A Model of Capital and Crises

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Introduction

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- We study the role of distressed intermediary capital in the crises of market with complex asset classes (e.g. MBS).

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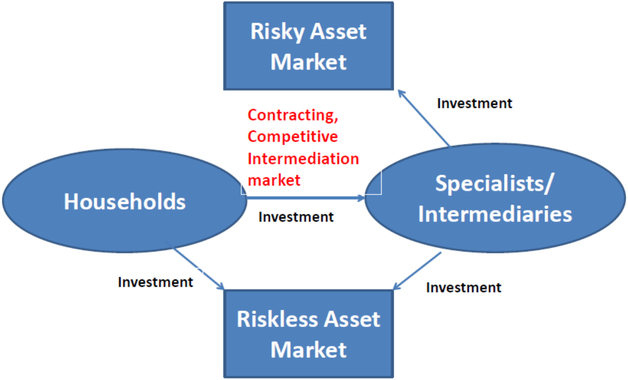
- We study the role of distressed intermediary capital in the crises of market with complex asset classes (e.g. MBS).
- A General Equilibrium (GE) model where intermediaries, rather than households, are marginal.
  - Frictions are endogenously derived based on optimal contracting considerations.
- Mechanism: Intermediation capital affects participation/risk-sharing.
- In normal times households participate through intermediation;
- When intermediaries suffer losses,
  - Distressed intermediary sector averse to hold risky positions, risk premium goes up.
  - Households "fly to quality," drive down interest rate.

Model Structure (1)

- Unit supply of **risky asset** with dividend $\frac{dD_t}{D_t} = gdt + \sigma dZ_t$, and **riskless asset** in zero-net supply.
  - Risky asset price P_t and interest rate r_t are determined in GE.
- **Households** $\mathbb{E}\left[\int_0^\infty e^{-\rho^h t} \ln c_t^h dt\right]$.
  - Limited participation in risky asset market. They invest in intermediaries.
- **Specialists** $\mathbb{E}\left[\int_0^\infty e^{-\rho t} \ln c_t dt\right]$, $\rho < \rho^h$. They run **intermediaries**.
  - Only intermediaries/specialists can invest in the risky asset. They are marginal investors.
  - Derive **Intermediation Constraint** from moral hazard primitives.

Model Structure (2)



The economy.

- **Intermediation**: 1) Short-term contracting between agents; 2) Equilibrium in competitive intermediation market;
- **Asset pricing**: 3) Optimal consumption/portfolio decisions; 4) GE.

The Heart of the Model: (equity) Capital Constraint

- Say household with wealth W_t^h, and specialist with wealth W_t.
  - Given specialist's contribution W_t in the intermediary, household contributes T_t^h as equity investment (for risk sharing).
  - Capital Constraint: T_t^h is capped at mW_t so risk sharing is capped at $1 : m$.
- Intermediation capacity mW_t is increasing in the specialist's contribution W_t, as reflection of agency friction.

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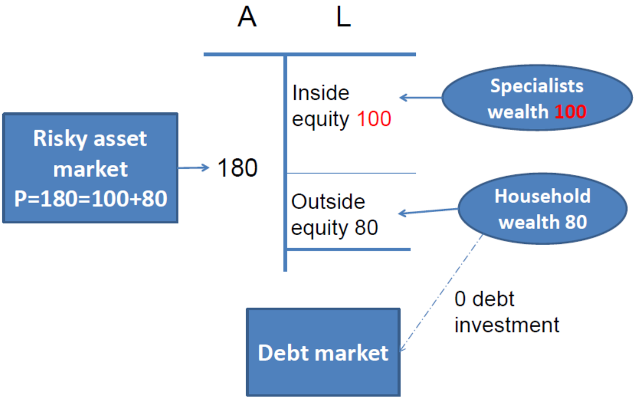
- How to interpret m?
  1. Intermediary capital requirement: outside/inside contribution ratio; (Holmstrom-Tirole, QJE)
     - Officers/Directors inside holdings in financial industry around 18%.
  2. Incentive contract—the performance share of hedge fund managers. Think of "2 and 20."
  3. Mutual funds' flow-performance sensitivity. Specialist's W_t tracks his past gains and losses (Shleifer-Vishny, JF)

