Tournaments as Response to Ambiguity Aversion in Incentive Contracts

Job Market Paper

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Abstract

We study a principal-agent problem with multiple identical agents, where the action-dependent stochastic relationship between actions and output is perceived to be ambiguous, and agents are ambiguity averse. We argue that ambiguity, and particularly ambiguity aversion, make it more attractive for the principal to choose a tournament: If agents are risk neutral, but ambiguity averse, we show that the set of optimal incentive schemes contains a tournament. Moreover, if ambiguity is rich enough, a wage contract can be optimal only if the total wage payment is independent of the realized output levels. When agents are both risk averse and ambiguity averse, tournaments need not be optimal, but ambiguity and ambiguity aversion still favor, in many cases, the use of tournaments over wage schemes that only depend on each agent's own output level.

1 Introduction

In principal agent problems with multiple agents, a variety of different kinds of incentive contracts can be observed. Sometimes the compensation of the agents is solely based on their own performance. Often, however, the relative performance of an agent in comparison to the other agents also determines the compensation. One example of this are tournaments, in which the payment awarded to an agent depends only on the rank of her output-contribution. In the context of wages in firms it is often argued (originally by Lazear & Rosen (1981)) that wage determination can be understood as the result of a firm-internal labor-market tournament, where promotions (which are accompanied by a wage increase) constitute the prize for winning the tournament. In fact, empirical evidence suggests that promotions explain variations in wages within firms to a large extent (e.g. McCue 1996, Baker, Gibbs & Holmstrom 1994*a*, Baker, Gibbs & Holmstrom 1994*b*, Lazear 1992), and Prendergast (1999) even claims that based on the available evidence, at least for white-collar workers, tournaments seem to be the optimal means of providing incentives in large firms.[1] Yet, as

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[1] Note, however, that the wage increase attributed to promotions could have other explanations besides being a reward for past performance to the winner of a tournament. For instance, Waldman (1984) argues that promotions are accompanied by wage increases since they send positive signals about the worker's ability to other potential employers.

we will argue in more detail below, the reasons for the popularity of tournaments are not necessarily well-understood.

We will contribute to explaining when a principal might prefer a tournament (or a tournament like contract) over other incentive schemes, including independent wage schemes (where wages depend only on the agent's own performance). We do so by allowing the agents to be ambiguity averse, which we will define after a short review of the theoretical setting of multiple-agent principal agent problems.

In general, incentive contracts, where pay varies with performance, are viewed as a reaction to the problem of moral hazard: The profit of a principal (typically the owner of a firm) depends on the actions of her agents (typically employees). However she cannot observe these actions directly, but can only condition wage payments on the agents' performance (their output contribution), which only depends in a noisy (stochastic) way on the effort exerted by the agent. In a setting with multiple agents the principal could design very complex wage contracts which depend on the performance level of every worker within the firm.

In analyzing this problem it has been typically assumed that the process governing the distribution of noise, conditional on the agent's actions, is well-understood, and that principal and the agents are expected-utility maximizers (and the latter are potentially risk averse). Instead, we assume that there is subjective uncertainty about the probability distributions that describe the relationship between effort and performance. In line with recent literature (e.g. Ghirardato, Maccheroni & Marinacci (2004) and Klibanoff, Marinacci & Mukerji (2005)), we use the term ambiguity in reference to such subjective uncertainty about probabilities. For most parts of the paper we will also assume that the agents (but not the principal) are ambiguity averse, which means that they prefer situations in which they have greater confidence in the probabilities of the payoff-relevant events. There is a lot of evidence, for instance obtained by experiments, that in fact many decision makers violate the expected utility assumptions in a way that is consistent with ambiguity aversion.[2]

One motivation for taking ambiguity aversion into account in the context of contracting is that in many contracting situations, it seems plausible to expect that the players perceive uncertainty to be ambiguous (so that ambiguity aversion could in fact make a difference): Certainly, a (noisy) performance measure representing the production of a worker cannot be considered to be objectively determined (like coin-flips or lottery tickets), but is better understood as an example of subjective uncertainty. For this case it has been argued (e.g. Marinacci 2002) that one might expect ambiguity to disappear in situations where the decision makers face the same problem a large number of times in a stationary environment, so that they might feel confident enough to describe uncertainty using a single probability distribution. Using this criterion, we would expect that ambiguity matters little in situations where the players routinely face the same task, like might be true for some manufacturing jobs. In the case of white collar workers, however, it is less often the case that employees routinely face unchanged tasks in an unchanged environment. Thus, ambiguity aversion could explain why contracts are different in such situations, and how this affects the severity of the moral hazard problem, and thus the principal's profits.

Before we will outline our findings how in fact ambiguity aversion affects the principal's choice between different kinds of incentive contracts, we will give a brief overview of the existing literature on contracts with multiple agents, and on tournaments in particular.

