Received: 10 December 2020 | Accepted: 10 January 2022

DOI: 10.1111/jpet.12572

ORIGINAL ARTICLE

JOURNAL OF PUBLIC ECONOMIC THEORY WILEY

# A theory of (de)centralization

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**Abstract**

This paper compares the efficacy of a centralized and a decentralized rights structure in determining the size of an externality-generating project. Consider a central authority and two localities. One locality can operate a variable-size project which produces an externality that affects the other locality. Each locality may have some private information concerning its own net benefit from the project. Under centralization, localities are vertically integrated with a benevolent central authority who effectively possesses all property rights. Under decentralization, localities are separate legal entities (endowed with property rights) who bargain to determine the project size. We examine the performance of these two regimes and show how one or the other may dominate depending on the distributions of private and external benefits from the project.

## 1 | INTRODUCTION

There are at least two classic approaches to the problem of externalities in the economic literature. One, Pigouvian taxation, solves the problem through centrally imposed taxes/subsidies on production. The Pigouvian solution specifies that a central authority imposes a tax, a subsidy, a quota, or a standard that must be obeyed by the agents. In a frictionless world with a benevolent central authority, this regulation leads to an efficient outcome.

Another, Coasian bargaining (Coase, 1960) offers a decentralized solution. The Coase Theorem states that, in the absence of transaction costs, the central authority only has to assign and/or enforce property rights of the concerned agents, and bargaining between the agents will generate an efficient outcome.

Both these approaches lead to efficiency when there are no market imperfections of any sort. If there are imperfections, however, the comparison between these two modes of regulation becomes more complicated. For example, if the central authority is imperfectly informed about the social costs and benefits of the project, it has to extract this information from the informed agents before putting in place its regulatory scheme. Similarly, if agents are asymmetrically informed, this will affect the outcome of bargaining between them. Since the two approaches may behave quite differently in the presence of asymmetric information, the problem of choosing the better one is not trivial.

The main difference between these two approaches is in whether property rights are assigned to the regulated agents. Under a centralized scheme, property rights are retained by the center who imposes a solution on the agents. Under a decentralized scheme, agents are endowed with property rights which they can trade or bargain with.

An example of a centralized rights structure is the portrait of a centralized regime as depicted by De Long and Shleifer (1993) who study the impact of centralization of power on economic growth in European cities between 1050 and 1800. They define a centralized (absolutist) regime as one where:

> *Subjects have no rights; they have privileges, which endure only as long as the prince wishes.*

In such a setting there are no enforceable agreements or bargains. The central authority can always break any promise. In a more modern example, consider two localities that have been merged or integrated and are now under the authority of a central municipal entity. The merged localities have no say in the central authority's decision making outside of communication channels and committees that may exist. In this example, the central authority may consult local officials but it retains all municipal decision-making power. The former localities have no standing as contracting parties with the center.

In contrast, in a decentralized setting, regulated agents are attributed rights and therefore have some scope for independent action. In the face of an externality on one agent generated by the actions of another, it is natural to suppose that these agents will bargain to try to internalize the externality. Because agents have rights and these rights can be traded, enforceable agreements and transfers are possible in this decentralized structure. Again, our view of decentralization finds expression in De Long and Shleifer (1993). They argue that, under decentralized (non-absolutist) regimes,

> *...the legal framework was, not an instrument of the prince's rule, but more of a semifeudal contract between different powers establishing the framework of their interactions. (...) Taxes could be raised only with the consent of feudal estates.*

Given a choice, would we expect a centralized or a decentralized regime to cope better with an externality in the presence of asymmetric information? Our model analyzes this problem by comparing a centralized/integrated Pigouvian setting, where no property rights exist and where centrally imposed quotas dictate the allocation of resources, as opposed to a decentralized Coasian environment, where local agents have property rights and bargain to determine the allocation of resources.

We now informally describe our model. The problem is to determine the proper size of a project affecting the welfare of two agents. We cast this problem in an environment where two neighboring localities are affected by a project. Each locality is privately informed of the benefit (harm) the project will provide to that locality.

In a centralized setting, a benevolent, but uninformed, central authority can impose any project size on the localities. The absence of property rights can be formalized by saying the two localities have no participation constraints that the central authority must respect.

In a decentralized setting, the two localities are legal entities and thus possess rights concerning the size of the project and taxation. The attribution of property rights can be formalized by the introduction of participation constraints for the localities. These constraints reflect their control over productive activity and right to refuse involuntary taxation. The decision about the project is by the localities through take-it-or-leave-it bargaining about project size and tax transfers subject to these participation constraints. In evaluating the performance of decentralization, we consider decentralized regimes corresponding to the four possible assignments of which locality gets to make the take-it-or-leave-it offer and which locality has the right to determine the project size if bargaining fails.

Without further assumptions, the fact that participation constraints (rights) are present under decentralization but not under centralization leads a welfare optimizing approach to always favor (at least weakly) centralization. Thus there would be no scope for a theory of decentralization versus centralization in dealing with externalities. To develop such a theory, we make two crucial assumptions.

First, we assume that there is a (small) social cost to taxation. This assumption is often found in the literature on public economics and may be based on inefficiencies resulting from the distortionary effects of taxes.

Second, in our model, no legal contract can be enforced between the localities and the central authority. This is another sense in which no rights are allocated to the localities under centralization. Localities effectively become internal divisions of the central authority's organization. They are vertically integrated and are part of the legal structure of the central authority. Within this organization or legal structure, divisions have no property or contracting rights. Though we recognize that this does not describe all "centralized" environments, it captures a salient aspect of many hierarchical governance structures in that there is some "highest level" (here, the central authority) that is not subject to enforcement in its dealings with the rest of the hierarchy. Recall our example of two localities that have been merged or integrated and are now under the authority of a central municipal entity.

An important consequence of these assumptions is that the central authority cannot commit to adjust transfers as a function of information revealed by the localities, rendering transfers useless for screening purposes under centralization. Without such commitment, given the social cost, the central authority does not undertake any transfers. Thus, the information of the localities is incorporated in the decision process under centralization only through lobbying or informal communication. Such lobbying is modeled as cheap talk.

The alternative is to allocate property rights to localities. Property rights give autonomy and legal means for signing and enforcing contracts. Localities are outside the central authority's organization. Localities can thus bargain and trade those rights under an enforceable legal framework. We call such regimes decentralization.

We thus look at two polar cases for the endogenous distribution of property rights. Under centralization, localities are vertically integrated and have no property rights. Under decentralization, localities are spun off and they have full property rights.

We shall see that outcomes under centralization may differ significantly from those under decentralization. One application of our model is to evaluate some "folk wisdom" about externalities and government control. Three statements often made (see e.g., Oates, 1972) are: (1) large externalities justify central control or regulation; (2) heterogeneity in localities' characteristics is disadvantageous for centralization; and (3) centralized policies tend to be insensitive to the preferences of localities or regions.

The flavor of our main findings is as follows. We show that within our model the characteristics (large externalities and local homogeneity) mentioned in (1) and (2) are necessary and jointly sufficient for centralization to be better than all of the decentralization regimes, while also providing precise measures of these characteristics that are relevant for this purpose. Furthermore, regarding the insensitivity of centralized policies, (3), we find that whenever centralization is better than decentralization the optimal centralized policy is a uniform one. The contractual limitations associated with central control endogenously generate constraints on the center's ability to discriminate according to ex post realized preferences. Thus, insensitivity to local preferences and the uniformity of optimal centralized policies is a result in our model. When decentralization is better than centralization, our findings also speak to which decentralized regime performs best. First, which locality should be given the privilege of making the offer depends on the extent of each locality's private information. Holding all else fixed, increasing the variance of one locality's information makes it more attractive to give that locality the bargaining power. Second, we show that rights to control the size of the project absent a negotiated agreement should go to the locality not given the bargaining power, that is, either locality 1 should be given bargaining power and locality 2 the rights to the project or vice-versa.

We provide detailed intuition for and discussion of our results in the main body of the paper. At the broadest level, compared to centralization, decentralization has the advantages that it is better able to incorporate private information (especially that of the offering locality) and that localities can be bound by negotiated agreements, and the disadvantages that the localities are self-interested and face individual rationality constraints generated by the control rights to the project absent agreement. Thus, the better screening in the decentralized outcome is balanced against its social distortions due to a trade-off between incentives and informational rents.

## 1.1 | Related literature

There are a number of papers that examine the problem of externalities in asymmetric information environments (see, e.g., Baliga & Maskin, 2003; Farrell, 1987; Klibanoff & Morduch, 1995; Rob, 1989). This strand of literature adopts a mechanism-design approach and emphasizes the crucial role of individual rationality constraints in hindering efficient solutions as pointed out by Laffont and Maskin (1979) and Myerson and Satterthwaite (1983).[1] These papers are interested in characterizing allocations in a setting where agents have individual rationality constraints, and therefore cannot have a project imposed upon them by the higher-authority principal. Thus, in our language, these papers all examine variations on decentralized environments. Their underlying theme can be characterized as inefficiencies caused by the trade-off between incentives and informational rents. Closely related to our formal modeling of centralization, but not specific to externalities, are models of communication such as Crawford and Sobel (1982) and Melumad and Shibano (1991). Our model draws on elements from both

[1]A notable exception to this is Greenwood and McAfee (1991) who focus on inefficiencies generated by incentive constraints alone. A different approach, that considers direct investment in changing preferences as an alternative to incentive mechanisms for internalizing externalities, may be found in Dutta et al. (2021).

these literatures to generate a comparison of centralized and decentralized structures in handling externalities.

An alternative, political-economy, approach to some of the questions in this paper has been taken by Lockwood (2002), Besley and Coate (2003), Loeper (2013), and others. These papers present models where the central authority's decisions do not aim to be welfare maximizing, but rather are the outcome of an explicit voting or legislative decision-making process. Like this paper, and unlike much of the earlier literature on centralization versus decentralization, these papers do not assume that a central policy must, by definition, be a uniform one.

A model that shares some of the features of the one we develop here is the limited communication model of Melumad et al. (1995) which compares two-tier and three-tier hierarchies under limited communication and asymmetric information. The limited communication in their model yields screening problems similar to those that come from limited commitment in our model. More broadly, our model can be viewed as part of the literature on organizational design under asymmetric information (see, e.g., Milgrom, 1988; Poitevin, 2000). Dessein (2002), Alonso et al. (2008) and Rantakari (2008) focus on the tradeoff between centralization and a form of decentralization when coordination is an issue. One major difference is that they do not allow for the allocation of rights and explicit contracts as in our model of decentralization. Loeper (2011) is a paper in this vein focused, like we are, on political decentralization, which he compares to centralized policies that are assumed uniform across localities. Koethenbuerger (2008) and Cho (2013) also compare decentralization versus centralization with externalities. Only the latter includes asymmetric information. Like the papers cited above on organizational design, both model decentralization without contracting or negotiation between localities—each locality makes its own decision in a simultaneous move game.

Similar to our result that when centralization is best the optimal policy is uniform, Dessein (2002) shows that centralization is only optimal when communication is at its minimum (a uniform policy requires no communication). His finding is in a setting where enforceable contracts are not feasible in either regime and there is only a single locality and the center. In our model, the central authority cannot contract, while under decentralization the localities can. Both our results and Dessein's with regard to the uniformity of centralization may be read as emphasizing that screening under centralization without enforceable contracts is very costly whenever transfers have any social cost.

In the next section, we formally describe the model. Section 3 briefly examines the symmetric information benchmark for centralization and decentralization. Section 4 solves for the optimal centralized and decentralized outcomes under asymmetric information. Section 5 gives a welfare comparison, and provides comparative statics. Section 6 concludes. Proofs are presented in an appendix.

## 2 | MODEL

There are two local governments (e.g., municipalities or counties), denoted localities 1 and 2, respectively, and a central government, denoted by $C$. In locality 1, there is a public project that can be undertaken with intensity $q \in [0, \infty)$. This project has some external effects on locality 2. We assume that the choice of $q$, once made, is extremely costly or impossible to change.

The public project might be the construction of an electric power plant. In this case $q$ would represent the capacity of the plant. Locality 1 would benefit from the increased generating capacity, and locality 2 might suffer from increased pollution. Or, the project might be the development of a new vocational training program or other improvement in the educational system. Here $q$ could be an indication of the size or quality of the program. This could provide direct benefits to the residents and businesses in locality 1 and may also result in benefits to businesses located in a neighboring locality through helping develop or attract a skilled workforce.

More formally, locality 1's utility function is given by

$$u_1(q, \theta, t) = \theta q - q^2/2 + t,$$

where $q$ is project intensity, $\theta (\geq 0)$ is a parameter which measures the desirability of the project to locality 1 and $t$ is a transfer from locality 2 to locality 1. The expression $\theta q$ represents the gross benefit to residents of locality 1. The expression $q^2/2$ represents the cost of the public project. We assume that it has to be financed in locality 1. Transfers, which can be associated with equalization payments or regional subsidies, are, however, possible to shift some of the cost burden to locality 2.

Locality 2's utility is given by

$$u_2(q, \gamma, t) = -\gamma q - t,$$

where $\gamma$ is a parameter which measures the degree to which the project hurts locality 2. Our formal analysis deals with the case of negative externalities (where the degree of hurt $\gamma$ is nonnegative). The case of positive externalities is entirely symmetric and thus our analysis (with appropriate absolute values inserted) applies to that case as well.

The central government maximizes equally weighted social utility:

$$u_C(q, \theta, \gamma, t) = u_1 + u_2 = (\theta - \gamma)q - q^2/2.$$

For most of the paper, $\theta$ and $\gamma$ will be assumed to be private information of localities 1 and 2, respectively. Since some parties may be uninformed, it is necessary to specify prior beliefs over these parameters. We assume that it is common knowledge that $\theta$ and $\gamma$ are independently distributed according to uniform distribution functions $F(\theta) = (\theta - \underline{\theta})/(\overline{\theta} - \underline{\theta})$ and $G(\gamma) = (\gamma - \underline{\gamma})/(\overline{\gamma} - \underline{\gamma})$ with densities $f(\theta)$ on $\Theta = [\underline{\theta}, \overline{\theta}]$ and $g(\gamma)$ on $\Gamma = [\underline{\gamma}, \overline{\gamma}]$, respectively. The assumption of uniform distributions is made for simplicity and to allow derivation of explicit solutions. Furthermore, we assume $\overline{\gamma} < \underline{\theta}$, so that a positive project intensity (or size) is always socially optimal.