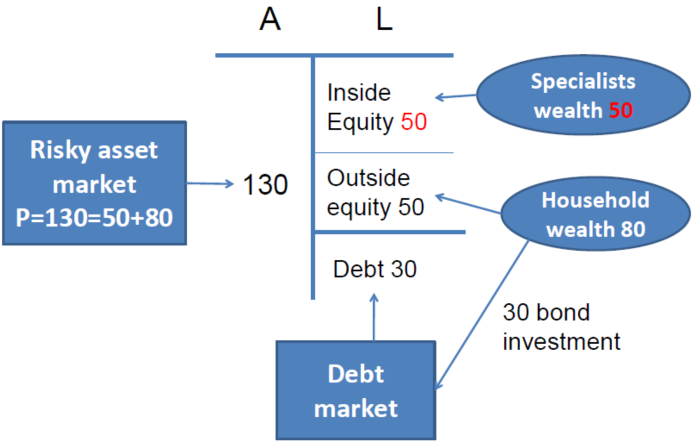
Intermediation Constraint: An Example (1)

- Say $m = 1$, $W_t^h = 80$. Comparing W_t^h to mW_t.
- **Unconstrained Region:** $W_t = 100$. Then $T_t^h = W_t^h = 80$;
  - Zero net debt. Risky asset price $P_t = W_t + W_t^h = 180$.
  - Fund's total equity 180. Intermediary holds risky asset without leverage, first-best risk sharing.



Intermediation Constraint: An Example (2)

- **Constrained Region:** $W_t = 50$. Then $T_t^h = mW_t = 50$;



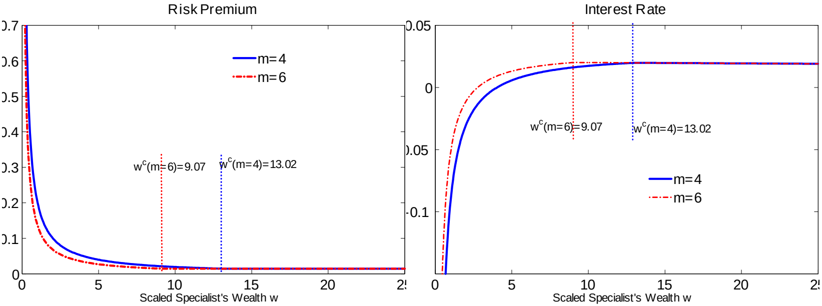
Intermediation Constraint: An Example (2)

- **Constrained Region:** $W_t = 50$. Then $T_t^h = mW_t = 50$;
- Intermediary's total equity is $50 + 50 = 100$. But $P_t = 130$.
- In equilibrium, the intermediary borrows 30 from the debt market;
  - It is supplied by households $W_t^h - T_t^h = 30$.
- Specialist and household have equal shares in the intermediary;
- Specialist's leveraged position in risky asset:

$$\alpha = \frac{\text{specialist's portion of asset}}{\text{specialist's equity}} = \frac{130/2}{50} = 130\%.$$

- Risk premium has to adjust to make this high leverage optimal.

Risk Premium and Interest Rate



Intermediation Stage Game

- **Short-term** contracts only. At time t, contract from t to $t + dt$.
- Household with wealth W_t^h, and specialist with wealth W_t.
  - Household contributes T_t^h, specialist T_t. $T_t^I = T_t^h + T_t$.

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- Specialist in charge of intermediary. **Moral Hazard:**
  1. Unobserved **due diligence action** $s_t = \{0, 1\}$.
     - Shirking ($s_t = 1$) reduce return by X_t but brings private benefit B_t.
  2. Unobserved **portfolio choice** $\mathcal{E}_t^I$ (dollar exposure to risky asset);
     - Undoing activity. Not crucial.
- Fund's return $\mathcal{E}_t^I \left(dR_t - r_t dt\right) + T_t^I r_t dt - s_t X_t dt$, private benefit $s_t B_t dt$. Focus on implementing working.

Intermediation Contract

- **Affine contracts** for sharing returns.
  - β_t: specialist's share; $\hat{K}_t dt$: transfer to specialist.
- Set $K_t \equiv \left(\beta_t T_t^I - T_t\right) r_t + \hat{K}_t$.
- Dynamic budget constraint

$$\begin{cases} dW_t = r_t W_t dt - c_t dt + \beta_t \mathcal{E}_t^I \left(dR_t - r_t dt\right) + K_t dt, \\ dW_t^h = r_t W_t^h dt - c_t^h dt + (1 - \beta_t)\mathcal{E}_t^I \left(dR_t - r_t dt\right) - K_t dt. \end{cases}$$

- Reduce contract to (β_t, K_t). **Sharing rule** and **fee**.
  - Specialist chooses $\mathcal{E}_t = \beta_t \mathcal{E}_t^I$. Household buys risk exposure $\mathcal{E}_t^h = (1 - \beta_t)\mathcal{E}_t^I$ from intermediary.
  - In competitive intermediation market, the fee will take some simple linear form.