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[2] The classic experiment suggesting that ambiguity aversion matters has been described by Ellsberg (1961).

The positive properties of tournaments have been documented in several papers, dating back to the seminal work of Lazear & Rosen (1981). Most notably, their paper finds that the prize awarded to the winner in a tournament should optimally increase in the level of noise in the performance measure. However, not many papers contribute to answering the more normative question: Why do tournaments exist in the first place? Lazear & Rosen (1981) show that tournaments achieve the first best solution if risk-aversion is absent. But in this case, many other incentive schemes, for instance linear piece rates, achieve this goal as well. An advantage of tournaments could be that sometimes only the rank of the agents' contribution can be observed, but not the level. This argument is often considered unsatisfactory (comp. e.g. Prendergast (1999)). Tournaments also seem to prevail in situations where a measure for individual performance is available.

Another argument is that that in the presence of risk aversion, a tournament may do better than (linear) piece rates if very specific assumptions about the agents' preferences are satisfied. (Piece rates, like tournaments, serve as an example of a very simple contract.) Lazear & Rosen (1981) discuss the case of declining absolute risk aversion: A tournament is preferred if the variance of the error term is small enough, though the threshold level varies with the agents' endowment. However, for the constant absolute risk aversion case, if errors are normally distributed and costs are quadratic, the best tournament is always worse than the best piece rate. They also argue that a common source of uncertainty that equally applies to all agents increases the number of cases where tournaments are better than piece rates. In fact, Green & Stokey (1983) show that if the restriction to piece rates is dropped, a common shock is necessary to make the principal prefer a tournament over the best independent wage scheme. They argue that tournaments are a way to filter common shocks (though, in general, they do this in an inefficient way), but this benefit comes at the cost of adding some of the other agents' idiosyncratic noise to the compensation scheme of any given individual.

Mookherjee (1984) and Holmstrom (1982) consider more general incentive contracts based on relative performance. Both papers confirm that an independent scheme is always optimal if outcome distributions are independent. Otherwise, the optimal incentive scheme should make use of all available information about the agent's choice of action. Tournaments typically fail to do so. Neither of the two papers, however, fully describes the optimal incentive scheme, since it may, in general, be very complicated.

Even if the optimal incentive scheme is more complex, it could still share one of the main characteristics of a tournament: Wages increase in the agent's own performance but decrease with the performance of other agents. Magill & Quinzii (2006) show that in an expected-utility context even this may not hold.

One could conclude that from an agency point of view, support for the use of tournaments or even tournament like schemes is actually not very strong. Prendergast (1999), who seems to share this view, discusses some (potential) advantages of tournaments that do not directly fit into the context of agency-models.[3]

Our own finding is that the use of tournaments (or at least tournament like incentive schemes) can be better justified, even in a classical principal-agent model, if ambiguity aversion is taken into

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[3] For instance, psychological consideration might prevent supervisors from discriminating between sub-ordinates and thus pay equal bonuses to everyone, while a tournament forces them to actually pick a winner. Alternatively, some have argued that tournaments make it harder for the principal to renege on paying the bonus.

account. Our baseline assumption is that there are identical agents who face the same tasks. We make this assumption as it allows us to isolate most clearly the potential advantages of tournaments in the case of ambiguity aversion. Specifically, we find that if agents are risk neutral but ambiguity averse, tournaments can implement the first-best outcome, while most other incentive contracts, including independent incentive schemes, typically do not. We establish that under a quite general assumption about the nature of ambiguity, all contracts will be in a certain sense similar to tournaments; when there are only two outcomes, no contract other than a tournament can be an optimal incentive scheme.

For the case of risk aversion, we first argue briefly that, in line with Magill & Quinzii (2006), the optimal incentive contract is not tournament like in certain situations. Even then, the optimal contract can become tournament like, however, if agents are sufficiently ambiguity averse. Second, we study and compare the properties of tournaments with independent wage schemes. As tournaments and independent wage schemes are the two most commonly considered wage schemes in a multiple-agent setting, it is interesting to compare their relative merits in the presence of ambiguity aversion even when neither of them is necessarily the optimal incentive scheme.[4] We focus on two questions in this context.

