Contractual Federalism and Strategy-proof Coordination[1]

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Abstract

This paper takes a mechanism design approach to federalism and assumes that local preferences are the private information of local jurisdictions. Contractual federalism is defined as a strategy-proof contract among the members of the federation supervised by a benevolent but not omniscient federal authority. We show that even if the size of the information to be elicited is minimal, the incentive compatibility constraint has a bite in terms of flexibility and welfare. Strategy-proof and efficient federal mechanisms are necessarily uniform. There exists inefficient and non-uniform strategy-proof mechanisms, but they are socially worse than non cooperative decentralization. Federal mechanisms which are neutral and robust to coalition manipulations are equivalent to voting rules on uniform policies.

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1 Introduction

Since the seminal work of Oates (1972), most models of federalism assume that under centralization. local policies cannot be differentiated to local conditions: centralized policies are necessarily uniform across local jurisdictions. Several informal justifications have been proposed for this ad-hoc assumption, among others equity considerations, constitutional constraints or organizational rigidities. As intuitive as it is, the uniformity assumption has been recently criticized by several authors.[1] Although there is some empirical evidence that more centralized federal systems have more uniform policies, the rationale underlying the rigidity of central interventions is far from clear.

The principles of contractual federalism suggest a more flexible approach: centralization should be thought as a general mechanism, a grand social contract among the members of a federation supervised by a neutral federal administration. A priori, there is no reason why its range should be restricted to uniform policies. However, as political scholars and economists have argued for long (e.g. Tocqueville 1835, Hayek 1945, Musgrave 1959), the information about local circumstances and preferences is typically decentralized. Local jurisdictions have an informational advantage over federal administrations and their private information is typically unverifiable.[2] Under those informational constraints, the decentralized information has to be elicited via the federal mechanism. The resulting incentive compatibility constraints restrict the set of feasible federal interventions. The aim of this paper is to assess the bite of the information asymmetry between the federal and the local level and investigates in particular if it can provide a rational for the rigidities of centralized policies.

Technically, the restrictions imposed by incentive compatibility constraints depend essentially on two factors: the notion of iincentive compatibility – the equilibrium concept – and the prior knowledge of the mechanism designer – the set of admissible preferences profile. In this paper, the notion of incentive compatibility that we use is strategy-proofness, because of its simplicity and robustness. The limited informational and rationality requirement of strategy-proofness are crucial features of the federal mechanism if the lat-

[1] See Seabright 1996, Crémer, Estache and Seabright 1996, Caillaud, Jullien and Picard, 1996, Qian and Weingast 1997, Oates 1999 and 2005 and Epple and Nechyba 2004 among others.

[2] For a recent empirical study of informational asymmetries between local and federal governments, see Azfar 2006.

ter is meant to be a transparent, simple, rule-based scheme understandable by all constituents of the federation. Moreover, if the federal mechanism has to be "played" by locally elected representatives, we show that any non strategy-proof mechanism will give incentives to voters to appoint local delegates whose preferences differ from their own, which will in turn distort the outcome of the federal mechanism.

Without restrictions on the set of admissible preference profiles, strategy-proof rules are either very rigid or dictatorial (Gibbard 1973, Satterthwaite 1975). However, satisfactory strategy-proof mechanisms can exist on well defined economic environments, i.e. when the mechanism designer can rule out ex-ante some preferences. In our case, the federal administration, although not completely omniscient, typically has some information on the structure of strategic interactions, depending on whether the problem at hand is the reduction of cross-border pollution or the coordination of regulatory regimes. The collective choice problem (i.e., the set of admissible preference profiles) considered in this paper features the usual ingredients of a typical model of federalism: different jurisdictions may have different ideal policies but local policies need to be coordinated because of inter-jurisdictional externalities. In line with the aforementioned literature on strategy-proofness in restricted domains, we assume that the federal administration knows the shape of externalities but does not know the ideal policy of each jurisdiction. In particular, the information to be elicited is only one-dimensional, which makes strategy-proofness less demanding.

In the efederalism literature, centralization is usually modelled as a voting rule over uniform policies (e.g. a common tax rate, law or level of public good).[3] The corresponding mechanisms are (group) strategy-proof because induced preferences over uniform policies are typically single-peaked (Moulin 1980) but a one-size-fits-all rule may generate preference frustration. A natural question is whether there exist more flexible strategy-proof mechanisms which could balance the coordination of local policies and the satisfaction of local preferences. Our environment leaves room for optimism: the relaxation of the policy uniformity constraint increases the number of degrees of freedom of the mechanism designer while keeping constant the number of incentive compatibility constraints.

However, our main result is that under some neutrality condition, group

[3] See e.g. Cremer and Palfrey 1996, Alesina and Spolaore 1997, Bolton and Roland 1997, Perotti 2001.

strategy-proof mechanisms are equivalent to the aforementioned voting rules on uniform policies. We then investigate whether one can get more flexible and satisfactory mechanisms by relaxing group strategy-proofness to strategy-proofness. We do get more flexibility: because of the small dimensionality of the set of parameters unknown to the central planner, there exists strategy-proof mechanisms which are neutral, anonymous, flexible and sensitive to local preferences. However, we show that non uniform strategy-proof mechanisms are necessarily inefficient.

In order to assess how inefficient they are, we compare their welfare performance to the welfare achieved under "non cooperative decentralization", i.e. at the equilibrium of a non-cooperative game where each jurisdiction controls its local policy. Decentralization is Pareto suboptimal – since externalities are not internalized – and thus leaves room for welfare improving federal interventions. Nevertheless, we show that decentralization is preferred to any strategy-proof federal mechanism by some jurisdiction. If the mechanism is furthermore neutral, then this conclusion holds at any profile of preferences and decentralization is uniformly socially preferred. The same conclusion holds if we allow for balanced transfers or if we restrict preferences to be arbitrarily homogeneous.

The heuristic intuition behind our results is that an efficient and flexible mechanism gives an incentive to local jurisdictions to over-report their preferences so as to pull the policy vector towards their ideal policy. Hence, to induce truthful report, the mechanism must be either completely rigid – as in the case of uniform rules – or entail excessive policy polarization so as to choke-off incentives to over-report preferences.

This paper shows that informational constraints imply a tension between the flexibility and efficiency of the federal intervention. The first-best may be theoretically achievable under permissive notions of implementability together with fully rational voters and unrestricted transfers. For more robust notions of incentive compatibility, the slightest amount of incomplete or unverifiable information prevents a benevolent federal administration from striking the optimal balance between policy coordination and local preferences matching. Hence, this paper provides a game theoretic rationale for the informal argument that an uninformed central administration may not do better than a uniform rule.

A few papers have looked at the rigidity of federal policy making and interjurisdictional mechanisms. Harstad 2007 considers a bargaining game in which two countries try to internalize spillovers. He shows that a uniform level of public good can reduce bargaining costs. Dreze, Le Breton and Weber 2007 analyze how to share the cost of a public good among heterogeneous residents. They show that to prevent a group of citizen to secede and form a new jurisdiction, the cost should be equalized among the users. Some recent papers have taken a more political economy approach to the cost of centralization by assuming away the benevolence of central administrations (e.g. Seabright 1996, Lockwood 2002 or Besley and Coate 2003). These models provide useful insights on the relative merit of centralization versus decentralization but do not explain its presumed rigidity.[4]

From a technical point of view, our paper uses a novel approach to characterize strategy-proof mechanisms. Following the Bayesian mechanism design literature, we use envelope theorems (Milgrom and Segal 2002) to prove the absolute continuity of the value function – without imposing any regularity condition on the mechanism itself as in Corchon and Rueda Llano 2004. This allows us to use standard calculus technics which are, to a great extent, independent of the fine details of the particular collective choice problem under consideration. As a result, contrary to most of the literature on strategy-proofness in small domains (e.g. Border and Jordan 1983), our results holds for a fairly large class of environments of minimal dimensionality. In particular, our domains need not contain a particular class of preferences (as in Schummer 1997 and 1999) nor preferences with arbitrary concavity or degree of complementarity (as in Zhou1991a and 1991b).[5] Finally, we do not restrict attention to Pareto efficient mechanisms.

Section 2 lays out the model. Section 3 characterizes neutral and group strategy-proof mechanisms and section 4 explores the welfare implications of strategy-proofness. The sketch of some proofs are provided in the text to illustrate the first-order approach, but complete proofs are relegated to an appendix.

[4]Indeed, a biased central government, or a minimal winning coalition of regions, should differentiate local policies even more than at the social optimum in order to extract maximal rents and/or favor their preferred constituency.

[5]One exception is proposition 1 for more than three jurisdictions, where the proof uses an arbitrarily “rightist” type for one jurisdiction.

2 The Model

We consider a federation composed of a finite number of local jurisdictions $I = \{1..N\}$. For all i, $x_i \in \mathbb{R}$ denotes the local policy of jurisdiction i and x_{-i} the policies of the other members of the federation. Voters are identified by their jurisdiction and a one-dimensional type: (i, t_i) denotes a resident of jurisdiction i with type t_i. The set of admissible types is an open interval of $\mathbb{R}$ denoted Θ. We assume that in each jurisdiction i, there is a unique local median type denoted θ_i which we call the type of jurisdiction i.

2.1 The Collective Choice Problem

The preferences, strict preferences and indifference relation of voter (i, t_i) are denoted respectively $\succeq_{i,t_i}$, $\succ_{i,t_i}$ and $\sim_{i,t_i}$. They are derived from the following utility function:

$$U_{i,t_i}(x) = -V(x_i - t_i) - \sum_{j \neq i} W_j(x_i - x_j). \tag{1}$$

Assumption 1 *The functions V and W_j for all $j \in I$ are continuous and quasi-convex, with a unique minimum at 0. V is continuously differentiable, $W_j(0) = 0$ for all j, and for $x \neq 0$, $\liminf_h \left| \frac{W_j(x+h) - W_j(x)}{h} \right| > 0$.*

The internal effect of local policies in their respective jurisdiction corresponds to the term $V(x_i - t_i)$ while $W_j(x_i - x_j)$ embodies the external cost imposed by the policy of jurisdiction j on jurisdictions i:[6] each jurisdiction suffers from the lack of homogeneity between its policy and the policies of its neighbors. The most preferred policy of voter (i, t_i) is the uniform policy $(t_i, .., t_i)$, hence the type can be viewed as the ideal local policy absent any coordination problems. The externalities generate a tension between the need for policy coordination and the heterogeneity of preferences: the members of the federation agree that some degree of harmonization is desirable but disagree on what the ideal policy should be. Examples of such coordination

[6] The fact that a given jurisdiction j imposes the same externality $W_j(x_j - x_0)$ on any two jurisdictions which have the same policy x_0 is immaterial when $N = 2$. When $N > 2$, it is necessary for proposition 1, since otherwise the fairness requirement (see definition 7) has no justification, and for proposition 4 as well since its proof uses the fact that the decentralized equilibrium is fair.

costs include conflicting laws, compliance costs imposed by heterogeneous regulations, inconsistent foreign policies, different time zones or incompatible technological standards (see Loeper 2011). Observe that we only require W_j to be single-peaked and have non zero derivative outside their peak, so these externalities can either be convex costs or network effects.

To sharpen our results or clarify the exposition, we will occasionally impose additional assumptions:

Definition 1 *Externalities are even if for all j, W_j is even.*

Definition 2 *A federation of two jurisdictions has reciprocal preferences if V is even, twice differentiable, quasi-convex and for all $x \in \mathbb{R}^2$, $W_1(x_1 - x_2) = W_2(x_2 - x_1)$.*

Definition 3 *A federation has quadratic preferences if there exists $\beta > 0$ such that*

$$U_{i,t_i}(x) = -|x_i - t_i|^2 - \frac{\beta}{N} \sum_j |x_i - x_j|^2 .$$

2.2 Contractual Federalism

2.2.1 Federal Mechanisms and Local Elections

A federal mechanism elicits local information via a game form which maps profiles of messages into policy vectors: $\Psi : M^N \to \mathbb{R}^N$. In our context, it is natural to let one delegate per jurisdiction represent the interests of her constituency at the federal level, rather than have each and every voter in the federation "play" the federal mechanism. We assume that local representatives are elected in their respective jurisdiction. Hence, local preferences are reported through local elections and are aggregated at the federal level via the mechanism.