The externality problem we investigate can be clearly seen by noting that there is a difference between the project size that is optimal for some locality and the project size that is optimal for the central government (socially optimal). The level $q(\theta, \gamma) = \theta$ is privately optimal for locality 1, the level $q(\theta, \gamma) = 0$ is privately optimal for locality 2, while $q(\theta, \gamma) = \theta - \gamma$ is socially optimal. Note that the social optimum depends on both localities' information.

We assume that raising funds for transfers is socially costly (due to inefficiencies in taxation, e.g.). In particular, we assume that the central government and both localities have a lexicographic dislike of giving transfers. The lexicographic dislike can be viewed as the limit case where these social costs are infinitely small compared to the effects of the choice of project size, $q$. We focus on this limit case mainly for simplicity: assuming some small social cost $\epsilon > 0$, so that giving a transfer $t$ would subtract $(1 + \epsilon)t$ from utility, would complicate the algebra without qualitatively affecting our results. That transfers have some social cost is a common assumption in the regulation literature. This assumption is important for our results, as without it or an alternative reason for the central government to care about transfers, our model of centralization could always implement the first-best outcome (and thus do at least as well as our model of decentralization).

We consider two different constitutional environments within which the problem of determining the project level can be tackled. Our task will be to compare the expected outcomes in these two settings from the point of view of the central government, that is, according to expected social welfare.

In the first environment, called centralization, all property rights over the public project and transfers reside with the central government. Localities are vertically integrated; they become internal divisions of the central government's organization. Specifically, the central government can mandate a project level, $q$, and may also require transfers between the two localities. The center's difficulty lies in inferring the localities' information so as to pick an appropriate project level $q$. Because localities are not legal entities, they have no legal rights and there can be no enforceable contract between the central government and its divisions (localities) that constrains the center's choice of project level or transfers. Localities thus communicate with the central government knowing that it will then make a unilateral decision about the project level and transfers. Without property rights, localities have no legal grounds for appealing the central government's decisions.

Accordingly, we model the implementation of the public project under centralization by the following (centralization ($C$)) game:

1. Locality 1 chooses an element in $\Theta = [\underline{\theta}, \bar{\theta}]$ and locality 2 chooses an element in $\Gamma = [\underline{\gamma}, \bar{\gamma}]$. These choices are communicated to the central government.
2. The central government chooses a project level, $q$, and transfer, $t$, to implement.

The central government's strategy is to choose a project level and a transfer contingent on the information reported by localities 1 and 2, respectively. The centralization game is thus a version of the cheap talk communication game of Crawford and Sobel (1982).

As an alternative to centralization, we consider an environment in which one locality is endowed with property rights over the public project and both localities have property rights over transfers. In this environment, localities are legal entities separate from the central government. We assume that the enforcement of property rights is achieved by a constitution that establishes a law of contract by which the localities may voluntarily agree to give up these rights. For example, the localities may agree to allow the transfer to be chosen as a function of locality 1's choice of $q$. Such agreements are enforceable by a court that may rule only on whether an action deprived a party of its constitutional rights. For example, if localities 1 and 2 write a contract in which 2 promises certain transfers as a function of the project level and then locality 2 refuses to pay up, locality 1 may argue before the court that, had 1 known locality 2 would renege on its promised transfer, 1 would have exercised its right to choose $q$ in a different way. The court would then rule in favor of locality 1 and force the transfer. (It is worth noting that such a court would be irrelevant under centralization because no rights belong to the localities there.) We refer to this environment, in which localities have property rights, as a decentralized environment.

Under decentralization, there are four basic regimes that we study. Bargaining power can rest with either locality 1 or 2; and property rights over the project can be allocated either to locality 1 or 2. If locality $i$ has property rights, then if bargaining fails, locality $i$ is entitled to choose $q$, and locality $j \neq i$ is not obligated to pay any transfers.

Thus a decentralized setting for choice of the project level is modeled by the following (decentralization ($D_{ij}$)) game where locality $i$ has rights to the project and locality $j$ has bargaining power:

1. Locality $j$ offers locality $j' \neq j$ a contract specifying $t(q)$, a (possibly negative) transfer from locality 2 to locality 1 as a function of the project level.
2. Locality $j'$ can accept or reject this offer. If locality $j'$ accepts locality $j$'s contract proposal, locality 1 chooses and implements a project level $q$ and locality 2 pays locality 1 the transfer $t(q)$. If locality $j'$ rejects the contract proposal, locality $i$ chooses and implements a project level and neither locality is obligated to make any transfers.

We are interested in comparing expected social welfare under centralization with that under decentralization. Which constitutional environment performs better will depend on the parameters of the problem, specifically the distributions of the direct benefit and externality, $\theta$ and $\gamma$. One building block along the way to this comparison is an analysis of how, given decentralization, the socially optimal assignment of property rights and bargaining power depends on these same parameters.

In our analysis, we use the standard solution concept of Perfect Bayesian Equilibrium as defined in Fudenberg and Tirole (1991).

## 3 | FULL INFORMATION

Before turning to the general case, we briefly analyze and compare centralization and decentralization according to expected social welfare, assuming that the realizations of $\theta$ (the private benefit parameter) and $\gamma$ (the externality parameter) become common knowledge before the project level and transfer are determined.

### 3.1 | Centralization

Consider the centralization game with $\theta$ and $\gamma$ being commonly known to all players. It is easy to show that the central authority chooses the socially optimal project level $q(\theta, \gamma) = \theta - \gamma$, and sets transfers equal to zero. Under full information, centralization entails no efficiency loss and the first-best expected social welfare is $\mathbb{E}_{\theta,\gamma}[(\theta - \gamma)^2/2]$.

### 3.2 | Decentralization

Consider the decentralization game with the private benefit and externality parameters being commonly known to all players. It is easy to show that the equilibrium contract when locality $j$ has bargaining power and locality $i$ has rights to the project solves the following maximization problem:

$$\max_{q,t} u_j(q, \alpha, t) \quad \text{s.t.} \quad u_{j'}(q, \beta, t) \geq u_{j'}(\bar{q}(i, \theta, \gamma), \beta, 0),$$

where $(\alpha, \beta) = (\theta, \gamma)$ if $j = 1$ (and $j' = 2$), and $(\alpha, \beta) = (\gamma, \theta)$ if $j = 2$ (and $j' = 1$), and where $\bar{q}(i, \theta, \gamma)$ is the quantity optimally chosen by locality $i$ if bargaining fails and no transfers are obligated. As was noted previously, $\bar{q}(1, \theta, \gamma) = \theta$ and $\bar{q}(2, \theta, \gamma) = 0$. The constraint is thus locality $j'$'s participation constraint, requiring that $j'$ does at least as well by accepting the offer as by rejecting it.

Solving this, locality $j$ chooses the socially optimal project level, $\theta - \gamma$, and sets transfers such that locality $j'$ accepts the project rather than having the default option being implemented.

Thus, under full information, centralization always does at least as well as decentralization with the difference vanishing in the social cost of transfers. This serves as a benchmark for the more interesting cases explored below.

This full information analysis supports Coase's (1960) intuition that, without transaction costs or bargaining imperfections, assigning property rights (to the project to some locality $i$ and to refuse transfers to both localities) yields an efficient outcome.

## 4 | ASYMMETRIC INFORMATION

Here we analyze centralization and decentralization assuming that the realizations of $\theta$ and $\gamma$, the benefit and externality parameters, are private information of Localities 1 and 2, respectively. In each environment, we solve for the social welfare maximizing equilibrium allocation of the associated game assuming that private information is realized before the first stage of the game.

### 4.1 | Centralization

In the last stage of the centralization game, the central authority optimally chooses the project intensity and the transfer conditional on whatever information may have been revealed by the two localities in the preceding stage. As in the case with full information, in equilibrium, no transfers will ever be made at this stage because they are disliked by the center.[2]

In the first stage, each locality chooses an element from its set of possible parameters taking into account how this may affect the project intensity through the information this conveys to the center.

Our analysis of centralization is closely related to Crawford and Sobel's (1982) seminal analysis of sender–receiver games. They study communication in a setting without transfers and are interested in the nature of communication for different parameterizations of preferences for the center and locality 1. In our model of centralization, the expected externality plays the role of the bias generating the conflict between the sender and receiver in Crawford and Sobel.

[2]In fact, the important feature is not that transfers are disliked, and that there are none, but rather that the central authority has *some* level of transfers that it strictly prefers for reasons external to and independent of the project and the externality. Without commitment, ex post the central authority would choose the transfer that maximizes its preferences. This implies that transfers will not provide incentives under centralization.

In equilibrium, two conditions—truth-telling for the two localities and conditional optimality of the central authority's choice of project level, $q$—restrict the amount of information that will be transmitted. A first implication is that, because transfers are constant in equilibrium, it is impossible to separate out the different possible externality values of locality 2. Locality 2's preferences are monotonically decreasing in the project size, $q$, and thus it will report whatever externality parameter would lead the center to lower the project level the most. In contrast, it may be possible to elicit some information from locality 1 since its preferences are not monotonic in the project level $q$. However, full revelation of the benefit parameter by locality 1 cannot be incentive compatible whenever the expected externality from the project is not zero (i.e., $\mathbb{E}\gamma > 0$).

As in Crawford and Sobel (1982), consider an incentive compatible partition of $\Theta$, $\mathcal{P}_\Theta = \{\Theta_1, \ldots, \Theta_J\}$, where $\Theta_j = [\theta_{j-1}, \theta_j)$ for $j = 1, \ldots, J-1$, $\Theta_J = [\theta_{J-1}, \theta_J]$ with $\theta_0 = \underline{\theta}$ and $\theta_J = \bar{\theta}$.[3] Suppose that in the first stage of the game, locality 1 selects the interval that includes the true benefit parameter, $\theta$. Then upon selection of $\Theta_j$ by locality 1, the central authority chooses the project level $q_j$ that maximizes its expected social welfare conditional on $\theta$ belonging to $\Theta_j$. This implies that

$$q_j = (\theta_j + \theta_{j-1})/2 - \mathbb{E}\gamma \text{ for } j = 1, \ldots, J. \tag{1}$$

Combining this conditional optimality requirement with incentive compatibility for locality 1 yields the following recursive characterization of the optimal partition of size $J$:

$$(\theta_{j+1} - \theta_j) = (\theta_j - \theta_{j-1}) + 4\mathbb{E}\gamma \text{ for } j = 1, \ldots, J-1 \tag{2}$$

whenever $J \geq 2$. If $J = 1$, there is no screening in $\theta$ and the project level is $\mathbb{E}\theta - \mathbb{E}\gamma$.

We follow the mechanism design literature in focusing on the unique Pareto-dominant equilibrium outcome. From Crawford and Sobel (1982, theorems 3 and 5), finer screening increases expected social welfare and results in Pareto improvements. There are no social costs but some benefits to increased screening by the central authority as this allows finer tuning of project levels to locality 1's private information.

**Proposition 4.1.**

(a) *There is an equilibrium outcome of the centralization game that ex ante Pareto-dominates all other equilibrium outcomes.*

*Under this equilibrium outcome,*

(b) *Locality 1's equilibrium reports and the center's corresponding equilibrium project levels in the centralization game are determined according to the following optimal partition:*

$$\theta_j^* = \frac{(J^* - j)\underline{\theta} + j\bar{\theta}}{J^*} - 2\mathbb{E}\gamma(J^* - j)j \quad \forall\, j = 0, \ldots, J^*$$

[3] That the restriction to partitions is without loss of generality follows from the results of Crawford and Sobel (1982). See also Melumad and Shibano (1991).

*where* $J^* = \lfloor 1/2 + \sqrt{1/4 + (\bar{\theta} - \underline{\theta})/(2\mathbb{E}\gamma)} \rfloor$ *and* $\lfloor x \rfloor$ *denotes the largest integer weakly smaller than* $x$.

(c) *The equilibrium expected social welfare loss (as compared to first-best expected social welfare) under the centralized regime is:*

$$SWL_C(J^*) = \frac{\sigma_\theta^2}{2(J^*)^2} + \frac{\sigma_\gamma^2}{2} + (\mathbb{E}\gamma)^2 \frac{((J^*)^2 - 1)}{6} \tag{3}$$

*where* $\sigma_z^2$ *denotes the variance of* $z$.

The expressions for the optimal $\theta_j$ and number of screening categories $J^*$ come from Equation (2) and finding the largest integer $J$ compatible with it.

The expected social welfare loss under centralization depends positively on the variances of both the benefit and externality parameters. For any fixed partition, the coarseness of the centralized policy due to the partitional structure is more socially costly the larger are the variances. Suppose, for example, that the equilibrium involves no screening ($J^* = 1$), and consider a mean-preserving increase in the variance of one of the parameters. It is clear that the efficiency of the centralized solution is reduced because there is now more weight on types further away from the average type, which is what determines the chosen project level. This same argument can be applied to any fixed level of screening, $J^*$. However, the impact on social welfare loss of variation in $\theta$ is attenuated when $J^*$ is larger since $\theta$ is then screened more closely. Since there is no effective screening of locality 2, the impact of the variance of $\gamma$ is independent of $J^*$.

If $J^* \geq 2$, the expected social welfare loss under centralization also depends positively on the mean of the externality parameter $\mathbb{E}\gamma$. The presence of truth-telling constraints for locality 1 forces a socially costly downward distortion in the cut-off types used for screening as the expected externality increases. This effect is captured by the term $(\mathbb{E}\gamma)^2$ multiplying the last term of the expression (3).

