i,1}}$$

Multiply by $m_o \cdot \hat{q}_{i,1}$

$$0 = \sigma_i \hat{q}_{i,1}\left(\eta_{i,1} + \eta_{i,2}\phi_{-i}\right) - \lambda_i \hat{\eta}_{i,1} + \lambda_i \hat{\eta}_{i,1}\sigma_i\left(q_{i,1} + \phi_{-i} q_{i,2}\right)$$

Isolate σ_i

$$\sigma_i = \frac{\lambda_i \hat{\eta}_{i,1}}{\hat{q}_{i,1}\left(\eta_{i,1} + \eta_{i,2}\phi_{-i}\right) + \lambda_i \hat{\eta}_{i,1}\left(q_{i,1} + \phi_{-i} q_{i,2}\right)}$$

To show that $\sigma_i < 0$ we show that the numerator of σ_i is positive, while the denominator is negative.

- The numerator is positive from claim 1.
- By $\phi_{-i} \le 1$ (lemma A.2) and $\eta_{i,2} \le 0 \le \eta_{i,1}$ and $\eta_{i,1} \ge -\eta_{i,2}$ (claim 1) :

  $$\eta_{i,1} + \eta_{i,2}\phi_{-i} \ge \eta_{i,1} + \eta_{i,2} \ge 0$$

  As $\hat{q}_{i,1} < 0$, this part of the denominator is negative.
- By $\phi_{-i} \le -\frac{q_{i,2}}{q_{i,1}} \le 1$ and $q_{i,2} \ge 0 > q_{i,1}$ (assumption 1):

  $$q_{i,1} + \phi_{-i} q_{i,2} \le q_{i,1} + q_{i,2} \le 0$$

  As $\hat{\eta}_{i,1} \ge 0$, this part of the denominator is also negative.
- The result implies that $\frac{\partial \sigma_i}{\partial m_i} \ge 0$, $\frac{\partial \sigma_i}{\partial m_o} \le 0$, $\frac{\partial \sigma_i}{\partial \phi_{-i}} \le 0$, which will be used in later proofs. These are proved here:
  - With respect to m_i, σ_i has the form $f(m_i) = \frac{s_1}{s_2 + s_3 m_i / m_o}$ where $s_1 = \hat{\eta}_{i,1}$, $s_2 = \hat{\eta}_{i,1}\left(q_{i,1} + \phi_{-i} q_{i,2}\right)$, and $s_3 = \hat{q}_{i,1}\left(\eta_{i,1} + \phi_{-i}\eta_{i,2}\right)$. As was proved above, s_2 and s_3 are negative. s_1 is positive by claim 1. Thus, increasing m_i increases the denominator in absolute terms and as $\sigma_i \le 0$, $\frac{\partial \sigma_i}{\partial m_i} \ge 0$.
  - The effect of m_o is the opposite of m_i.
  - For $\frac{\partial \sigma_i}{\partial \phi_{-i}} \le 0$: σ_i has the form $f(\phi_{-i}) = \frac{s_1}{\tilde{s}_2 \phi_{-i} + \tilde{s}_3}$ where s_1 is as before and $\tilde{s}_2 = \frac{m_i}{m_o}\eta_{i,2}\hat{q}_{i,1} + q_{i,2}\hat{\eta}_{i,1}$ and $\tilde{s}_3 = \frac{m_i}{m_o}\eta_{i,1}\hat{q}_{i,1} + q_{i,1}\hat{\eta}_{i,1}$. It is easy to verify that $\tilde{s}_2 \ge 0$. Taking derivatives: $f'(\phi_{-i}) = \frac{-s_1 \tilde{s}_2}{\left(\tilde{s}_2 \phi_{-i} + \tilde{s}_3\right)^2} \le 0$.

- As $\hat{q}_{i,1}\left(\eta_{i,1}+\eta_{i,2}\phi_{-i}\right) \leq 0$ and $\sigma_i \leq 0$, the result also implies that

$$\sigma_i \geq \frac{\lambda_i \hat{\eta}_{i,1}}{\lambda_i \hat{\eta}_{i,1}\left(q_{i,1}+\phi_{-i} q_{i,2}\right)}=\frac{1}{q_{i,1}+\phi_{-i} q_{i,2}}$$

Q.E.D.

Proposition 5 *The equilibrium of the game is characterized by the following equations*

$$c-\eta_i=c-\hat{\eta}_i=-\left(\frac{\partial q_i}{\partial p_{-i}} / \frac{\partial q_i}{\partial p_i}\right) \cdot q_i \cdot \phi_{-i} \cdot \sigma_i<0 \qquad i \in\{A, B\} .$$

At the extreme cases:

- *If MMC is large for both firms, $\lambda_i \rightarrow \infty$, the equilibrium in the overlapping markets is the two-stage benchmark*
- *If MMC is small for both firms, $\lambda_i \rightarrow 0$, the equilibrium in the overlapping markets is the one-shot benchmark*

Proof: The objective function in the first stage is

$$\begin{aligned}
\pi_i &= R_i(k_i, k_{-i})-c k_i \\
&= q_i(p_i, p_{-i}) \cdot p_i+\hat{q}_i(\hat{p}_i) \cdot \hat{p}_i-c k_i
\end{aligned}$$

where all prices are functions of k_i and k_{-i}. Taking the partial with respect to k_i gives:

$$\frac{\partial \pi_i}{\partial k_i} = m_o\left(\left(q_i+p_i q_{i,1}\right) \frac{d p_i}{d k_i}+p_i q_{i,2} \frac{d p_{-i}}{d k_i}\right)+m_i\left(\hat{q}_i+\hat{p}_i \hat{q}_{i,1}\right) \frac{d \hat{p}_i}{d k_i}-c$$

