Collateral Constraints and the Amplification Mechanism

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Abstract

Macroeconomic models of credit market imperfections have been offered as a theory for how common shocks to the balance sheets of credit constrained firms are amplified through changes in the value of collateral and transmitted as fluctuations in output. This paper clarifies and extends these models by first showing that they are not robust to the introduction of markets which allow these firms to hedge against common shocks. A theory of incomplete hedging is then proposed in which the supply of hedging available in the economy is constrained by the aggregate value of collateral. In aggregate, the capacity of banks and other financial intermediaries to provide finance is limited by the aggregate collateral constraint. I find that the constraint introduces a skewed response of the economy to shocks. While the constraint may not affect activity in many states of the world, if shocks are sufficiently adverse, the constraint binds and financial market imperfections amplify the downturn.

Keywords: Collateral, Credit Market Imperfections, Financial Crises, Incomplete markets.

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1 Introduction

Collateral – buildings, machinery, etc. – is typically used to secure business loans. It has been pointed out that when this same collateral is a productive input for business, then aggregate conditions have a direct effect on the operations of business by altering collateral values and therefore the debt capacity of individual firms. This point has been made most forcefully in the literature by Kiyotaki and Moore (1997).[1] The central idea is that bad times for the economy will also be times when the liquidation value of collateral will be low since potential buyers of these assets will be cash-strapped. This naturally leads to low debt capacity in bad times, which can then turn bad times into even worse times as limited financing forces firms to curtail production. This further reinforces the bad times, causing collateral values to fall, and so on. Kiyotaki and Moore (1997) describe this as a collateral amplification mechanism. Their model helps to address the important macroeconomic question of how small shocks can lead to large enough effects to account for empirically observed business cycle fluctuations.

A troubling aspect of the model of Kiyotaki and Moore (1997) (henceforth, KM) is that firms are excluded from hedging or insurance opportunitities. This is troubling because if given a choice, firms in these model would in fact choose to hedge (see Myers and Majluf (1984) and Froot, Scharfstein, and Stein (1993)). Of course, in practice there has been an explosive growth in both the availability and use of hedging instruments by corporate risk managers. My aim in this paper is to understand the importance of these hedging markets, both for the theory under question and more generally for transmission mechanisms involving collateral values. The baseline model I propose, under the constraint that firms are unable to participate in these insurance markets, provides results that are qualitatively similar to the Kiyotaki and Moore (1997) model.[2] However when insurance markets are introduced, I show that firms will find it optimal to hedge and that doing so completely decouples any linkages between between collateral values and real investment decisions.[3] Thus, I arrive at the conclusion that *a model of the collateral amplification mechanism*

[1] These ideas can be traced back to Fisher(1933)'s debt-deflation theory of the Great Depression. More recently, Bernanke and Gertler (1989), Kashyap, Scharfstein and Weil (1990) and Shleifer and Vishny (1992) have also explored this channel.

[2] The question of robustness to insurance is raised by KM in the conclusions of their paper. However, since agents in their model do not anticipate shocks, they are not able to address the question. In the model we construct, shocks are fully anticipated; thus, we are able to clarify this issue.

[3] The insurance markets I introduce are insurance against common shocks. This may puzzle the reader since in a standard general equilibrium framework (say, Lucas (1978)) insurance against aggregate shocks is not possible. Models of credit market imperfections emphasize the aggregate effects of the distribution of wealth rather than the level of wealth itself. The insurance markets are used to insure shocks to the distribution of wealth.

must incorporate a theory of incomplete hedging. I suggest that an important limitation to hedging lies in the credibility of the suppliers of insurance. Insurance, like credit, is a contract subject to credibility concerns. Under the constraint that insurance suppliers must also post collateral to guarantee any obligations, the depth of insurance markets is limited to the aggregate value and quantity of collateral. The main result of the paper is that amplification effects are preserved even in the presence of insurance markets, however the relevant collateral constraint shifts from borrowers to suppliers of insurance.

An example will help to fix ideas regarding the collateral constraint. A bank typically has contingent credit lines that it extends to firms. Likewise, it issues contingent deposit contracts to its depositors. Both of these are insurance-like contracts that require the banks to provide liquidity in some states of the world.[4] Against this commitment to provide liquidity, most banks hold assets such as mortgages, corporate debt and government bonds. In the US, most banks are sufficiently collateralized that they are credible in providing this liquidity.[5] Now, consider the following experiment. Suppose that there was only one bank in the economy and that it held all of the collateral in the economy - i.e. all of the corporate debt and mortgages. Take a state of the world, say z, and suppose that the value of all of this collateral in this state is $V(z)$. Then, liquidity claims sold against this state of the world, because of the collateral constraint, cannot exceed $V(z)$. $V(z)$ is referred to as the *aggregate collateral constraint.* Amplification effects arise only when the aggregate collateral constraint is the factor limiting insurance use. Take a "bad" state of the world - a notion that will be made precise in the model. For emphasis, call this a depression and note that $V(z)$ is low in this state. Insurance against a depression is limited to $V(z)$. The supply of insurance may be limited in precisely the states in which it is most desired. In addition to restoring amplification effects, this channel also generates implications for equilibrium interest rates, the volume of hedging, and the volatility of asset prices that are in accord with the empirical findings in a credit crunch.

There is a growing literature in business cycle theory on the role of financial factors in propagating and amplifying external shocks.[6] However, the bulk of this literature suppresses insurance/hedging markets against the external shock. An exception to this is the work of Suarez and Sussman (1997), and indeed, the goals of my paper parallel theirs. Suarez and Sussman (1997) note that indexation/hedging would be a preferred choice of both borrowers

[4]See Kashyap, Stein and Rajan (1999) for a theoretical rationale for tying the deposit and asset side of banks together. The argument hinges on the insurance like features of both sides; when combined the insurance is enhanced by diversification.

[5]Regulations to increase credibility such as deposit insurance or capital constraints can be viewed as government creation of collateral. See Gorton and Pennacchi (1993) and Holmstrom and Tirole (1998) for a related argument on this point.

[6]See Bernanke, Gertler, and Gilchrist (1998) for a comprehensive survey.

and lenders in the models of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). They then construct a model that delivers propagation effects despite the fact that agents are able to index their financial contracts. While our goals are similar, the analysis of this paper differs from theirs in many respects. First of all my interest is in understanding the collateral amplification mechanism. There is no collateral in the Suarez and Sussman model, thus collateral amplification of the kind that KM emphasize has no role in their model. The analytical approach of this paper allows me to clarify the importance of hedging markets for the collateral channel of KM. Second, my study of collateral leads in a rather different direction, as the conclusion of this paper is that the supply side of finance is an extremely important part of the collateral amplification mechanism.

There are a number of other papers which have focused on the supply side of finance by explicitly modeling financial intermediation (see for example, Bernanke (1983), Bernanke and Blinder (1988), Kashyap and Stein (1994), and Holmstrom and Tirole (1997)).[7] However, these papers, which focus on the effects of shocks to the capital of banks and other financial intermediaries, also assume that intermediaries cannot hedge against aggregate shocks. Insurance markets in my model are open to both intermediaries and firms, thus the model is free of this criticism. My modeling of limited insurance is much closer to Holmstrom and Tirole (1998) who present a model of firms that demand collateral (insurance) to guard against a stochastic liquidity shock. Collateral consists of the aggregate value of equity claims on these same firms. Holmstrom and Tirole study whether the aggregate supply of collateral will be sufficient for demand, concluding that it may not be and hence motivating a role for the state in the creation of these assets. Aggregate sufficiency of collateral is also at the heart of this paper, but the central question is studying how changes in the value of collateral affects this sufficiency.[8]

The paper is arranged as follows. The next section lays out the basic model and its assumptions. Sections 3 and 4 studies an economy in which firms have private collateral constraints, but are able to hedge against aggregate shocks. We show that that they will have a desire to do so, and that doing so eliminates amplification effects. Section 5 introduces the aggregate collateral constraint into the supply of hedging and shows how amplification effects are reintroduced. Section 6 simplifies the model and illustrates the collateral amplification channel of the paper via an example. Section 7 contains conclusions.

[7] There is also a growing literature on capacity constraints in the insurance industry. See Gron and Lucas (1995), Froot and O'Connell (1997).

[8] The modeling structure I employ is formally akin to that of the general equilibrium literature on collateral and incomplete markets introduced by Dubey, Geanakoplos and Zame (1995). However the goals of my paper are vastly different in that I am specifically interested in the macroeconomic implications of collateral considerations.

2 The Model

Overview and Structure

I present a stylized model of an economy in which a physical asset (land) serves as both an input into production as well as collateral to secure contracts. Farmers and bankers each produce corn using land and seed as an input. Farmers are short of funds and so must borrow from the more wealthy bankers in order to produce. This borrowing, because of contracting issues, will require the security of collateral. We are particularly interested in the interaction between collateral values and real investment decisions. To highlight this, it is sufficient to focus on a three period world. The first period is an investment period; shocks affect the economy at the second, and thereby affect collateral values and reinvestment; in the final period all production ends and only consumption takes place. The middle period, in which collateral values are uncertain, will deserve most of our attention. The amplification effect, insurance, and the effect of the aggregate collateral constraint will be visible here.

The Economy

The economy has three dates, $t = 0, 1, 2$. There are two goods, one perishable ("corn") and the other durable ("land"). The durable good is valued only as an input into production, and has no consumption value. It is in fixed supply, $\bar{K}$, and does not depreciate. The perishable good is both an input into production and can be consumed.

Firms

There are two types of firms, labeled B and F. There is a continuum of unit measure of each type. Firms are run by entrepreneurs who have preferences over their consumption of corn, (c_0, c_1, c_2)[9]

$$U = E[c_0 + c_1 + c_2], \qquad\qquad c_t \geq 0 \tag{1}$$

c_t are the dividends of the firm at date t.[10]

Production Opportunities

Firms are characterized by their endowments and their production functions. We refer to F as the *farming sector*. F has access to a production technology at dates 0 and 1 which allows it to produce corn with an input of a combination of land and corn (really, seed,

[9]The consumption good is corn rather than money for the simple reason that money is a financial asset that can also serve as collateral. For the sake of clarity it is essential to define consumption over real goods.

[10]The use of linear preferences, in addition to simplifying the exposition, will highlight the demand for insurance as being production rather than preference related.

to follow the farming analogy). To produce an output of $zf(k_0)$ units of corn at date 1, a farmer must combine exactly k_0 units of land with αk_0 units of corn. Thus if actual inputs are k_0 land and C_0 corn, output is,

$$zf(min[k_0, C_0/\alpha])$$

Similarly, an input of k_1 units of land and C_1 units of corn at date 1 yields $f(min[k_1, C_1/\alpha])$ units of corn at date 2. While date 1 production is non-stochastic, date 0 production is stochastic. z is a random variable which is realized after date 0 investment is made and is a common shock affecting all F firms. It is a discrete random variable taking on values in a finite set, $z \in \mathcal{Z} \equiv \{z_1, z_2, ..., z_N\}$, with corresponding probabilities $\pi(z)$. The shock is normalized so that $E[z] = 1$. With a slight abuse of terminology, z will be referred to as both the state of the world and as the aggregate shock.

B also uses land in production. It produces via a non-stochastic technology, $b(min[k, C/\alpha)$

The two technologies are as follows,

$$\begin{aligned} f'(\frac{\bar{K}}{2}) &= \rho, \qquad f(0) = 0, f'(k) > 0, f''(k) < 0, f'(0) < \bar{B} < \infty && (2) \\ b'(\frac{\bar{K}}{2}) &= \rho, \qquad b(0) = 0, b'(k) > 0, b''(k) < 0, b'(0) < \bar{B} < \infty \qquad \forall k && (3) \\ \rho &> 1 + \alpha \end{aligned}$$

Both functions have continuous first and second derivatives. There is decreasing returns to scale with marginal product equalized when sharing land evenly.

