

Intermediary Asset Pricing

Zhiguo He Arvind Krishnamurthy *

February 27, 2012

Abstract

We model the dynamics of risk premia during crises in asset markets where the marginal investor is a financial intermediary. Intermediaries face an equity capital constraint. Risk premia rise when the constraint binds, reflecting the capital scarcity. The calibrated model matches the nonlinearity of risk premia during crises, and the speed of reversion in risk premia from a crisis back to pre-crisis levels. We evaluate the effect of three government policies: reducing intermediaries borrowing costs, injecting equity capital, and purchasing distressed assets. Injecting equity capital is particularly effective because it alleviates the equity capital constraint that drives the model's crisis.

JEL Codes: G12, G2, E44

Keywords: Liquidity, LTCM Crisis, Subprime Crisis, Delegated Portfolio Management, Financial Institutions, Asset Pricing, Collateral, Credit.

*Respectively: University of Chicago, Booth School of Business; Northwestern University, Kellogg School of Management and NBER. E-mails: zhiguo-he@chicagobooth.edu, a-krishnamurthy@northwestern.edu. We thank Patrick Bolton, Markus Brunnermeier, Doug Diamond, Andrea Eisfeldt, Vadim Linetsky, Mark Loewenstein, Pablo Kurlat, Annette Vissing-Jorgensen, Amir Sufi, Neng Wang, Hongjun Yan and seminar participants at UC-Berkeley, Boston University, UCSB-LAEF conference, University of Chicago, Columbia University, ESSFM Gerzensee, FDIC, University of Maryland, NBER Asset Pricing, NBER Monetary Economics, NBER EFG, NY Fed, SF Fed, and Yale for helpful comments.

1 Introduction

The performance of many asset markets – e.g., prices of mortgage-backed securities, corporate bonds, etc. – depend on the financial health of the intermediary sector, broadly defined to include traditional commercial banks as well as investment banks and hedge funds. The 2007-2009 subprime crisis and the 1998 hedge fund crisis are two compelling data points in support of this claim.¹ However, traditional approaches to asset pricing ignore intermediation by invoking the assumption that intermediaries’ actions reflect the preferences of their client-investors. With this assumption, the traditional approach treats intermediaries as a “veil,” and instead posits that a representative household is marginal in pricing all assets. Thus, the pricing kernel for the S&P500 stock index is the same as the pricing kernel for mortgage-backed securities. Yet, many crises, such as the subprime crisis and the 1998 episode, play out primarily in the more complex securities that are the province of the intermediary sector. The traditional approach cannot speak to this relationship between financial intermediaries and asset prices. It sheds no light on why “intermediary capital” is important for asset market equilibrium. It also does not allow for a meaningful analysis of the policy actions, such as increasing intermediaries’ equity capital or discount window lending, which are commonly considered during crises.

We offer a framework to address these issues. We develop a model in which the intermediary sector is not a veil, and in which its capital plays an important role in determining asset market equilibrium. We calibrate the model to data on the intermediation sector and show that the model performs well in replicating asset market behavior during crises.

The striking feature of financial crises is the sudden and dramatic increase of risk premia. For example, in the hedge fund crisis of the fall of 1998, many credit spreads and mortgage-backed security spreads doubled from their pre-crisis levels. Our baseline calibration can replicate this dramatic behavior. When intermediary capital is low, losses within the intermediary sector have significant effects on risk premia. However, when capital is high, losses have little to no effect on risk premia. The asymmetry in our model captures the non-linearity that is present in asset market crises. Simulating the model, we find that the average risk premium

¹There is a growing body of empirical evidence documenting the effects of intermediation constraints (such as capital or collateral constraints) on asset prices. These studies include, research on mortgage-backed securities (Gabaix, Krishnamurthy, and Vigneron, 2005), corporate bonds (Collin-Dufresne, Goldstein, and Martin, 2001), default swaps (Berndt, et. al., 2004), catastrophe insurance (Froot and O’Connell, 1999), and index options (Bates, 2003; Garleanu, Pedersen, and Poteshman, 2005). Adrian, Etula, and Muir (2010) show that an intermediary pricing kernel based on intermediary balance sheet information can explain the cross-section of asset returns.

when intermediaries' capital constraint is slack is 3.07%. Using this number to reflect a pre-crisis normal level, we find that the probability of the risk premium exceeding 6%, which is about twice the "normal" level, is 1.33%.

Another important feature of financial crises is the pattern of recovery of spreads. In the 1998 crisis, most spreads took about 10 months to halve from their crisis-peak levels to pre-crisis levels. In the subprime crisis, the half-life of most bond market spreads was about 6 months. As we discuss later in the paper, half-lives for recovery of between 6 months and extending over a year have been documented in a variety of asset markets and crisis situations. We note that these types of recovery patterns are an order of magnitude slower than the daily mean reversion patterns documented in the market microstructure literature (e.g., Campbell, Grossman, and Wang, 1993). A common wisdom among many observers is that this recovery reflects the slow movement of capital into the affected markets (Froot and O'Connell, 1999, Berndt, et. al., 2004, Mitchell, Pedersen, and Pulvino, 2007, Duffie and Strulovici, 2010). Our baseline calibration of the model can replicate these speeds of capital movement. We show that simulating the model starting from an extreme crisis state (risk premium of 12%), the half-life of the risk premium back to the unconditional average risk premium is 8 months. From a risk premium of 10%, the half-life is 11 months.

We also use the model as a laboratory to quantitatively evaluate government policies. Beginning from an extreme crisis state with risk premium of 12%, we trace the crisis recovery path conditional on three government policies: (1) Infusing equity capital into the intermediaries during a crisis; (2) Lowering borrowing rates to the intermediary, as with a decrease in the central bank's discount rate; and, (3) Direct purchase of the risky asset by the government, financed by debt issuance and taxation of households. These three policies are chosen because they are among those undertaken by central banks in practice. In comparing \$205bn of equity infusion to \$1.8tn of risky asset purchase, we find that the equity infusion is far more effective in reducing the risk premium. This occurs in our model because the friction in the model is an equity capital constraint. Thus infusing equity capital attacks the problem at its heart. The interest rate policy is also highly effective, uniformly increasing the speed of crisis recovery.

The contribution of our paper is to work out an equilibrium model of intermediation that is dynamic, parsimonious, and can be realistically calibrated. The paper is related to a large literature in banking studying disintermediation and crises (see Diamond and Dybvig (1983), Holmstrom and Tirole (1997), Diamond (1997), and Diamond and Rajan (2005)). We differ from this literature in that our model is dynamic, while much of this literature is static. The paper is also related to the literature in macroeconomics studying effects of

collateral fluctuations on aggregate activity (Kiyotaki and Moore (1997)). In much of the macro literature, equilibrium is derived by log-linearizing around the steady-state. As a result, there is almost no variation in equilibrium risk premia, which does not allow the models to speak to the behavior of risk premia in crises. We solve a fully stochastic model that better explains how risk premia varies as a function of intermediary capital. In Bernanke and Gertler (1989), credit spreads are linked to the net worth of the entrepreneurial sector. However, the action in credit spreads is due to default risk and bankruptcy costs, rather than due to changes in economic risk premia. Brunnermeier and Sannikov (2010) is another recent paper that develops a macroeconomic model that is fully stochastic and links intermediaries' financing position to risk premia. Our paper is also related to the literature on limits to arbitrage studying how impediments to arbitrageurs' trading strategies may affect equilibrium asset prices (Shleifer and Vishny (1997)). One part of this literature explores the effects of margin or debt constraints for asset prices and liquidity in dynamic models (see Aiyagari and Gertler (1999), Gromb and Vayanos (2002), Geanakoplos and Fostel (2008), Adrian and Shin (2010), and Brunnermeier and Pedersen (2008)). Our paper shares many objectives and features of these models. The principal difference is that we study a constraint on raising equity capital, while these papers study a constraint on raising debt financing. Xiong (2001) and Kyle and Xiong (2001) model the effect of arbitrageur capital on asset prices by studying an arbitrageur with log preferences, where risk aversion decreases with wealth. The effects that arise in our model are qualitatively similar to these papers. An advantage of our paper is that intermediaries and their equity capital are explicitly modeled allowing our paper to better articulate the role of intermediaries in crises.² Finally, many of our asset pricing results come from assuming that some markets are segmented and that households can only trade in these markets by accessing intermediaries. Our paper is related to the literature on asset pricing with segmented markets (see Allen and Gale, 1994, Alvarez, Atkeson, and Kehoe, 2002, and Edmond and Weill, 2009).³

Our paper is closely related to a companion paper, He and Krishnamurthy (2011). We solve for the optimal intermediation contract in that paper, while we assume the (same) form of contract in the current analysis. That paper also solves for the equilibrium asset prices in closed form, while we rely on numerical solutions in the present paper. On the other hand, that paper has a degenerate steady state distribution which does not allow for a meaningful

²The paper is also related to Vayanos (2005) who studies the effect of an open-ending friction on asset-demand by intermediaries. We study a capital constraint rather than an open-ending friction.

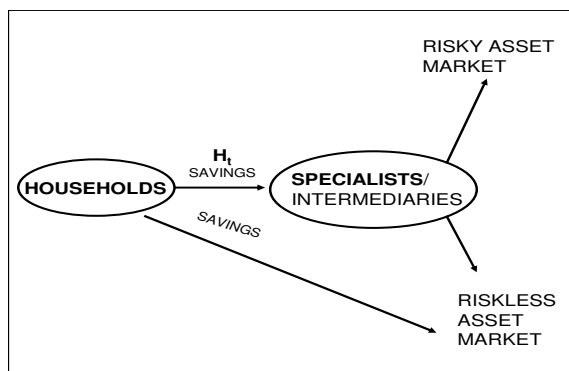
³Our model is also related to the asset pricing literature with heterogeneous agents (see Dumas (1989) and Wang (1996)).

simulation or the other quantitative exercises that we perform in the present paper. In addition, the present paper models households with labor income and an intermediation sector which always carries some leverage. Both aspects are important in realistically calibrating the model. Apart from these differences, the analysis in He and Krishnamurthy (2011) provides theoretical underpinnings for some of the assumptions we make in this paper.

The paper is organized as follows. Sections 2 and 3 outline the model and its solution. Section 4 explains how we calibrate the model. Section 5 presents the results of the crisis calibration. Section 6 studies policy actions. Section 7 concludes and is followed by an Appendix with details of the model solution.

2 The Model: Intermediation and Asset Prices

Figure 1: The Economy



This figure depicts the agents in the economy and their investment opportunities.

Figure 1 lays out the building blocks of our model. There is a risky asset that represents complex assets where investment requires some sophistication. In our calibration, we match the risky asset to the market for mortgage-backed securities, as a representative large asset class that fits this description. Investment in the mortgage-backed securities market is dominated by financial institutions rather than households, and sophisticated prepayment modeling is an important part of the investment strategy. The calibration is also appropriate for analyzing the financial crisis that began in 2007, where mortgage-backed securities have a prominent role.

There are two groups of agents in the economy, households and specialists. We assume that **households** cannot invest directly in the risky asset market. There is limited market participation, as in Mankiw and Zeldes (1991), Allen and Gale (1994), Basak and Cuoco (1998), or Vissing-Jorgensen (2002). **Specialists** have the knowledge to invest in the risky assets, and unlike in the limited market participation literature, the specialists can invest in the risky asset on behalf of the households. This investment conduit is the intermediary of our model. In our model, the households demand intermediation services while the specialists supply these services. We are centrally interested in describing how this intermediation relationship affects and is affected by the market equilibrium for the “intermediated” risky asset.

We assume that if the household does not invest in the intermediary, it can only invest in a riskless short-term bond. This is clearly counterfactual (i.e. households invest in the S&P 500 index), but simplifies the analysis considerably.

Households thus face a portfolio choice decision of allocating funds between the intermediaries and the riskless bond. The intermediaries accept H_t of the household funds and then allocate their total funds under management between the risky asset and the riskless bond. We elaborate on each of the elements of the model in the next sections.

2.1 Assets

The assets are modeled as in the Lucas (1978) tree economy. The economy is infinite-horizon, continuous-time, and has a single perishable consumption good, which we will use as the numeraire. We normalize the total supply of intermediated risky assets to be one unit. The riskless bond is in zero net supply and can be invested in by both households and specialists.

The risky asset pays a dividend of D_t per unit time, where $\{D_t\}$ follows a geometric Brownian motion (GBM),

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t \quad \text{given } D_0. \quad (1)$$

$g > 0$ and $\sigma > 0$ are constants. Throughout this paper $\{Z_t\}$ is a standard Brownian motion on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. We denote the processes $\{P_t\}$ and $\{r_t\}$ as the risky asset price and interest rate processes, respectively. We also define the total return on the risky asset as,

$$dR_t = \frac{D_t dt + dP_t}{P_t}. \quad (2)$$

2.2 Specialists and intermediation

There is a unit mass of identical specialists who manage the intermediaries in which the households invest. The specialists represent the insiders/decision-makers of a bank, hedge fund, or mutual fund. We collapse all of an intermediary's insiders into a single agent, following the device of modeling entrepreneur-managers of firms in the corporate finance literature (e.g. Holmstrom and Tirole, 1997). The specialists are infinitely-lived and maximize an objective function,

$$E \left[\int_0^\infty e^{-\rho t} u(c_t) dt \right] \quad \rho > 0; \quad (3)$$

where c_t is the date t consumption rate of the specialist. We consider a CRRA instantaneous utility function with parameter γ for the specialists, $u(c_t) = \frac{1}{1-\gamma} c_t^{1-\gamma}$.

Each specialist manages one intermediary. At every t , each specialist is randomly matched with a household to form an intermediary. These interactions occur instantaneously and result in a continuum of (identical) bilateral relationships.⁴ The household allocates some funds H_t to the intermediary. We denote the date t wealth of specialists as w_t and assume that this is wholly invested in the intermediary. Specialists then execute trades for the intermediary in a Walrasian risky asset and bond market, and the household trades in only the bond market. At $t + dt$ the match is broken, and the intermediation market repeats itself.

Consider one of the intermediary relationships between specialist and household. The specialist manages an intermediary whose total capital is the sum of the specialist's wealth, w_t , and the wealth that the household allocates to the intermediary, H_t . The specialist makes all investment decisions on this capital and faces no portfolio restrictions in buying or short-selling either the risky asset or the riskless bond. Suppose that the specialist chooses to invest a fraction α_t^I of the portfolio in the risky asset and $1 - \alpha_t^I$ in the riskless asset. Then, the return delivered by the intermediary is,

$$\widetilde{dR}_t = r_t dt + \alpha_t^I (dR_t - r_t dt), \quad (4)$$

⁴Why the matching structure instead of a Walrasian intermediation market? We study the Walrasian case in He and Krishnamurthy (2011) and find that when intermediation is supply constrained, specialists charge the households a fee for managing the intermediary that depends on the tightness of the intermediation constraint. In particular the fees rise during financial crises because households see through the intermediaries and determine that economic risk premia are high and as a result compete to give funds to the intermediary to manage. This is clearly counterfactual. We find it unnatural to assume that households who do not participate in the risky asset markets, likely because of informational costs, would be so sophisticated as to be able to compute expected returns as a function of the state. Thus in the current setting, to keep the model realistic, we adopt the matching structure.

where dR_t , defined in (2), is the total return on the risky asset. In this notation, $\alpha_t^I > 1$ means that the specialist invests more than 100% of the intermediary equity capital in the risky assets. In other words, the specialist borrows $(\alpha_t^I - 1)(w_t + H_t)$ via the riskless short-term bond market and makes a leveraged investment in the risky asset.

2.3 Capital Constraint

The key assumption of our model is that the household is unwilling to invest more than mw_t in the equity of the intermediary, where $m > 0$ is a constant that parameterizes the financial constraint. If the specialist has one dollar of wealth invested in the equity of the intermediary, the household will only invest up to m dollars of his own wealth in the intermediary. The capital constraint implies that the *supply of intermediation* facing a household is at most,

$$H_t \leq mw_t. \tag{5}$$

If either m is small or w_t is small, the household's ability to indirectly participate in the risky asset market will be restricted with equilibrium effects on risk premia and asset prices.