We assume that local elections are open, competitive and decided by local majority rule. We assume furthermore that voters are forward-looking: their preferences over representatives are derived from their preferences over the resulting equilibrium policies. More precisely, if t_i^r denotes the type of the representative of jurisdiction i and if the equilibrium outcome of the game induced by the mechanism Ψ among the representatives is denoted $\Psi^e(t^r)$ (we will be more concrete on the equilibrium concept later on), we define the equilibria of the delegation game as follows:

Definition 4 *A delegation equilibrium is a vector of representatives* $(t_1^r, .., t_N^r)$ *such that in all jurisdictions* i*,* $\Psi^e\left(t_i^r, t_{-i}^r\right)$ *is preferred by a majority of residents to* $\Psi^e\left(t_i, t_{-i}^r\right)$ *for any* $t_i \in \Theta$*.*

This equilibrium concept is standard in the literature on federalism (e.g. Persson and Tabelini 1992 or Besley and Coate 2003).[7]

2.2.2 Incentive Compatibility and Delegation-proofness

Since local representatives are meant to represent the preferences of their constituency at the federal level, a desirable property of the mechanism is to ensure that strategic voters will not appoint delegates with preferences different from their own in order to manipulate the outcome of the federal mechanism. A mechanism Ψ is delegation-proof if

$$\forall t \in \mathbb{R}^N, \forall i \in I, \forall s_i \in \mathbb{R}, \Psi^e\left(t\right) \succeq_{i,t_i} \Psi^e\left(s_i, t_{-i}\right).$$

A mechanism Φ is called direct if its message space M^N is the set of profiles of types Θ^N. A direct mechanism Φ is strategy-proof if

$$\forall t \in \mathbb{R}^N, \forall i \in I, \forall t_i' \in \mathbb{R}, \Phi\left(t\right) \succeq_{i,t_i} \Phi\left(t_i', t_{-i}\right).$$

Clearly, a strategy-proof mechanism is delegation-proof and a direct delegation-proof mechanism is strategy-proof. Moreover, any non direct delegation proof mechanism Ψ is equivalent to the direct strategy-proof mechanism Φ defined by $\Phi\left(t\right) \equiv \Psi^e\left(t\right)$ for all $t \in \Theta^N$. This paper characterizes delegation-proof mechanisms. From what precedes, there is no loss of generality in restricting attention to direct strategy-proof mechanisms.

From our specification in (1), the conditions of the representative voter theorem are met within each jurisdiction (Gans and Smart 1996) so the majority preferences in each jurisdiction i coincide with the preferences of its median voter (i, θ_i). Therefore, under a strategy-proof mechanism, it is always a delegation equilibrium for all jurisdictions to elect their local median voter (i.e. $t^r = \theta$) and we shall focus on this equilibrium throughout the paper.[8]

[7] The aforementioned papers have a different approach in the sense that they analyze the consequences of strategic delegation on a particular mechanism.

[8] Since it is a dominant strategy for local majority preferences, this equilibrium selection hypothesis is not too demanding. If local jurisdiction were considered as single players, this would be equivalent to restricting attention to truthful implementation rather than full implementation. See e.g. Dasgupta, Hammond Maskin 1979.

If the mechanism is strategy-proof, elected representatives have no individual incentive to misreport their preferences at the federal level, but jointly profitable coalitional deviations may still exist. Group strategies are hardly avoidable in our federal context. Indeed, federalism is precisely about bringing the members of the federation to cooperate in order to avoid coordination failures. To rule out such manipulations, the mechanism has to be group strategy-proof:

$$\forall \theta \in \mathbb{R}^N, \forall C \subset I, \forall \theta'_C \in \mathbb{R}^{|C|}, \exists i \in C : \Phi(\theta) \succeq_{i,\theta_i} \Phi(\theta'_C, \theta_{-C}).$$

2.2.3 A Remark on Strategic Delegation

In this subsection, we briefly explain why non strategy-proof mechanisms and strategic delegation are problematic in our context. If we were to consider Bayesian incentive-compatible mechanisms instead, the delegation equilibrium would depend on the fine details of the high-order beliefs among voters and representatives within and across jurisdictions, which would be problematic for practical purposes. As shown in Bergeman and Morris 2005, strategy-proof mechanisms are the only allocation rules which are implementable for a sufficiently rich set of information structures. Second, even if information is known to be complete among voters, the usual integer games and denouncing methods used in the Nash implementation literature to exploit the information that participants have on each other are neither satisfactory nor realistic in our political context.[9] On the contrary, the simplicity of direct mechanisms ensures the transparency of the federal intervention. Third, the delegation equilibrium of a non strategy-proof mechanism will be sensitive to the degree of sophistication of voters, and the mechanism designer may not know who will vote strategically or truthfully. This compounds the mechanism design problem since, as we shall see in section 4, the optimal mechanism under truthful voting may be quite bad under strategic voting and vice versa. With a strategy-proof mechanism, truthful and strategic voting coincide and voters need not know anything beyond their own interests. Finally, we want the incentive compatibility requirement to be consistent with the usual definition of centralization, which is strategy-proof as we shall see in subsection 2.2.5.

[9] See for instance Moore1990 for a review on Nash implementation.

2.2.4 Properties of Federal Mechanisms

To characterize reasonable incentive compatible mechanisms, we will occasionally impose additional normative properties. The latter are important ingredients of contractual federalism. Indeed, by the very act of designing a collective mechanism, members of a political federation express their ethical preferences. Therefore, the normative content of the mechanism can be an important determinant of its popular support.

Definition 5 *A mechanism Φ is efficient if for all distributions of preferences, no policy is strictly preferred to $\Phi(\theta)$ by a majority of voters in all jurisdictions.*

As noted above, the majority preferences in each jurisdiction are the preferences of its median voters, so efficiency is equivalent to Pareto efficiency among local median voters.

Definition 6 *A mechanism is neutral if for all $\theta \in \Theta^N$, $u \in \mathbb{R}$,*

$$\Phi(\theta + (u, .., u)) = \Phi(\theta) + (u, .., u).$$

In words, a neutral mechanism should be independent of the choice of origin with which policies and types are labelled.[10] The neutrality condition guarantees that a particular policy, or a political orientation such as liberal or conservative is not favored ex-ante by the mechanism. It can only be so on the basis of the preferences reported at the local elections. This property guarantees that the federal intervention will not be exploited to legitimate biased outcomes.

Definition 7 *A mechanism Φ is fair if for all $\theta \in \Theta^N$, for all $i, j \in I$, $\theta_i = \theta_j$ implies $\Phi_i(\theta) = \Phi_j(\theta)$.*

Fairness means that any two jurisdictions whose representatives have the same type should have the same policy.[11] It can be viewed as some form of equal treatment of equals.

[10] As for all $i \in I, u \in \mathbb{R}$, $x, \theta \in \mathbb{R}^N$, $U_{i,\theta_i}(x) = U_{i,\theta_i+u}(x + (u, .., u))$, our definition of neutrality is implied by the usual condition of neutrality with respect to the alternatives which states that the mechanism should depend on ordinal preferences only and not on the labelling of the alternatives. See e.g. Moulin 1998 for a formal definition.

[11] It is justified by the fact that from our specification in (1), if two jurisdiction have the same type, conditionally on having the same preferences they have exactly the same preferences.

2.2.5 Uniform Voting Rules

The public economy literature has focused on unitarian centralization, typically modelled as a voting rule (usually majority rule) on uniform policies. To allow different jurisdictions and coalitions to carry different vote weights, uniform voting rules are defined as follows:

Definition 8 *A family of winning coalitions $W \subset 2^I$ is a non empty collection of coalitions which is*
(i) monotonic: $C \in W$ and $C \subset C'$ imply $C' \in W$
(ii) proper: if $C \in W$, $I \setminus C \notin W$
A mechanism is a uniform voting rule if there exists a family of winning coalitions W such that for any $\theta \in \mathbb{R}^N$ with $\theta_{i_1} \leq .. \leq \theta_{i_N}$, if $\{i_1, .., i_p\} \in W$ and $\{i_p, .., i_N\} \in W$ then $\Phi(\theta) = \left(\theta_{i_p}, .., \theta_{i_p}\right)$.

Uniform voting rules are neutral, fair and most importantly group strategy-proof (Moulin 1980). From a welfare point of view, they remove external effects and are Pareto efficient since they are locally dictatorial. However, they are not completely satisfactory for a unique policy may generate preferences frustration.[12] The main goal of this paper is to investigate whether there exists more flexible mechanisms which satisfy the same incentive compatibility requirement.

2.3 The Information Structure

Since we restrict our attention to dominant strategy mechanisms, we do not have to specify what voters and representatives know about each other. We could either assume that each voter knows only its type, or that local median types are common knowledge among all voters but cannot be verified by the federal administration.

On the contrary, the information of the mechanism designer is a crucial determinant of the bite of strategy-proofness. Throughout, we assume that the federal planner knows the functions V and W_j but not the local median types: the federal administration knows (or can verify) the shape of external effects but has to elicit local preferences. Contrary to most of the literature on

[12] See Loeper 2008a for a more formal statement of the social cost of uniform policies with coordination externalities.

strategy-proofness, the asymmetric information has finite dimension, which simplifies the mechanism design exercise.[13]

The ex-ante knowledge of the mechanism designer can be further refined by restricting the domain of admissible types. Since the proofs are based on calculus technics, our results (with the exception of proposition 1 for $N \geq 3$) hold for any type space of the form Θ^N for some open interval Θ of $\mathbb{R}$.

3 Group Strategy-proofness and Uniformity

As argued earlier, uniform voting rules (see definition 8) are robust to coalition manipulations. However, as the next two propositions show, nothing more flexible can be expected when group strategy-proofness is required.

Proposition 1 *A mechanism is neutral, fair and group strategy-proof if and only if it is a uniform voting rule.*

Proof. (Sketch for $N = 2$, see the appendix for the complete proof) If Φ is strategy-proof, from the envelope theorem (Milgrom and Segal 2002) $U_{i,\theta_i}(\Phi(\theta))$ is absolute continuous in θ_i. Neutrality implies $U_{i,\theta_i}(\Phi(\theta)) = U_{i,\theta_i+u}(\Phi(\theta + (u,u)))$. Combining the latter two properties with the differentiability of $U_{i,\theta_i}(x)$ in θ_i, we have that $U_{i,\theta_i}(\Phi(\theta + h))$ is totally differentiable in h at $h = (0,0)$ for almost all $\theta \in \Theta^2$.

Using the Taylor expansion of $U_{i,\theta_i}(\Phi(\theta + h))$ at $h = (0,0)$, and the fact that by strategy-proofness $\frac{\partial U_{i,\theta_i}(\Phi(\theta+h))}{\partial h_i} = 0$ for $i = 1,2$, we see that if both $\frac{\partial U_{1,\theta_1}(\Phi(\theta+h))}{\partial h_2} \neq 0$ and $\frac{\partial U_{2,\theta_2}(\Phi(\theta+h))}{\partial h_1} \neq 0$, there exists a joint deviation whose first order effect is positive for both jurisdictions. Group strategy-proofness implies then that the gradient of $U_{i,\theta_i}(\Phi(\theta + h))$ is $(0,0)$ at $h = (0,0)$ for some i, i.e. i is a local dictator on the range of the mechanism. Using the continuity of $U_{i,\theta_i}(\Phi(\theta))$, there is a unique local dictator on each side of the

[13] In this sense, our environment is comparable to the single-peaked domain of Border and Jordan 1983 in which types are one-dimensional as well. However, in BJ the mechanism can use only one policy to elicit preferences. Heuristically, if we replace the strategy-proofness constraints by the corresponding first-order conditions (i.e. $\frac{\partial U_{i,\theta_i}(\Phi(t_i,\theta_{-i}))}{\partial t_i} = 0$ at $t_i = \theta_i$) both problems reduce to a system of N partial differential equations. But our N local policies gives more instruments to the mechanism designer. While BJ has N degrees of freedom – the gradient of N partial derivatives of the single policy – while our setup has N^2 degrees of freedom – the $N \times N$ matrix of partial derivatives of the policy vector.

diagonal $\theta_1 = \theta_2$, and by fairness, the local dictator i can impose a uniform policy (θ_i, θ_i). ■

The welfare properties of mechanisms which are not neutral, fair or group strategy-proof are analyzed in the next section. In the remainder of this section, we discuss the tightness of proposition 1, i.e. whether the relaxation of its hypotheses allows for non uniform mechanisms.

The term "group" cannot be dropped as will be shown in section 4. However, the proof uses manipulations by coalitions of at most two groups of unanimous jurisdictions, i.e. each group having the same type. Hence, it does not require an unreasonable amount of coordination.

Relaxing fairness allows for non uniform mechanisms. However, the next proposition shows (in the case of even externalities, see definition 1) that these mechanisms, on top of being unfair, are still excessively rigid.

Proposition 2 *Let Φ be a neutral and group strategy-proof mechanism in a federation of two jurisdictions with even externalities, then $|\Phi_1 - \Phi_2|$ is constant and for all $\theta \in \Theta^2$, $\Phi_i(\theta) = \theta_i$ for some jurisdiction i.*

Hence, the distance between policies is independent of the heterogeneity of local preferences, and there is always one jurisdiction which is a local dictator on the range of the mechanism. In particular, if the mechanism is not uniform, it is not efficient even when preferences are unanimous, i.e. when types are identical.

4 Individual Strategy-proofness and Federal Welfare

In this section, we investigate whether strategy-proofness alone leads to more flexible and satisfactory mechanisms. The set of strategy-proof mechanisms obviously depends on the functions V and W. In any case, as argued in subsection 2.3 and footnote 12, it may be quite large and should contain non uniform mechanisms.