Endowments

The basic problem is a borrowing/lending relationship. On the one hand, B is endowed with a large amount of corn at all three dates (exogenous to the specification) and all of the stock of land, $\bar{K}$. On the other hand, F only has a small date 0 endowment of corn, w_0^f. B will be the source of credit in this economy and we refer to it as the *banking sector.*

3 E0: The Benchmark Economy

In the economy described so far, the assumptions behind Modigliani-Miller hold. F can borrow from B to finance any positive net present value investment. Thus, consumption and production decisions can be treated separately and the equilibrium is fairly easy to characterize. However, let us go through the individual agent optimization problems. The purpose in doing this is to introduce the notation for prices and quantities that will be used in the rest of the paper.

First, given the Leontief production technology, it is clear that firms will always choose inputs of corn and land in proportion to each other. That is, if a firm uses k units of land, it will always use $C = \alpha k$ units of corn. We therefore write the production choices purely in terms of k.

Let X^a be the vector of consumption and investment choices, and θ^a be a security plan for each of $a \in \{F, B\}$. Then,

$$\begin{aligned} X^a &= (c_0^a, c_1^a(z), c_2^a(z), k_0^a, k_1^a(z)) \\ \theta^a &= (\theta_0^a(z), \theta_1^a(z)) \end{aligned}$$

Define P as the equilibrium price vector: $P = (u_0, u_1(z), \phi_0(z), \phi_1(z))$. u_0 and $u_1(z)$ are the rental price of land at date 0 and in each state z at date 1. The rental prices are expressed as relative to the price of corn in that date and state. Corn at date 0 is taken to be the numeraire, and its price is set to one. $\phi_0(z)$ is the date 0 price of contingent claim that pays one unit of corn at date 1 in state z. $\phi_1(z)$ is the date 0 price of a claim that pays one unit of corn at date 2 in state z.

Notice that a price for ownership of land has not been specified. It is unnecessary to do so because there is a market for rental of land. Since the only use of land is as a productive input, and ownership carries no special rights, specifying a rental market is sufficient.

The optimization problem for F(similar for B) is,

$$\begin{aligned} \max_{\{X^f \in \Re_+^{3N+2}, \theta^f \in \Re^{2N}\}} \quad & E[c_0^f + c_1^f(z) + c_2^f(z)] \\ s.t. \quad & c_0^f + \alpha k_0^f + u_0 k_0^f + \sum_{z \in \mathcal{Z}} (\phi_0(z)\theta_0^f(z) + \phi_1(z)\theta_1^f(z)) \leq w_0^f \\ & c_1^f(z) + \alpha k_1^f(z) + u_1 k_1^f(z) \leq z f(k_0^f) + \theta_0^f(z) \\ & c_2^f(z) \leq f(k_1^f(z)) + \theta_1^f(z) \end{aligned}$$

The first constraint is the date 0 budget constraint for F. F purchases date 0 corn, rents land, and purchases financial securities. At date 1 the harvest from date 0 plus the payoff on financial securities becomes the resources for date 1 consumption and production. At date 2, there is no further production, so all proceeds from investment are consumed as dividends.

In equilibrium, markets for land use and financial securities must clear,

$$\sum_{a \in \{F,B\}} k_0^a = \bar{K} \tag{4}$$

$$\sum_{a \in \{F,B\}} k_1^a(z) = \bar{K} \tag{5}$$

$$\sum_{a \in \{F,B\}} \theta_0^a(z) = 0 \tag{6}$$

$$\sum_{a \in \{F,B\}} \theta_1^a(z) = 0 \tag{7}$$

Definition 1 *The competitive equilibrium of E0 consists of consumption and investment choices, $X = (X^f, X^b)$, a security plan, $\theta = (\theta^f, \theta^b)$ and prices P. X and θ are optimal choices for each agent given P, and given X and θ, the market clearing conditions, $(4), (5), (6)$ and (7) hold at P.*

Since preferences are linear and utility transferable, the equilibrium is easily characterized. The first best consists of date 0 and date 1 production choices which maximize expected output.[11]

$$\max_{\{k^f, k^b\}} \quad f(k^f) + b(k^b)$$
$$s.t. \quad k^f + k^b = \bar{K}$$

Proposition 1 (First Best) *In the first best, B and F each produce with exactly half the land. $k^{f*} = k^{b*} = \bar{K}/2$. Rental prices and security prices are,*

$$u_0 = u_1(z) = \rho - \alpha$$
$$\phi_0(z) = \phi_1(z) = \pi(z)$$

The marginal product of land use is equalized when sharing land equally. At this point the rental on land, its one period use-value, must be equal to the marginal product of $\rho - \alpha$. Since all firms are risk neutral and there is no discounting, the prices of financial securities are equal to the probabilities.

We can also define a purchase price of land as q_t. At date 0, a firm that purchases land enjoys the benefit of the rental stream for two periods, therefore $q_0 = 2(\rho - \alpha)$. At date 1, since there is no production at date 2, purchase and rental are equivalent and q_1 equals u_1.

Neither the uncertainty in production nor the distribution of initial endowments affect investment and prices. Uncertainty is irrelevant as firms are risk neutral. Initial endowments do not matter as firms can always raise funds for a positive net present value investment. Finally, collateral has no use in this case.

[11]In writing this optimization, we have also used the fact that B has a large endowment of corn at each date, and has linear preferences with no discounting.

4 E1: Private Collateral Constraints with Hedging Markets

Contracting Assumptions

In E0, F borrows from B to finance production. It short sells financial securities($\theta^f < 0$) and uses these proceeds to purchase inputs into production. This section introduces contracting assumptions that restrict the firms from short selling financial securities. These assumptions also create a role for collateral.

Assumption 1 (Non-observability of Output): *The yield on production undertaken at either date 0 or date 1 is private information of the farmer and is not observable and verifiable by an outsider.*

Assumption 2 (Collateral): *Ownership of land is observable and verifiable.*

Assumption 3 (Access): *A farmer with sufficient funds to purchase land cannot be barred from purchasing land, regardless of credit history*

Discussion of Assumptions:

The first is a stark assumption that implies that none of output can be promised to outsiders. With just this assumption, a borrower can never seek outside funds for investment as it cannot commit to repay these funds. Any promised repayment to an outsider will not be made, as the farmer will simply claim to have no output. Since output is not verifiable (i.e. verification is infinitely costly), an outsider will never be able to prove otherwise.[12] This means that prices and investment levels of a firm will be a function of initial endowments. This is a common theme in most asymmetric information based corporate finance models.[13]

The first assumption also creates a need for collateral. The second assumption identifies this collateral to be land. For instance, a farmer who seeks funds to purchase land may take out a collateralized mortgage. In this case land will be the collateral to make the farmer's promise of repayment credible.

Finally the last assumption says that history cannot be used against a farmer in any new contract. Basically this means that if a farmer defaults on a loan, he can always walk

[12]KM assume that contracts are incomplete and hence a promise to repay corn is not enforceable. I find that it is sufficient to assume complete contracts but require that output is *neither* observable nor verifiable – as opposed to the usual incomplete contracts assumption of observable but not verifiable. Under my assumption, the promise to make a repayment is also not enforceable.

[13]see Hubbard (1995) for a survey and exposition of these ideas as applied to firm level investment.

away from the land, take his funds and purchase a plot of land in the market. Seizure of collateral is the only threat on default. Unlike Bolton and Scharfstein (1996), the threat of denial of access to the means of production is not possible and cannot be used to extract repayments from the farmer.

The three assumptions have basically been chosen to parallel those of KM. The upshot of the assumptions is that a farmer is credit constrained in obtaining funds since any promised repayments of corn output are not enforceable. Land can serve as collateral and therefore collateralized debt becomes a viable option.

However I shall make one last assumption that differs from KM, and embodies the main issue of hedging against aggregate shocks.

Assumption 4 *The aggregate state, z, is both observable and verifiable. Contracts can be written contingent on z.*

The Contract between B and F

F is short of funds to control land for production and will sign a contract at date 0 with B in order to secure enough land. Define the following,

- (i) $r_0, r_1(z), r_2(z)$ are state contingent transfers (corn) from F to B at each date and,
- (ii) $k_0, k_1(z)$ are F's production scale in units of land.

A contract consists of (r_0, k_0, S). $S : \mathcal{Z} \to \Re^3_+$ is defined by $S(z) = (r_1(z), k_1(z), r_2(z))$. S is a function that maps outcomes to allocations of corn and land.

A contract, (r_0, k_0, S) is *feasible* if it satisfies the restrictions imposed by Assumption's 1 through 4 and satisfies B's breakeven constraint,

$$r_0 + \sum \phi_0(z) r_1(z) + \sum \phi_1(z) r_2(z) = k_0 q_0 + \sum \phi_0(z) u_1(z) k_1(z)$$

Lemma 1 *Consider a feasible contract, (r_0, k_0, S). Define a financial security with date 1 state contingent payoffs given by,*

$$\theta(z) = (u_1(z)k_1(z) - r_1(z)) 1_{\{u_1(z)k_1(z) > r_1(z)\}} \geq 0 \tag{8}$$

Any feasible contract can be implemented by F renting and farming k_0 units of land at date 0 and saving the balance by purchasing this financial security.

Proof: see appendix

A contract consists of primitives (r_0, k_0, S). There are many possible implementations of these primitives, ranging from rental of land plus hedging contracts to purchase of land plus borrowing. The lemma states that, given feasibility conditions, all of these can be restated in terms of rental and farming of land, k_0, and financial investment, $\theta(z)$. The first part of this, land rental, is just a spot transaction. The need for a long term contract stems from the second part, the desire to save. Note that we have not yet described what an optimal contract is, we shall do so in the next section. Compared to economy E0, the main restriction imposed by the contracting assumptions is that $\theta(z) \geq 0$.

Remark : A collateralized debt contract with face amount, D, on k_0 units of land can be represented as, $\theta(z) = Max[k_0 u_1(z) - D, 0]$, and a rental of k_0 units of land. In states of the world in which the collateral is worth less than the face of debt D, F defaults on the debt and gives up the collateral. This implies that $\theta(z) = 0$. In states of the world in which the collateral is worth more than the face, the borrower does not default and is left with the excess of collateral value over face, $k_0 u_1(z) - D$. KM derive the collateralized debt contract as optimal in their setting. The main substantial difference in our assumptions vis-a-vis KM is that we assume that while a firm's holding of corn is not verifiable, the aggregate state is verifiable. Thus, contracts can be written contingent on the aggregate state. The justification for this is two-fold. First, it seems reasonable to treat the aggregate state as verifiable. More compellingly, (as will be shown) the date 1 price of land, in equilibrium, will reflect the aggregate state. Thus if the price of land is verifiable, the aggregate state becomes verifiable.

The contracting assumptions imply that F cannot short sell financial securities and therefore faces credit constraints. We next analyze the general equilibrium with credit constraints. This will clarify the role of the financial investment, $\theta(z)$ and the form that it takes. We will interpret $\theta(z)$ as a hedge contract that F enters into with B.

Control Problem for F

At date 0, F chooses an amount of productive investment, k_0^f, and financial investment $\theta^f(z)$, out of initial resources w_0^f. These choices, and the realization of the production shock will result in F beginning date 1 with resources w_1.

$$w_1 = zf(k_0^f) + \theta^f(z)$$

Again, a productive investment choice is made resulting in output at date 2. F's program is,

$$(E1F) \qquad \max_{\{c_0^f, k_0^f, \theta^f(z), k_1^f(z), c_1^f(z)\}} \quad E_z[c_0^f + c_1^f(z) + f(k_1^f(z))]$$
$$s.t. \quad c_0^f + (\alpha + u_0)k_0^f + \sum \phi(z)\theta^f(z) \leq w_0^f$$
$$(\alpha + q_1)k_1^f(z) + c_1^f(z) \leq zf(k_0^f) + \theta^f(z)$$
$$c_0^f, c_1^f(z), \theta^f(z) \geq 0$$

$f(k_1^f(z))$ has been substituted in for $c_2^f(z)$. Otherwise the objective is exactly as defined in E0. Constraints are altered in only one substantial way. $\theta^f(z)$ is restricted to be non-negative. This implies that F firms face a sequential budget constraint. The first constraint is the date 0 budget constraint. F divides resources between productive investment and financial securities. The second constraint is that at date 1 in each state of the world, consumption plus productive investment must equal resources from date 0 investment plus security holdings.