Since $\eta_i=p_i+\frac{q_i}{q_{i,1}}$, then $p_i q_{i,1}+q_i=\eta_i q_{i,1}$. Substituting this in and applying the values for $\frac{dp_{-i}}{dk_i}$ and $\frac{d\hat{p}_i}{dk_i}$ as in the proof for lemma A.2:

$$\frac{\partial \pi_i}{\partial k_i} = \eta_i q_{i,1} \sigma_i+p_i q_{i,2} \sigma_i \phi_{-i}+m_i \hat{\eta}_i \hat{q}_{i,1} \frac{1-\sigma_i q_{i,1}-\sigma_i \phi_{-i} q_{i,2}}{m_i \hat{q}_{i,1}}-c$$

Simplifying:

$$\frac{\partial \pi_i}{\partial k_i} = \eta_i q_{i,1} \sigma_i+p_i q_{i,2} \sigma_i \phi_{-i}+\hat{\eta}_i\left(1-q_{i,1} \sigma_i-q_{i,2} \sigma_i \phi_{-i}\right)-c$$

Collecting terms and using $\eta_i=\hat{\eta}_i$ in equilibrium:

$$\begin{aligned}
\frac{\partial \pi_i}{\partial k_i} &= \eta_i+(p_i-\eta_i) q_{i,2} \sigma_i \phi_{-i}-c \\
&= \eta_i-q_i \frac{q_{i,2}}{q_{i,1}} \sigma_i \phi_{-i}-c
\end{aligned}$$

Equating the FOC to zero obtains the result.

To show that $-q_i \frac{q_{i,2}}{q_{i,1}} \sigma_i \phi_{-i}<0$: $q_{i,1}$ and σ_i are negative while $q_{i,2}$ and ϕ_{-i} are positive.

- If MMC is large for both firms, $\lambda_i \rightarrow \infty$, the equilibrium in the overlapping markets is by definition the two-stage benchmark – the firms compete only in the overlapping market using the two stage structure.
- If MMC is small for both firms, $\lambda_i \rightarrow 0$, $\sigma_i \rightarrow 0$ and so the one shot benchmark is obtained ($\eta_i=c$).

Q.E.D.

PROPOSITION 6 *If prices in the overall game are strategic complements, then any increase in MMC increases prices for both firms.*

$$\begin{array}{lll} \frac{dp_i}{dm_i} < 0 & \frac{dp_i}{dm_{-i}} \le 0 & \dfrac{dp_i}{dm_o} \ge 0 \\ \frac{d\hat{p}_i}{dm_i} < 0 & \frac{d\hat{p}_i}{dm_{-i}} \le 0 & \dfrac{d\hat{p}_i}{dm_o} \ge 0 \end{array}$$

- *Firms are always less aggressive than in the one-shot benchmark and more aggressive that in the two-stage benchmark.*
- *Consumer welfare and total surplus decrease in MMC*

PROOF: The proof proceeds similar to that of proposition 3. The first part of the proof also establishes that it is sufficient to prove the comparative static result for p_i and the same holds for $\hat{p}_i$. The comparative statics imply that the most aggressive outcome is when $\frac{m_o}{m_i} \to 0$ and the least aggressive is when $\frac{m_i}{m_o} \to \infty$. Applying the statements of proposition 5 for these two extreme cases obtains the first bullet statement. The second bullet follows as prices increase in MMC and are always above cost.

Step 1. Change of variables.

The four equations that characterize the second stage equilibrium provide a mapping between $(p_i, \hat{p}_i)$ and (k_i, k_{-i}). These four equations are the two equations equating marginal revenue (one for each firm) and the two constraints (one for each firm). Apply the implicit function theorem on these four equations to express k_i, $\hat{p}_i$, and p_{-i} as a function of (p_i, k_{-i}). Denote these functions by κ_i ,$\hat{\vartheta}_i$, and ϑ_{-i} so that $\kappa_i(p_i, k_{-i}) = k_i$, $\tilde{\vartheta}_i(p_i, k_{-i}) = \hat{p}_i$, and $\vartheta_{-i}(p_i, k_{-i}) = p_{-i}$.

The definition of flexibility (ϕ) implies that $\frac{d\vartheta_{-i}}{dp_i} = \phi_{-i}$.

The definition of commitment (σ) implies that $\frac{d\kappa_i}{dp_i} = \frac{m_o}{\sigma_i}$.

By construction

$$\frac{d\hat{\vartheta}_i}{dp_i} = \frac{dk_i}{dp_i} \cdot \frac{d\hat{p}_i}{dk_i}$$

To determine $\frac{\partial\hat{\vartheta}_i}{\partial p_i}$, start from the result in the proof for lemma A.2:

$$\begin{aligned} \frac{d\hat{\vartheta}_i}{dk_i} &= \frac{m_o}{\sigma_i} \cdot \frac{1 - \sigma_i q_{i,1} - \sigma_i \phi_{-i} q_{i,2}}{m_i \hat{q}_{i,1}} \\ &= \lambda_i \frac{\frac{1}{\sigma_i} - q_{i,1} - q_{i,2}\phi_{-i}}{\hat{q}_{i,1}} \end{aligned}$$

From lemma A.2,

$$\frac{1}{\sigma} = \frac{1}{\lambda_i} \frac{\hat{q}_{i,1}}{\hat{\eta}_{i,1}} (\eta_{i,1} + \eta_{i,2}\phi_{-i}) + q_{i,1} + q_{i,2}\phi_{-i}$$