IC Constraint and Maximum Household's Exposure

- **IC constraint**: specialist bears at least a certain fraction of risk.
  - Incentive provision. Skin in the game.
  - No shirking: $\beta_t X_t - B_t \geq 0 \Rightarrow \beta_t \geq \frac{B_t}{X_t} \equiv \frac{1}{1+m}$.
  - A lower bound on β_t.
- Specialist always chooses $\beta_t \mathcal{E}_t^I = \mathcal{E}_t^*$ independent of β_t.
  - In the paper we show $\mathcal{E}_t^*$ is independent of K.

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- $\mathcal{E}_t^I$ fund's total risk exposure. S: $\mathcal{E}_t = \beta_t \mathcal{E}_t^I$, H: $\mathcal{E}_t^h = (1-\beta_t)\,\mathcal{E}_t^I$.
  - Household exposure from the contract, or **exposure supply**:

$$\mathcal{E}_t^h = (1-\beta_t)\mathcal{E}_t^I = \frac{1-\beta_t}{\beta_t}\mathcal{E}_t^*.$$

- As $\beta_t \geq \frac{1}{1+m}$, **households maximum exposure** $\mathcal{E}_t^h \leq m\mathcal{E}_t^*$.

Key Intuition and Equity Implementation

- The households exposure is capped due to agency frictions $\mathcal{E}_t^h \leq m\mathcal{E}_t^*$.
- It caps a risk-sharing rule between households and specialists.
  - Incentive provision implies that specialists have to bear sufficient risk.
- In bad times this friction kicks in.
  - Even if specialists wealth is low, they still have to bear disproportionally large risk.

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- In bad times this friction kicks in.
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- **Equity implementation**: Households (outsiders) cannot hold more than $\frac{m}{1+m}$ (equity) shares.
- **Equity capital constraint:** Given specialist's equity W_t, households can make at most mW_t equity contributions.
- Recall contract (β_t, K_t). We have derived equilibrium β_t. What determines fee K_t?
  - Households pay competitive fees in the intermediation market.

Competitive Intermediation Market

At time t, specialists make offers (β_t, K_t) to households who can accept/reject offers. The intermediation market reaches equilibrium if: 1) β_t is incentive compatible; 2) no profitable deviation coalitions.

Lemma: *In equilibrium, households face a per-unit-exposure price of $k_t \geq 0$: to purchase $\mathcal{E}_t^h$, he has to pay $K_t = k_t \mathcal{E}_t^h$.*

- Idea: equivalence between core and Walrasian equilibrium.
  - Households and specialists form coalitions to chop off the exposure linearly.
- Now we start studying agents' consumption/portfolio problems.

Households' Consumption/Portfolio Rules

- Log investors. Simple consumption rule; myopic mean-variance portfolio choice.
- Risky asset excess return $dR_t - r_t dt = \pi_{R,t} dt + \sigma_{R,t} dZ_t$.
- Household $\max_{\{c_t, \mathcal{E}_t\}} \mathbb{E}\left[\int_0^\infty e^{-\rho^h t} \ln c_t^h dt\right]$ subject to

$$dW_t^h = W_t^h r_t dt - c_t^h dt + \mathcal{E}_t^h (dR_t - r_t dt) - k_t \mathcal{E}_t^h dt.$$

- Standard problem; households achieve exposure $\mathcal{E}_t^h$ by paying per-unit-cost of k_t.
- Optimal consumption $c_t^{h*} = \rho^h W_t^h$, optimal exposure $\mathcal{E}_t^{h*} = \frac{\pi_{R,t} - k_t}{\sigma_{R,t}^2} W_t^h$.
    - Optimal risk exposure is decreasing in exposure price k_t.

Specialists' Consumption/Portfolio Rules

- The specialist supplies an exposure $\frac{1-\beta_t}{\beta_t}\mathcal{E}_t^*$. Given exposure price k_t, he gets intermediation fees $K_t dt = k_t\left(\frac{1-\beta_t}{\beta_t}\mathcal{E}_t^*\right)dt$.
- The specialist is solving: $\max\limits_{\{c_t,\mathcal{E}_t,\beta_t\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\ln c_t dt\right]$ subject to

$$dW_t = \mathcal{E}_t\left(dR_t - r_t dt\right) + \max_{\beta_t\in\left[\frac{1}{1+m},1\right]}\left(\frac{1-\beta_t}{\beta_t}\right)k_t\mathcal{E}_t^* dt + W_t r_t dt - c_t dt.$$

- $\beta_t^* = \frac{1}{1+m}$ if $k_t > 0$, otherwise $\beta_t^* \in \left[\frac{1}{1+m},1\right]$ if $k_t = 0$. **Exposure supply schedule**.
- $\mathcal{E}_t^*$ is the exposure **expected** by households, and must coincide with the specialist's **actual** optimal choice in REE.