Additionally, there are effects of the expected externality and the variance of $\theta$ on $J^*$. Increasing $\mathbb{E}\gamma$ may result in such a large distortion of cut-off types that using $J^*$ categories to screen becomes impossible, forcing $J^*$ to be reduced. In constrast, as the support of $\theta$ increases (thus increasing $\sigma_\theta^2 = (\bar{\theta} - \underline{\theta})^2/12$), $J^*$ may increase.

These arguments show that increasing the expected size of the externality ($\mathbb{E}\gamma$) or the variance of locality 2's private information ($\sigma_\gamma^2$) has an unambiguously negative effect on social welfare under centralization, while the variance of locality 1's private information ($\sigma_\theta^2$) has an ambiguous effect (though negative for fixed $J^*$).

## 4.2 | Decentralization

There are four basic regimes under decentralization. Rights over the project can be granted to locality 1 or 2, and bargaining power over these rights can rest with either locality. One might imagine more complex, hybrid regimes where rights and bargaining power over the project are somehow shared between the two localities. By analyzing the four possible polar cases, we aim to capture the scope of decentralization while maintaining some tractability and institutional realism.

We first consider the regimes in which locality 2 has the bargaining power. The solutions under these regimes are more easily characterized (than when locality 1 makes the offer) because locality 1 does not care about any private information that might be revealed through locality 2's offer.

### 4.2.1 | Locality 2 has bargaining power

When locality 2 has bargaining power, it makes a take-it-or-leave-it offer to locality 1 in the decentralization game. This offer must satisfy participation and incentive constraints for locality 1, denoted by ($\mathrm{IR}_\theta$) and ($\mathrm{IC}_\theta$) respectively. Since locality 2's private information does not enter locality 1's payoff, we do not need to consider incentive constraints for locality 2. Each $\gamma$-type is facing the same constraint set, and thus each type selects its preferred allocation subject to these constraints. Locality 2 solves the following maximization problem:[4]

$$\begin{array}{lll} & \max\limits_{\{q(\theta,\gamma),t(\theta,\gamma)\}_{\theta=\underline{\theta}}^{\bar{\theta}}} & \int_{\underline{\theta}}^{\bar{\theta}} u_2(q(\theta,\gamma),\gamma,t(\theta,\gamma))f(\theta)d\theta \\ \text{s.t.} & (\mathrm{IR}_\theta) & u_1(q(\theta,\gamma),\theta,t(\theta,\gamma)) - u_1(\bar{q}(i,\theta,\gamma),\theta,0) \geq 0 \quad \forall\ \theta \\ & (\mathrm{IC}_\theta) & u_1(q(\theta,\gamma),\theta,t(\theta,\gamma)) \geq u_1(q(\theta',\gamma),\theta,t(\theta',\gamma)) \quad \forall\ \theta,\theta' \end{array}$$

where, recall, $\bar{q}(1,\theta,\gamma) = \theta$ if locality 1 has rights, and $\bar{q}(2,\theta,\gamma) = 0$ if locality 2 has rights. The ($\mathrm{IR}_\theta$) constraints say that each type $\theta$ of locality 1 must get at least as much from accepting the contract as it can get by rejecting it. The ($\mathrm{IC}_\theta$) constraints ensure that locality 1 does not want to misrepresent its benefit parameter. Locality 2 of type $\gamma$ chooses its offer to maximize its expected utility subject to these constraints.

The next proposition characterizes the solution to this problem.

**Proposition 4.2.**

(i) *Suppose locality 2 has rights and has the bargaining power. Then the solution is:*

$$\begin{aligned} q_{D_{22}}(\theta,\gamma) &= \max\left\{0, \theta - \gamma - \frac{1-F(\theta)}{f(\theta)}\right\} \\ &= \max\{0, 2\theta - \gamma - \mathbb{E}\theta - \sqrt{3}\sigma_\theta\} \end{aligned} \tag{4}$$

$$t_{D_{22}}(\theta,\gamma) = \int_{\underline{\theta}}^{\theta} q_{D_{22}}(x,\gamma)dx - \theta q_{D_{22}}(\theta,\gamma) + q_{D_{22}}(\theta,\gamma)^2/2 \leq 0;$$

*yielding expected social welfare loss (as compared to first-best expected social welfare) of*

[4]Note that the objective function instead could be written as expectation over $\gamma$ of the stated objective function. This would not affect the solution since the constraint set is independent of $\gamma$ so that the pointwise solution for each $\gamma$ would also be a solution under this other objective function.

$$SWL_{D_{22}} = \mathbb{E}_{\theta,\gamma}[(\theta - \gamma - q_{D_{22}}(\theta,\gamma))^2/2].$$

(ii) *Suppose locality* 1 *has rights and locality* 2 *has the bargaining power. Then the solution is:*

$$q_{D_{12}}(\theta,\gamma) = \min\left\{\theta, \theta - \gamma + \frac{F(\theta)}{f(\theta)}\right\} = \min\{\theta, 2\theta - \gamma - \mathbb{E}\theta + \sqrt{3}\sigma_\theta\}$$

$$t_{D_{12}}(\theta,\gamma) = \int_\theta^{\bar{\theta}} (x - q_{D_{12}}(x,\gamma))dx - \left(\theta q_{D_{12}}(\theta,\gamma) - q_{D_{12}}(\theta,\gamma)^2/2\right) + \theta^2/2 \geq 0; \tag{5}$$

*yielding expected social welfare loss* (*as compared to first-best expected social welfare*) *of*

$$SWL_{D_{12}} = \mathbb{E}_{\theta,\gamma}[(\theta - \gamma - q_{D_{12}}(\theta,\gamma))^2/2].$$

From this proposition, it follows that the project level lies between locality 2's privately optimal level, 0, and locality 1's privately optimal level, $\theta$. Locality 2 trades off between its private benefit from decreasing the project level below $\theta$ and its cost in terms of increased payment to or decreased payment from locality 1. The max and min terms in (4) and (5) occur because this tradeoff may be extreme enough to push the project level to these boundaries (and the corresponding transfers to zero) for some parameter realizations. Away from these boundaries, as is standard, this tradeoff results in distortion of the project level away from the social optimum, $\theta - \gamma$, by a hazard-rate term, the exact form of which depends on where ($\mathrm{IR}_\theta$) binds, which, in turn, depends on which locality has rights. The locality with rights receives a transfer payment for allowing the project level to differ from its private optimum. This transfer also incorporates an informational rent for locality 1.

These solutions are an equilibrium outcome of the corresponding decentralization game. To see this, note that these outcomes are supported by the following strategies:

1. Locality 2 of type $\gamma$ offers a schedule of transfers as a function of the project level, denoted by $\hat{t}_\gamma(\hat{q}) \equiv t(\hat{\theta},\gamma)$ such that $\hat{q} = q(\hat{\theta},\gamma)$.[5]
2. Following any offer $t(\hat{q})$, locality 1 of type $\theta$ best responds by either selecting a project level and being remunerated according to the offered schedule, or rejecting the schedule. Note that locality 1's best response is independent of its beliefs about $\gamma$. On path, locality 1's best response is to accept $\hat{t}_\gamma(\hat{q})$ and choose $\hat{q} = q(\theta,\gamma)$.
3. If locality 1 were to reject the schedule, the locality $i$ having the rights then unilaterally selects the project level $\bar{q}(i,\theta,\gamma)$.

[5]One can verify, for the solutions given in Proposition 4.2, that $q$ and $\gamma$ are together sufficient to determine the transfer $t$.

### 4.2.2 | Locality 1 has bargaining power

When locality 1 has bargaining power, we cannot nest the two problems corresponding to the different allocations of rights. We first solve for the case where locality 2 has property rights.

When locality 2 has rights, the constraints ($\mathrm{IR}_\gamma$) and ($\mathrm{IC}_\gamma$) do not depend directly on $\theta$. Each $\theta$-type is maximizing over the same constraint set, and thus each type selects its preferred allocation. The problem is to solve the following maximization problem:[6]

$$\begin{array}{lll} & \max_{\{q(\theta,\gamma),t(\theta,\gamma)\}_{\gamma=\underline{\gamma}}^{\overline{\gamma}}} & \int_{\underline{\gamma}}^{\overline{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma \\ \text{s.t.} & (\mathrm{IR}_\gamma) & u_2(q(\theta,\gamma),\gamma,t(\theta,\gamma)) \geq 0 \quad \forall\ \gamma \\ & (\mathrm{IC}_\gamma) & u_2(q(\theta,\gamma),\gamma,t(\theta,\gamma)) \geq u_2(q(\theta,\gamma'),\gamma,t(\theta,\gamma')) \quad \forall\ \gamma,\gamma' \end{array}$$

The next proposition characterizes the solution to this problem.

**Proposition 4.3.** *Suppose locality 2 has rights and locality 1 has the bargaining power. Then the solution is:*

$$q_{D_{21}}(\theta,\gamma) = \max\left\{0, \theta - \gamma - \frac{G(\gamma)}{g(\gamma)}\right\} = \max\{0, \theta - 2\gamma + \mathbb{E}\gamma - \sqrt{3}\sigma_\gamma\} \tag{6}$$

$$t_{D_{21}}(\theta,\gamma) = -\gamma q_{D_{21}}(\theta,\gamma) - \int_\gamma^{\overline{\gamma}} q_{D_{21}}(\theta,x)dx \leq 0;$$

*yielding expected social welfare loss (as compared to first-best expected social welfare) of*

$$SWL_{D_{21}} = \mathbb{E}_{\theta,\gamma}[(\theta - \gamma - q_{D_{21}}(\theta,\gamma))^2/2].$$

Again it follows that the project level lies between 0 and $\theta$. Now it is locality 1 trading off between its private benefit from increasing the project level above 0 and its cost in terms of increased payment to locality 2. The hazard-rate distortion now depends on locality 2's information and the fact that ($\mathrm{IR}_\gamma$) binds at $\overline{\gamma}$. Locality 2 receives a transfer payment. By the same reasoning used following Proposition 4.2, this solution is an equilibrium outcome of the corresponding decentralization game.

We now turn to the case where locality 1 has both rights to the project and bargaining power. Observe that the constraints ($\mathrm{IR}_\gamma$) now depend on locality 2's beliefs about $\theta$. The problem is then one of an informed principal, which implies that (interim) incentive constraints for locality 1, denoted ($\mathrm{IIC}_\theta$), must be satisfied as well. Note that each $\theta$ type offers a schedule $t_\theta(q)$ to locality 2. Locality 2, knowing the equilibrium offer of each $\theta$, updates its beliefs about $\theta$. If an out-of-equilibrium offer is observed, locality 2 is assumed to believe that locality 1 is of the lowest type $\underline{\theta}$.[7] We limit attention to separating allocations, implying that

[6]Analogously to Footnote 4, the objective function could alternatively be written as expectation over $\theta$ of the stated objective function without affecting the solution.

[7]This assumption, by making ($\mathrm{IR}_\gamma$) as tight as possible following an out-of-equilibrium offer, potentially supports more favorable equilibrium offers for locality 1. This is in the spirit of locality 1 having the bargaining power.

locality 2's updated beliefs on the equilibrium path after receiving the offer are degenerate on $\theta$. The right-hand side of ($\mathrm{IR}_\gamma$) reflects these beliefs.[8] Thus, the solution to the following problem identifies the separating equilibrium allocation corresponding to locality 1 having rights and bargaining power:

$$\begin{array}{lll}
 & \max_{\{q(\theta,\gamma),t(\theta,\gamma)\}_{\gamma,\theta}} & \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{\gamma}}^{\bar{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma f(\theta)d\theta \\
\text{s.t.} & (\mathrm{IR}_\gamma) & u_2(q(\theta,\gamma),\gamma,t(\theta,\gamma)) \geq -\gamma\theta \quad \forall\,\gamma,\theta \\
 & (\mathrm{IC}_\gamma) & u_2(q(\theta,\gamma),\gamma,t(\theta,\gamma)) \geq u_2(q(\theta,\gamma'),\gamma,t(\theta,\gamma')) \quad \forall\,\gamma,\gamma',\theta \\
 & (\mathrm{IIC}_\theta) & \int_{\underline{\gamma}}^{\bar{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma \\
 & & \quad \geq \int_{\underline{\gamma}}^{\bar{\gamma}} u_1(q(\theta',\gamma),\theta,t(\theta',\gamma))g(\gamma)d\gamma \quad \forall\,\theta,\theta'
\end{array}$$

The next proposition characterizes the solution to this problem under the assumption that $0 \leq q(\theta,\gamma) \leq \theta$ for all $\theta$ and $\gamma$. This assumption ensures that the equilibrium realized surplus over the outside option for locality 2 is increasing in the externality $\gamma$, and thus ($\mathrm{IR}_\gamma$) if binding anywhere is binding only at $\underline{\gamma}$. Furthermore, we assume that ($\mathrm{IR}_\gamma$) is binding at $\underline{\gamma}$ for all $\theta$. Without these assumptions, we do not know the form of the solution. It is worth noting that in the relaxed problem without the constraints ($\mathrm{IC}_\gamma$) or without the constraints ($\mathrm{IIC}_\theta$) these assumptions are satisfied. Under these assumptions, the previous problem is equivalent to the following one:

$$\begin{array}{lll}
 & \max_{\{0\leq q(\theta,\gamma)\leq\theta,t(\theta,\gamma)\}_{\gamma,\theta}} & \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{\gamma}}^{\bar{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma f(\theta)d\theta \\
\text{s.t.} & (\mathrm{IR}_{\underline{\gamma}}) & u_2(q(\theta,\underline{\gamma}),\underline{\gamma},t(\theta,\underline{\gamma})) = -\underline{\gamma}\,\theta \quad \forall\,\theta \\
 & (\mathrm{IR}_\gamma) & u_2(q(\theta,\gamma),\gamma,t(\theta,\gamma)) \geq -\gamma\theta \quad \forall\,\gamma,\theta \\
 & (\mathrm{IC}_\gamma) & u_2(q(\theta,\gamma),\gamma,t(\theta,\gamma)) \geq u_2(q(\theta,\gamma'),\gamma,t(\theta,\gamma')) \quad \forall\,\gamma,\gamma',\theta \\
 & (\mathrm{IIC}_\theta) & \int_{\underline{\gamma}}^{\bar{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma \\
 & & \quad \geq \int_{\underline{\gamma}}^{\bar{\gamma}} u_1(q(\theta',\gamma),\theta,t(\theta',\gamma))g(\gamma)d\gamma \quad \forall\,\theta,\theta'
\end{array} \tag{7}$$

The next proposition characterizes the solution to this problem.