So:

$$\begin{aligned}
\frac{\partial\hat{\vartheta}_i}{\partial p_i} &= \frac{\lambda_i}{\hat{q}_{i,1}}\left(\frac{1}{\lambda_i}\frac{\hat{q}_{i,1}}{\hat{\eta}_{i,1}}\left(\eta_{i,1}+\eta_{i,2}\phi_{-i}\right)+q_{i,1}+q_{i,2}\phi_{-i}-q_{i,1}-q_{i,2}\phi_{-i}\right)\\
&= \frac{\lambda_i}{\hat{q}_{i,1}}\left(\frac{1}{\lambda_i}\frac{\hat{q}_{i,1}}{\hat{\eta}_{i,1}}\left(\eta_{i,1}+\eta_{i,2}\phi_{-i}\right)\right)\\
&= \frac{\eta_{i,1}+\eta_{i,2}\phi_{-i}}{\hat{\eta}_{i,1}}
\end{aligned}$$

By $\phi_{-i}\in[0,1]$ and $\eta_{i,1}\geq-\eta_{i,2}$, we have that $\frac{\partial\hat{\vartheta}_i}{\partial p_i}\geq 0$ and so it is sufficient to prove the comparative static result in the proposition for p_i and the same holds for $\hat{p}_i$.

The objective function can now be expressed in terms of p_i:

$$\pi_i(p_i)=m_o q_i(p_i,\vartheta_{-i})p_i+m_i\hat{\vartheta}_i\hat{q}_i(\hat{\vartheta}_i)-c\kappa_i$$

The FOC is

$$\frac{\partial\pi_i}{\partial p_i}=\pi_{p_i} \quad = \quad m_o\left(q_{i,1}p_i+q_i+p_iq_{i,2}\frac{d\vartheta_{-i}}{dp_i}\right)+m_i\left(\hat{q}_{i,1}\hat{p}_i+\hat{q}_i\right)\frac{d\hat{\vartheta}_i}{dp_i}-c\frac{d\kappa_i}{dp_i}$$

Using $q_{i,1}p_i+q_i=\eta_iq_{i,1}$ and substituting in the results for $\frac{d\vartheta_{-i}}{dp_i}$, $\frac{d\hat{\vartheta}_i}{dp_i}$, and $\frac{d\kappa_i}{dp_i}$ the FOC is:

$$\pi_{p_i} \quad = \quad m_o\eta_iq_{i,1}+m_op_iq_{i,2}\phi_{-i}+m_i\hat{\eta}_i\hat{q}_{i,1}\left(\frac{\eta_{i,1}+\eta_{i,2}\phi_{-i}}{\hat{\eta}_{i,1}}\right)-c\frac{m_o}{\sigma_i}$$

From lemma A.2, the following relationship holds:

$$m_i\hat{q}_{i,1}\left(\eta_{i,1}+\eta_{i,2}\phi_{-i}\right)/\hat{\eta}_{i,1}=\frac{m_o}{\sigma_i}-m_o\left(q_{i,1}+q_{i,2}\phi_{-i}\right)$$

Substituting this into the FOC gives

$$\begin{aligned}
\pi_{p_i} &= m_o\eta_iq_{i,1}+m_op_iq_{i,2}\phi_{-i}+\hat{\eta}_i\left(\frac{m_o}{\sigma_i}-m_o(q_{i,1}+q_{i,2}\phi_{-i})\right)-c\frac{m_o}{\sigma_i}\\
&= \frac{m_o}{\sigma_i}\left(\eta_iq_{i,1}\sigma_i+p_iq_{i,2}\phi_{-i}\sigma_i+\hat{\eta}_i-\hat{\eta}_iq_{i,1}\sigma_i-\hat{\eta}_iq_{i,2}\phi_{-i}\sigma_i-c\right)\\
&= \frac{m_o}{\sigma_i}\left(q_{i,2}\phi_{-i}\sigma_i\left(p_i-\hat{\eta}_i\right)+q_{i,1}\sigma_i\left(\eta_i-\hat{\eta}_i\right)+\hat{\eta}_i-c\right)
\end{aligned}$$

In equilibrium $\pi_{p_i}=0$ and so

$$\tilde{\pi}_{p_i}=q_{i,2}\phi_{-i}\sigma_i\left(p_i-\hat{\eta}_i\right)+q_{i,1}\sigma_i\left(\eta_i-\hat{\eta}_i\right)+\hat{\eta}_i-c=0$$

In addition, in equilibrium $\hat{\eta}_i=\eta_i$ and

$$p_i-\hat{\eta}_i=-\frac{q_i}{q_{i,1}}$$

Placing these in these in $\tilde{\pi}_{p_i}$ yields the same solution as in Proposition 5:

$$c-\eta_i==-q_{i,2}\phi_{-i}\sigma_i\cdot\frac{q_i}{q_{i,1}}$$

For the following steps, we also use $S_i = q_{i,2}\phi_{-i}\sigma_i\left(p_i - \hat{\eta}_i\right)$.

Step 2. The FOC is decreasing in m_i when holding p_{-i} fixed.

Since $\sigma_i \leq 0$, the sign of $\frac{\partial \pi_{p_i}}{\partial m_i}$ when holding p_{-i} fixed is given by the sign of $\frac{\partial}{\partial m_i}(-\tilde{\pi}_{p_i})$.