To solve this program, work backwards from date 1. At date 1 in state z, given resources w_1, F chooses an amount of land to use out of this wealth to solve,

$$\max_{\{k_1^f(z)\}} \quad w_1 - (\alpha + u_1(z))k_1^f(z) + f(k_1^f(z))$$
$$s.t \quad (\alpha + u_1(z))k_1^f(z) \leq w_1$$

This can be broken down into two regions depending on whether the firm is wealth constrained. Assume the constraint does not bind, then the solution is given by $k_1^f(z) = f'^{-1}(\alpha + u_1)$. We verify later that, in equilibrium, $u_1(z)$ is always less than $\rho - \alpha$ so that $k_1^f(z) < \frac{\bar{K}}{2}$. The constraint will bind if $f'^{-1}(\alpha + u_1(z))(\alpha + u_1(z)) > w_1$. Call this critical value $w_1(z)^*$ and write the date 1 land demand and F's value function over w_1 as,[14]

$$k_1^f(z) = f'^{-1}(\alpha + u_1(z)) - \frac{1}{\alpha + u_1(z)}(w_1(z)^* - w_1)^+$$
$$J^z(w_1) = w_1 + f(\frac{w_1(z)^*}{\alpha + u_1(z)}) - w_1(z)^* - Max\left[f(\frac{w_1(z)^*}{\alpha + u_1(z)}) - w_1(z)^* - (f(\frac{w_1(z)}{\alpha + u_1(z)}) - w_1), 0\right]$$

The expression allows for a simple interpretation which we illustrate in figure 1. In a world without contracting problems, F would be risk neutral - this is the first part of the expression for the value function and the linear part of the graph. The contracting problem introduces the second term - the strictly concave part of the graph - the firm

[14] That is,
$$w_1(z)^* = f'^{-1}(\alpha + u_1(z))(\alpha + u_1(z)).$$

is short levered "put" options on w_1 struck at $w_1(z)^*$. Note that J is concave since f is concave. F is effectively risk averse at date 0 with respect to date 1 wealth. Because of decreasing returns in the reinvestment technology, low wealth results in foregoing high marginal product investments. This risk averse behavior arises due to the interaction of contracting considerations and the dynamic investment problem.[15]

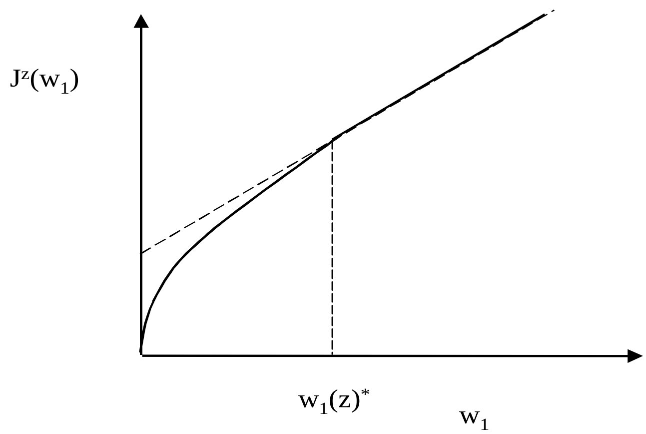

Figure 1: Value Function

The date 0 control problem is (c_0^f must be 0 since a little algebra shows that $J'(\cdot) \geq 1$),

$$\begin{array}{ll} \max\limits_{\{k_0^f, \theta^f(z)\}} & E_z[J^z(zf(k_0^f) + \theta^f(z))] \\ s.t & \sum \phi(z)\theta^f(z) + (\alpha + u_0)k_0^f \leq w_0^f \\ & \theta^f(z) \geq 0 \end{array}$$

G chooses a pattern of payments $\theta^f(z)$ that best matches its value function. Because $J^z(\cdot)$ is concave and production is risky, there is scope for insurance. In particular, F will choose to guard against low realizations of z, as the following proposition shows.

Proposition 2 (Insurance) *In economy E1, F's optimal financial security provides insurance against low realizations of z and takes the form of put options.*[16]

[15]See Froot, Scharfstein, and Stein (1993) for a similar discussion. Also see Grossman and Vila (1992) for an analysis in the context of a wealth constrained investor.

[16]Clearly the prediction of put options is due to the specifics of this model and the desire for insurance

Proof: see appendix.

Let $\theta^f(z)$ be the payoff of the optimal financial security in state z. Then,

$$\theta^f(z) = Max[\theta^f(z^*) + f(k_0^f)(z^* - z), 0]$$

The strike of the put option is z^*, and the payments increase linearly in the difference, $z^* - z$. Basically, F would like to protect against having too little resources available for investment at date 1. Hedging means setting the shadow value of these resources equal in all states, or equalizing these resources themselves. Transfers required to accomplish this turn out to be a linear function of the state, z. However, above some critical level, F would like to commit to paying out funds. As this cannot be done (since F's promise of payment is not credible), payments in these states are zero. This gives the strike of the put option.

Remark: A debt contract will have payments, $\theta^f(z) = Max[k_0^f u_1(z) - D, 0]$. In other words, payments are highest in the best states. This is the opposite of what an optimal choice of $\theta^f(z)$ will dictate.

Control Problem for B

B is not constrained in investment choices at any date. The optimization is,

$$(E1B) \qquad \max_{\{c_0^b, \theta_0^b(z), k_0^b, c_1^b(z), k_1^b(z)\}} \quad E[c_0 + c_1(z) + b(k_1^b(z))]$$
$$s.t. \quad c_0 + \sum \phi(z)(c_1(z) + \theta^b(z))$$
$$+(\alpha + u_0)k_0^b + (\alpha + u_1(z))k_1^b(z) \leq b(k_0^b)$$

Note that we have not imposed non-negativity on c_t. This is a simple way of representing that B is not wealth constrained. Because B is not constrained, optimality for land use means setting the marginal product of land equal to its rental price,

$$b'(k_t^b) \quad = \quad u_t + \alpha$$

Equilibrium

The equilibrium conditions are market clearing for land at date 0 and in each state z at date 1 and market clearing for financial securities.

$$k_0^f + k_0^b \quad = \quad \bar{K} \tag{9}$$
$$k_1^f(z) + k_1^b(z) \quad = \quad \bar{K} \tag{10}$$
$$\theta^f(z) + \theta^b(z) \quad = \quad 0 \tag{11}$$

therein.

Definition 2 *Equilibrium in E1 consists of* (X, θ, P)*: consumption and investment choices,* $X = ((c_0^j, c_1^j(z), c_2^j(z), k_0^j, k_1^j(z))^{j=b,f})$*, a security plan,* $\theta = (\theta^b(z), \theta^f(z))$ *and prices,* $P = (u_0, u_1(z), \phi(z))$*.* X *and* θ *are solutions to* $E1F$ *and* $E1B$*, given* P*, and given* X *and* θ*,* $(9), (10)$ *and* (11) *are satisfied at* P*.*

Lemma 2 *An equilibrium of E1 exists under the maintained assumptions.*[17]

Proof: see appendix.

Closing the model in general equilibrium allows us to characterize land prices and the prices of financial securities. It is easy to see that we must have,

$$\phi(z) = \pi(z)$$

F purchases a hedge which pays off in bad states of the world. B provides this hedge at an actuarially fair price. In a standard general equilibrium model of insurance(for example, Lucas (1978)), insurance against states in which the aggregate endowment is low carries a premium. The key distinction is that, while a low realization of z does result in a low aggregate endowment of corn at date 1, since B is risk neutral and is assumed to have a large date 1 endowment of corn, the price of insurance is actuarially fair.

Proposition 3 (Equilibrium Land Prices) *Land rentals in economy E1 are as follows: in states where* $z \leq z^*$*, the equilibrium rental price of land,* $u_1(z)$ *is constant. For* $z > z^*, u_1(z)$ *is weakly increasing in* z*.*

Proof: see appendix

Since resources in the hedging states are equalized, the date 1 demand for land from F in these states is equalized. This implies that the price of land in these states is constant. High realizations of z are high realizations of date 0 investment. Thus, F is less constrained and is able to use more land at date 1. Since F is more productive with the land, rental and land prices rise. Hedging insures against swings in the distribution of wealth. As it is this distribution which affects asset prices, the volatility of prices is reduced through the holding of financial securities.

In the region where agents choose not to hedge $(z > z^*)$, one of two things can happen. First, if shocks are large enough then agents are unconstrained in their investment at date 1. The economy efficiently allocates capital between B and F leading to an equilibrium price of ρ. There can be an intermediate region where $u_1(z^*) < u_1(z) < \rho$, where agents

[17] Unfortunately, results on uniqueness of equilibrium have not been established.

are constrained in their reinvestment, however choose not to hedge as the shocks are not adverse enough to justify hedging. In these sense, hedging cuts off the lower tail of the distribution of shocks and equalizes asset prices and investment in these states.

4.1 Amplification and Hedging

The main difference between E1 and E0 is that F firms are restricted from taking short positions in financial securities. In E0, F short sold output from production to finance this production. When short sales are prohibited, firms actually go long financial securities. This arises because of the dynamic investment problem. Production at date 1 is dependent on output from date 0. If output from date 0 production was very low (suppose $z = 0$), the firm will have to shut down date 1 production. As this can be very costly, the firm will seek to hold insurance that buffers against a low realization of z.[18]

The restriction on short sales also leads to a violation of the conditions necessary for Modigliani-Miller to hold. In particular, production choices and land prices in E1 are functions of the shock z and the endowments of the F firms. Output is a function of the distribution of wealth. This is the basic mechanism of credit channel models, as presented, for example, by Bernanke and Gertler (1989). In E1, however, there is no additional amplification mechanism at work. Collateral values and output are correlated, but there is no feedback of changes in collateral values into output.

Let us contrast these results with the amplification mechanism that KM illustrate. By their assumptions, the aggregate shock is not verifiable and firms are forced to borrow via debt contracts. Under these restrictions, $\theta(z)$ would be $Max[k_0^f u_1(z) - D, 0]$. A firm, after the date 0 production shock, is left at date 1 with resources $zf(k_0^f) + \theta(z)$. The collateral amplification mechanism of KM is that in states in which z is high, not only is $zf(k_0^f)$ high, but also $\theta(z)$ is high. In their infinite horizon model they show that the second term can be quite large. Thus a shock (z) to the balance sheet of a firm feeds back via a general equilibrium change in the value of collateral as an amplified shock to the firm's balance sheet.

As I have shown, the crucial restriction in arriving at this result is that the aggregate shock cannot be verifiable. If contracts can be written contingent on this shock, firms will

[18]The demand for insurance by F would arise even if production was constant returns to scale. Suppose $f(k) = \rho k$, then, $J'^z(w) = \frac{\rho}{\alpha + u_1(z)}$. The marginal valuation depends inversely on $u_1(z)$. Since it is the marginal value that must be hedged, differences in prices across two states will lead to a firm shifting all of its hedge to the high marginal value state. In equilibrium, this will lead to equal prices in hedging states and a hedge security that resembles a put option.

enter into hedge contracts that will undo any amplification effects.[19] In fact, $\theta(z)$ will resemble put options whose payoffs are highest in low z states. This not only eliminates amplification effects, it also dampens the effect of shifts in the distribution of wealth on activity.

5 E2: Aggregate Collateral Constraints

Amplification arising purely from private collateral constraints must rely on incomplete hedging by firms. I introduce, in this section, a theory of incomplete hedging based on limited supply of aggregate collateral and show that this constraint reinstates amplification effects.

Assumption 5 (Aggregate Collateral): *Sale of financial securities must be collateralized by land.*

F holds put options to insure its production against bad realizations of z. In order for F to buy this security, B must sell it - or must promise to make payments to F in these states of the world. The question I ask is *what ensures that B is credible in making this promise.* In the same way that F must post collateral to guarantee a debt contract, B must post collateral to guarantee its sale of put options.[20]

A state z Arrow-Debreu claim is a financial asset promising one unit of corn in state z. It must be backed by an amount of collateral, L (land), that is required to be held by a seller of the claim.[21] Non-performance on an asset promise constitutes default and results in seizure of collateral by the owner of the financial asset. Thus in any state z, the actual delivery is given by $d(z) = min(1, Lq_1(z))$.

