Experiments on compound risk in relation to simple risk and to ambiguity

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Web appendix W

W.1

Additional analysis for Study 1

Pattern of moving from seeking to aversion for simple risk when the winning probability increases

Winning probability $\longrightarrow$	**Sample size** n	**1/12**	**1/2**	**11/12**	**Page's L test p-value for within-subject increasing trend**
Average risk premia	94	-5.75	2.35	8.09	0.000***
R presented first	49	-5.68***	0.93	5.93***	0.000***
CR presented first	45	-5.82***	3.90**	10.45***	0.000***

Table 1: **Average risk premia by probability treatment and order effect**

W.2

Additional analysis and robustness checks related to Study 2

Comparison with coarsened data

One difference between our observed certainty equivalents and Halevy's is that ours are observed on a discrete scale with one euro increments while his are to the nearest US cent. The fineness of scale could potentially increase or decrease correlation. Here we provide a comparison with a "coarsened" version of Halevy's data.

Individuals exhibiting slight non-reduction at a finer scale might be classified as reducing at coarser scale, while individuals classified as slightly ambiguity non-neutral might be ambiguity neutral according

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			Compound risk attitudes Reduce all compound risks		
			Reduce	**Do not reduce**	**Total**
Ambiguity attitudes	**Neutral**	Count	25	11	36
		Expected	6.8	29.2	
	Non-neutral	Count	2	104	106
		Expected	20.2	85.8	
Total			27	115	142
Fisher's exact test p-value (2-tailed)			0.000		

Table 2: **Coarsening the Halevy (2007) Data: contingency table**

to coarser scale. To test the effect, if any, we went back to Halevy's data reconstructed his table as it would have appeared if he had used a similarly coarse classification (in theory, the maximum difference in certainty equivalents that our measure might classify as the same is 1 euro, i.e., 2% of 50 euros. Since Halevy's winning prizes were either \$2 or \$20, 2% is \$0.04 and \$0.40 respectively). The result is recorded in Table 2. This does move his data closer to our pooled data, however, the evidence for neutrality given reduction in his data remains extreme – conditional on reducing compound risk, 2 out of 27 subjects are non-neutral toward ambiguity. The evidence for reduction given neutrality is less stark in his data under coarsening – conditional on ambiguity neutrality, 11 out of 36 subjects do not reduce compound risk. Finally, note that these calculations reflect a "worst-case scenario" in coarseness as coarse measurement will typically pick up some of the differences of less than 2%. At worst, the p-values for the results reported after Table 5 increase to $p = 0.13$ and $p = 0.033$ respectively.

Does the type of compound risk matter for the relationship?

A plausible cognitive process behind the hypothesis, as in Halevy (2007) and Halevy and Ozdenoren (2008), that reduction of compound risk implies ambiguity neutrality is the following: when faced with ambiguity, an individual forms a "mental model" of the ambiguity in the form of a compound risk and then evaluates this model as they would an objective compound risk. Given such a hypothesis, an implication is that evaluation of compound risks that are more plausible mental models of a given ambiguous bet should be more strongly related to evaluation of the ambiguity than compound risks that are unlikely to be seen as similar to the ambiguous bet. In both Halevy (2007) and our Study 2, the compound risks used were chosen in large part because they are at least plausible models of the ambiguous bet. In Study 3 (see W.3 for a description), we used a greater variety of compound risks, some of which were far less plausible as mental models of the ambiguity. For example, the compound risks CR low and CR high (see Figure 2 in W.3) would be unlikely to be models of the ambiguous bet. Do we see reduction of these compound risks having a weaker association with ambiguity neutrality? To make this comparison cleanly, we compare the advanced engineering students in Study 2 to those in Study 3.[1] This gives rise to the following contingency table:

[1] As there was only a small proportion of non-engineers in Study 3, and their background was quite different from the non-engineers in Study 2, comparing non-engineers across these studies seems less appropriate. For completeness, we report here that of the non-engineers reducing compound risks, in Study 2 (from Table 6) 6 out of 6 were ambiguity neutral while in Study 3, considering compound risks CR low and CR high, 0 out of 1 were ambiguity neutral.

			Compound risk attitudes					
			Engineers Study 2			Engineers Study 3		
			Reduce means R=CRU=CRD=CRG			Reduce means R=CR low=CR high		
			Reduce	**Do not reduce**	**Total**	**Reduce**	**Do not reduce**	**Total**
Ambiguity attitudes	**Neutral**	Count	7	8	15	3	13	16
		Expected	2.2	12.8		1.8	14.2	
	Non-neutral	Count	4	56	60	3	33	36
		Expected	8.8	51.2		4.2	31.8	
		Total	11	64	75	6	46	52
Fisher's exact test p-value (2-tailed)			0.001			0.357		

Table 3: **Contingency table relating ambiguity and compound risk attitudes in Study 2 vs Study 3, engineers only**

Contingent on reducing the less plausible compound risks, 3 out of 6 engineers in Study 3 are non-neutral to ambiguity. This compares to the ambiguity non-neutral 4 out of 11 engineers in study 2 who reduce compound risks. Thus, the observed proportion moves in the direction predicted by the "less plausible models" hypothesis, but not significantly so (two-sample test of proportions, $p = 0.59$). With regard to the other direction of the association, entailed in Seo (2009), contingent on ambiguity neutrality, 3 out of 16 engineers reduce the less plausible compound risks in Study 3 while 7 out of 15 engineers reduce the compound risks in Study 2 (two-sample test of proportions, $p < 0.1$). This indicates a weaker association with the less plausible compound risks.