As $\hat{\eta}_i = \eta_i$:

$$\frac{\partial}{\partial m_i}(-\tilde{\pi}_{p_i}) = \frac{\partial}{\partial m_i}(-S_i) = q_i \frac{q_{i,2}}{q_{i,1}}\left(\sigma_i \frac{\partial \phi_{-i}}{\partial m_i} + \phi_{-i}\frac{\partial \sigma_i}{\partial m_i}\right) \leq 0$$

By lemma A.2, $\frac{\partial \phi_{-i}}{\partial m_i} = 0$. By lemma A.2 $\frac{\partial \sigma_i}{\partial m_i} \geq 0$. As $q_i \frac{q_{i,2}}{q_{i,1}}$ is negative and ϕ_{-i} is positive, π_{p_i} is decreasing in m_i.

Step 3. The FOC is decreasing in m_{-i} when holding p_{-i} fixed.

Since $\sigma_i \leq 0$, the sign of $\frac{\partial \pi_{p_i}}{\partial m_{-i}}$ when holding p_{-i} fixed is given by the sign of $\frac{\partial}{\partial m_{-i}}(-\tilde{\pi}_{p_i})$.

As $\hat{\eta}_i = \eta_i$:

$$\begin{aligned}\frac{\partial}{\partial m_{-i}}(-\tilde{\pi}_{p_i}) = \frac{\partial}{\partial m_{-i}}(-S_i) &= q_i \frac{q_{i,2}}{q_{i,1}}\left(\sigma_i \frac{\partial \phi_{-i}}{\partial m_{-i}} + \phi_{-i}\left(\frac{\partial \sigma_i}{\partial m_{-i}} + \frac{\partial \sigma_i}{\partial \phi_{-i}}\frac{\partial \phi_{-i}}{\partial m_{-i}}\right)\right) \\ &= q_i \frac{q_{i,2}}{q_{i,1}}\frac{\partial \phi_{-i}}{\partial m_{-i}}\left(\sigma_i + \phi_{-i}\frac{\partial \sigma_i}{\partial \phi_{-i}}\right)\end{aligned}$$

The second line follows from lemma A.2: $\frac{\partial \sigma_i}{\partial m_{-i}} = 0$.

Since $\sigma_i \leq 0$, $\phi_{-i} \geq 0$ and $\frac{\partial \sigma_i}{\partial \phi_{-i}} \leq 0$ (see proof of lemma A.2), the term in parenthesis is negative. $\frac{\partial \phi_{-i}}{\partial m_{-i}} \leq 0$ (see proof of lemma A.2) and $q_i \frac{q_{i,2}}{q_{i,1}}$ is negative so the term outside the parenthesis is positive and thus $\frac{\partial}{\partial m_i}(-S_i)$ is negative.

Step 4. The FOC is increasing in m_o when holding p_{-i} fixed.

Since $\sigma_i \leq 0$, the sign of $\frac{\partial \pi_{p_i}}{\partial m_o}$ when holding p_{-i} fixed is given by the sign of $\frac{\partial}{\partial m_o}(-\tilde{\pi}_{p_i})$.

As $\hat{\eta}_i = \eta_i$:

$$\frac{\partial}{\partial m_o}(-\tilde{\pi}_{p_i}) = \frac{\partial}{\partial m_o}(-S_i) = q_i \frac{q_{i,2}}{q_{i,1}}\left(\sigma_i \frac{\partial \phi_{-i}}{\partial m_o} + \phi_{-i}\left(\frac{\partial \sigma_i}{\partial m_o} + \frac{\partial \sigma_i}{\partial \phi_{-i}}\frac{\partial \phi_{-i}}{\partial m_o}\right)\right) \geq 0$$

To prove the sign:

- $q_i \frac{q_{i,2}}{q_{i,1}} \leq 0$
- $\frac{\partial \phi_{-i}}{\partial m_o} \geq 0$ (see proof of lemma A.2) and $\sigma_i \leq 0$ so the first term in the parenthesis is negative.
- $\phi_{-i} \geq 0$, $\frac{\partial \sigma_i}{\partial m_o} \leq 0$ and $\frac{\partial \phi_{-i}}{\partial m_o} \geq 0$ and $\frac{\partial \sigma_i}{\partial \phi_{-i}} \leq 0$ (see proof of lemma A.2). Thus the second term in the parenthesis is also negative.

Step 5. Strategic complementarity is preserved: $\frac{\partial^2 \pi_i}{\partial p_i \partial p_{-i}} \geq 0$

This is part of the proposition's statement.

However, as with quantity competition, it can be proved by adding similar structure on flexibility and commitment. See next Lemma. *Q.E.D.*

LEMMA 5 *If in equilibrium:* $\frac{\partial \phi_i}{\partial k_i} \leq 0$, $\frac{\partial \sigma_i}{\partial k_{-i}} \geq 0$, *then* $\frac{\partial^2 \pi_i}{\partial p_i \partial p_{-i}} \geq 0$

PROOF: As in the previous section, for the rival to change p_{-i}, it must change k_{-i} by a factor of $\sigma_{-i} < 0$. Thus, for strategic complementarity to be preserved we need to show that $\frac{\partial^2 \pi_i}{\partial p_i \partial k_{-i}} \leq 0$

Since $\sigma_i \leq 0$, the sign of $\frac{\partial \pi_{p_i}}{\partial k_{-i}}$ is given by the sign of $\frac{\partial}{\partial k_{-i}}(-\tilde{\pi}_{p_i})$.

$$\frac{\partial}{\partial k_{-i}}(-\tilde{\pi}_{p_i}) = \frac{\partial}{\partial k_{-i}}(-q_{i,2}\phi_{-i}\sigma_i (p_i - \hat{\eta}_i) - \frac{\partial}{\partial k_{-i}} (q_{i,1}\sigma_i (\eta_i - \hat{\eta}_i)) - \frac{\partial \hat{\eta}_i}{\partial k_{-i}})$$