One can look at this in two ways. The financial asset can be thought of simply as a derivative security. B acts as an investment bank by purchasing land and then issuing a land-backed security against this. Alternatively, one can think of the financial asset as part of the long term contract between B and F. F demands a particular pattern of payments $(\theta^f(z))$ from B in the optimal contract. We can interpret B as an insurance company that makes state contingent promises which are backed by its holdings of land.

[19] If shocks were not common across all F firms, there would be no general equilibrium change in collateral values. As such hedging, while it would still be desirable, would not affect amplification.

[20] This is similar to the treatment in Dubey, Geanakoplos, and Zame (1996).

[21] Collateral is held by the seller of the asset in this model. That is, if there are any benefits from using the asset, they accrue to the seller. Dubey, et al. consider more general asset structures.

The description of the financial market has the following implication: while a full set of Arrow-Debreu claims are traded, the supply of these claims is limited by collateral considerations. In this sense, the financial market has limited depth.

There are two sets of equilibrium conditions: market clearing for land at date 0 and in each state z at date 1, and market clearing for financial securities at date 0.

$$k_0^f + k_0^b = \bar{K} \tag{12}$$

$$k_1^f(z) + k_1^b(z) = \bar{K} \tag{13}$$

$$\theta^f(z) + \theta^b(z) = 0 \tag{14}$$

In case E1, it was assumed that B was unconstrained in its sale of financial assets. Market clearing for financial assets and the introduction of collateral constraints in this market, requires that,

$$\theta^f(z) = -\theta^b(z) \leq Lq_1(z)$$

In other words, B will have to back its sale of insurance by holdings of collateral L. Since there is no cost of increasing L, B may as well pledge all of its land as collateral, and since B is wealthy, assume without loss of generality that B owns all of the land in the economy. Thus, rewrite the collateral constraint as,

$$\theta^f(z) = -\theta^b(z) \leq \bar{K}q_1(z)$$

In this expression, the price of land, $q_1(z)$, rather than the rental on land, $u_1(z)$, has been used purely for expositional purposes. In fact, $q_1(z) = u_1(z)$, since land has no use after date 2. If the constraint binds, a premium is introduced into the price of collateralized insurance. Call this premium $\eta(z) = \frac{\phi(z)}{\pi(z)}$, and write the market clearing condition as,

$$\theta^f(z, \eta(z)) \leq \bar{K}q_1(z) \tag{15}$$

Definition 3 *The equilibrium of E2 consists of choices,* $X = ((c_t^j)_{t=0,1,2}^{j=b,f}, (k_t^b, k_t^f)_{t=0,1})$*, a security plan,* $\theta = (\theta^j)^{j=b,f}$ *and prices* $P = (u_0, (u_1(z), \eta(z))_{z\in\mathcal{Z}})$*. Choices are optimal given prices, and given choices, market clearing conditions (12), (13) and (15) hold at these prices.*

Lemma 3 *An equilibrium of E2 exists.*

Proof: see appendix.

If $\eta(z) > 1$, insurance against state z carries a premium. Since the quantity of collateral is fixed at $\bar{K}$, the supply of insurance will be determined by the value of collateral, $q_1(z)$. When this is low, the supply is constrained and insurance may carry a premium. Note that the collateral constraint may bind for only a small number of states. There is a non-linearity in insurance provision that implies that only with a low probability will the economy get hit by a sufficiently negative shock that the constraint binds. But it is precisely in these low z states that the availability of insurance is constrained.

The premium on insurance against aggregate shocks is analogous to one in the usual pure-exchange consumption based models (see Lucas (1978)). In these, insurance against states of the world in which the aggregate endowment is low is expensive because insurance is constrained by the endowment. However, in our case there is no constraint on the endowment, the premium on insurance arises purely through the introduction of the collateral constraint.

Definition 4 *The economy is collateral sufficient if* $\eta(z) = 1, \forall z$

The question of sufficiency is a determination at equilibrium prices if the supply of financial assets exceeds the demand for them. In short, is there enough collateral in the economy to satisfy insurance demand. There is no reason to expect this to hold. However, it is worth noting that in one special case the economy is sufficient.

Proposition 4 (Collateral Sufficiency) *Suppose* $\alpha = 0$*, then economy E2 is collateral sufficient.*

Proof:

The collateral constraint is,

$$\theta^f(z) \leq \bar{K} u_1(z)$$

The LHS is the demand in state z, and the RHS is the aggregate supply of financial assets. Both sides are determined in equilibrium. In any state, optimal production requires that F use exactly half the land, $k^{f*} = \bar{K}/2$. Now, suppose that $L = \bar{K}$ (this is all of B's land) and there is default. F would be left with $k^f = \bar{K}$, which is greater than k^{f*} and would clearly be sub-optimal. At date 0, F's demand for insurance is a desire to protect against having too few resources at date 1. If, in a state of world with default, F is left with more land than it needs it can clearly do better by cutting back on insurance purchase in that state and spending this on insurance in another state. Thus, there can never be default as F would never demand an amount of insurance that is not possible to provide.

Proposition 5 (Amplification) *There exist parameters $\alpha \geq \underline{\alpha} > 0$ and $w_0^f > \underline{w}_0^f > 0$ such that E2 is collateral constrained. Consider this case. The collateral constrained states are as follows. There exists z_* such that for $z \leq z_*$,*

- *the aggregate collateral constraint binds:*

$$\theta^f(z) = \bar{K} q_1(z),$$

- *$\eta(z)$ is strictly decreasing in z,*
- *and, the price of land $q_1(z)$ is strictly increasing in z.*

There exists $z^* \geq z_*$ such that for $z \geq z^*$, $q_1(z)$ is weakly increasing in z, and $\eta(z) = 1$. In the case when $z^* > z_*$, for $z_* < z < z^*$, $q_1(z)$ is constant and $\eta(z) = 1$.

Proof: see appendix.

As α rises, F's demand for insurance rises. In a bad realization of z, F must hold enough insurance to cover both the purchase of land as well as the purchase of corn. Since land is collateral, insurance to ensure the purchase of land is always possible. However, the insurance to purchase corn must also be backed by the collateral of land. Land serves double duty as collateral for insurance to purchase both inputs into production.

As α rises the supply of collateral becomes insufficient to support the demand for it, and the aggregate collateral constraint binds for the low realizations of z. In these cases, the price of insurance carries a premium above the actuarially fair level. As a result, agents hedge incompletely against these bad realizations. This depresses the price of land in these states, which feeds back into an even tighter aggregate collateral constraint, further increasing the price of insurance and reducing hedging. The shock is amplified through this collateral insurance channel.

The realizations of $z > z_*$ is the collateral sufficient states. This follows exactly from proposition 3: in the states where agents hedge, the completely hedge away the shock so that land prices are constant. Above a certain level, agents would like to short sell financial securities. However, lacking collateral to do so, they choose no hedging and the price of land is weakly increasing in these states.

The non-linear behavior of asset prices (and output as is easily seen) around z_* is certainly reminiscent of episodes of financial crises. The response of the economy to shocks is muted in the collateral sufficient region. However, as soon as shocks become large enough, the aggregate collateral constraint binds and amplifies the shock.

5.1 Private versus Aggregate Collateral Constraints

It useful to pause at this juncture and contrast proposition 5 with the results of KM. KM draw two associations in order to arrive at their amplification mechanism. First, the credit constrained agent - F in this case - must bear the full burden of a fall in collateral values. Second, it is the tightness of the financial constraint of this agent - since he also uses the collateral as a productive input - that ultimately determines the value of collateral. The introduction of insurance markets, but no aggregate collateral constraint (economy E1) breaks the first association. With full insurance markets, there is no reason not to, and indeed good reason to, insure against a drop in collateral values. E2 introduces a somewhat different association that restores the amplification mechanism. As long as F has to rely on B for payments at date 1, and B's ability to make payments directly depends on collateral values, F will indirectly be affected by a fall in collateral values. Someone has to bear the risk of falling collateral values. KM align this perfectly with the agent who affects the value of collateral to arrive at their amplification mechanism. E2 breaks this perfect alignment, but shows that amplification can still arise.

The truth of who bears the burden of falling collateral values probably lies somewhere between KM and E2. Lack of indexation of debt contracts and incomplete hedging seem evident from casual empiricism. However it is clear that it is not simply the agents who use collateral as a productive input who are affected by falling collateral values. History has repeatedly provided examples - from the Great Depression to New England in the early '90's and mostly recently Japan and East Asia - that falling real estate and stock prices directly affect the capital of banks and compromise their ability to lend.

In this regard, B is most like a bank with whom F has an ongoing relationship. Credit lines and liquidity facilities for customers account for close to 80% of C&I loans from US banks (Shockley and Thakor (1997)). The put options of proposition 2 closely resemble prearranged financing for firms. The risk to firms is then that this financing is curtailed or disappears in the event that falling collateral values leaves its bank in distress. There is much evidence that firms do suffer when their bank is distressed. Slovin, Sushka and Polonchek (1993) present evidence that the impending insolvency of Continental Illinois in 1984 resulted in negative stock returns of 4.2% for their borrowing firms. Kashyap, Stein and Wilcox (1993) outline and present evidence for a channel through which a monetary contraction decreases loan supply from banks and thereby hampers financing and investment by firms. That the capital of banks affects firms seems clear. E2 demonstrates that this connection is an important part of the mechanism through which downturns are amplified and reinforced through falling collateral values.

5.2 Economy E3: Contagion and Efficiency

Shifting the relevant constraint in the amplification mechanism from the demand side of finance (F) to the supply side (B) raises the possibility of contagion and the question of whether such contagion is efficient. F affects the value of collateral through its production, but the collateral of F is the capital for B. If there are other sectors of the economy whose operation depends on B - and indirectly on the value of collateral - then two possibilities arise. First, it easy to see that such a shock will be transmitted and amplify the downturn in the output of other sectors. More interestingly, the model suggests that F, as the collateral operating sector, should be treated favorably in the event of an aggregate downturn. The model prescribes a combination of industrial and stabilization policy in the event of a financial crisis.

To investigate these possibilities, I introduce a third sector into the economy, that of H firms. Otherwise continue with the specification as in economy E2.

H firms have date 0 endowments of w_0^h and have access to a production technology yielding $yh(C_0)$ at date 1 and $h(C_1)$ at date 2. The production shock, y, is taken to be an idiosyncratic shock to the output of each H firm and is assumed to be non-contractible. $y \in [0, \infty]$ with cumulative distribution function $F(y)$. This shock structure is assumed purely for simplicity of exposition and to highlight the amplification and contagion effects in the model.[22] $h(C_t)$ uses only corn as input (and produces corn as output). $h(C_t)$ is strictly increasing and concave, satisfying $h(0) = 0, h'(C) > 0, h''(C) < 0$.

The technology specification is in contrast to that of the farming sector. H firms use no physical capital in production, but require inputs in units of corn to secure "*human capital*." In keeping with the pleistocene motif, I shall refer to H as hunters.