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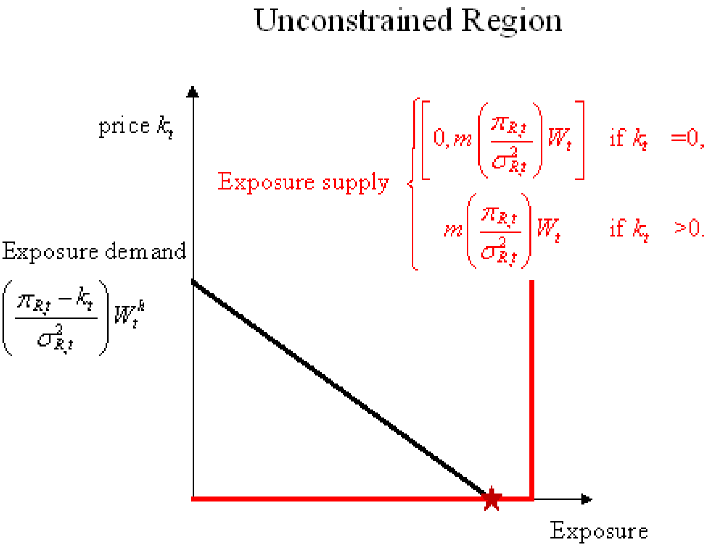
- $\mathcal{E}_t^*$ is the exposure **expected** by households, and must coincide with the specialist's **actual** optimal choice in REE.
- **Solution:** $c_t^* = \rho W_t$ and $\mathcal{E}_t^* = \frac{\pi_{R,t}}{\sigma_{R,t}^2} W_t$, and specialists receive fee of $K_t = q_t W_t$ where $q_t = \left(\frac{1-\beta_t^*}{\beta_t^*}\right) k_t \frac{\pi_{R,t}}{\sigma_{R,t}^2}$.

Unconstrained vs. Constrained Regions (1)



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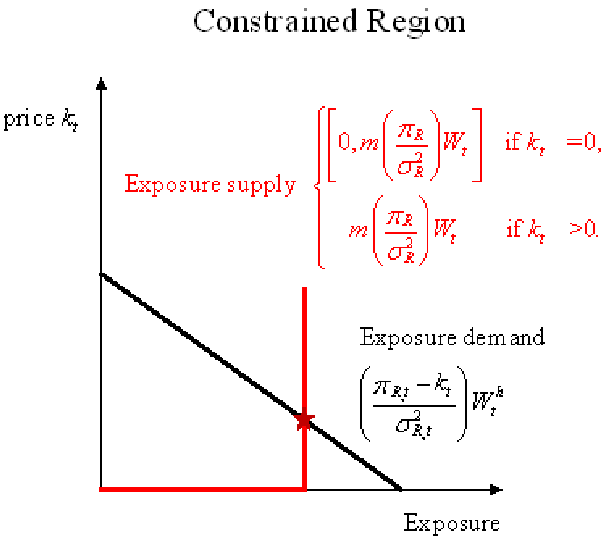

Equilibrium Asset Prices: Solution

- We derive everything in closed form.
- State variables (D_t, W_t). Scales with D_t.
- Uni-dimensional state variable $w_t \equiv W_t / D_t$ captures wealth distribution.

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- State variables (D_t, W_t). Scales with D_t.
- Uni-dimensional state variable $w_t \equiv W_t / D_t$ captures wealth distribution.
- Consumption rules $c_t^* = \rho W_t^h$, $c_t^{h*} = \rho^h W_t^h$.
- Zero net debt $W_t + W_t^h = P_t$, goods clearing $c_t^* + c_t^{h*} = D_t$. So

$$\frac{P_t}{D_t} = \frac{1}{\rho^h} + \left(1 - \frac{\rho}{\rho^h}\right) w_t.$$

- Specialist's risky (percentage) position $\alpha_t = \frac{P_t}{(1+m)W_t} > 1$ in constrained region.