**Proposition 4.4.** *Suppose locality* 1 *has rights and has the bargaining power. If* $\bar{\gamma} - \underline{\gamma}(2 + \ln((\bar{\gamma} - \underline{\gamma})/\underline{\gamma})) \geq \bar{\theta} - \underline{\theta}$, *then a solution to* (7) *is:*

[8] We do not include a formal analysis of equilibria involving pooling or partial pooling by locality 1. However, we can show that no full-pooling equilibrium exists because there would always be at least one $\theta$ that would strictly prefer to exercise its right to set the project level at its private optimum.

$$q_{D_{11}}(\theta,\gamma) = \theta - \gamma + \frac{1-G(\gamma)}{g(\gamma)} + \underline{\gamma}(1 + \mathrm{ProductLog}(-e^{-(\theta+\underline{\gamma}-\underline{\theta})/\underline{\gamma}}))$$
$$= \theta - 2\gamma + 2\mathbb{E}\gamma + (\mathbb{E}\gamma - \sqrt{3}\sigma_\gamma)\left(1 + \mathrm{ProductLog}\left(-e^{-\frac{\theta+\mathbb{E}\gamma-\mathbb{E}\theta-\sqrt{3}\sigma_\gamma+\sqrt{3}\sigma_\theta}{\mathbb{E}\gamma-\sqrt{3}\sigma_\gamma}}\right)\right) \tag{8}$$
$$t_{D_{11}}(\theta,\gamma) = -\gamma q_{D_{11}}(\theta,\gamma) + \gamma\theta - \int_{\underline{\gamma}}^{\gamma}(\theta - q_{D_{11}}(\theta,x))dx \geq 0$$

*where* $w = \mathrm{ProductLog}(z)$ *is the principal real-valued solution to* $we^w = z$*. At this solution, expected social welfare loss (as compared to first-best expected social welfare) is*

$$SWL_{D_{11}} = \mathbb{E}_{\theta,\gamma}[(\theta - \gamma - q_{D_{11}}(\theta,\gamma))^2/2].$$

In interpreting Proposition 4.4, it is useful to note that the assumption

$$\bar{\gamma} - \underline{\gamma}(2 + \ln((\bar{\gamma} - \underline{\gamma})/\underline{\gamma})) \geq \bar{\theta} - \underline{\theta}$$

is equivalent to $0 \leq q_{D_{11}}(\theta,\gamma) \leq \theta$ for all $\theta$ and $\gamma$. This explains why there is no max or min term appearing in $q_{D_{11}}$. Notice that the first three terms in the expression for $q_{D_{11}}$ are analogous to the expressions for $q$ in the previous propositions. The last term, which is non-negative and increasing in $\theta$, is an additional distortion of the project level induced by the ($\mathrm{IIC}_\theta$) constraints.

The solutions presented in Propositions 4.3 and 4.4 are equilibrium outcomes of the corresponding decentralization games. With the exception of the additional specification of out-of-equilibrium beliefs used in the analysis of the $D_{11}$ regime, the construction demonstrating this is analogous to that at the end of Section 4.2.1.

## 5 | COMPARISON OF THE VARIOUS REGIMES

We now have solved for the equilibrium centralization allocation as well as for the four equilibrium decentralized allocations. In this section, we compare these allocations to characterize the socially optimal regime as a function of the parameters of the model. This forms the basis for our main results about the choice and trade-offs between centralization and decentralization. One caveat to keep in mind is the possibility that the efficiency of decentralization might be underestimated if more complex allocations of rights and bargaining power were both feasible and helpful.

We have seen that the solution to the various decentralization regimes may involve some bunching either at $q = 0$ or at $q = \theta$. This results in potentially complicated expressions for the social welfare loss. To simplify comparisons, we focus on two cases for each decentralization regime: the no bunching (*nb*) case where $q$ is strictly between 0 and the private optimum $\theta$ for all interior values of $\theta$ and $\gamma$, and the "always" bunching (*b*) case where the 0 and/or the $\theta$ bounds are hit for all types of the locality making the offer for at least some types of the other locality.[9]

[9]We are able to include the $D_{11}$ regime in our comparisons only in the no bunching case, as we do not know the solution under this regime in the always bunching case.

We first define the conditions on the underlying parameters for no bunching, one for each decentralization regime:[10]

$$
\begin{aligned}
&c_{12}^{nb}: \mathbb{E}\gamma - 2\sqrt{3}\sigma_\theta \geq \sqrt{3}\sigma_\gamma;\\
&c_{21}^{nb}: \mathbb{E}\theta - \mathbb{E}\gamma - 3\sqrt{3}\sigma_\gamma \geq \sqrt{3}\sigma_\theta;\\
&c_{22}^{nb}: \mathbb{E}\theta - \mathbb{E}\gamma - 3\sqrt{3}\sigma_\theta \geq \sqrt{3}\sigma_\gamma;\\
&c_{11}^{nb}: -\mathbb{E}\gamma + 3\sqrt{3}\sigma_\gamma - 2\sqrt{3}\sigma_\theta - (\mathbb{E}\gamma - \sqrt{3}\sigma_\gamma)\ln\left[\frac{2\sqrt{3}\sigma_\gamma}{\mathbb{E}\gamma - \sqrt{3}\sigma_\gamma}\right] \geq 0 \quad \text{and}\\
&\qquad \mathbb{E}\gamma - 3\sqrt{3}\sigma_\gamma > 0.
\end{aligned}
$$

We similarly introduce the "always" bunching condition for each decentralization regime except the regime where locality 1 both offers and has rights ($D_{11}$):[11]

$$
\begin{aligned}
&c_{12}^{b}: \mathbb{E}\gamma - 2\sqrt{3}\sigma_\theta \leq -\sqrt{3}\sigma_\gamma;\\
&c_{21}^{b}: \mathbb{E}\theta - \mathbb{E}\gamma - 3\sqrt{3}\sigma_\gamma \leq -\sqrt{3}\sigma_\theta;\\
&c_{22}^{b}: \mathbb{E}\theta - \mathbb{E}\gamma - 3\sqrt{3}\sigma_\theta \leq -\sqrt{3}\sigma_\gamma.
\end{aligned}
$$

For the $D_{11}$ regime, we do not know the solution when $c_{11}^{nb}$ is violated.

We can now state our main results on the socially optimal regime as a function of the parameters of the model. We first assume that $c_{11}^{nb}$ holds, allowing the analysis to include all of the decentralization regimes.

**Theorem 5.1.** *Suppose $c_{11}^{nb}$ is satisfied and that, for each $ij \neq 11$, either $c_{ij}^{nb}$ or $c_{ij}^{b}$ is satisfied.*

(a) *Centralization is better than all of the decentralization regimes if and only if $1/3 < \sigma_\theta^2/\sigma_\gamma^2 < 3$. When centralization is best, the equilibrium policy under centralization is uniform (i.e., $J^* = 1$).*

(b) *When $\sigma_\theta^2/\sigma_\gamma^2 > 3$, decentralization where 2 has rights and 1 offers is better than centralization and the other decentralization regimes.*

(c) *When $\sigma_\theta^2/\sigma_\gamma^2 < 1/3$, decentralization where 2 offers is better than centralization and the other decentralization regimes. Whether the rights belong to 1 or 2 in this case makes no difference to social welfare.*

When $c_{11}^{nb}$ is violated, we compare centralization to the decentralization regimes known for that case (i.e., all but 1 has rights and offers).

[10]The notation $c_{ij}^{nb}$ refers to the condition for the no bunching case when locality $i$ has rights and locality $j$ has bargaining power.

[11]The notation $c_{ij}^{b}$ refers to the condition for the "always" bunching case when locality $i$ has rights and locality $j$ has bargaining power.

**Theorem 5.2.** *Suppose $c_{11}^{nb}$ is violated and that, for each $ij \neq 11$, either $c_{ij}^{nb}$ or $c_{ij}^{b}$ is satisfied.*

(a) *Centralization is better than all the decentralization regimes if and only if $1/3 < \sigma_\theta^2/\sigma_\gamma^2 < 3$ and $\mathbb{E}\gamma$ is greater than the middle of the three real roots of $x^3 - 3\sqrt{3}\sigma_\theta x^2 + 3\sigma_\gamma^2 x + 3\sqrt{3}\sigma_\theta^3$. When centralization is best, the equilibrium policy under centralization is uniform (i.e., $J^* = 1$).*

*If $c_{12}^{nb}$ is satisfied, then either centralization is best or $1/3 < \sigma_\theta^2/\sigma_\gamma^2 < 3$ does not hold, and:*

(b) *If $\sigma_\theta^2/\sigma_\gamma^2 > 3$, decentralization where 2 has rights and 1 offers is better than centralization and the other decentralization regimes;*

(c) *If $\sigma_\theta^2/\sigma_\gamma^2 < 1/3$, then decentralization where 2 offers is better than centralization and the other decentralization regimes. Whether the rights belong to 1 or 2 in this case makes no difference to social welfare.*

*If instead $c_{12}^{b}$ is satisfied:*

(d) *When centralization is not best, either $D_{21}$ or $D_{12}$ is best. The comparison between $D_{21}$ and $D_{12}$ depends on the parameters, with comparative statics as follows:*
  (i) *Fixing $\mathbb{E}\theta$, $\sigma_\gamma^2$ and $\sigma_\theta^2$, increasing $\mathbb{E}\gamma$ favors $D_{21}$ relative to $D_{12}$;*
  (ii) *Fixing $\mathbb{E}\gamma$, $\sigma_\gamma^2$ and $\sigma_\theta^2$, increasing $\mathbb{E}\theta$ has no effect on the comparison;*
  (iii) *Fixing $\mathbb{E}\gamma, \mathbb{E}\theta$ and $\sigma_\theta^2$, increasing $\sigma_\gamma^2$ favors $D_{12}$ relative to $D_{21}$;*
  (iv) *Fixing $\mathbb{E}\gamma, \mathbb{E}\theta$ and $\sigma_\gamma^2$, increasing $\sigma_\theta^2$ favors $D_{21}$ relative to $D_{12}$.*

We provide intuition for these theorems in pieces. We first discuss the uniformity of centralization when it is socially optimal, then the comparisons among the various decentralization regimes, and finally the comparison between centralization and decentralization.

Theorems 5.1(a) and 5.2(a) tell us that whenever centralization is socially optimal, the central authority implements the same project level, $\mathbb{E}\theta - \mathbb{E}\gamma$, for all parameter realizations. Key to this is the observation that, whenever centralization is best, the expected externality $\mathbb{E}\gamma$ is sufficiently large. This is stated explicitly in Theorem 5.2(a) and the same bound on $\mathbb{E}\gamma$ is always satisfied under the assumptions of Theorem 5.1. From the results on centralization in Proposition 4.1, an expected externality this large implies that cheap-talk screening on $\theta$ is not incentive compatible because of the substantial difference between the central authority's objective and locality 1's objective it generates. There is also no cheap-talk screening on $\gamma$ because locality 2's preferences are monotonically decreasing in the project size.

An implication of this uniformity of project size under centralization when it is socially optimal is that the expected social welfare loss under centralization (as compared to the first-best expected social welfare) is:

$$SWL_C(1) = \frac{\mathbb{E}[(\theta - \gamma - (\mathbb{E}\theta - \mathbb{E}\gamma))^2]}{2} = \frac{\sigma_\theta^2}{2} + \frac{\sigma_\gamma^2}{2}. \tag{9}$$

Observe that the variances of both parameters contribute to social welfare loss and do so symmetrically.

Now consider decentralization. Theorem 5.1(b) and (c) tell us that in the cases where we can analyze all four decentralization regimes, whenever decentralization is better than centralization, bargaining power is better allocated to the locality with the higher variance of its privately known parameter. This is intuitive, as the party who offers is better able to incorporate its private information in the offer, and, with uniform distributions, the information with the higher variance is the more important of the two to incorporate from a social welfare perspective.[12]

Next turn to Theorem 5.2. Parts (b) and (c) tell us that if there is no bunching under the $D_{12}$ regime, the same result about the relative variances determining the optimal allocation of bargaining power under decentralization when it is better than centralization holds. If instead there is bunching under the $D_{12}$ regime, this optimal allocation of bargaining power becomes more complex. There are cases where locality 2 is optimally given the bargaining power despite having the smaller variance. Part (d) of Theorem 5.2 tells us that when decentralization is best and bunching occurs under $D_{12}$, either $D_{21}$ or $D_{12}$ can be best and it gives comparative statics in the parameters that collectively determine which of the two is better. These comparative statics reveal that, in addition to the effects of increasing the respective variances ((iii) and (iv)) that push in the intuitive directions discussed just above, there is an additional effect, described in (i), of the average externality. All else equal, smaller average externalities favor $D_{12}$ relative to $D_{21}$ and this can outweigh the variance effects. All these comparative static effects are also present in Theorem 5.1 when there is bunching under the $D_{12}$ regime, but the condition $c_{11}^{nb}$ imposed in that result restricts the parameter space in such a way that effect (i) does not outweigh the variance effects there.

