Regulating Exclusion from Financial Markets*

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Abstract

We study optimal enforcement in credit markets in which the only threat facing a defaulting borrower is restricted access to financial markets. We solve for the optimal level of exclusion, and link it to observed institutional arrangements. Regulation in this environment must accomplish two objectives. First, it must prevent borrowers from defaulting on one bank and transferring their resources to another bank. Second, and less obviously, it must give banks the incentive to make sizeable loans, and to honor their promises of future credit. We establish that the optimal regulation resembles observed laws governing default on debt. Moreover, if debtors have the right to a "fresh start" after bankruptcy then this must be balanced by enforceable provisions against fraudulent conveyance. Our optimal regulation is robust, in that it can be implemented in a way that does not require the regulator to have information about either the borrower or lender. Finally, restricting the availability of credit to a defaulted borrower is not a threat, in and of itself, that motivates borrowers to repay loans.

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1 Introduction

We study credit markets in which borrowers have no collateral. A significant amount of lending in developing countries is unsecured by collateral, for the simple reason that borrowers often lack collateral. Even in developed countries (where collateralizable assets are more plentiful) consumer credit and educational loans are typically unsecured. In the sovereign debt context it has long been recognized that since creditors have no ability to seize the vast majority of a defaulting debtor's assets, loans are effectively unsecured. In light of recent discussions of the IMF-proposed sovereign bankruptcy court[1] this distinction vis-a-vis corporate debt takes on new importance. In each case, accounts of lending typically emphasize that the primary inducement for repayment is the restricted access to financial markets that follows default.[2]

Squaring these accounts with economic theory has long proved surprisingly challenging. Observationally, the clearest restriction placed on defaulted borrowers is a loss of access to credit markets. But writing in the context of sovereign lending Bulow and Rogoff (1989b) establish that the threat of losing access to credit markets, in and of itself, can play no role in ensuring loan repayment.[3] At the other extreme of the spectrum of possible financial threats lies the prospect of complete and indefinite exclusion from financial markets. Following Kehoe and Levine (1993), a number of recent papers have explored the financial arrangements that can be supported by this threat. But while *theoretically* this much more extreme threat can (unsurprisingly) support credit markets, *empirically* this threat appears uninterpretable. Clearly no institution exists to deny a debtor all future access to financial markets upon default.[4]

In this paper, we tackle these issues by considering the following pair of closely related questions. First, what level of financial exclusion is required to support active credit markets? Second, how is the required level of financial exclusion implemented

[1]See, for example, Rogoff and Zettelmeyer (2002) or Bolton (2002).

[2]In group lending in developing countries, such as by the Grameen Bank in Bangladesh, it is the whole group that faces financial exclusion. Other lending programs in developing countries offer only individual loans. Indonesia's Bank Rakyat Indonesia is a prominent example. Again, borrowers have little or no collateral, and repayment incentives are perceived to stem from the "carrot" of continued access to financial services. Morduch (1999) provides an excellent overview of both group- and individual-based approaches in developing country microfinance.

[3]Specifically, they show that if the borrower can save at the same rate as the lender then credit denial is useless as a means of ensuring loan repayment. The reason is simple: rather than repay one dollar today to obtain a future loan, the borrower would rather just save the dollar and effectively self-finance the promised future loan. The same argument can be found, in different contexts, in Chari and Kehoe (1993a) and Krueger and Uhlig (2000).

[4]Even with regard to exclusion from credit markets, which at first seems easier to interpret, credit records often have only limited "memory"; in the United States credit records cover only the most recent seven years. Motivated in part by this observation, authors such as Azariadas and Lambertini (forthcoming) have examined the effects of financial market exclusion of shorter durations.

institutionally?

Answering these questions should advance our understanding of credit markets in a number of contexts. Many recent papers study risk sharing in models in which the threat facing defaulting borrowers is financial exclusion. Ligon *et al* (forthcoming), Fafchamps (1999), and Attanasio and Ríos-Rull (2000) are examples that study risk sharing in village economies. There is also a sizeable (and growing) literature that studies the implications for risk sharing and asset pricing in developed economies, where the credit constraint is derived from the threat of financial exclusion (see, e.g., Kocherlakota 1996 and Alvarez and Jermann 2000, 2001). In assessing this research, one inevitably asks how broadly applicable these results are. Are they most applicable when the threats are self-enforcing, as in small and tightly knit communities (see, e.g., Fafchamps)?[5] Does this mean that looking to the future of these village economies, credit markets will break down? In this paper, we provide an implementation of the threat of financial exclusion that we argue is both theoretically and empirically satisfying. Our results suggest that the threat of financial exclusion is more broadly applicable. This has implications for both the scope of the risk-sharing analyses, as well as legal reforms in debt markets.

We study a multi-period environment in which a single borrower without collateral seeks credit from one of multiple banks. As discussed above, the only punishment that the borrower can be threatened with is some level of exclusion from financial markets. Instead of restricting study to a single specification of exclusion, we start by describing the complete class of exclusions with which a borrower can be threatened. Credit denial and complete exclusion forever following default are just two members of this large class of exclusions. We establish that many members of this class implement the constrained optimal solution.

We then focus on a specification of exclusion that we term the "debt-default" rule. In words, the debt-default rule is that a delinquent borrower is excluded from holding cash deposits with any bank other than his creditor bank. This restriction is lifted once he has repaid his creditor bank. Over the course of the paper, we make the following three arguments for why this form of exclusion is of special interest.

First, although our debt-default exclusion rule appears very lenient compared to extremes such as complete exclusion forever, in fact it provides an inducement to repay that is just as strong. The debt-default rule prohibits a delinquent borrower from placing savings with another bank. On the other hand, full exclusion is a prohibition against both placing savings and accepting credit from other banks. Loosely speaking, since the prohibition against future credit has no bite in our environment – for reasons similar to that in Bulow and Rogoff (1989b) – the prohibition against savings is as stringent a

[5]See also the work of Chari and Kehoe (1993b) and Kletzer and Wright (2000), who establish a folk-theorem result that lenders can together implement the desired level of exclusion in an infinite horizon setting. We discuss our relation to these papers in more detail on page 4 below.

punishment as full exclusion.

Second, the optimal pattern of lending and repayment entails banks making an initial loan, followed by new loans in subsequent periods. An effective specification of exclusion must also provide banks with the incentive to make all of these loan payments. This leads to another desirable property of the debt-default exclusion rule. Specifications of exclusion such as that of complete and indefinite exclusion do not in general provide such incentives: a borrower who makes one repayment remains under the threat of full exclusion, and so banks receive subsequent payments *whether or not* they make additional loans. In contrast, under the debt-default specification of exclusion a borrower who repays faces no subsequent threat of exclusion unless he receives a new loan. Given this, a bank receives no subsequent payments from the borrower unless it extends a new loan.

These last two points relate to the theoretical desirability of the debt-default exclusion rule relative to other specifications of exclusion. To state our results in more precise terms, we show that the debt-default rule lies in the subclass of rules that provides optimal repayment incentives, and in the even smaller subclass of rules that also provide optimal incentives for banks to grant sizeable loans. Moreover, we establish that the only specifications of exclusion which meet both these criteria are those that are close to our debt-default rule in the sense that a borrower must have the right to repay his debt and regain full access to financial markets. Loosely speaking, we show that effective exclusion rules grant both creditor rights — i.e. the right to impose effective sanctions following non-repayment — as well as debtor rights — i.e. the right of the borrower to escape from the threat of sanctions by repaying his loan.

Our third main argument relates to the interpretability of the debt-default exclusion rule. We have already observed that it entails a much less extreme threat of exclusion than some other possibilities. The debt-default rule also has the advantage that it resembles laws governing default on debt contracts. At a basic level, the restriction that a delinquent borrower cannot hold savings with other banks corresponds to the court-enforced ability of a lender to garnish any assets such a borrower holds elsewhere in the formal financial sector, up to the point where the debt is repaid. In practice, however, many countries grant borrowers the right to declare bankruptcy and gain a "fresh start."[6] The prohibition against fraudulent conveyance that is encoded in U.S. bankruptcy law prevents abuse of the fresh start provision. For instance, Baird (1993, page 143) summarizes prohibitions against fraudulent conveyance as follows: "Transfers made and obligations incurred with the intent to delay, hinder, or defraud creditors are fraudulent and void as against creditors." Thus, if the borrower declares bankruptcy and hides some assets in bankruptcy proceedings which he subsequently transfers back into the formal financial sector, then this constitutes a fraudulent conveyance. Lenders

[6] In the U.S. this right is available to individual borrowers under Chapter 13.

are permitted to seize these fraudulently hidden assets, even after bankruptcy. A defaulted borrower is thereby prohibited from saving within the financial system. To summarize, the debt-default rule is consistent with the fact that laws governing default and bankruptcy strip a borrower of savings but place no formal restriction on future access to credit.

Our paper belongs to the literature on dynamic credit relationships (see e.g., Diamond 1989, Thomas and Worrall 1994, DeMarzo and Fishman 2000, Albuquerque and Hopenhayn 2000, Bulow and Rogoff 1989a, Kehoe and Levine 1993, Kocherlakota 1996 and Alvarez and Jermann 2000). Since we study an environment in which the only threat facing a defaulted borrower is a restriction on access to borrowing and lending markets, we are closest to Kehoe and Levine (1993) and Kocherlakota (1996). Relative to these papers our focus is on the implementation and interpretation of the exclusion threat in terms of standard debt contracts, rather than on questions concerning risk-sharing and asset-pricing. Our results on debt contracts are related to the work of Hart and Moore (1994, 1998), Aghion and Bolton (1992), Bolton and Scharfstein (1990) and Gromb (1999). In particular, we share with the last two of these papers the focus on credit denial. In the Bolton and Scharfstein model, the borrower has a future project which the lender commits to funding if the borrower repays the original loan (Gromb[7] extends the Bolton and Scharfstein model to to a multi-period setting). If the borrower does not repay the loan, he is restricted from using his limited collateral to borrow from another lender. This effectively means that he is denied access to his project, and this threat enables repayment. In contrast, in our environment, there is no collateral, and the only punishment involves regulating access to financial markets.

A chief goal of our work is to provide some insight as to how exclusion is institutionally implemented. Reflecting this, our approach is fundamentally different from the related work of Chari and Kehoe (1993b) and Kletzer and Wright (2000). These papers establish folk-theorem style results showing that full exclusion can be implemented in an infinite horizon game; basically, if a lender deals with a borrower following default, other lenders then punish the deviating lender. While important theoretically, by their nature these results do not lend themselves to an interpretation in terms of observed legal arrangements.

The paper proceeds as follows. Section 2 describes the economy to be analyzed, and computes the constrained efficient outcome for the case where banks can completely coordinate their actions to enforce financial exclusion. Section 3 shows that when banks compete, and there is no central enforcement, then there is no possibility of credit. This generalizes the result of Bulow and Rogoff (1989b). Section 4 defines a general class of enforcement rules, and looks at four leading examples in detail. We show

[7]Like us, Gromb is also concerned with commitment on the lender side. However, while we are concerned that the lender is tempted to punish too often, the lender in Gromb's paper faces the opposite problem in that it is tempted to punish too little.

that a range of different enforcement rules give the borrower optimal incentives repay — though some are more easily interpretable than others. In Sections 5 and 6 we examine the provision of lending incentives to the banks in our environment. We characterize the class of enforcement rules that provide optimal incentives to both the borrower and the banks. In Section 7 we discuss the potential applicability of our analysis.

2 A benchmark: Constrained efficient outcomes

Time periods are indexed $t = 0, 1, \ldots, T$, where $T \geq 2$. There is a borrower B and M banks, $1, \ldots, M$. The borrower has access to a valuable production technology, but has limited funds of his own with which to invest. His resources entering date t are denoted W_t (endogenously determined), while resources at date 0 are exogenously given as W_0. Banks face no resource constraint and have outside investment opportunities that always yield a return of $r > 0$. We assume that this is in the form of a storage technology with return of r. Importantly, storage at rate r is only an option for banks. The production opportunity for borrowers is constant returns to scale and deterministically yields R_t between periods t and $t+1$. Let ρ_t denote the ratio of these return rates, $\rho_t \equiv R_t/r$. Finally, both the borrower and bank are risk neutral and take as their objective the maximization of date T wealth.

We refer to any period $t < T$ in which the borrower's return exceeds the bank's return (i.e. $R_t \geq r$) as an *investment* period, and any period $t < T$ in which the bank's return exceeds the borrower's return (i.e. $r > R_t$) as a *payment* period. Since the only incentive compatible payment in period T is from the bank to the borrower, we refer to this period as an investment period also. We assume throughout that period 0 is an investment period.

During each period t, first each bank m simultaneously makes a payment to the borrower, $L_t^m \geq 0$. Second, the borrower simultaneously makes a payment to each bank m, $P_t^m \geq 0$. We refer to the bank payments L_t^m as "loans" and the borrower payments P_t^m as "payments" or "repayments". We require that all payments P_t^m be feasible for the borrower, in the sense of being less than his resources, $\sum_m P_t^m \leq W_t + \sum_m L_t^m$. Note that we require all payments to be weakly positive — no agent can unilaterally seize resources from another agent. Let $\mathbf{P}_t$ and $\mathbf{L}_t$ respectively denote the M-vectors of payments P_t^m and loans L_t^m.

In order to examine the borrower's repayment incentives we begin by focusing on the situation where only the borrower faces a commitment problem. In Sections 5 and 6 we turn to the problem of providing banks with lending incentives, and relax the commitment assumption. That is, for now we assume while the borrower can always choose not to make a promised payment P_t^m, each bank m can fully commit to make a future payment L_t^m. Thus, at date 0 each bank m can commit to make a payment at time $t = 0, \ldots, T$ of $l_t^m (\mathbf{P}_0, \ldots, \mathbf{P}_{t-1})$.