For instance, in the case of quadratic preferences (see definition 3), simple algebra shows that the following mechanism

$$\Phi^{SP}(\theta) = \left(\omega\theta_i + (1-\omega)\overline{\theta}\right)_{i\in I} \text{ with } \omega = \left(\frac{N-2+N\sqrt{1+4\beta}}{2(N-1+N^2\beta)}\right), \quad (2)$$

is strategy-proof. Intuitively, Φ^{SP} has the right "form": each policy is a weighted sum of the jurisdictions' types. For the sake of comparison, the social choice function Φ^* which maximizes the sum of utilities of local median voters has the same expression as (2), but with $\omega^* = 1/(1+2\beta)$. The mechanism Φ^{SP} balances local preferences matching (through the weight ω each local policy Φ_i^{SP} puts on its respective type θ_i) and policy coordination (through the weight $1-\omega$ each policy Φ_i^{SP} puts on the federal mean type $\overline{\theta}$). Moreover, the trade-off between flexibility and coordination is sensitive to the magnitude of externalities, since ω is decreasing in the magnitude of externality β. Contrary to most of the strategy-proof mechanisms exhibited in the literature, Φ^{SP} is furthermore neutral, fair, has full range, is nowhere locally dictatorial nor constant.[14] Nevertheless, Φ^{SP} is Pareto inefficient. The reason is that Φ^{SP} puts too little weight on policy coordination, i.e. ω is too large for Φ^{SP} to be efficient: contrary to uniform voting rules, Φ^{SP} is too sensitive to local preferences!

The intuition for this inefficiency is that a flexible and efficient mechanism gives incentives to voters to over-report their preferences in order to pull the policy vector towards their own ideal policy. Consider for instance a mechanism which implements the policy which mazimizes the sum of utilities of local representatives, i.e. $\Phi^*(t^r) = \left(\frac{t_i^r + 2\beta \overline{t^r}}{1+2\beta}\right)_{i\in I}$. Such a mechanism would be optimal under sincere voting, i.e. if all jurisdictions were to elect their local median voters. However Φ^* is not strategy-proof and hence not delegation-proof. Simple algebra shows that the representatives elected at the delegation equilibrium (see definition 4) will be more extreme than the local median types: $t^r = \left(\theta_i + \frac{\beta(1+4\beta/N)}{1+2\beta/N+\beta}\left(\theta_i - \overline{\theta}\right)\right)_{i\in I}$. Furthermore, the resulting equilibrium policy, $\Phi^*(t^r) = \left(\omega^r\theta_i + (1-\omega^r)\overline{\theta}\right)_{i\in I}$ with $\omega^r = \frac{1+2\beta/N}{1+2\beta/N+\beta}$, is socially worse than $\Phi^{SP}(\theta)$: $\omega^r > \omega$, so at the delegation equilibrium, the "optimal" mechanism Φ^* puts even less weight on policy coordination than the strategy-proof mechanism Φ^{SP}.[15] To avoid this race to the extreme, the

[14] As one can see from definition 3, quadratic preferences have an additive separable form, but the coordinates used in the decomposition are not the same for each jurisdiction, so the mechanism is not locally dictatorial component-wise as in Border and Jordan 1983, Barbera, Sonnenschein and Zhou 1991 or Moulin and Sen 1999.

[15] This result relates to the argument in subsection 2.2.3 that the optimal mechanism given truthful voting may be quite bad under strategic voting. Similarly, one can show in the quadratic case that the optimal utilitarian mechanism given strategic voting is worse than Φ^{SP} under truthful voting. On the contrary, by definition, a strategy-proof

mechanism must be either inflexible (as uniform voting rules) or be excessively flexible (as Φ^{SP}) so as to choke-off incentives to strategically elect more extreme delegates.

The next proposition formalizes in the two jurisdictions case the tension between the flexibility and efficiency of strategy-proof federal mechanisms.[16]

Proposition 3 *A strategy-proof mechanism on a federation of two jurisdictions is efficient (see definition 5) if and only if it is a uniform voting rule.*

Proof. (Sketch) If Φ is efficient, the welfare of jurisdiction 1 at the equilibrium must be the value of the following utilitarian convex program

$$P(\theta) = \max_{U_{2,\theta_2}(x) - U_{2,\theta_2}(\Phi(\theta)) \geq 0} U_{1,\theta_1}(x).$$

The maximand of P is constant in θ_2. Strategy-proofness for jurisdiction 2 implies that the derivative of the constraint with respect to θ_2 must be 0 at the maximum $x = \Phi(\theta)$. Applying the envelope theorem to $P(\theta) = U_{1,\theta_1}(\Phi(\theta))$, we have $\frac{\partial U_{1,\theta_1}(\Phi(\theta))}{\partial \theta_2} = 0$ whenever the choice set of P is not a singleton. This implies that some jurisdiction i is a local dictator at any θ. Using efficiency, one can show that this jurisdiction can impose its most preferred policy (θ_i, θ_i). ■

Nevertheless, proposition 3 does not rule out that some nonuniform strategy-proof mechanisms may be quite satisfactory although not fully efficient. To gauge more precisely the welfare cost of strategy-proofness, we need a "reasonably" inefficient social choice function to be used as a benchmark for welfare comparison. A natural candidate is non-cooperative decentralization, i.e. the equilibrium of the game in which each jurisdiction chooses its policy by local majority rule taking the other policies as given.[17] It is generically Pareto inefficient since the cross-border external effects $(W_j)_{j \in I}$ are not internalized and can be interpreted as what the jurisdictions could expect

mechanism has the same outcome under truthful and strategic voting.

[16] The proof, which assumes that utility functions are strictly quasi-concave, is a corollary of a more general result in Loeper 2008 but is provided for the sake of self-containment.

[17] A decentralized equilibrium is a policy vector x such that in each jurisdiction i, x is preferred by a majority of voters to (y_i, x_{-i}) for all $y_i \in \mathbb{R}$. As argued earlier, majority preferences in all jurisdiction are exactly the preferences of their respective median voter, so decentralization can be viewed as a Nash equilibrium between local median voters.

without federal intervention.[18]

In the sequel, Φ^{dec} refers to the decentralized equilibrium and to guarantee its uniqueness, we assume that V and W are twice differentiable and strictly convex.

Proposition 4 *Let Φ be a strategy-proof mechanism, then for at least all jurisdictions i but one, there exists a profile of local median types $\theta \in \Theta^N$ at which (a majority of voters in) jurisdiction i is strictly better-off under $\Phi^{dec}(\theta)$ than under $\Phi(\theta)$.*[19]

Proposition 4 yields a negative result under the sole requirement of strategy-proofness. It formalizes the trade-off between the incentive compatibility and the distributional welfare gains of a central intervention. Notice that contrary to the common wisdom on the relative advantage of centralization, this result holds however homogeneous local preferences are, i.e. however small the set of admissible type Θ is.

We now turn to the aggregate welfare performance of strategy-proof mechanisms using the Benthamite social welfare function among local median voters $B(x) = \sum_i U_{i,\theta_i}(x)$.[20]

Proposition 5 *Let Φ be a strategy-proof, neutral mechanism in a federation of two jurisdictions with reciprocal preferences (see definition 2), then for all $\theta \in \Theta^N$, Φ is Benthamite dominated by Φ^{dec}, strictly so whenever $\theta_1 \neq \theta_2$.*

As an immediate corollary, if Φ is strategy-proof and neutral, for all distributions of preferences there is a majority of voters in some jurisdiction which are better-off under decentralization. Notice that the neutrality requirement

[18] At this point, we should stress that decentralization is described as a complete information game, which may seem surprising since we haven't specified the information structure among voters. However, it is the unique equilibrium of a supermodular game, so it is a reasonable prediction even if information is only approximately complete (see Kajii and Morris 1997). Moreover, even if information is not near complete, most tatonement process would converge to it (Milgrom and Roberts 1990). In any case, to the extent that decentralization is used as a benchmark for welfare assessment, the information structure is immaterial.

[19] It should be clear from the proof of proposition 4 that, at least in the two jurisdictions case, the set of such profile of types is non negligible, i.e. of positive Lebesgue measure.

[20] Observe that the Benthamite criterion among local median voters can be strictly equivalent to the Benthamite criterion for the whole confederation, for instance if V is even and if types are symmetriclaly distributed in each jurisdiction.

is indispensable: since decentralization is suboptimal, it can be dominated by a constant mechanism on a non negligible parameter constellation.

Let us conclude this section on a remark about transfers. To the extent that they are feasible and free of stigma, the use of unrestricted inter-jurisdictional monetary transfers would allow more flexible and efficient outcomes. However, we show in the appendix that if such transfers are restricted to be balanced, proposition 5 carries over in the usual quasi-linear setting. Since uniform rules are not efficient in a transferable setup, proposition 3 should be rephrased as follows: there is no efficient and strategy-proof mechanism with balanced transfers.

5 Conclusion

This paper takes an informational approach to contractual federalism and endogenizes the cost of centralization via incentive compatibility constraints. We show that the later have a bite even if the dimensionality of private information is minimal. Our model highlights a trade-off between the welfare performance, the flexibility and the robustness to manipulations of the federal mechanism.

We first provide a characterization of uniform voting rules in terms of group strategy-proofness and neutrality. We then consider individual strategy-proof (or equivalently delegation-proof) mechanisms and show that the only such mechanisms which are efficient are uniform. There exists non uniform strategy-proof mechanisms but they are even worse than non cooperative decentralization in terms of welfare, even if (balanced) transfers are allowed.

Our results suggest that a flexible federal intervention which could balance efficiently policy coordination and local preferences satisfaction is hanging upon assumptions which are not innocuous. It may be achievable under permissive notions of incentive compatibility together with fully rational voters, unrestricted transfers and no budget constraints. But for robust notions of implementation, a benevolent but not omniscient central administration may not do better than a uniform policy. The fact that more centralized federations are empirically associated with more uniform policies can be interpreted as an evidence of the informational constraints faced by federal administrations.

6 Proofs in Section 3

Induced preferences over uniform rules are single-peaked. Therefore, from Moulin 1980, strategy-proof uniform rules must be min-max rules. By neutrality, the min-max rules must have no (finite) phantom voters. One can easily show that such min-max rules are equivalent to the uniform voting rules described in definition 8. Reciprocally all uniform voting rules are neutral, fair and group strategy-proof. Hence, to prove proposition 1, it suffices to show that the range of a group-strategy-proof, neutral and fair mechanisms is restricted to uniform policies. It will be done successively for two, three and more than three jurisdictions. In the two jurisdictions case, we shall prove proposition 2 as well.

6.1 Preliminary Lemmas

Lemma 1 *Let Φ be a strategy-proof mechanism, then for all $i \in I$, $\Phi_i(\theta)$ is non decreasing in θ_i, $U_{i,\theta_i}(\Phi(\theta))$ is absolutely continuous*[21] *in θ_i and*

$$U_{i,\theta_i}\left(\Phi\left(\theta_i', \theta_{-i}\right)\right) - U_{i,\theta_i'}\left(\Phi\left(\theta_i, \theta_{-i}\right)\right) = \int_{\theta_i}^{\theta_i'} V'\left(\Phi_i\left(t, \theta_{-i}\right) - t\right) dt. \quad (3)$$

Proof. The monotonicity of $\Phi_i(\theta)$ in θ_i comes from the single crossing property of U_i in (x_i, θ_i) and the usual revealed preferences argument. As V is continuously differentiable, U_i is absolutely continuous and differentiable in θ_i and (3) follows directly from the envelope theorem in Milgrom and Segal 2002 (theorem 2). ■

Lemma 2 *Let Φ be a strategy-proof, neutral mechanism in a federation of two jurisdictions, then $U_\theta(\Phi(\theta))$ is continuous in θ, $\Phi(\theta)$ has a left and right limit everywhere in θ_1 and θ_2 and is almost everywhere continuous in θ. For $i = 1, 2$, at any continuity point θ of Φ_i, $U_{i,\overline{\theta}_i}(\Phi(\theta))$ is totally differentiable*[22] *in θ at $\theta = \overline{\theta}$ and*

$$\begin{aligned} \frac{\partial U_{i,\overline{\theta}_i}\left(\Phi\left(\theta\right)\right)}{\partial \theta_i}\left(\theta = \overline{\theta}\right) &= 0, \\ \frac{\partial U_{i,\overline{\theta}_i}\left(\Phi\left(\theta\right)\right)}{\partial \theta_j}\left(\theta = \overline{\theta}\right) &= -V'\left(\Phi_i\left(\theta\right) - \theta_i\right). \end{aligned}$$

[21] See Royden 1988 for a definition of absolute continuity.

[22] See Royden 1988 for a definition of absolute continuity and total differentiability.

Proof. As Φ is neutral, $U_{i,\theta_i}(\Phi(\theta)) = U_{i,\theta_i+u}(\Phi(\theta+(u,u)))$ for $i = 1, 2$. From lemma 1, $U_{i,\theta_i}(\Phi(\theta))$ is continuous in θ_i. By letting $u \to 0$, it is continuous in θ_j.

From lemma 1, Φ_i is monotonic. As such, it is continuous almost everywhere in θ_i and has a right and left limit everywhere. Using neutrality, $\Phi(\theta+\epsilon) = \Phi(\theta_1+\epsilon_1-\epsilon_2, \theta_2) + (\epsilon_2, \epsilon_2)$. Therefore, Φ_i is continuous in θ whenever it is continuous in θ_i.