This type of constraint linking “net worth” and external financing is by now standard fare in the literature on financial frictions (e.g., Kiyotaki and Moore (1997)) and can be rationalized by a variety of agency or informational frictions.⁵ One point worth noting is that the constraint in our model is on the intermediary's ability to raise outside equity financing rather than outside debt financing. In Kiyotaki and Moore (1997), for example, firms cannot raise any equity financing, while they can raise some debt financing subject to a constraint. An easy way to see that our model describe a constraint on raising equity financing is to observe that both the household and specialist receive the return \widetilde{dR}_t (see (4)) on their contributions to the intermediary and hence both hold equity investments in the intermediary. As noted in the introduction, our focus is on an equity capital constraint rather than a debt constraint. Additionally, we place no constraint on the intermediary's ability to borrow in the debt market; that is, total debt financing is equal to $\alpha_t^I - 1$ and we place no constraint on this quantity.

When we calibrate the model, we interpret the equity capital requirement in one of two ways. First, the managers of a hedge fund typically have much of their wealth tied up in terms of the returns of the hedge fund, both because these managers invest some of their wealth in the hedge fund and also because they earn future income based on the returns of the fund.

⁵In a setting that is close to this paper, He and Krishnamurthy (2011) derive this capital constraint by assuming moral hazard by the specialist.

Our constraint is that outside investors require that the manager’s stake be sufficient to align incentives. If a hedge fund loses a lot of money then the stake of the managers of the hedge fund will be depleted. In this case, investors will be reluctant to contribute their own capital to the hedge fund, fearing mismanagement or further losses. A hedge fund “capital shock” is one phenomena that we can capture with our model. In our calibration, we interpret the 20% of returns that are typically paid to the hedge fund manager in terms of an incentive contract and the model’s m . Second, the ownership stake interpretation also applies more broadly to the banking sector. Holderness, Kroszner and Sheehan (1999) report that the mean equity ownership of officers and directors in the Finance, Insurance, and Real Estate sector was 17.4% in 1995. This stake can also be related to the fraction of the intermediary that the specialist owns, $\frac{w_t}{w_t+H_t}$.⁶

The specialist chooses his consumption rate and the portfolio decision of the intermediary to solve,

$$\max_{\{c_t, \alpha_t^I\}} E \left[\int_0^\infty e^{-\rho t} u(c_t) dt \right] \quad s.t. \quad dw_t = -c_t dt + w_t r_t dt + w_t \left(\widetilde{dR}_t(\alpha_t^I) - r_t dt \right), \quad (6)$$

where the intermediary return $\widetilde{dR}_t(\alpha_t^I)$ as a function of intermediary portfolio choice α_t^I is given by (4). We can also rewrite the budget constraint in terms of the underlying return:

$$dw_t = -c_t dt + w_t r_t dt + \alpha_t^I w_t (dR_t - r_t dt). \quad (7)$$

Note that the intermediary’s portfolio choice of α_t^I is effectively the specialist’s portfolio share in the risky asset.

2.4 Households: The demand for intermediation

We model the household sector as an overlapping generation (OG) of agents. This keeps the decision problem of the household fairly simple.⁷ On the other hand, we enrich the

⁶Another interpretation, which is more in keeping with regularities in the mutual fund industry, is that the wealth of a specialist summarizes his past success in making investment decisions. Low wealth then reflects poor past performance by a mutual fund, which makes households reluctant to delegate investment decisions to the specialist. The relation between past performance and mutual fund flows is a well-documented empirical regularity (see, e.g., Warther (1995)). As w_t falls, reflecting poor past performance, investors reduce their portfolio allocation to the mutual fund. Shleifer and Vishny (1997) present a model with a similar feature: the supply of funds to an arbitrageur in their model is a function of the previous period’s return by the arbitrageur.

⁷Note the specialists are infinitely lived while households are modeled using the OG structure. As we will see, specialists play the key role in determining asset prices. Our modeling ensures that their choices reflect the

model to include household labor income and introduce heterogeneity within the household sector. Without household income it is possible to reach states where the household sector vanishes from the economy, rendering our analysis uninteresting (see Dumas (1989) for more on this problem in two-agent models). Heterogeneity within the household sector is useful in realistically calibrating the model.

For the sake of clarity in explaining the OG environment in a continuous time model, we index time as $t, t + \delta, t + 2\delta, \dots$ and consider the continuous time limit when δ is of order dt . A unit mass of generation t agents are born with wealth w_t^h and live in periods t and $t + \delta$. They maximize utility:

$$\rho\delta \ln c_t^h + (1 - \rho\delta) E_t[\ln w_{t+\delta}^h]. \quad (8)$$

c_t^h is the household's consumption rate in period t and $w_{t+\delta}^h$ is a bequest for generation $t + \delta$. Note that both utility and bequest functions are logarithmic.

In addition to wealth of w_t^h , we assume that generation t households receive labor income at date t of $lD_t\delta$. Here, $l > 0$ is a constant, and recall that D_t is the dividend rate on the risky asset at time t . Labor income is assumed proportional to dividends in order to preserve some useful homogeneity properties of the equilibrium.

It is easy to verify that in the continuous time limit, i.e., when $\delta \rightarrow dt$, the household's consumption rule is,

$$c_t^h = \rho w_t^h. \quad (9)$$

In particular, note that the labor income does not affect the consumption rule because the labor income flow is of order dt . Interpreting $\rho > 0$ as the household's rate of time preference, we note that this is the standard consumption rule for logarithmic agents. The household is "myopic" and his rule does not depend on his investment opportunity set.

A household invests its wealth from t to $t+\delta$ in financial assets. As noted earlier, households are not directly able to save in the risky asset and can only directly access the riskless bond market. We assume that the household can choose any positive level of bond holdings when saving in the riskless bond (note that short-selling of the bond is ruled out). The household must use an intermediary when accessing the risky asset market.

We consider a further degree of heterogeneity in the intermediation investment restriction. We assume that a fraction λ of the households can ever only invest in the riskless bond. The remaining fraction, $1 - \lambda$, may enter the intermediation market and save a fraction of their

forward-looking dynamics of the economy. We treat households in a simpler manner for tractability reasons. We deem the cost of the simplification to be low since households play a secondary role in the model.

wealth with intermediaries which indirectly invest in the risky asset on their behalf. We refer to the former as “debt households” and the latter as “risky asset households.”⁸

The heterogeneity among households is realistic. Clearly, there are many households that only save in a bank account. In the literature cited earlier on limited market participation, all households are “debt households.” The demand for intermediation in our model stems from the risky asset households. Introducing this degree of heterogeneity is not necessary to illustrate the main results of the model but allows for a better model calibration.

To summarize, a debt and risky asset household are born at generation t with wealth of w_t^h . The households receive labor income, choose consumption, and make savings decisions, respecting the restriction on their investment options. The debt household’s consumption decision, given wealth of w_t^h , is described by (9). The savings decision is to invest w_t^h in the bond market at the interest rate r_t . The risky asset household with wealth $(1 - \lambda)w_t^h$ will consume according to (9), and decide how much to allocate to intermediaries. Denote $\alpha_t^h \in [0, 1]$ for the fraction of his wealth invested in the intermediary and recall that the intermediary’s return is \widetilde{dR}_t in (4). The remaining $1 - \alpha_t^h$ of the risky asset household wealth is invested in the riskless bond to earn the interest rate of $r_t dt$. Given the log objective function in (8), the risky asset household chooses α_t^h to solve (where we have taken the limit as $\delta \rightarrow dt$),

$$\max_{\alpha_t^h \in [0, 1]} \alpha_t^h E_t[\widetilde{dR}_t - r_t dt] - \frac{1}{2} (\alpha_t^h)^2 \text{Var}_t[\widetilde{dR}_t - r_t dt] \quad s.t. \quad \alpha_t^h (1 - \lambda) w_t^h \equiv H_t \leq m w_t. \quad (10)$$

The constraint here corresponds to the intermediation constraint (5) that we have discussed earlier.

Given the decisions by the debt household and the risky asset household, the evolution of w_t^h across generations is described by,

$$dw_t^h = (lD_t - \rho w_t^h) dt + w_t^h r_t dt + \alpha_t^h (1 - \lambda) w_t^h (\widetilde{dR}_t - r_t dt). \quad (11)$$

⁸The wealth of the debt household and risky asset household evolve differently between t and $t + \delta$. We assume that this wealth is pooled together and distributed equally to all agents of generation $t + \delta$. The latter assumption ensures that we do not need to keep track of the distribution of wealth over the households when solving for the equilibrium of the economy.

2.5 Equilibrium

Definition 1 *An equilibrium is a set of price processes $\{P_t\}$ and $\{r_t\}$, and decisions $\{c_t, c_t^h, \alpha_t^I, \alpha_t^h\}$ such that,*

1. *Given the price processes, decisions solve the consumption-savings problems of the debt household, the risky asset household (10) and the specialist (6);*
2. *Decisions satisfy the intermediation constraint of (5);*
3. *The risky asset market clears:*

$$\frac{\alpha_t^I(w_t + \alpha_t^h(1 - \lambda)w_t^h)}{P_t} = 1; \quad (12)$$

4. *The goods market clears:*

$$c_t + c_t^h = D_t(1 + l). \quad (13)$$

Given market clearing in risky asset and goods markets, the bond market clears by Walras' law. The market clearing condition for the risky asset market reflects that the intermediary is the only direct holder of risky assets and has total (equity) funds under management of $w_t + \alpha_t^h(1 - \lambda)w_t^h$, and the total holding of risky asset by the intermediary must equal the supply of risky assets.

Finally, an equilibrium relation that proves useful when deriving the solution is that,

$$w_t + w_t^h = P_t.$$

That is, since bonds are in zero net supply, the wealth of specialists and households must sum to the value of the risky asset.

3 Solution

We outline the main steps in deriving the solution in this section, highlighting the economic mechanism linking intermediary equity capital and risk premia for the special case of log utility. Detailed derivations are in Appendix A.

3.1 Equilibrium risk premium

We look for a stationary Markov equilibrium where the state variables are (x_t, D_t) , where $x_t \equiv \frac{w_t}{P_t} \in (0, 1)$ is the fraction of wealth in the economy owned by the specialists. We refer to the fraction x_t as specialist capital. As standard in any CRRA/GBM economy, our economy is homogeneous in dividends D_t . We conjecture that the equilibrium risky asset price is,

$$P_t = D_t p(x_t), \quad (14)$$

where $p(x)$ is the price/dividend ratio of the risky asset.

While the household faces investment restrictions on his portfolio choices, the specialist (intermediary) is unconstrained in his portfolio choices. This important observation implies that the specialist is always the marginal investor in determining asset prices, while the household may not be. Standard arguments then tell us that we can express the pricing kernel in terms of the specialist's equilibrium consumption process. Optimality for the specialist gives us the standard consumption-based asset pricing relations (Euler equation):⁹

$$-\rho dt - \gamma E_t \left[\frac{dc_t}{c_t} \right] + \frac{1}{2} \gamma (\gamma + 1) \text{Var}_t \left[\frac{dc_t}{c_t} \right] + E_t [dR_t] = \gamma \text{Cov}_t \left[\frac{dc_t}{c_t}, dR_t \right]; \quad (15)$$

and for interest rate, we have

$$r_t dt = \rho dt + \gamma E_t \left[\frac{dc_t}{c_t} \right] - \frac{\gamma (\gamma + 1)}{2} \text{Var}_t \left[\frac{dc_t}{c_t} \right]. \quad (16)$$

Combining these equations gives an expression for the risk premium,

$$E_t [dR_t] - r_t dt = \gamma \text{Cov}_t \left[\frac{dc_t}{c_t}, dR_t \right].$$

The risk premium depends on the covariance of the asset return with the specialist's consumption growth.

3.2 Log-utility special case

Consider the special case where $\gamma = 1$ as this case offers a clear characterization of the solution. With log utility, the consumption-to-wealth ratio is constant, implying that consumption

⁹The Euler equation is a necessary condition for optimality. In Appendix B, we prove sufficiency.

growth is equal to wealth growth:

$$E_t[dR_t] - r_t dt = Cov_t \left[\frac{dw_t}{w_t}, dR_t \right]. \quad (17)$$

The specialist's wealth growth is given in (7). The key term driving the return volatility of specialist wealth dw_t/w_t , is the specialist's leveraged exposure to the risky asset, $\alpha_t^I dR_t$. Combined with (17), this observation implies that,

$$E_t[dR_t] - r_t dt = \alpha_t^I Var_t[dR_t].$$

There are two terms affecting the risk premium: α_t^I is the intermediary's exposure to the risky asset, while $Var_t[dR_t]$ is the variance of returns. In our calibration, most of the action in the risk premium is driven by the exposure term rather than the variance. Therefore we consider the exposure term α_t^I in further detail.

Suppose that the equity capital constraint binds and the intermediaries raise total equity capital of $w_t + mw_t$. We refer to this case as being in the *constrained region*. Since all risky assets are held through the intermediary, the equilibrium market clearing condition (12) in the constrained region gives,

$$\alpha_t^{I,const}(w_t + mw_t) = P_t.$$

Rewriting, we find that,

$$\alpha_t^{I,const} = \frac{1}{x_t} \frac{1}{1+m}. \quad (18)$$

Thus as specialist capital, x_t , falls in the constrained region, the risk premium rises. Moreover, when m is larger so that the specialist is able to raise more equity capital from the household for a given amount of his own equity stake, the risk premium effect is dampened.

Next consider the *unconstrained region*. Total capital of the intermediary sector is equal to the specialist's capital plus the risky asset household's capital contribution to the intermediary sector. This gives the market clearing condition for the risky asset:

$$\alpha_t^{I,unconst}(w_t + (1-\lambda)w_t^h \alpha_t^h) = P_t.$$

We make an assumption that implies that $\alpha_t^h = 1$:

Parameter Assumption 1 *We focus on parameters of the model such that in the absence of any portfolio restrictions, the risky asset household will choose to have at least 100% of his*

wealth invested in the intermediary, i.e., $\alpha_t^h = 1$.

Although we are unable to provide a precise mathematical statement for this parameter restriction, in our calibration it appears that $\gamma \geq 1$ is a sufficient condition.¹⁰ We then have that,

$$\alpha_t^{I,unconst} = \frac{1}{x_t + (1 - \lambda)(1 - x_t)} = \frac{1}{1 - \lambda(1 - x_t)}. \quad (19)$$

In the case where $\lambda = 0$ (no debt households in the economy), $\alpha_t^{I,unconst}$ is constant and equal to one.

This result implies that the risk premium in the unconstrained region is constant. In contrast, as shown above, the risk premium rises in the constrained region when the intermediary capital falls. This pattern is the central economic feature of our model. There is an asymmetry whereby when the equity capital constraint binds, further reductions in specialist capital cause a large increase in risk premia.

For the case where $\lambda > 0$ which we consider in the calibration, the risk premium also rises in the unconstrained region because of a leverage effect. However, in our calibration, the latter effect is small compared to the effect in the constrained region. Analytically, we can see that the effect of falling x_t in the constrained region is more significant than that of falling x_t in the unconstrained region by looking at the ratio,

$$\frac{\alpha_t^{I,const}}{\alpha_t^{I,unconst}} = \frac{1}{1 + m} \left(1 + (1 - \lambda) \frac{1 - x_t}{x_t} \right). \quad (20)$$

If there was no capital constraint in our model, then $\alpha_t^{I,unconst}$ would describe the portfolio share as a function of x_t . Thus the ratio describes how much higher α_t^I is relative to this benchmark. The key term is $\frac{1-x_t}{x_t}$ which is the ratio of household wealth to specialist wealth. As specialist wealth falls relative to household wealth $\frac{1-x_t}{x_t}$ rises and we see that the equity capital constraint causes the risk premium to rise.

The model also has an amplification effect in the constrained region. Since $\alpha_t^{I,const}$ is high in the constrained region, the specialist has a large exposure to the risky asset. Then, a negative dividend shock translates to a large fall in x_t , which further increases the risk premium and lowers asset prices.