In this section we characterize the maximal level of borrower consumption subject to the banks collectively making a return of at least r (in per-period terms). For this exercise, we can assume without loss that if the borrower ever fails to make a promised payment he suffers the maximal punishment available in our environment, namely that he receive no further payment from any bank.[8] Moreover, we can further assume without loss that the borrower deals only with one of the banks. Consequently, we omit the superscript m from our notation whenever possible.

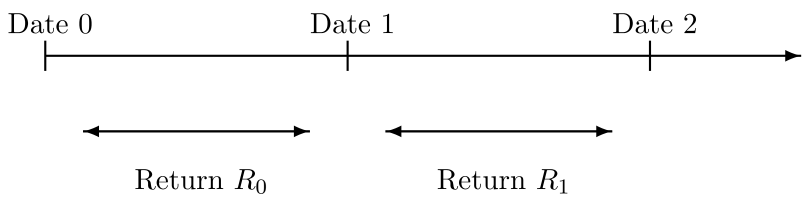

Figure 1: Timeline for $T = 2$

To build intuition, we begin by considering the simplest possible version of our environment — just three periods (i.e. $T = 2$, see Figure 1). Note that since date 2 is the terminal date, the borrower will clearly never hand over any resources at that date. On the other hand, at date 1 the banks can induce the borrower to make a payment by promising to make a payment in return (a new "loan") at date 2. That is, the borrower can be induced to make a date 1 payment by the threat that he will otherwise be denied "credit" at date 2.

Suppose the borrower arrives at date 1 with a wealth level of W_1. We denote by V_1^M the highest level of profits (in present value terms) that the bank can collectively make from date 1 onwards. There are two distinct cases to consider.

On the one hand, if $R_1 \geq r$ then the banks will have to promise $\$R_1$ for every \$1 they want to the borrower to pay them at date 1. Since the banks' cost of funds is $r \leq R_1$, they will lose money doing this. So the banks cannot make a profit in this case, i.e. $V_1^M = 0$.

On the other hand, if $R_1 < r$ then promising $\$R_1$ for every \$1 the borrower hands over at date 1 is attractive for the banks. Their profits are maximized when they induce the borrower to hand over all his wealth at date 1, i.e. $P_1 = W_1$, in exchange for receiving $L_2 = W_1 R_1$ at date 2. In this case $V_1^M = r^{-1} W_1 (r - R_1) = W_1 (1 - \rho_1)$.

[8] i.e. complete exclusion from financial markets.

Given the bank profits available at date 1, we now consider how large a loan the banks will be prepared to make at date 0. In the case when $R_1 \geq r$, they will not make any loan at all, since as discussed above they cannot extract any repayment from the borrower at date 1 without losing still more money. In constrast, in the case where $R_1 < r$ the banks will receive some repayment at date 1. Since the borrower's date 1 wealth following a loan of L_0 is $W_1 = (W_0 + L_0) R_0$, the largest loan the banks will be prepared to make is given by the solution L_0 to

$$r^{-1} (W_0 + L_0) R_0 (1 - \rho_1) = L_0 \tag{1}$$

Summarizing, the banks will be prepared to make a loan of

$$L_0^* = \frac{\rho_0 \max\{0, 1 - \rho_1\}}{1 - \rho_0 \max\{0, 1 - \rho_1\}} W_0 \tag{2}$$

The borrower's final consumption from such a loan is

$$\frac{R_0 R_1}{1 - \rho_0 (1 - \rho_1)} W_0 \text{ if } r > R_1$$

and $R_0 r W_0$ if $r \leq R_1$.

In this simple environment Bulow and Rogoff's (1989b) result that credit denial alone cannot support lending if the borrower has full access to asset markets is very clear. Full access to asset markets implies that $R_1 \geq r$, and thus the maximal loan size is 0.

Unsurprisingly, the maximal loan size and the borrower's final consumption are increasing in both the period 0 rate of return R_0 and the borrower initial wealth W_0. Less obvious is that they are also increasing as the borrower's date 1 return R_1 decreases. The reason is that when R_1 is lower, the borrower is prepared to make a larger date 1 repayment in exchange for a "loan" at date 2. Since the date 1 repayment is higher, it is then possible to increase the size of the original loan.

Up to now, we have been referring to the transfers from the banks to the borrower as loans, and transfers from the borrower to the bank as repayments. While this terminology makes the transfers easiest to interpret, for many of our results it is in fact easier to think of the transfers slightly differently. Recall that the transfers that maximize the borrower's final consumption when $R_1 < r$ are $L_0 = L_0^*, P_1 = W_1$ and $L_2 = R_1 W_1$. Effectively, the banks make a *gift* of L_0^* to the borrower at date 0. They then recover this gift by taking deposits between dates 1 and 2, when the borrower's return is low. Their profits from date 1 onwards stem from paying a return of only R_1 on these deposits, while investing funds at a rate of return r. As (1) makes clear, the maximal loan size is determined by setting the profits from acting as a "savings" bank for the borrower equal to the original transfer L_0.[9]

[9] A similar discussion can be found in Gromb (1999).

Note that we have been assuming that $\rho_0 (1 - \rho_1) < 1$. If this is not the case, a gift of \$1 to the borrower increases his date 1 wealth by enough that the consequent increase in the banks' date 1 profits more than compensates them for the cost of the gift. If this inequality does not hold there is no sense in which the borrower is credit-constrained, and the problem ceases to be of any economic interest.

All of the above remarks generalize to the T-period version of our environment. Proceeding more formally than before, we compute the maximal level of profits attainable by the coalition of M banks from time t onwards, subject to the borrower obtaining transfers with a present value of v (at the borrower's opportunity cost R_t), the borrower payments being incentive compatible and feasible.

Take any date t, borrower wealth W and present value of transfers to the borrower v (at the borrower's interest rate). Let $V_t^M(W, v)$ denote the maximal profits that the M banks can collectively make between date t and the terminal date T, subject to the constraint that over this period they make/receive transfers to the borrower with a present value of v (at the borrower's discount rate). Any transfers received must of course be incentive compatible. Measuring the banks' profits $V_t^M(W, v)$ in date t terms, we have the following recursive formulation:[10]

$$V_t^M(W, v) = \max_{P \geq 0, L \geq 0, v' \geq 0} -(L - P) + \frac{1}{r} V_{t+1}^M(W', v') \tag{7}$$

$$\text{subject to} \quad W' = (W + L - P) R_t \tag{8}$$

[10] In non-recursive terms, $V_t^M(W, v)$ is the solution to

$$\max_{\{P_s \geq 0, L_s \geq 0 : s = t, \ldots T\}} -\frac{1}{r^{T-t}} \sum_{s=t}^{T} r^{T-s} (L_s - P_s) \tag{3}$$

subject to the transfers to the borrower having a present value of v

$$\sum_{s=t}^{T} \left(\prod_{\tilde{s}=s}^{T-1} R_{\tilde{s}} \right) (L_s - P_s) = \prod_{\tilde{s}=t}^{T-1} R_{\tilde{s}} v \tag{4}$$

and such that at all dates $t' = t, \ldots, T$ the borrower prefers to make the payment $P_{t'}$ over defaulting and receiving no further payments from any bank,

$$\sum_{s=t'}^{T} \left(\prod_{\tilde{s}=s}^{T-1} R_{\tilde{s}} \right) (L_s - P_s) \geq \left(\prod_{\tilde{s}=t'}^{T-1} R_{\tilde{s}} \right) L_{t'} \tag{5}$$

and finally subject to these payments being feasible,

$$P_{t'} - L_{t'} \leq W_{t'} \equiv \prod_{\tilde{s}=t}^{t'-1} R_{\tilde{s}} W + \sum_{s=t}^{t'-1} \left(\prod_{\tilde{s}=s}^{t'-1} R_{\tilde{s}} \right) (L_s - P_s) \tag{6}$$

Since the constraint set is closed and bounded (details omitted but available from authors) this problem has a solution provided the constraint set is non-empty. This maximization problem is then easily shown to reduce to the recursive problem given in the text.

$$v' = R_t (v - (L - P)) \quad (9)$$
$$v \geq L \quad (10)$$
$$W \geq P - L \quad (11)$$

Constraint (8) is the law of motion for the borrower's wealth. Constraint (9) is the "promise-keeping" constraint on bank transfers. Constraint (10) is the borrower's incentive constraint, representing the fact that the harshest possible threat for non-payment is complete exclusion from dealing with the banks. Finally, constraint (11) is just the feasibility constraint on borrower payments.

Exactly as in the $T = 2$ case, the banks are able to obtain loan repayments in periods t when the borrower's rate of return is lower than the bank's ($R_t < r$, payment periods) by threatening to deny future loans if the payment is not made. The value of loan repayments that can be obtained is proportional to the wealth of the borrower, and the shortfall of R_t below r. This is easily seen from the "savings market" intuition discussed above — a bigger gap between R_t and r, and a higher level of borrower wealth, allow larger bank profits in the saving market

In the two-period problem, if the bank increases its initial loan by one dollar, then it is able to collect $\rho_0 (1 - \rho_1)$ present-value dollars more in repayments during a payment period. From (1) this allows us to calculate the maximum initial loan size. In the T-period problem, if the bank increases its initial loan by one dollar, it collects more in repayments during *each* payment period in the future. Working out the maximum loan size requires us to sum up this increase over the entire future.

Define α_t iteratively by

$$\begin{aligned} \alpha_T &= 0 \\ \alpha_{t-1} &= \begin{cases} \rho_{t-1}\alpha_t & \text{if } t-1 \text{ is an investment period} \\ 1 - \rho_{t-1}(1 - \alpha_t) & \text{if } t-1 \text{ is a payment period} \end{cases} \quad \text{for } t = 1, \ldots, T \end{aligned} \quad (12)$$

The quantity α_t is the present value of future repayments that a lender will collect by increasing his loan at time t by one dollar. So the present value of the banks' profits at date 0 is $-L_0 + (W_0 + L_0)\alpha_0$. As before, the largest initial loan can be found by choosing L_0 so that the banks just break even, i.e. $L_0 = W_0\alpha_0 / (1 - \alpha_0)$.

If $\alpha_0 > 1$, the bank's optimal strategy is to make an infinite transfer to the borrower, since each dollar transferred increases bank profits by more than a dollar. Under these conditions, the borrower is essentially no longer credit-constrained — he can raise as much financing as he wants. Consequently, to keep the problem of economic interest we assume that $\alpha_0 < 1$ for the remainder of the paper. This is the exact analogue of the condition $\rho_0 (1 - \rho_1) < 1$ discussed in the $T = 2$ case. Note for future use that if $\alpha_0 < 1$ then $\alpha_t < 1$ for all $t > 0$.[11] In general, the requirement that $\alpha_0 < 1$ will be

[11] For suppose otherwise, i.e. $\alpha_{t^*} \geq 1$ for some t^*. Then from (12) it would follow that $\alpha_t \geq 1$ for all

more stringent when T, the total number of time periods, is larger.

Proposition 1 ***(An upper bound on borrower welfare)***
Assume $\alpha_0 < 1$. The borrower's maximal consumption at date T (subject to the banks collectively making non-negative profits) is $\prod_{t=0}^{T-1} R_t W_0 / (1 - a_0)$. It can be achieved by the loans and payments $\{L_t^*, P_t^*\}$ defined as follows:

1. *The borrower receives a loan $L_0^* = W_0 \alpha_0 / (1 - \alpha_0)$ at date 0.*[12]
2. *In any payment period t that follows an investment period, the borrower pays to the banks his entire wealth, i.e. $P_t^* = W_t$.*
3. *In any investment period $t > 0$ that follows a payment period, the banks make a loan to the borrower equal to the borrower's last payment cumulated at the borrower's rate of return R_s. That is, $L_t^* = \prod_{s=\tau+1}^{t-1} R_s P_{\tau+1}^*$ where τ is the investment period prior to t.*
4. *All other payments are 0.*
5. *$V_t^M (W, v) = \alpha_t W - (1 - \alpha_t) v$*

Proof: The proof is straightforward and omitted. The details are available from the authors.

3 Borrower welfare in the absence of enforcement

In the previous section we characterized the maximal utility level attainable by the borrower when the M banks can fully coordinate their decisions. This is a (simple) version of the problem that has been studied by numerous previous papers (see, for example, Kehoe and Levine 1993 and Kocherlakota 1996). The existing literature has had little to say about how this coordination actually occurs. This is the focus of the remainder of the current paper.

$t < t^*$, contradicting $\alpha_0 < 1$.

[12] The analogue of Bulow and Rogoff's (1989b) in this context is that no lending will occur if the borrower always has access to an investment opportunity equal to or better than the banks' opportunity cost of funds, r. Formally, if $R_t > r$ for all $t = 0, \ldots, T-1$, then the maximum loan size the lender can make while breaking even is zero. The proof is straightforward: an iterative application of (12) implies $\alpha_0 = 0$, and so $\alpha_0 W_0 / (1 - \alpha_0) = 0$.

As is widely appreciated, competition among banks undercuts their ability to coordinate on excluding a defaulting borrower.[13] Formally, we consider competition as occurring as follows. At date 0, each bank m simultaneously announces a lending policy

$$\mathcal{L}^m \equiv \left\{ l_0^m, l_1^m \left(\mathbf{P}_0 \right), \ldots, l_T^m \left(\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_{T-1} \right) \right\}.$$

The borrower then chooses $\{P_t^m\}$ to maximize his final consumption,

$$W_0 \prod_{t=0}^{T-1} R_t + \sum_m \sum_{t=0}^{T} \prod_{s=t}^{T-1} R_s \left(L_t^m - P_t^m \right).$$

Note that since at this point we are assuming that banks are able to commit to future payments, we are allowing for the possibility that each bank's commitment depends not only on the transfer the borrower makes to that bank, but on the entire vector of transfers the borrower makes to all M banks. We show that even with this ability to make interdependent promises it is still impossible for the banks to successfully coordinate their actions and profitably lend any funds.