Let θ be a continuity point of Φ_i, say $i = 1$ for concreteness. Using successively the neutrality of Φ, lemma 1 and the continuity of V' and Φ_1 at θ, we have for all $h \in \mathbb{R}^2$,

$$\begin{aligned}
&U_{1,\theta_1+h_1}(\Phi(\theta+h)) - U_{1,\theta_1}(\Phi(\theta)) \\
&\qquad = U_{1,\theta_1+h_1-h_2}(\Phi(\theta_1+h_1-h_2,\theta_2)) - U_{1,\theta_1}(\Phi(\theta)), \\
&= \int_{\theta_1}^{\theta_1+h_1-h_2} V'(\Phi_1(t,\theta_2)-t)\,dt = (h_1-h_2)(V'(\Phi_1(\theta)-\theta_1)) + o(h).
\end{aligned}$$

From (1),

$$\begin{aligned}
U_{1,\theta_1}(\Phi(\theta+h)) - U_{1,\theta_1+h_1}(\Phi(\theta+h)) &= \int_{\theta_1+h_1}^{\theta_1} V'(\Phi_1(\theta+h)-t)\,dt, \\
&= -h_1(V'(\Phi_1(\theta)-\theta_1)) + o(h).
\end{aligned}$$

Summing the equations above, we have

$$U_{1,\theta_1}(\Phi(\theta+h)) - U_{1,\theta_1}(\Phi(\theta)) = -h_2(V'(\Phi_1(\theta)-\theta_1)) + o(h), \qquad (4)$$

which establishes total differentiability of $U_{1,\theta_1}(\Phi(\theta+h))$ in h at any continuity point θ of Φ_1. The partial derivatives are by definition the coefficients of h_1 and h_2 in the linear part of the right hand-side of (4). ■

Definition 9 *A mechanism Φ is unanimity respecting for jurisdictions i and j if for all $\theta \in \Theta^N$, $\theta_i = \theta_j$ implies $\Phi(\theta) = (\theta_i, .., \theta_i)$.*

Lemma 3 *If Φ is group strategy-proof and unanimity respecting for for jurisdictions i and j, then $\theta_i \leq \theta_j \Rightarrow \Phi_i(\theta) \leq \Phi_j(\theta)$. If Φ is strategy-proof, neutral and fair on a federation of two jurisdictions, it is unanimity respecting for jurisdiction 1 and 2.*

Proof. Let $\overline{\theta} \in \Theta^N$ and suppose $\theta_1 \leq \theta_2$ and $\Phi_1(\theta) > \Phi_2(\theta)$. If $\Phi_2(\theta) \leq \theta_1$, then both jurisdictions strictly prefer the policy $(\theta_1, .., \theta_1)$ to $\Phi(\theta)$. If $\Phi_1(\theta) \geq \theta_2$, then both jurisdiction strictly prefer the policy $(\theta_2, .., \theta_2)$ to $\Phi(\theta)$. Finally, if $\theta_1 < \Phi_2(\theta)$ and $\Phi_1(\theta) < \theta_2$, then both jurisdictions prefer $(\Phi_2(\theta), .., \Phi_2(\theta))$ to $\Phi(\theta)$. For all $u \in [\theta_1, \theta_2]$, $(u, .., u) \in \Theta^N$ so the deviations above belongs to the range of Φ, a contradiction to group strategy-proofness.

To show the second part, let $u \in \Theta$. As Φ is fair, $\Phi(u, u) = (v, v)$ for some v. As Θ is open, $(u + \epsilon, u + \epsilon) \in D_\Theta$ for ϵ small enough. By neutrality, $\Phi(u + \epsilon, u + \epsilon) = (v + \epsilon, v + \epsilon)$. The later policy should not be preferred by both jurisdictions for $\epsilon < 0$ and $\epsilon > 0$, which implies $u = v$. ■

6.2 Two Jurisdictions

Throughout subsection 6.2, Φ is a neutral, group strategy-proof mechanism on a domain Θ^2 for some open interval Θ of $\mathbb{R}$. We shall make it explicit when we assume furthermore that externalities are even (to prove proposition 2) or that Φ is fair (to prove proposition 1 for $N = 2$).

By neutrality, Φ is completely characterized by the function $\theta_1 \to \Phi(\theta_1, \theta_2)$ for a given $\theta_2 \in \Theta$. Hence, to simplify notations, unless otherwise mentioned, we will fix θ_2, and consider Φ as a function of θ_1 only.

Claim 1 *Φ_1 (resp. Φ_2) can only have upward (resp. downward) jumps in θ_1.*

Proof. From lemma 1, Φ_i is non decreasing in θ_i, and by neutrality $\Phi_2(\theta) = \Phi_2(0, \theta_2 - \theta_1) + \theta_1$. ■

Definition 10 *For $i = 1, 2$, $D_i = \{\theta_1 \in \Theta : \Phi_i(\theta_1) = \theta_i\}$.*

The strategy of the proof is to show that D_1 and D_2 cover the whole type space Θ and that on D_i, jurisdiction i is a local dictator on the range of the mechanism. In the sequel, for any subset A of Θ, $A^c, \overline{A}$ and A^o denote respectively its complement, topological closure and interior for the usual metric topology in Θ.

Claim 2 *$\overline{D_1} \cup \overline{D_2} = \Theta$ and $\left(\overline{D_1}^o \cup \overline{D_2}^o\right)^c \subset \overline{D_1} \cap \overline{D_2}$.*

Proof. Let θ be a continuity point of Φ. Suppose $V'(\Phi_i(\theta) - \theta_i) \neq 0$ for $i = 1, 2$ and set $h_i = -V'(\Phi_j(\theta) - \theta_j)$. From equation (4) in lemma 2, reporting $\theta + \epsilon h$ is a profitable deviation for both jurisdictions at θ for ϵ small enough. So group strategy-proofness implies $V'\left(\Phi_i\left(\overline{\theta}\right) - \overline{\theta}_i\right) = 0$, i.e. $\Phi_i\left(\overline{\theta}\right) = \overline{\theta}_i$ for some i. From lemma 2, Φ is continuous almost everywhere, so $\overline{D_1} \cup \overline{D_2} = \Theta$. Therefore, $\overline{D_1}^c \subset \overline{D_2}$ so $\overline{D_1}^c \subset \overline{D_2}^o$ and $\left(\overline{D_2}^o\right)^c \subset \overline{D_1}$ which proves the second point. ■

Claim 3 *For $i = 1, 2$, $\overline{D_i}^o \subset D_i$ and $U_{i,\theta_i}(\Phi(\theta))$ is locally constant in θ_1 on $\overline{D_i}^o$.*

Proof. Let $\theta_1 \in \overline{D_i}^o$, there exists two sequences s^n and t^n in D_i respectively non decreasing and non increasing which tend to θ_1. From claim 1, Φ_i has either upward or downward jump in θ_1, so by definition of D_i, $\theta_1 \in D_i$.

Let $\theta_1 \in \overline{D_1}^o$, there exists $\epsilon > 0$ such that $[\theta_1 - \epsilon, \theta_1 + \epsilon] \subset \overline{D_1}^o$. From lemma 1, $U_{1,\theta_1}(\Phi(\theta))$ is absolutely continuous in θ_1 and since $\overline{D_1}^o \subset D_1$ its derivative is null on $[\theta_1 - \epsilon, \theta_1 + \epsilon]$. Therefore it is constant on this interval. If $\theta_1 \in \overline{D_2}^o$, from lemma 2, $\frac{\partial}{\partial \theta_1} U_{2,\theta_2}(\Phi(\theta)) = 0$ and the same reasoning applies. ■

Claim 4 *For all θ_1, the left and right limit of Φ in θ_1, denoted $\Phi\left(\theta_1^-\right)$ and $\Phi\left(\theta_1^+\right)$, exist and $\Phi_i\left(\theta_1^-\right) = \theta_i$ and $\Phi_j\left(\theta_1^+\right) = \theta_j$ for some i, j. Moreover, if $\theta_1 \in \overline{D_1} \cap \overline{D_2}$, this must be true for some $i \neq j$.*

Proof. The existence of the limits is established in lemma 2. The equalities $\Phi_i\left(\theta_1^-\right) = \theta_i$ and $\Phi_j\left(\theta_1^+\right) = \theta_j$ follow from $\overline{D_1} \cup \overline{D_2} = \Theta$ (claim 2).

Suppose the second part of the claim is not true. This implies that there exist $\theta_1 \in \overline{D}_i$, $\Phi_i\left(\theta_1^-\right) \neq \theta_i$ and $\Phi_i\left(\theta_1^+\right) \neq \theta_i$ for some i, say 2. This implies that there exists $\epsilon > 0$ such that $]\theta_1 - \epsilon, \theta_1[\cap D_2 = \emptyset$ and $]\theta_1, \theta_1 + \epsilon[\cap D_2 = \emptyset$. Since $\theta_1 \in \overline{D_2}$, it must be that $\theta_1 \in D_2$. From claim 2, D_1 is dense in $]\theta_1 - \epsilon, \theta_1 + \epsilon[$. From claim 1, $]\theta_1 - \epsilon, \theta_1 + \epsilon[$ is included in D_1, and so in $\overline{D_1}^o$. From claim 3, $U_{1,\theta_1}(\Phi(t_1, \theta_2))$ is constant for $t_1 \in]\theta_1 - \epsilon, \theta_1 + \epsilon[$ and $\Phi_1(t_1, \theta_2) = t_1$. From assumption 1, given $x_1 = \theta_1$, $U_{1,\theta_1}(x)$ is strictly quasi-concave in $x_1 - x_2$. Hence, on $]\theta_1 - \epsilon, \theta_1 + \epsilon[$, $\Phi_1 - \Phi_2$ can take at most two values. From claim 1, $\Phi_1 - \Phi_2$ can only have upward jumps in θ_1, so it must be continuous to the right or to the left. As $\theta_1 \in \overline{D_1}^o$, Φ_1 is continuous at θ_1, so Φ_2 must be continuous to the right or to the left. Since $\theta_1 \in D_2$, $\Phi_2\left(\theta_1^-\right) = \theta_2$ or $\Phi_2\left(\theta_1^+\right) = \theta_2$, a contradiction. ■

Claim 5 *Suppose that Φ is fair or externalities are even and $\theta_1 \neq \theta_2$. If $\theta_1 \in \overline{D_1}^o \cup \overline{D_2}^o$, $\Phi_1 - \Phi_2$ is locally constant in θ_1 if $\theta_1 \in \overline{D_1} \cap \overline{D_2}$, Φ is continuous in θ_1 and $\Phi(\theta) = \theta$.*

Proof. Suppose to fix ideas that $\overline{\theta}_1 < \overline{\theta}_2$. If $\overline{\theta}_1 \in \overline{D_i}^o$ for some i, then from claim 3, $U_{i,\theta_i}(\Phi(\theta))$ is constant and $V(\Phi_i(\theta) - \theta_i) = 0$ on a neighborhood N of $\overline{\theta}_1$. So for $U_{i,\theta_i}(\Phi(\theta))$ to be constant, it must be the case that $W_j(\Phi_i - \Phi_j)$ is constant. Under assumption 1, it means that $\Phi_i - \Phi_j$ can take only two values of opposite sign on N. If Φ is fair, from lemma 3, it can take only one value on N since $\overline{\theta}_1 < \overline{\theta}_2$. If externalities are even, $|\Phi_i - \Phi_j|$ must be constant on N. From lemma 2, $U_{j,\theta_j}(\Phi(\theta))$ is continuous, so $\Phi_j(\theta) - \theta_j$ must be continuous. Since $\overline{\theta}_1 < \overline{\theta}_2$, this implies that $\Phi_i - \Phi_j$ cannot discontinuously change sign and is therefore constant on N as well.

Suppose for the rest of the proof that $\overline{\theta}_1 \in \overline{D_1} \cap \overline{D_2}$. From claim 4, $\Phi_i\left(\overline{\theta}_1^-\right) = \overline{\theta}_i$ and $\Phi_j\left(\overline{\theta}_1^+\right) = \overline{\theta}_j$ for some $i \neq j$, say $i = 1$ and $j = 2$. If $\Phi_1\left(\overline{\theta}_1^+\right) \neq \overline{\theta}_1$, $V\left(\Phi_1\left(\theta_1^-\right) - \theta_1\right) < V\left(\Phi_1\left(\theta_1^+\right) - \theta_1\right)$. From lemma 2, $U_{1,\theta_1}(\Phi(\theta))$ is continuous, so it must be that

$$W_2\left(\Phi_1\left(\theta_1^-\right) - \Phi_2\left(\theta_1^-\right)\right) > W_2\left(\Phi_1\left(\theta_1^+\right) - \Phi_2\left(\theta_1^+\right)\right). \tag{5}$$

If $\Phi_2\left(\overline{\theta}_1^-\right) \neq \overline{\theta}_2$, a symmetric argument using the continuity of U_2 gives

$$W_1\left(\Phi_2\left(\theta_1^-\right) - \Phi_1\left(\theta_1^-\right)\right) < W_1\left(\Phi_2\left(\theta_1^+\right) - \Phi_1\left(\theta_1^+\right)\right). \tag{6}$$

Equations (5) and (6) are incompatible if externalities are even ($W_i(.)$ can be replaced by $|.|$) or if Φ is fair (from lemma 3 $\Phi_1 - \Phi_2$ does not change sign on N).