¹⁰Loosely speaking, if the specialist is weakly more risk averse than the household, the household will hold more risky assets than the specialist. But given market clearing in the risky asset market, the specialist always holds more than 100% of his wealth in the risky asset. Recall that we assume that the household cannot short bonds. Thus, the equity household allocates the maximum of 100% of his wealth to the intermediary.

3.3 $\gamma > 1$ case

For the general case where $\gamma > 1$, specialist consumption is not proportional to wealth and the simple characterization is no longer exact. We solve the model in a different manner in the general case. The market clearing condition for goods (from (13)) is,

$$c_t + c_t^h = D_t(1 + l).$$

Since the household with log utility sets consumption $c_t^h = \rho w_t^h = \rho(1 - x_t)P_t$, in equilibrium, the specialist consumes:

$$c_t = D_t(1 + l) - \rho(1 - x_t)P_t = D_t [(1 + l) - \rho(1 - x_t)p(x_t)]. \quad (21)$$

Using (21) and (14), we can express dR_t and $\frac{dc_t}{c_t}$ in terms of the price/dividend ratio $p(x)$ and its derivatives. We also need the drift and diffusion of x_t . To find these terms, note that $x_t = \frac{w_t}{P_t}$, and the specialist's wealth w_t evolves according to

$$dw_t = -c_t dt + w_t r_t dt + \alpha_t^I w_t (dR_t - r_t dt). \quad (22)$$

The key term driving the wealth evolution of the specialist is his portfolio exposure to the risky asset, i.e., α_t^I . Recall that in the previous section, we derive α_t^I for the constrained and the unconstrained regions as (18) and (19). The derivation is based on market clearing conditions and does not depend on the value of γ . Combining all of these results, we arrive at a differential equation that must be satisfied by $p(x)$ (see Appendix A).

The differential equation is solved for the constrained and unconstrained regions. The intermediation constraint binds at the point where $\frac{\alpha_t^{I, const}}{\alpha_t^{I, unconst}} = 1$ for (20). Solving, we find that the critical value of x^c at which the constraint binds is,

$$\frac{1}{x^c} = 1 + \frac{m}{1 - \lambda} \Leftrightarrow x^c = \frac{1 - \lambda}{1 - \lambda + m}. \quad (23)$$

When $x < x^c$, the intermediation constraint binds, while if $x > x^c$ the constraint does not bind. If m is high, then x^c is low, and hence the constraint binds for less of the state space. If the debt households fraction λ is high, there are less risky asset households looking to invest in the equity of the intermediation sector, and as a result the capital constraint binds for less of the state space.

3.4 Boundary Condition

In equilibrium the fraction of specialist capital x_t moves in the range $(0, 1)$. When $x_t \rightarrow 1$, the economy behaves as if comprised of specialists only. We derive an expression for the boundary condition for $p(1)$ in the Appendix A.3. On the other hand, as x_t approaches zero, the economy is comprised mostly of households. The boundary condition in this case is as follows. Consider the specialist's consumption in (21). We must have that specialist consumption approaches zero as x_t approaches zero.¹¹ Thus, using (14), we have

$$D(1 + l) = \rho P(x = 0, D) \quad \Rightarrow \quad p(0) = \frac{1 + l}{\rho}. \quad (24)$$

In Appendix B, we show that $x = 0$ is an entrance-no-exit boundary and that x_t never reaches $x = 0$.

4 Calibration

Table 1 provides data on the main intermediaries in the US economy. Households hold wealth through a variety of intermediaries including banks, retirement funds, mutual funds, and hedge funds.

4.1 Choice of m

The m of the model parameterizes the equity capital constraint of the intermediaries. We set m equal to 4, which matches both ownership data of banks and compensation data from hedge funds. Holderness, Kroszner and Sheehan (1999) report that the mean equity ownership of officers and directors in the Finance, Insurance, and Real Estate sector was 17.4% in 1995. This translates to an m of $4.7 (= \frac{1-0.174}{0.174})$. Hedge fund contracts typically pay the manager 20% of the fund's return in excess of a benchmark, plus 1 – 2% of funds under management (Fung and Hsieh, 2006). The choice of m dictates how much of the return of the intermediary goes to the specialist ($\frac{1}{1+m}$) and how much go to equity investors ($\frac{m}{1+m}$). A value of $m = 4$ implies that the specialist's share $1/5 = 20\%$. The 20% that is common in hedge fund contracts is an option contract so it is not a full equity stake as in our model, suggesting that perhaps we

¹¹In the argument for verification of optimality of the specialist's equilibrium strategy which is detailed in Appendix B, we see that this condition translates to the transversality condition for the specialist's budget equation. Therefore the boundary condition (24) is sufficient for the equilibrium presented in this paper to be well-defined.

Table 1: Intermediation Data (\$ Billions) ^a

Group	Assets	Debt	Debt/Assets
Commercial banks	11,800	10,401	0.88
S&L and Credit Unions	2,574	2,337	0.91
Property & Casualty Insurance	1,381	832	0.60
Life Insurance	4,950	4,662	0.94
Private Pensions	6,391	0	0.00
State & Local Ret Funds	3,216	0	0.00
Federal Ret Funds	1,197	0	0.00
Mutual Funds (excluding Money Funds)	7,829	0	0.00
Broker/Dealers	2,519	2,418	0.96
Hedge Funds	6,913	4,937	0.71

^a Most data are from the Flow of Funds March 2010 Level Tables, corresponding to the year 2007. The broker/dealer and hedge fund total assets are as computed in He, Khang, and Krishnamurthy (2010), who use data from SEC filings for the broker/dealer sector and data from Barclay's Hedge for the hedge fund sector. We assume that the average broker/dealer runs a leverage of 25, based on Adrian and Shin (2010). We assume the average hedge fund leverages up its capital base 3.5 times (taken from McGuire, Remolona and Tsatsaronis (2005))

should use a larger value of m . However, to balance this, note that the $1 - 2\%$ fee is on funds under management and therefore grows as the fund is successful and garners more inflows. We thus settle on a value of $m = 4$ as representative, in a linear scheme, of the payoff structure of the hedge fund.¹²

4.2 Choice of λ

The choice of m only plays a role in driving the intermediary sector's capital structure when the equity capital constraint binds. In particular, if we set $\lambda = 0$, then the leverage of the

¹²The m in our calibration applies to the entire intermediation sector, and as is evident in Table 1, there is functional heterogeneity across the modes of intermediation. In particular, it is not obvious what the m for the mutual fund or pension fund sector should be, which may lead one to worry about our choice of m based solely on considering the leveraged sector. Our justification for m is as follows. When the intermediation constraint (5) binds, losses among intermediaries lead households to reduce their equity exposure to these intermediaries. If the intermediaries scale down their asset holdings proportionately, the asset market will not clear – i.e. the intermediary sector's assets still have to be held in equilibrium. In the model, the equilibrium is one where the [identical] intermediaries take on debt and hold a riskier position in the asset. Asset prices are then set by the increased risk/leverage considerations of the intermediaries. In practice, if households withdraw money from mutual funds, then mutual funds do not take on debt. Rather, they reduce their holdings of financial assets and some other entity buys their financial assets. In practice, the other entity will be a trading desk at a bank or a hedge fund that temporarily provides liquidity to the mutual fund rather than a non-leveraged investor. Thus we see that to model asset price behavior we want m to correspond to the equity capital constraints of banks/hedge funds rather than features of the broad intermediary sector. This is because it is the marginal pricing condition of these intermediaries that is most relevant during a liquidation crisis.

intermediary sector will be zero when the equity capital constraint does not bind. This is likely counterfactual, as banks always carry debt, even during boom periods when capital constraints are likely slack. Across all of the intermediaries of Table 1, the Total Debt/Total Assets ratio in 2007 is 0.52. We assume that this was a period when the equity capital constraint was slack and choose λ to target the 0.52 number. We set $\lambda = 0.6$, which produces an unconditional average debt-to-asset ratio of 0.55, and a ratio of 0.50 in the unconstrained region.

4.3 σ and g

We calibrate the intermediated asset to the market for mortgage-backed securities (MBS) as a representative large intermediated asset class. The Securities Industry and Financial Markets Association (SIFMA) reports that the total outstanding MBS securities (Agency-backed MBS, private-label MBS, commercial MBS) totaled \$8.9tn in 2007. SIFMA reports that the outstanding amount of asset-backed securities (auto, credit card, etc.) totaled \$2.5tn in 2007. We are unaware of data that allow us to know precisely who holds these securities. However, the pattern of losses as reported by financial institutions in the subprime crisis, and most analyses of losses (e.g., see the IMF's Global Financial Stability Report of October 2008) suggests these securities are mostly held in intermediary portfolios.

The Barclays Capital U.S. MBS Index (formerly the Lehman Brothers U.S. MBS index) tracks the return on the universe of Agency-backed MBS from 1976 onwards. The annual standard deviation of the excess return of this index over the Treasury bill rate, using data from 1976 to 2008, is 8.1%. Note that this index measures the returns on Agency-backed MBS which is the least risky (although largest) segment of the MBS market. As another benchmark, the annual standard deviation of the excess return on Barclays index of commercial MBS over the period 1999 (i.e. inception of the index) to 2008 is 9.6%.

We choose σ to be 9%. With this choice, the standard deviation of the excess return on the intermediated asset in our model is 9.2%. This number is in the range between the low risk Agency MBS and the higher risk commercial MBS.

We choose $g = 2\%$ to reflect average per-capita growth in US GDP. We would expect that the payouts on mortgage assets should grow with the economy. The choice of g has a minor effect on results. On the other hand, σ is critical because it is closely related to the amount of risk borne by the specialist and the volatility of the intermediary pricing kernel.

Table 2: Parameters and Targets

Panel A: Intermediation			
m	Intermediation multiplier	4	Compensation data for financial managers
λ	Debt ratio	0.6	Average debt/assets of intermediary sector
Panel B: Preferences and Cashflows			
g	Dividend growth	2%	Growth of economy
σ	Dividend volatility	9%	Volatility of MBS portfolio
ρ	Time discount rate	4%	Short-term interest rate
γ	RRA of specialist	2	Risk premium on MBS portfolio
l	Household labor income ratio	1.84	Share of labor income in total income

4.4 γ , l , and ρ

We choose $\gamma = 2$ as risk aversion of the specialist. This choice of γ produces an average excess return on the intermediated asset of 3.36%. Over the 1976 to 2008 period, the average excess return on the Barclay's Agency MBS index was 2.6%. Over the 1999 to 2008 period, the return on the commercial MBS index was 0.32%. However, the latter sample is quite short and heavily weighted by a large -22.9% return in 2008.

We choose l based on the share of labor income to total income for the US, which is 66% from aggregate income statistics. In the model, households receive labor income of lD_t . We classify the capital gains and dividend income that specialists receive from running intermediaries also as labor income. That is for a hedge fund, the labor income of the manager comes from his 20% stake in the returns of the fund. We set $l = 1.84$, which produces an average household-plus-specialist labor to total income in the model of 64.5%.

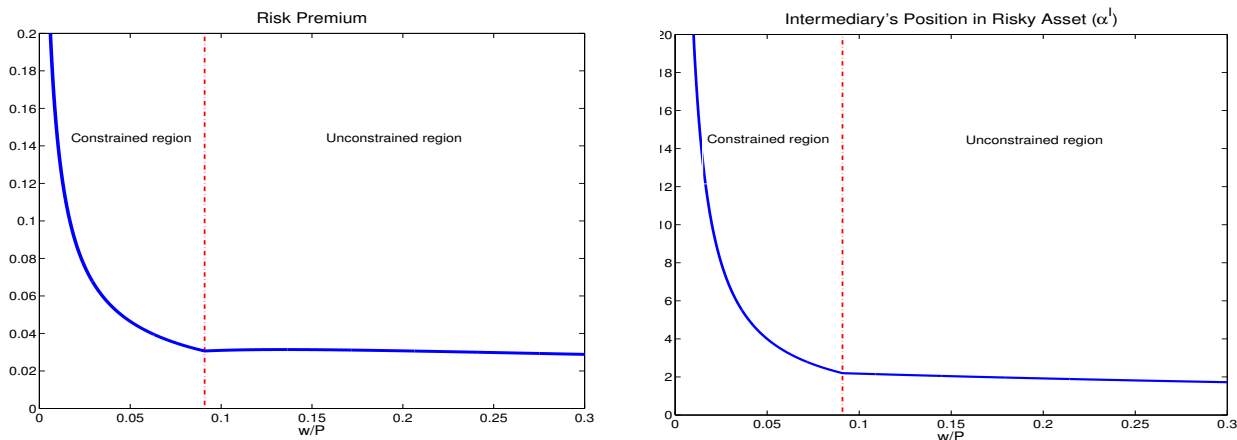
We choose $\rho = 0.04$. This choice produces an average riskless interest rate of 0.06% which is lower than the typical numbers in the literature (0.5%). Our parameter choices are also dictated by the restriction that, $\rho + g(\gamma - 1) - \frac{\gamma(\gamma-1)\sigma^2}{2} - \frac{l\gamma\rho}{1+l} > 0$. This restriction is necessary to ensure that the economy is well-behaved at $x = 1$ (see Appendix A.3). Given our other parameter choices, setting ρ higher violates this condition.

5 Results

This section presents results from solving the model, beginning by showing how risk premia are related to specialist capital, and then showing measures from simulating the model. The appendix describes details on the solution and simulation method.

5.1 Risk Premium as a Function of Specialist Capital

Figure 2: Risk Premium and Portfolio Share



Risk premium (left panel) and intermediary's portfolio share in the risky asset (right panel) are graphed against $x = w/P$, the specialist wealth as a percentage of the assets held by the intermediation sector. Parameters are those given in Table 2.

Figure 2 graphs the risk premium for the calibration of Table 2 as a function of x , the specialist capital relative to the value of the risky asset (w/P). The prominent feature of our model, clearly illustrated by the graphs, is the asymmetric behavior of the risk premium. The right hand side of the graph represents the unconstrained states of the economy, while the left hand side represents the constrained states. The cutoff for the constrained region in the figures is $x^c = \frac{1-\lambda}{1-\lambda+m} = 0.091$. In words, the constrained region arises when specialists own no more than equity equivalent to 9.1% of the assets held by intermediaries. Note that this number refers to the equity ownership of the entire intermediation sector; there may be some sectors where the specialists own far less than 9.1%, and some where the specialists own more. Risk premia rise as specialist wealth falls in the constrained region, while being relatively constant in the unconstrained region.

As discussed in Section 3.2, the asymmetry in the risk premium is driven by the rising portfolio share α_t^I . The right panel of the figure graphs this portfolio share as a function of specialist capital. Note the close relation between this figure and that for the risk premium. Finally, note that the effect in the constrained region is non-linear. If capital halves from $x = 0.091$ to $x = 0.045$ the risk premium only rises modestly from about 4% to 5%. The risk premium rises much more as capital falls below $x = 0.045$.

An interesting point of comparison for our results is to the literature on state-dependent risk premia, notably, Campbell and Cochrane (1999), Barberis, Huang, and Santos (2001), and Kyle and Xiong (2001). In these models, as in ours, the risk premium is increasing in the adversity of the state. Campbell and Cochrane and Barberis, Huang, and Santos modify the utility function of a representative investor to exhibit state-dependent risk aversion. We work with a standard CRRA utility function, but generate state dependence endogenously as a function of the frictions in the economy. For empirical work, our approach suggests that measures of intermediary capital/capacity will help to explain risk premia. In this regard, our model is closer in spirit to Kyle and Xiong who generate a risk premium that is a function of “arbitrageur” wealth. The main theoretical difference between Kyle and Xiong and our model is that the wealth effect in their model comes from assuming that the arbitrageur has log utility, while in our model it comes because the intermediation constraint is a function of intermediary capital. For empirical work, our model suggests that measures of intermediary capital will explain risk premia. One notable distinction of our model is the sharp asymmetry of our model’s risk premia: a muted dependence on capital in the unconstrained region and a strong dependence in the constrained region. In Kyle and Xiong, the log utility assumption delivers a risk premium that is a much smoother function of arbitrageur wealth. Plausibly, to explain a crisis episode, one needs the type of asymmetry delivered by our model.