We make this argument in two steps. First, competition among banks precludes the possibility of them collectively making positive profits from any date t onwards. The basic argument is as follows. Suppose to the contrary that banks' combined profits were strictly positive. Then any bank m' whose profits are the lowest could offer a lending policy that replicates the transfers currently made by the M banks together, but which delivers slightly more to the borrower. This raises the profits of bank m', and so is a profitable deviation. (Lemma 2 in the Appendix formalizes this argument, and in particular takes care of the details relating to the possible interdependence in the banks' lending policies.)

Second, given that the banks cannot make positive profits from any date t onwards, it follows that they can never obtain a repayment on any loans previously made to the borrower. So competition completely under-cuts the loan market, and implies that the only useful service the banks can offer is to accept deposits and pay an interest rate r on them. Consequently the borrower's final period consumption is at most the amount corresponding to taking his initial wealth W_0, investing it at rate R_t in his own project in investment period and investing it with the bank at rate r in payment period. That is, the borrower's welfare is bounded above by $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$.

Because this upper bound on borrower welfare is obtained without the benefit of any loans, it is less than the benchmark level derived in Proposition 1.[14] As discussed above, it is competition between banks which, by under-cutting their ability to recover

[13] See, for example, Bulow and Rogoff (1989b), Chari and Kehoe (1993a) and Krueger and Uhlig (2000). We generalize their results in that our setting allows a borrower to simultaneously deal with multiple banks and allows for interdependencies in their lending policies.

[14] To establish this formally, note that it is sufficient to prove that $\prod_{s=t}^{T-1} \min\{1, \rho_s\} > 1 - \alpha_t$ at $t = 0$.

any funds lent, eliminates the credit market. This raises the question of whether the borrower's consumption would actually be higher if the banking sector was monopolistic, i.e. $M = 1$. The answer is no. Although a monopoly bank *could* make a loan and then obtain repayment by threatening the borrower with exclusion from saving at rate r, it does even better simply by proposing the lending policy $\mathcal{L}^1$ given by $\tilde{l}_t^1(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = R_{t-1}P_{t-1}^1$. That is, the monopoly bank simply makes monopoly profits in the deposit market, without making any loan. The following Proposition summarizes these observations:

Proposition 2 ***(Borrower's welfare absent enforcement)***
The borrower's final period consumption is $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$ if there are two or more banks ($M \geq 2$) and $W_0 \prod_{t=0}^{T-1} R_t$ if there is only a single bank ($M = 1$).

Proof: Consider first the case where $M \geq 2$. We first show there is an equilibrium in which the borrower's final consumption is as claimed. Consider the set of lending policies $\tilde{\mathcal{L}}^m$ consisting simply of $\tilde{l}_t^m(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = rP_{t-1}^m$, i.e. each bank pays a return r on any funds deposited. Clearly the borrower's final period consumption is $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$, and all banks make zero profits. Moreover, there is no profitable deviation. For if there were, we would effectively have one deviating bank m' making strictly positive profits while dealing with a borrower with a technology paying $\max\{r, R_t\}$. From Proposition 1 we know that the present value of a bank's profits in this situation is no more than $\alpha_0(W_0 + v) - v$ where v is the present value (at the borrower's rate of return) of transfers to be made to the borrower. Since $\alpha_0 = 0$ when the borrower's return is always weakly above r (see footnote 12 on page 10 above) and $v \geq 0$, then bank m' must have weakly negative profits.

Next, we note that there cannot exist any equilibrium in which the borrower's final consumption is not as claimed. We observed in the text that this expression is certainly an upper bound on the borrower's welfare (see Lemmas 2 and 3 in the Appendix for the formal argument), and so the only possibility is that there is an equilibrium in which it is strictly less. But in this case any bank m could profit be proposing a lending policy $\tilde{\mathcal{L}}^m$ consisting simply of $\tilde{l}_t^m(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = (r - \varepsilon) P_{t-1}^m$ for ε sufficiently small.

Finally, we deal with the case $M = 1$. From Proposition 1 the bank's profits are bounded above by $\alpha_0(W_0 + v) - v$. Since $\alpha_0 < 1$, this upper bound is maximized by $v = 0$. We claim that this profit level is attainable by the monopoly bank $m = 1$ offering a lending policy $\tilde{\mathcal{L}}^1$ defined by $\tilde{l}_t^1(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}) = R_{t-1}P_{t-1}^1$. We establish this claim by inductively showing that under this lending policy the bank obtains profits $\alpha_t W_t$ from each date t onwards. The base case of $t = T$ is trivial. For $t - 1$ an investment period,

This follows easily by an inductive argument. The strict inequality is obtaining whenever there is at least one period with $R_t > r$ (i.e. a "strict" investment period) that is followed by another period with $R_t < r$ (payment period).

the borrower does not deposit funds with the bank and so the bank's profits from $t-1$ on are $\alpha_t R_{t-1} W_{t-1}/r = \alpha_{t-1} W_{t-1}$. For $t-1$ a payment period, the borrower deposits his entire wealth and the bank's profits from $t-1$ onwards are $(r - R_{t-1}) W_{t-1}/r + \alpha_t R_{t-1} W_{t-1}/r = \alpha_{t-1} W_{t-1}$. Given that the bank can obtain the upper bound on profits corresponding to setting $v = 0$, it then follows that the borrower's final period consumption is simply $W_0 \prod_{t=0}^{T-1} R_t$. **QED**

4 Enforcement

We now introduce third-party enforcement into the environment in order that loans may be supported in equilibrium. By assumption, borrowers have no collateral that can be seized. Thus, enforcement boils down to regulating transfers between borrowers and banks. We assume that a central authority ("court") exists and can (1) Observe the net payments made in each period between the borrower and each of the M banks, and (2) Seize some or all these net payments.[15]

Given these assumptions, one obvious form of enforcement – the most stringent – is the punishment whereby the central authority seizes all of the transfers between the borrower and any of the M banks if the borrower ever misses a repayment on a loan. In fact, exactly this punishment is considered in the literature initiated by Kehoe and Levine (1993) and Kocherlakota (1996). For example, Alvarez and Jermann (2001) assume that "default is punished by permanent exclusion from the asset markets." Effectively, banks coordinate by committing to not trade with a defaulting borrower.

Notationally, let $\mathbf{NP}_t \equiv \mathbf{P}_t - \mathbf{L}_t$ denote the vector of net payments in period t. An enforcement rule $\mathcal{B}$ is then a specification of how much of each net payment is seized, where the amount seized can potentially depend on the history of prior net payments. That is, $\mathcal{B}$ is a set of vector-valued functions

$$\mathcal{B} = \left\{ \beta_t : \Re^{(t+1)M} \to [0,1]^M : t = 0, 1, \ldots, T \right\}$$

where $\beta_t^m (\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t)$ is the fraction of the net payment NP_t^m that is *not seized*, given the history of net payments $\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t$.

[15] We assume that any funds seized in this way are simply destroyed. Allowing the central authority to distribute funds seized from a transfer between bank m and the borrower to the remaining $M-1$ banks would add nothing of substance, and would serve only to complicate the notation.

4.1 Three enforcement rules that implement the constrained optimum

A rule that specifies full-exclusion from the asset market upon a borrower's failure to pay less than some pre-specified amount (i.e. as in the quote from Alvarez and Jermann 2001) can be represented as follows. Suppose that we want to implement the constrained efficient aggregate payments $\{L_t^*, P_t^*\}$ of Proposition 1 as an equilibrium. Without loss, we can assume that all these payments are made to and from bank 1 — only the borrower faces an incentive problem at this point in the paper.

Definition 1 (Full-exclusion rule)
The full-exclusion enforcement rule of Kehoe and Levine (1993) and Kocherlakota (1996) is the rule $\mathcal{B}_{KLK}$, defined by

$$\beta_t^m(\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t) = \begin{cases} 1 & \text{if } NP_s^1 \geq P_s^* \text{ for all } s \leq t \\ 0 & \text{if } NP_s^1 < P_s^* \text{ for some } s \leq t \end{cases}$$

for all all $m \in M$ and $0 \leq t \leq T$.

That is, if the net payment from the borrower to bank 1 is strictly less than P_t^* in any period, then this payment and all future transfers between the borrower and the whole banking sector are seized in entirety.

Although this rule has been invoked widely in the literature, and is the most stringent in our setting, it should be clear that the rule is only one in a large class of enforcement rules. Moreover, although the results of Proposition 2 imply that some enforcement is required to achieve the constrained efficient outcome, there is no *a priori* reason why enforcement should take the form of $\mathcal{B}_{KLK}$.

The main difficulty with the $\mathcal{B}_{KLK}$ rule is that it is hard to identify in observed institutions. Indeed, although this literature commonly refers to "debt" constraints, the implementation is in fact a complex state- and date-contingent specifications of payments. Exclusion occurs upon failure to make the payment that is specified by a central authority, the computation of which requires intimate knowledge of an agent's production and consumption possibilities and is information intensive. It is difficult to see how this information arises in decentralized settings and becomes available to courts who enforce contracts. As such, let us consider two alternative rules that are easier to interpret.

Referring back to the intuition we provided when characterizing the efficient outcome as a gift followed by exclusive trading rights, another possibility is to stipulate that once the borrower has accepted a loan from one bank (i.e. a negative net payment), he is not allowed to deal with any other bank. Effectively we are thus granting each bank the possibility of acquiring a complete monopoly (i.e. *exclusive* trading rights) over the borrower by making a loan.

Definition 2 (Exclusivity rule)
The exclusivity enforcement rule $\mathcal{B}_{excl}$ is defined by

$$\beta_t^m(\mathbf{NP}_0, \mathbf{NP}_1, \ldots, \mathbf{NP}_t) = \begin{cases} 0 & \text{if } NP_s^{m'} < 0 \text{ for some } s \leq t \text{ and } m' \neq m \\ 1 & \text{otherwise} \end{cases}$$

all $m \in M$ and $0 \leq t \leq T$.

While we find the full-exclusion rule $\mathcal{B}_{KLK}$ hard to interpret in terms of observed institutional arrangements, the exclusivity rule $\mathcal{B}_{excl}$ resembles the sale of the borrower's project to a financier. That is, in exchange for a sale price L_0^*, the buyer of the project obtains property rights over any cash thrown off by the project that is deposited in the financial sector, and has a veto right on any future loans.[16]

Finally consider a third enforcement rule that (as we make clear below) resembles laws surrounding default on debt contracts.

Definition 3 (Debt-default rule)
Let D_t^m denote the borrowers indebtedness to bank m: $D_t^m = r(D_{t-1}^m - NP_{t-1}^m)$. Then the debt-default rule, $\mathcal{B}_{DD}$, is defined by

$$\beta_t^m(\mathbf{NP}_0, \ldots, \mathbf{NP}_t) = \begin{cases} \min\left\{\frac{D_t^m}{NP_t^m}, 1\right\} & \text{if } \exists m' \neq m \text{ s.t. } D_t^{m'} - NP_t^{m'} > 0 \\ & \quad \text{and } NP_t^m > 0, D_t^m > 0 \\ 0 & \text{if } \exists m' \neq m \text{ s.t. } D_t^{m'} - NP_t^{m'} > 0 \\ & \quad \text{and } NP_t^m > 0, D_t^m \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

for all $m \in M$ and $0 \leq t < T$.

That is, whenever the borrower is indebted to bank m' at the end of period t, he cannot deposit funds with any other bank $m \neq m'$ in excess of the amount he owes to this second bank. Less formally, a borrower in default to one bank is prohibited from saving with any other bank.

We call this a debt-default rule because it resembles laws that regulate a borrower's defaulting on debt and declaring bankruptcy. First, by definition most legal systems

[16]Of course, we continue to respect the fundamental friction of our model that no outsider can unilaterally seize the cash flows produced by the borrower's project.

dictate that if a borrower is in default, his creditors have the right to seize any of his assets.[17] By assumption, in our model only assets transferred between the borrower and the financial system are seizable. Our enforcement rule $\mathcal{B}_{DD}$ allows for these transfers to be seized until the borrower has repaid his debt. The rule grants a creditor special rights, until the borrower has repaid his debt.

Second, bankruptcy proceedings require the borrower to disclose and surrender all of his assets. In our model the borrower always has the option of instead hiding the portion of his assets represented by his ongoing project. But the stipulation that he should declare them still matters, because it implies that he cannot in the future deposit the proceeds of this project into the financial system — for to do so would reveal to his creditors that he acted fraudulently in the bankruptcy proceedings and did not disclose all his assets. In legal terms, his hiding of his project would constitute a fraudulent conveyance, and the funds thus hidden are still subject to seizure *even after* the discharge of his debts in bankruptcy.

The following recent example illustrates well the manner in which prohibitions against fraudulent conveyance[18] prevent a defaulting debtor from continuing to hold formal sector financial assets. In 1991 Harry and Margie Rosholm were failing to service a refinancing loan they had taken from Southwest Savings and Loan Association. In February of that year they transferred a $100,000 certificate of deposit (C.D.) that they held with Citibank to their daughter Jacque. In November Southwest served a writ of garnishment to Citibank. In subsequent legal proceedings Southwest learnt that the Rosholms had transferred the C.D. to Jacque Rosholm, and obtained a further writ of garnishment against her. When Jacque Rosholm claimed that the money was truly hers', the trial court found against her. This verdict was upheld on appeal.[19]

Thus, although most legal systems grant a borrower the right to save in the financial system after bankruptcy, this right only applies to assets acquired after bankruptcy. A borrower *does not* have the right to hide assets during bankruptcy and then deposit these in the financial sector post-bankruptcy. This is exactly the punishment that our enforcement rule $\mathcal{B}_{DD}$ imposes.