Table 4 provides some evidence that comparing across Studies 2 and 3 is reasonable. When considering only the hypergeometric compound risk and the ambiguous bet common across the two studies, both the proportions of engineers reducing the hypergeometric compound risk and the proportion of those reducing who are ambiguity non-neutral are virtually the same across studies. Moreover, equality of the proportion of those who are ambiguity neutral who fail to reduce hypergeometric compound risk is not rejected (two-sample test of proportions, $p = 0.35$).

			Compound risk attitudes					
			Engineers Study 2			Engineers Study 3		
			Reduce means R=CRG					
			Reduce	**Do not reduce**	**Total**	**Reduce**	**Do not reduce**	**Total**
Ambiguity attitudes	**Neutral**	Count	10	5	15	8	8	16
		Expected	4.0	11.0		4.6	11.4	
	Non-neutral	Count	10	50	60	7	29	36
		Expected	16.0	44.0		10.4	25.6	
Total			20	55	75	15	37	52
Fisher's exact test p-value (2-tailed)			0.000			0.044		

Table 4: **Contingency table relating ambiguity and hypergeometric compound risk attitudes in Study 2 vs Study 3, engineers only**

Order effects

Are the results discussed above relating ambiguity and compound risk attitudes sensitive to the order in which bets are presented? To address this, we make use of the fact that subjects in Study 2 were placed in one of six order treatments corresponding to the six possible orderings of simple risk, compound risk and ambiguity. Arguably, since simple risk is not directly involved, the purest measurement of the association between ambiguity and compound risk attitudes comes from the orders in which simple risk is presented last. Thus we begin by examining a pooled contingency table using only these orders (Table 5). Comparing this to the overall results from Study 2 in Table 5, conditional on reducing compound risks, 2 out of 4 subjects are ambiguity neutral versus 13 out of 17 (two-sample test of proportions, $p = 0.29$). Therefore, there is no evidence that our finding of a weaker association than in Halevy (2007) is driven by simple risk interfering with the relationship between ambiguity and compound risk. Furthermore, within the "simple risk last" orderings, we find no evidence that ambiguity first versus compound risk first makes a difference.

			Compound risk attitudes when risk is last		
			Reduce	**Do not reduce**	**Total**
Ambiguity attitudes	**Neutral**	Count	2	7	9
		Expected	1.0	8.0	
	Non-neutral	Count	2	26	28
		Expected	3.0	25.0	
		Total	4	33	37
Fisher's exact test p-value (2-tailed)			0.244		

Table 5: **Contingency table relating ambiguity and compound risk attitudes when simple risk is last, pooled**

Another order effect we investigate is whether orders where compound risk comes before ambiguity are much different than those where ambiguity comes before compound risk. The answer is no, they are not. Specifically, when compound risk comes before ambiguity, conditional on reducing compound risk, 2 of 7 subjects are ambiguity non-neutral, while under the reverse ordering, 2 of 10 are non-neutral (two-sample test of proportions, $p = 0.68$). In the other direction, conditional on ambiguity neutrality, 5 of 13 reduce compound risks, while under the reverse ordering, 8 of 17 reduce (two-sample test of proportions, $p = 0.64$).

The examination of order effects above was targeted precisely at evaluating whether our main findings are artifacts of order. We conclude this section by testing for order effects in the more general sense of effects on the central tendency of reported certainty equivalents. Specifically, for each of the five bets in Study 2, we tested whether the mean and/or median certainty equivalents varied by order and found that none did (all five MANOVA p-values for equality of means are at least 0.69, all five k-sample ranksum p-values for equality of medians are at least 0.33).

W.3

Description of Study 3

Study 3 was conducted for an earlier version of the present paper (Abdellaoui et al. (2011)) prior to Studies 1 and 2. The data from Study 3 is used for some results in Web Appendix W.2.

Sample

64 subjects were recruited, 51 from Arts et Métiers ParisTech (an elite French graduate engineering school) and 13 from a Masters degree program in quantitative economics at Université Paris 1.

Procedure

The procedure was much like that in Studies 1 and 2, thus we describe here mainly the differences. First, subjects in Study 3 participated one at a time and the subject and the experimenter sat in front of a computer together. For each screen where choices were needed, the subject verbally indicated his choices to the experimenter who then entered them onto the screen. Second, subjects were faced with thirty-two bets, including simple risks, ambiguities and compound risks. There were no order treatments in this study, and all subjects saw simple risks first, followed by ambiguities and then compound risks. Also, the color(s) associated with the high (50 euro) payoff varied from bet to bet, were the same for all subjects, and were fixed in advance rather than chosen by subjects. A training phase using the first bet from the category coming next was used at the beginning of each of these three sections of the experiment to check whether subjects had a correct understanding of the design and of the type of uncertainty faced.