As $\hat{p}_i$ and k_i are functions only of k_{-i} and p_i so that $\eta_i = \hat{\eta}_i$, so $\frac{\partial \eta_i}{\partial p_{-i}} = \frac{\partial \hat{\eta}_i}{\partial p_{-i}}$. Thus

$$\frac{\partial}{\partial k_{-i}} (q_{i,1}\sigma_i (\eta_i - \hat{\eta}_i)) = 0 \quad \text{and} \quad \frac{\partial \hat{\eta}_i}{\partial k_{-i}} = \sigma_{-i}\eta_{i,2} \text{ and } \; p_i - \hat{\eta}_i = -\frac{q_{i,2}}{q_{i,1}}$$

Plugging these in:

$$\begin{aligned}
\frac{\partial}{\partial k_{-i}}(-\tilde{\pi}_{p_i}) &= \frac{\partial}{\partial k_{-i}}(-q_{i,2}\phi_{-i}\sigma_i (p_i - \eta_i)) - \sigma_{-i}\eta_{i,2} \\
&= \frac{\partial}{\partial k_{-i}} \left(q_i \frac{q_{i,2}}{q_{i,1}} \phi_{-i}\sigma_i \right) - \sigma_{-i}\eta_{i,2} \\
&= q_i \frac{q_{i,2}}{q_{i,1}} \frac{\partial}{\partial k_{-i}} (\phi_{-i}\sigma_i) + \phi_{-i}\sigma_i \frac{\partial}{\partial k_{-i}} \left(q_i \frac{q_{i,2}}{q_{i,1}} \right) - \sigma_{-i}\eta_{i,2} \\
&= q_i \frac{q_{i,2}}{q_{i,1}} \left(\phi_{-i} \frac{\partial \sigma_i}{\partial k_{-i}} + \sigma_i \frac{\partial \phi_{-i}}{\partial k_{-i}} \right) + \phi_{-i}\sigma_i\sigma_{-i} \left(q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2} \right) - \sigma_{-i}\eta_{i,2}
\end{aligned}$$

The last equality follows from $\eta_{i,2} = \frac{\partial}{\partial p_{-i}} \left(p_i + \frac{q_i}{q_{i,1}} \right) = \frac{\partial}{\partial p_{-i}} \left(\frac{q_i}{q_{i,1}} \right)$.

Both $\frac{\partial \sigma_i}{\partial k_{-i}} \geq 0$ and $\frac{\partial \phi_{-i}}{\partial k_{-i}} \leq 0$ by assumption. Since $q_i \frac{q_{i,2}}{q_{i,1}}$ is negative, the whole first term is negative and:

$$\frac{\partial}{\partial p_{-i}}(-\tilde{\pi}_{p_i}) \leq \sigma_{-i} \cdot \left[\phi_{-i}\sigma_i \left(q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2} \right) - \eta_{i,2} \right]$$

As σ_{-i} is negative ,$\sigma_i \leq 0$ and $\phi_{-i} \geq 0$, if $q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2} \leq 0$ the first term is positive and as $\eta_{i,2} \leq 0$ the proof is complete.

If $q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2} \geq 0$ then, as proved at the end of lemma A.2

$$\sigma_i \geq \frac{1}{q_{i,1} + \phi_{-i} q_{i,2}}$$

Then

$$\frac{\partial}{\partial p_{-i}}(-\tilde{\pi}_{p_i}) \leq 0 \iff \phi_{-i} \frac{q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2}}{q_{i,1} + \phi_{-i} q_{i,2}} - \eta_{i,2} \geq 0$$

We now show the second inequality holds:

LEMMA *If* $q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2} \geq 0$ *then* $\phi_{-i} \frac{q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2}}{q_{i,1} + \phi_{-i} q_{i,2}} - \eta_{i,2} \geq 0$

PROOF: We prove that multiplying both sides by $q_{i,1} + \phi_{-i} q_{i,2}$ results in a negative number:

$$\phi_{-i} \left(q_{i,22} \frac{q_i}{q_{i,1}} + q_{i,2}\eta_{i,2} \right) - \eta_{i,2} (q_{i,1} + \phi_{-i} q_{i,2}) \quad \leq 0$$

Collect terms, we need to show that

$$\phi_{-i}\left(q_{i,22}\frac{q_i}{q_{i,1}}+q_{i,2}\eta_{i,2}-\eta_{i,2}q_{i,2}\right)-\eta_{i,2}q_{i,1}\leq 0$$

Thus, it is sufficient to show that

$$\phi_{-i}q_{i,22}\frac{q_i}{q_{i,1}}-\eta_{i,2}q_{i,1}\leq 0$$

By definition,

$$\eta_{i,2}=\frac{q_{i,2}}{q_{i,1}}-\frac{q_iq_{i,12}}{\left(q_{i,1}\right)^2}.$$

Plugging in, we need to show that

$$\phi_{-i}q_{i,22}\frac{q_i}{q_{i,1}}-q_{i,2}+\frac{q_iq_{i,12}}{q_{i,1}}\leq 0$$

As $q_{i,2}\geq 0$, $q_{i,12}\geq 0$, $q_{i,1}\leq 0$ and $\phi_{-i}\in[0,1]$ the inequality holds for $\phi_{-i}=0$ directly. So, it remains to show that the inequality holds for $\phi_{-i}=1$. In this case the inequality is

$$-q_{i,2}+\frac{q_i}{q_{i,1}}\left(q_{i,12}+q_{i,22}\right)$$

As $q_{i,2}\geq 0$ and by assumption 3 $q_{i,12}+q_{i,22}\geq 0$, the proof is complete. *Q.E.D.*

Q.E.D.