[17] Different legal systems place different constraints on the ability of a creditor to seize assets or garnish wages without judicial enforcement, and indeed even within the same legal system different constraints are placed on the seizure of different types of asset (see, e.g., White, 1998).

[18] The legal prohibition against fraudulent conveyances in the U.S. can be traced back to sixteenth century England. Most commentators regard *Twyne's Case* (1601) as the point at which the law was first clearly articulated. In brief, Pierce owed Twyne 400 pounds and a second creditor C 200 pounds. Pierce transferred his assets — mostly sheep, and worth about 300 pounds — to Twyne. In spite of the legal change of ownership however, Pierce remained in full possession of the sheep, to the extent of shearing them and marking them as his own. Having received nothing from Pierce, creditor C protested against this arrangement. The court found in his favor.

[19] 185 Arizona 80.

Proposition 3 ***(Enforcement rules)***
If there are two or more banks ($M \geq 2$), then all three enforcement rules, $\mathcal{B}_{KLK}$, $\mathcal{B}_{excl}$, and $\mathcal{B}_{DD}$ achieve the constrained efficient outcome of Proposition 1.

Proof: (A) The proof for the $\mathcal{B}_{KLK}$ rule is tedious but very straightforward — and is thus omitted (details are available from the authors). Basically, full exclusion from all financial trade is the harshest punishment in our environment. Moreover, the upper bound on borrower welfare of Proposition 1 is constructed precisely by using the harshest punishment available.

(B) The proof for the $\mathcal{B}_{excl}$ rule is also straightforward. Consider the set of lending policies $\mathcal{L}^m = \left\{L_0^m = L_0^*, L_t^m (\mathbf{P}_0, \ldots, \mathbf{P}_t) = R_{t-1} P_{t-1}^m\right\}$ where L_0^* is as defined in Proposition 1. Given our exclusion rule $\mathcal{B}_{excl}$, the borrower will only ever accept a loan (i.e. negative net payment) from at most one bank — bank 1, say. By construction, accepting the initial loan payment L_0^* from bank 1 and then depositing his entire wealth in each payment period ($R_t < r$) gives the borrower a final consumption as in Proposition 1. All banks makes zero profits under this borrower strategy. The borrower cannot profitably deviate to another strategy. And finally, no bank can unilaterally and profitably deviate to an alternative lending policy. This last point follows since by construction a bank cannot make strictly positive profits while delivering the borrower a final consumption as in Proposition 1. Since $M \geq 2$, this rules out the possibility of a profitable deviation.

Note that the $\mathcal{B}_{excl}$ rule is similar in spirit to the decentralization considered in Prescott and Townsend (1984) or Atkeson and Lucas (1992). In these papers, financial intermediaries compete ex-ante for exclusive trading rights. As long are there are more than two banks, competition ensures that the constrained efficient outcome obtains.

(C) The proof for the $\mathcal{B}_{DD}$ rule follows next. Consider the following set of lending policies. One of the banks — without loss, bank 1 — offers the lending policy $\mathcal{L}^1$ defined by

$$L_t^1 (\mathbf{P}_0, \ldots, \mathbf{P}_t) = \begin{cases} L_t^* & \text{if } P_s^1 = P_s^* \text{ and } P_s^m = 0 \text{ for all } s < t \text{ and } m \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\{L_t^*, P_t^*\}$ are as defined in Proposition 1. That is bank 1 commits to making loans of the efficient size as long as the borrower repays these loans to bank 1 (and no other bank).

Each of the remaining banks $m \neq 1$ offers the "savings" lending policy $\mathcal{L}^m$ defined by $L_t^m (\mathbf{P}_0, \ldots, \mathbf{P}_t) = r P_{t-1}^m$. We claim that these polices, along with the borrower paying P_t^* in period t, constitute an equilibrium. Note the borrower's final consumption in this equilibrium is $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$, and that all banks make zero profits.

First, we claim that given the lending policies the borrower's behavior is a best-response. Suppose to the contrary that the borrower has a strictly welfare-increasing deviation and that the resulting payments are $\left\{\tilde{L}_t^m, \tilde{P}_t^m\right\}$ between the borrower and the M banks. Given the lending policies of banks $m \neq 1$, $D_{T+1}^m = 0$. So for the deviation to strictly increase the borrower's final consumption, by Proposition 1 we know that $D_{T+1}^1 > 0$. Let τ be the date at which the borrower first deviates. Bank 1 makes no payments to the borrower after this date, so D_t^1 must be weakly decreasing over dates $t \geq \tau + 1$. The debt-default exclusion rule $\mathcal{B}_{DD}$ then implies that at dates $t \geq \tau + 1$ the borrower can never deposit funds with any other bank $m \neq 1$ in the deviation. But then the deviation must be unprofitable, by the definition of $\{L_t^*, P_t^*\}$.

Second, no bank $m \neq 1$ can profitably deviate. Suppose to the contrary that bank 2 deviates to $\tilde{\mathcal{L}}^2$, leading to equilibrium payments $\left\{\tilde{L}_t^m, \tilde{P}_t^m\right\}$ between the borrower and the M banks. For the deviation to strictly increase bank 2's utility, $\tilde{P}_t^2$ must be strictly positive for some date. Let τ_0 be the first such date. Moreover, let τ_1 be the last period in which the net payment between the borrower and bank 2 is non-zero. Note that it must be the case that $NP_{\tau_1}^2 < 0$ (i.e. the last payment must be from bank 2 to the borrower). For bank 2 to make strictly positive profits, we need $D_{\tau_1}^2 - NP_{\tau_1}^2 < 0$. The borrower's final consumption must be equal to at least $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$, his payoff from staying with bank 1. Since bank 1 makes no payments to the borrower after the deviation date τ_0, D_t^1 must be weakly decreasing over dates $t \geq \tau_0 + 1$. By Proposition 1 we know that $D_{T+1}^1 > 0$. So $D_t^1 > 0$ for all dates $t \geq \tau_0 + 1$. But $D_{t+1}^2 = r(D_t^2 - \beta_t^2 NP_t^2)$, so $D_{t+1}^2 \geq 0$ whenever $D_t^2 \geq 0$ for any $t \geq \tau_0$. But since $D_{\tau_0}^2 \geq 0$, this contradicts $D_{\tau_1}^2 < 0$ completing the claim.

Third, we claim that bank 1 cannot deviate and make strictly positive profits. For suppose to the contrary that there exists a deviation $\tilde{\mathcal{L}}^1$ under which bank does make strictly positive profits. Let $\left\{\tilde{L}_t^m, \tilde{P}_t^m\right\}$ be the resulting payments between the borrower and the M banks. Since bank 1 makes strictly positive profits, $\tilde{D}_{T+1}^1 < 0$. Let τ be the last date t at which $\tilde{D}_t^1 \geq 0$. So $\tilde{P}_\tau^1 - \tilde{L}_\tau^1 \geq \tilde{D}_\tau^1 > 0$. But this cannot be an equilibrium, since the borrower could instead pay just $\tilde{D}_\tau^1 + \tilde{L}_\tau^1$ to bank 1 (thus repaying his debt), and make all future deposits to one of the competitor banks, say bank 2. This must be welfare improving for the borrower since all payments to bank 1 in excess of $\tilde{D}_\tau^1 + \tilde{L}_\tau^1$ at date τ, along with all payments to bank 1 after date τ, constitute deposit payments on which bank 1 pays an interest rate of strictly less than r. **QED**

Although our debt-default rule appears very lenient compared to the full-exclusion rule, the proposition shows that in fact it provides an inducement to repay that is just as strong. The debt-default rule embeds a prohibition on *saving* with another bank. The exclusion from savings is the crucial part of the penalty in the full-exclusion rule.

Consider a debtor contemplating defaulting on a loan. If default triggers complete and indefinite exclusion, he compares the cost of repaying with the cost of indefinite financial autarchy. Now, consider the borrower's calculation in the case where the punishment for non-payment is that implied by the debt-default rule. If he does not pay, then he cannot save with any other bank until he has done so. Suppose for a moment that our defaulting borrower intends *never* to repay the debt — and so consequently he faces indefinite exclusion from holding savings in the formal financial sector forever. Moreover, he will certainly not receive a loan from another bank (since he has no incentive to repay new loans). So in the case where a defaulting borrower plans never to repay, the debt-default rule in fact replicates the full threat of apparently much harsher forms of exclusion.

In the other case, the defaulting debtor plans to repay the debt after a few periods. However, provided the loan contract written by the bank is itself optimal, the borrower is only called upon to make repayments in periods when his private rate of return is low. So by delaying debt repayment the borrower only makes himself worse off: the debt continues to grow at the bank's interest rate, while the borrower's wealth grows at his lower private rate of return.

4.2 Does credit denial implement the constrained optimum?

Our debt-default enforcement rule $\mathcal{B}_{DD}$ places restrictions on the ability of indebted borrowers to place savings with other banks. As we discussed above, this restriction on savings is effectively embedded in observed institutions. On the other hand, there is no direct restriction on access to credit either in our rule $\mathcal{B}_{DD}$, nor in the law.[20]

Indeed, we next show that altering our rule to restrict a borrower from receiving credit, as opposed to placing savings with the financial system, renders enforcement useless. The equilibrium reverts to the no-enforcement outcome of Proposition 2. This highlights that it is savings denial rather than credit denial that is at the heart of enforcement in our environment.

Formally, a prohibition on credit to indebted borrowers would be achieved by a *credit-*

[20]Of course, in practice, lenders may be unwilling to advance credit to a borrower that has declared bankruptcy. However, this is not legally enforced credit denial. It is presumably a consequence of reputation effects, which are absent in our environment.

prohibition enforcement rule $\mathcal{B}_{CP}$ by

$$\beta_t^m(\mathbf{NP}_0,\ldots,\mathbf{NP}_t) = \begin{cases} \min\left\{\frac{D_t^m}{NP_t^m}, 1\right\} & \text{if } \exists m' \neq m \text{ s.t. } D_t^{m'} - NP_t^{m'} > 0 \\ & \qquad\qquad \text{and } NP_t^m < 0, D_t^m < 0 \\ 0 & \text{if } \exists m' \neq m \text{ s.t. } D_t^{m'} - NP_t^{m'} > 0 \\ & \qquad\qquad \text{and } NP_t^m < 0, D_t^m \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

The rule $\mathcal{B}_{CP}$ is exactly analogous to the debt-default rule $\mathcal{B}_{DD}$: Where $\mathcal{B}_{DD}$ prevented an indebted borrower making a payment to another bank that leaves him out of debt, the rule $\mathcal{B}_{CP}$ prevents an indebted borrower receiving a payment from another bank that leaves him indebted.

Lemma 1 ***(The credit-prohibition enforcement rule)***
Suppose there are two or more banks ($M \geq 2$). Then in every equilibrium under the credit–prohibition enforcement rule $\mathcal{B}_{CP}$, the borrower's final consumption is $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$.

Proof: See Appendix.

The intuition underlying Lemma 1 is essentially the same as that for Proposition 2 (as is the proof). Competition among banks under-cuts the possibility of positive future profits. In any equilibrium the final payments have to be from the banks to the borrower. Consider the first date prior to this (date t_0 say) at which the future profits of the banks are positive. Then over the period from t_0 to the final date, the borrower is effectively saving with the banking sector. Moreover, he must be saving at a rate below r, since the banks are making positive profits. So any one of the banks can undercut the others by offering a higher rate of return on savings. And since this deviation does not involve any new loans, the enforcement rule $\mathcal{B}_{CP}$ never has any effect.

Finally, we should note that despite the fact that the debt-default rule only restricts the savings options of an indebted borrower, it endogenously restricts his borrowing options. That is to say, a bank only extracts repayments in periods where the borrower has a low R_t, at which points the bank effectively offers the borrower low returns on his savings. However, if a borrower is in debt with one bank, his savings options with another bank are restricted. As a result, the other bank will never receive repayments on any loans made to the borrower. Thus, outcomes under the debt-default rule will look as if a credit-prohibition rule was also in effect. Of course, if the borrower repays the debt of the lending bank, he is free to save/borrow from another bank. This distinction in the treatment of saving and credit is exactly what we observe in formal laws governing default and bankruptcy: as discussed, a defaulting borrower is prevented from investing his existing assets in the formal financial sector, while he faces no (formal) restrictions on borrowing activity.

5 Limited bank commitment

In the previous section we established that the debt-default enforcement rule provides the borrower with as great an incentive to repay as do forms of exclusion that appear to represent much stronger threats. In fact, the debt-default rule provides a sufficiently large punishment that it belongs in the subclass of enforcement rules that implement the constrained optimum as an equilibrium.

In this section and the next we turn to a second property that effective forms of exclusion must possess, namely that they must provide lenders with the incentive to make efficient loans at both the initial and subsequent dates. Thus far we have been assuming that banks can completely commit to lending policies as of date 0. Competition at date 0 then ensures that all loans, both at the initial date and in subsequent periods, are of the efficient size. In order to address the issue of lender incentives we now relax our assumption of full bank commitment.