5.2 Discussion: Leverage and Heterogeneity

Figure 2 indicates that the rise in the risk premium in the constrained region is closely related to the rise in leverage of the intermediary sector. The ratio of total intermediary sector assets to intermediary equity, or accounting leverage, is equal to α^I . Thus, our model implies that the leverage of the intermediary sector rises during a crisis.

In practice, many intermediary sectors during a crisis reduce leverage, while other sectors increase leverage. There is heterogeneity within the intermediation sector that our single intermediary model cannot capture. Adrian and Shin (2010) document that the leverage of the broker/dealer sector is procyclical, suggesting that it falls during recessions and crises. However, the Adrian and Shin evidence is based on data from the Federal Reserve’s flow of funds which is measured in terms of book values. The α^I of our model corresponds to market value leverage.¹³

¹³The average amount of credit extended by households to intermediaries in the constrained region, measured as $(\lambda w^h + (1 - \lambda)(1 - \alpha^h)w^h)/D$, is 83% of the amount of credit extended by households in the unconstrained region. That is, as a fraction of GDP our model predicts that the household sector extends less credit to the

Ang, Gorovyy and van Inwegen (2011) show that if one uses market value of equity in computations then leverage of the broker/dealer sector rises during the 2008 crisis. They also document that the leverage of the hedge fund sector falls during the crisis. He, Khang, and Krishnamurthy (2010) document that in the period from the fourth quarter of 2007 to the first quarter of 2009, spanning the worst episode of the subprime crisis, the hedge fund sector sheds assets, consistent with Adrian and Shin (2010). He, Khang, and Krishnamurthy also show that the commercial banking sector increased asset holdings over this period significantly. Moreover, the leverage of the top 19 commercial banks sector rises from 10.4 at the end of 2007 to near 30 at the start of 2009. The differential behavior of the banking sector in 2008 is reflective of a broader pattern of reintermediation during financial downturns, as documented by Gatev and Strahan (2006) and Pennacchi (2006). Importantly for the present analysis, in accord with our model the intermediaries that are the buyers during the crisis (i.e. banks) do so by borrowing and increasing leverage. Our model does not capture the other aspect of this process, as reflected in the behavior of the hedge fund sector, that some parts of the financial sector reduce asset holdings and deleverage.

5.3 Steady State Risk Premia

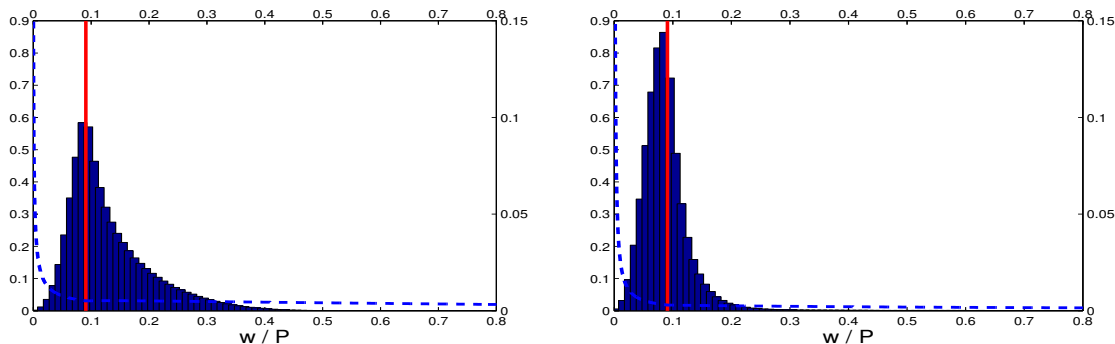
Quantitatively, as one can read from Figure 2, the calibration produces a risk premium in the unconstrained region of approximately 3%.¹⁴ The numbers for the risk premium are higher in the constrained region. To provide some sense for the values of the risk premium we may be likely to observe in practice, we simulate the model and compute the equilibrium probability of each state. The resulting steady state distribution over x is graphed in Figure 3. Also superimposed on the figure in a dashed line is the risk premium from the previous graph.

There are two forces driving the center-peaked distribution in Figure 3. First, as x falls, the risk premium rises. This in turn means that the specialist's wealth, due to the levered position in the risky asset, expect to increase as time passes, which tends to push x back to the original level. This force is stronger as the risk premium rises, which is why the distribution places almost no weight on risk premia as high as 30%. At the other end, when x is large so that w^h is small, the households are poor and consuming little but still receive labor income.

intermediary sector during crises. However, the market value of intermediary equity falls more than this reduction in debt borrowing, which therefore drives up market value leverage in the model. If we instead held market value of equity fixed – loosely corresponding to an accounting book measure that responds slowly to market prices – then our model implies that book leverage falls in a crisis. In this way, our model can be made consistent with the Adrian and Shin (2010) evidence.

¹⁴More precisely, the average risk premium conditional on being in the unconstrained region is 3.07%.

Figure 3: Steady State Distribution



The steady state distribution of x is graphed. Left panel is for the baseline parameters, while right panel is for $\gamma = 1$. The vertical line at $x^c = \frac{1-\lambda}{1-\lambda+m}$ gives the state where the intermediation constraint starts binding. The dashed line graphs the risk premium in order to illustrate the actual range of variation of the risk premium. Risk premium is indicated on the left scale, while the distribution is indicated on the right scale.

Thus, their wealth grows as they save the labor income, which shifts the wealth distribution back towards the center of the distribution.

The left panel of the figure is for the baseline parameters, while the right panel is for the $\gamma = 1$ case. For this latter case, the entire steady-state distribution is shifted to the left. As we explain in the next section, this result is due to greater risk-taking by the intermediary sector.

5.4 Measurements from simulation

The first column of Table 3 corresponds to the baseline parameterization of Table 2. Parameters have been chosen to match the risk premium, return volatility, interest rate, labor-to-total income ratio for the economy, and the debt/assets ratio of the intermediary sector when the capital constraint does not bind. As a result, the fit of the model in these dimensions is as expected. On the other hand, none of the parameters are explicitly chosen to match patterns during crises. The last two rows focuses on tails of the simulated distribution when the risk premium exceeds twice its unconditional average. In the next sections, we will further explore the behavior of the model in this tail.

Before doing so, we focus in this section on explaining how the model’s parameters affect the results, which are presented in columns 2 through 6 in Table 3, where we change a single parameter relative to the baseline.

Table 3: Measurements

We present a number of key moments from the model. We report the unconditional average risk premium, Sharpe ratio, volatility, interest rate, and labor-to-total income ratio. We also report the unconditional probability of the capital constraint not binding, and the Debt/Asset ratio of the intermediary sector conditional on the capital constraint not binding. The last two rows provide information on the tails of the distribution, where the risk premium double its unconditional average. In the first column, we report measure for the baseline parameterization. The rest of the columns consider variations where we change a single parameter relative to the baseline given in Table 2.

	Base	$\sigma = 6\%$	$\gamma = 1$	$m = 8$	$\lambda = .05$	$l = 1$
Risk Premium (%)	3.36	1.96	2.35	3.38	3.25	3.19
Sharpe Ratio (%)	36.46	32.62	27.34	37.11	35.72	34.81
Return volatility (%)	9.25	6.12	10.60	9.17	9.23	9.18
Interest Rate (%)	0.06	1.42	0.95	0.02	0.14	0.83
Labor income ratio	0.64	0.52	0.54	0.62	0.55	0.61
Price/Dividend	70.50	70.02	71.00	71.00	70.22	49.50
Probability(Unconstrained) (%)	65.50	12.48	39.40	78.95	0.91	78.35
Debt/Assets Ratio (Unconstrained)	0.50	0.49	0.52	0.52	0.00	0.48
Prob($RiskPremium > 2 \times \overline{RiskPremium}$)	0.87	1.99	3.49	0.55	1.43	0.57
E($RiskPremium > 2 \times \overline{RiskPremium}$)	8.89	5.23	7.41	9.40	8.60	8.41

Risk prices: γ and σ The first two variations consider changes in the risk-aversion γ of the specialist and the fundamental risk of the economy σ .

As expected, relative to the baseline of $\gamma = 2$, setting $\gamma = 1$ decreases the risk premium and Sharpe ratio. It also increases interest rates because of the dampening of the precautionary savings effect. The interesting result is a “risk-taking” effect. With lower risk-aversion, the intermediary sector is willing to hold a more risky portfolio. Hence, the same fundamental volatility is translated to a greater volatility for the specialist’s wealth, and the economy is more likely to enter the constrained region. The effect is reinforced through a general equilibrium effect. While the total risk in our economy is relatively stable due to the endowment structure, when the specialist has lower risk aversion, the price of risk falls and causes the intermediary to be compensated less per unit of risk.¹⁵ Hence, the intermediary sector on

¹⁵Although when $\gamma = 1$, we find that the return volatility rises relative to the baseline $\gamma = 2$, which leads

average earns and retains less profits, which in turn leads the capital constraint to bind more frequently.

In Table 3, this “risk-taking” effect is reflected in a smaller Prob(unconstrained) of 39.4% (relative to 65.5% in the baseline). We can also see this effect in the tails of the distribution, which are in the last two rows of Table 3. There, the probability of hitting the double-average-risk-premium states rises. Graphically, this effect is apparent from Figure 3, where we see that the $\gamma = 1$ case causes the entire distribution to shift towards crisis-states.

The above general equilibrium effect also plays a role in the results for a lower fundamental volatility $\sigma = 6\%$. Not surprisingly, reducing σ decreases the risk premium, Sharpe ratio, and return volatility. The surprising result in this case is that the probability of the unconstrained region decreases. That is, a lower cash flow volatility leads to the capital constraint binding more frequently. This result is due to the lower average risk premium demanded by the economy with relatively low fundamental risk, which again lowers average intermediary sector profits.

Intermediation: m and σ The next two variations focus on changing the intermediation parameters. The case with $m = 8$ shows that most of the unconditional asset market measures do not depend on m . The main effect is on the probability of hitting the constraint. With a higher m , the capital constraint is loosened, and hence the probability of hitting the constraint falls. The probability of doubling the risk premium also falls. But note that with $m = 8$, the average risk premium in these states rises. This latter effect is due to a “sensitivity” effect of m . With a higher m , a one dollar fall in specialist capital leads to larger withdrawal of household capital, which increases the sensitivity of the risk premium to the specialist capital.¹⁶

The next column considers the effect of setting λ near zero. This variation has the expected effect of lowering the debt/assets ratio in the unconstrained region to near zero. Like the variation with m , changing λ has little effect on the unconditional asset market measures. The main effect again is on the probability of hitting the constraint. From (23) we see that lowering λ increases x^c .¹⁷ Effectively, the entire steady-state distribution shifts towards constrained regions, with the $\lambda = 0.05$ case presented in the table giving an unconstrained

to a small compounding effect beyond the direct effect of a lower price of risk.

¹⁶This “sensitivity” effect is formally analyzed in He and Krishnamurthy (2011).

¹⁷A related effect is that when there is a greater fraction of risky asset households, the household sector earns a higher return, which shifts the wealth distribution toward the households sector, and thus increasing the probability of being constrained even further.

probability of near zero. We opt in our baseline for a higher λ primarily because doing so implies that the equity capital constraint only binds occasionally.

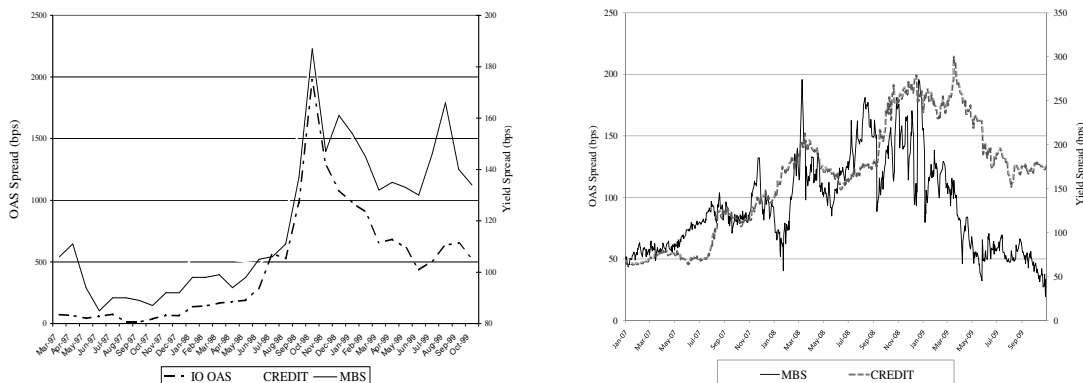
Labor income l The last case in the table demonstrates the effect of lowering l . With a lower l , the probability of being in the constrained region falls. Intuitively, because households have less income per period, the specialist wealth tends to grow faster relative to the household wealth, and as a result the economy spends more time away from the capital constrained region. We also see this effect in the last lines of the table. The probability of hitting the high risk premium states falls when l falls. These effects in turn lower the average risk premium. The most significant effect of l is on the average price/dividend ratio. With lower labor income, households save less money every period which lowers their demand for assets, leading to lower asset prices. This effect is most evident from the boundary condition, (24); there, we see that asset values are scaled down commensurately with a lower l . The effect of l on the labor-income ratio is relatively small. In our economy, the total income is $(1+l)D_t$; while the labor income is lD_t for households plus the specialist’s portion of the dividend income which we count as the specialist’s labor income. Ignoring this latter piece (unconditionally it accounts for about 9% of the total labor income), the labor income ratio $l/(1+l)$ falls from 0.65 to 0.5 when l goes from 1.84 to one. The effect is further dampened because the specialist’s portion of labor income, dependent on capital gains and dividends, does not change much when we lower l .

To summarize, these results suggest that changing γ and σ can significantly affect the simulation. The γ variation highlights an economically interesting “risk-taking” effect in the model. The table also indicates that our results are relatively insensitive to the choice over m , λ , and l . For brevity, we focus only on variations based on γ and m in the rest of the paper.

5.5 Crisis Episodes

Figure 4, left-panel, graphs the behavior of the high grade credit spread (AAA bonds minus Treasuries), the spread on FNMA mortgage backed securities relative to Treasuries, and the option adjusted spread on volatile interest-only mortgage derivative securities (data are from Gabaix, Krishnamurthy, and Vigneron, 2007). The spreads are graphed over a period from 1997 to 1999 and includes the fall of 1998 hedge fund crisis. During 1997 and upto the middle of 1998 spreads move in a fairly narrow range. If we interpret the unconstrained states of our model as this “normal” period, then the muted response of risk premia to the state

Figure 4: Crisis Spreads



The left panel graphs the spreads between the Moody’s index of AAA corporate bonds and the 10 year Treasury rate (grey line, “credit”), the spreads between FNMA 6% TBA mortgage-backed securities and the 10 year Treasury rate (black line, “MBS”), and the option-adjusted spreads on a portfolio of interest-only mortgage-backed securities relative to Treasury bonds (dashed line, “IO OAS”) from 1997 to 1999. The right panel graphs the same credit spread as well as the OAS on the FNMA 6% MBS from 2007 to 2009.

can capture this pre-crisis period. In a short period around October 1998 spreads on these securities increase sharply. The credit spreads and MBS spreads double from their pre-crisis level. The mortgage derivative spread increases by many multiples. The right-panel graphs the credit spread and the FNMA mortgage spread from 2007 to 2009. The subprime crisis begins in the summer of 2007, escalating until the fall of 2008. From the pre-crisis period to the fall of 2008, the MBS spread quadruples, while the credit spread rises six-fold. It is hard to say precisely how much Sharpe ratios increase during these episodes, because the underlying default risk in these bonds rises during the same time. However, a doubling or tripling is plausibly within the range of estimates.

Table 4 provides information on the likelihood of events such as those of 1998 and 2007 within our model. We simulate the model and determine the probability that the risk premium exceeds a given threshold. We also report the Sharpe ratio, interest rate, and the intermediary sector’s debt/assets ratio at the relevant risk premium.