Asset Pricing (1)

- The economy is in constrained region whenever

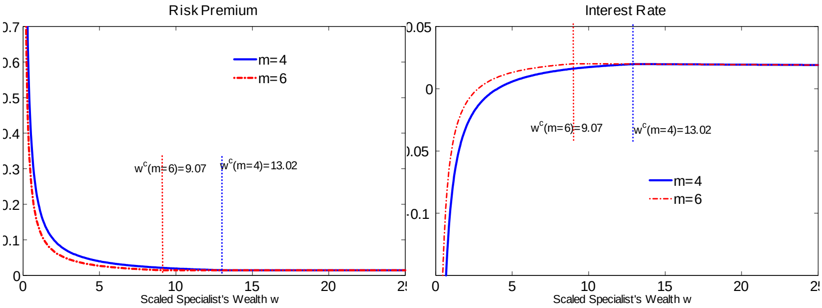
$$w_t = W_t / D_t < w^c \equiv \frac{1}{m\rho^h + \rho}.$$

- In unconstrained region, w_t increases deterministically toward w^c.
  - Perfect risk sharing rule. Relative patience level $\rho < \rho^h$ matters.
- In constrained region, specialists take a higher leverage than households. Therefore w_t becomes stochastic and drops when fundamental falls.
- When (scaled) intermediary capital w_t falls in constrained region,
  - Risk premium rises;
  - Interest rate falls;
  - Volatility rises;
  - Correlation endogenously rises.

Asset Pricing (2)

	Uncon. Region	Con. Region
$\mathcal{E}_t^*$	W_t	$\frac{1}{1+m} P_t$
α_t	1	$\frac{1+(\rho^h-\rho)w_t}{(1+m)\rho^h w_t} > 1$
$\sigma_{R,t}$	σ	$\frac{\sigma}{1+(\rho^h-\rho)w_t}\left(\frac{(1+m)\rho^h}{m\rho^h+\rho}\right) > \sigma$
$\pi_{R,t}$	σ^2	$\frac{\sigma^2}{w_t(m\rho^h+\rho)}\left(\frac{(1+m)\rho^h}{m\rho^h+\rho}\right)\left(\frac{1}{1+(\rho^h-\rho)w_t}\right) > \sigma^2$
k_t	0	$\frac{1-\left(\rho+m\rho^h\right)w_t}{(1-\rho w_t)\left(1+(\rho^h-\rho)w_t\right)}\frac{(1+m)\rho^h\sigma^2}{w_t(m\rho^h+\rho)^2} > 0$
r_t	$\rho^h + g - \sigma^2 + \rho\left(\rho - \rho^h\right) w_t$	$\rho^h + g + \rho\left(\rho - \rho^h\right) w_t - \sigma^2 \frac{\left[\rho\left((1+m)\left(\frac{1}{w_t}-\rho\right)-m^2\rho^h\right)+\left(m\rho^h\right)^2\right]}{(1-\rho w_t)\left(\rho+m\rho^h\right)^2}$

Risk Premium and Interest Rate



- Asymmetry. Crisis like.
- When constraint binds $w_t < w^c$, specialist bears disproportionally large risk, causing more volatile pricing kernel.
- Flight to quality. 1) Specialists precautionary savings. 2) Household fly to debt market.

Comovement

- Consider an infinitesimal asset with

$$\frac{d\hat{D}_t}{\hat{D}_t} = \frac{dD_t}{D_t} + \hat{\sigma} d\hat{Z}_t.$$

- The correlation between dR_t and $d\hat{R}_t$ is:

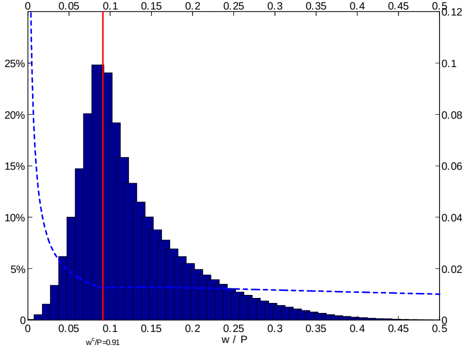
$$corr(dR_t, d\hat{R}_t) = \frac{1}{\sqrt{1 + (\hat{\sigma}/\sigma_{R,t})^2}}.$$

- Unconstrained region, since σ_R is constant, the correlation is constant.
- Constrained region, rising correlation.
  - Market return volatility $\sigma_{R,t}$ rises, magnifying the common component of returns.

Concluding Remarks (1)

- Canonical intermediation friction meets canonical GE asset pricing models.
- Calibratable, easy to quantify effects.
- We have another paper where specialists have general CRRA power utility, with capital constraint as given.
  - Calibrate the model to the MBS market;
  - Add in labor income, debt households (create leverage in unconstrained region), and other necessary twists...
  - Study the crisis dynamics (especially recovery), government liquidity injection policies, etc.

Concluding Remarks (2): Calibration result



	Crisis Recovery	
Transit from 12%	W/o Capital Infusion	W. Capital Infusion (48bn)
10%	0.17	0
7.5%	0.66	0.31
5%	2.72	2.20
4%	5.88	5.06