Formally, we now assume that banks can only commit to future payments at the penultimate date[21] $T-1$. That is, the timing is now as follows. At all dates $0 \leq t \leq T-2$, the each bank m simultaneously chooses a payment L_t^m to make to the borrower. The borrower then chooses a payment P_t^m to make to each bank. In contrast, at date $T-1$ each bank m simultaneously chooses a payment L_{T-1}^m to make today, and a payment $\mathcal{L}_{T-1}^m(\mathbf{P}_{T-1})$ that will be made at date T, where this payment can be contingent on the vector of date $T-1$ payments from the borrower to the banks. After the M banks have announced $\left(L_t^m, \mathcal{L}_{T-1}^m(\mathbf{P}_{T-1})\right)$ the borrower then chooses the vector of payments $\mathbf{P}_{T-1}$. Finally, at date T each bank honors its commitment $\mathcal{L}_{T-1}^m(\mathbf{P}_{T-1})$.[22]

Once we no longer allow banks to commit at date 0, it is clear that any equilibrium set of payments $\{L_t^m, P_t^m\}$ must possess that property that

$$\sum_{s=t'}^{T} r^{-s}\left(L_s^m - P_s^m\right) \geq 0 \text{ for all dates } 0 \leq s \leq T-1 \tag{13}$$

i.e. each bank must be making non-negative future profits at all dates s prior to the terminal date T. Proposition 1 of Section 2 characterized the maximum possible level of the borrower's final consumption when the bank's were able to commit (i.e. when constraint (13) is required to hold only at $s=0$). The analogue under constraint (13) is:

Proposition 4 ***(An upper bound on borrower welfare under limited bank commitment)***

[21] For instance, it may be the case that banks only possess pledgeable assets at date $T-1$. The bank then uses these assets to commit to payments at date T.

[22] Without loss, we can obviously assume that the borrower will not make any payments in period T.

For any $0 \le t \le T-1$ *define* $\gamma_t = (1-\rho_{T-1}) \prod_{s=t}^{T-2} \max\{1, \rho_s\}$. *Then when constraint (13) is required to hold, the borrower's date* T *consumption is bounded above by* $\prod_{t=0}^{T-1} R_t W_0 / (1-\gamma_0)$.[23] *This level can be achieved by the loans and payments* $\{L_t^*, P_t^*\}$ *defined as follows:*

1. *The borrower receives a loan* $L_0^* = W_0 \gamma_0 / (1-\gamma_0)$ *at date* 0.
2. *In any payment period* t *that follows an investment period, the borrower pays off his entire debt (i.e. his last loan cumulated at rate* r*).*
3. *In any investment period* $t > 0$ *that follows a payment period, the banks make a loan to the borrower equal to* $W_t \gamma_t / (1-\gamma_t)$

Proof: See Appendix.

The loans and payments described in Proposition 4 have the property that the borrower is paying less than 100% of his wealth to the banking sector in payment periods. In terms of the savings market intuition of Section 2, this is because the banks are no longer able to credibly promise to repay an arbitrarily large deposit made by the borrower. In other words, limited commitment of banks places an endogenous constraint on the size of "deposits" that the banking sector can accept. This, in turn, means that the maximal consumption level characterized in Proposition 4 is lower than that achievable when banks can fully commit (see Proposition 1).

We now turn to a simple example in order to illustrate the importance of bank commitment for distinguishing our rules. We contrast the results between having the debt-default rule in effect and the exclusivity enforcement rule in effect.

There are 5 periods ($T = 4$) — see Figure 2. The banks' interest rate is $r = 1$. The borrower's rate of return (R_t) alternates between $\frac{4}{3}$ ($t = 0, 2$), and $\frac{3}{4}$ ($t = 1, 3$). The borrower's initial is wealth is $W_0 = 100$.

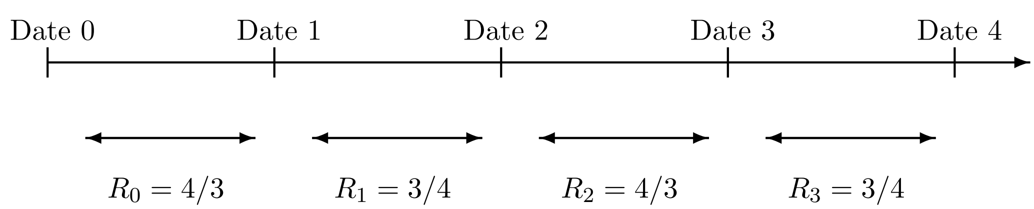

Figure 2: Numerical example

[23]Note that we previously assumed that $\alpha_0 < 1$. It is straightforward to verify that this implies that $\gamma_t < 1 \ \forall \ t$.

If the exclusivity enforcement rule is in effect, the borrower's final consumption is no more than 150.[24] One equilibrium to achieve this is bank 1 making an initial loan of 50, followed by the borrower saving all his wealth with bank 1 between dates 3 and 4 at an interest rate of 3/4. In this case,

- At date 0, bank 1 lends $L_0^1 = 50$.
- At date 1, the borrower enters with wealth $W_1 = (100 + 50) \times \frac{4}{3} = 200$ and debt $D_1^1 = 50$. He invests all of this in his own technology at return $\frac{3}{4}$.
- At date 2, the borrower enters with wealth $W_2 = 200 \times \frac{3}{4} = 150$ and debt $D_2^1 = 50$. Bank 1 extends 0 new loans.
- At date 3, the borrower enters with wealth $W_3 = 150 \times \frac{4}{3} = 200$ and debt $D_3^1 = 50$. He places all 200 as savings with bank 1, where the bank takes advantage of its monopoly position and offers a return of just $\frac{3}{4}$ (the minimum the borrower will accept since his private rate of return is $\frac{3}{4}$). His final consumption at date 4 is then 150. Bank 1 earns the monopoly spread of $\frac{1}{4}$ on the 200 of savings and obtains a profit of 50. This compensates bank 1 for the initial loan of 50.

Under the exclusivity enforcement rule $\mathcal{B}_{excl}$, a bank is happy to make a loan because after doing so the borrower can deal with no other bank. We have commented before that an alternative way to think about a sequence of loans and repayments is instead as an initial gift followed by the borrower saving with the bank in periods when $R_t < r$. When the borrower is restricted from dealing with all but one of the banks, the remaining bank can effectively make monopoly profits in the savings periods. These profits compensate the bank for the initial loan.

Sticking with this intuition, the reason a bank only makes a single loan (before the final date) under the exclusivity enforcement rule $\mathcal{B}_{excl}$ is that after the initial loan, he has already gained all the monopoly benefits available. There is no point in him transferring more funds to the borrower, since doing so in no way increases his ability to profit from his position as the only bank the borrower is able to trade with. This absence of lending at date 2 implies that when the exclusivity rule $\mathcal{B}_{excl}$ is in effect the borrower's equilibrium welfare must fall short of the upper bound established in Proposition 4. In the example, $\gamma_0 = 4/9$ and the upper bound on the borrower's final period consumption from Proposition 4 is 180.

[24] The formal proof that this is the upper bound on the borrower's consumption is straightforward, and is omitted. Informally, banks collectively can obtain a repayment of at most $(1 - 3/4)\, W_3$ at date 3. Moreover, banks will only lend at dates 0 or 2, since the borrower's date 1 rate of return is low. Because the borrower's two-period private rate of return in this example between dates 0 and 2 is simply unity, a borrower in receipt of total loans L has wealth $W_3 = \frac{4}{3}(100 + L)$ at date 3. Solving for the maximal loan banks can advance and still be repaid gives $L = 50$.

Let us contrast this with the result when the debt-default enforcement rule is in effect. The following is an equilibrium:

- At date 0, bank 1 lends $L_0^1 = 80$.
- At date 1, the borrower enters with wealth $W_1 = (100 + 80) \times \frac{4}{3} = 240$ and debt $D_1^1 = 80$. He repays $P_1^1 = 80$.
- At date 2, the borrower enters with wealth $W_2 = 160 \times \frac{3}{4} = 120$ and debt $D_2^1 = 0$. Bank 1 lends $L_2^1 = 60$.
- At date 3, the borrower enters with wealth $W_3 = (120 + 60) \times \frac{4}{3} = 240$ and debt $D_3^1 = 60$. He repays $P_3^1 = 60$. He then places the rest of his wealth (180) as savings with any bank at the return of one. His final consumption at date 4 is simply 180.

At date 3 the borrower repays 60. If he does not, then he will be restricted to dealing with bank 1, in which case bank 1 will offer to save for the borrower at the return of $\frac{3}{4}$. This leaves the borrower with final wealth of 180. Thus the borrower is indifferent between repaying and paying the 60. Note that if the borrower had wealth W_1 any higher than 240 or debt D_3^1 any lower than 60, the borrower would strictly prefer repayment.

Also, note that if instead of debt-default, if the exclusivity enforcement rule was in place, the borrower would repay nothing, and would save with bank 1 at the return of $\frac{3}{4}$. The resulting consumption levels for the borrower and bank would be identical to that under debt-default.

Although seemingly irrelevant at date 3, the fact that the borrower has some incentive to repay the loan in full is relevant for the lender incentives at date 2.

If the exclusivity enforcement rule is in place, the bank will never make a loan payment to the borrower at date 2. This is because regardless of his making a loan, the borrower would make no repayments at date 3, and the lender's profits would come solely from offering the below-market savings rate of $\frac{3}{4}$ to the borrower at date 3. So the bank is strictly better off if it does not make a new loan.

In contrast, under the debt-default enforcement rule the bank is happy to make a new loan at date 2.

If the lender makes a loan of L_2^1 between 0 and 60, the borrower will strictly prefer to repay the loan at date 3 as opposed to defaulting. In this interval, the marginal loan is fully repaid. In contrast, for loans greater than 60 a larger loan has no marginal impact on the borrower's repayment. This, of course, is the same logic we appealed to when stating that under the exclusivity enforcement rule the lender will make no loans beyond an initial one.

All of these considerations only apply if the borrower is out of debt at the start of date 2. If instead he is in debt, bank 1 is already in a monopoly situation, and would be strictly worse off if it made a new loan at date 2.

Finally, note the borrower is persuaded to make the date 1 repayment precisely because there is an equilibrium of the subgame starting at date 2 in which bank 1 makes a new loan if and only the borrower arrives out of debt. The borrower's date 1 repayment incentives are maximized if we choose the subgame equilibrium in which the borrower's consumption is highest, which the above example does in choosing $L_2^1 = 60$.

To summarize, the key property of the debt-default exclusion rule $\mathcal{B}_{DD}$ relative to the exclusivity exclusion rule $\mathcal{B}_{excl}$ is that while both reward a bank for making an initial loan, only the former also gives a lending bank the incentive to make a second loan. Moreover, note that the final consumption level achieved by the borrower in this example is in fact equal to the upper bound derived in Proposition 4. That is, the debt-default enforcement rule $\mathcal{B}_{DD}$ is strictly better than the exclusivity rule $\mathcal{B}_{excl}$ precisely because only the former supplies banks with the incentive to make a loan at date 2.

All important aspects of this example extend to the general T-period environment of the paper. Because of the need to keep track of the value functions of the lending bank and the borrower at all nodes, both in- and out-of-equilibrium, the formal construction of the equilibrium is somewhat lengthy and is (reluctantly!) omitted. Details are available from the authors.

Proposition 5 ***(Limited commitment and the debt-default rule)***
Suppose that the debt-default enforcement rule $\mathcal{B}_{DD}$ is in effect. Then there exists an equilibrium in which the payments between the borrower and one of the banks m^* are $L_t^{m^*} = L_t^*$ and $P_t^{m^*} = P_t^*$, where L_t^* and P_t^* are as defined in Proposition 4. No other transfers occur between the borrower and the other banks $m \neq m^*$. That is, under the debt-default enforcement rule $\mathcal{B}_{DD}$ the constrained efficient outcome exists as an equilibrium.

One final point is worth noting. The equilibrium we constructed in the above example involves the same bank making loans and taking repayments in every period. In fact, it is straightforward to show that there are also efficient equilibria in which a different bank makes the loan in different periods — e.g. bank 1 makes the loan at the initial date and is repaid in the first payment period, bank 2 makes the loan at the next investment date and is then repaid at the following payment date, etc.

6 Robust enforcement rules

In the previous section we showed that while the debt-default enforcement rule $\mathcal{B}_{DD}$ gives banks the incentives to make efficient-sized loans, other specifications of exclusion do not satisfy this same property. In particular, we showed that the exclusivity rule $\mathcal{B}_{excl}$ completely fails to provide banks with the incentive to make a loan at any more than a single date. So even though the $\mathcal{B}_{excl}$ rule provides a harsh enough punishment to provide the borrower with the required level of repayment incentives, it is inferior to other possible specifications of exclusion when it comes to providing lender incentives.

In general, we would like to be able to characterize the set of exclusion specifications that satisfy the two criteria of supplying the borrower with sufficient repayment inducements and the banks with sufficient lending inducements. To address this question in a transparent (and tractable) way, we consider a variant of our environment in which there are only three periods ($T = 2$) and only one bank has funds to make a loan at date 0. The advantage of this setting is that banks need to be given lending incentives at exactly one date (date 0), while similarly the borrower needs to be given repayment incentives at exactly one date (date 1). Essentially this is a snapshot of the general T-period limited commitment case in which banks and the borrower need to be given incentives at multiple dates.

Specifically, we assume that only one of the M banks — without loss, bank 1 — has funds to lend to the borrower at date 0. That is, while all banks are present in the lending game at date 0, only one bank has funds available to actually make a date 0 loan. So bank 1 has a limited monopoly over the borrower and must be provided incentives to make a large enough initial loan. Additionally, we assume that $R_0 > r$ and $R_1 < r$, so that it is beneficial for the borrower to receive a loan at date 0 and to repay at date 1. An enforcement rule regulates financial transfers at dates 1 and 2.