Hence, $\Phi_i\left(\overline{\theta}_1^-\right) = \overline{\theta}_i$ and $\Phi_i\left(\overline{\theta}_1^+\right) = \overline{\theta}_i$ for some i, and from claim 1, $\Phi_i\left(\overline{\theta}_1\right) = \overline{\theta}_i$ and Φ_i is continuous at $\overline{\theta}_1$. If Φ is fair, from lemma 2, $U_i(\Phi)$ is continuous and from lemma 3, $\Phi_1 - \Phi_2$ cannot change sign, so Φ_j must be continuous and $\Phi_j\left(\overline{\theta}_1\right) = \theta_j$. If externalities are even, the continuity of $U_i(\Phi)$ and Φ_i implies that $|\Phi_i - \Phi_j|$ is continuous, and the continuity of $U_j(\Phi)$ implies that Φ_j must be continuous as well and $\Phi(\theta) = \theta$. ■

Claim 6 *$\overline{D_1} \cap \overline{D_2}$ has no accumulation point $\theta_1 \neq \theta_2$.*

Proof. Let $\overline{\theta}_1$ be an accumulation point of $\overline{D_1} \cap \overline{D_2}$, then $\overline{\theta}_1 \in \overline{D_1} \cap \overline{D_2}$. From claim 5, $\Phi(\theta) = \theta$ on $\overline{D_1} \cap \overline{D_2}$, so if $\frac{\partial \left[U_{1,\theta_1}(\Phi(\theta))\right]}{\partial \theta_1}\left(\overline{\theta}_1\right)$ exists, it must be equal to $\frac{\partial [W_1(\theta_1 - \theta_2)]}{\partial \theta_1}\left(\overline{\theta}_1\right)$, which from assumption 1 is non zero whenever $\overline{\theta}_1 \neq \theta_2$. From claim 5, Φ is continuous at θ_1 and from lemma 1, $\frac{\partial \left[U_{1,\theta_1}(\Phi)\right]}{\partial \theta_1} = V'(\Phi_1(\theta) - \theta_1) = 0$, a contradiction. ■

To conclude the proof, observe that from claim 2, $\overline{D_1}^o \cup \overline{D_2}^o$ and $\overline{D_1} \cap \overline{D_2}$ cover the whole domain Θ. So claim 5 shows that on each side of the diagonal, $\Phi_1 - \Phi_2$ is locally constant except possibly on $\overline{D_1} \cap \overline{D_2}$ where Φ must be continuous. From claim 6, $\overline{D_1} \cap \overline{D_2}$ has only isolated points outside the diagonal. By continuity $\Phi_1 - \Phi_2$ must be constant on each side of the diagonal. From lemma 2, the value function is continuous. So if Φ is fair, $\Phi_1 = \Phi_2$ on the diagonal, and thus everywhere. If externalities are even, $|\Phi_1 - \Phi_2|$ must be continuous on the diagonal, and hence constant on Θ^2.

6.3 Coalitions of Unanimous Jurisdictions

For any partition $\Gamma = (C_1, .., C_P)$ of the original federation I, we define the reduced federation $I_\Gamma = \{1, .., P\}$. Given a group strategy-proof, neutral and fair mechanism Φ on I, we shall construct a mechanism Φ^Γ on I^Γ which inherits the properties of Φ. Let $\Lambda^\Gamma : \mathbb{R}^P \to \mathbb{R}^N$ be defined by:

$$\forall \theta \in \mathbb{R}^P, \forall k \in I_\Gamma, i \in C_k \Rightarrow \Lambda_i^\Gamma(\theta) = \theta_k.$$

If the preferences of the members of the federation I are given by the functions V and W as in (1), the preferences of jurisdictions $k \in I_\Gamma$ with type θ_k are defined by:

$$\forall x \in \mathbb{R}^P,\ U_{k,\theta_k} = -V(x_k - \theta_k) - \sum_{j \in I} W_j\left(\left(x_k - \Lambda_j^\Gamma(x)\right)_{i \in I}\right).$$

For any profile of type $\theta \in \mathbb{R}^P$ of the reduced federation, $\Lambda^\Gamma(\theta)$ is a profile of type of the original federation I such that types are identical within each coalition of Γ. As Φ is fair, $\Phi\left(\Lambda^\Gamma(\theta)\right)$ prescribes the same policy to any two members of the same coalition in Γ. Therefore we can define Φ^Γ as $\Phi^\Gamma = \left(\Lambda^\Gamma\right)^{-1} \circ \Phi \circ \Lambda^\Gamma$. It is immediate to see that Φ^Γ inherits the properties of group strategy-proofness, neutrality and fairness of Φ.

Definition 11 *A coalition $C \subset I$ is a right (resp. left) winning coalition for Φ if whenever $\theta_C = (a, .., a)$ and $\theta_{I \backslash C} = (b, .., b)$ for some $a > b$ (resp. $a < b$) then $\Phi(\theta) = (a, .., a)$.*

Lemma 4 *For all $C \subset I$, either C is left (resp. right) winning or $I \setminus C$ is right (resp. left) winning and I is always right and left winning.*

Proof. Consider the partition $\Gamma = (C, I \backslash C)$. Let I^{Γ} and Φ^{Γ} be the corresponding reduced federation and mechanism. From what precedes, Φ_{Γ} is neutral, fair and group strategy-proof. As there are only two coalitions, we know from subsection 6.2 that Φ_{Γ} is a uniform voting rule. In the case of two jurisdictions, there are only four such voting rules, the min, the max and the two dictatorship, which all have a unique dictator on each side of the diagonal. ■

6.4 Three Jurisdictions

Let Φ be a fair, neutral, group strategy-proof mechanism in a federation of three jurisdictions on a domain $\mathbb{R}^3$. We assume w.l.o.g. $\theta_1 \leq \theta_2 \leq \theta_3$. We consider two cases: either $\{1\}$ or $\{3\}$ are respectively left and right winning, and both $\{1, 2\}$ and $\{2, 3\}$ are respectively left and right winning coalitions (see definition 11). By lemma 4, the two cases cover all possibilities.

Claim 7 *If there is at least two identical types, then:*
$\Phi(\theta) = (\theta_1, \theta_1, \theta_1)$ if $\{1\}$ is a left winning coalition,
$\Phi(\theta) = (\theta_3, \theta_3, \theta_3)$ if $\{3\}$ is a right winning coalition,
$\Phi(\theta) = (\theta_2, \theta_2, \theta_2)$ if $\{1, 2\}$ and $\{2, 3\}$ are respectively left and right winning coalitions.

Proof. If there is at least two identical types, there is at most two different types and lemma 4 applies. ■

Claim 8 *If $\theta_1 < \theta_2 < \theta_3$ and if $\{1\}$ (resp.$\{3\}$) is a left (resp. right) winning coalition then $\Phi(\theta) = (\theta_1, \theta_1, \theta_1)$ (resp. $(\theta_3, \theta_3, \theta_3)$).*

Proof. Consider the case where $\{1\}$ is a left winning coalition, the other case is identical. From claim 7, jurisdiction 2 can get the policy $(\theta_1, \theta_1, \theta_1)$ by reporting θ_1, so $\Phi(\theta) \succeq_{2,\theta_2} (\theta_1, \theta_1, \theta_1)$, which implies $\Phi_2(\theta) \geq \theta_1$. For the same reason, $\Phi_3(\theta) \geq \theta_1$.

If only one of the latter two inequalities is strict, say $\Phi_3(\theta) > \Phi_2(\theta) = \theta_1$, then $(\theta_1, \theta_1, \theta_1) \succ_{2,\theta_2} \Phi(\theta)$, which contradicts strategy-proofness. If $\Phi_2(\theta) > \theta_1$ and $\Phi_3(\theta) > \theta_1$, then for $i = 2, 3$, as $\Phi(\theta) \succeq_{i,\theta_i} (\theta_1, \theta_1, \theta_1)$ and as $\succeq_i$ satisfies the strict single-crossing condition in (θ_i, x_i), $\Phi(\theta) \succ_{i,\theta_3+1} (\theta_1, \theta_1, \theta_1)$. It is a contradiction to group strategy-proofness since from claim 7, $\Phi(\theta_1, \theta_3 + 1, \theta_3 + 1) = (\theta_1, \theta_1, \theta_1)$. Finally, we must have $\Phi_2(\theta) = \Phi_3(\theta) = \theta_1$ and strategy-proofness for jurisdiction 2 implies $\Phi_1(\theta) = \theta_1$ as well. ■

Let us now assume for the rest of the subsection that both $\{1, 2\}$ and $\{2, 3\}$ are respectively left and right winning coalitions. To prove $\Phi(\theta) = (\theta_2, \theta_2, \theta_2)$, the strategy is to let $\theta_3 \to +\infty$, in other words to consider the case where jurisdiction 3 has purely "ideological" preferences: it cares only about having a policy as far as possible to the right. We shall show that this extreme type induces a mechanism for jurisdiction 1 and 2 which is group strategy-proof, neutral and fair (claim 9 to 13).[23] We shall then show that it must be uniform using a reasoning similar to the two jurisdiction case (claim 14 to 16). We first define the correspondence of the accumulation points of Φ as $\theta_3 \to +\infty$:

$$A : \left\{(\theta_1, \theta_2) \in \mathbb{R}^2 : \theta_1 \leq \theta_2\right\} \rightrightarrows \mathbb{R}^3$$
$$(\theta_1, \theta_2) \rightrightarrows \cap_{\overline{\theta}_3} \left\{\Phi(\theta_1, \theta_2, \theta_3) : \theta_3 \geq \overline{\theta}_3\right\}.$$

Claim 9 *The correspondence A is non empty and neutral, i.e.:*

$$\forall u \in \mathbb{R},\ A(\theta_1 + u, \theta_2 + u) = A(\theta_1, \theta_2) + (u, u, u).$$

Proof. From claim 7, jurisdiction 1 can secure the policy $(\theta_2, \theta_2, \theta_2)$ by reporting θ_2. Therefore, strategy-proofness implies that $\Phi(\theta_1, \theta_2, \theta_3)$ is bounded in θ_3 for $\theta_3 \geq \theta_2$. From Bolzano-Weierstrass, $A(\theta_1, \theta_2)$ is non empty. The neutrality of A follows from the neutrality of Φ. ■

Claim 10 *For all $\overline{\theta}_3$ and $\theta_1 \leq \theta_2$, the two following finite limits exist:*

$$\lim_{\theta_3 \to \infty} U_{3,\overline{\theta}_3}(\Phi(\theta_1, \theta_2, \theta_3)),$$
$$\lim_{\theta_3 \to \infty} \Phi_3(\theta_1, \theta_2, \theta_3).$$

[23] We cannot simply fix the type of jurisdiction 3 to a finite value since the resulting mechanism for jurisdiction 1 and 2 would not be neutral.

Proof. From lemma 1, $\Phi_3(\theta_1,\theta_2,\theta_3)$ is non decreasing in θ_3. To show that $U_{3,\bar{\theta}_3}(\Phi(\theta_1,\theta_2,\theta_3))$ is non increasing for $\theta_3 \geq \theta_2$, let $\theta_3' > \theta_3 > \bar{\theta}_3$. By strategy-proofness, $\Phi(\theta_1,\theta_2,\theta_3) \succeq_{3,\theta_3} \Phi(\theta_1,\theta_2,\theta_3')$ and from lemma 1, $\Phi_3(\theta_1,\theta_2,\theta_3) \leq \Phi_3(\theta_1,\theta_2,\theta_3')$. As $\succeq_3$ satisfies the single-crossing condition in (θ_3,x_3), $\Phi(\theta_1,\theta_2,\theta_3) \succeq_{3,\bar{\theta}_3} \Phi(\theta_1,\theta_2,\theta_3')$. As argued in the proof of claim 9, $\Phi(\theta_1,\theta_2,\theta_3)$ is bounded in θ_3, so by monotonicity the two limits exist. ■

Claim 11 *Let $\theta_1, t_1, \theta_2 \in \mathbb{R}$, if for all increasing sequence $\theta_3^n \to \infty$ and $t_3^n \to \infty$, there exists N such that for $n \geq N$, $\Phi(\theta_1,\theta_2,\theta_3^n) \succ_{3,t_3^n} \Phi(t_1,\theta_2,t_3^n)$ then $\lim_{\theta_3\to\infty} U_{1,\theta_1}(\theta_1,\theta_2,\theta_3)$ exists. The same claim holds by switching the role of 1 and 2.*

Proof. Under the hypotheses of the claim, group strategy-proofness for jurisdictions $(3,t_3^n)$ and $(1,t_1)$ implies that for $n \geq N$,

$$\Phi(\theta_1,\theta_2,\theta_3^n) \preceq_{1,\theta_1} \Phi(t_1,\theta_2,t_3^n). \tag{7}$$

As it is true for any sequences θ_3^n and t_3^n, letting $n \to \infty$ in (7), the continuity of $U_1(x)$ implies

$$x \in A(\theta_1,\theta_2), y \in A(t_1,\theta_2) \Rightarrow x \preceq_{1,\theta_1} y. \tag{8}$$