The following is the intuition for the comparative statics in Theorem 5.2(d)(i)–(iv):

(i) Under $D_{21}$, locality 1 offers and needs to give information rents to 2. However, when there is no bunching under 1's offer (as is always true when $c_{12}^{b}$ holds), the required information rents, once $\sigma_\gamma$ is fixed, naturally do not vary with the *level* of the average externality, as shifting a fixed-length interval of $\gamma$ values up or down does not change the trade-off between the desired project level and information rents. Mathematically, this can be seen from (6) which shows that the project size deviates from the socially efficient level $\theta - \gamma$ by $\mathbb{E}\gamma - \gamma - \sqrt{3}\sigma_\gamma$, and thus perfectly accounts for shifts in $\mathbb{E}\gamma$. Under $D_{12}$, when there is bunching, (5) shows that bunching occurs at the privately optimal project level for locality 1 (i.e., at level $\theta$). This happens because pushing locality 1 to produce above that level is undesirable to both localities and thus too expensive from 2's perspective. Thus, where bunching occurs, project size deviates from the socially efficient level by $\gamma$ and thus loses efficiency as $\mathbb{E}\gamma$ grows.

[12]In Dessein (2002), there is only the center and one agent, but he also finds that delegation to the informed agent is optimal when the variance of its information is large enough.

(ii) Since locality 1 offers under $D_{21}$ and there is no bunching, locality 1 is able to fully incorporate its private information (see 6) so that there is no distortion in project size along the $\theta$ dimension. Under $D_{12}$, when bunching (at $\theta$) occurs, as explained in (i) above, $\theta$ (and thus $\mathbb{E}\theta$) is perfectly incorporated. When bunching does not realize under $D_{12}$, the intuition for why the distortion away from efficiency is not affected by changes in $\mathbb{E}\theta$ is analogous to the intuition explained in (i) for why, under $D_{21}$, changes in $\mathbb{E}\gamma$ are fully incorporated.

(iii) and (iv) These comparative statics in the standard deviations of $\gamma$ and $\theta$ respectively have the same intuition as was explained in discussing parts (b) and (c) of Theorems 5.1 and 5.2—since the offering party can more easily incorporate their own private information, all else equal, greater variance of this information pushes towards that party making the offer being more efficient.

Another implication for decentralization from Theorems 5.1(b) and (c) and 5.2(b), (c) and (d) is that *the same locality having both rights and bargaining power is never the strictly socially best regime*. Suppose locality 1 offers. Intuitively, giving rights to 1 imposes an extra set of incentive constraints for 1 since then 1's information affects the project level that 2 expects if it were to reject 1's offer. This is socially costly compared to giving rights to 2 when 1 offers.

Similarly, suppose locality 2 offers. First consider the region where $c_{12}^{nb}$ and $c_{22}^{nb}$ hold. The allocation of rights simply determines which of two "mirror image" distortions of output away from the first best occurs, as can be seen from Equations (4) and (5). This implies that the $D_{22}$ and $D_{12}$ regimes create the same expected social welfare loss, so that $D_{22}$ cannot be strictly best. Second, consider the remaining cases, that is, either $c_{12}^{b}$ or $c_{22}^{b}$ holds. It can be shown that, in such cases, $\sigma_\theta^2 > \sigma_\gamma^2$ and $c_{21}^{nb}$ holds. Comparison of social welfare losses then shows, similar to the logic underlying Theorems 5.1(b) and 5.2(b), that the regime $D_{22}$, since it gives bargaining power to the locality with the smaller variance, is worse than the regime $D_{21}$.

We now say a few words about the dependence of our results relating the allocation of bargaining power to the relative variances of the two localities on our assumption that private information is uniformly distributed.[13] This discussion focuses on parameter regions where bunching does not occur. In these regions, as can be seen from Equations (4), (5), and (6), output is distorted away from the socially optimal level by inverse-hazard-rate-like expressions concerning the information of the locality without bargaining power $\left(\text{e.g., } \frac{1-F(\theta)}{f(\theta)}, \frac{F(\theta)}{f(\theta)} \text{ or } \frac{G(\gamma)}{g(\gamma)}\right)$. This results in social welfare losses proportional to the expectation of the square of these expressions. For uniform distributions, the ranking of these losses under $F$ compared to $G$ is the same as the inverse ranking of the variances of $\theta$ compared to $\gamma$, which explains the role of variances in our decentralization results. For more general distributions, the ranking of these losses may differ from the inverse ranking of the variances. When such a difference occurs, bargaining power under decentralization should be assigned according to the expectations of the square of the appropriate inverse-hazard-rate expressions rather than according to the variances. For example, suppose that $\gamma$ is distributed according to a Beta distribution with parameters $\alpha = 4$ and $\beta = 1$ with support $[1, 2]$, while $\theta$ is uniformly distributed on $[4, 4.5]$. Calculation shows that $\sigma_\gamma^2 = 2/75 > 1/48 = \sigma_\theta^2$, while $SWL_{D_{21}} < SWL_{D_{12}} = SWL_{D_{22}}$, and thus bargaining power should be given to locality 1 even though it has the smaller variance.

[13]The following discussion ignores the $D_{11}$ regime that we do not know how to analyze with general distributions.

Theorems 5.1(a) and 5.2(a) give conditions that jointly are necessary and sufficient for centralization to be the best regime. One condition bounds the ratio of the variances of the private information of the two localities ($1/3 < \sigma_\theta^2/\sigma_\gamma^2 < 3$). Recall from our earlier discussion that all social welfare loss under centralization comes from the pooling of all types at a common project size, which (see Equation 9) implies that this loss is symmetric in the two variances. Because of the uniform distributions and quadratic form of the social utility function, this loss is half the sum of the variances. Under decentralization (except for $D_{11}$), when there is no bunching, social welfare loss arises only from the private information of the locality that does not make the offer (as was explained above). Thus, when the asymmetric information of one locality is much smaller than the other, that is, has a much smaller variance, the best of the decentralization regimes performs better than centralization. This explains why the variance ratio being neither too small nor too large is a necessary condition for centralization to be better than decentralization.

Why is this variance-ratio condition also sufficient for the optimality of centralization in many cases, for example, when there is no bunching under decentralization? Since the equilibrium output under decentralization (again excepting $D_{11}$ and assuming no bunching) differs from the social optimum, $\theta - \gamma$, by an inverse-hazard-rate term, monotonicity of these terms ($F/f$ and $G/g$ increasing, and $(1 - F)/f$ decreasing) implies that the equilibrium output is more sensitive than the socially optimal output to the information of the locality not making the offer. Under our uniform distribution assumptions, these hazard rates are linear with unit slope. Given this unit slope, the expected social welfare losses under decentralization are twice the variance of the locality not making the offer. Therefore, whenever twice the smaller variance exceeds the average of the two variances, centralization is strictly socially best. This is what the variance-ratio condition reflects.

Although the above discussion of the variance-ratio bounds assumes that no bunching is involved under decentralization, the same reasoning applied to the comparison between centralization and some decentralization regime(s) where no bunching is present is enough for the argument to carry through in many cases. The exception occurs when $c_{11}^{nb}$ is violated and $c_{12}^{b}$ holds (Theorem 5.2(d)). Under these conditions, the variance bound remains necessary for the superiority of centralization, but is not sufficient.

The other necessary condition that, when joined to the variance bounds, becomes necessary and sufficient (as stated in Theorem 5.2(a)) is that the expected externality is large enough relative to a function of both localities' asymmetric information. Except when $c_{11}^{nb}$ is violated and $c_{12}^{b}$ holds, this expected-externality bound is implied by the variance-ratio bounds (which explains why this condition does not appear in Theorem 5.1(a)). When this additional bound matters, it does so only through the comparison of centralization with $D_{12}$ since the variance bound alone ensures that centralization improves on the $D_{21}$ and $D_{22}$ regimes. While centralization perfectly incorporates changes in the expected externality (see Equation 9), the $D_{12}$ regime, as we described in the discussion of Theorem 5.2(d)(i), loses efficiency as $\mathbb{E}\gamma$ grows. This explains why a larger expected externality favors centralization over $D_{12}$ and consequently the role of this additional condition.

## 6 | CONCLUSION

There are at least two ways to interpret our results. First, we have shown that there are environments in which a central authority may desire to decentralize power (by conferring rights to localities) to provide incentives for greater incorporation of localities' ex post preferences in policies. Second, despite the limited contractual ability of our central authority, we have shown that, in some circumstances, centralization can still be better than all of our decentralization regimes. This implies that it may be better not to confer property rights to either locality. In contrast to the Coase Theorem, decentralization of rights may not be the optimal thing to do.

As with any comparison of centralization and decentralization, the findings presented here should be understood in the context of the specific manner in which these organizational forms are modeled, and in particular the possibilities allowed. For example, our modeling of decentralization limits attention to take-it-or-leave-it bargaining rather than allowing arbitrary bargaining protocols. To the extent that such concerns lead to underestimating the performance of decentralization, this should be taken as a caveat for our comparison results in those instances where our paper finds that centralization is socially best. A possible direction for future research would be to consider the full universe of bargaining protocols and allocations of bargaining power including, for example, the case where bargaining power is equally shared between the localities where the maximand in the associated maximization problem is the sum of the two localities' utility. We note that the relevance of such generalized bargaining may be less clear in the absence of specific implementing protocols.

What do we learn about centralization and decentralization from our analysis? In Section 1 we presented three pieces of "folk wisdom" about externalities and government structure. Consider the first two. It is often suggested that, first, large externalities justify central control, and second, centralization is disadvantaged in situations of local heterogeneity. Our model provides support for these two characteristics (large externalities and local homogeneity) as necessary and jointly sufficient for centralization to be better than all of our decentralization regimes. Furthermore we provide the precise measures of these characteristics that are relevant for this purpose: regarding large externalities, it is the *expected* externality that matters, and, regarding local homo/heterogeneity, the key dimension is the ratio of the variances of the private benefit of the project and the externality ($\sigma_\theta^2/\sigma_\gamma^2$). In particular, this variance ratio being close enough to one and the expected externality exceeding a bound (which is increasing in both variances) taken together are necessary and sufficient for centralization to be better than all of our decentralization regimes (Theorems 5.1 and 5.2).

Regarding the third piece of "folk wisdom," much of the prior literature takes for granted (i.e., assumes) that central policies are insensitive to (ex post) local preferences. Our model shows that the contractual limitations associated with central control endogenously generates limits on the center's ability to discriminate according to ex post realized preferences. We show that whenever centralization is better than decentralization, the equilibrium centralized policy is a uniform one (Theorems 5.1(a) and 5.2(a)). Thus, insensitivity to local preferences and the uniformity of socially optimal centralized policies is a result rather than an assumption. For the preferences of locality 2, this follows rather directly from our model of centralization. For the preferences of locality 1, however, the comparison of centralization with decentralization is crucial in establishing the uniformity of optimal centralized policies.

When decentralization is better than centralization, our findings also speak to which decentralized regime performs best. First, which locality should be given the bargaining power depends on the extent of each locality's private information as measured by the variance of their respective parameters. Holding all else fixed, increasing the variance of one locality makes it more attractive to give that locality the bargaining power. In many cases, comparing the two variances is sufficient: the locality with the higher variance should be given the bargaining power (i.e., get to make the offer). In the remaining cases (see Theorem 5.2(d)), an additional factor is present—holding all else fixed, increasing the expected externality makes it more attractive to give locality 1 the bargaining power. In these cases, the combination of both effects is determinative. Second, we show that rights should go to the locality not given the bargaining power. Thus, when better than centralization, either locality 1 should be given bargaining power and locality 2 the rights to the project or vice-versa.

## ACKNOWLEDGMENTS

We thank Robin Boadway, Deniz Dizdar, Faruk Gul, Lu Hu, Eric Maskin, Dilip Mookherjee, Stefan Reichelstein, Mike Peters, Patrick Rey, Jacques Robert, Lars Stole, Tymon Tatur, and François Vaillancourt for comments and thank a number of seminar and conference audiences. The first author thanks the Department of Economics of the Université de Montréal for its hospitality during several visits. The second author thanks the MEDS Department at Northwestern University for its hospitality during several visits. He is grateful to C.I.R.A.N.O., C.R.S.H. and F.Q.R.S.C. for financial support. We are grateful to the Editor and the anonymous reviewers for their helpful advice.

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**How to cite this article:** Klibanoff, P., Poitevin, M. (2022). A theory of (de) centralization. *Journal of Public Economic Theory*, 24, 417–451.
https://doi.org/10.1111/jpet.12572

## APPENDIX A

Before proving Proposition 4.1, we state and prove the following lemma.

**Lemma A.1.** *Fix the partition size at $J$. A solution to the central authority's problem of optimally screening locality 1 using $J$ categories exists if and only if the expected externality is small enough. Specifically, it exists if and only if $\mathbb{E}\gamma \le (\bar{\theta} - \underline{\theta})/(2J(J-1))$ holds.*

*Proof of Lemma* A1. Applying Equation (2) recursively to solve for $\theta_j$ yields

$$\theta_j = \frac{(J-j)\underline{\theta} + j\bar{\theta}}{J} - 2\mathbb{E}\gamma(J-j)j \quad \forall j = 0, \ldots, J.$$

This solution is feasible if the intervals constructed from the $\theta_j$'s form a partition of $\Theta$ and all the project levels, $q_j = (\theta_j - \theta_{j-1})/2 - \mathbb{E}\gamma$, are nonnegative. A necessary condition for this is that $\underline{\theta} = \theta_0 \le \theta_1$. This condition amounts to:

$$\frac{(J-1)\underline{\theta} + \bar{\theta}}{J} - 2\mathbb{E}\gamma(J-1) \ge \underline{\theta}.$$

It reduces to:

$$\mathbb{E}\gamma \leq \frac{\bar{\theta} - \underline{\theta}}{2J(J-1)},$$

which is the inequality in the lemma.