First, we compare tournaments to independent wage schemes. We show that in many cases, ambiguity and ambiguity aversion relatively favor the use of tournaments. We compare the ambiguity neutral case to existing findings in the literature. For instance, we point out that the addition of an additive common shock (as in Green & Stokey 1983) could be interpreted as increasing both ambiguity and riskiness at the same time. Moreover, we show that ambiguity aversion favors tournaments even in cases where ambiguity alone does not. Second, we study the design of the optimal tournament. For the case of two outcomes we show that there are two different scenarios, depending on the parameters of the model. In the first case, tournaments create more ambiguity about the equilibrium wage scheme than they do for deviations from the equilibrium. In this case, the principal reacts to ambiguity aversion by choosing a tournament that reduces ambiguity about the equilibrium wage scheme. However, when tournaments create less ambiguity about equilibrium wages in comparison to wages resulting from deviations, this need not be true. In fact, under ambiguity aversion it could even be optimal to choose a tournament that induces more ambiguity about equilibrium wages (compared to the ambiguity neutral case). Finally, we investigate how robust our results are to the specifications of our model, such as our assumptions about the nature of ambiguity or the symmetry of the agents. The case of infinitely many outcomes (with atomless outcome distributions) deserves special attention. Since ties become irrelevant, all tournaments result in an ambiguity-free wage scheme. Thus, in this case, it follows quite generally that ambiguity aversion makes the best independent wage scheme less attractive in comparison to the best tournament.

2 The Model

We study the problem of providing incentives to multiple workers. The owner of a firm, the principal, employs more than one worker, the agents, to undertake a certain task. The effort of the agents determines, to a large extent, the payoff-relevant output of the firm, but it is also influenced by random events that are beyond the control of the workers. Inefficiencies may arise as the agents'

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[4] See Prendergast (1999) for evidence on the use of these two kinds of contracts.

effort cannot be observed (or otherwise inferred) by the principal. In the case of multiple agents the nature of the interactions between the agents deserves some attention. We focus on the case of two identical agents, who are working on exactly the same task, and any output that is produced can be clearly associated with the agent who produced it.[5] We rule out any considerations typically related to team work (synergies, free riding).

We will assume that there are only finitely many output levels that every worker might possibly achieve, which are elements of the finite set $Q = \{q_1, \ldots, q_I\}$.[6] As a convention we index the output levels so that $q_i < q_j$ if $i < j$. As the principal can reward each agent based on the output contribution achieved by the agent herself, as well as the output of the other worker, agent one's wage scheme can be represented by a function $w : Q \times Q \to (\underline{w}, \infty)$. Here, $\underline{w}$ represents the minimum wage that the principal needs to pay to the workers (but we allow for the possibility that $\underline{w}$ is $-\infty$). As both workers are identical, we focus on the case of symmetric wage functions, where the labels of the two agents do not matter. Hence, if agent one produces output q_i and agent two produces q_j, the wage for agent one is given by $w(q_i, q_j)$ and agent two's wage is given by $w(q_j, q_i)$. For simplicity, each of the two agents has only two actions available: The high action, H represents high effort, and results in effort cost c^H to the agent. Choosing the low action, L incurs lower effort costs, c^L, but will also, on average, result in a lower productivity level.

If we did not allow for ambiguity, we would conclude this discussion by stating that in addition to the effort cost, each action $a \in A = \{H, L\}$ is identified with a probability distribution $p^a \in \Delta Q$ over outcomes, which applies equally to both agents.

To introduce ambiguity, we assume that both the principal and the agents have a common but imperfect understanding of the (stochastic) relationship between the actions and the outcomes. We represent this subjective uncertainty about the outcome distributions using the smooth ambiguity model (Klibanoff et al. 2005).[7]

Decision makers who satisfy the axioms imposed in this model appear to evaluate uncertain prospects in the following way: They are unable to assign a single probability distribution to describe the likelihood of each outcome. Instead, a set of probabilities might better describe the implications of each action. However, they do not necessarily consider all members of that set equally likely, but assign different likelihoods to them. To evaluate prospects, like wage schemes, that depend on the possible realization of the various outcomes, they first compute the expected utility corresponding to any outcome distribution that they deem possible. Before they aggregate these expected utilities using the likelihood they attribute to the underlying probability distribution, they discount higher expected utility levels, just as an expected utility maximizer discounts higher payoffs, using a concave transformation function. This reflects their aversion to ambiguity.

Formally, let f denote any act which specifies the decision makers payoff for any possible con-

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[5] Note that the assumption that agents are identical might apply particularly well in cases where choosing the high-cost action over the low cost action stands for choosing a higher quality of inputs, which are equally available to both agents. This could be the case if the agent is in fact the head of a organizational unit. Tournaments between various kinds of organizational units can be observed widely. For instance university departments are often funded based on their ranking. In Europe, many regional governments and interest groups award prizes to towns rewarding the quality of public services, such as "most bicycle friendly town".

[6] This is the same modeling approach taken by Grossman & Hart (1983) for the single-agent case and Mookherjee (1984) for the case of multiple agents. Lazear & Rosen (1981), Holmstrom (1982) and Green & Stokey (1983) allow for infinitely many outcomes. For our main results we briefly address this case as well.