The baseline parameters put the probability of the risk premium exceeding 6% at 1.33%, while the probability of the risk premium exceeding 12% is 0.07%. Recall that the risk premium is near 3% in the unconstrained region of the baseline model. These numbers indicate that crises are unlikely within the model. Moreover, even conditioning on the risk premium

Table 4: Probability of Crisis

Risk Premium (RP^*)	3.00%	6%	9%	12%
We condition on the state, or specialist capital, corresponding to given value of the risk premium, denoted RP^* . We report the probability that the risk premium exceeds RP^* , as well as the model's Sharpe ratio, interest rate, and intermediary debt/assets ratio at that value of RP^* . Panel A is for the baseline parameters, while the other panels are for variations where we change a single parameter.				
Panel A: Baseline				
Probability(Risk Premium $> RP^*$)	89.77	1.33	0.22	0.07
Sharpe Ratio	32.31	66.26	103.59	144.04
Interest Rate	0.48	-2.35	-5.47	-8.81
Debt/Assets	45.63	82.66	90.28	93.57
Panel B: $\gamma = 1$				
Probability(Risk Premium $> RP^*$)	14.54	1.61	0.46	0.02
Sharpe Ratio	33.34	66.68	100.02	133.36
Interest Rate	0.41	-2.59	-5.59	-8.59
Debt/Assets	73.03	86.53	91.03	93.28
Panel C: $m = 8$				
Probability(Risk Premium $> RP^*$)	93.16	1	0.19	0.06
Sharpe Ratio	32.17	66.48	101.82	138.58
Interest Rate	0.39	-2.5	-5.55	-8.7
Debt/Assets	44.59	82.81	89.92	93.03

being larger than 6%, the probability that it then exceeds 12% is only 5.3% (i.e. 0.07/1.33).

The interest rate falls dramatically in the crisis states. There are two intuitions behind this fall in interest rates. First, as the specialist's consumption volatility rises with the tightness of the intermediation constraint, the precautionary savings effect increases specialist demand for the riskless bond. Second, as specialist wealth falls, households withdraw equity from intermediaries, increasing their demand for the riskless bond. To clear the bond market, the equilibrium interest rate has to fall. Both the behavior of the interest rate and the disintermediation-driven demand for bonds are consistent with a flight to quality. However, in our model the interest rate, which is falling to -8.81% in the 12% risk premium scenario, is over-sensitive to the aggregate state.

The main reason for this over-sensitive interest rate is that we are pushing the general equilibrium of our model too far. Our model-economy consists of only an intermediation sector and therefore ascribes all movements in interest rates to shocks within that sector. In practice, part of the demand for bonds in the economy is from sectors that are unaffected

by the intermediation constraint, so that it is likely that our model overstates the interest rate effect. However, it also does not seem appropriate to fix the interest rate exogenously, since interest rates do fall during a crisis episode. Thus, while the qualitative prediction of our model for interest rates seems correct, the quantitative implications are the least credible results of our analysis.

The model also implies that the debt/assets ratio of the intermediary sector rises in the crisis states. This implies that the market value leverage of the financial sector rises in these states. As noted earlier in Section 5.2, this result is driven by the falling market value of assets, which then leads to a falling market value of equity (i.e. assets minus debts), and hence increased leverage.

Panels B and C provide results for the two variations of the model. The most interesting variation is in Panel B where we report results for the $\gamma = 1$ case. We find that the probabilities of the risk premium exceeding RP^* are higher in this case than the baseline. Note that this is despite the fact that the average risk premium in the $\gamma = 1$ case is lower than that of the $\gamma = 2$ case (see Table 3). This is due to the “risk-taking” effect we have alluded to earlier. If we interpret the comparative static of going from $\gamma = 1$ to $\gamma = 2$ as akin to increasing the average capital requirements on banks, then this result indicates that higher capital requirements can reduce the probability of a crisis. Doubling capital requirements roughly halves the probabilities of the 9% and 12% crisis-states. Panel C indicates that allowing the intermediaries to raise more equity capital from households by increasing m reduces the probability of crises, consistent with intuition.

5.6 Capital Movement and Recovery from Crisis

Referring to Figure 4, left-panel, the corporate bond spread and MBS spread widen from 90 bps in July 1998 to a high of 180 bps in October 1998 before coming down to 130 bps in June 1999. Thus, the half-life — that is, the time it takes the spread to fall halfway to the pre-crisis level — is about 10 months. The interest-only mortgage derivative spread, which is very sensitive to market conditions, widens from 250 bps in July 1998 to a high of 2000 bps before coming back to 500 bps in June 1999. In the right-panel, the MBS spread recovers back to its pre-crisis level by June 2009, while the credit spread remains elevated through the end of the period. We note that this timescale for mean reversion, on the order of months, is much slower than the daily mean-reversion patterns commonly addressed in the market micro-structure literature (e.g., Campbell, Grossman, and Wang, 1994).

A common wisdom among many observers is that this pattern of recovery reflects the slow movement of capital into the affected markets (Duffie, 2010). Our model captures this slow movement. We will show in this section that our baseline calibration can also replicate these speeds of capital movement.

In the crisis states of our model, risk premia are high and the specialists hold leveraged positions on the risky asset. Over time, profits from this position increase w_t , thereby increasing the capital base of the intermediaries. The increase in specialist capital is mirrored by an m -fold increase in the allocation of households' capital to the intermediaries. Together these forces reflect a movement of capital back into the risky asset market, leading to increased risk-bearing capacity and lower risk premia. Note, however, that one dimension of capital movement that plausibly occurs in practice but is not captured by our model is the entry of “new” specialists into the risky asset market.

We can use the model simulation to gauge the length and severity of a crisis within our model. Table 5 presents data on how long it takes to recover from a crisis in our model. We fix a state (x, D) corresponding to an instantaneous risk premium in the “Transit from” row. Simulating the model from that initial condition, we compute and report the first passage time that the state hits the risk premium corresponding to the “Transit to” column. The time is reported in years.

Consider Panel A which corresponds to the baseline parameters. If we start from the extreme crisis state of 12% and compute how long it takes to recover to 7.5% — i.e. halfway back to the unconditional average levels we report earlier of around 3% — the time is 0.65 years (7.9 months). From the 10% crisis state to the 6.5% state (halfway to 3%) takes 0.93 years (this number is not reported in the table). For the fall of 1998 episode, the half-life we suggested was around 10 months. The model half-life from 10% is of the same order of magnitude of the empirical observation.

Panel B is for the $\gamma = 1$ case, fixing the same crisis states measured by the risk premium. Recovery times are faster in this case because a less risk-averse intermediary sector takes on more risk to reach the same risk premium. Essentially, for the same risk premium across Panel A and B, the intermediary sector in Panel B holds a riskier portfolio and thus earns higher profits from its portfolio. Additionally, the unconditional average risk premium in the $\gamma = 1$ case is 2.35% compared to the 3.36% of the baseline. Thus, even at a risk premium of 3.5%, the economy in Panel B is still recovering from the crisis.

Panel C is for the $m = 8$ case. Here also we find faster recovery times. The reason is that specialist capital growth leads to more injections of household capital for a higher m .

Table 5: Crisis Recovery

This table presents transition time data from simulating the model. We fix a state corresponding to an instantaneous risk premium of 12% (“Transit from”). Simulating the model from that initial condition, we compute and report the first passage time that the state hits the risk premium corresponding to that in the “Transit to” column. Time is reported in years. The column “Increment time” reports the time between incremental “Transit to” rows. Data is presented for the baseline parameters and two variations.

Panel A: Baseline						
Transit to	10	7.5	6	5	4	3.5
Transit time from 12	0.18	0.65	1.42	2.67	5.56	9.34
Increment time	0.18	0.47	0.77	1.25	2.90	3.78
Panel B: $\gamma = 1$						
Transit time from 12	0.12	0.37	0.71	1.15	2.02	2.85
Increment time	0.12	0.25	0.34	0.43	0.88	0.82
Panel C: $m = 8$						
Transit time from 12	0.16	0.60	1.31	2.41	5.28	8.78
Increment time	0.16	0.44	0.71	1.10	2.87	3.49

The slow adjustment of risk premia, in timescales of many months, during the 1998 episode is also consistent with other studies of crisis episodes. Berndt, et. al. (2005) study the credit default swap market from 2000 to 2004 and note a dramatic market-wide increase in risk premia (roughly a quadrupling) in July 2002 (see Figures 1 and 2 of the paper). Risk premia gradually fall over the next two years: From the peak in July 2002, risk premia halve by April 2003 (9 months). The authors argue that dislocations beginning with the Enron crisis led to a decrease in risk-bearing capacity among corporate bond investors. Mirroring the decreasing risk-bearing capacity, risk premia rose before slowly falling as capital moved back into the corporate bond market and expanded risk bearing capacity. Gabaix, Krishnamurthy, and Vigneron (2007) note a dislocation in the mortgage-backed securities in late 1993 triggered by an unexpected wave of consumer prepayments. A number of important hedge fund players suffered losses and went out of business during this period, leading to a reduction in risk bearing capacity. Figure 3 in the paper documents that risk premia reached a peak in December 1993 before halving by April 1994 (5 months). Froot and O’Connell (1999) study the catastrophe insurance market and demonstrate similar phenomena. When insurers suffer losses that deplete capital they raise the price of catastrophe insurance. Prices then gradually fall back to long-run levels as capital moves back into the catastrophe insurance market. Froot

and O’Connell show that the half-life in terms of prices can be well over a year.¹⁸

Each of these markets are intermediated markets that fit our model well. Investors are institutions who have specialized expertise in assessing risk in their markets. Our theory explains the slow movement of risk bearing capacity and risk premia documented in these case-studies. The calibrated model also captures the frequency of the slow adjustment of risk premia.

6 Crisis Policy Experiments

We study the effect of policy interventions in the crisis of the model. We study three policies: (1) Lowering borrowing rates to the intermediary, as with a decrease in the central bank’s discount rate; (2) Direct purchase of the risky asset by the government, financed by debt issuance and taxation of households; and (3) Infusing equity capital into the intermediaries during a crisis. These three policies are chosen because they are among those undertaken by central banks in practice. Our aim is to quantify the effects of these policies based on our model. The analysis is purely positive, and we make no claims as to optimality.

Our policy experiments correspond to the following exercise. Suppose we are in a crisis state currently, with a given asset/liability position for the households and specialists. From this initial condition, suppose that the government conducts a policy that was not anticipated by the agents. We trace the effects of this policy on the recovery of the economy from that crisis state.¹⁹

To be more precise, we compute two equilibria: one without the policy (which is the equilibrium we have studied thus far), and one with the policy. For example, the first policy that we consider is giving the intermediary sector a borrowing subsidy as long as the economy is in the constrained region. Under such a policy, we can write the subsidy as a function of the primitive state variables and solve the with-policy equilibrium of our model. We then consider the following thought experiment. Suppose that the economy is currently in a given crisis state

¹⁸Mitchell, Pedersen, and Pulvino (2007) document similar effects in the convertible bond market in 1998 and again in 2005. In both cases, crisis recovery times are in the order of months. They also note that spreads in merger arbitrage strategies took several months to recover following the October 1987 risky asset-market crash. Duffie (2010) discusses these and other cases in his presidential address to the American Finance Association.

¹⁹The government policy is a zero-probability event in our exercise. Another experiment would be to study a policy that is expected to be enacted given some value of the state variable – say the government infuses equity capital if the risk premium touches 12%. Such a policy would be anticipated by agents within the equilibrium of the model. Analyzing such a policy does not pose any difficulty for our modeling structure, but it adds an extra layer of complexity. For the sake of brevity, we have opted to focus on the simpler experiment.

of the no-policy equilibrium (12% risk premium state in the simulations). The government announces the policy which is unanticipated by agents, and its action causes asset prices to jump immediately. From that point on, the dynamics of the economy are described by the solution to our model under the with-policy equilibrium.²⁰

6.1 Borrowing Subsidy

During financial crises, the central bank lowers its discount rate and its target for the overnight interbank interest rate. Financial intermediaries rely heavily on rolling over one-day loans for their operation (see, for example, Adrian and Shin (2010) on the overnight repurchase market). Because of this dependence, intermediaries are perhaps the most sensitive sector within the economy to overnight interest rates. Commercial and investment banks have access to overnight funds at the discount window of the central bank. Thus, to the extent that the central bank lowers overnight rates, including the discount rate, it reduces the borrowing costs of financial intermediaries.

While our model does not have a monetary side within which to analyze how a central bank alters the equilibrium overnight interest rate, we can go some way towards examining the effect of this policy by studying the following transfer. The debt position of intermediaries at date t is $(\alpha_t^I - 1)w_t$. Suppose that the government makes a lumpsum transfer of $\Delta r \times (\alpha_t^I - 1)w_t dt$ from households to intermediaries, where Δr measures the size of the transfer. The transfer is proportional to the debt of the intermediary.

The subsidy experiment can be thought of as a reduction in the central bank's discount rate. In practice, when the central bank makes funds available more cheaply to the financial sector through the discount window, it can be viewed as transferring real resources from taxpaying households to the financial sector.

Formally, we examine the policy in which the subsidy Δr is provided to the intermediaries as long as the economy is in the constrained region. That is, Δr is paid only if $x < x^c$, and there is no subsidy if $x > x^c$. We express this transfer of $\Delta r \times (\alpha_t^I - 1)w_t dt$ in terms of the primitive state variables x_t and D_t . Then, the dynamic budget constraints of household and specialist are altered to account for the transfer (see equation (11)), and this change is traced through to rederive the ODE for the price/dividend ratio (see Appendix C for details).

Table 5 presents the results for the baseline parameters. We start the economy in the

²⁰The initial condition from which we simulate the with-policy equilibrium is chosen to match the portfolio holdings of the household in the 12% risk premium state of the no-policy equilibrium.

Table 5: Borrowing Subsidy

This table presents transition time data from simulating the model. We begin in the 12% risk premium state and report the first passage time for the state to reach that in the first column of the table (“Transit to” column). Time is reported in years. We report the case of no subsidy ($\Delta r = 0$), as well as subsidies of 0.01, 0.02 and 0.045. A subsidy of 0.01 corresponds to 100 *bps*. The first row of the table reports the instantaneous jump downwards in the risk premium when the government initiates the policy. The simulation is for the baseline parameters.

Transit to	$\Delta r = 0$	$\Delta r = 0.01$	$\Delta r = 0.02$	$\Delta r = 0.045$
		10.85%	9.82%	7.94%
10.00%	0.18	0.08		
7.50%	0.65	0.45	0.18	0.08
6.00%	1.42	1.04	0.65	0.37
5.00%	2.67	1.85	1.42	0.70
4.00%	5.56	3.74	2.67	1.29

state corresponding to the 12% risk premium. The table reports the recovery times from the 12% extreme crisis state for different levels of Δr . Consistent with intuition, a higher subsidy speeds up the recovery process. The 200 bps subsidy speeds up the recovery to 7.5% by 0.57 years. Note that from August 2007 to October 2008, the discount rate decreased by 450 bps. The last column in the table indicates the effect of this policy within our model.

6.2 Direct Asset Purchase

In both the subprime crisis as well as the Great Depression the government directly entered the asset market to purchase distressed assets. The Federal Reserve and GSEs purchased nearly \$1.8tn of mortgage-backed securities over the period from August 2007 to August 2009 (\$1.25tn by the Federal Reserve and \$550bn by the GSEs). We can evaluate the impact of this policy as follows. Suppose that the government purchases a fraction s of the risky asset in states $x < x^c$, financing this purchase by issuing sP_t of short-term debt (recall that P_t is the price of the risky asset). The cash-flow from this transaction is $sP_t(dR_t - r_t dt)$, and we assume that the government raises lumpsum taxes from (or rebates to) the households to balance this cash-flow.

Table 5 reports the results for three values of s , which is the share of the intermediated

risky asset market that the government purchases. If we take the stock of intermediated assets to be \$15tn, then the \$1.8tn number cited above is 12% of this stock. We assume that the policy is initiated in the state corresponding to 12% risk premium and not removed until the economy recovers back to the unconstrained region. We trace the recovery path from this state.