We conducted three probability treatments – for most types of uncertainty, certainty equivalents were elicited for 3 different probabilities $(1/12, 1/2, 11/12)$ of winning 50 euros. For ambiguity, instead of probability levels, we varied the fraction of winning colors: $(1/12, 1/2, 11/12)$. The specific simple risk, ambiguous and compound risk bets used for each probability treatment are listed in Tables 6 and 7. The notation $(p,\, 50; 0)$ represents a simple lottery with probability p of winning 50 and $(1-p)$ of winning 0. Similarly, $(q_1, (r_1, 50; 0); ...; q_m, (r_m, 50; 0))$ represents a two-stage compound lottery with first stage probability q_i of a second stage lottery giving 50 with probability r_i and 0 with probability $1 - r_i$. Additionally, $(q, (r, 50; 0); c)$ represents a two-stage compound lottery with first stage probability q of a second stage lottery giving 50 with probability r and 0 with probability $1 - r$ and first stage probability $1-q$ of giving amount $c \in \{0, 50\}$. Finally, $(k\ colors,\ 50; n-k\ colors,\ 0)$ represents a bet on an ambiguous urn that yields 50 if one of k colors is drawn and 0 if one of the other $(n-k)$ colors is drawn. Figures

1-2 present the visual depiction of the full set of bets for the probability level 1/2. Those for the other probability treatments are analogous.

Probability ↓	**Simple risk**		**Ambiguity**	
Urn(s)⟶	12 ball	2 ball	12 ball	2 ball
1/12	(1/12, 50; 0)	-	$(1\ color, 50; 11\ colors, 0)$[a]	-
1/2	(1/2, 50; 0)	(1/2, 50; 0)	$(6\ colors, 50; 6\ colors, 0)$	$(1\ color, 50; 1\ color, 0)$[b]
11/12	(11/12, 50; 0)	-	$(11\ colors, 50; 1\ color, 0)$	-

Table 6: **Simple risk and ambiguity bets**

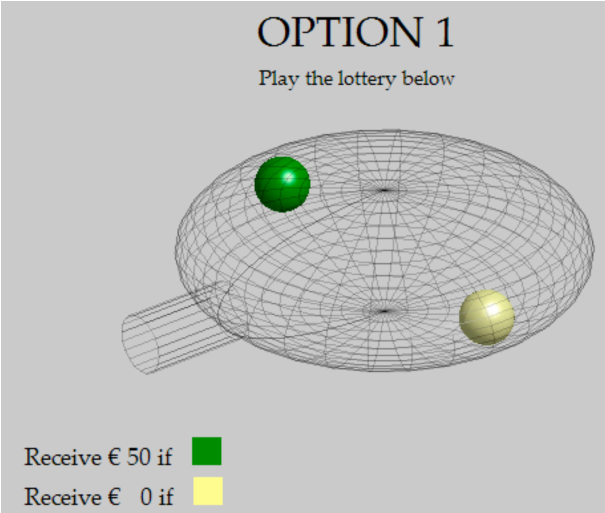
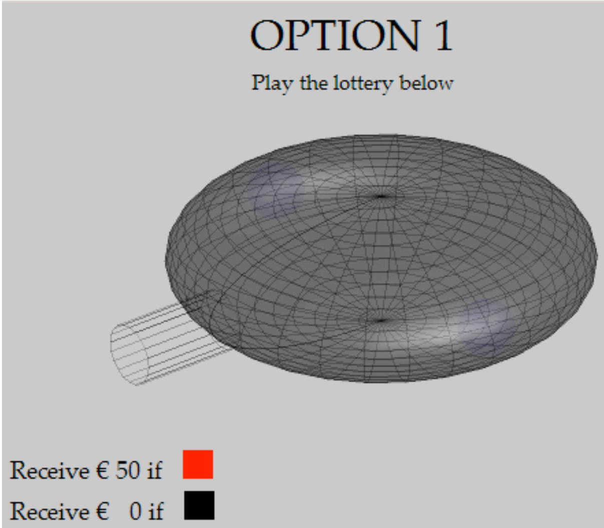
[a]Subjects faced this bet three times with the winning color varied.
[b]Subjects faced this bet two times with the winning color varied.

Probability ↓	**Diverse uniform CR**	**Degenerate uniform CR**	**Hypergeometric CR**
Urn(s)⟶	12 ball in the first stage and 2 ball in the second stage		
1/12	-	-	(1/6, (1/2, 50; 0); 5/6, (0, 50; 0))
1/2	(1/4, (1, 50; 0); 1/4, (1/2, 50; 0); 1/4, (1/2, 50; 0); 1/4, (0, 50; 0))	(1/2, (1, 50; 0); 1/2, (0, 50; 0))	(5/22, (1, 50; 0); 12/22, (1/2, 50; 0); 5/22, (0, 50; 0))
11/12	-	-	(5/6, (1, 50; 0); 1/6, (1/2, 50; 0))