A.3. *Merger Model (section 4)*

A.3.1. *Premerger Benchmarks*

- The pre-merger benchmark is as derived by setting $m_i=0$ for both firms in the linear solution (appendix B.2)
- If one firm (A) has a cost of $(1-\rho)\,c$, re-solve the linear solution with the different costs. The resulting equilibrium quantities and profits are

$$k_A=\frac{3}{10}\left(2a-bc+\frac{4}{3}bc\rho\right)\quad;\quad k_C=\frac{3}{10}\left(2a-bc-\frac{1}{3}bc\rho\right)$$
$$\pi_A=\frac{3}{b}\left(\frac{2a-bc+\frac{4}{3}bc\rho}{5}\right)^2\quad;\quad \pi_C=\frac{3}{b}\left(\frac{2a-bc-\frac{1}{3}bc\rho}{5}\right)^2$$

- If both firms obtain the lower cost, simply replace c in the pre-merger benchmark with $(1-\rho)\,c$.

A.3.2. *Proposition 7*

Proposition *If firms A and B merge, total market quantity, total welfare and consumer surplus increases. However, per market:*

- *The merged-firm produces more post-merger iff cost savings (ρ) satisfy $\rho>\frac{1}{135}\underline{\pi}$*
- *The merged-firm produces more than the non-merged firm iff cost savings (ρ) satisfy $\rho>\frac{1}{34}\underline{\pi}$*

- *Each non-merged firm produces **less** post-merger iff cost savings (ρ) satisfy $\rho > \frac{4}{35}\underline{\pi}$*

PROOF: Premerger market quantity per firm is $\frac{3}{10}bc\underline{\pi}$. Market shares are $\frac{1}{2}$ per firm. Post merger quantities are given in Lemma 4. *Q.E.D.*

- Total market quantity is

$$q_{AB} + q_C = \frac{3}{202} bc \left(41\underline{\pi} + 20\rho\right)$$

As quantity increases with ρ it is sufficient to show for $\rho = 0$ which is simple algebra $\left(\frac{123}{202} - \frac{6}{10} > 0\right)$.

- Total welfare and CS are

$$\begin{aligned} W\left(q_{AB}, q_C\right) &= U\left(q_{AB}, q_C\right) - c\left(1-\rho\right) q_{AB} - cq_C \\ CS\left(q_{AB}, q_C, p_{AB}, p_C\right) &= U\left(q_{AB}, q_C\right) - p_{AB}q_{AB} - p_C q_C \end{aligned}$$

Prices are

$$p_{AB} = \frac{80a + 61bc - 47bc\rho}{101b} \qquad ; \qquad p_C = \frac{78a + 62bc - 13bc\rho}{101b}$$

Plugging in all the values shows that both W and CS increase in ρ and are larger at $\rho = 0$ than the values pre-merger. The values were obtained using the Mathematica algebra solver. We reproduce the values here for reference:

$$\begin{aligned} CS^{OneMerger} &= \frac{3}{204020b}\left(5044a^2 - 2abc\left(2522 - 1213\rho\right) + (bc)^2\left(1261 - 1213\rho + 589\rho^2\right)\right) \\ W^{OneMerger} &= \frac{3}{204020b}\left(11520a^2 - 2abc\left(5760 - 2827\rho\right) + (bc)^2\left(2880 - 2827\rho + 2138\rho^2\right)\right) \end{aligned}$$

$$\begin{aligned} CS^{OneMerger}\left(\rho = 0\right) &= \frac{3783}{2b}\left(\frac{2a - bc}{101}\right)^2 \\ W^{OneMerger}\left(\rho = 0\right) &= \frac{4320}{b}\left(\frac{2a - bc}{101}\right)^2 \end{aligned}$$

$$\begin{aligned} CS^{NoMerger} &= \frac{9}{2b}\left(\frac{2a - bc}{5}\right)^2 \\ W^{NoMerger} &= \frac{21}{2b}\left(\frac{2a - bc}{5}\right)^2 \end{aligned}$$

It is easy to verify that

$$CS^{OneMerger}\left(\rho = 0\right) - CS^{NoMerger} = \frac{1383}{255025b}\left(2a - bc\right)^2 > 0$$

and

$$W^{OneMerger}\left(\rho = 0\right) - W^{NoMerger} = \frac{1779}{510050b}\left(2a - bc\right)^2 > 0$$

- Merged firm produces more post-merger iff $\rho \geq \frac{\underline{\pi}}{135}$. Simplify

$$\frac{3}{202} bc \left(20\underline{\pi} + 27\rho\right) \geq \frac{3}{10} bc\underline{\pi}$$

- Merged firm's market share increases iff $\rho \geq \frac{\underline{\pi}}{34}$. Simplify

$$\frac{3}{202}bc\left(20\underline{\pi}+27\rho\right) \geq \frac{3}{202}bc\left(21\underline{\pi}-7\rho\right)$$

- Nonmerged firm produces less iff $\rho \geq \underline{\pi}\frac{4}{35}$. Simplify

$$\frac{3}{202}bc\left(21\underline{\pi}-7\rho\right) \geq \frac{3}{10}bc\underline{\pi}$$

A.3.3. *Proposition 8:*

PROPOSITION *If firms A and B merge:*

- *The merger increases the merged-firm's profits (relative to the pre-merger joint profits) iff cost savings satisfy* $\rho > \frac{1}{135}\underline{\pi}$
- *The non-merged firms' profits increase iff* $.00535\underline{\pi}$