In this setting, the debt-default rule continues to obtain the constrained efficient outcome of Proposition 1. This can easily be seen from the proof of Proposition 3. There we constructed an equilibrium in which only one of the banks offers to make a loan at date 0, while all other banks offered a savings contract, and showed that the efficient outcome obtained.

However, not all rules still deliver the efficient outcome. For consider the exclusivity rule ($\mathcal{B}_{excl}$). If the borrower only traded with banks $m \neq 1$, the maximum consumption that the borrower obtains is $W_0 R_0 r$, since the borrower receives no loan in period 0. This is strictly less than the constrained efficient outcome, since $r < R_1 / (1 - \alpha_0)$. The borrower can only obtain a higher consumption level if he received funds from bank 1 at date 0. However, since bank 1 receives an exclusive right to trade with the borrower *regardless* of the size of the date 0 loan, the bank can reduce the initial loan size and the borrower would continue to accept the loan. This implies that the equilibrium requires the borrower to have consumption that is no more than $W_0 R_0 r$.

Another alternative is the full-exclusion rule $\mathcal{B}_{KLK}$. This rule requires a central authority to compute and dictate specific payments to be made by agents. An obvious application of this rule to the current setting is that the central authority computes the efficient loan size, L_0^*, and dictates that the monopoly bank must make this initial loan or be restricted from all future trade. Although this delivers the efficient outcome, as we observed earlier, we find it unappealing because it is information intensive and is not robust to small errors in the information possessed by the central authority. That is, suppose the central authority mistakenly believes that the true value of L_0^* is $\tilde{L}_0^* > L_0^*$. In this case it will always be unprofitable for bank 1 to make *any* loan at date 0.

To formalize what it means for a rule to be robust to errors in information, we proceed as follows. Write $\mathcal{X}$ for the set of all possible parameter configurations,

$$\mathcal{X} \equiv \{(W_0, R_0, R_1) : W_0 > 0, R_0 > r, 0 \leq R_1 < r, R_0 (r - R_1) < r^2\}$$

For use below, for any parameter choice $x \in \mathcal{X}$ let $L_0^*(x)$ denote the efficient loan size, i.e.

$$L_0^*(x) = \frac{R_0 (r - R_1)}{r^2 - R_0 (r - R_1)} W_0$$

We regard an exclusivity rule as robust if it does not depend on the borrower-specific parameters x:

Definition 4 ***(Robust enforcement rule)***
For parameters, $x \in \mathcal{X}$*, net payments,* $(\mathbf{NP}_0, \mathbf{NP}_1)$*, and bank return* r*, a general enforcement rule* $\mathcal{B}$ *is* ***robust*** *if,*

$$\beta_1^m(\mathbf{NP}_0, \mathbf{NP}_1, r, x) = \beta_1^m(\mathbf{NP}_0, \mathbf{NP}_1, r, x') \ \forall \ x, x' \in \mathcal{X}.$$

In other words, courts only need to observe the net payments at date 0 and date 1 as well as the interest rate r in order to implement a robust enforcement rule. While it may be that x indirectly affects repayments – since $\mathbf{NP}_0$ and $\mathbf{NP}_1$ are endogenous – this effect is summarized in the values of the net payments.

Clearly our debt-default enforcement rule is robust in this sense, and we have already shown that it achieves the constrained efficient level of lending for all $x \in \mathcal{X}$. Likewise, the full-exclusion rule $\mathcal{B}_{KLK}$ is not robust because it is dependent on knowledge of x. In general, what are the properties of an enforcement rule that delivers efficient outcomes and is robust?

Proposition 6 ***(Creditor and debtor rights)***
Let $\mathcal{B}$ be a robust enforcement rule such that for all $x \in \mathcal{X}$ the constrained efficient outcome exists as an equilibrium. Then:

1. ***Creditor rights:*** *Suppose at date* 1 *the borrower has failed to repay a loan to bank* 1*, i.e.* $NP_1^1 < rNP_0^1$*. Then the rule $\mathcal{B}$ must be such that the borrower is punished with a restriction on his ability to deposit funds with the other $M-1$ banks. More precisely, there exists a $\lambda < 1$ such that for any choice of net payments to the remaining $M-1$ banks $\{NP_0^m, NP_1^m : m \neq 1\}$ such that $NP_0^m = 0$ and $\sum_{m \neq 1} NP_1^m > rNP_0^1 - NP_1^1$, then*
$$\sum_{m \neq 1} \beta_1^m \left(\mathbf{NP}_0, \mathbf{NP}_1\right) NP_1^m \leq \lambda \sum_{m \neq 1} NP_1^m \tag{14}$$

2. ***Debtor rights:*** *For all date* 0 *loan values $L_0 > 0$ and any value of $\mu < 1$, the borrower must possess a choice a payments $\mathbf{P}_0$ and $\mathbf{P}_1$ such that*
$$\sum_{m \neq 1} P_1^m > r\left(L_0 - P_0^1\right) - P_1^1$$
(i.e. the borrower attempts to save more than the shortfall in his loan repayment) and
$$\sum_{m \neq 1} \beta_1^m \left(\mathbf{NP}_0, \mathbf{NP}_1\right) P_1^m \geq \mu \sum_{m \neq 1} P_1^m \tag{15}$$
(i.e. at most a fraction $1-\mu$ of these payments are seized) where $\mathbf{L}_0 = (L_0, 0, \ldots, 0)$ and $\mathbf{L}_1 = (0, 0, \ldots, 0)$. Moreover, whenever μ is sufficiently high the payments $\mathbf{P}_0$ and $\mathbf{P}_1$ must satisfy either $P_1^1 \geq r\left(L_0 - P_0^1\right)$ or $P_0^m \neq 0$, some bank $m \neq 1$.

Proposition 6 establishes that all optimal robust enforcement rules must share the following two features: (1) A borrower in default is punished by restricting his access to saving within the financial system; and (2) A borrower must always have the ability to repay his debt so as to have unlimited access to savings.

The reasoning behind the creditor rights part of this Proposition is fairly intuitive: banks must possess the right to punish defaulting borrowers. The intuition behind the debtor rights is as in the example of the previous section. A borrower must have the right to pay off his loan and regain the right to trade freely with all M banks. If he does not possess this right, bank 1 will not have the incentive to make the socially efficient loan at the initial date. The payments that correspond to the borrower paying off his loan must be such that (with a caveat[25]) the loan is actually repaid — i.e. bank 1

[25]The caveat reflects the fact that although we have ruled out message games *per se*, there remains the possibility of the borrower using payments to banks $m \neq 1$ as messages. We have not been able to rule out the case where the borrower has the possibility of making a non-zero payment to a bank $m \neq 1$ to signal that bank 1 made an inefficient loan.

receives a payment that is r times the original loan.

In practice, these are characteristics of virtually all laws that govern debt, default and bankruptcy. Of the rules formally defined in this paper — the full-exclusion rule $\mathcal{B}_{KLK}$, the exclusivity rule $\mathcal{B}_{excl}$, the debt-default rule $\mathcal{B}_{DD}$, and the credit prohibition rule $\mathcal{B}_{CP}$ — only the debt-default rule $\mathcal{B}_{DD}$ possesses both these properties.

Finally, we should note that if we were to make the additional restriction that the fraction of the transfer between the borrower and a bank $m \neq 1$ (i.e. NP_1^m) that is seized by the exclusion rule must be constant with respect to the transfer NP_1^m (though not, of course, with respect to the transfers to bank 1, NP_0^1 and NP_1^m), then we could establish the even stronger result that the debt-default rule is the *only* robust rule to achieve the efficient outcome for all $x \in \mathcal{X}$.

7 Conclusion

We have shown that the threat of financial exclusion can be implemented via the widely-observed right of creditors to garnish the formal sector assets of delinquent borrowers. When a fresh start following bankrupcty is available to borrowers, additional restrictions against fraudulent conveyance are both necessary and sufficient. These findings are perhaps most relevant for the applied development literature and the debate on sovereign bankruptcy reform.

In village economies, where collateral is scarce, it is commonly thought that credit and insurance arrangements are supported by the threat of financial exclusion. Indeed, many authors have analyzed risk sharing in these settings in precisely these terms (for example, Ligon *et al*, forthcoming, use this framework). When implementation is discussed, the common view is that these threats are self-enforcing (see Fafchamps 1999). Our results suggest that these types of analyses may be more broadly applicable. Even in communities in which the close ties necessary to support a self-enforcing arrangement may not exist, as long as financial exclusion can be legally enforced in the ways we have described its threat remains applicable. For example, analyses of the scope for regional risk sharing might plausibly be conducted in the same framework.

This is even more relevant in thinking about the future, as the process of development inevitably results in formal financial institutions entering these village economies. For example, the success of the Grameen Bank has led to NGO's and micro-credit institutitions broadening their lending base. Many observers have expressed concerns with this practice as it undercuts pre-existing informal insurance arrangements (see Attanasio and Ríos-Rull 2000 for a formal analysis of this point). On the one hand, our analysis supports this view. Our results of Section 3 show that if all borrowers have access to savings at rate of return r, then the lending market breaks down. On the other hand, our results on implementation show that the welfare loss associated with this breakdown can be avoided. If financial development is accompanied by improvement in formal legal arrangements – i.e. enforceable creditor rights of the type we have described – then the effects of the loss of informal enforcement can be softened. We speculate that there may be a broader lesson in this that formal financial development and legal development need to go hand-in-hand.

In reaction to a growing perception that sovereign debt default is handled inefficiently, many observers have called for the creation of some form of international bankruptcy court. Most reform proposals draw parallels to U.S. bankruptcy procedure, and particularly to Chapter 9 (municipalities) and Chapter 11 (corporations). However, there is also recognition that sovereigns cannot be "liquidated" and that there is no collateral that can be seized in the event of default.

Although it seems clear that a direct application of U.S. bankruptcy procedure is not appropriate, it is not clear what regulations are optimal in the sovereign setting. Here, our analysis provides a starting point. We have explicitly modeled the punishment of financial exclusion and solved for the optimal procedure to implement this punishment. We think that a similar analysis, suitably enriched, can be conducted for sovereign bankruptcy. While we model the loss of access to financial markets, which most observers view as the main threat underpinning sovereign debt, we ignore important details. For example, in practice, threats of financial exclusion depend on the identity of the lender. The IMF or Citibank may be much more effective in extracting repayment from sovereigns than a small lender. At this point, threats in our model are symmetric because all lenders have the same commitment power. Similarly, our assumption of constant returns to scale in the production technology implicitly rules out grab-races following default, which are an important consideration in sovereign default. We plan on investigating these modifications to our model in the future.

We have tried to conduct our analysis in a general setting, but have clearly simplified on many dimensions. Of these, the most glaring omission is uncertainty. Our investigations confirm that in the stochastic case the debt-default rule ceases to achieve the constrained optimal outcome. A trade-off then arises. On the one hand, increasing return uncertainty makes the debt-default rule less desirable. On the other hand, increasing heterogeneity in the population of borrowers increases the cost of non-robust rules. While we leave an analysis of this trade-off for future research, we conjecture that debt-like arrangements are likely to remain optimal whenever return uncertainty is low.

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A Mathematical proofs

A.1 Two technical results omitted from Section 3

Lemma 2 ***(Non-positive profits)***
For any sequence of loans and payments $\{L_t^m, P_t^m\}$, let V_t^m denote the present value of bank m's (at rate r) loans and payments to the borrower from date t onwards,

$$V_t^m = \sum_{s=t}^{T} r^{-(s-t)} \left(P_t^m - L_t^m\right).$$

Suppose there are at least two banks ($M \geq 2$). Then at any date t the combined net future profits of the banks must be non-positive, $\sum_m V_t^m \leq 0$, in any equilibrium.

Proof of Lemma 2: Suppose to the contrary that $\sum_m V_t^m > 0$ for some t. Denote the equilibrium loans and payments by $\{L_t^{*m}\}$ and $\{P_t^{*m}\}$. Let t_0 be the last date at which this is true. Also, let t_1 be the first date after t_0 such that the net payment to the borrower is weakly positive, i.e. $L_t \geq P_t$. Note that t_1 is well-defined, since at date T the borrower will not make any payments to the banks ($P_T = 0$), so that certainly $\sum_m V_T^m \leq 0$. Thus $t_0 < t_1 \leq T$.

Let m' be a bank whose future profits at date t_0 are minimal, i.e. $V_{t_0}^{m'} \leq V_{t_0}^m$ for all $m \in M$. Then consider the following deviation for bank m', in which basically this bank proposes replacing the aggregate existing payments between the borrower and all banks after date t_0 with identical payments made only to this one bank, and with an extra ϵ delivered to the borrower at date t_1. Formally, define bank m''s deviating lending policy $\tilde{\mathcal{L}}^{m'}$ as follows. For dates $t < t_0$, let $\tilde{\mathcal{L}}^{m'}$ be as before. For dates $t \geq t_0$ other than t_1, let

$$\tilde{l}_t^{m'}\left(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1}\right) = \max\left\{0, \sum_m \left(L_t^{*m} - P_t^{*m}\right)\right\}$$

provided $P_s^{m\prime} = P_s^{*m'}$ for $0 \leq s < t_0$ and $P_s^{m'} = \max\{0, \sum_m (P_s^{*m} - L_s^{*m})\}$ for $t_0 \leq s < t$. Otherwise let $\tilde{l}_t^{m'}\left(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1}\right) = 0$. Finally, let

$$\tilde{l}_{t_1}^{m'}\left(\mathbf{P}_0, \ldots, \mathbf{P}_{t_1-1}\right) = \max\left\{0, \sum_m \left(L_{t_1}^{*m} - P_{t_1}^{*m}\right)\right\} + \varepsilon$$

provided $P_s^{m'} = P_s^{*m'}$ for $0 \leq s < t_0$ and $P_s^{m'} = \max\{0, \sum_m (P_s^{*m} - L_s^{*m})\}$ for $t_0 \leq s < t_1$. Otherwise let $\tilde{l}_{t_1}^{m'}\left(\mathbf{P}_0, \ldots, \mathbf{P}_{t-1}\right) = 0$.