By strategy-proofness, for all θ_3, $\Phi(\theta_1,\theta_2,\theta_3) \succeq_{1,\theta_1} \Phi(t_1,\theta_2,\theta_3)$. Letting $\theta_3 \to \infty$,

$$\forall x \in A(\theta_1,\theta_2), \exists y \in A(t_1,\theta_2) : x \succeq_{1,\theta_1} y. \tag{9}$$

From (8) and (9), $U_{1,\theta_1}(A(\theta_1,\theta_2))$ must be a singleton. ■

Claim 12 *For all $\theta_1 \leq \theta_2$, for $i = 1,2$, the following finite limit exists:*

$$\lim_{\theta_3\to\infty} U_{i,\theta_i}(\Phi(\theta_1,\theta_2,\theta_3)).$$

Proof. Let (θ_1,θ_2) be such that $(\theta_2,\theta_2,\theta_2) \notin A(\theta_1,\theta_2)$. Let $\theta_3^n \to \infty$ be an increasing sequence bounded below by θ_2. Claim 7 implies

$$\Phi(\theta_1,\theta_2,\theta_3^n) \succeq_{3,\theta_3^n} (\theta_2,\theta_2,\theta_2). \tag{10}$$

From (1), (10), $(\theta_2,\theta_2,\theta_2) \notin A(\theta_1,\theta_2)$ and $\theta_3^n > \theta_2$, we have that for all n, $\Phi_3^\infty(\theta_1,\theta_2,\theta_3^n) > \theta_2$. So, by the strict single-crossing condition, for all sequence t_3^n such that for all n, $t_3^n > \theta_3^n$, we have

$$\Phi(\theta_1,\theta_2,\theta_3^n) \succ_{3,t_3^n} (\theta_2,\theta_2,\theta_2) = \Phi(\theta_1,\theta_2,t_3^n). \tag{11}$$

where the last equality comes from claim 7. Claim 11 and (11) implies that $\lim_{\theta_3\to\infty} U_{1,\theta_1}(\theta_1,\theta_2,\theta_3)$ exists.[24]

Since $\theta_1 \le \theta_2 < t_3^n$, $(\theta_2,\theta_2,\theta_2) \succ_{3,t_3^n} (\theta_1,\theta_1,\theta_1)$, so from (11), $\Phi(\theta_1,\theta_2,\theta_3^n) \succ_{3,t_3^n} \Phi(\theta_1,\theta_2,t_3^n)$, which together with claim 11 proves that $\lim_{\theta_3\to\infty} U_{2,\theta_2}(\theta_1,\theta_2,\theta_3)$ exists.

If $(\theta_2,\theta_2,\theta_2) \in A(\theta_1,\theta_2)$, from claim 10, for all $x \in A(\theta_1,\theta_2)$, $x_3 = \theta_2$ and as $U_{3,\theta_3}(x) = U_{3,\theta_3}(\theta_2,\theta_2,\theta_2)$ for all θ_3, one can easily see from (1) that necessarily $x = (\theta_2,\theta_2,\theta_2)$. ■

Claim 13 *Let Φ^∞ be a selection[25] of A which is neutral, then Φ^∞ is a group strategy-proof mechanism and $\theta_1 \le \theta_2 \Rightarrow \Phi_1^\infty(\theta) \le \Phi_2^\infty(\theta) \le \Phi_3^\infty(\theta)$.*

Proof. Let $\Phi^\infty(\theta_1,\theta_2)$ be a neutral selection from $A(\theta_1,\theta_2)$ (such a selection exists from claim 9). Let $\theta_1,\theta_2,t_1,t_2 \in \mathbb{R}$, and let $t_3^n \to \infty$ be such that $\lim \Phi(t_1,t_2,t_3^n) \to \Phi^\infty(t_1,t_2)$. Group strategy-proofness for Φ implies

$$U_{i,\theta_i}(\Phi(\theta_1,\theta_2,t_3^n)) \ge U_{i,\theta_i}(\Phi(t_1,t_2,t_3^n))$$

for $i = 1$ or 2. Letting $n \to \infty$, from claim 12 and the continuity of $U_i(x)$ we get

$$U_{i,\theta_i}(\Phi^\infty(\theta_1,\theta_2)) \ge U_{i,\theta_i}(\Phi^\infty(t_1,t_2))$$

for $i = 1$ or 2, which establishes group strategy-proofness for Φ^∞. Individual strategy-proofness can be established similarly.

From claim 7 and lemma 3, $\Phi_1(\theta) \le \Phi_2(\theta) \le \Phi_3(\theta)$ for all $\theta_1 \le \theta_2 \le \theta_3$. Letting $\theta_3 \to \infty$ completes the proof. ■

The strategy for the reminder of the proof is to consider $\Phi^\infty(\theta_1,\theta_2)$ as a mechanism for jurisdiction 1 and 2 only and adapt the proof in subsection 6.2. From (1), $U_{i,\theta_i}(x)$ is additively separable in x_3 and θ_i for $i = 1,2$. Hence, lemma 1 and its proof hold unchanged for $U_{i,\theta_i}(\Phi^\infty)$. From claim 13, Φ^∞ is neutral so $U_{i,\theta_i+u}(\Phi^\infty(\theta_1+u,\theta_2+u))$ is constant in u, and lemma 2 and its proof hold unchanged as well.

Having this in mind, claims 1, 2, 3 and 4 hold for Φ^∞ since their proofs are based solely on lemmas 1 and 2.

[24] Clearly, claim 11 still holds if the sequences θ_3^n and t_3^n are required to be bounded below by θ_2 and such that for all n, $\theta_3^n < t_3^n$.

[25] A selection s of a correspondence C with domain D_C is a function such that for all $x \in D_C$, $f(x) \in C(x)$.

Claims 5 cannot be proven as in the two jurisdictions case: the additional term $W_3(\Phi_i^\infty - \Phi_3^\infty)$ in the utility of jurisdictions 1 and 2 gives more instruments to the mechanism designer to satisfy incentive compatibility. However, we shall show that group strategy-proofness implies that $\Phi_3^\infty - \Phi_2^\infty$ is increasing in θ_1, which implies that it is null since $\Phi_3^\infty \geq \Phi_2^\infty$ for all $\theta_1 < \theta_2$ and $\Phi_3^\infty = \Phi_2^\infty$ when $\theta_1 = \theta_2$.

As in the two jurisdiction case, in the sequel we fix θ_2, consider Φ^∞ as a function of θ_1 only and assume $\theta_1 < \theta_2$.

Claim 14 *If $\theta_1 \in \overline{D_1}^o \cup \overline{D_2}^o$, then $\Phi_3^\infty - \Phi_2^\infty$ is locally non decreasing in θ_1.*

Proof. From claim 13, $\Phi_1^\infty \leq \Phi_2^\infty \leq \Phi_3^\infty$. If $\overline{\theta}_1 \in \overline{D_1}^o$ from claim 3, there is a neighborhood N of $\overline{\theta}_1$ such that on N, $U_{1,\theta_1}(\Phi^\infty(\theta_1,\theta_2))$ and $V(\Phi_1^\infty(\theta_1,\theta_2) - \theta_1)$ are constant in θ_1 on N. From claim 1, $\Phi_1^\infty - \Phi_2^\infty$ is non decreasing in θ_1 and non positive on N, so $W_2(\Phi_1^\infty - \Phi_2^\infty)$ is non increasing in θ_1. Therefore, $W_3(\Phi_1^\infty - \Phi_3^\infty)$ must be non decreasing. Since $\Phi_1^\infty \leq \Phi_3^\infty$ this implies that $\Phi_1^\infty - \Phi_3^\infty$ is non increasing, and thus $\Phi_3^\infty - \Phi_2^\infty$ must be non decreasing.

If $\overline{\theta}_1 \in \overline{D_2}^o$, a similar reasoning shows that on a neighborhood N of $\overline{\theta}_1$, $U_{2,\theta_2}(\Phi^\infty)$ and $V(\Phi_2^\infty - \theta_2)$ are locally constant and $W_1(\Phi_2^\infty - \Phi_1^\infty)$ is non increasing, so $W_3(\Phi_2^\infty - \Phi_3^\infty)$ and thus $\Phi_3^\infty - \Phi_2^\infty$ must be non decreasing. ■

For any function f, in the sequel we shall write $f(\theta_1^+) > f(\theta_1)$ for "$\exists\epsilon > 0 : \forall t_1 \in]\theta_1, \theta_1 + \epsilon[,\ f(t_1) > f(\theta_1)$." We shall say that f has an upward jump (to the right of θ_1) if $\exists\epsilon, \delta > 0 : \forall t_1 \in]\theta_1, \theta_1 + \epsilon[,\ f(t_1) > f(\theta_1) + \delta$. Downward jumps and $f(\theta_1^+) < f(\theta_1)$ are defined similarly.

Claim 15 *Either $(\Phi_3^\infty - \Phi_2^\infty)(\theta_1^+) > (\Phi_3^\infty - \Phi_2^\infty)(\theta_1)$ or Φ^∞ is continuous and $\Phi_{1,2}^\infty(\theta_1, \theta_2) = (\theta_1, \theta_2)$.*

Proof. Observe first that to obtain the first conclusion of the claim, it suffices to show that $W_3(\Phi_2^\infty - \Phi_3^\infty)$ or $W_3(\Phi_1^\infty - \Phi_3^\infty)$ has an upward jump. Indeed, from claim 13, $\Phi_2^\infty \leq \Phi_3^\infty$ so from assumption 1, if $W_3(\Phi_2^\infty - \Phi_3^\infty)$ has an upward jump, necessarily $(\Phi_3^\infty - \Phi_2^\infty)(\theta_1^+) > (\Phi_3^\infty - \Phi_2^\infty)(\theta_1)$. From claim 1, $\Phi_2^\infty - \Phi_1^\infty$ is decreasing so the same conclusion holds if $W_3(\Phi_1^\infty - \Phi_3^\infty)$ has an upward jump.

Suppose first that $\Phi_2^\infty - \Phi_1^\infty$ is discontinuous to the right of θ_1. By claim 1, $\Phi_2^\infty - \Phi_1^\infty$ can only have downward jumps. Since $\Phi_1^\infty \leq \Phi_2^\infty$, from assumption 1, $W_1(\Phi_2^\infty - \Phi_1^\infty)$ has a downward jump. If $\Phi_2^\infty(\theta_1^+) = \theta_2$, $V(\Phi_2^\infty - \theta_2)$ is either continuous at θ_1 or has a downward jump. From lemma 2, $U_{2,\theta_2}(\Phi^\infty)$ is continuous, so $W_3(\Phi_2^\infty - \Phi_3^\infty)$ must have an upward jump. If $\Phi_2^\infty(\theta_1^+) \neq \theta_2$, from claim 4, $\Phi_1^\infty(\theta_1^+) = \theta_1$ and a similar reasoning for jurisdiction 1 shows that for $U_{2,\theta_2}(\Phi^\infty)$ to be continuous, it must be that $W_3(\Phi_1^\infty - \Phi_3^\infty)$ has an upward jump.

Now if Φ_1^∞ and Φ_2^∞ are continuous to the right of θ_1, the continuity of $U(\Phi^\infty)$ implies the continuity of Φ_3^∞. If $\Phi_i(\theta_1^+) \neq \theta_i$ for some i, then by continuity, $]\theta_1, \theta_1 + \epsilon[\cap D_j = \emptyset$ for some $\epsilon > 0$. So from claim 2, $]\theta_1, \theta_1 + \epsilon[$ is included in D_j and so in $\overline{D_j}^o$ and from claim 14, $\Phi_3^\infty - \Phi_2^\infty$ is non decreasing on $]\theta_1, \theta_1 + \epsilon[$. By continuity, it is non decreasing on $[\theta_1, \theta_1 + \epsilon[$ which implies $(\Phi_3^\infty - \Phi_2^\infty)(\theta_1^+) > (\Phi_3^\infty - \Phi_2^\infty)(\theta_1)$. If $\Phi_{1,2}(\theta_1^+) = (\theta_1, \theta_2)$, by continuity the other conclusion follows.

The only case left is $\Phi_2^\infty - \Phi_1^\infty$ continuous but Φ_1^∞ (and hence Φ_2^∞) discontinuous to the right of θ_1. From claim 4, $\Phi_i^\infty(\theta_1^+) = \theta_i$ for some i, say $i = 2$, the other case is identical. $V(\Phi_2^\infty - \theta_2)$ has a downward jump in θ_1 and $W_1(\Phi_2^\infty - \Phi_1^\infty)$ is continuous by hypothesis. From lemma 2, $U_{2,\theta_2}(\Phi^\infty)$ is continuous so $W_3(\Phi_2^\infty - \Phi_3^\infty)$ must have an upward jump in θ_1. ■

Claim 16 $\Phi_3^\infty - \Phi_2^\infty$ *is non decreasing in* θ_1.

Proof. Let us denote

$$D(\theta_1) = \lim \sup_{t_1 \to \theta_1, t_1 \geq \theta_1} \frac{(\Phi_3^\infty - \Phi_2^\infty)(t_1) - (\Phi_3^\infty - \Phi_2^\infty)(\theta_1)}{t_1 - \theta_1}.$$

We shall show that $D(\theta_1) \geq 0$ for all $\theta_1 < \theta_2$, which implies the claim (see e.g. Royden 1988 p. 99).