Table 6: Asset Purchase

This table presents transition time data from simulating the model. We begin in the 12% risk premium state and report the first passage time for the state to reach that in the first column of the table (“Transit to” column). Time is reported in years. We report the case of no purchase ($s = 0$), as well as purchases of 0.04, 0.08, and 0.12. A purchase with $s = 0.04$ corresponds to the government buying 4% of the outstanding stock of intermediated risky assets. The first row of the table reports the instantaneous jump downwards in the risk premium when the government begins its purchase.

Transit to	$s = 0$	$s = 0.04$	$s = 0.08$	$s = 0.12$
10.00%	0.18	0.14	0.10	0.05
7.50%	0.65	0.61	0.58	0.52
6.00%	1.42	1.39	1.32	1.27
5.00%	2.67	2.51	2.48	2.40
4.00%	5.56	5.50	5.48	5.37

The policy causes a downward jump in the risk premium. The asset purchase policy indirectly increases the household’s exposure to the risky asset because future taxes now depend on the returns to the risky asset. In turn, this means that specialists bear less risk in equilibrium and hence the risk premium falls. Effectively this policy puts less risk on the limited risk-bearing capacity of the intermediary sector. After this initial jump the recovery path is almost the same as the case of no intervention. For example, if we compare the incremental time it takes the economy to move from 7.5% to 6%, we see that the time for the no intervention case is 0.76 years, while it is 0.73 years for the case of $s = 0.12$. Intuitively the purchase has no further effect because of the following countervailing force. The specialist holds a smaller position in the risky asset, as the taxpayer holds a larger share. Hence, less of the risk premium accrues to the intermediary capital, leading to a slower recovery.

6.3 Capital infusion

A number of crisis interventions are aimed at increasing the equity capital of intermediaries. For example, in the Great Depression, the government directly acquired preferred shares in banks, thereby increasing their equity capital. In the subprime crisis, the U.S. Treasury purchased \$205 bn of preferred shares in the intermediary sector through the capital purchase program.

We examine a policy in which m is increased to $\bar{m} > 4$ in the constrained region with $x < x^c$. The higher m indicates that the intermediary increases its equity capital proportionate to $\bar{m} - m$. The extra equity capital is purchased by the government, and paid for by lumpsum taxes on the households. Returns on the government investment are rebated in a lumpsum fashion to the households. We think of the increase in m as a temporary relaxation of the equity capital constraint. For example, one may imagine that the government is temporarily able to monitor intermediaries better than households during a crisis and can thus relax the capital constraint. Our aim is to quantify the effect of the relaxation of the constraint on the crisis recovery.

We choose \bar{m} to represent the Treasury's purchase of \$205 bn of bank capital. Note that capital in our model refers to common shares, while in practice, the Treasury purchased preferred shares. The distinction is important because our model works through the sharing of risk between the specialist and the household/government, rather than directly through the amount of funds that are transferred to the intermediary sector. When the government invests in the intermediary and shares some of the risk in the specialist's investment, the specialist bears less risk in equilibrium and the risk premium adjusts downwards. The returns on common shares are more sensitive to the returns on intermediary investment than are the returns on preferred shares, indicating that common shares allow for more risk sharing than preferred shares. Franks and Torous (1994), based on a sample of distressed firms over the period 1983 to 1988, document that in a bankruptcy/reorganization preferred shares are repaid 42% of face value.²¹ We translate the preferred share purchase in terms of common shares using the 42% number of Franks and Torous—that is, we assume that an injection of \$1 of preferred shares is equal to an injection of \$0.58 ($= 1 - 0.42$) of common shares.

The Treasury's capital injection was distributed across many banks, from pure lending institutions to trading institutions. Since our model is primarily about securities markets

²¹In our model, the value of common shares approaches zero as the value of assets falls towards the value of liabilities (the bankruptcy threshold). Likewise, as the value of assets rises, preferred shares received a relatively fixed dividend, while the value of common shares increases.

and trading institutions, we apportion the \$205 bn capital to reflect the injection of capital to support securities trading. We multiply the injection by 0.40, which is the fraction of securities in total bank assets, as computed from the Flow of Funds 2007 data, to give an equity injection of \$48bn ($=205 \times 0.58 \times 0.40$). Finally, we interpret the government’s equity injection as an increase of the specialist’s inside capital which is then matched by a four-fold increase in capital from households. That is, we interpret the events of 2008/2009 as, first the government does TARP, and second, this enabled the financial sector to raise further equity from private investors.

Within the model we evaluate the effect of raising m to \bar{m} in the crisis states, where \bar{m} is chosen so that the implied increase in specialist equity capital (as a fraction of total assets under intermediation) in the 12% crisis state is 1.28% (i.e., $4 \times \$48$ bn divided by \$15 tn). The equity-asset ratio of the intermediary sector in the constrained region is $\frac{(1+m)w}{P}$. We choose \bar{m} so that $\frac{(1+\bar{m})w}{P} - \frac{(1+m)w}{P}$ is equal to 1.28%, based on the w and P of the no-policy equilibrium in the 12% risk premium state. The results are in Table 7. We also present results for a \$38 bn and \$58 bn equity injection.

Table 7: Equity Injection

This table presents transition time data from simulating the model. We begin in the 12% risk premium state and report the first passage time for the state to reach that in the first column of the table (“Transit to” column). Time is reported in years. We report the case of a purchase of equity capital of \$38 bn, \$48 bn and \$58 bn, which is reversed in roughly one year. The first row of the table reports the instantaneous jump downwards in the risk premium when the government injects the equity capital.				
Transit to	Baseline	\$38bn	\$48bn	\$58bn
		9.57%	9.05%	8.57%
10.00%	0.18			
7.50%	0.65	0.43	0.37	0.27
6.00%	1.42	1.19	1.10	0.99
5.00%	2.67	2.35	2.24	2.14
4.00%	5.56	5.23	5.01	4.95

The effects of policy are qualitatively similar to the other cases: there is a jump downwards in the risk premium and a gradual adjustment afterwards. It is most interesting to compare the effects of the three policies. Compared to the asset purchase case, we see that a relatively small amount of funds used towards equity purchase produces a much faster recovery. The

reason that the equity injection has such a large effect is because the fundamental friction in our model is an equity capital constraint. The equity capital injection of \$48 bn, corresponding to the actually policy enacted in 2008/2009, leads to a recovery time to the 6% state of 1.10 years. Compare this to the asset purchase policy, where buying \$1.8 tn of assets leads a recovery time of 1.27 years. Finally, the 450 basis point borrowing subsidy, corresponding to actual policy, leads to the fastest recovery time of 0.37 years. This policy has a large effect because the intermediaries are very leveraged in our model, carrying a debt/asset ratio of 94% in the 12% risk premium state. These numbers concern the benefits of these policies. To provide a more comprehensive assessment of the policies, one also needs to evaluate the costs of these interventions.

7 Conclusion

We have presented a model to study the dynamics of risk premia in a crisis episode where intermediaries' equity capital is scarce. We calibrate the model and show the model does well in matching two aspects of crises: the nonlinearity of risk premia in crisis episodes; and, the recovery from crises in the order of many months. We also use the model to evaluate the effectiveness of central bank policies, finding that infusing equity capital into intermediaries is the most effective policy in our model.

A limitation of our model is that it does not shed any light on the connection between the performance of intermediated asset markets we model (i.e. the mortgage-backed securities market) and the aggregate stock market. Yet, as we have seen during the subprime crisis, the deterioration in intermediation does spillover to the S&P500. It will be interesting to explore such a connection by introducing a second asset, in positive supply, that the households invest in directly. Such an asset can represent the S&P500 and may shed light on the equity premium puzzle.

References

- [1] Adrian, Tobias and Shin, Hyun Song, 2010, “Liquidity and Leverage,” *Journal of Financial Intermediation*, forthcoming.
- [2] Adrian, Tobias, Erkko Etula, and Tyler Muir, 2010, “Financial Intermediaries and the Cross-Section of Asset Returns,” working paper, Northwestern University and Federal Reserve Bank of NY.
- [3] Allen, Franklin and Douglas Gale, 1994, Limited Market Participation and Volatility of Asset Prices, *American Economic Review* 84, 933-955.
- [4] Allen, Franklin and Douglas Gale, 2005, “From Cash-in-the-Market Pricing to Financial Fragility,” *Journal of the European Economic Association*, 3(2-3), 535-546.
- [5] Aiyagari, Rao and Mark Gertler, 1999, “Overreaction of Asset Prices in General Equilibrium,” *Review of Economic Dynamics* 2(1), 3-35.
- [6] Alvarez, Fernando, Andrew Atkeson, and Patrick Kehoe, 2002, “Money, interest rates, and exchange rates with endogenously segmented markets,” *Journal of Political Economy* 110, 73–112.
- [7] Barberis, N., M. Huang, and T. Santos, 2001, Prospect theory and asset price, *The Quarterly Journal of Economics*, 116, 1-53.
- [8] Basak, Suleyman and Domenico Cuoco, 1998, An Equilibrium Model with Restricted risky asset Market Participation, *Review of Financial Studies*, 11, 309-341.
- [9] Bates, David, 2003, Empirical Option Pricing: A Retrospection, *Journal of Econometrics*, 116, 387–404.
- [10] Bernanke, Ben and Mark Gertler, 1989, Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review*, 79, pp. 14-31.
- [11] Berndt, Antje, Rohan Douglas, Darrell Duffie, Mark Ferguson, and David Schranz, 2004, Measuring Default Risk Premia from Default Swap Rates and EDFs, mimeo, Stanford University.
- [12] Borodin, A.N. and P. Salminen, 1996, *Handbook of Brownian Motion*. Birkhauser, Boston, MA.

- [13] Boudoukh, Jacob, Matthew Richardson, Richard Stanton, and Robert Whitelaw, 1997, Pricing mortgage-backed securities in a multifactor interest rate environment: A multivariate density estimation approach, *Review of Financial Studies* 10, 405-446.
- [14] Brunnermeier, Markus and Lasse Pedersen, 2009, Market Liquidity and Funding Liquidity, *Review of Financial Studies*, 22(6), 2201-2238.
- [15] Brunnermeier, Markus and Yuliy Sannikov, 2010, "A Macroeconomic Model with a Financial Sector," working paper, Princeton University.
- [16] Campbell, John and John Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205-251.
- [17] Campbell, John, Sanford Grossman and Jiang Wang, 1993, "Trading Volume and Serial Correlation in risky asset Returns," *Quarterly Journal of Economics*, 108, 905-940.
- [18] Collin-Dufresne, Pierre, Robert Goldstein, and J. Spencer Martin, 2001, The Determinants of Credit Spread Changes, *The Journal of Finance*, 66, 2177-2207.
- [19] Diamond, Douglas, 1997, "Liquidity, Banks, and Markets," *Journal of Political Economy*, 105(5), pp. 928-56.
- [20] Diamond, Douglas and Philip Dybvig, 1983, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91(5), pp. 401-19.
- [21] Diamond, Douglas and Raghuram Rajan, 2005, "Liquidity Shortages and Banking Crises," *Journal of Finance* 60 (2), pp. 615-647.
- [22] Duffie, Darrell, 2010, "Presidential Address: Asset Price Dynamics with Slow-Moving Capital," *Journal of Finance* 65(4), pp. 1237-1266.
- [23] Duffie, Darrell and Bruno Strulovici, 2009, "Capital Mobility and Asset Pricing," working paper, Stanford University.
- [24] Dumas, Bernard, 1989, "Two-Person Dynamic Equilibrium in the Capital Market," *Review of Financial Studies*, 2(2), 157-188.
- [25] Edmond, Chris and Pierre-Olivier Weill, 2009, "Aggregate Implications of Micro Asset Market Segmentation," NBER Working Paper 15254.

- [26] Franks, Julian and Walter Torous, 1994, "A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganizations," *Journal of Financial Economics*, 35, 349-370.
- [27] Froot, Kenneth A., and Paul G. O'Connell, 1999, The Pricing of US Catastrophe Reinsurance in *The Financing of Catastrophe Risk*, edited by Kenneth Froot, University of Chicago Press.
- [28] Fung, William, and David Hsieh, 2006, "Hedge Funds: An Industry in Its Adolescence," *Federal Reserve Bank of Atlanta Economic Review*, (Fourth Quarter), 91, 1-33
- [29] Gabaix, Xavier, Arvind Krishnamurthy and Olivier Vigneron, 2007, Limits of Arbitrage: Theory and Evidence from the MBS market, *Journal of Finance*, 62(2), 557-596.
- [30] Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M. Poteshman, 2005, Demand-Based Option Pricing, Wharton mimeo.
- [31] Gatev, Evan, and Phillip Strahan, 2006, "Banks' Advantage in Supplying Liquidity: Theory and Evidence from the Commercial Paper Market," *Journal of Finance*, Vol. 61, No. 2, pp. 867-92.
- [32] Geanakoplos, John, and Ana Fostel, 2008, "Leverage Cycles and the Anxious Economy," *American Economic Review*, Vol. 98, No. 4, pp. 1211-244.
- [33] Gromb, Denis and Dimitri Vayanos, 2002, Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs, *Journal of Financial Economics* 66, 361-407.
- [34] He, Zhiguo, and Arvind Krishnamurthy, 2011, "A Model of Capital and Crisis," forthcoming in *Review of Economic Studies*.
- [35] He, Zhiguo, InGu Khang and Arvind Krishnamurthy, 2010, "Balance Sheet Adjustments during the 2008 Crisis," *IMF Economic Review* 1, 118-156.
- [36] Holmstrom, Bengt and Jean Tirole, 1997, Financial Intermediation, Loanable Funds, and the Real Sector, *Quarterly Journal of Economics* 112, 663-691.
- [37] Huang, Jennifer and Jiang Wang, 2007, "Liquidity and Market Crashes," forthcoming, *Review of Financial Studies*.

- [38] Karatzas, I. and S. Shreve, 1998, *Methods of Mathematical Finance*. New York: Springer-Verlag.
- [39] Kiyotaki, N. and J. Moore, 1997, Credit Cycles, *Journal of Political Economy*, 105 (2), 211-248.
- [40] Kyle, Albert and Wei Xiong, 2001, "Contagion as a Wealth Effect," *Journal of Finance* 56, 1401-1440.
- [41] Lucas, Robert E., Jr, 1978, Asset Prices in an Exchange Economy, *Econometrica*, 46, 1429-1445,
- [42] Mankiw, Gregory N, and Stephen Zeldes, 1991, The Consumption of risky assetholders and Non-risky assetholders, *Journal of Financial Economics*, 29, pp. 97-112.
- [43] Mitchell, Mark, Lasse Pedersen, and Todd Pulvino, 2007, "Slow Moving Capital," *American Economic Review Papers and Proceedings*, forthcoming.
- [44] Pennacchi, George, 2006, "Deposit insurance, bank regulation and financial system risk," *Journal of Monetary Economics*, Vol. 53, No. 1, pp. 1-30.
- [45] Shleifer, Andrei and Robert W. Vishny, 1997, The Limits of Arbitrage, *Journal of Finance* 36, 35-55.
- [46] Vayanos, Dimitri, 2004, Flight to Quality, Flight to Liquidity, and the Pricing of Risk, working paper, LSE.
- [47] Vissing-Jorgensen, Annette, 2002, Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy*, 110(4), 825-853.
- [48] Wang, Jiang, 1996, "The Term Structure of Interest Rates in a Pure Exchange Economy with Heterogeneous Investors," *Journal of Financial Economics*, 41(1), 75-110.
- [49] Warther, Vincent, 1995, Aggregate mutual fund flows and security returns, *Journal of Financial Economics*, 39, 209-235.
- [50] Xiong, Wei, 2001, Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markets, *Journal of Financial Economics* 62, 247-292

A ODE Solution

In this appendix, we detail the ODE that characterizes the equilibrium.