The control problem for H at date 0 is as follows,

$$(E3H) \qquad \begin{aligned} \max_{\{c_0^h, C_0^h, \theta^h(z), C_1^h(z), c_1^h(z)\}} \quad & E_{z,y}[c_0^h + c_1^h(z) + h(C_1^h(z))] \\ s.t. \quad & c_0^h + C_0^h + \sum \phi(z)\theta^h(z) \leq w_0^h \\ & C_1^h(z) + c_1^h(z) \leq yh(C_0^h) + \theta^h(z) \\ & c_0^h, c_1^h(z)\theta^h(z) \geq 0 \end{aligned}$$

We can again solve this problem by backward induction. The value function at date 1 over

[22] We have studied a variety of other shock structures including: cases in which the shocks include both idiosyncratic and aggregate components; cases in which H and F have uncorrelated shocks; and a case in which shocks occur at both dates and are correlated across time. While interesting in their own right, the results from these cases do not affect the substance of what is being presented in this paper. Details are available upon request.

resources is,

$$J_h(w_1) = \begin{cases} h(w_1), & w_1 \le w_*^h \equiv h'^{-1}(1) \\ w_1, & w_1 > w_*^h \end{cases}$$

The program at date 0 is then,

$$\begin{array}{rl} \max_{\{c_0^h, C_0^h, \theta^h(z)\}} & E_{z,y}[c_0^h + J_h(w_1)] \\ s.t. & c_0^h + C_0^h + \sum \phi(z)\theta^h(z) \le w_0^h \\ & w_1 = yh(C_0^h) + \theta^h(z) \\ & c_0^h, \theta^h(z) \ge 0 \end{array}$$

Let us turn next to the equilibrium. While the market clearing conditions for land are not altered, the introduction of H does affect market clearing for financial securities.

$$\theta^f(z) + \theta^h(z) = -\theta^b(z) \le \bar{K}q_1(z) \tag{16}$$

Definition 5 *The equilibrium of E3 consists of choices,* $X = ((c_t^j)_{t=0,1,2}^{j=h,b,f}, (k_t^b, k_t^f)_{t=0,1}, C_t^h)$, *a security plan,* $\theta = (\theta^j)^{j=h,b,f}$ *and prices* $P = (u_0, (u_1(z), \eta(z))_{z\in\mathcal{Z}})$. *Choices are optimal given prices, and given choices, market clearing conditions (12), (13) and (16) hold at these prices.*

Since the value function over date 1 resources is concave, H will have a desire for insurance to protect against poor y realizations of date 1 output. However since the price of insurance is dependent on $\eta(z)$, H's demand for insurance will be state dependent. This can be summarized as follows.

Lemma 4 *Consider E3 in which parameters are such that the economy is collateral constrained. As long as* w_0^h *is large enough, H will choose to hold a positive amount of financial securities. H's desire for insurance takes the form of precautionary savings. For states* $z > z_*$, *where the aggregate collateral constraint does not bind (*$\eta(z) = 1$*),* $\theta^h(z)$ *is constant. For states* $z \le z_*$, *the demand for insurance* $\theta^h(z)$ *is increasing in the state as* $\eta(z)$, *the price of insurance, is decreasing in the state.*

Proof: see appendix.

The key point of this lemma is that H's holding of financial securities will be state dependent and hence its resources at date 1 will be dependent on z when the economy is collateral constrained. Investment at date 1 by H is,

$$C_1^h(w_1) = \begin{cases} w_1, & w_1 \le w_*^h \\ w_*^h, & w_1 > w_*^h \end{cases}$$

This implies that the aggregate investment of sector H is,

$$I_1^h(z) = E_y[C_1^h(yh(C_0^h) + \theta_h(z))] \tag{17}$$

Proposition 6 *(**Contagion**) The investment and output of H is increasing in z in the states where the aggregate collateral constraint binds despite y and z being uncorrelated.*

Proof: This follows from inspection of (17)

It is worth re-emphasizing the non-linearity in this correlation structure. As long as $z > z_*$ there is no contagion from the F sector to the H sector. The correlation is induced by falling collateral values in the states where the aggregate collateral constraint binds.

Correlation across sectors when the underlying uncertainty is uncorrelated is by no means surprising and should not be taken as a sign of inefficient contagion. After all, in many standard general equilibrium models of asset pricing we find that all agents should bear some risk of the aggregate endowment and hence the stochastic discount factor reflects all aggregate uncertainty. In turn, this would imply that the investment and consumption choices of all agents should be correlated. However, in our context contagion can be inefficient.

Definition 6 *Suppose markets clear competitively at date 1. Fix the date 0 production choices, (k_0^b, k_0^f, C_0^h), and consider only date 0 choices over financial securities, θ. The competitive equilibrium is constrained financially efficient (CFE) if there is no perturbation to the security holdings, $d\theta$, that is feasible and results in a Pareto improvement.*

Proposition 7 **(Efficiency)** *The competitive equilibrium is CFE as long as either,*

- *the economy is collateral sufficient($\eta(z) = 1 \ \forall z$), or*
- *the F sector is not credit constrained($\theta^f(z) = 0, q_1(z) = \rho$).*

If both conditions are violated, the equilibrium is constrained financially inefficient.

Proof: see appendix.

The appropriate notion of efficiency is constrained efficiency - can a central planner subject to the same trading restrictions as the agents reallocate portfolios to make everyone better off. In the first two cases, efficiency is not surprising. Agents face a short sale constraint, as they cannot borrow from the future. Forced to reallocate portfolios along the same dimensions as agents, a central planner cannot improve allocations either.

In the third case, *prices* become important. The inefficiency is caused by a pecuniary externality. Insurance use by the F sector affects the wealth distribution and asset prices. In particular, increased use of insurance raises the value of collateral. This relaxes the aggregate collateral constraint, allowing for more insurance to all sectors of the economy. The farmer's action therefore creates a pecuniary externality on others.

The two cases in which efficiency is restored highlight this. When the aggregate collateral constraint does not bind, relaxing it has no effect. If the farming sector is not credit constrained, collateral values are constant. Thus, even if the aggregate collateral constraint binds, a farmer's actions do not get transmitted to the rest of the economy.

6 An Example of Amplification

In this section, I construct an example to illustrate amplification due to the aggregate collateral constraint. A bad external shock interacts with the aggregate collateral constraint to create two equilibria. The bad equilibrium is one in which collateral values are expected to be low, implying a tighter aggregate collateral constraint and less contingent insurance from bankers. This in turns feeds back into the production of farmers by constraining their investment, reducing their demand for land, and validating the expectation of low collateral values.

6.1 Objectives and Equilibrium

Let us return to the economy of E2, with a few simplifications.[23] First suppose that production functions take on the following functional form:

$$f(k) = b(k) = k(A - k).$$

Note that this means that,

$$f'(k) = b'(k) = A - 2k,$$

so that the marginal product of capital diminishes linearly with k. Since prices can always be determined from the banker's marginal product, we will have that,

$$q = A - 2k^b$$

[23]The inefficiency in the previous section means that there is an externality in farmer's production decisions. The inefficiency was highlighted relative to the hunter's production decisions. However it turns out that even without the H sector there is an externality in each farmer's production decision. The multiple equilibria of this section are due to this externality.

Rewriting this expression, noting that $k^f + k^b = \bar{K}$, we arrive at the supply curve for land facing a farmer,

$$q = A - 2(\bar{K} - k^f) \tag{18}$$

I shall make the following technical assumption,

$$2\bar{K} \geq A + \alpha > A > \bar{K}$$

Since it will always be that $k^b \leq \frac{\bar{K}}{2}$, this ensures that $q > 0$ and $f(k^f), b(k^b) > 0$, so that our production function is well specificed. This technical assumption will also generate exactly two equilibria.

Next suppose that, output in the farming sector (F) is subject to one of two aggregate shocks. In the boom, which occurs with probability π, the aggregate shock, z_H, equals $\frac{1}{\pi}$. In the recession, the aggregate shock z_L is zero (probability $1 - \pi$). Assume that in the boom no constraints are binding; that is, F is not credit constrained and the aggregate collateral constraint does not bind. On the other hand, in the recession both the credit constraint for F and the aggregate collateral constraint bind.

It is easy to see what happens in the boom. Since neither farmer nor banker is constrained and they have identical decreasing returns production functions, they will set $k_H^f = k_H^b$. Market clearing is that $k_H^f + k_H^b = \bar{K}$, so that,

$$\begin{aligned} k_H^f &= \frac{\bar{K}}{2} \\ q_H &= A - \bar{K} \end{aligned}$$

In the bad state the farmer has no output from date 0 production. Any production he wishes to undertake must come from the insurance that he purchases at date 0. Thus if the farmer wishes to use k_L^f land, he must set,[24]

$$\theta_L^f = (\alpha + q_L)k_L^f.$$

The supply of insurance however is limited by the aggregate collateral constraint,

$$\theta_L^f \leq q_L\bar{K}. \tag{19}$$

[24]More precisely, F's problem is as follows:

$$\begin{aligned} \max_{\{k_0^f, k_L^f\}} \quad & f(k_0^f) + (1-\pi)f(k_L^f) \\ s.t. \quad & k_0^f(u_0 + \alpha) + k_L^f(q_L + \alpha)\phi_L = w_0 \end{aligned}$$

ϕ_L is the price of insurance in the bad state, while u_0 is the rental price of land at date 0, and q_L is the value of collateral at date 0 in state L.

From the supply curve of (18) we can see that the price of land, q_L will be increasing in the equilibrium amount of land use by F, k_L^f. Thus, we have the association that more land use relaxes the collateral constraint, further cheapening the price of insurance, which can give rise to multiple equilibria. Algebraically, taking the insurance market clearing condition from (19) (as an equality now),

$$(\alpha + q_L)k_L^f = q_L \bar{K}.$$

and rewriting this as,[25]

$$k_L^f = \frac{q_L}{\alpha + q_L}\bar{K}$$

gives us the effective demand for land by F – now determined purely off the aggregate collateral constraint of (19). Combining this with (18) describes equilibrium in state L.

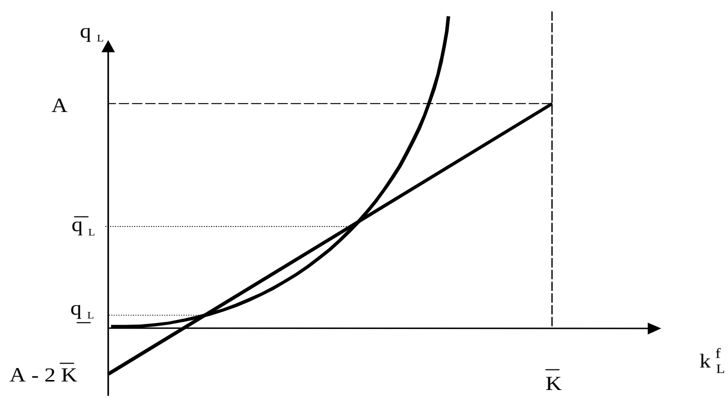

Figure 2: Multiple Equilibria

Graphically, the equilibrium is represented in figure 2. First, the supply curve is linearly increasing in F's use of land – this is the line beginning at $A - 2\bar{K}$. Second, the insurance market clearing describes a curve beginning from the origin and approaching the vertical line at $\bar{K}$. As q_L rises, insurance becomes cheaper and F is able to purchase more insurance,

[25] It should also be clear why collateral sufficiency is assured if $\alpha = 0$. In this case the right hand side would be $\bar{K}$ and since in an equilibrium $k_L^f < \bar{K}$, the aggregate collateral constraint of (19) would never bind.

and hence more land. This gives us a land demand curve that is "upward-sloping" in price. When, $A \leq 2\bar{K}$ we can have exactly two equilibria.

Multiple equilibria are only a possibility when (19) is the binding constraint limiting F's land purchase at date 1. If, for example, the output in the bad state, z_L, was not zero, and indeed was high enough that the aggregate collateral constraint did not bind, the land would be a normal good for F.[26] That is F would have a downward sloping demand for land, and there would be only one equilibrium at some level q_L. While it may be the case that $q_H > q_L$, this fact would have no effect on investment decisions as it would simply be a reflection of a bad realization of the state. The feedback into investment only arises when the aggregate collateral constraint binds, raising the price of insurance and creating the possibility of the bad equilibrium.

6.2 Implications

Amplification effects only arise when the aggregate collateral constraint binds – this is largely the point of the example. Additionally, the example generates some fairly realistic empirical implications by comparing across the good and bad equilibrium.