Let S_+ denotes the set of types at which $(\Phi_3^\infty - \Phi_2^\infty)(\theta_1^+) > (\Phi_3^\infty - \Phi_2^\infty)(\theta_1)$ and S_c the set of types at which Φ^∞ is continuous and $\Phi_{1,2}^\infty(\theta_1, \theta_2) = (\theta_1, \theta_2)$. From claim 2, S_+ and S_c cover all possible situations. By definition, $\theta_1 \in S_+ \Rightarrow D(\theta_1) \geq 0$. If $\theta_1 \in S_c$ and $]\theta_1, \theta_1 + \epsilon[\subset S_+$ for some $\epsilon > 0$, $\Phi_3^\infty - \Phi_2^\infty$ must be non decreasing on $]\theta_1, \theta_1 + \epsilon[$ so by continuity $D(\theta_1) \geq 0$.

The only case left is $\theta_1 \in S_c$ and there is a decreasing sequence θ_1^n in S_c which converges to θ_1. From claim 15, $\Phi^\infty(\theta) = (\theta_1, \theta_2, \Phi_3^\infty(\theta))$. Strategy-proofness for jurisdiction 3 and claim 7 implies $\Phi_3^\infty(\theta) > \theta_2$. Moreover, $\theta_1 < \theta_2$ so assumption 1 implies that at $x = \Phi^\infty(\theta)$, for $i = 1, 2$,

$$\begin{aligned}
\lim \inf_{y_1 \to x_1} \frac{U_{i,\theta_i}(y_1, x_2, x_3) - U_{i,\theta_i}(x_1, x_2, x_3)}{y_1 - x_1} &> 0, \\
\lim \sup_{y_3 \to x_3} \frac{U_{i,\theta_i}(x_1, x_2, y_3) - U_{i,\theta_i}(x_1, x_2, x_3)}{y_3 - x_3} &< 0.
\end{aligned}$$

As $\theta_1^n \in S_c$, from claim 15, $\Phi_{1,2}^{\infty}(\theta_1^n, \theta_2) = (\theta_1^n, \theta_2, \Phi_3^{\infty}(\theta_1^n, \theta_2))$ and since $\theta_1^n > \theta_1$, from what precedes $\Phi_{1,2}^{\infty}(\theta_1^n, \theta_2)$ is strictly preferred by jurisdictions 1 and 2 to $\Phi^{\infty}(\theta)$ for n sufficiently large whenever $\Phi_3^{\infty}(\theta_1^n, \theta_2) \leq \Phi_3^{\infty}(\theta)$. Hence, group strategy-proofness implies that $\Phi_3^{\infty}(\theta_1^n, \theta_2) > \Phi_3^{\infty}(\theta_1, \theta_2)$ for n sufficiently large, so $D(\theta_1) \geq 0$. ■

Claim 17 *If $\theta_1 \leq \theta_2 \leq \theta_3$ and $\{1,2\}$ and $\{2,3\}$ are respectively left and right winning coalitions, $\Phi(\theta) = (\theta_2, \theta_2, \theta_2)$.*

Proof. From claim 13, $\Phi_2^{\infty} = \Phi_3^{\infty}$ at $\theta_1 = \theta_2$, so claim 16 implies $\Phi_2^{\infty} = \Phi_3^{\infty}$ for all $\theta_1 \leq \theta_2$. Under our specification, given $x_2 = x_3$, the preferences of jurisdiction 1 and 2 over (x_1, x_2) have exactly the same form as in the standard 2 jurisdictions case. Hence, from subsection 6.2, Φ^{∞} is uniform.

Since the policy $(\theta_2, \theta_2, \theta_2)$ can be secured both by coalitions $\{1,2\}$ and $\{2,3\}$ and since this policy is strictly preferred to any other uniform policy by either coalition, $\Phi^{\infty}(\theta_1, \theta_2) = (\theta_2, \theta_2, \theta_2)$. Since Φ^{∞} is any neutral selection from A, $A(\theta_1, \theta_2) = \{(\theta_2, \theta_2, \theta_2)\}$ for all $\theta_1 \leq \theta_2$.

By monotonicity of Φ_3 in θ_3, $\Phi_3(\theta) \leq \Phi_3^{\infty}(\theta_1, \theta_2) = \theta_2$ for all $\theta_1 \leq \theta_2 \leq \theta_3$. Since jurisdiction $(3, \theta_3)$ strictly prefers $(\theta_2, \theta_2, \theta_2)$ to any other policy such that $\Phi_3(\theta) \leq \theta_2$, strategy-proofness and claim 7 implies $\Phi(\theta) = (\theta_2, \theta_2, \theta_2)$. ■

6.5 More than Three Jurisdictions

Let Φ a neutral, fair and group strategy-proof mechanism on a federation of N jurisdictions.

Definition 12 *A jurisdiction $i \in I$ is pivotal for Φ if $\{1, .., i\}$ and $\{i, .., N\}$ are respectively left and right winning coalitions.*

Claim 18 *There exists $p \in I$ such that p is pivotal for Φ at θ.*

Proof. Let p be the smallest integer such that $\{1, .., p\}$ is left winning. Such a p exists since by lemma 4, $\{1, .., N\}$ is left winning. By construction, $\{1, .., p-1\}$ is not left winning. Using lemma 4 again, $\{p, .., N\}$ is right winning, and p is pivotal. ■

Let $\theta \in \mathbb{R}^N$ and suppose w.l.o.g. $\theta_1 \leq .. \leq \theta_N$. Let $p \in I$ such that p is pivotal.

Claim 19 $\Phi(\theta) = (\theta_p, ..., \theta_p)$.

Proof. Consider the following sequence of profiles of type: $\theta^o = \theta$, and for all $n \geq 0$, if $\Phi(\theta^n) \neq (\theta_p, ..., \theta_p)$ and if there exists $i \in I$ such that $\theta_i \neq \theta_p$ and $\Phi(\theta_p, \theta^n_{-i}) \neq (\theta_p, ..., \theta_p)$, then $\theta^{n+1} = (\theta_p, \theta^n_{-i})$. Otherwise $\theta^{n+1} = \theta^n$. Clearly, θ^n is stationary after a finite number of step. Let λ denotes its limit, and let $\underline{C}$ (resp. C, resp. $\overline{C}$) denotes the set of jurisdiction i such that $\lambda_i < \theta_p$ (resp. $\lambda_i = \theta_p$, resp. $\lambda_i > \theta_p$). By construction, $\Phi(\lambda) = (\theta_p, ..., \theta_p)$ i.f.f. $\Phi(\theta) = (\theta_p, ..., \theta_p)$.

The collection of coalitions $\Gamma = (\underline{C}, C, \overline{C})$ defines a reduced federation of three jurisdiction as explained in subsection 6.3, and Φ induces a mechanism Φ^Γ on $I^\Gamma = \{1, 2, 3\}$ which is group strategy-proof, neutral and fair. By construction, $p \in C$ so by definition of p, $\underline{C} \cup C$ and $C \cup \overline{C}$ are respectively left and right winning coalitions in I, and so do $\{1, 2\}$ and $\{2, 3\}$ in I^Γ. From claim 17, $\Phi^\Gamma(a, \theta_p, b) = (\theta_p, \theta_p, \theta_p)$ for any $a \leq \theta_p \leq b$. By definition of Φ^Γ, for all t such that $\forall i \in \underline{C}$, $t_i = a$, $\forall i \in C$, $t_i = \theta_p$, and $\forall i \in \overline{C}$, $t_i = b$, we have $\Phi(t) = (\theta_p, .., \theta_p)$.

Suppose $\Phi(\lambda) \neq (\theta_p, ..., \theta_p)$. By construction of θ^n, $\forall i \in \underline{C} \cup \overline{C}$, $\Phi(\theta_p, \lambda_{-i}) = (\theta_p, ..., \theta_p)$. Strategy-proofness implies that $\forall i \in \underline{C}$, $\Phi(\lambda) \succeq_{i,\theta_i} (\theta_p, ..., \theta_p)$ and since $\Phi(\lambda) \neq (\theta_p, ..., \theta_p)$ and $a < \theta_p$, $\Phi_i(\lambda) < \theta_p$. The strict single crossing condition in (θ_i, x_i) implies that $\Phi(\lambda) \succ_{i,a} (\theta_p, .., \theta_p)$ for any $a < \lambda_i$. Similarly, for $i \in \overline{C}$, $\Phi(\lambda) \succ_{i,b} (\theta_p, .., \theta_p)$ for any $b > \lambda_i$. This implies that at t, the jurisdictions from $\underline{C}$ and $\overline{C}$ can strictly benefit by reporting their respective type in λ instead of a and b, a contradiction. ■

7 Proofs in Section 4

The next lemma derives some properties of the non cooperative decentralized equilibrium. As a reminder, for the proof of the following lemma and propositions 4 and 5 we assume that V and W are twice differentiable and strictly convex.

Lemma 5 *For all profile of type* θ
(i) there is a unique decentralized equilibrium $\Phi^{dec}(\theta)$.
(ii) Φ^{dec} *is differentiable almost everywhere in* θ_i *for all* $i \in I$.
(iii) Φ^{dec} *is increasing in* θ
(iv) $\theta_i < \theta_j \Rightarrow \Phi^{dec}_i(\theta) < \Phi^{dec}_j(\theta)$

(v) for all $J \subset I$, if θ_J is uniform, $x_J = \Phi_J^{dec}(\theta)$ is the unique most-preferred response to $x_{I \setminus J} = \Phi_{I \setminus J}^{dec}(\theta)$ for any jurisdiction $j \in J$.

Proof. Since W_j is convex for all j, the game is supermodular which proves existence (see Milgrom and Roberts 1990 and Vives 1990). Uniqueness comes from the fact that the vector $\left(\frac{\partial U_{i,\theta_i}}{\partial x_i}\right)_{i=1..N}$ has a diagonally dominant differential (see Rosen 1965). Since U_{i,θ_i} has increasing differences in (x_i, θ_i) and the game is supermodular, we know that Φ^{dec} is non-decreasing in θ. To show that it is increasing, observe that in equilibrium, each jurisdiction i minimizes

$$V(x - \theta_i) + \sum_j W_j\left(x - \Phi_j^{dec}(\theta)\right). \tag{12}$$

The strict convexity of V and W implies that (12) has increasing marginal returns in (x, θ_i) and in $\left(x, \Phi_j^{dec}\right)$ for all $j \neq i$ (see Edlin and Shannon 1998). So the maximand of (12) is increasing in $\left(\theta_i, \Phi_{-i}^{dec}(\theta)\right)$. Since $\Phi_{-i}^{dec}(\theta)$ is non decreasing in θ_i, $\Phi_i^{dec}(\theta)$ must be increasing in θ_i. This implies in turn that $\Phi_j^{dec}(\theta)$ must be increasing in θ_i which proves (iii). Moreover, as (12) depends on i only through the type θ_i, point (iv) follows.

If θ_J is uniform for some $J \subset I$, given $\Phi_{I \setminus J}^{dec}(\theta)$ the jurisdictions in J have a common most preferred uniform policy x_J. One can easily see from (1) that at $\left(x_J, \Phi_{I \setminus J}^{dec}\right)$ no jurisdiction in J has an incentive to unilaterally change its local policy. So $\left(x_J, \Phi_{I \setminus J}^{dec}(\theta)\right)$ is the equilibrium which implies point (v). ■

7.1 Proof of proposition 3

Since Φ is efficient (i.e. Pareto efficient among local median voters), a discontinuity of Φ in θ_i corresponds to a jump of $U_{i,\theta_i}(\Phi(\theta))$ in θ_i, which is incompatible with strategy-proofness.[26] Since Φ is continuous in θ_i, from assumption 1, $V'(\Phi_i(\theta) - \theta_i)$ is continuous in θ_i and from lemma 1, $U_{i,\theta_i}(\Phi(\theta))$ is continuously differentiable in θ_i.

[26] Suppose $\theta_1^n \to \theta_1$ and $\Phi(\theta_1^n, \theta_2) \to l \neq \Phi(\theta_1, \theta_2)$. By continuity of $U_{1,\theta_1}(\Phi(\theta_1, \theta_2))$ and $U_{1,\theta_1}(x)$ in θ_i, $U_{1,\theta_1}(\Phi(\theta_1, \theta_2)) = U_{1,\theta_1}(l)$. By efficiency, $U_{2,\theta_2}(\Phi(\theta_1, \theta_2)) = U_{2,\theta_2}(l)$. By strict quasi-concavity, there exists a policy x inbetween $\Phi(\theta_1, \theta_2)$ and l which is strictly preferred by both jurisdictions, a contradiction.