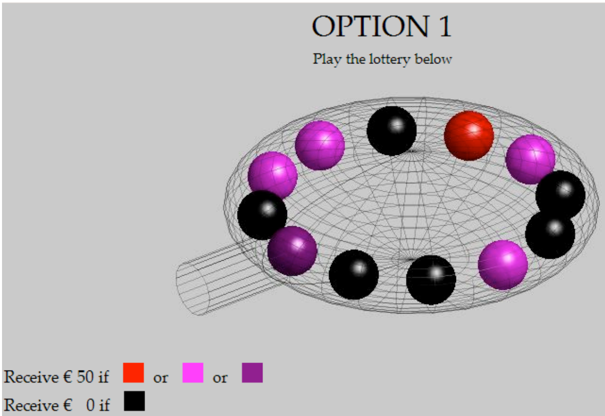
Probability ↓	**CR high**	**CR low**	**CR high with explicit degenerate urn**	**CR low with explicit degenerate urn**
Urn(s)⟶	12 ball in both the first and second stage			
1/12	(1/2, (1/6, 50; 0); 0)	(1/6, (1/2, 50; 0); 0)	(1/2, (1/6, 50); 1/2, (1, 0))	(1/6, (1/2, 50; 0); 5/6, (1, 0))
1/2	(3/4, (2/3, 50; 0); 0)	(2/3, (3/4, 50; 0); 0)	(3/4, (2/3, 50; 0); 1/4, (1, 0))	(2/3, (3/4, 50; 0); 1/3, (1, 0))
11/12	(5/6, (1/2, 50; 0); 50)	(1/2, (5/6, 50; 0); 50)	(5/6, (1/2, 50; 0); 1/6, (1, 50))	(1/2, (5/6, 50; 0); 1/2, (1, 50)

Table 7: **Compound risk bets**

Below, the bets corresponding to probability 1/2 are displayed in urn form (the form in which subjects saw them).