We have just shown that this condition is necessary for existence. We now show that it is also sufficient by showing that, under this condition, $\theta_{j-1} \leq \theta_j$ and $q_j \geq 0, \forall j = 1, ..., J$. Observe that

$$\theta_j - \theta_{j-1} = \frac{(\bar{\theta} - \underline{\theta})}{J} - 2\mathbb{E}\gamma(J - 2j + 1).$$

It follows that $\theta_j - \theta_{j-1}$ is increasing in $j$. Since the necessary condition yields $\theta_1 - \theta_0 \geq 0$, it must be that $\theta_j - \theta_{j-1} \geq 0$ for all $j = 2, ..., J$ as well. Furthermore, $q_j = (\theta_{j-1} + \theta_j)/2 - \mathbb{E}\gamma \geq \underline{\theta} - \mathbb{E}\gamma \geq 0$, where the first inequality follows from $\theta_j$ nondecreasing and the second inequality follows from our assumption that $\underline{\theta} \geq \bar{\gamma}$. The condition is thus also sufficient for existence. □

*Proof of Proposition* 4.1.

(a) From Equation (1), the expected output is $\mathbb{E}_{\theta,\gamma} q = \sum_j q_j(\theta_j - \theta_{j-1})/(\bar{\theta} - \underline{\theta}) = \mathbb{E}\theta - \mathbb{E}\gamma$. Next, note that locality 2's utility is linear in output and independent of $\theta$, and there are no transfers, thus locality 2 is ex ante indifferent between all equilibrium outcomes. As mentioned in the text, from Crawford and Sobel (1982, theorems 3 and 5), finer screening increases ex ante expected social welfare which, recall, is the sum of the two localities' ex ante utility. Thus, it must be that locality 1 and the central authority both prefer the equilibrium outcome with the finest revelation of information about $\theta$. Part (b) of this proposition characterizes this finest partition consistent with incentive compatibility and conditional optimality.

(b) From Crawford and Sobel (1982, theorems 3 and 5), we know that the finest partition, that is, the largest number of elements $J$, that is consistent with incentive compatibility and conditional optimality is socially optimal. By Lemma A.1, a $J$ partition is consistent with incentive compatibility and conditional optimality if and only if $\mathbb{E}\gamma \leq (\bar{\theta} - \underline{\theta})/(2J(J-1))$. Inverting this expression and taking into account that $J$ is an integer yields that $J^* = \lfloor 1/2 + \sqrt{1/4 + (\bar{\theta} - \underline{\theta})/(2\mathbb{E}\gamma)} \rfloor$. Applying Equation (2) recursively to solve for $\theta_j$ and setting $J = J^*$ yields the expression for the optimal partition cutoffs given in the proposition. The optimal project levels are $q_j^* = \left(\theta_{j-1}^* + \theta_j^*\right)/2 - \mathbb{E}\gamma$.

The following strategies and beliefs support this allocation as an equilibrium outcome of the centralization game.

1. Locality 1 of type $\theta$ chooses its most preferred element of the optimal partition, that is, the element of the partition containing $\theta$. Locality 2 always reports $\mathbb{E}\gamma$.
2. The central authority believes that locality 1's type is distributed uniformly on the selected element of the optimal partition, and it implements $q_j^* = \left(\theta_{j-1}^* + \theta_j^*\right)/2 - \mathbb{E}\gamma$. The central authority sets the transfer equal to zero.

*These strategies and beliefs form a PBE of the centralization game. In the last stage of the game, given the two localities' strategies and its own beliefs, the central authority maximizes expected social welfare when choosing the project levels and transfers. The central authority updates its prior beliefs and chooses the conditionally optimal project level.*

(c) For $J = J^*$, the expected social welfare loss is:

$$\mathbb{E}_{\theta,\gamma}\left\{\frac{(\theta-\gamma)^2}{2}\right\} - \sum_{j=1}^{J^*}\int_{\theta_{j-1}^*}^{\theta_j^*}\left[(\theta-\mathbb{E}\gamma)\left(\frac{\left(\theta_j^*+\theta_{j-1}^*\right)}{2}-\mathbb{E}\gamma\right) - \frac{\left(\left(\theta_j^*+\theta_{j-1}^*\right)/2-\mathbb{E}\gamma\right)^2}{2}\right]\frac{1}{\bar{\theta}-\underline{\theta}}d\theta$$

where the $\theta_j^*$'s are defined in part (b). This expression was simplified to the expression in (3) using *Mathematica*[14] and induction. □

*Proof of Proposition* 4.2. Define

$$U(\theta) = u_1(q(\theta,\gamma),\theta,t(\theta,\gamma)) - u_1(\bar{q}(i,\theta,\gamma),\theta,0) \quad \forall\ \theta,\gamma, \text{ and } i = 1, 2.$$

Recall that $\bar{q}(i,\theta,\gamma) = \theta$ if $i = 1$, and that $\bar{q}(i,\theta,\gamma) = 0$ if $i = 2$. Using $\mathrm{IC}_\theta$, we have

$$\frac{dU(\theta)}{d\theta} = q(\theta,\gamma) - \bar{q}(i,\theta,\gamma).$$

First, we know that $q(\theta,\gamma) \geq 0$ (by assumption). Second, we know that $q(\theta,\gamma) \leq \theta$. Suppose that $q(\theta,\gamma) > \theta$ over some interval $I$ of $\Theta$. Over this interval, set $q(\theta,\gamma) = \theta$ and $t(\theta,\gamma) = 0$. This satisfies $\mathrm{IR}_\theta$. It also satisfies local $\mathrm{IC}_\theta$. If $q$ is continuous, then $q = \theta$ at the boundaries of $I$. Local $\mathrm{IC}_\theta$ then still hold since $\theta$ is private optimum. The reduction in $q$ over the interval $I$ increases the value of the objective function. Hence, $0 \leq q(\theta,\gamma) \leq \theta$.

This implies that $dU(\theta)/d\theta \geq 0$ if locality 2 has rights, which means that $\mathrm{IR}_\theta$ is binding at $\underline{\theta}$. If locality 1 has rights, $dU(\theta)/d\theta = q(\theta,\gamma) - \theta \leq 0$ and $\mathrm{IR}_\theta$ is binding at $\bar{\theta}$. We can then write

$$U(\theta) = U(\hat{\theta}) + \int_{\hat{\theta}}^{\theta}(q(x,\gamma) - \bar{q}(i,x,\gamma))dx$$

where $\hat{\theta} = \underline{\theta}$ if $i = 2$, and $\hat{\theta} = \bar{\theta}$ if $i = 1$. Since $\mathrm{IR}_\theta$ is binding at $\hat{\theta}$, we have $U(\hat{\theta}) = 0$.

Transfers are then

$$t(\theta,\gamma) = \int_{\hat{\theta}}^{\theta}(q(x,\gamma) - \bar{q}(i,x,\gamma))dx - \theta q(\theta,\gamma) + q(\theta,\gamma)^2/2 + u_1(\bar{q}(i,\theta,\gamma),\theta,0). \tag{A1}$$

[14] *Mathematica* is a registered trademark of Wolfram Research, Inc.

We can substitute into locality 2's objective function to obtain

$$\int_{\underline{\theta}}^{\bar{\theta}}\left[-\gamma q(\theta,\gamma)-\int_{\hat{\theta}}^{\theta}(q(x,\gamma)-\bar{q}(i,x,\gamma))dx+\theta q(\theta,\gamma)-q(\theta,\gamma)^2/2\right.$$
$$\left.-\,u_1(\bar{q}(i,\theta,\gamma),\theta,0)\right]f(\theta)d\theta.$$

This simplifies to

$$\int_{\underline{\theta}}^{\bar{\theta}}[(\theta-\gamma)q(\theta,\gamma)-q(\theta,\gamma)^2/2-u_1(\bar{q}(i,\theta,\gamma),\theta,0)]f(\theta)d\theta$$
$$+\int_{\underline{\theta}}^{\bar{\theta}}\left[-f(\theta)\cdot\int_{\hat{\theta}}^{\theta}(q(x,\gamma)-\bar{q}(i,x,\gamma))dx\right]d\theta.$$

We integrate by parts the last integral of the objective function with

$$u=\operatorname{sign}\{\theta-\hat{\theta}\}\int_{\hat{\theta}}^{\theta}(q(x,\gamma)-\bar{q}(i,x,\gamma))dx \text{ and } u'$$
$$=\operatorname{sign}\{\theta-\hat{\theta}\}(q(\theta,\gamma)-\bar{q}(i,\theta,\gamma)),$$

and

$$v=1-F(\hat{\theta})-\operatorname{sign}\{\theta-\hat{\theta}\}F(\theta)\quad\text{and}\quad v'=-\operatorname{sign}\{\theta-\hat{\theta}\}f(\theta).$$

The last integral is then equal to

$$\left\{\operatorname{sign}\{\theta-\hat{\theta}\}\int_{\hat{\theta}}^{\theta}(q(x,\gamma)-\bar{q}(i,x,\gamma))dx\cdot(1-F(\hat{\theta})-\operatorname{sign}\{\theta-\hat{\theta}\}F(\theta))\right\}_{\theta=\underline{\theta}}^{\theta=\bar{\theta}}$$
$$-\int_{\underline{\theta}}^{\bar{\theta}}\operatorname{sign}\{\theta-\hat{\theta}\}(q(\theta,\gamma)-\bar{q}(i,\theta,\gamma))(1-F(\hat{\theta})-\operatorname{sign}\{\theta-\hat{\theta}\}F(\theta))d\theta.$$

If $\hat{\theta}=\underline{\theta}$, then the first term vanishes. It does as well when $\hat{\theta}=\bar{\theta}$. Locality 2's objective function reduces to

$$\int_{\underline{\theta}}^{\bar{\theta}}\left[(\theta-\gamma)q(\theta,\gamma)-q(\theta,\gamma)^2/2-u_1(\bar{q}(i,\theta,\gamma),\theta,0)\right.$$
$$\left.-\operatorname{sign}\{\theta-\hat{\theta}\}(q(\theta,\gamma)-\bar{q}(i,\theta,\gamma))\frac{1-F(\hat{\theta})-\operatorname{sign}\{\theta-\hat{\theta}\}F(\theta)}{f(\theta)}\right]f(\theta)d\theta$$

Pointwise maximization for each $\theta$ type yields

$$q(\theta,\gamma)=\min\left\{\theta,\max\left\{0,\theta-\gamma-\operatorname{sign}\{\theta-\hat{\theta}\}\frac{1-F(\hat{\theta})-\operatorname{sign}\{\theta-\hat{\theta}\}F(\theta)}{f(\theta)}\right\}\right\},$$

where the min and max operators account for the corner solutions of $\theta$ and 0.

(i) If locality 2 has rights, we have $\hat{\theta}=\underline{\theta}$, and

$$q_{D_{22}}(\theta,\gamma)=\max\left\{0,\theta-\gamma-\frac{1-F(\theta)}{f(\theta)}\right\}.$$

Under our uniform assumption, this simplifies to $q_{D_{22}}(\theta,\gamma)=\max\{0,2\theta-\gamma-\bar{\theta}\}=\max\{0,2\theta-\gamma-\mathbb{E}\theta-\sqrt{3}\sigma_\theta\}<\theta$. Using the expression for transfers (A1), we obtain:

$$\begin{aligned}t_{D_{22}}(\theta,\gamma)&=\int_{\underline{\theta}}^{\theta}q_{D_{22}}(x,\gamma)dx-\theta q_{D_{22}}(\theta,\gamma)+q_{D_{22}}(\theta,\gamma)^2/2=\\&\left(\int_{\underline{\theta}}^{\theta}q_{D_{22}}(x,\gamma)dx-(\theta-\underline{\theta})q_{D_{22}}(\theta,\gamma)/2\right)\\&+\left(q_{D_{22}}(\theta,\gamma)^2/2-\theta q_{D_{22}}(\theta,\gamma)/2\right)-\underline{\theta}\,q_{D_{22}}(\theta,\gamma)/2\le 0.\end{aligned}$$

The inequality follows since $q_{D_{22}}(\theta,\gamma)$ weakly increasing in $\theta$ and $0\le q_{D_{22}}(\theta,\gamma)\le\theta$ imply that the two expressions in parentheses are nonpositive.

We can now compute the expected social welfare loss of this regime compared to first-best expected social welfare. It is easy to show that under our quadratic formulation, the expected social welfare loss of any regime is given by

$$\mathbb{E}_{\theta,\gamma}\{(\theta-\gamma-q_{D_{22}}(\theta,\gamma))^2/2\}.$$

(ii) If locality 1 has rights, we have $\hat{\theta}=\bar{\theta}$, and

$$q_{D_{12}}(\theta,\gamma)=\min\left\{\theta,\theta-\gamma+\frac{F(\theta)}{f(\theta)}\right\}.$$

Under our uniform assumption, this simplifies to $q_{D_{12}}(\theta,\gamma)=\min\{\theta,2\theta-\gamma-\underline{\theta}\}=\min\{\theta,2\theta-\gamma-\mathbb{E}\theta+\sqrt{3}\sigma_\theta\}>0$. Using the expression for transfers (A1), we obtain:

$$\begin{aligned}t_{D_{12}}(\theta,\gamma)=&\int_{\theta}^{\theta}(x-q_{D_{12}}(x,\gamma))dx-\theta q_{D_{12}}(\theta,\gamma)+q_{D_{12}}(\theta,\gamma)^2/2\\&+\theta^2/2\ge 0.\end{aligned}$$

The expected social welfare loss when locality 2 has bargaining power and locality 1 has rights is

$$SWL_{D_{12}} = \mathbb{E}_{\theta,\gamma}\{(\theta - \gamma - q_{D_{12}}(\theta,\gamma))^2/2\}. \qquad \square$$

*Proof of Proposition* 4.3. Define

$$V(\gamma) = -\gamma q(\theta,\gamma) - t(\theta,\gamma) \quad \forall\ \theta,\gamma.$$

Using $\mathrm{IC}_\gamma$, we have

$$\frac{dV(\gamma)}{d\gamma} = -q(\theta,\gamma).$$

Since we assume that $0 \leq q(\theta,\gamma)$, $V$ is decreasing, and hence $\mathrm{IR}_\gamma$ is binding at $\gamma = \overline{\gamma}$. We can then write

$$V(\gamma) = V(\overline{\gamma}) - \int_\gamma^{\overline{\gamma}} - q(\theta,x)dx,$$

with $V(\overline{\gamma}) = 0$ by $\mathrm{IR}_\gamma$. Transfers are

$$t(\theta,\gamma) = -\gamma q(\theta,\gamma) - \int_\gamma^{\overline{\gamma}} q(\theta,x)dx. \tag{A2}$$