Suppose $\Phi(\theta) \neq (\theta_2, \theta_2)$. Since $\Phi(\theta)$ is efficient, it solves the following program:

$$P(\theta) = \max_{x \,:\, U_{2,\theta_2}(x) - U_{2,\theta_2}(\Phi(\theta)) \geq 0} U_{1,\theta_1}(x). \tag{13}$$

By hypothesis, $U_{i,\theta_i}(x)$ is quasi-concave and continuous in x. The maximand of P is constant in θ_2. From what precedes, the constraint is differentiable in θ_2 and its derivative $V'(x_2 - \theta_2) - V'(\Phi_2(\theta) - \theta_2)$ is continuous in (θ_2, x). Moreover, since $\Phi(\theta) \neq (\theta_2, \theta_2)$, the choice set has a non empty interior. Therefore, the conditions of theorem 5 in Milgrom and Segal 2002 are met and $P(\theta)$ is absolutely continuous in θ_2.[27] Whenever it is differentiable,

$$\frac{\partial P}{\partial \theta_2} = \lambda \left[\frac{\partial \left[U_{2,\theta_2}(x)\right]}{\partial \theta_2} (x = \Phi(\theta)) - \frac{\partial \left[U_{2,\theta_2}(\Phi(\theta))\right]}{\partial \theta_2} \right] \tag{14}$$

for some $\lambda \geq 0$. From lemma 1, the right hand-side of (14) is null for almost all θ_2 so by absolute continuity, P is constant in θ_2.

Let O be an open subset of Θ^2 such that $\Phi(\theta) \notin \{(\theta_1, \theta_1), (\theta_2, \theta_2)\}$ for all $\theta \in O$. From what precedes, for all $i \neq j$ $U_{i,\theta_i}(x)$ is constant in θ_j so its derivative $V'(\Phi_i(\theta) - \theta_i)$ must be constant in θ_j as well. Since V' is one to one, the mechanism can be decomposed as $\Phi(\theta) = (\Phi_1(\theta_1), \Phi_2(\theta_2))$. However, one can easily check that such a mechanism cannot be efficient since the externalities are not internalized. Therefore, the set of points where $\Phi(\theta) = (\theta_i, \theta_i)$ is dense in Θ^2. As noted above, $\Phi(\theta)$ and hence $U_{i,\theta_i}(\Phi(\theta))$ must be continuous in θ, and there must be a dictator at any point of Θ^2. Finally, since preferences over uniform policies are single-peaked, from Moulin 1980, Φ must be a uniform voting rule.

7.2 Proof of proposition 4

It suffices to show that Φ^{dec} is preferred by the local median voter (i, θ_i) of all but one jurisdiction. Let $u \in \Theta$. As $\Phi^{dec}(u, ..., u) = (u, ..., u)$ is unanimously the best policy at the profile of types $(u, ..., u)$, for any $i \in I$, $\Phi(u, ..., u) \preceq_{i,\theta_i} \Phi^{dec}(u, ..., u)$. Let $\theta \in \Theta^N$ be such that $\theta_{-i} = (u, .., u)$ and $\theta_i \neq u$ for some $i \in I$. If jurisdiction i is strictly better-off at θ under Φ^{dec}

[27] Observe that concavity assumption in the theorem 5 of Milgrom and Segal 2002 is not necessary: from the min-max theorem, quasi-concavity is enough to guarantee the existence of a saddle point of the Lagrangean, which is the only role of the concavity assumption.

than under Φ and such a type θ exists for all i, the proposition follows. If not, $\Phi(\theta) \succeq_{i,\theta_i} \Phi^{dec}(\theta)$ for some i and some $\theta \in \Theta^N$ such that $\theta_{-i} = (u, .., u)$ and $\theta_i \neq u$. From what precedes,

$$U_{i,\theta_i}(\Phi(\theta)) - U_{i,u}(\Phi(u,...,u)) \geq U_{i,\theta_i}\left(\Phi^{dec}(\theta)\right) - U_{i,u}\left(\Phi^{dec}(u,...,u)\right). \quad (15)$$

Using lemma 5 (ii) together with the envelope theorem,

$$U_{i,\theta_i}\left(\Phi^{dec}(\theta)\right) - U_{i,u}\left(\Phi^{dec}(u,...,u)\right) = \quad (16)$$
$$\int_u^{\theta_i}\left(V'\left(\Phi_i^{dec} - t\right) + \sum_{j\neq i}\frac{\partial \Phi_j^{dec}}{\partial \theta_i} W_j'(\Phi_i^{dec} - \Phi_j^{dec})\right) dt,$$

where the argument of Φ^{dec} in the integrand is (t, θ_{-i}). If we combine (15), (16) and lemma 1 we get

$$\int_u^{\theta_i}\left[V'(\Phi_i - t) - V'\left(\Phi_i^{dec} - t\right) - \sum_{j\neq i}\frac{\partial \Phi_j^{dec}}{\partial \theta_i} W_j'(\Phi_i^{dec} - \Phi_j^{dec})\right] dt \geq 0, \quad (17)$$

Suppose for instance that $\theta_i > u$. From lemma 5, for all $i \neq j$, $\frac{\partial \Phi_j^{dec}}{\partial \theta_i} > 0$ a.e. and for all $t > u$, $\Phi_i^{dec}(t, \theta_{-i}) > \Phi_j^{dec}(t, \theta_{-i})$. Hence, the sum in the integrand in (17) is positive a.e. Therefore, for (17) to hold, it must be that $V'(\Phi_i(t, \theta_{-i}) - t) > V'\left(\Phi_i^{dec}(t, \theta_{-i}) - t\right)$, i.e. $\Phi_i(t, \theta_{-i}) > \Phi_i^{dec}(t, \theta_{-i})$ for a set of type t of positive Lebesgue measure.

As $\theta_{-i} = (u, ..., u)$, from lemma 5 (v), for all $j \neq i$,

$$\Phi_{-i}^{dec}(t, \theta_{-i}) = \arg\max_{x_{-i}} U_{j,u}\left(\Phi_i^{dec}(t, \theta_{-i}), x_{-i}\right)$$

From what precedes, $u < \Phi_i^{dec}(t, \theta_{-i}) < \Phi_i(t, \theta_{-i})$: Φ_i^{dec} is closer to the ideal policy of the other jurisdictions than Φ_i. So from our specification in (1), it is clear that

$$\max_{x_{-i}} U_{j,u}\left(\Phi_i^{dec}(t, \theta_{-i}), x_{-i}\right) > \max_{x_{-i}} U_{j,u}(\Phi_i(t, \theta_{-i}), x_{-i}).$$

Therefore, if $\Phi_i(t, \theta_{-i}) > \Phi_i^{dec}(t, \theta_{-i})$, whatever $\Phi_{-i}(t, \theta_{-i})$, all jurisdictions $j \neq i$ prefer Φ^{dec} to Φ. A symmetric reasoning yields the same result if $\theta_i < u$.

7.3 Proof of proposition 5

We denote $B_\theta(x) = U_{1,\theta_1}(x) + U_{2,\theta_2}(x)$ and $\Delta B(\theta) = B(\Phi(\theta)) - B\left(\Phi^{dec}(\theta)\right)$ for all $\theta \in \Theta^N$. Throughout the proof, we assume $\theta_1 > \theta_2$ and we omit θ unless confusion is possible. Since preferences are reciprocal, $W_1(x) = W_2(-x)$ for all x and W refers to W_2. Combing the F.O.C. of the decentralized equilibrium with lemmas 1 and 2, we have that for almost all θ,

$$\frac{\partial \Delta B}{\partial \theta_1} = V'(\Phi_1 - \theta_1) - V'(\Phi_2 - \theta_2) - V'\left(\Phi_1^{dec} - \theta_1\right) + V'\left(\Phi_2^{dec} - \theta_2\right) \quad (18)$$

$$- \frac{\partial \Phi_2^{dec}}{\partial \theta_1} W'(\Phi_1^{dec} - \Phi_2^{dec}) + \frac{\partial \Phi_1^{dec}}{\partial \theta_2} W'(\Phi_2^{dec} - \Phi_1^{dec}). \quad (19)$$

From lemma 5 (iv), $\Phi_1^{dec} > \Phi_2^{dec}$. As Φ^{dec} is increasing in θ and as W is strictly convex with a maximum in 0, (19) is negative. Therefore, for $\frac{\partial \Delta B}{\partial \theta_1}$ to be non negative, it must be that the right hand-side of (18) is positive. We shall show that this implies that ΔB is negative.

Consider the following maximization program:

$$\max_{x_1, x_2} B_\theta(x) \quad (20)$$

$$\text{such that } V'(x_1 - \theta_1) - V'(x_2 - \theta_2) \geq C \quad (21)$$

Suppose first that $C = V'\left(\Phi_1^{dec} - \theta_1\right) - V'\left(\Phi_2^{dec} - \theta_2\right)$. We shall show that the constraint (21) is binding. The F.O.C. of the unconstrained program (20) is

$$V'(x_1^* - \theta_1) = -V'(x_2^* - \theta_2) = -2W'(x_1^* - x_2^*) < 0. \quad (22)$$

The F.O.C. of decentralization yields

$$V'\left(\Phi_1^{dec} - \theta_1\right) = -V'\left(\Phi_2^{dec} - \theta_2\right) = -W'(\Phi_1^{dec} - \Phi_2^{dec}) < 0. \quad (23)$$

The first equality of (22) and (23) together with the evenness of V implies that $x_1 - \theta_1 = -(x_2 - \theta_2)$ and $\frac{x_1 + x_2}{2} = \frac{\theta_1 + \theta_2}{2}$ both for $x = x^*$ and $x = \Phi^{dec}$. The second equality of (22) and (23) implies then $\Phi_1^{dec} - \Phi_2^{dec} > x_1^* - x_2^* > 0$,[28] which in turns implies that $|V'(x_i^* - \theta_i)| > \left|V'\left(\Phi_i^{dec} - \theta_i\right)\right|$ for $i = 1, 2$. This shows that x^* does not satisfy (21).

[28] For any $v \in \mathbb{R}^2$ we denote $\Delta v = v_1 - v_2$. From what precedes, Δx^* and $\Delta \Phi^{dec}$ are solutions of $V'\left(\frac{\Delta x - \Delta t}{2}\right) + \alpha W_2'(\Delta x) = 0$ respectively for $\alpha = 1$ and $\alpha = 2$. As both $V'\left(\frac{\Delta x - \Delta t}{2}\right)$ and $\alpha W_2'(\Delta x)$ are increasing in Δx and are respectively negative and positive, it must be the case that the solution $\Delta x(\alpha)$ is deceasing in α.

Since the maximand in (20) is strictly concave, from what precedes, the constraint (21) must be binding. Denoting λ its multiplier, the necessary first order conditions yields

$$-V'(x_1-\theta_1)+\lambda V''(x_1-\theta_1) = 2W'(x_1-x_2), \tag{24}$$
$$-V'(x_2-\theta_2)-\lambda V''(x_2-\theta_2) = -2W'(x_1-x_2). \tag{25}$$

Under our hypothesis, V' is odd, increasing and V'' is even, decreasing on $\mathbb{R}_-$ and increasing on $\mathbb{R}_+$. Hence, the left hand-side of (24) (resp. (25)) is strictly monotonic in $x_1-\theta_1$ on $\mathbb{R}_-$ (resp. in $x_2-\theta_2$ on $\mathbb{R}_+$). As the right hand-side of (24) and (25) are opposite, the two equations imply $x_1-\theta_1=-(x_2-\theta_2)$ which, together with the binding constraint (21), yields $x=\Phi^{dec}(\theta)$ as the only solution of the program (20) for $C=V'\left(\Phi_1^{dec}-\theta_1\right)-V'\left(\Phi_2^{dec}-\theta_2\right)$.

Therefore, for any $C>V'\left(\Phi_1^{dec}-\theta_1\right)-V'\left(\Phi_2^{dec}-\theta_2\right)$, the value of program (20) is strictly less than $B_{\theta_1,\theta_2}\left(\Phi^{dec}\right)$. In particular, if the right hand-side of (18) is positive, $\Delta B(\theta)$ is negative.

We have argued earlier that if $\frac{\partial \Delta B}{\partial \theta_1}(\theta)\geq 0$, the right hand-side of (18) must be positive. From what precedes, this implies in turn implies that $\Delta B(\theta)<0$. A symmetric argument yields that for $\theta_1<\theta_2$, $\frac{\partial \Delta B}{\partial \theta_1}(\theta)\leq 0$ implies $\Delta B(\theta)<0$. Finally as $\Delta B(\theta)\leq 0$ whenever $\theta_1=\theta_2$, this implies that $\Delta B(\theta)\leq 0$ for all $\theta\in\Theta^N$, the inequality being strict whenever $\theta_1\neq\theta_2$.

7.4 Balanced Transfers

We show how propositions 3 and 5 change if we extend "quasi-linearly" our environment to allow for balanced payments. Observe first that since transfers are additively separable from the type in the utility function, the derivative of the utility function w.r.t. the type are not affected by the transfers. Therefore, the envelope theorems in lemmas 1 and 2. Since transfers are balanced, they cancel out in the Benthamite criterion and one can easily check that proposition 5 and its proof are unaffected. The envelope theorem in proposition 3 is unchanged as well. But the conclusion that the mechanism can be decomposed as $\Phi(\theta)=(\Phi_1(\theta_1),\Phi_2(\theta_2))$ implies an impossibility, since such a mechanism cannot maximize the sum of utilities, as efficiency requires in a transferable setup.

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