Simple risk, 2 balls	Ambiguity, 2 balls
	

Simple risk, 12 balls



Ambiguity, 12 balls

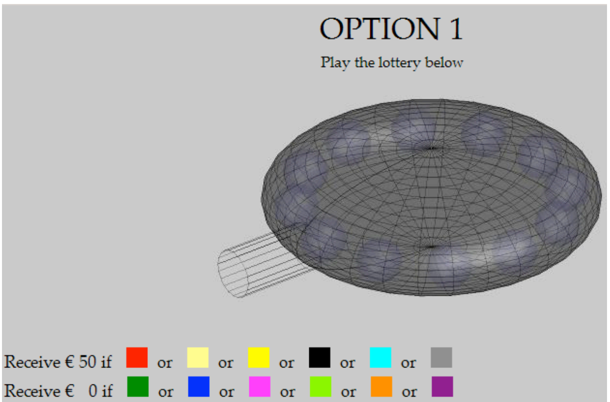
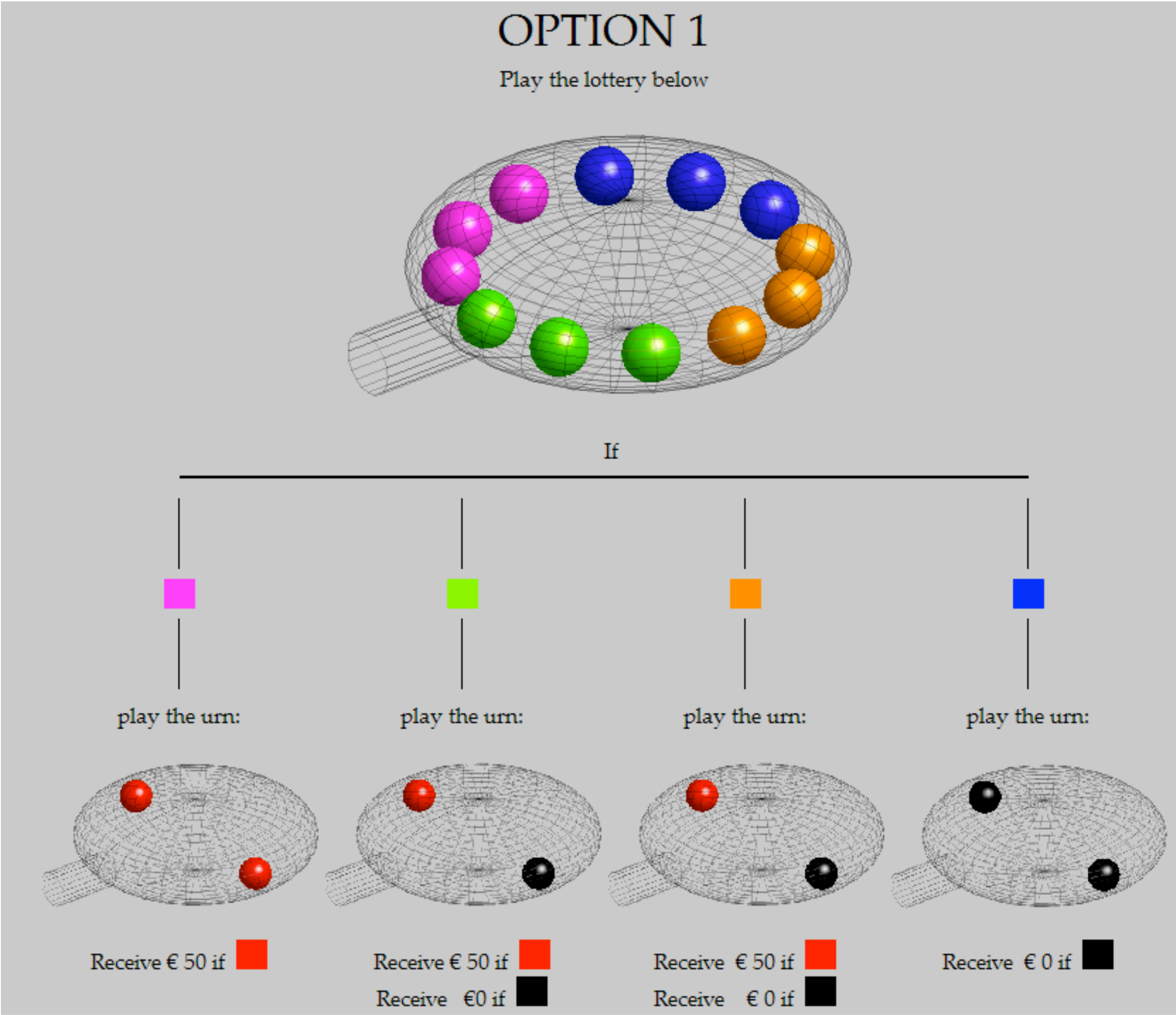
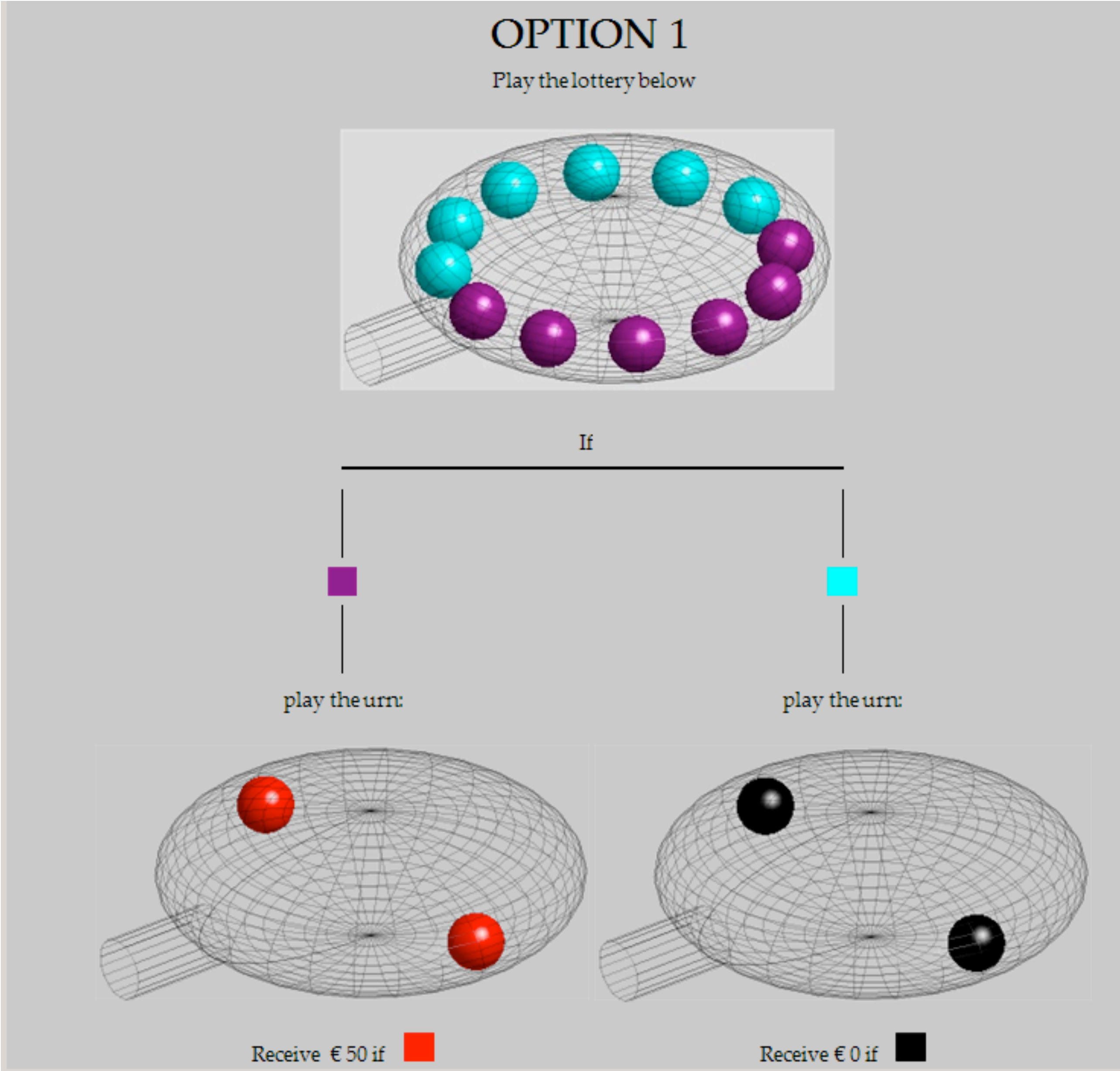