Substituting for transfers, locality 1's objective function becomes

$$\int_{\underline{\gamma}}^{\overline{\gamma}} \left[\theta q(\theta,\gamma) - q(\theta,\gamma)^2/2 - \gamma q(\theta,\gamma) - \int_\gamma^{\overline{\gamma}} q(\theta,x)dx\right] g(\gamma)d\gamma,$$

or, equivalently,

$$\int_{\underline{\gamma}}^{\overline{\gamma}} [(\theta - \gamma)q(\theta,\gamma) - q(\theta,\gamma)^2/2]g(\gamma)d\gamma - \int_{\underline{\gamma}}^{\overline{\gamma}} g(\gamma) \cdot \int_\gamma^{\overline{\gamma}} q(\theta,x)dxd\gamma.$$

We integrate by parts the last integral of the objective function with

$$u = \int_\gamma^{\overline{\gamma}} q(\theta,x)dx \quad \text{and} \quad u' = -q(\theta,\gamma),$$

and

$$v = G(\gamma) \quad \text{and} \quad v' = g(\gamma).$$

The last integral is then equal to

$$\left\{\int_\gamma^{\overline{\gamma}} q(\theta,x)dx \cdot G(\gamma)\right\}_{\gamma=\underline{\gamma}}^{\overline{\gamma}} - \int_{\underline{\gamma}}^{\overline{\gamma}} - q(\theta,\gamma)G(\gamma)d\gamma.$$

The first term vanishes to 0. Locality 1's objective function reduces to

$$\int_{\underline{\gamma}}^{\overline{\gamma}} [(\theta-\gamma)q(\theta,\gamma) - q(\theta,\gamma)^2/2]g(\gamma)d\gamma - \int_{\underline{\gamma}}^{\overline{\gamma}} q(\theta,\gamma)G(\gamma)d\gamma,$$

that simplifies to

$$\int_{\underline{\gamma}}^{\overline{\gamma}} \left[\left(\theta-\gamma-\frac{G(\gamma)}{g(\gamma)}\right)q(\theta,\gamma) - q(\theta,\gamma)^2/2\right]g(\gamma)d\gamma.$$

Pointwise maximization for each $\gamma$ type yields

$$q_{D_{21}}(\theta,\gamma) = \max\left\{0, \theta-\gamma-\frac{G(\gamma)}{g(\gamma)}\right\}.$$

Under our uniform assumption, this simplifies to $q_{D_{21}}(\theta,\gamma) = \max\{0, \theta-2\gamma+\underline{\gamma}\} = \max\{0, \theta-2\gamma+\mathbb{E}\gamma-\sqrt{3}\sigma_\gamma\} \leq \theta$. Using the expression for transfers (A2), we obtain:

$$t_{D_{21}}(\theta,\gamma) = -\gamma q_{D_{21}}(\theta,\gamma) - \int_{\gamma}^{\overline{\gamma}} q_{D_{21}}(\theta,x)dx \leq 0.$$

The expected social welfare loss when locality 1 has bargaining power and locality 2 has rights is

$$SWL_{D_{21}} = \mathbb{E}_{\theta,\gamma}\{(\theta-\gamma-q_{D_{21}}(\theta,\gamma))^2/2\}. \qquad \square$$

*Proof of Proposition* 4.4. Define

$$V(\theta,\gamma) = -\gamma q(\theta,\gamma) - t(\theta,\gamma) + \gamma\theta \quad \forall \ \theta,\gamma.$$

$IC_\gamma$ implies that

$$\frac{dV(\theta,\gamma)}{d\gamma} = -q(\theta,\gamma) + \theta.$$

The assumption that $0 \leq q(\theta,\gamma) \leq \theta$ implies that $V(\theta,\cdot)$ is increasing. We can then write

$$V(\theta,\gamma) = V(\theta,\underline{\gamma}) + \int_{\underline{\gamma}}^{\gamma} (\theta - q(\theta,x))dx.$$

Transfers are

$$t(\theta,\gamma) = -\gamma q(\theta,\gamma) + \gamma\theta - V(\theta,\underline{\gamma}) - \int_{\underline{\gamma}}^{\gamma} (\theta - q(\theta,x))dx.$$

We can substitute for transfers in the expected payoff of locality 1.

$$\int_{\underline{\gamma}}^{\overline{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma$$
$$= \int_{\underline{\gamma}}^{\overline{\gamma}} \left[(\theta-\gamma)q(\theta,\gamma) - q(\theta,\gamma)^2/2 + \gamma\theta - \int_{\underline{\gamma}}^{\gamma}(\theta - q(\theta,x))dx\right] g(\gamma)d\gamma$$
$$- V(\theta,\underline{\gamma}).$$

Totally differentiating this equation yields

$$\frac{d\int_{\underline{\gamma}}^{\overline{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma}{d\theta}$$
$$= \int_{\underline{\gamma}}^{\overline{\gamma}} \left[q(\theta,\gamma) + (\theta-\gamma-q(\theta,\gamma))\frac{dq(\theta,\gamma)}{d\theta} + \gamma - \int_{\underline{\gamma}}^{\gamma}\left(1 - \frac{dq(\theta,x)}{d\theta}\right)dx\right] g(\gamma)d\gamma - \frac{dV(\theta,\underline{\gamma})}{d\theta}.$$

The envelope condition from $\mathrm{IIC}_\theta$ is

$$\frac{d\int_{\underline{\gamma}}^{\overline{\gamma}} u_1(q(\theta,\gamma),\theta,t(\theta,\gamma))g(\gamma)d\gamma}{d\theta} = \int_{\underline{\gamma}}^{\overline{\gamma}} q(\theta,\gamma)g(\gamma)d\gamma.$$

This implies that

$$\int_{\underline{\gamma}}^{\overline{\gamma}} \left[(\theta-\gamma-q(\theta,\gamma))\frac{dq(\theta,\gamma)}{d\theta} + \gamma - \int_{\underline{\gamma}}^{\gamma}\left(1 - \frac{dq(\theta,x)}{d\theta}\right)dx\right] g(\gamma)d\gamma$$
$$= \frac{dV(\theta,\underline{\gamma})}{d\theta},$$

or, equivalently,

$$\int_{\underline{\gamma}}^{\overline{\gamma}} \left[(\theta-\gamma-q(\theta,\gamma))\frac{dq(\theta,\gamma)}{d\theta} + \underline{\gamma} + \int_{\underline{\gamma}}^{\gamma}\frac{dq(\theta,x)}{d\theta}dx\right] g(\gamma)d\gamma = \frac{dV(\theta,\underline{\gamma})}{d\theta}.$$

Integrating the last term by parts yields

$$\int_{\underline{\gamma}}^{\overline{\gamma}} \left[(\theta-\gamma-q(\theta,\gamma))\frac{dq(\theta,\gamma)}{d\theta} + \underline{\gamma} + \frac{(1-G(\gamma))}{g(\gamma)}\frac{dq(\theta,\gamma)}{d\theta}\right] g(\gamma)d\gamma$$
$$= \frac{dV(\theta,\underline{\gamma})}{d\theta}.$$

This simplifies to

$$\int_{\underline{\gamma}}^{\overline{\gamma}} \left[ \left( \theta - \gamma - q(\theta, \gamma) + \frac{(1 - G(\gamma))}{g(\gamma)} \right) \frac{dq(\theta, \gamma)}{d\theta} + \underline{\gamma} \right] g(\gamma) d\gamma = \frac{dV(\theta, \underline{\gamma})}{d\theta}.$$

Using the uniform distribution for $\gamma$, we get

$$\frac{1}{\overline{\gamma} - \underline{\gamma}} \cdot \int_{\underline{\gamma}}^{\overline{\gamma}} \left[ (\theta - \gamma - q(\theta, \gamma) + \overline{\gamma} - \gamma) \frac{dq(\theta, \gamma)}{d\theta} + \underline{\gamma} \right] d\gamma = \frac{dV(\theta, \underline{\gamma})}{d\theta}.$$

Observe that ($\mathrm{IR}_{\underline{\gamma}}$) implies $V(\theta, \underline{\gamma}) = 0$ for all $\theta$. Given this, one solution to the differential equation is

$$q(\theta, \gamma) = \theta + \underline{\gamma} + \overline{\gamma} - 2\gamma + \underline{\gamma} \cdot \text{ ProductLog}\left( \frac{-e^{-\theta/\underline{\gamma}} C(\gamma)}{\underline{\gamma}} \right),$$

where $C(\gamma)$ is some function of $\gamma$, and where $w = \text{ProductLog}(z)$ is the principal real-valued solution to $we^w = z$. We need to choose the function $C$ such that it solves locality 1's optimization problem. Optimization implies that $q(\underline{\theta}, \overline{\gamma}) = \underline{\theta} - \overline{\gamma}$, that is, there is no distortion at the top. For this to hold, we have that

$$C(\gamma) = \underline{\gamma}\, e^{\frac{\underline{\theta} - \gamma}{\underline{\gamma}}}.$$

Substituting for $C$ yields

$$q_{D_{11}}(\theta, \gamma) = \theta + \underline{\gamma} + \overline{\gamma} - 2\gamma + \underline{\gamma} \cdot \text{ ProductLog}(-e^{-(\theta + \underline{\gamma} - \underline{\theta})/\underline{\gamma}}).$$

Using our uniform distributions, this may be rewritten as

$$\begin{aligned} q_{D_{11}}(\theta, \gamma) &= \theta - \gamma + \frac{1 - G(\gamma)}{g(\gamma)} + \underline{\gamma}(1 + \text{ProductLog}(-e^{-(\theta + \underline{\gamma} - \underline{\theta})/\underline{\gamma}})) \\ &= \theta - 2\gamma + 2\mathbb{E}\gamma + (\mathbb{E}\gamma - \sqrt{3}\sigma_\gamma)\left( 1 + \text{ProductLog}\left( -e^{-\frac{\theta + \mathbb{E}\gamma - \mathbb{E}\theta - \sqrt{3}\sigma_\gamma + \sqrt{3}\sigma_\theta}{\mathbb{E}\gamma - \sqrt{3}\sigma_\gamma}} \right) \right). \end{aligned}$$

This solution is valid only for parameter values such that $q_{D_{11}}(\theta, \gamma) \leq \theta$ for all $\theta$. This is satisfied when

$$\overline{\gamma} - \underline{\gamma} + \underline{\gamma} \cdot \text{ ProductLog}(-e^{-(\overline{\theta} - \underline{\theta} + \underline{\gamma})/\underline{\gamma}}) \leq 0. \tag{A3}$$

One can further show that (A3) is equivalent to the assumed inequality in the statement of the proposition. Finally, the expected social welfare loss is

$$SWL_{D_{11}} = \mathbb{E}_{\theta,\gamma}\{(\theta - \gamma - q_{D_{11}}(\theta, \gamma))^2/2\}. \qquad \square$$

*Proof of Theorem* 5.1. We begin by computing the following expressions for expected social welfare loss under the various regimes (using the expressions stated in Propositions 4.1, 4.2, 4.3, and 4.4 substituting in for the optimal project levels and simplifying):

$$
\begin{aligned}
SWL_C(J^*) &= \frac{\sigma_\theta^2}{2(J^*)^2} + \frac{\sigma_\gamma^2}{2} + (\mathbb{E}\gamma)^2 \frac{((J^*)^2-1)}{6} \\
SWL_{D_{11}}^{nb} &= -\frac{\underline{\gamma}^2\left(\text{ProductLog}\left[-e^{-\frac{\underline{\gamma}+\bar{\theta}-\underline{\theta}}{\underline{\gamma}}}\right]+1\right)^2\left(3\bar{\gamma}+2\underline{\gamma}\,\text{ProductLog}\left[-e^{-\frac{\underline{\gamma}+\bar{\theta}-\underline{\theta}}{\underline{\gamma}}}\right]+2\underline{\gamma}\right)}{12(\bar{\theta}-\underline{\theta})} \\
&\quad +2\sigma_\gamma^2 + \frac{\bar{\gamma}\,\underline{\gamma}}{2} \\
SWL_{D_{12}}^{nb} &= \frac{\mathbb{E}((\mathbb{E}\theta-\theta)-(\bar{\theta}-\underline{\theta})/2)^2}{2} = \frac{\sigma_\theta^2}{2} + \frac{(\bar{\theta}-\underline{\theta})^2}{8} = 2\sigma_\theta^2 \\
SWL_{D_{22}}^{nb} &= \frac{\mathbb{E}((\mathbb{E}\theta-\theta)+(\bar{\theta}-\underline{\theta})/2)^2}{2} = \frac{\sigma_\theta^2}{2} + \frac{(\bar{\theta}-\underline{\theta})^2}{8} = 2\sigma_\theta^2 \\
SWL_{D_{21}}^{nb} &= \frac{\mathbb{E}((\gamma-\mathbb{E}\gamma)-(\bar{\gamma}-\underline{\gamma})/2)^2}{2} = \frac{\sigma_\gamma^2}{2} + \frac{(\bar{\gamma}-\underline{\gamma})^2}{8} = 2\sigma_\gamma^2 \\
SWL_{D_{21}}^{b} &= \frac{1}{96(\bar{\gamma}-\underline{\gamma})}(16\bar{\gamma}^3 - (24\bar{\gamma}^2 - 6\underline{\gamma}^2)(\bar{\theta}+\underline{\theta}) \\
&\quad + (16\bar{\gamma}-4\underline{\gamma})(\bar{\theta}^2+\bar{\theta}\,\underline{\theta}+\underline{\theta}^2) \\
&\quad -4\underline{\gamma}^3 - 3(\bar{\theta}^3+\bar{\theta}^2\underline{\theta}+\bar{\theta}\,\underline{\theta}^2+\underline{\theta}^3)) \\
SWL_{D_{12}}^{b} &= \frac{2(\bar{\gamma}^3-\underline{\gamma}^3)(\bar{\theta}-\underline{\theta})-(\bar{\gamma}^4-\underline{\gamma}^4)}{12(\bar{\gamma}-\underline{\gamma})(\bar{\theta}-\underline{\theta})} \\
SWL_{D_{22}}^{b} &= \frac{1}{96(\bar{\theta}-\underline{\theta})}((\bar{\gamma}^2+\underline{\gamma}^2)(3(\bar{\gamma}+\underline{\gamma})+4(\bar{\theta}-4\underline{\theta}))+4\bar{\gamma}\,\underline{\gamma}(\bar{\theta}-4\underline{\theta}) \\
&\quad -6(\bar{\gamma}+\underline{\gamma})(\bar{\theta}^2-4\underline{\theta}^2)+4(\bar{\theta}^3-4\underline{\theta}^3)).
\end{aligned}
$$

Consider the case where $c_{12}^{nb}$ holds. Under this assumption, we first prove that the optimal policy under centralization is uniform (i.e., $J^*=1$). We know from Proposition 4.1 that $J^* = \lfloor 1/2 + \sqrt{1/4 + (\bar{\theta}-\underline{\theta})/(2\mathbb{E}\gamma)} \rfloor$. This implies that $J^*=1$ if and only if $1 \le 1/2 + \sqrt{1/4 + (\bar{\theta}-\underline{\theta})/(2\mathbb{E}\gamma)} < 2$. The first inequality is always satisfied since $(\bar{\theta}-\underline{\theta})/(2\mathbb{E}\gamma) > 0$. The second inequality reduces to $\bar{\theta}-\underline{\theta} \le 4\mathbb{E}\gamma$ which, under uniform distribution, is equivalent to $\mathbb{E}\gamma \ge \sqrt{3}\sigma_\theta/2$. Condition $c_{12}^{nb}$ says $\mathbb{E}\gamma - 2\sqrt{3}\sigma_\theta \ge \sqrt{3}\sigma_\gamma > 0$. So, if condition $c_{12}^{nb}$ is satisfied, $J^*=1$.