Clearly asset prices in the $\underline{q}_L$ bad equilibrium are lower than they would be in the $\overline{q}_L$ equilibrium.[27] In addition, there is a large fall in asset prices from date 0 to the recession state of date 1.

$$q_0 - \underline{q}_L = u_0 + \pi q_H - \underline{q}_L(1 - (1-\pi)\underline{\eta}_L) > q_0 - \overline{q}_L > 0$$

[26]That is to say, we would have,

$$k_L^f = \min[\frac{z_L f(k_0^f) + \theta_L}{q_L}, \frac{\bar{K}}{2}]$$

which is downward sloping in price in the region where F is still credit constrained, and flat at $\frac{\bar{K}}{2}$ in the region where the constraint does not bind (as in state H).

[27]Combining (19) and (18), we get

$$q_L = A - 2\bar{K}\frac{\alpha}{\alpha + q_L}$$

or,

$$h(q_L) \equiv q_L^2 - q_L(A - \alpha) + \alpha(2\bar{K} - A) = 0.$$

The roots of this quadratic are,

$$q_L = \frac{A-\alpha}{2} \pm \sqrt{(\frac{A-\alpha}{2})^2 - \alpha(2\bar{K} - A)}.$$

In the high state, $q_H = A - \bar{K}$, we can also evaluate $h(\cdot)$ at q_H. Doing this shows that $h(q_H) > 0$ which means that we can order,

$$q_H > \overline{q}_L > \underline{q}_L$$

A shift in expectations to the bad equilibrium (at date 0) creates a scramble for collateralizable assets that pushes up the collateral premium, $\underline{\eta}_L > \overline{\eta}_L$. The real interest rate in this economy is the yield on a bond that pays one unit of corn in each state at date 1.

$$R = \frac{1}{\pi + (1-\pi)\eta_L}$$

Thus, $\underline{R} < \overline{R}$, or interest rates are lower in the bad equilibrium. While this behavior of the interest rate is consistent with many episodes, an alternate interpretation of this result is in terms of rising premia on collateralizable versus non-collateralizable assets. Even in non-crises times, short term Treasury Bills, which are among the most actively used financial collateral, carry a significant premium over similar financial assets which are not used as collateral. An empirical regularity in financial crises is the flight to quality, manifest as an increase in this premium. For example, the result is consistent with the well documented widening of the Commercial Paper - Treasury Bill spread going into a downturn (Friedman and Kuttner (1992)).

The fall in asset prices causes a reduction in the supply of insurance. In practice this will be manifest as a loss of collateral leading to bankruptcies or downgrades in the credibility of financial institutions and a drop in the volume of services provided by these institutions. This contraction in supply also leads to a lower volume of insurance/hedging (θ) by F. This in turn implies that the wealth distribution at date 1 will be more dissimilar across the low and high states which will translate to higher volatility of land prices. A little algebra shows show that the volatility of land prices when the land price is q_L in the low state is given by,

$$\sigma(q_L) = (q_H - q_L)\left(\pi(1-\pi)\right)^{\frac{1}{2}}.$$

Comparing across the two equilibria we find that since $\underline{q}_L < \overline{q}_L$,

$$\sigma(\underline{q}_L) > \sigma(\overline{q}_L)$$

or the volatility of land prices is higher in the bad equilibrium.

7 Conclusion

What is the mechanism through which changes in collateral values feed through into changes in output? To be sure, some of this mechanism must be due to private collateral constraints, as illustrated by KM. Lack of indexation of debt contracts, and/or the lack of hedging by firms are evident from even casual empiricism. Why this occurs is somewhat a puzzle and is left open for future research. The goal of this paper has been to show that amplification effects are preserved even in the presence of insurance markets. However the relevant constraint shifts from the private to the aggregate collateral constraint. The supply of collateral limits the extent of insurance provision. Thus, unlike other models of market incompleteness (for example, Allen and Gale (1991)), the model endogenizes the *depth* of insurance markets.

The most natural interpretation of our insurance providers are banks. Likewise the put options that are purchased by firms is similar to the prearranged liquidity facilities that banks offer firms. Thus the channel I outline is most similar to a credit crunch. A fall in collateral values causes the supply of financing to be constrained leading to constrained investment by all sectors and, in turn, falling collateral values. Thus I arrive at amplification through an insurance channel.

The model I have considered is a three period one which qualitatively captures both the static and dynamic implications of the KM model. However, the stress in my modeling choices has been on the qualitative rather than the quantitative. As KM argue the power of the collateral channel really is in the quantitatively large dynamic multiplier that emerges in the infinite horizon model. Following the logic outlined in this paper it seems clear that complete insurance markets (economy E1) will eliminate the dynamic multiplier as well. It is harder to say how much of this will be restored in an infinite horizon version of our model via the insurance channel outlined in economy E2. This is an important question and I leave its answering for future research.

A Appendix

Proof of Lemma 1

F and B will sign an initial contract that specifies, (1) transfers, $r_0, r_1(z), r_2(z)$ from F to B at each date and (2) investment levels $k_0, k_1(z)$ in land.

Since land is worthless at date 2, assumption 1 implies that F cannot commit to make payments at this date. Hence,

$$r_2(z) = 0$$

F is assumed to be "poor," but has high marginal product for low levels of production. This means that F will always find it profitable to invest all of its initial wealth in the contract. Therefore,

$$r_0 = w_0$$

At date 1, resources available to F are,

$$zf(k_0) + u_1(z)k_1(z)$$

This is the sum of output from date 0 production and the value of land under its control. Out of this F can choose to make the payment of $r_1(z)$ to B. If F chooses to default, B seizes the collateral and receives an effective payment of $u_1(z)k_1(z)$. In this case, F walks away with output $zf(k_0)$. If F does not default, the contractual payment of $r_1(z)$ is made, and control of $k_1(z)$ units of land is retained. Let $m(z)$ be the probability measure over events in which default occurs.

As there is a continuum of B, lenders are competitive and make zero profits. The breakeven constraint for lenders can be written as,

$$w_0 + \sum \phi_0(z)[m(z)u_1(z)k_1(z) + (1 - m(z))r_1(z)] = (\alpha + q_0)k_0 + \sum \phi_0(z)[u_1(z)(k_1(z) - k_0)]$$

The LHS of this expression is expected receipts by B, and the RHS is the value of physical assets that are transferred to F. This can be rewritten as,

$$w_0 = (\alpha + u_0)k_0 + \sum \phi_0(z)[(1 - m(z))(u_1(z)k_1(z) - r_1(z))]$$

where, $u_0 = q_0 - \sum \phi_0(z)u_1(z)$ is the date 0, one period rental on land. This expression allows for a very simple interpretation of the contract. At date 0, k_0 units of land are rented, αk_0 units of seed corn are purchased and an option to purchase another $k_1(z)$ units at a price of $r_1(z)$ is granted to F in state z. The default rule follows easily from this. The option that is granted is the option to default. If the value of the land $u_1(z)k_1(z)$ is less than the exercise price, $r_1(z)$, F will always default, and $m(z) = 1$. In the case where $u_1(z)k_1(z) > r_1(z)$, default never occurs and $m(z) = 0$.

$$w_0 = (\alpha + u_0)k_0 + \sum \phi_0(z)[(u_1(z)k_1(z) - r_1(z))1_{\{u_1(z)k_1(z)>r_1(z)\}}]$$

Since the second expression in the RHS is non-negative, we have that $w_0 \geq (\alpha + u_0)k_0$. In other words, not all of the initial wealth is used for date 0 real investment, some of it is applied towards the date 1 payment. One can think of this simply as savings. Define a financial security with state contingent payoffs given by,

$$\theta(z) = (u_1(z)k_1(z) - r_1(z))1_{\{u_1(z)k_1(z)>r_1(z)\}}$$

The optimal contract between B and F can be implemented by F renting k_0 units of land at date 0, purchase αk_0 units of corn, and saving the balance by purchasing this financial security.

At date 0, F divides its resources between rental and savings. There is no lending from B to F (in the sense of B contributing extra resources towards F's production.) This is an importance difference from other models of collateralized borrowing (for example Hart and Moore (1998) and Bolton and Scharfstein (1996)). It occurs because the asset used by one firm in production is a perfect substitute for that of another firm in this model. Thus the entrepreneur has full bargaining power and he can always let the investor seize the assets of the firm, as he can buy new and equivalent ones in the marketplace. Most other models assume some form of asset specificity which breaks this substitutability. The reason we have not done this is to shy away from firm specific effects and focus on the aggregate effects of collateral.[28]

Proofs of Propositions 2 and 3

The claim is twofold : the optimal financial security is a put option, and land prices are constant for $z < z^*$ but increasing for $z > z^*$. To show this, we solve F's problem in a world with a full set of state z contingent claims, but assuming a short sale constraint for F. Note from the text that $\phi(z) = \pi(z)$.

$$\begin{aligned} \max_{\{k_0,\theta(z)\}} \quad & E_z J^z(zf(k_0) + \theta(z)) \\ s.t \quad & (\alpha + u_0)k_0 + E_z[\theta(z)] \leq w_0^f \\ & \theta(z) \geq 0 \end{aligned}$$

[28]It may also be clear from this paragraph that assumption 3 is fairly important. A firm that defaults on its obligation to one agent at date 1 walks with its output and purchases new land from another agent. We have assumed that this new purchase cannot be ruled out in the original contract. Thus, there is full substitutability of assets.

The second constraint is the short sale constraint for F. The FOC's are,

$$f'(k_0)E_z[zJ'^z(zf(k_0)+\theta(z))] = \lambda(u_0+\alpha)$$
$$\pi(z)J'^z(zf(k_0)+\theta(z)) = \lambda\pi(z)+\mu_z$$

Consider $z^1, z^2 \in \mathcal{Z}' \subseteq \mathcal{Z}$, where $\mathcal{Z}'$ represents states in which the short sale constraint does not bind. In these two states,

$$J'^{z1}(z^1 f(k_0)+\theta(z^1)) = J'^{z2}(z^2 f(k_0)+\theta(z^2))$$

Optimality requires that the marginal product be equalized over all states where the short sale constraint does not bind.

Now, we have claimed that in equilibrium, land prices in these two states, $q_1(z^1), q_1(z^2)$, are equal. Given this, it must be that $J^{z1}(w) = J^{z2}(w)$. Then, since the marginal product function is invertible, equating marginal product must mean equating wealth.

$$f(k_0)(z^1-z^2) = \theta(z^2)-\theta(z^1)$$

This implies, that if $z < z^*$ and $z^* \in \mathcal{Z}'$ then, $z \in \mathcal{Z}'$. Call this critical value z^*, then,

$$\theta(z) = Max[\theta(z^*)+f(k_0)(z^*-z), 0]$$

Note that this is just a put option. The budget constraint gives,

$$k_0(u_0+\alpha)+E_z[\theta(z)] = w_0^f$$

Combining these equations, we can solve for k_0 and z^*. To complete the argument, we need to verify that in equilibrium, land prices in the two states z^1 and z^2 are indeed the same. Or more generally, that land prices for states, $z < z^*$ are constant.

$$k_z^f = min[f'^{-1}(q_1(z)), \frac{w}{\alpha+q_1(z)}]$$

In the two states, z^1, z^2, the optimal hedging strategy prescribes setting wealth equal. If in addition, land prices are constant in these two states, it must be $k_{z^1}^f = k_{z^2}^f$. Then, from the market clearing condition it is clear that land prices are in fact equal in these two states. By equating wealth in hedging states, the dependence of k_1^f is eliminated. This in turn means that land prices are equalized across these states, resulting in decreased volatility of land prices and returns.

The last claim is that for $z > z^*$, land prices are weakly increasing in z. First note that for these states $\theta(z) = 0$. The proof is by contradiction. For two states $\hat{z}$ and z, with $\hat{z} > z$, suppose that $\hat{q}_1 < q_1$.

