1. Appendix for "Slotting Allowances and New Product Introductions"

Proof of Lemma 1

The retailer's objective function is given by:

$$\max_{p,e} \pi^R(p,e) = (p-w)(\tau + f(e) - \beta p) - e.$$

The first order conditions yield

$$\begin{aligned} p^* &= \frac{\tau + f(e) + \beta w}{2\beta} \\ f'(e) &= \frac{2\beta}{\tau + f(e) - \beta w}. \end{aligned}$$

We assume that $-2\beta(p-w)f''(e) - [f'(e)]^2 > 0$ so second order conditions hold.

Totally differentiating the first order conditions and solving simultaneously, we have

$$\frac{\partial e}{\partial w} = \frac{3\beta f'(e)}{[f'(e)]^2 + 2\beta(p-w)f''(e)} < 0 \tag{1.1}$$

$$\frac{\partial p}{\partial w} = \frac{[f'(e)]^2 + \beta(p-w)f''(e)}{[f'(e)]^2 + 2\beta(p-w)f''(e)}. \tag{1.2}$$

Since the denominator in (1.2) is negative, $[f'(e)]^2 + \beta(p-w)f''(e)$ must be less than zero for p^* to be increasing in w. Note that this automatically implies that the second order conditions for maximization hold. We can similarly show that

$$\begin{aligned} \frac{\partial e}{\partial \tau} &= \frac{-f'(e)}{[f'(e)]^2 + 2\beta(p-w)f''(e)} > 0 \\ \frac{\partial p}{\partial \tau} &= \frac{f''(e)(p-w)}{[f'(e)]^2 + 2\beta(p-w)f''(e)} > 0. \end{aligned}$$

Part (3) of the lemma follows from the fact that if A were not restricted to being non-negative, the optimal contract would be $w = c$ and

$$A = -\frac{(\tau + f(e) - \beta c)^2}{4\beta} + e.$$

In other words, A becomes a franchise fee.

For part (4) of the lemma, note that if the participation constraint binds, the optimal full information wholesale price w_F is found from:

$$\frac{(\tau+f(e)-\beta w_F)^2}{4\beta}-(K+e)=0$$

yielding $w_F=\left(\tau+f(e)-2\sqrt{\beta\,(K+e)}\right)/\beta$. The wholesale price is clearly decreasing in K, and manufacturer profits must also fall since the wholesale price must be below its unconstrained level.

Proof of Proposition 2

Given that $f(e)=\alpha\sqrt{e}$, the retailer's objective is given by:

$$\pi^R(p,e)=(p-w)(\tau+\alpha\sqrt{e}-\beta p)-e.$$

It straightforward to show the following:

$$\begin{aligned} e^* &= \frac{\alpha^2\,(\tau-\beta w)^2}{(4\beta-\alpha^2)^2} \\ p^* &= \frac{2\tau+w\,(2\beta-\alpha^2)}{4\beta-\alpha^2} \\ \pi^R\,(w) &= \pi^R(p^*,e^*)=\frac{(\tau-\beta w)^2}{4\beta-\alpha^2}. \end{aligned}$$

In addition, the signaling constraint (9) can be written as

$$(w-c)\left(L+\tfrac{\alpha^2(H-\beta w)}{4\beta-\alpha^2}-\tfrac{\beta\left(2\tau+w\left(2\beta-\alpha^2\right)\right)}{4\beta-\alpha^2}\right)-A<0. \tag{1.3}$$

For the first part of the proposition, (1.3) binds at $w_1=\frac{H\left(\alpha^2-2\beta\right)+L\left(4\beta-\alpha^2\right)}{2\beta^2}$ without the payment of a slotting allowance. As $\pi^R\,(w_1)=K^*$, the retailer's participation constraint will not bind if $K\le K^*$. The high demand manufacturer's profits for a given (w,A) are:

$$\Pi^H\,(w,A)=\frac{2\beta\,(w-c)\,(H-\beta w)}{4\beta-\alpha^2}-A.$$

Substituting $(w_1, 0)$ yields the expression for the manufacturer's profits.

For $K > K^*$, Proposition 1 gives that both the signaling and participation constraints must bind. From the latter, the equilibrium value of the slotting allowance must be $A_2 = K - \pi^R(w_2)$. We can then write (1.3) as:

$$-w^2\frac{\beta^2}{4\beta-\alpha^2} - w\left((H-L)\left(4\beta-\alpha^2\right) - 2\beta^2 c\right) + \frac{H^2 - Hc\left(\alpha^2 - 2\beta\right)}{4\beta-\alpha^2} - Lc - K \leq 0. \tag{1.4}$$

Solving for the wholesale price such that the (1.4) binds yields w_2. One can then derive the slotting allowance from the participation constraint as described above. The retailer's profits are fixed at K by construction. The manufacturer's profits are founds from $\Pi^H(w_2, A_2)$.

The comparative statics of part (3) follow from simple differentiation.

Proof of Proposition 3

The proof of parts (1) to (3) are essentially identical to corresponding parts of Proposition 1. If the retailer participation constraint does not bind, one can use an argument similar to the one employed Proposition 1 to show that no slotting allowance is offered to show that no wasteful advertising is used. Again the manufacturer charges $w_1 = \frac{2L-H+f(e)}{\beta}$. If participation constraint binds, the Lagrangian of the manufacturer's problem is

$$\begin{aligned}
\mathcal{L} &= \frac{(w-c)(H+f(e)-\beta w)}{2} - D \\
&+ \lambda\left\{\frac{(H+f(e)-\beta w)^2}{4\beta} - (K+e)\right\} \\
&+ \mu\left[D - \frac{(w-c)(2L-H+f(e)-\beta w)}{2}\right].
\end{aligned}$$

From the Kuhn-Tucker conditions, we have

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial D} &= -1+\mu = 0 \quad \Rightarrow \quad \mu = 1 \\
\frac{\partial\mathcal{L}}{\partial w} &= \frac{1}{2}\left[H + f(e) - \beta w + (w-c)\left(f'(e)\frac{\partial e}{\partial w} - \beta\right)\right] \\
&\quad -\frac{\mu}{2}\left[2L - H + f(e) - \beta w + (w-c)\left(f'(e)\frac{\partial e}{\partial w} - \beta\right)\right] \\
&\quad -\frac{\lambda}{2}(H + f(e) - \beta w) = 0,
\end{aligned}$$

which implies

$$\lambda = \frac{2(H-L)}{H+f(e)-\beta w}.$$

The wholesale price can be obtained from participation constraints as

$$w_R = H + f(e) - 2\sqrt{\beta(K+e)}.$$

and the optimal dissipative advertising level is

$$\begin{aligned} D^* &= \frac{(w_R - c)(2L - H + f(e) - \beta w_c)}{2} \\ &= \frac{\left(H + f(e) - 2\sqrt{\beta(K+e)}\right)\left(2L - 2\sqrt{\beta(K+e)}\right)}{2}. \end{aligned}$$

For part (4), we note that if retailer participation constraint does not bind, the solutions to P1 and P2 are identical and manufacturer is indifferent. If the participation constraint does bind in P2, then a contract with w_R and slotting allowance of $A = D^*$ is feasible for P1. However, it is not optimal. Hence, the manufacturer would strictly prefer paying a slotting allowance to dissipative advertising.