In the main text we use x , which is the ratio of the specialist's wealth w to the price of risk asset price P , as the state variable. In writing down the ODE, it turns out the expressions are simpler by changing variables to an alternative state variable $y \equiv w^h/D$, which is households' wealth w^h scaled by the current dividend level D . Denote the price-dividend ratio with the alternative state variable as $F(y)$. Once we solve for the equilibrium price-dividend ratio $F(y)$, we can convert back to the original state variable x by

$$x = 1 - y/F(y), \quad (25)$$

and the price-dividend ratio $p(x)$ as a function of the fraction of specialists wealth x satisfies

$$F(y) = p(1 - y/F(y)).$$

When x ranges from 0 to 1, y takes value from $F(y_b)$ to 0, where the maximum households wealth $y_b = \frac{1+l}{\rho}$ as discussed in the text.

Denote the dynamics of y_t as,

$$dy_t = \mu_y dt + \sigma_y dZ_t, \quad (26)$$

for unknown functions μ_y and σ_y . We write $\frac{dc_t}{c_t}$ and dR_t as functions of μ_y, σ_y and the derivatives of $F(y)$. Due to market clearing, $c_t = D_t(1 + l - \rho y_t)$ (the simple expression for the equilibrium specialist's consumption is the reason why we work with the new state variable y), thus we have

$$\begin{aligned} \frac{dc_t}{c_t} &= \frac{dD_t}{D_t} - \frac{\rho dy}{1 + l - \rho y} - \frac{\rho}{1 + l - \rho y} \text{Cov}_t \left[dy, \frac{dD}{D} \right] \\ &= \left(g - \frac{\rho}{1 + l - \rho y} (\mu_y + \sigma_y \sigma) \right) dt + \left(\sigma - \frac{\rho \sigma_y}{1 + l - \rho y} \right) dZ_t. \end{aligned}$$

We also have

$$dR_t = \frac{dP_t + D_t dt}{P_t} = \left[g + \frac{F'}{F} \mu_y + \frac{1}{2} \frac{F''}{F} \sigma_y^2 + \frac{1}{F} + \frac{F'}{F} \sigma_y \sigma \right] dt + \left(\sigma + \frac{F'}{F} \sigma_y \right) dZ_t.$$

Substituting these expressions into (15) we obtain the following ODE,

$$\begin{aligned} &g + \frac{F'}{F} (\mu_y + \sigma_y \sigma) + \frac{1}{2} \frac{F''}{F} \sigma_y^2 + \frac{1}{F} \\ &= \rho + \gamma g - \frac{\gamma \rho}{1 - \rho y} (\mu_y + \sigma_y \sigma) + \gamma \left(\sigma - \frac{\rho \sigma_y}{1 - \rho y} \right) \left(\sigma + \frac{F' \sigma_y}{F} \right) - \frac{\gamma(\gamma + 1)}{2} \left(\sigma - \frac{\rho \sigma_y}{1 - \rho y} \right)^2. \end{aligned} \quad (27)$$

A.1 Derivation of μ_y and σ_y

We rewrite equation (11) which describes the wealth dynamics (budget constraint) of the household sector as:

$$dw^h = \theta_s dP + D \theta_s dt + r \hat{\theta}_b dt + l D_t dt - \rho w^h dt. \quad (28)$$

In this equation,

$$\theta_s = \alpha^I \alpha^h (1 - \lambda) \frac{w^h}{P} \quad (29)$$

are the number of shares that the risky asset household owns, and

$$\theta_b D = w^h - \theta_s P \quad (30)$$

is the amount of funds that the households have invested in the riskless bond, which includes the direct investment of debt households, the investment of risky asset households (when constraint is binding), and the indirect investment through intermediaries who short on bond in equilibrium. The portfolio choices α^h and α^I are defined in Section 3.2 in the main text and depends on whether the economy is constrained or not. Note that we have $\theta_b + F(y) \theta_s = y$ from (30).

We apply Ito's Lemma to $P = DF(y)$ to find expressions for the drift and diffusion of dP . We can then substitute back into equation (28) to find expressions for the drift and diffusion of dw^h .

Also, recall that we have defined $w^h = Dy$. We apply Ito's Lemma to this equation to arrive at a second expression for the drift and diffusion of dw^h . Matching the drift and diffusion terms from these two ways of writing dw^h , we solve to find μ_y and σ_y .

The result of this algebra is that:

$$\sigma_y = -\frac{\theta_b}{1 - \theta_s F'} \sigma,$$

and,

$$\mu_y + \sigma \sigma_y = \frac{1}{1 - \theta_s F'} \left(\theta_s + l + (r - g) \theta_b - \rho y + \frac{1}{2} \theta_s F'' \sigma_y^2 \right). \quad (31)$$

A.2 ODE

Substituting for $\mu_y + \sigma \sigma_y$ derived in (31) into (27), we find,

$$\begin{aligned} & \left(\frac{F'}{F} + \frac{\gamma \rho}{1 + l - \rho y} \right) \left(\frac{1}{1 - \theta_s F'} \right) (\theta_s + l + \theta_b (r - g) - \rho y) + \frac{1}{F} \\ & + \frac{1}{2} F'' \sigma_y^2 \left(\frac{1}{1 - \theta_s F'} \right) \left(\frac{1}{F} + \theta_s \frac{\gamma \rho}{1 + l - \rho y} \right) \\ = & \rho + g(\gamma - 1) + \gamma \left(\sigma - \frac{\rho}{1 + l - \rho y} \sigma_y \right) \left(\sigma + \frac{F'}{F} \sigma_y \right) - \frac{1}{2} \gamma (\gamma + 1) \left(\sigma - \frac{\rho}{1 + l - \rho y} \sigma_y \right)^2, \end{aligned}$$

where the endogenous interest rate r is determined by the following equation:

$$r = \rho + g\gamma - \frac{\rho\gamma}{1+l-\rho y} \frac{\theta_s + l + (r-g)\theta_b - \rho y + \frac{\sigma^2}{2}\theta_s F'' \frac{\theta_b^2}{(1-\theta_s F')^2}}{1-\theta_s F'}$$

$$- \frac{\gamma(\gamma+1)\sigma^2}{2} \left(1 + \frac{\rho\theta_b}{1+l-\rho y} \frac{1}{1-\theta_s F'} \right)^2.$$

We define a function, $G(y) \equiv \frac{1}{1-\theta_s F'}$, and hence $\sigma_y = -\frac{\theta_b}{1-\theta_s F'}\sigma = -\theta_b\sigma G$. Therefore we can rewrite our ODE as

$$F''\theta_s G^2 \frac{(\theta_b\sigma)^2}{2} G \left(\frac{1}{\theta_s F} + \frac{\gamma\rho}{1+l-\rho y} \right) = \rho + g(\gamma-1) - \frac{1}{F}$$

$$+ \frac{1}{2}\gamma\sigma^2 \left(1 + \frac{\rho}{1+l-\rho y}\theta_b G \right) \left(\frac{2(y-G\theta_b)}{\theta_s F} - (1+\gamma) \frac{1+l-\rho y + \rho G\theta_b}{1+l-\rho y} \right)$$

$$- \left(\frac{G-1}{\theta_s F} + \frac{\gamma\rho}{1+l-\rho y} G \right) (\theta_s + l + \theta_b(r-g) - \rho y),$$

and solve for r as

$$r = \frac{1}{1 + \frac{\rho\gamma G\theta_b}{1+l-\rho y}} \left[\rho + g\gamma - \frac{\rho\gamma G}{1+l-\rho y} \left(\theta_s + l - g\theta_b - \rho y + \frac{\sigma^2}{2}\theta_s G^2 F'' \theta_b^2 \right) - \frac{\gamma(\gamma+1)\sigma^2}{2} \left(1 + \frac{\rho\theta_b G}{1+l-\rho y} \right)^2 \right].$$

We combine these two pieces, use the relation $\theta_b = y - \theta_s F$ and $\theta_b \left(\frac{G-1}{\theta_s F} + \frac{\gamma\rho G}{1+l-\rho y} \right) = -\frac{y-G\theta_b}{\theta_s F} + \frac{1+l-\rho y + \gamma\rho G\theta_b}{1+l-\rho y}$, and arrive at the final expression of the ODE:

$$F''\theta_s G^2 \frac{(\theta_b\sigma)^2}{2} \frac{G}{\theta_s F} \left(\frac{1+l+\rho y(\gamma-1)}{1+l-\rho y + \rho\gamma G\theta_b} \right) \tag{32}$$

$$= \rho + g(\gamma-1) - \frac{1}{F} + \frac{\gamma(1-\gamma)\sigma^2}{2} \left(1 + \frac{\rho G\theta_b}{1+l-\rho y} \right) \frac{y-G\theta_b}{\theta_s F} \left[\frac{1+l-\rho y - \rho G\theta_b}{1+l-\rho y + \rho\gamma G\theta_b} \right]$$

$$- \left(\frac{(1+l-\rho y)(G-1)}{\theta_s F} + \gamma\rho G \right) \frac{\theta_s + l + \theta_b(g(\gamma-1) + \rho) - \rho y}{1+l-\rho y + \rho\gamma G\theta_b}.$$

The expressions for the bond holding θ_b and risky asset holding θ_s depend on whether the economy is constrained or not. In the unconstrained region, as shown in Section 3.2, $\alpha^h = 1$, and $\alpha^I = \frac{1}{x+(1-\lambda)(1-x)} = \frac{F}{F-\lambda y}$ using (25). Utilizing (30) and (29), we have $\theta_s = \frac{(1-\lambda)y}{F-\lambda y}$, and $\theta_b = \lambda y \frac{F-y}{F-\lambda y}$. In the constrained region $\alpha^h = \frac{m}{1-\lambda} \frac{x}{1-x} = \frac{m(F-y)}{(1-\lambda)y}$, $\alpha^I = \frac{1}{1+m} \frac{1}{x} = \frac{1}{1+m} \frac{F}{F-y}$, therefore $\theta_s = \frac{m}{1+m}$, and $\theta_b = y - \frac{m}{1+m}F$. Finally, as illustrated in Section 3.2, the cutoff for the constraint satisfies $\frac{1}{x^c} = 1 + \frac{m}{1-\lambda}$. Translating to y we have $y^c = \frac{m}{1-\lambda+m}F(y^c)$, and the economy is in the unconstrained region if $0 < y \leq y^c$.

A.3 Boundary conditions and technical parameter restriction

The upper boundary condition is described in Section 3.4. A lower boundary condition occurs when $y \rightarrow 0$. This case corresponds to one where specialists hold the entire financial wealth of the economy. Using L'Hopital's rule, it is easy to check that $\frac{G-1}{\theta_s F} \rightarrow \frac{F'(0)}{F(0)}$. Plugging this result into (32), and noting that both θ_s and θ_b go to zero as y goes to zero, we obtain,

$$F(0) = \frac{1 + F'(0)l}{\rho + g(\gamma - 1) + \frac{\gamma(1-\gamma)\sigma^2}{2} - \frac{l\gamma\rho}{1+l}}. \quad (33)$$

When $l = 0$, one can check that $F(0)$ is the equilibrium Price/Dividend ratio for the economy with the specialists as the representative agent. However because in our model the growth of the household sector affects the pricing kernel, this boundary P/D ratio $F(0)$ also depends on the household's labor income l . For the P/D ratio to be well defined at $y = 0$ (and thus, at $x = 1$) we require that parameters satisfy,

$$\rho + g(\gamma - 1) + \frac{\gamma(1-\gamma)\sigma^2}{2} - \frac{l\gamma\rho}{1+l} > 0.$$

Furthermore, a straightforward calculation yields that $F'(y^b) = 1$ if $F(y^b) = y^b$. This result also ensures that at the boundary y_b the mapping from the scaled household's wealth y to the fraction of the specialists wealth $x = w/P = 1 - y/F(y)$ is monotonely decreasing, as $x'(y) = (F(y) - yF'(y))/F^2(y)$ attaches zero when $y \rightarrow y^b$.

A.4 Numerical Method

In our ODE (32) both boundaries are singular, causing difficulties in directly applying the built-in ODE solver *ode15s* in *Matlab*. To overcome this issue, we approximate the upper-end boundary $(y^b, F(y^b) = y^b)$ by $(y^b - \eta, y^b - \eta)$ (where η is sufficiently small), and adopt a ‘‘forward-shooting and line-connecting’’ method for the lower-end boundary. Take a small $\epsilon > 0$ and call \tilde{F} as the attempted solution. For each trial $\phi \equiv \tilde{F}'(\epsilon)$, we set $\tilde{F}'(0) = \phi$, solve $\tilde{F}(0)$ based on (33), and let $\tilde{F}(\epsilon) = \tilde{F}(0) + \phi\epsilon$. Since $(\epsilon, \tilde{F}(\epsilon))$ is away from the singularity, by trying different ϕ 's we apply the standard shooting method to obtain the desired solution F that connects at $(y^b - \eta, y^b - \eta)$. For $y < \epsilon$, we simply approximate the solution by a line connecting $(0, F(0))$ and $(\epsilon, F(\epsilon))$. In other words, we solve F on $[\epsilon, y^b]$ with a smooth pasting condition for $F'(\epsilon) = \frac{F(\epsilon) - F(0)}{\epsilon}$ and a value matching condition for $F(y^b) = y^b$.

We use $\epsilon = 0.25$ and $\eta = 0.001$ which give ODE errors bounded by 3×10^{-5} for $y > \epsilon$. Different ϵ 's and η 's deliver almost identical solutions for $y > 1$. Because we are mainly interested in the solution behavior near y^c (which takes a value of 63.53 in the benchmark case) and onwards, our main calibration results are free of the approximation errors caused by the choice of ϵ and η . Finally we find that, in fact, these errors are at the same magnitude as those generated by the capital

constraint around y^c (3.5×10^{-5}).

A.5 Simulation

With the ODE solutions in hand, we numerically simulate the model to obtain the steady state distribution of the state variable x as well as a number of asset price measurements. We begin the economy at a state ($x_0 = x^c, D_0 = 1$) and simulate the economy for 5000 years. That is we obtain a sequence of independent draws from the normal distribution and use these draws to represent innovations in our shock process Z_t . The path of Z_t can then be mapped into a path of the state variable. We compute the time-series averages of a number of relevant asset price measurements from years 1000 to 5000 of this sample. The simulation unit is monthly, and based on those monthly observations we compute annual averages. We repeat this exercise 5000 times, averaging across all of the simulated Z_t paths. We find that changing the starting value x_0 does not affect the computed distribution or any of the asset price measurements, indicating that the distribution is an accurate representation of the steady state distribution of the economy.

B Verification of optimality

In this section we take the equilibrium Price/Dividend ratio $F(y)$ as given, and verify that the specialist's consumption policy $c = D_t(1 + l - y_t)$ is optimal subject to his budget constraint. Our argument is a variant of the standard one: it uses the strict concavity of $u(\cdot)$ and the specialist's budget constraint to show that the specialist's Euler equation is necessary and sufficient for the optimality of his consumption plan.

Specifically, fixing $t = 0$ and the starting state (y_0, D_0) , define the pricing kernel as

$$\xi_t \equiv e^{-\rho t} c_t^{-\gamma} = e^{-\rho t} D_t^{-\gamma} (1 + l - \rho y_t)^{-\gamma}.$$

Consider another consumption profile \hat{c} which satisfies the budget constraint $E \left[\int_0^\infty \hat{c}_t \xi_t dt \right] \leq \xi_0 D_0 (F_0 - y_0)$ (recall that the specialist's wealth is $D_0 (F_0 - y_0)$; here we require that the specialist's feasible trading strategies be well-behaved, e.g., his wealth process remains non-negative). Then we have

$$\begin{aligned} E \left[\int_0^\infty e^{-\rho t} u(c_t) dt \right] &\geq E \left[\int_0^\infty e^{-\rho t} u(\hat{c}_t) dt \right] + E \left[\int_0^\infty e^{-\rho t} u'(c_t) (c_t - \hat{c}_t) dt \right] \\ &= E \left[\int_0^\infty e^{-\rho t} u(\hat{c}_t) dt \right] + E \left[\int_0^\infty \xi_t c_t dt \right] - E \left[\int_0^\infty \xi_t \hat{c}_t dt \right]. \end{aligned}$$

If the specialist's budget equation holds in equality for the equilibrium consumption process c , i.e., if

$$E \left[\int_0^\infty \xi_t c_t dt \right] = \xi_0 D_0 (F_0 - y_0),$$

then the result follows. Somewhat surprisingly, for our model this seemingly obvious claim requires an involved argument because of the singularity at $y^b = \frac{1+l}{\rho}$.