When $c_{11}^{nb}$ and $c_{12}^{nb}$ hold, there are exactly three nonempty subcases: (i) $c_{21}^{nb}$ and $c_{22}^{nb}$ hold; (ii) $c_{21}^{nb}$ and $c_{22}^{b}$ hold; and (iii) $c_{21}^{b}$ and $c_{22}^{nb}$ hold.

For (i), observe that $SWL_C(1) < \min\left\{SWL_{D_{12}}^{nb}, SWL_{D_{21}}^{nb}, SWL_{D_{22}}^{nb}\right\}$ if and only if $1/3 < \sigma_\theta^2/\sigma_\gamma^2 < 3$. Furthermore, $SWL_{D_{21}}^{nb} < \min\left\{SWL_C(1), SWL_{D_{12}}^{nb}, SWL_{D_{22}}^{nb}\right\}$ if and only if $\sigma_\theta^2/\sigma_\gamma^2 > 3$; and $SWL_{D_{12}}^{nb} = SWL_{D_{22}}^{nb} < \min\left\{SWL_C(1), SWL_{D_{21}}^{nb}\right\}$ if and only if $\sigma_\theta^2/\sigma_\gamma^2 < 1/3$.

To complete the proof for subcase (i), we show that $SWL_{D_{21}}^{nb} < SWL_{D_{11}}^{nb}$. One can show that there is an expression with the same sign as $SWL_{D_{21}}^{nb} - SWL_{D_{11}}^{nb}$ depending only on two variables: $y := (\bar{\gamma}-\underline{\gamma})/\underline{\gamma}$ and $z := (\bar{\theta}-\underline{\theta})/\underline{\gamma}$. Given $c_{11}^{nb}$ holds, we know that $(\bar{\gamma}-\underline{\gamma}) < \underline{\gamma}$, so that $0 < y < 1$. Next, we observe that, for all $0 < y < 1$, $SWL_{D_{21}}^{nb} - SWL_{D_{11}}^{nb} = 0$ when $z = 0$. Furthermore, this difference is decreasing in $z$ so that it is negative for all positive $z$. This implies that $SWL^{nb}_{D_{21}} < SWL^{nb}_{D_{11}}$. Note that this result is always valid when the conditions $c^{nb}_{11}$ and $c^{nb}_{21}$ are satisfied.

For (ii), the argument just above for $SWL^{nb}_{D_{21}} < SWL^{nb}_{D_{11}}$ remains true. Furthermore, the combination of $c^{nb}_{21}$ and $c^{b}_{22}$ holding implies that $\sigma^2_\theta/\sigma^2_\gamma > 3$. Finally, for this subcase one can verify that $SWL^{nb}_{D_{21}} < \min\left\{SWL_C(1), SWL^{nb}_{D_{12}}, SWL^{b}_{D_{22}}\right\}$.

For (iii), observe that $SWL^{nb}_{D_{12}} = SWL^{nb}_{D_{22}}$ so that who has rights does not matter when locality 2 offers. Furthermore, the combination of $c^{b}_{21}$ and $c^{nb}_{22}$ holding implies that $\sigma^2_\theta/\sigma^2_\gamma < 1/3$. Calculation shows that $SWL^{nb}_{D_{12}} < \min\left\{SWL_C(1), SWL^{b}_{D_{21}}\right\}$. To complete the proof for subcase (iii), we show that $SWL^{nb}_{D_{12}} < SWL^{nb}_{D_{11}}$. One can show that there is an expression with the same sign as $SWL^{nb}_{D_{12}} - SWL^{nb}_{D_{11}}$ depending only on two variables: $y := (\bar{\gamma} - \underline{\gamma})/\underline{\gamma}$ and $z := (\bar{\theta} - \underline{\theta})/\underline{\gamma}$. Given $c^{nb}_{11}$ holds, we know that $(\bar{\gamma} - \underline{\gamma}) < \underline{\gamma}$, so that $0 < y < 1$. Given $c^{nb}_{12}$, $c^{b}_{21}$ and $c^{nb}_{22}$ hold, $0 < z \le y/2$. Next, one can show that, for all $0 < y < 1$, the derivative of $SWL^{nb}_{D_{12}} - SWL^{nb}_{D_{11}}$ with respect to $y$ is negative when $0 < z \le y/2$. Furthermore, when $y = 2z$ (i.e., the smallest $y$ in the relevant region), $SWL^{nb}_{D_{12}} - SWL^{nb}_{D_{11}} = 0$ when $z = 0$ and is decreasing in $z$ and is therefore negative for $0 < z \le y/2$. This implies that $SWL^{nb}_{D_{12}} < SWL^{nb}_{D_{11}}$.

Now we turn to the case where $c^{b}_{12}$ holds. When $c^{nb}_{11}$ and $c^{b}_{12}$ hold, there are exactly two nonempty subcases: (iv) $c^{nb}_{21}$ and $c^{b}_{22}$ hold and (v) $c^{nb}_{21}$ and $c^{nb}_{22}$ hold.

For both (iv) and (v), our earlier argument shows that $c^{nb}_{11}$ and $c^{nb}_{21}$ holding implies $SWL^{nb}_{D_{21}} < SWL^{nb}_{D_{11}}$.

For (iv), the combination of $c^{nb}_{21}$ and $c^{b}_{22}$ holding implies that $\sigma^2_\theta/\sigma^2_\gamma > 3$. Finally, for this subcase one can verify that $SWL^{nb}_{D_{21}} < \min\left\{SWL_C(J^*), SWL^{b}_{D_{12}}, SWL^{b}_{D_{22}}\right\}$.

For (v), the combination of $c^{nb}_{11}$ and $c^{b}_{12}$ holding implies that $\sigma^2_\theta/\sigma^2_\gamma > 3$. Finally, for this sub-case one can verify that $SWL^{nb}_{D_{21}} < \min\left\{SWL_C(J^*), SWL^{b}_{D_{12}}, SWL^{nb}_{D_{22}}\right\}$. □

*Proof of Theorem* 5.2. Given that $c^{nb}_{11}$ is violated, the following are the nonempty combinations of $c_{ij}$ conditions: (i) $c^{nb}_{12}$, $c^{nb}_{22}$ and $c^{nb}_{21}$ hold; (ii) $c^{nb}_{12}$, $c^{nb}_{22}$ and $c^{b}_{21}$ hold; (iii) $c^{nb}_{12}$, $c^{b}_{22}$ and $c^{nb}_{21}$ hold; (iv) $c^{b}_{12}$, $c^{b}_{22}$ and $c^{nb}_{21}$ hold; and (v) $c^{b}_{12}$, $c^{nb}_{22}$ and $c^{nb}_{21}$ hold.

For combinations (i), (ii), and (iii), since $c^{nb}_{12}$ holds, the same argument used in the proof of Theorem 5.1 to show that the optimal policy under centralization is uniform, that is, $J^* = 1$, remains valid, and therefore the social welfare loss under centralization is given by $SWL_C(1)$.

Consider combination (i). Comparison of the social welfare loss expressions for the various regimes shows that, when $\sigma^2_\theta/\sigma^2_\gamma > 3$, $SWL^{nb}_{D_{21}}$ is lower than for the other regimes. Similarly, when $\sigma^2_\theta/\sigma^2_\gamma < 1/3$, $SWL^{nb}_{D_{12}} = SWL^{nb}_{D_{22}}$ are lower than for the other regimes, and, when $1/3 < \sigma^2_\theta/\sigma^2_\gamma < 3$, $SWL_C(1)$ is lower than for the other regimes. Finally, under combination (i), calculation shows that $\mathbb{E}\gamma$ is always greater than the middle of the three real roots of the cubic polynomial $x^3 - 3\sqrt{3}\sigma_\theta x^2 + 3\sigma^2_\gamma x + 3\sqrt{3}\sigma^3_\theta$.

Next, consider combination (ii). This combination implies that $\sigma^2_\theta/\sigma^2_\gamma < 1/3$. Comparison of the social welfare loss expressions for the various regimes shows that $SWL^{nb}_{D_{12}} = SWL^{nb}_{D_{22}}$ are lower than for the other regimes.

Combination (iii) implies that $\sigma_\theta^2/\sigma_\gamma^2 > 3$. Comparison of the social welfare loss expressions for the various regimes shows that $SWL_{D_{21}}^{nb}$ is lower than for the other regimes.

For these three combinations (i, ii, and iii), this proves parts (a), (b), and (c) of the theorem. Part (d) does not apply to these combinations.

Next, consider combination (iv). This combination implies that $\sigma_\theta^2/\sigma_\gamma^2 > 3$. Comparison of the social welfare loss expressions for the various regimes (without assuming that $J^* = 1$ under centralization) shows that either $SWL_{D_{21}}^{nb}$ or $SWL_{D_{12}}^{b}$ is lower than for the other regimes. Which one is lower varies depending on specific parameter values. Examples show that either $D_{21}$ or $D_{12}$ can be strictly optimal under these assumptions. If $\underline{\gamma} = 2, \bar{\gamma} = 3, \underline{\theta} = 4$ and $\bar{\theta} = 8$, then $SWL_{D_{21}}^{nb} < SWL_{D_{12}}^{b}$. If $\underline{\gamma} = 1/16, \bar{\gamma} = 17/16, \underline{\theta} = 33/16$ and $\bar{\theta} = 81/16$, then $SWL_{D_{12}}^{b} < SWL_{D_{21}}^{nb}$.

Finally, consider combination (v). Calculation shows that $SWL_{D_{21}}^{nb}$ is strictly lower than $SWL_{D_{22}}^{nb}$, and that, when $J^* \geq 2$, centralization gives an expected social welfare loss strictly greater than under the better of $D_{12}$ and $D_{21}$. Further calculation shows that centralization is strictly optimal if and only if $1/3 < \sigma_\theta^2/\sigma_\gamma^2 < 3$ and $\mathbb{E}\gamma$ is larger than the middle of the three real roots of the cubic polynomial $x^3 - 3\sqrt{3}\sigma_\theta x^2 + 3\sigma_\gamma^2 x + 3\sqrt{3}\sigma_\theta^3$. The latter bound on $\mathbb{E}\gamma$ is also sufficient to ensure that $J^* = 1$. Examples show that either $D_{12}, D_{21}$ or centralization with $J^* = 1$ can be strictly optimal under combination (v). If $\underline{\gamma} = 1/64, \bar{\gamma} = 17/64, \underline{\theta} = 1$ and $\bar{\theta} = 3/2$, then $SWL_{D_{12}}^{b}$ is lowest. If $\underline{\gamma} = 12/64$, $\bar{\gamma} = 28/64, \underline{\theta} = 1$ and $\bar{\theta} = 3/2$, then $SWL_{D_{21}}^{nb}$ is lowest. If $\underline{\gamma} = 8/64, \bar{\gamma} = 32/64, \underline{\theta} = 1$ and $\bar{\theta} = 3/2$, then $SWL_C(1)$ is lowest.

What remains for combinations (iv) and (v) is to prove the comparative statics stated in part (d) of the theorem. To this end, observe that simplification shows

$$SWL_{D_{12}}^{b} - SWL_{D_{21}}^{nb} = \frac{(\mathbb{E}\gamma)^2}{2} - \frac{3\sigma_\gamma^2}{2} - \frac{(\mathbb{E}\gamma)^3}{6\sqrt{3}\sigma_\theta} - \frac{\mathbb{E}\gamma\sigma_\gamma^2}{2\sqrt{3}\sigma_\theta}. \tag{A4}$$

To prove the comparative-static result (d)(i), one can show that the partial derivative of (A4) with respect to $\mathbb{E}\gamma$ is positive when either combination (iv) or combination (v) holds. Result (d)(ii) follows from the fact that $\mathbb{E}\theta$ does not appear in (A4). Result (d)(iii) follows since all terms involving $\sigma_\gamma^2$ are decreasing in that parameter. Finally, result (d)(iv) follows since all terms involving $\sigma_\theta$ are increasing in that parameter. □