PROOF: Pre-merger profits are

$$\pi = \frac{3}{b}\left(bc\right)^2\left(\frac{\underline{\pi}}{5}\right)^2$$

Post merger profits are

$$\begin{aligned} \pi_{AB} &= \frac{3}{b}\left(bc\right)^2\left(\frac{20\underline{\pi}+27\rho}{101}\right)^2 \\ \pi_C &= \frac{3}{b}\cdot\left(bc\right)^2\cdot\frac{91}{2}\left(\frac{3\underline{\pi}-\rho}{101}\right)^2 \end{aligned}$$

Comparing the terms obtains the results. *Q.E.D.*

A.3.4. *Proposition 9:*

PROPOSITION *The second merger is profitable iff* $\rho \geq 0.00134\underline{\pi}$*. In particular, if cost efficiencies are large enough such that the first merger is profitable, then so is the second merger. Moreover, the firm that merged first benefits from the second merger iff the first merger induced a decrease in the merged-firm's market share; i.e.:*$\rho < \frac{1}{34}\underline{\pi}$.

PROOF: The condition for $\pi_C < \pi_{CD}$ is:

$$\frac{3}{b}\cdot\left(bc\right)^2\cdot\frac{91}{2}\left(\frac{3\underline{\pi}-\rho}{101}\right)^2 < \frac{3}{b}\left(bc\right)^2\left(\frac{\underline{\pi}+\rho}{5}\right)^2$$

The condition for $\pi_{AB}^{OneMerger} < \pi_{AB}^{TwoMergers}$ is:

$$\frac{3}{b}\left(bc\right)^2\left(\frac{20\underline{\pi}+27\rho}{101}\right)^2 < \frac{3}{b}\left(bc\right)^2\left(\frac{\underline{\pi}+\rho}{5}\right)^2$$

Isolating ρ in each term obtains the result. *Q.E.D.*

A.3.5. *Proposition 10*

PROPOSITION *The second merger increases consumer surplus iff it decreases profits for the first merged firm (*$\rho > \frac{1}{34}\underline{\pi}$*). If* $\rho < 0.0082\underline{\pi}$ *the second merger decreases total welfare as well as consumer surplus.*

PROOF: The values for CS and total welfare after the second merger are the same as for no merger, with c replaced by $(1-\rho)\,c$. These are provided in the proof for proposition 7. Comparing to the values after one merger (also provided in the same proof) obtains two rather complicated terms. The values were obtained using the Mathematica algebra solver. For both CS and W, the difference from the single merger is increasing in ρ and is negative for $\rho = 0$. Thus, the proposition is obtained by finding the ρ values for which CS or total welfare exactly equal before and after the second merger. *Q.E.D.*

APPENDIX B: EXACT SOLUTIONS FOR LINEAR DEMAND

We provide here the exact solution to the prices and quantities in both models with linear demand.

B.1. *Cournot*

The benchmark is the standard $q^* = \frac{a-c}{3b}$ for the quantity in an overlapping market and $\hat{q}^* = \frac{a-c}{2b}$ for the quantity in a private market.

As the marginal revenues are equal, for each firm:

$$2q_i + q_{-i} \quad = \quad 2\hat{q}_i$$

Placing the values ofϕ and σ in the foc yields four equations (a foc and the equation above for each firm) in four unknowns (the market quantities). The result is

$$q_A = q^* \frac{(9+6\lambda_B+12\lambda_A+12\lambda_A\lambda_B)(3+4\lambda_B+4\lambda_A(1+\lambda_B))}{16\lambda_A^2(1+\lambda_B)(2+3\lambda_B)+20\lambda_A(1+\lambda_B)(3+4\lambda_B)+(3+4\lambda_B)(9+8\lambda_B)}$$

$$\hat{q}_A = \hat{q}^* \cdot \frac{(9+8\lambda_B+10\lambda_A+12\lambda_A\lambda_B)(3+4\lambda_B+4\lambda_A(1+\lambda_B))}{16\lambda_A^2(1+\lambda_B)(2+3\lambda_B)+20\lambda_A(1+\lambda_B)(3+4\lambda_B)+(3+4\lambda_B)(9+8\lambda_B)}$$

Note that the only difference between the two fractions is the first element in the numerator, and that these are equal when $\lambda_A = \lambda_B$, in which case we have that

$$q_i \quad = \tfrac{2}{3}\hat{q}_i \quad = \frac{a-c}{3b} \cdot \frac{9+24\lambda+12\lambda^2}{9+22\lambda+12\lambda^2} \quad .$$

B.2. *Price Competition*

B.2.1. *Benchmarks*

The private market benchmark solution is the solution from equating marginal revenue to marginal cost.

The one shot benchmark for the overlapping market is the simultaneous solution to

$$\max_{p_i} (p_i - c) \cdot \left(a - bp_i + \frac{b}{2}p_{-i}\right)$$

The f.o.c. is

$$2\frac{a}{b} + 2c + p_{-i} = 4p_i$$

Solving for both simultaneously:

$$p_i^1 = \frac{2}{3}\left(\frac{a}{b}+c\right)$$
$$q_i^1 = \frac{1}{3}\left(2a-b\cdot c\right)$$

The two-stage benchmark is solved by setting $m_i = 0$ for both firms, which yields

$$\tilde{p}_i = \frac{1}{5}\left(4\frac{a}{b}+3c\right)$$
$$\tilde{q}_i = \frac{3}{10}\left(2a-b\cdot c\right) = \frac{9}{10}q_i^1$$

B.2.2. *Solution*

With linear demand

$$\eta_i = p_i - \frac{q\left(p_i, p_{-i}\right)}{b}$$
$$\hat{\eta}_i = \hat{p}_i - \frac{q\left(\hat{p}_i\right)}{b}$$