Date 1 wealth is given by,

$$w_1 = zf(k_0)$$

Given this wealth, F will choose a date 1 production level, k_1^f of,

$$k_1^f = \frac{w_1}{\alpha + q_1} - \left(\frac{w_1}{\alpha + q_1} - f'^{-1}(\alpha + q_1)\right)^+$$

Then,

$$\begin{aligned}\hat{k}_1^f &= \frac{\hat{z}}{\alpha + \hat{q}_1} f(k_0) - \left(\frac{\hat{z}}{\alpha + \hat{q}_1} - f'^{-1}(\alpha + \hat{q}_1)\right)^+ \\ k_1^f &= \frac{z}{\alpha + q_1} f(k_0) - \left(\frac{z}{\alpha + q_1} - f'^{-1}(\alpha + q_1)\right)^+\end{aligned}$$

Since $\frac{\hat{z}}{\alpha+\hat{q}_1} > \frac{z}{\alpha+q_1}$ and $f'^{-1}(\alpha + \hat{q}_1) > f'-1(\alpha + q_1)$, we have that $\hat{k}_1^f \geq k_1^f$. In effect, F demands more land if its wealth is higher and land prices are lower. Market clearing in these two states are,

$$\begin{aligned}k_1^b(\hat{q}_1) + \hat{k}_1^f &= \bar{K} \\ k_1^b(q_1) + k_1^f &= \bar{K}\end{aligned}$$

However, as $k_1^b(\hat{q}_1) > k_1^b(q_1)$, both of these conditions cannot hold and we have a contradiction. Therefore, prices are strictly increasing in the state.

Proof of Proposition 5

The problem is as follows:

$$\begin{aligned}\max_{\{k_0,\theta(z)\}} \quad & E_z J^z(zf(k_0) + \theta(z)) \\ s.t \quad & (\alpha + u_0)k_0 + E_z[\eta(z)\theta(z)] \leq w_0^f \\ & \theta(z) \geq 0\end{aligned}$$

The FOC's are,

$$\begin{aligned}f'(k_0)E_z[zJ'^z(zf(k_0) + \theta(z))] &= \lambda(u_0 + \alpha) \\ \pi(z)J'^z(zf(k_0) + \theta(z)) &= \lambda\phi(z) + \mu_z\end{aligned}$$

In all regions where the short sale constraint does not bind, $\mu_z = 0$. In these regions,

$$J'^z(zf(k_0) + \theta(z)) = \lambda\eta(z)$$

Begin by considering the region in which the aggregate collateral constraint binds. First note that in an region where insurance is purchased, it must also be that we are in the concave part of the value function J^z. Then,

$$f'(\frac{zf(k_0) + \theta(z)}{\alpha + q_1(z)}) = \lambda\eta(z)$$

Consider $z^1 < z^2$, where $z^1, z^2 \in \mathcal{Z}_* \subseteq \mathcal{Z}$, and $\mathcal{Z}_*$ represents states in which the aggregate collateral constraint does bind. In these two states,

$$\frac{\eta(z^1)}{\eta(z^2)} = \frac{f'(k_1^f(z^1))}{f'(k_1^f(z^2))}$$

Note that,

$$\frac{\eta(z^1)}{\eta(z^2)} > 1 \Rightarrow \frac{f'(k_1^f(z^1))}{f'(k_1^f(z^2))} > 1 \Rightarrow \frac{k_1^f(z^1)}{k_1^f(z^2)} < 1$$

where the last implication follows from strict concavity of $f(k)$. Now since in equilibrium, $k_1^b(z) = \bar{K} - k_1^f(z)$ and $q_1(z) = b'(k_1(z))$, we have that,

$$\frac{k_1^f(z^1)}{k_1^f(z^2)} < 1 \Rightarrow \frac{k_1^b(z^1)}{k_1^b(z^2)} > 1 \Rightarrow \frac{q_1(z^1)}{q_1(z^2)} < 1$$

This is all to say that if insurance prices are higher in one state than in another, it must be that land prices are lower in that state than the other.

To close the proof of the first part of the proposition, we simply need to show that insurance prices are in fact higher in state z^1 than z^2.

If the aggregate collateral constraint binds,

$$\theta(z) = q_1(z)\bar{K}$$

Then,

$$k_1^f(z) = \frac{zf(k_0) + q_1(z)\bar{K}}{\alpha + q_1(z)}$$

Let us just differentiate this expression with respect to z, noting that $q_1(z)$ solves,

$$q_1(z) = b'(\bar{K} - k_1^f(z))$$

We find that,

$$\frac{\partial k_1^f(z)}{\partial z} = \frac{1}{\alpha + q_1(z)}\left(f(k_0) + k_1^b(z)\frac{\partial q(z)}{\partial z}\right) > 0$$

Since the derivative is always positive, it follows that in the region where the aggregate collateral constraint binds, land prices are increasing in the state and $\eta(z)$ is decreasing in the state.

The proof of the rest of the proposition follows exactly from the proof of propositions 2 and 3.

Proof of Lemma 4

An H firm that is credit constrained solves the following program at date 0,

$$\begin{array}{rl} \max_{\{C_0^h,\theta^h(z)\}} & E_{z,y}[J_h(yh(C_0^h)+\theta^h(z))] \\ s.t. & C_0^h+\sum\phi(z)\theta^h(z)\le w_0^h \\ & \theta^h(z)\ge 0 \end{array}$$

The FOC's give,

$$h'(C_0^h)E_{z,y}[yJ_h'(yh(C_0^h)+\theta^h(z))]=\lambda$$
$$\pi(z)E_y[J_h'(yh(C_0^h)+\theta(z))]=\lambda\phi(z)+\mu_z$$

where,

$$J_h(w_1)=\begin{cases} h(w_1), & w_1\le w_*^h\equiv h'^{-1}(1) \\ w_1, & w_1>w_*^h \end{cases}$$

In all regions where the short sale constraint does not bind, $\mu_z=0$. In these regions,

$$E_y[J_h'(yh(C_0^h)+\theta(z))]=\lambda\eta(z)$$

In any two states z^1 and z^2 where $\eta(z^1)=\eta(z^2)$,

$$E_y[J_h'(yh(C_0^h)+\theta(z^1))]=E_y[J_h'(yh(C_0^h)+\theta(z^2))]$$

The solution to this is $\theta(z^1)=\theta(z^2)$.

Consider regions $z^1<z^2<z_*$ where the price of insurance is increasing as z falls. We wish to show that $\theta(z^1)<\theta(z^2)$. From the FOC we must have that,

$$E_y[J_h'(yh(C_0^h)+\theta(z^1))]>E_y[J_h'(yh(C_0^h)+\theta(z^2))]$$

The proof is by contradiction. Suppose $\theta(z^1)\ge\theta(z^2)$. In the case of equality, it is clear that we have a contradiction. Suppose $\theta(z^1)>\theta(z^2)$. We can write,

$$\begin{aligned} E_y[J_h'(yh(C_0^h)+\theta(z^1))] &= \int_0^\infty J_h'(yh(C_0^h)+\theta(z^1))dF(y) \\ &= \int_0^\infty J_h'(yh(C_0^h)+\theta(z^2))dF(y)+ \\ &\qquad \int_0^\infty \Big(J_h'(yh(C_0^h)+\theta(z^1))-J_h'(yh(C_0^h)+\theta(z^2))\Big)\,dF(y) \\ &< \int_0^\infty J_h'(yh(C_0^h)+\theta(z^2))dF(y)=E_y[J_h'(yh(C_0^h)+\theta(z^2))] \end{aligned}$$

Where the last step follows since $J_h'(\cdot)$ is monotone decreasing in its argument. This gives us the contradicion and hence $\theta(z^1)<\theta(z^2)$.

Proof of Proposition 6

The competitive equilibrium has each of sectors B, F and H choosing consumption and production levels, $X = \{X^b, X^f, X^h\}$ and a security plan $\theta = \{\theta^b, \theta^f, \theta^h\}$, given prices, $P = \{u_0, \{q_1(z), \eta(z)\}_{z \in \mathcal{Z}}\}$.

H's problem can be written as,

$$\begin{array}{cl} \max\limits_{\{(C_0^h, c_0^h, \theta^h(z)) \in \Re_+^{N+2}\}} & c_0^h + E_{z,y}[J_h(yh(C_0^h) + \theta^h(z))] \\ s.t. & w_0^h = C_0^h + c_0^h + E_z[\eta(z)\theta^h(z)] \end{array}$$

where,

$$J_h(w_1) = \begin{cases} h(w_1), & w_1 \leq w_*^h \equiv h'^{-1}(1) \\ w_1, & w_1 > w_*^h \end{cases}$$

B's problem is as follows.

$$\begin{array}{cl} \max\limits_{\{c_0^b, c_1^b(z), k_0^b, k_1^b(z), \theta^b(z)\}} & c_0^b + E_z[c_1^b(z) + b(k_1^b(z))] \\ s.t. & u_0 k_0^b + c_0^b + E_z[\eta(z)\theta^b(z)] = 0 \\ & c_1^b(z) + u_1(z)k_1^b(z) = \theta^b(z) + b(k_0^b) \\ & \theta^b(z) + q_1(z)\bar{K} \geq 0 \qquad \forall z \end{array}$$

Notice that we have not imposed non-negativity on consumption in this case. This is equivalent to saying that B is wealthy.

Finally, the optimization problem for F is,

$$\begin{array}{cl} \max\limits_{\{c_0^f, c_1^f(z), k_0^f, k_1^f(z), \theta^f(z)\}} & c_0^f + E_z[c^f(z) + f(k_1^f(z))] \\ s.t & c_0^f + u_0 k_0^f + E_z[\eta(z)(c^f(z) + u_1(z)k^f(z))] = w_0 + E_z[\eta(z) z f(k_0^f)] \\ & \theta(z) + z f(k_0^f) = c^f(z) + u_1(z)k_1^f(z) \\ & \theta(z), c^f(z) \geq 0 \end{array}$$

Market clearing is,

$$\begin{array}{rcl} c_0^b + c_0^f + c_0^h + C_0^h & = & w_0^h + w_0^f \\ c_1^b(z) + c_1^f(z) + c_1^h(z) + C_1^h(z) & = & E[y]h(C_0^h) + b(k_0^b) + z f(k_0^f) \\ k_0^f + k_0^b & = & \bar{K} \\ k_1^f(z) + k_1^b(z) & = & \bar{K} \\ \theta^h(z) + \theta^b(z) + \theta^f(z) & = & 0 \end{array}$$

Let (X, θ, P) be a competitive equilibrium.

Case 1: Suppose the aggregate collateral constraint, which in this case is part of B's budget constraint, does not bind in any state. That is, suppose,

$$\begin{aligned} \theta^b(z) + u_1(z)\bar{K} &> 0 \qquad \forall z \\ \theta^h(z) + \theta^f(z) &< q_1(z)\bar{K} \end{aligned}$$

We wish to show that the competitive equilibrium is CFE. Our proof proceeds in two steps. First, fix the date 0 production decisions, (C_0^h, k_0^f, k_0^b). Next, consider a feasible perturbation of the security holdings, $d\theta$. We wish to show that there does not exist $d\theta$ that results in a Pareto improvement.

WLOG, drop the short sale constraint for B. This strictly enlarges the set of feasible perturbations. Then, a feasible perturbation must satisfy,

$$\begin{aligned} d\theta &= (d\theta^h, d\theta^f, d\theta^b) \\ d\theta_z^h &\leq 0 \\ d\theta^f &= (d\theta_l^f, d\theta_h^f) \\ d\theta_h^f &\geq 0 \\ d\theta^f \cdot \pi &= \sum d\theta_z^f \pi_z \leq 0 \\ d\theta_z^f + d\theta_z^h + d\theta_z^b &= 0 \end{aligned}$$

Since H is not consuming at date 1, it can only decrease its security holding (assuming these are greater than zero, otherwise it cannot do anything).

It is easy to show that in the competitive equilibrium F will have $\theta_z = 0$ for $z > z^l$ and $\theta_z > 0$ for $z < z^l$. Then, since F has a short sale constraint, the only feasible perturbations are an increase in security holdings in the high states and a decrease in the low states. Finally, B has no constraints. However, the perturbation to security holdings must net to zero in each state.

H's security holdings has no effect on date 1 land prices. Therefore, the $d\theta^h$ has no effect on F or B at date 1. This is not so with $d\theta^f$, since the change in wealth affects relative prices at date 1, and therefore production possibilities at date 1. We can therefore treat $d\theta^h$ as separate from $d\theta^f$. It is then easy to show that the perturbation to θ^h has no effect on utilities (envelope theorem). F's perturbation is a little harder since it triggers a change in date 1 land prices, and the proof focuses on this.