Given this deviation by bank m', the borrower can now obtain an improvement of ε in his utility if he pays $\max\{0, \sum_m (P_s^{*m} - L_s^{*m})\}$ to bank m' at all dates from t_0

onwards. So he certainly deviates to some alternative sequence of payments $\left\{\tilde{P}_t^m\right\}$. Moreover, whatever his deviation it must entail $\tilde{P}_t^m = P_t^m$ for all $t < t_0$, along with $\tilde{P}_t^{m\prime} = \max\{0, \sum_m (P_s^{*m} - L_s^{*m})\}$ for $t_0 \leq t < t_1$, since otherwise there is no way for him to strictly increase his welfare over the maximum previously attainable. Given that the borrower must be transferring to bank m' an amount equal to his previous aggregate payment to all banks in all dates between t_0 and some date $\tau \geq t_1$, the date 0 profits of bank m' under the deviation lending policy are

$$-\varepsilon r^{-t_1} + \sum_{s=0}^{t_0-1} r^{-s}\left(P_s^{*m'} - L_s^{*m\prime}\right) + \sum_{s=t_0}^{\tau}\sum_m r^{-s}\left(P_s^{*m} - L_s^{*m}\right)$$

This expression rewrites to

$$\begin{aligned} & -\varepsilon r^{-t_1} + \sum_{s=0}^{t_0-1} r^{-s}\left(P_s^{*m'} - L_s^{*m\prime}\right) + r^{-t_0}\sum_m V_{t_0}^m - r^{-(\tau+1)}\sum_m V_{\tau+1}^m \\ \geq \quad & -\varepsilon r^{-t_1} + \sum_{s=0}^{t_0-1} r^{-s}\left(P_s^{*m'} - L_s^{*m\prime}\right) + r^{-t_0}\sum_m V_{t_0}^m \end{aligned}$$

where the inequality follows from $\sum_m V_{\tau+1}^m \leq 0$ since $\tau + 1 > t_0$. Since the bank m' was chosen such that its date t_0 future profits $V_{t_0}^{m'}$ were minimal, it follows that the lending policy $\tilde{\mathcal{L}}^{m'}$ results in strict increase in date 0 profits for ε sufficiently small. This completes the proof. **QED**

Lemma 3 ***(Bound on borrower's welfare)***
Suppose there are at least two banks ($M \geq 2$). Then the borrower's final consumption can be no more than $W_0 \prod_{t=0}^{T-1} \max\{r, R_t\}$. That is, the borrower simply invests in his own project when $R_t \geq r$, and saves with the banks at a rate r when $R_t < r$.

Proof of Lemma 3: Consider any date t, with the borrower's wealth level given by W and such that the present value (at the borrower's interest rate) of future payments to the borrower is v. When $M \geq 2$, Lemma 2 implies that the present value of future bank profits (at rate r) must be non-positive at all future dates $t+1, \ldots, T$. So an upper bound on the combined present value is given by the solution $V_t^M(W, v)$ to the maximization problem described in Section 2 consisting of maximizing (3) subject to the constraints (4), (5) and (6), and to the additional constraint that at all $t' > t$

$$-\frac{1}{r^{T-t'}}\sum_{s=t'}^{T} r^{T-s}\left(L_s - P_s\right) \leq 0$$

As in Section 2 we can write this problem recursively as the problem of maximizing the objective (7) subject to the constraints (8) - (11), but now with the additional constraint

$$V_{t+1}^M\left(R_t\left(W + L - P\right), R_t\left(v - \left(L - P\right)\right)\right) \leq 0 \qquad (16)$$

This recursive problem then simplifies to

$$V_t^M(W,v) = \max_{P\in[0,W+L],L\in[0,v]} -(L-P) + \frac{1}{r}V_{t+1}^M\left(R_t(W+L-P), R_t(v-(L-P))\right)$$

again subject to the additional constraint (16). Clearly $V_T^M(W,v) = -v$. We guess (and will then verify) that $V_t^M(W,v)$ is linear in W and v, and of the form $V_t^M(W,v) = \gamma_t W - (1-\gamma_t)v$ with $\gamma_t < 1$. Note that $\gamma_T = 0$. Substituting this guess into the problem, we have

$$V_t^M(W,v) = \max_{P\in[0,W],L\in[0,v]} (\rho_t - 1)(L-P) + \rho_t(\gamma_{t+1}W - (1-\gamma_{t+1})v) \tag{17}$$

subject to

$$L - P \le -\gamma_{t+1}W + (1-\gamma_{t+1})v \tag{18}$$

In payment periods, $\rho_t < 1$ and we want to set $L-P$ as low as possible. So set $L=0$, $P=W$, and the additional constraint (18) does not bind since $\gamma_{t+1} < 1$. Thus for t a payment period, $V_t^M(W,v) = W(1-(1-\gamma_{t+1})\rho_t) - v(1-\gamma_{t+1})\rho_t$ and so $\gamma_t = (1-(1-\gamma_{t+1})\rho_t) < 1$.

In investment periods, $\rho_t \ge 1$ and so it is optimal to set $L-P$ as high as possible. This time the constraint (18) will bind, so we set $L-P = -\gamma_{t+1}W + (1-\gamma_{t+1})v$. Substituting into the objective (17) then gives $V_t^M(W,v) = \gamma_{t+1}W - (1-\gamma_{t+1})v$ so that for any investment period we have $\gamma_t = \gamma_{t+1} < 1$.

Our guess as to the form of $V_t^M(W,v)$ is thus verified, and we have $V_0^M(W,v) = \gamma_0 W_0 - (1-\gamma_0)v$. Since certainly $V_0^M(W,v) \ge 0$ if the banks are to collectively break even, we must have $v \le W_0\gamma_0/(1-\gamma_0)$. Thus the borrower's final period consumption can be no more than

$$\prod_{t=0}^{T-1} R_t\left(W_0 + \frac{W_0\gamma_0}{(1-\gamma_0)}\right) = \frac{W_0}{1-\gamma_0}\prod_{t=0}^{T-1} R_t$$

To complete the proof it is sufficient to establish that $\frac{1}{1-\gamma_t}\prod_{s=t}^{T-1} R_s = \prod_{s=t}^{T-1}\max\{r, R_s\}$ for all t. We proceed inductively. Since $\gamma_T = 0$ the relation is clearly satisfied for $t = T$. Let $t < T$ be a payment period, i.e. $\rho_t < 1$. Then $\frac{1}{1-\gamma_t} = \frac{1}{1-\gamma_{t+1}}\frac{1}{\rho_t}$ so that applying the inductive step we have

$$\frac{1}{1-\gamma_t}\prod_{s=t}^{T-1} R_s = \frac{R_t}{\rho_t}\frac{1}{1-\gamma_{t+1}}\prod_{s=t+1}^{T-1} R_s = r\prod_{s=t+1}^{T-1}\max\{r, R_s\} = \prod_{s=t}^{T-1}\max\{r, R_s\}.$$

Finally, let $t < T$ be an investment period, i.e. $\rho_t \ge 1$. Then $\gamma_t = \gamma_{t+1}$ and so again applying the inductive step,

$$\frac{1}{1-\gamma_t}\prod_{s=t}^{T-1} R_s = \frac{R_t}{1-\gamma_{t+1}}\prod_{s=t+1}^{T-1} R_s = R_t\prod_{s=t+1}^{T-1}\max\{r, R_s\} = \prod_{s=t}^{T-1}\max\{r, R_s\},$$

completing the proof. **QED**

A.2 Proof of Lemma 1

The proof is exactly analogous to the proof of Proposition 2. The only non-trivial issue to show that the analogue to Lemma 2 continues to hold. First note that a straightforward inductive argument implies that under rule $\mathcal{B}_{CP}$ a borrower is indebted to at most a single bank at any date t. Lemma 2 established that the banks cannot be collectively making strictly positive future profits at any date t. The key step in the proof was to consider a deviation in which one of the banks (bank m' say) deviated to a lending policy in which *all* the payments between the borrower and the M banks from a date t_0 onwards are made through bank m'. There are two cases to consider.

First, suppose that the borrower is not indebted to a bank with minimal future profits at date t_0. In this case, under the deviation

$$\tilde{D}_t^{m'} = D_{t_0}^{m'} r^{t-t_0} - \left(\sum_{m \in M} V_{t_0}^m \right) r^{t-t_0} + \sum_{m \in M} V_t^m$$

where $\tilde{D}_t^{m'}$ is the debt level under the deviation. Since by the deviation date t_0 was chosen to be the last date at which aggregate future bank profits are positive, and $D_{t_0}^{m'}$, it follows that $\tilde{D}_t^{m'} < 0$. The enforcement rule $\mathcal{B}_{CP}$ can then have no impact on this deviation, since the borrower is never indebted to the deviating bank m'.

Second, suppose that there is a unique bank with minimal future profits at date t_0, and the borrower is indebted to this bank. Denote this bank m'. Consequently the borrower cannot be indebted to any bank $m \neq m'$. We can then select bank m' as the deviating bank. The enforcement rule $\mathcal{B}_{CP}$ has not impact on the payments between the borrower and bank m' under the deviation The rest of the proof is as before.

A.3 Proof of Proposition 4

Let $V_t^M(W, v)$ be the maximum present value of payments achievable by the coalition of M banks facing a borrower with wealth W, subject to the constraint that the present value of transfers to the borrower v (at the borrower's interest rate). Thus $V_t^M(W, v)$ is the solution to the maximization problem described in Section 2 consisting of maximizing (3) subject to the constraints (4), (5) and (6), along with the additional constraint (13) that the banks' future profits are non-negative at all dates

As in the proof of Lemma 3, we can write this problem recursively as

$$V_t^M(W, v) = \max_{P \in [0, W+L], L \in [0, v]} -(L-P) + \frac{1}{r} V_{t+1}^M \left(R_t (W + L - P), R_t (v - (L-P)) \right)$$

subject to the additional constraint

$$V_{t+1}^{M}\left(R_{t}\left(W+L-P\right),R_{t}\left(v-\left(L-P\right)\right)\right)\geq 0$$

for all $t\leq T-2$. We will guess (and then verify) that $V_{t}^{M}\left(W,v\right)$ is linear in W and v, and of the form $V_{t}^{M}\left(W,v\right)=\gamma_{t}W-\left(1-\gamma_{t}\right)v$ with $\gamma_{t}<1$. From Proposition 1, we know $V_{T-1}^{M}\left(W,v\right)=\gamma_{T-1}W-\left(1-\gamma_{T-1}\right)v$, where $\gamma_{T-1}=\rho_{T-1}$.

Substituting the guess $V_{t}^{M}\left(W,v\right)=\gamma_{t}W-\left(1-\gamma_{t}\right)v$ into the problem, we have

$$V_{t}^{M}\left(W,v\right)=\max_{P\in[0,W+L],L\in[0,v]}\left(\rho_{t}-1\right)\left(L-P\right)+\rho_{t}\left(\gamma_{t+1}W-\left(1-\gamma_{t+1}\right)v\right)\qquad(19)$$

subject to

$$L-P\geq-\gamma_{t+1}W+\left(1-\gamma_{t+1}\right)v\qquad(20)$$

In investment periods, $\rho_{t}\geq 1$ and so it is optimal to set $L-P$ as high as possible. In this case the constraint (20) is not binding. Thus we set $L=v$, $P=0$. Substituting into the objective (19) then gives $V_{t}^{M}\left(W,v\right)=\rho_{t}\gamma_{t+1}W-\left(1-\rho_{t}\gamma_{t+1}\right)v$. So in any investment period t we have $\gamma_{t}=\rho_{t}\gamma_{t+1}$.

In payment periods, $\rho_{t}<1$ and we want to set $L-P$ as low as possible. This time the constraint (20) is binding. Substituting (20) at equality into the objective (19) then gives $V_{t}^{M}\left(W,v\right)=\gamma_{t+1}W-\left(1-\gamma_{t+1}\right)v$. So in any payment period $\gamma_{t}=\gamma_{t+1}$. Without loss, we can assume this is achieved by $L=0$ and $P=\gamma_{t+1}W-\left(1-\gamma_{t+1}\right)v$.

Our guess as to the form of $V_{t}^{M}\left(W,v\right)$ is thus verified, and we have $V_{0}^{M}\left(W,v\right)=\gamma_{0}W_{0}-\left(1-\gamma_{0}\right)v$. Since certainly $V_{0}^{M}\left(W_{0},v\right)\geq 0$ if the banks are to collectively break even, we must have $v\leq W_{0}\gamma_{0}/\left(1-\gamma_{0}\right)$. Thus the borrower's final period consumption can be no more than

$$\prod_{t=0}^{T-1}R_{t}\left(W_{0}+\frac{W_{0}\gamma_{0}}{1-\gamma_{0}}\right)=\frac{W_{0}}{1-\gamma_{0}}\prod_{t=0}^{T-1}R_{t}$$

where $\gamma_{0}=\left(1-\rho_{T-1}\right)\prod_{t=0}^{T-2}\max\left\{1,\rho_{t}\right\}$.