One can easily check that, for $\forall T > 0$, we have

$$\xi_0 D_0 (F_0 - y_0) = \int_0^T c_t \xi_t dt + \int_0^T \sigma (D_t, y_t) dZ_t + \xi_T D_T (F_T - y_T), \quad (34)$$

where $\sigma (D_t, y_t)$ corresponds to the specialist's equilibrium trading strategy (which involves terms such as $(1 + l - \rho y)^{-\gamma-1}$ and is NOT uniformly bounded as $y \rightarrow y^b$). Our goal in the following steps is to show that in expectation, the latter two terms vanishes when $T \rightarrow \infty$.

Step 1: Limiting Behavior of y at y^b The critical observation regarding the evolution of y is that when y approaches y^b , it approximately follows a Bessel process with a dimension $\delta = \gamma + 2 > 2$. (Given a δ -dimensional Brownian motion Z , a Bessel process with a dimension δ is the evolution of $\|Z\| = \sqrt{\sum_{i=1}^{\delta} Z_i^2}$, which is the Euclidean distance between Z and the origin.) According to standard results on Bessel processes, y^b is an entrance-no-exit point, and is not reachable if the starting value $y_0 < y^b$ (if $\delta > 2$). Intuitively, when y is close to y^b , the dominating part of μ_y is proportional to $\frac{1}{y-y^b} < 0$, while the volatility σ_y is bounded— therefore a drift that diverges to negative infinity keeps y away from the singular point y^b . This result implies that our economy never hits y^b .

To show that for y close to y^b , y 's evolution can be approximated by a Bessel Process, one can easily check that when $y \rightarrow y^b$,

$$r \simeq -\frac{(\gamma + 1) \sigma^2}{2} \frac{\rho^h \hat{\theta}_b G}{1 + l - \rho^h y}, \mu_y \simeq -\frac{(\gamma + 1) \sigma^2}{2} \frac{\rho^h \hat{\theta}_b^2 G^2}{1 + l - \rho^h y}, \sigma_y = -G \sigma \hat{\theta}_b;$$

and therefore

$$dy = -\frac{(\gamma + 1) \sigma^2}{2} \frac{\rho \hat{\theta}_b^2 G^2}{1 + l - \rho y} dt - G \sigma \hat{\theta}_b dZ_t.$$

Utilizing the result $F'(y^b) = 1$ established in Section 3.4, we know that when $y \rightarrow y^b$, $\hat{\theta}_b \simeq F - \theta_s y \simeq \frac{1}{1+m} y^b = \frac{1}{1+m} \frac{1+l}{\rho}$, and $G \simeq 1 + m$. Let

$$z_t = 1 + l - \rho y_t;$$

then it is easy to show that $q = \frac{z}{G \sigma \hat{\theta}_b \rho} = \frac{z}{\sigma(1+l)}$ evolves approximately according to

$$dq_t = -\frac{1}{G \sigma \hat{\theta}_b} dy_t = \frac{(\gamma + 1)}{2q_t} dt + dZ_t,$$

which is a standard Bessel process with a dimension $\delta = \gamma + 2$. Therefore, z is also a scaled version of a Bessel process, and can never reach 0 (or, y cannot reach y^b). In the following analysis, we focus on the limiting behavior of z .

Step 2: Localization Note that in (34), due to the singularity at $y = y^b$, both the local martingale part $\int_0^T \sigma(D_t, y_t) dZ_t$ and the terminal wealth part $\xi_T D_T (F_T - y_T)$ are not well-behaved. To show our claim, we have to localize our economy, i.e., stop the economy once either y is sufficiently close to y^b or D is sufficiently close to 0. Specifically, we define

$$T_n = \inf \left\{ t : \text{either } z_t = n^{-1} \text{ or } D_t = n^{-h} \right\}$$

where h is a positive constant (as we will see, the choice of h , which is around 1, gives some flexibility for γ other than 2). Here, because y and z have a one-to-one relation ($z = 1 + l - \rho y$), for simplicity we localize z instead.

Clearly this localization technique ensures that the local martingale part $\int_0^{T_n} \sigma(D_t, y_t) dZ_t$ is a martingale (one can check that $\sigma(D_t, y_t)$ is continuous in D_t and y_t , in turn D_t and x_t ; therefore $\sigma(D_t, y_t)$ is bounded). As $T_n \rightarrow \infty$ when $n \rightarrow \infty$, for our claim we need to show

$$\lim_{n \rightarrow \infty} E [\xi_{T_n} D_{T_n} (F_{T_n} - y_{T_n})] = 0$$

We substitute from the definition of ξ :

$$E \left[e^{-\rho T_n} D_{T_n}^{1-\gamma} z_{T_n}^{-\gamma} (F(y_{T_n}) - y_{T_n}) \right] \leq E \left[e^{-\rho T_n} n^{h(\gamma-1)} z_{T_n}^{-\gamma} (F(y_{T_n}) - y_{T_n}) \right].$$

Since the analysis will be obvious if $z^{-\gamma} (F(y) - y)$ is uniformly bounded (notice here $z = 1 + l - \rho y$), it is sufficient to consider $z_{T_n} = \frac{1}{n}$ (or, $y_{T_n} = y^b - \frac{1}{n\rho}$). Because $F(y^b) = y^b$ and $F'(y^b) = 1$, by Taylor expansion we know that $F\left(y^b - \frac{1}{n\rho}\right) - \left(y^b - \frac{1}{n\rho}\right)$ can be written as $\psi(n) \frac{1}{n}$ when n is sufficiently large, and $\psi(n) \rightarrow 0$ as $n \rightarrow \infty$. Therefore we have to show that, as $n \rightarrow \infty$,

$$E \left[e^{-\rho T_n} n^{(\gamma-1)(1+h)} \right] \psi(n) \rightarrow 0$$

and a sufficient condition is that there exists some constant M so that

$$E \left[e^{-\rho T_n} \right] n^{(\gamma-1)(1+h)} \rightarrow M.$$

We apply existing analytical results in the literature to show our claim. To do so, we have to separate our two state variables. We define

$$T_n^D = \inf \left\{ t : D_t = n^{-h} \right\}, T_n^x = \inf \left\{ t : z_t = n^{-1} \right\}.$$

We want to bound $E[e^{-\rho T_n}]$ by the sum of $E[e^{-\rho T_n^D}]$ and $E[e^{-\rho T_n^x}]$. The Laplace transform of T_n is simply

$$E[e^{-\rho T_n}] = \int_0^\infty e^{-\rho T} d\mathbf{F}(T) = \rho \int_0^\infty e^{-\rho T} \mathbf{F}(T) dT,$$

where the bold \mathbf{F} denotes the distribution function of T_n . The similar relation also holds for T_n^D or

T_n^z . Denote $\mathbf{F}^D(\cdot)$ (or $\mathbf{F}^z(\cdot)$) as the distribution function for T_n^D (or T_n^z), and notice that

$$\begin{aligned} 1 - \mathbf{F}(T) &= \Pr(T_n > T) = \Pr(T_n^D > T, T_n^z > T) > \Pr(T_n^D > T) \Pr(T_n^z > T) \\ &= 1 - \mathbf{F}^D(T) - \mathbf{F}^z(T) + \mathbf{F}^D(T) \mathbf{F}^z(T), \end{aligned}$$

because $\mathbf{1}_{\{T_n^D > T\}}$ and $\mathbf{1}_{\{T_n^z > T\}}$ are positively correlated (both take the value 1 when the Brownian Z is high).²² Therefore $\mathbf{F}(T) < \mathbf{F}^D(T) + \mathbf{F}^z(T)$, or

$$E[e^{-\rho T_n}] n^{(\gamma-1)(1+h)} < E[e^{-\rho T_n^D}] n^{(\gamma-1)(1+h)} + E[e^{-\rho T_n^z}] n^{(\gamma-1)(1+h)}.$$

Our goal is to show the right hand side of the above inequality goes to zero when $n \rightarrow \infty$.

There are two terms in the right hand side of the above inequality. For the first term, we can use the standard result of the Laplace transform of the first-hitting time distribution for a GBM process (e.g., Borodin and Salminen (2002), page 622):

$$E[e^{-\rho T_n^D}] = n^{-\frac{h}{\sigma^2}} \left(\sqrt{2\rho\sigma^2 + (g-0.5\sigma^2)^2} + g - 0.5\sigma^2 \right).$$

Thus, by choosing some appropriate h so that

$$\frac{h}{\sigma^2} \left(\sqrt{2\rho\sigma^2 + (g-0.5\sigma^2)^2} + g - 0.5\sigma^2 \right) > (\gamma-1)(1+h),$$

the first term $E[e^{-\rho T_n^D}] n^{(\gamma-1)(1+h)}$ vanishes as $n \rightarrow \infty$. For instance, this condition holds when $h = 0.9$ under our parameterization. The next step is for the second term.

Step 3: Regulated Bessel Process For the second term $E[e^{-\rho T_n^z}] n^{(\gamma-1)(1+h)}$, because our economy (i.e., evolution of z) differs from the evolution of a Bessel process when z is far away from 0, an extra care needs to be taken. We consider a regulated Bessel process which is reflected at some positive constant \bar{z} . Intuitively, by doing so, we are putting an upper bound for $E[e^{-\rho T_n^z}]$, as the reflection makes z_t to hit n^{-1} more likely (therefore, a larger \mathbf{F}^z). Also, for a sufficiently small $\bar{z} > 0$, when $z \in (0, \bar{z}]$, z follows a Bessel process with a dimension $\gamma + 2 - \varepsilon$. Therefore, \mathbf{F}^z must be bounded by the first-hitting time distribution of a Bessel process with a dimension δ , where δ takes value from $\gamma + 2 - \varepsilon$ to $\gamma + 2$, where ε is sufficiently small. Finally, note that by considering a Bessel process we are neglecting certain drift for z . However, one can easily check that when z is close to 0, the adjustment term for μ_y is $-\frac{1+l}{\rho}\gamma\sigma^2 < 0$. This implies that we are neglecting a positive drift

²²Technically, using the technique of Malliavian derivatives, we can show that both z_s and D_s have positive diffusions in the martingale representations for all s . Then, the running minimum $\underline{z}_T = \min\{z_t : 0 < t < T\}$ and $\underline{D}_T = \min\{D_t : 0 < t < T\}$ have positive loadings always on the martingale representations (using the technique in *Methods of Mathematical Finance*, Karatzas and Shreve (1998), Page 367). The same technique can be applied to $\mathbf{1}_{\{T_n^z > T\}} = \mathbf{1}_{\{\underline{z}_T > T\}}$ and $\mathbf{1}_{\{T_n^D > T\}} = \mathbf{1}_{\{\underline{D}_T > T\}}$, as an indicator function can be approximated by a sequence of differentiable increasing functions.

for z —which potentially makes hitting less likely—thereby yielding an upper-bound estimate.

We have the following Lemma from the Bessel process.

Lemma 1 *Consider a Bessel process x with $\delta > 2$ which is reflected at $\bar{x} > 0$. Let $\nu = \frac{\delta}{2} - 1$. Starting from $x_0 \leq \bar{x}$, we consider the hitting time $T_n^x = \inf \{t : x_t = \frac{1}{n}\}$. Then we have*

$$E \left[e^{-\rho T_n^x} \right] \propto n^{-2\nu} \text{ as } n \rightarrow \infty$$

Proof. Due to the standard results in Bessel process and the Laplace transform of the hitting time (e.g., see Borodin and Salminen (1996), Chapter 2), we have

$$E \left[e^{-\rho T_n^z} \right] = \varphi(z_0) / \varphi(n^{-1}),$$

where

$$\varphi(u) = c_1 u^{-\nu} I_\nu(\sqrt{2\rho}u) + c_2 u^{-\nu} K_\nu(\sqrt{2\rho}u),$$

and $I_\nu(\cdot)$ (and $K_\nu(\cdot)$) is modified Bessel function of the first (and second) kind of order ν . Because R is a reflecting barrier, the boundary condition is

$$\varphi'(\bar{z}) = 0,$$

which pins down the constants c_1 and c_2 (up to a constant multiplication; notice that this does not affect the value of $E[e^{-\rho T_n^z}]$). Therefore the growth rate of $E[e^{-\rho T_n^z}]$ is determined by $n^\nu K_\nu(\sqrt{2\rho}n^{-1})$ as K_ν dominates I_ν near 0. Since $K_\nu(z)$ has a growth rate $z^{-\nu}$ when $z \rightarrow 0$, the result is established.

■

For any y_0 , redefine starting point as $z_0 = \min(1 + l - y_0, \bar{z})$; clearly this leads to an upper-bound estimate for $E[e^{-\rho T_n^z}]$. However, since for all $\delta \in [\gamma + 2 - \epsilon, \gamma + 2]$, the above Lemma tells us that for any $\epsilon \in [0, \epsilon]$, given $\gamma = 2$ and $h = 0.9$, when $n \rightarrow \infty$, for some sufficiently small $\epsilon > 0$ we have

$$n^{(\gamma-1)(1+h)} E[e^{-\rho T_n^z}] \propto n^{(\gamma-1)(1+h)} n^{-2\nu} = n^{(\gamma-1)(1+h) - \gamma + \epsilon} \rightarrow 0 \text{ uniformly.}$$

Therefore we obtain our desirable result.

Finally $c_t \xi_t > 0$ implies that $\int_0^\infty c_t \xi_t dt$ converges monotonically, and therefore the specialist's budget equation $\lim_{T \rightarrow \infty} E \left[\int_0^T \xi_t c_t dt \right] = \xi_0 D_0 (F_0 - y_0)$ holds for all stopping times that converge to infinity. Q.E.D.

C Appendix for Section 6

C.1 Borrowing Subsidy

We have the same ODE as in Appendix A. The only difference is that

$$\mu_y = \frac{1}{1 - \theta_s F'} \left(\theta_s + l + (r + \sigma^2 - g) \hat{\theta}_b - \hat{\theta}_b \Delta r - \rho y + \frac{1}{2} \theta_s F'' \sigma_y^2 \right).$$

C.2 Direct Asset Purchase

In this case, the intermediary holds $1 - s$ of the risky asset (where s is a function of (y, D)). In the unconstrained region, $\alpha^h = 1$, and

$$\frac{\alpha^I (w + \alpha^h (1 - \lambda) w^h)}{P} = 1 - s$$

which implies that $\alpha^I = \frac{(1-s)F}{F-\lambda y}$. Therefore the households' holding of the risky asset through intermediaries is

$$\theta_s^I = \frac{(1-s)(1-\lambda)y}{F-\lambda y},$$

and the total holding is $\theta_s = \theta_s^I + s = \frac{(1-s)(1-\lambda)y}{F-\lambda y} + s(y, D)$.

In the constrained region, $\alpha^h = \frac{m(F-y)}{(1-\lambda)y}$ and $\alpha^I = \frac{1}{1+m} \frac{(1-s)F}{F-y}$. So

$$\theta_s^I = \frac{m(F-y)}{(1-\lambda)y} \frac{1}{1+m} \frac{(1-s)F}{F-y} (1-\lambda) \frac{y}{F} = \frac{m}{1+m} (1-s)$$

and the total holding is

$$\theta_s = \frac{m}{1+m} (1-s) + s = \frac{m+s}{1+m}.$$

The same constraint cutoff applies $y^c = \frac{m}{1-\lambda+m} F^c$.

Finally, the expressions for the case of capital infusion (i.e., changing m) is isomorphic to the case of $s > 0$. This is because given s we can find some appropriate $m'(s) = \frac{s+m}{1-s}$ such that $\frac{m'}{1+m'} = \frac{m+s}{1+m}$.