Let (dU_z^b, dU_z^f) be the date 1 changes in welfare from the perturbation. For $d\theta^f$ to result in a Pareto gain, $dU^b \cdot \pi > 0$ and $dU^f \cdot \pi > 0$.

From the competitive equilibrium, we can identify levels of the aggregate shock, z^c and z^l, such that, for $z > z^c$, F is unconstrained in date 1 production, and for $z < z^l$, F's land use is constant (z^l is the strike of the put option). Then, we have that, for $z > z^c > z^l$,

$$\begin{aligned} dU_z^f &= d\theta_z^f \\ dU_z^b &= 0 \end{aligned}$$

for $z < z^c$,

$$\begin{aligned} dU_z^f &= f'(k_z^f)dk_z^f \\ dU_z^b &= d\theta_z^f - b'(\bar{K} - k_z^f)dk_z^f \end{aligned}$$

where, dk_z^f is the change in date 1 land use that the perturbation results in.

$$\begin{aligned} dU^b &= \sum_{z<z^c} d\theta_z^f \pi_z - \sum_{z<z_c} b'(\bar{K} - k_z^f)dk_z^f \pi_z \\ dU^f &= \sum_{z>z^c} d\theta_z^f \pi_z + \sum_{z<z_c} f'(k_z^f)dk_z^f \pi_z \end{aligned}$$

The first expression on the RHS is just a transfer from B to F. Call this $t_c \geq 0$. Then, for $d\theta^f$ to be a Pareto improvement,

$$\begin{aligned} dU^b &= -t_c - \sum_{z<z_c} b'(\bar{K} - k_z^f)dk_z^f \pi_z > 0 \\ dU^f &= t_c + \sum_{z<z_c} f'(k_z^f)dk_z^f \pi_z > 0 \end{aligned}$$

For $z > z^l$, $d\theta_z^f \geq 0$ and $dk_z^f \geq 0$. For $z < z^l$, $k_z^f = k_l^f$, implying that, $f'(k_z^f) = f'(k_l^f)$ and $b'(\bar{K} - k_z^f) = b'(\bar{K} - k_l^f)$. Then we can rewrite the two inequalities as,

$$\begin{aligned} -\frac{t_c}{b'(\bar{K} - k_l^f)} - \sum_{z^l<z<z^c} \frac{b'(\bar{K} - k_z^f)}{b'(\bar{K} - k_l^f)} dk_z^f \pi_z &> \sum_{z<z^l} dk_z^f \pi_z \\ +\frac{t_c}{f'(k_l^f)} + \sum_{z^l<z<z^c} \frac{f'(k_z^f)}{f'(k_l^f)} dk_z^f \pi_z &> \sum_{z<z^l} dk_z^f \pi_z \end{aligned}$$

However, since $f'(k_l^f) > b'(\bar{K} - k_l^f), t_c \geq 0$ and $\frac{f'(k_z^f)}{f'(k_l^f)} < \frac{b'(\bar{K}-k_z^f)}{b'(\bar{K}-k_l^f)}$, these inequalities cannot be simultaneously satisfied.

Case 2: If F is not credit constrained, effectively the non-negativity restriction on consumption is lifted.

Since markets are assumed to clear competitively at date 1, regardless of date 0 choices,

$$k_1^b(z) = k_1^f(z) = \frac{\bar{K}}{2}, \qquad q_1(z) = \rho$$

Given this, we can remove the date 1 production decision and rewrite the date 0 problem for B and F as,

$$\begin{aligned} \max_{\{c_0^b, c_1^b(z), \theta^b(z)\}} \quad & c_0^b + E_z[c_z^b] \\ s.t. \quad & c_0^b + E_z[\eta(z)\theta^b(z)] = 0 \\ & c_1^b(z) = \theta^b(z) \\ & \theta^b(z) + \rho\bar{K} \geq 0 \end{aligned}$$

and,

$$\begin{aligned} \max_{\{c_0^f, c_1^f(z), \theta^f(z)\}} \quad & c_0^f + E_z[c_z^f] \\ s.t. \quad & c_0^f + E_z[\eta(z)\theta^f(z)] = w_0^f \\ & c_1^f(z) = \theta^f(z) \\ & \theta^f(z) \geq 0 \end{aligned}$$

The problem is recast as a 2 period, 1 good economy. The proof of constrained efficiency is fairly standard in this case. Write H's problem,

$$\begin{aligned} \max_{\{(C_0^h, c_0^h, \theta^h(z)) \in \Re_+^{N+2}\}} \quad & c_0^h + E_z[V(zh(C_0^h) + \theta^h(z))] \\ s.t. \quad & w_0^h = C_0^h + c_0^h + E_z[\eta(z)\theta^h(z)] \end{aligned}$$

There are $N+1$ market clearing conditions,

$$\begin{aligned} c_0^b + c_0^f + c_0^h + C_0^h &= w_0^h + w_0^f \\ \theta^h(z) + \theta^b(z) + \theta^f(z) &= 0 \end{aligned}$$

The competitive equilibrium consists of an allocation, $X = (c_0^b, c_0^f, c_0^h, C_0^h)$, a security plan, $\theta = (\theta^b(z), \theta^f(z), \theta^h(z))$, and prices, $P = (\eta(z))$.

Now, suppose there exists another feasible allocation, $(\hat{X}, \hat{\theta})$, that is strictly better for all agents. Then, it must have cost more at prices P, or,

$$\begin{aligned} \hat{c}_0^b + E_z[\eta(z)\hat{\theta}^b(z)] > 0 \\ \hat{c}_0^f + E_z[\eta(z)\hat{\theta}^f(z)] > w_0^f \\ \hat{c}_0^h \hat{C}_0^h + E_z[\eta(z)\hat{\theta}^h(z)] > w_0^h \end{aligned}$$

But, summing these budget constraints and using the fact that $\sum \theta^j = 0$, we find that,

$$\hat{c}_0^b + \hat{c}_0^f + \hat{c}_0^h + \hat{C}_0^h > w_0^f + w_0^h$$

which is a violation of the aggregate resource constraint, and hence, a contradiction.

Case 3: If neither of cases 1 nor 2 applies, the allocation is inefficient. Thus, a central planner can reallocate portfolios to make all agents better off. We show this via a perturbation argument.

Consider two states $\hat{z}$ and z in which the aggregate collateral constraint binds, and F is using a positive amount of insurance. Suppose that prices in these states in the competitive equilibrium are (WLOG), $\hat{q}_1 > q_1$ and $\hat{\eta} < \eta$. Since, $q_1 = b'(\bar{K} - k_1^f)$, this means that $\hat{k}_1^f > k_1^f$. Now market clearing for insurance is,

$$\begin{aligned}\hat{\theta}^h + \hat{\theta}^f &= \hat{q}_1 \bar{K} \\ \theta^h + \theta^f &= q_1 \bar{K}\end{aligned}$$

Consider transfers at market prices, $(d\theta^h, d\theta^f)$ and $(d\hat{\theta}^h, d\hat{\theta}^f)$. These must satisfy,

$$\begin{aligned}d\theta^h + d\theta^f &= 0 \\ d\hat{\theta}^h + d\hat{\theta}^f &= 0 \\ d\theta^f &= \frac{\hat{\eta}}{\eta} d\hat{\theta}^f\end{aligned}$$

Note that these transfer are feasible (since security holdings are positive for both agents in these states) and leave agents no worse off (envelope condition). Now, in each of these cases, the date 1 price of land is affected.

$$\frac{dq}{dk^f} = -b''(\bar{K} - k^f)$$

thus, the perturbation causes land prices to change,

$$\begin{aligned}d\hat{q} &= \frac{b''(\bar{K} - \hat{k}^f)}{b'(\bar{K} - \hat{k}^f)} d\hat{\theta}^h \\ dq &= -\frac{b''(\bar{K} - k^f)}{b'(\bar{K} - k^f)} \frac{\hat{\eta}}{\eta} dd\hat{\theta}^h\end{aligned}$$

The value of this change in liquidity at market prices is,

$$d\hat{\theta}^h \hat{\eta} \bar{K} \left(\frac{b''(\bar{K} - \hat{k}^f)}{b'(\bar{K} - \hat{k}^f)} - \frac{b''(\bar{K} - k^f)}{b'(\bar{K} - k^f)} \right)$$

As long as this is not zero, we can choose $d\hat{\theta}^h$ to make this expression positive, and redistribute the additional liquidity to all agents making everyone better off.

Proof of Existence

We shall prove existence of equilibrium for economy E2; the other cases follow closely. The proof proceeds by defining and establishing properties of relevant sets and then applying Kakutani's fixed point theorem.

Define the price simplex,

$$P = \{(u_0, \phi_0, (u_1(z), \phi(z))_{z\in\mathcal{Z}}) \in \Re_+^{2(N+1)} : \phi_0 + u_0 + \sum(\phi(z) + u_1(z)) = 1\}$$

Here, ϕ_0 is the date 0 price of corn, which is no longer the numeraire good. Note that P is compact and convex.

Define the budget sets for $a \in \{F, B\}$,

$$B^a = \{(c_0^a, k_0^a, k_1^a(z), \theta^a(z)) : 0 \leq c_0^a \leq \sum w_0^a, 0 \leq k_0^a \leq \bar{K}, 0 \leq k_1^a(z) \leq \bar{K}, -\rho\bar{K} \leq \theta^a(z) \leq \rho\bar{K}\}$$

where, this time we haven take B's date 0 endowment to be w_0^b and assumed that this is large but bounded above.

Let, $B^A \equiv B^f \times B^b$. Then, the competitive equilibrium consists of $(x \in B^A, p \in P)$.

Date 0 optimization for F is,

$$\begin{aligned} \Phi^f(p) = argmax_{\{(c_0^f, k_0^f, k_1^f(z), \theta^f(z))\in B^f\}} \quad & c_0^f + E_z[J^z(zf(k_0^f) + \theta^f(z))] \\ s.t. \quad & \phi_0 w_0^f = (\alpha + u_0)k_0^f + \phi_0 c_0^f + E_z[\eta(z)\theta^f(z)] \\ & k_1^f(z) = min[\frac{zf(k_0^f) + \theta^f(z)}{\alpha + u_1(z)}, f'^{-1}(\alpha + u_1(z))] \end{aligned}$$

where $J^z(\cdot)$ is concave and continuous. Then, since the choice set is compact and convex and the objective is concave and continuous, $\Phi^f(p)$ is non-empty, convex, and upper semi-continuous.

Date 0 optimization for B is,

$$\begin{aligned} \Phi^b(p) = argmax_{\{(c_0^b, k_0^b, k_1^b(z), \theta^b(z))\in B^b\}} \quad & c_0^b + E_z[zb(k_0^b) + \theta^b(z)] \\ s.t. \quad & \phi_0 w_0^b = (\alpha + u_0)k_0^b + \phi_0 c_0^b + E_z[\eta(z)\theta^b(z)] \\ & k_1^b(z) = b'^{-1}(\alpha + u_1(z)) \end{aligned}$$

Then, since the choice set is compact and convex and the objective is concave and continuous, $\Phi^b(p)$ is non-empty, convex, and upper semi-continuous.

Next, define the correspondence, $\Phi^0 : B^A \to P$ by,

$$\Phi^0(x) = argmax_{p\in P} \qquad u_0(\sum_a k_0^a - \bar{K}) - \sum_z u_1(z)(\sum_a k_1^a(z) - \bar{K}) + \sum_z \phi(z)\sum_a \theta_z^a$$

The choice set is compact and convex, the objective is continuous and concave, implying that Φ^0 is upper semi-continuous. Last define $\Phi : P \times B^A \to P \times B^A$ by,

$$\Phi(p, x) = (\Phi^0 \times \Phi^f \times \Phi^b)$$

Since this too is upper semi-continuous, by Kakutani's fixed point theorem, Φ has a fixed point.

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