Finally, we characterize a set of payments that lead to this upper bound. Let date t be an investment period and date τ the following payment period. Assume that at date t, $L_{t}=W_{t}\gamma_{t}/\left(1-\gamma_{t}\right)$. At all dates $s=t+1,\ldots,\tau-1$ we have $L_{t}=P_{t}=v=0$. So at date τ we have $v=0$, and so $P_{\tau}=\gamma_{\tau}W_{\tau}$. Note that $W_{\tau}=\prod_{s=t}^{\tau-1}R_{s}W_{t}/\left(1-\gamma_{t}\right)$ and $\gamma_{t}=\gamma_{\tau}\prod_{s=t}^{\tau-1}\rho_{s}$. Then

$$P_{\tau}=\gamma_{\tau}\prod_{s=t}^{\tau-1}R_{s}W_{t}/\left(1-\gamma_{t}\right)=\frac{\gamma_{t}}{\prod_{s=t}^{\tau-1}\rho_{s}}\prod_{s=t}^{\tau-1}R_{s}W_{t}/\left(1-\gamma_{t}\right)=r^{\tau-t}L_{t}$$

i.e. the borrower repays the loan in full.

Next, let $\hat{\tau}$ be the first investment period following the payment period τ. At all dates $s = t+1, \ldots, \tau - 1$ we have $L_t = P_t$. At date $\tau + 1$ we have $v = R_\tau \gamma_\tau W_\tau$, so at date $\hat{\tau}$ we have $v = \prod_{s=\tau}^{\hat{\tau}-1} R_s \gamma_\tau W_\tau$. The borrower's wealth at date $\hat{\tau}$ is $W_{\hat{\tau}} = (W_\tau - \gamma_\tau W_\tau) \prod_{s=\tau}^{\hat{\tau}-1} R_s$. So the loan size at date $\hat{\tau}$ is

$$v = \frac{\gamma_\tau}{1-\gamma_\tau} W_{\hat{\tau}} = \frac{\gamma_{\hat{\tau}}}{1-\gamma_{\hat{\tau}}} W_{\hat{\tau}}$$

A.4 Proof of Proposition 6

Part 1 (Creditor rights): Suppose that contrary to the Proposition's statement there exist values of $\widehat{NP}_0^1$ and $\widehat{NP}_1^1$ such that $\widehat{NP}_1^1 < r\widehat{NP}_0^1$ but with the property that for any $\mu < 1$, there exists a set of payments $\left\{\widehat{NP}_0^m, \widehat{NP}_1^m : m \neq 1\right\}$ such that $\widehat{NP}_0^m = 0$ and $\sum_{m \neq 1} \widehat{NP}_1^m > r\widehat{NP}_0^1 - \widehat{NP}_1^1$ but inequality (14) fails to hold. The proof consists of showing that under these assumptions, there always exists at least some parameter value $x \in \mathcal{X}$ for which the borrower can do strictly better by not repaying bank 1's loan, so that the constrained efficient outcome does not exist as an equilibrium.

Let $\hat{X}$ denote the subset of the parameter space $\mathcal{X}$ for which $L_0^*(x) = \widehat{NP}_0^1$. Suppose $\{NP_0^m, NP_1^m\}$ is an equilibrium that does achieve the constrained efficient outcome. This means that $NP_0^1 = L_0^*\left(\hat{X}\right)$, $NP_0^m = 0$ for $m \neq 1$, and the borrower's final consumption is

$$\hat{U}(x) = \left(W_0 + L_0^*\left(\hat{X}\right)\right) R_0 r - L_0^*\left(\hat{X}\right) r^2$$

and all banks make zero profits.

Suppose for now that the payments $\left\{\widehat{NP}_0^m, \widehat{NP}_1^m : m \neq 1\right\}$ have the property that they completely exhaust the borrower's date 1 wealth, i.e.

$$\sum_{m \neq 1} \widehat{NP}_1^m = \left(W_0 + \widehat{NP}_0^1\right) R_0 - \widehat{NP}_1^1 \tag{21}$$

Choose $\varepsilon > 0$ and $\lambda \in [0,1]$ to be such that the inequality

$$\lambda\left(\left(W_0 + \widehat{NP}_0^1\right) R_0 r - \widehat{NP}_1^1 r\right) - \varepsilon > \left(W_0 + \widehat{NP}_0^1\right) R_0 r - \widehat{NP}_0^1 r^2 = U\left(\hat{X}\right) \tag{22}$$

holds. That is, inequality (22) says that if the borrower can transfer a proportion λ of his date 1 wealth to banks $m \neq 1$ and earn an interest rate r, then he will be strictly better off than repaying bank 1 in full. Note that such choice is always possible, since the left-hand side of (22) is equal to

$$\lambda\left(W_0 + \widehat{NP}_0^1\right) R_0 r - \widehat{NP}_0^1 r^2 + r\left(r\widehat{NP}_0^1 - \lambda\widehat{NP}_1^1\right) - \varepsilon$$

and by supposition $\widehat{NP}_0^1 - \widehat{NP}_1^1 > 0$.

Given inequality (22), it follows that the banks $m \neq 1$ can strictly increase their collective profits to ε by offering to accept saving between dates 1 and 2 at a rate just less than r. Inequality (22) guarantees that the borrower will accept this offer, since doing so yields a utility level strictly greater than $U\left(\hat{X}\right)$. Thus we have established that there cannot be an equilibrium of the type described if (21) holds.

To complete the proof of the lemma it remains only to show that (21) holds for at least some parameter value $x \in \hat{X}$. Consider the line in $\hat{X}$ given by

$$x(\delta) = (W_0, R_0, R_1) = \left(\frac{\delta^2}{r^2 - \delta^2} L_0^*\left(\hat{X}\right), r + \delta, \delta\right) \text{ where } \delta \in (0, r)$$

Since $\widehat{NP}_0^1 = L_0^*\left(\hat{X}\right)$, the borrower's date 1 wealth under the parameter $x(\delta)$ is $\frac{r^2}{r-\delta} L_0^*\left(\hat{X}\right)$. By assumption, $\sum_m \widehat{NP}_1^m > r L_0^*(X)$. So we can always find a value of $\delta \in (0, r)$ such that $\sum_m \widehat{NP}_1^m = \frac{r^2}{r-\delta} L_0^*(X)$, and so (21) holds. This completes the first part of the proof.

Part 2 (Debtor rights): Suppose that contrary to the Proposition's statement there exists an $\hat{L}_0 > 0$ and a $\hat{\mu} < 1$ such that for all $(\mathbf{P}_0, \mathbf{P}_1)$ with $\sum_{m \neq 1} P_1^m > r\left(\hat{L}_0 - P_0^1\right) - P_1^1$ the inequality (15) does not hold. Fix $W_0 = \hat{W}_0$ arbitrarily, and define the set $\hat{X} \subset \mathcal{X}$ to be set of all parameter values x with wealth level $\hat{W}_0$ and such that $L_0^*(x) = \hat{L}_0$. Observe that $(r - R_1) R_0$ is constant over the subset $\hat{X}$.

Note that for the constrained efficient outcome to be an equilibrium at x, bank 1 must use a lending policy with $L_0^1 = \hat{L}_0 = L_0^*\left(\hat{X}\right)$. For any $\varepsilon, \delta > 0$, consider the deviation by bank 1 to a lending policy $\tilde{\mathcal{L}}^1$ with $\tilde{L}_0^1 = \hat{L}_0 - \varepsilon$ and

$$\tilde{l}_2^1(\mathbf{P}_1) = \begin{cases} \left(W_0 + \tilde{L}_0^1\right)(R_1 + \delta) R_0 & \text{if } P_1^1 = \left(W_0 + \tilde{L}_0^1\right) R_0 \\ 0 & \text{otherwise} \end{cases}$$

i.e. at date 1, bank 1 offers to pay a return of $R_1 + \delta$ if the borrower deposit all his wealth. The proof consists of showing that the lending policy $\tilde{\mathcal{L}}^1$ is a profitable deviation for bank 1.

First, assume that given the policy $\tilde{\mathcal{L}}^1$ that the borrower's best response is $\mathbf{P}_0 = \mathbf{0}$ and $\mathbf{P}_1 = \left(\left(W_0 + \tilde{L}_0^1\right) R_0, 0, \ldots, 0\right)$, i.e. the borrower deposits all his date 1 wealth with bank 1. Then at any $x \in \hat{X}$ bank 1 gets

$$-\tilde{L}_0^1 r^2 + (r - R_1 - \delta)\left(W_0 + \tilde{L}_0^1\right) R_0 = \varepsilon\left(r^2 - (r - R_1) R_0\right) - \delta R_0\left(W_0 + \tilde{L}_0^1\right) \quad (23)$$

where we are using the fact that at any $x \in \hat{X}$ we know $-\hat{L}_0 r^2 + (r - R_1)\left(W_0 + \hat{L}_0\right) R_0 = 0$.

It is sufficient to show that the deviation to $\tilde{\mathcal{L}}^1$ is profitable for some $x \in \hat{X}$ (since we require the rule $\mathcal{B}$ to be robust) and some values $\varepsilon, \delta > 0$. We select values of x, ε, δ as follows. First, choose $\mu \geq \hat{\mu}$ such that $(1-\mu) r^2 < (r - R_1) R_0$ for all $x \in \hat{X}$. Choose $\varepsilon \in \left[0, \hat{L}_0\right]$ such that for all $x \in X$

$$(1-\mu) r^2 < \frac{\varepsilon \left(r^2 - (r - R_1) R_0\right)}{W_0 + \hat{L}_0 - \varepsilon} \tag{24}$$

Such a choice is always possible since as $\varepsilon \to L_0$ the RHS tends to $\frac{\hat{L}_0}{W_0}\left(r^2 - (r - R_1) R_0\right) = (r - R_1) R_0$. Let $\hat{x} \in \hat{X}$ be such that

$$\mu r < R_1 \tag{25}$$

(clearly such a choice is always possible by setting R_1 high enough). Finally, choose δ so that

$$(1-\mu) r^2 < \delta R_0 \tag{26}$$

$$\delta R_0 < \varepsilon \frac{\left(r^2 - (r - R_1) R_0\right)}{W_0 + L_0 - \varepsilon} \tag{27}$$

where such a choice is possible by inequality (24).

The right-hand side of (23) is strictly positive given inequality (27), and so bank 1's deviation from $\mathcal{L}^1$ to $\tilde{\mathcal{L}}^1$ is strictly profitable *provided that the borrower responds by depositing all his date* 1 *wealth with bank* 1, i.e., $\mathbf{P}_0 = \mathbf{0}$ and $\mathbf{P}_1 = \left(\left(W_0 + \tilde{L}_0^1\right) R_0, 0, \ldots, 0\right)$. Note that the borrower's final consumption under this choice of $\mathbf{P}_0, \mathbf{P}_1$ given $\tilde{\mathcal{L}}^1$ is

$$\left(W_0 + \tilde{L}_0^1\right) R_0 (R_1 + \delta) = \left(W_0 + \tilde{L}_0^1\right) R_0 R_1 + \left(W_0 + \tilde{L}_0^1\right) R_0 \delta$$

Next, consider any other choice of $\mathbf{P}_0, \mathbf{P}_1$. Necessarily it must feature either $P_0^m > 0$ for some m, and/or $P_1^1 < \left(W_0 + \tilde{L}_0^1\right) R_0$. However, if $P_0^m > 0$ for some m then the payment $P_1^1 = \left(W_0 + \tilde{L}_0^1\right) R_0$ is not feasible,[26] and so either way we have $P_1^1 < \left(W_0 + \tilde{L}_0^1\right) R_0$. The borrower's final consumption is then at most

$$\left(\left(W_0 + \tilde{L}_0^1 - \sum_m P_0^m\right) R_0 - P_1^1\right) R_1 + \sum_{m \neq 1} \left(\beta_1^m r - R_1\right) P_1^m$$

[26] This is true provided that no bank $m \neq 1$ offers a lending policy $\mathcal{L}^m$ in which deposits earn a rate of return strictly higher than r. Such a lending policy would generate strictly negative profits. Ruling out bank $m \neq 1$ strategies that yield negative out-of-equilibrium profits is consistent with our assumption that only bank 1 has surplus funds available.

For the case

$$\sum_{m\neq 1} P_1^m > r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1 \tag{28}$$

then by supposition the borrower's consumption is less than

$$\left(\left(W_0 + \tilde{L}_0^1 - \sum_m P_0^m\right) R_0 - P_1^1\right) R_1 + (\mu r - R_1) \sum_{m\neq 1} P_1^m$$

By inequality (25) this expression must be strictly less than $\left(W_0 + \tilde{L}_0^1\right) R_0 R_1$. On the other hand, if (28) does not hold then the borrower's consumption is certainly less than

$$r\left(r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1\right) + R_1\left(W_1 - P_1^1 - \left(r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1\right)\right)$$

where $W_1 = \left(W_0 + \tilde{L}_0^1 - \sum_m P_0^m\right) R_0$ is the borrower's date 1 wealth. Since

$$\left(W_0 + \tilde{L}_0^1\right) R_0 \geq W_1 \geq r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1$$

the borrower's consumption under the deviation is less than his consumption from sticking to $\mathbf{P}_0, \mathbf{P}_1$ by at least $(R_1 - r + \delta)\left(r\left(\tilde{L}_0^1 - P_0^1\right) - P_1^1\right)$. Conditions (25) and (26) imply that $R_1 - r + \delta > 0$, again establishing that the borrower will indeed stick to repayments $\mathbf{P}_0, \mathbf{P}_1$. Thus we can conclude that $\mathbf{P}_0 = \mathbf{0}$ and $\mathbf{P}_1 = \left(\left(W_0 + \tilde{L}_0^1\right) R_0, 0, \ldots, 0\right)$ is indeed a strict best response for the borrower, completing the second part of the proof.