Figure 1: Urn representation of the bets described in Table 6 for probability one-half

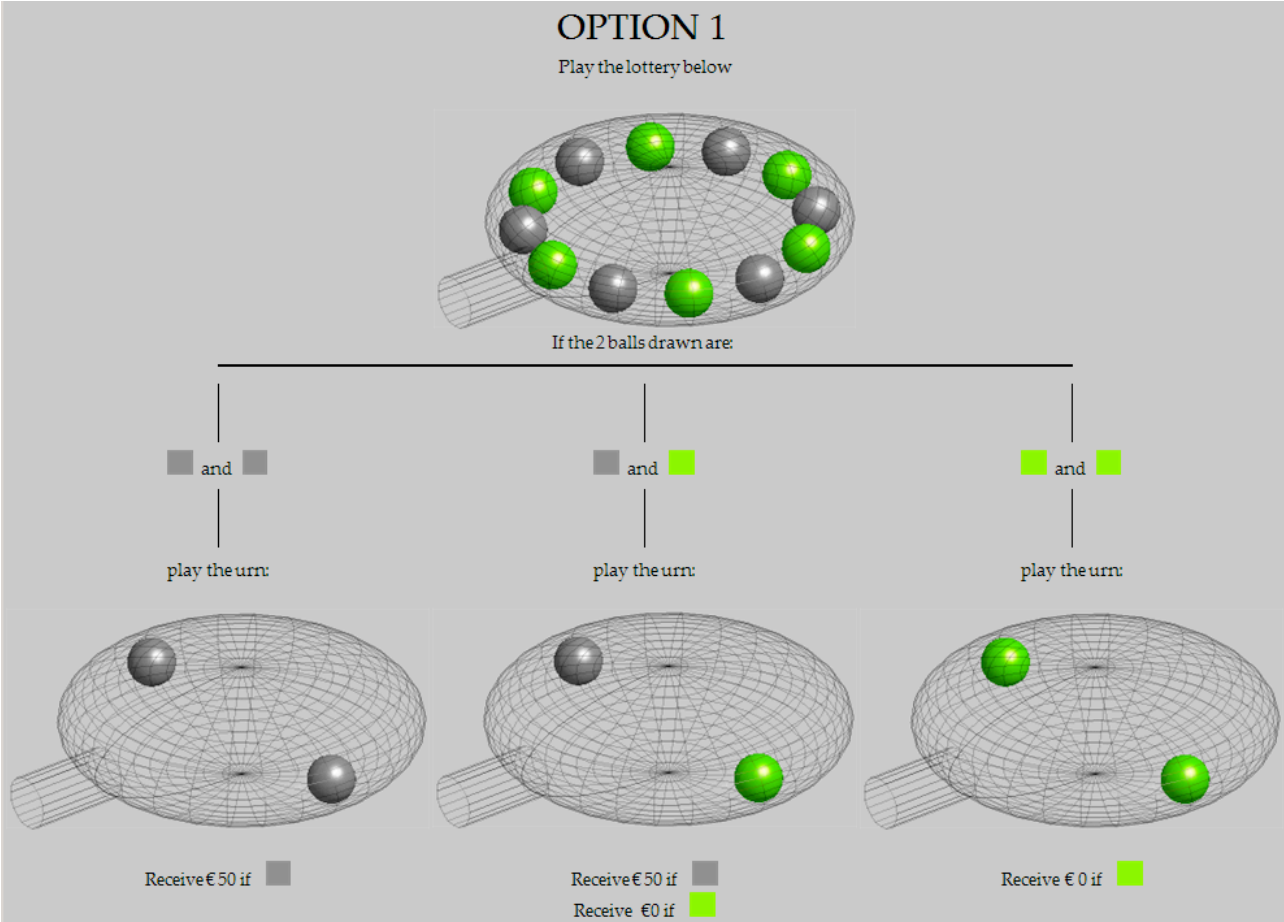
Diverse uniform CR

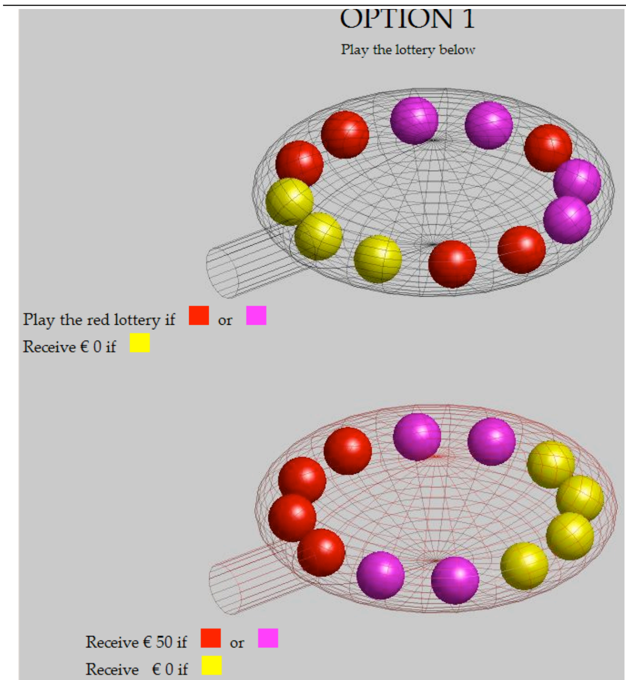
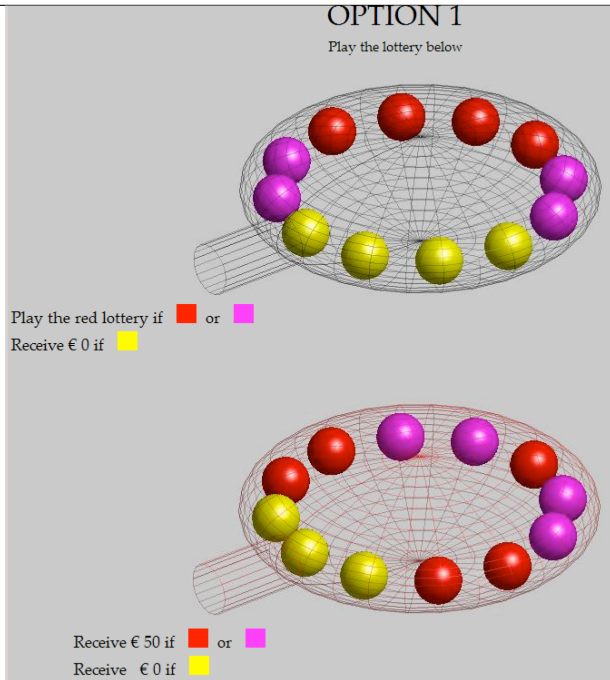