Setting $\hat{\eta}_i = \eta_i$, plugging in the values for ϕ and σ and using the foc for η_i yields four equations in four unknowns. The solution is

$$\begin{aligned} p_i &= \frac{2c}{\xi_A}\left(75+74\lambda_B+72\lambda_A+72\lambda_A\lambda_B\right)\left(15+14\lambda_B+2\lambda_A(7+6\lambda_B)\right) \\ &\quad +\frac{2a}{b\xi_A}(15+14\lambda_B+16\lambda_A+16\lambda_A\lambda_B)(75+76\lambda_B+\lambda_A(70+72\lambda_B)) \\ \hat{p}_i &= \frac{c}{2\xi_A}\left(225+214\lambda_A+224\lambda_B+216\lambda_A\lambda_B\right)\left(15+14\lambda_B+2\lambda_A(7+6\lambda_B)\right) \\ &\quad +\frac{a}{2b\xi_A}\left((15+14\lambda_B)(225+224\lambda_B)+8\lambda_A^2(427+6\lambda_B(143+72\lambda_B))+6\lambda_A(1135+6\lambda_B(373+184\lambda_B))\right) \end{aligned}$$

$$\xi_A = 3375+14\lambda_A(465+224\lambda_A)+6510\lambda_B+4\lambda_A(3131+1504\lambda_A)\lambda_B+64(1+\lambda_A)(49+45\lambda_A)\lambda_B^2$$

If $\lambda_A = \lambda_B = \lambda$

$$p = \frac{a\left(30+60\lambda+32\lambda^2\right)+2bc\left(15+28\lambda+12\lambda^2\right)}{b\left(45+86\lambda+40\lambda^2\right)}$$

Quantities are

$$\begin{aligned} q_A &= \tilde{q}\frac{10}{3}\frac{1}{\xi_A}\left(1125+2190\lambda_B+1064\lambda_B^2+4\lambda_A^2\left(245+462\lambda_B+216\lambda_B^2\right)+4\lambda_A\left(525+1006\lambda_B+480\lambda_B^2\right)\right) \\ \hat{q}_A &= \frac{a}{2\xi_A}\left(3(15+14\lambda_A)(75+68\lambda_A)+2\left(3255+5810\lambda_A+2584\lambda_A^2\right)\lambda_B+32(98+\lambda_A(169+72\lambda_A))\lambda_B^2\right) \\ &\quad -\frac{bc}{2\xi_A}(225+214\lambda_A+8(28+27\lambda_A)\lambda_B)(15+14\lambda_B+2\lambda_A(7+6\lambda_B)) \end{aligned}$$

If the firms are symmetric ($\lambda_A = \lambda_B = \lambda$)

$$q = \tilde{q}\frac{10}{3}\cdot\frac{15+28\lambda+12\lambda^2}{45+86\lambda+40\lambda^2}$$

Note that if $\lambda \to \infty$, $q \to \tilde{q}$ and if $\lambda = 0$, $q = \frac{10}{9}\tilde{q} = q^1$.

APPENDIX C: N FIRMS

Consider an industry with many markets in which all firms are active (overlapping markets) and in addition each firm has some markets that it dominates (private markets). As in the two firm case, we let m_i denote the number of

i's private markets and m_o the number of overlapping markets. For simplicity we assume that each firm has a private market $\left(m^i > 0\right)$ and all markets have identical inverse demand $P(Q)$ where Q is the total quantity in the market, and $P()$ satisfies the demand regularity assumption (1). As in the two firm case, we use q_i,$\hat{q}_i$, η_i and $\hat{\eta}_i$ to denote i's quantities in the private and overlapping markets and it's marginal revenues.

For the results, it is sufficient to focus on *rival* flexibility and *own* commitment. Thus, we let ϕ_{-i} denote the second stage reaction curve of all of i's rivals for a marginal change in i's quantity in all overlapping markets:

$$\phi_{-i} \equiv \sum_{n \neq i} \frac{\partial q_n}{\partial q_i}$$

Commitment is the same as in the two firm case:

$$\sigma_i = m_o \frac{dq_i}{dk_i}$$

The intermediate results established for the two firm case extend directly, as the arguments do not depend on the two firm structure. These are stated without proof:

LEMMA 6 *For any I-firm equilibrium, the following hold:*

- *Capacity binds for each firm.*
- *Marginal revenue for i is identical in all markets i serves.* $\eta_i = \hat{\eta}_i$.
- $\frac{d\phi_{-i}}{dm_{j\neq i}} \leq 0$. *That is, as i's rivals have better outside options for their extra capacity, they become more accommodating (flexible).*
- $\frac{d\sigma_i}{dm_i} \leq 0$ *and* $\frac{d\sigma_i}{dm_{j\neq i}} \geq 0$. *That is, as i's private markets can accommodate more quantity, i's commitment to all of its overlapping markets decreases and i's rivals' commitment power in the overlapping markets increases.*

For the comparative static result, we also directly assume that, as in the two firm case, strategic substitution is maintained – if firm i increases quantity in all of its markets, its rivals weakly decrease their quantity. We note that in the two firm case, this made use of assumption 2 and thus the parallel assumption for N firms is required. These are

- Rivals become less flexible when their capacity increases: $\frac{\partial \phi_{-i}}{\partial k_{j\neq i}} \geq 0$
- Commitment to overlapping markets decreases in the rival's capacity: $\frac{\partial \sigma_i}{\partial k_j} \leq 0$.

The proof of proposition 3 can now be replicated directly.