Degenerate uniform CR

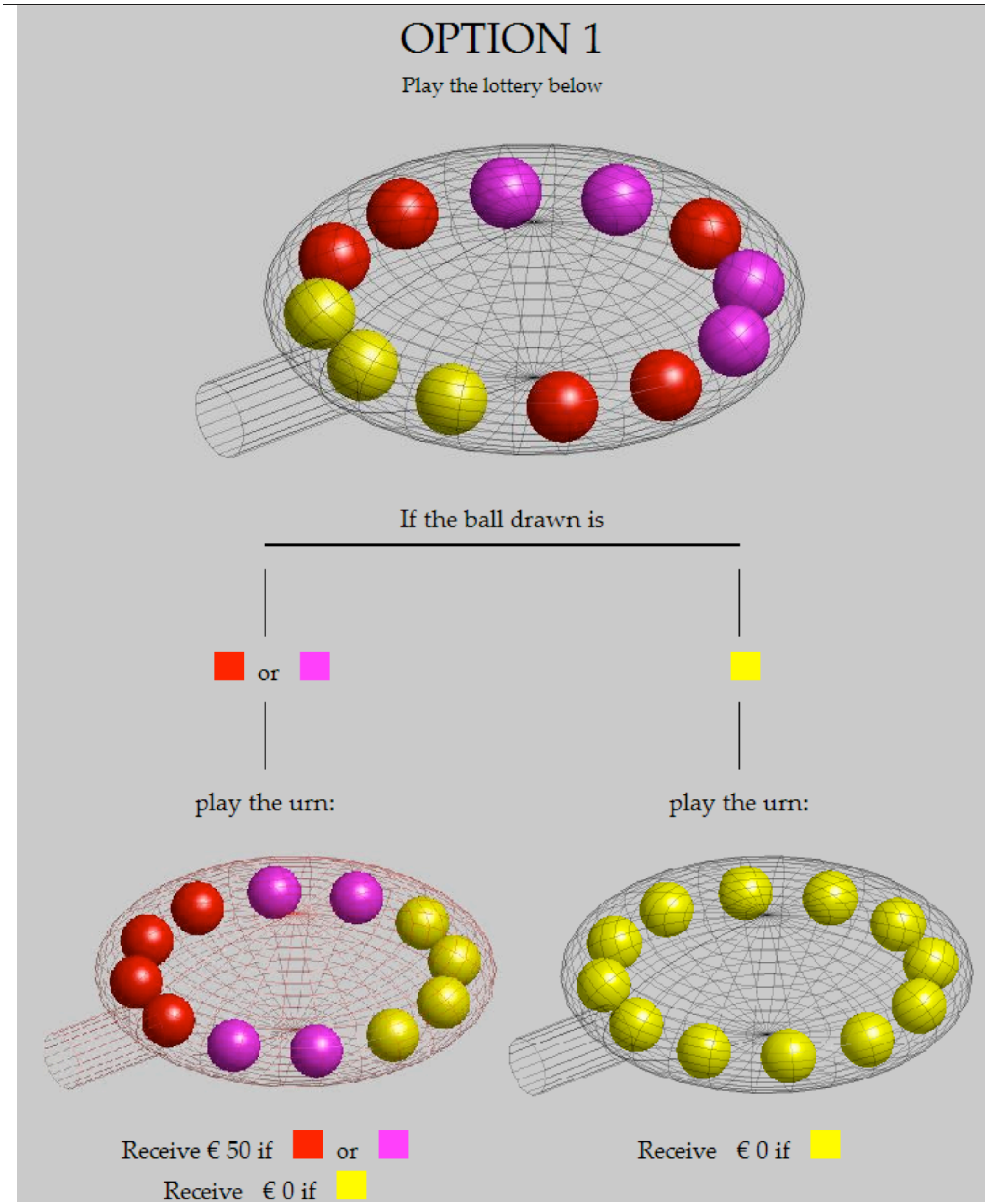


Hypergeometric CR



CR high	CR low
	

CR high with explicit degenerate second stage



CR low with explicit degenerate second stage

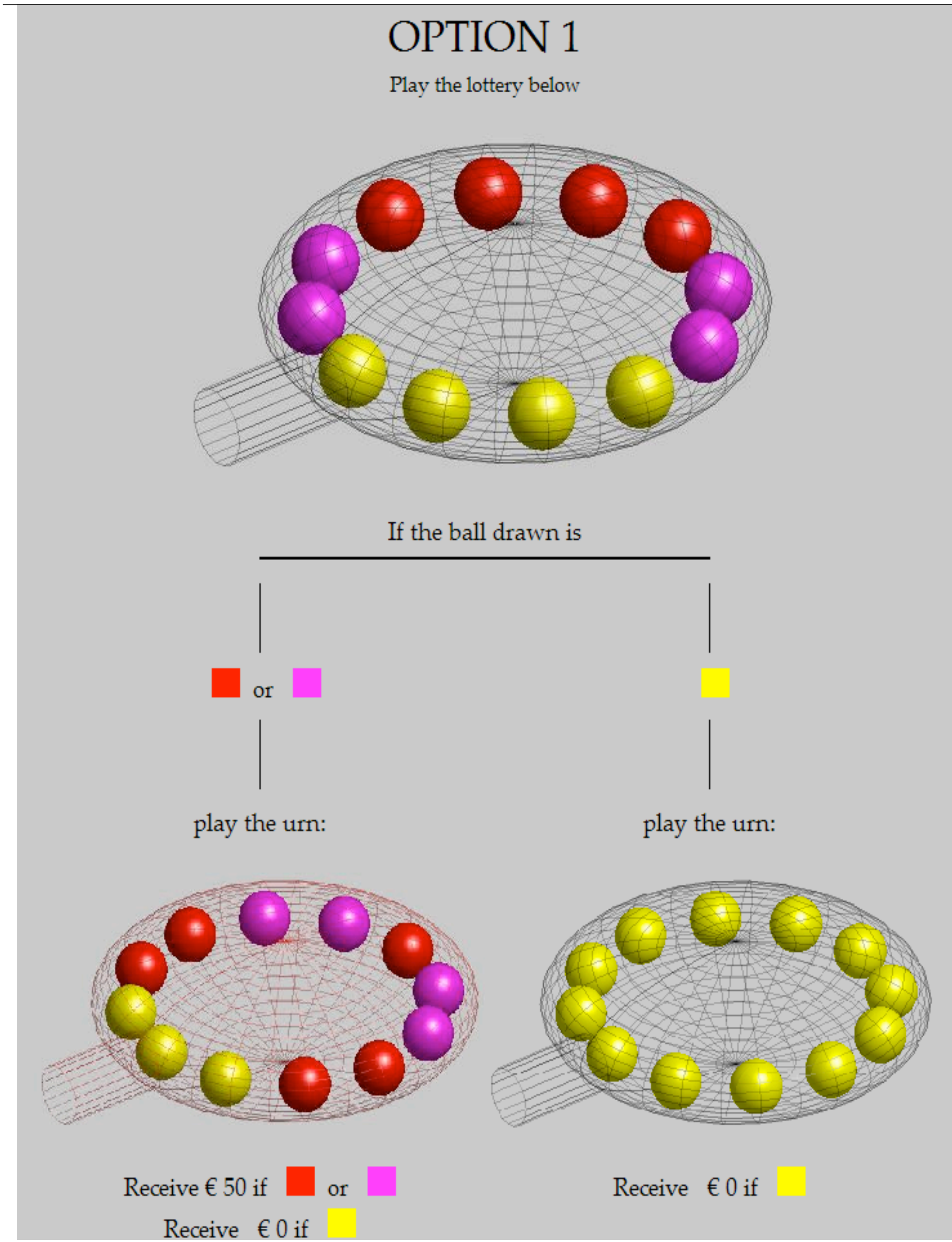


Figure 2: Urn representation of the bets described in Table 7 for probability one-half

References

Abdellaoui, M., Klibanoff, P., and Placido, L. (2011). Ambiguity and compound risk attitudes: An experiment. *Mimeo.*