[7] A future version of this paper may explore to what extent our results carry over to other ambiguity models.

tingency (i.e. f is a real valued function over a state space, S). If preferences obey the axioms of the smooth ambiguity model, they can be represented by the function

$$U(f) = \int_{\Delta} \phi \left(\int_{S} u(f) dp \right) d\mu \tag{1}$$

where p is a probability measure over S, Δ the set of all such probabilities, u is a von-Neumann-Morgenstern utility function, representing risk attitude, and ϕ is a real-valued transformation which represent ambiguity attitude. In analogy to risk aversion, ambiguity aversion corresponds to a concave ϕ; if the function ϕ is linear the decision maker is ambiguity neutral, and thus an Expected-Utility maximizer.

When we formalize the perception of ambiguity, it turns out to be sometimes more practical to describe ambiguity using the likelihoods the players attribute to certain possible deviations around an average output distribution. Thus we assume that every action is identified by a triple $(\bar{p}^a, M^a, \mu^a)$. The probability distribution $\bar{p}^a \in \Delta(Q)$ stands for the average output distribution (and $\bar{p}^a_i$ represents the average probability of outcome i). The set $M^a \subset \Delta(Q)$ represents all possible deviations around the average output distribution. We write $\hat{p}$ for a typical member of M^a and $\hat{p}_i$ represents the deviation of the probability assigned to output i. Finally, the measure μ^a indicates the likelihood that the agent attributes to the deviations in M^a. Again, the fact that the decision makers consider these deviations can be understood to model the fact that they are not confident in using a single probability distribution to measure the likelihood of the various output levels following from an effort level. (Together with the average probability, M^a still describes, indirectly, a set of probabilities over output levels that are considered possible by the agent.)

To close our description of ambiguity, we need to specify how the outcome distributions of the two agents and the two actions are perceived to vary together. Thus we let μ over $M \subset M^H \times M^L$ denote the joint probability distribution describing the likelihood of possible deviations for both actions and we view μ^a as the marginal distribution for action a. Note that we have assumed that which deviations are possible, and how likely they are, only depends on the action, but not the agent. In doing so, want to capture the idea that all ambiguity about the outcome distributions is solely about the link between effort and output, but not about whether or not the two agents and the problems they are facing are in fact are identical. In particular, this implies that everyone understands that when they contemplate a deviation around the average outcome distribution, this deviation needs to apply for both agents, if they choose the same action. Specifying the joint distribution is still necessary since we need to know the likelihood attributed to the deviations if the two agents decide not to choose the same action.

We assume that agent one's preferences over wage schemes, dependent on both agents' action choices, can be represented by the utility function $U : A \times A \times (\underline{w}, \infty)^{I \times I} \to \mathbb{R}$ defined by

$$U(a, a', w) = -\int \exp \left[-\alpha \left(\sum_{i \in Q, j \in Q} (\bar{p}^a_i + \hat{p}^a_i)(\bar{p}^{a'}_j + \hat{p}^{a'}_j)[u(w(q_i, q_j)) - c^a] \right) \right] d\mu.$$

Here $a \in A$ is the action chosen by agent one, while agent two chose action $a' \in A$; $u : (\underline{w}, \infty) \to \mathbb{R}$ is an increasing real-valued and concave function, and $\alpha > 0$.[8] For a fixed pair of actions, these preferences fit into the smooth ambiguity model introduced in equation (1) as follows: The wage

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[8] To rule out corner solutions we require that $\lim_{w \to \underline{w}} u(w) = -\infty$.

scheme w can be understood as an act over the possible set of states $Q \times Q$, which stands for all possible combinations of output levels. For ϕ we chose the specific function $\phi(u) = -\exp[-\alpha u]$.

Note that the term in the sum resembles the utility of an expected utility maximizer who thinks that the *independent* probability distributions $\bar{p}^a + \hat{p}^a$ and $\bar{p}^{a'} + \hat{p}^{a'}$ determine the likelihood of any combination of output levels in $Q \times Q$. We make the usual assumption that effort costs contribute in an additive way to expected utility, and since u is weakly concave, agents are risk neutral or risk averse.

Ambiguity enters this utility function since the agent does not only evaluate the expected utility level that a wage scheme provides according to a single possibility distribution (for each agent) over the output levels, but instead she computes various expected utility levels corresponding to a set of probability distributions (indirectly described by their difference from the respective average probability distribution).

When modeling ambiguity attitude, we chose the negative exponential form for ϕ for convenience. We we will replace it by a linear function to model the case of ambiguity neutrality, the limiting case as α approaches zero. Klibanoff et al. (2005) classify such preferences as constant absolute ambiguity aversion (CAAA, in analogy to CARA risk aversion). Such preferences are also members of the class of variational preferences studied by Maccheroni, Marinacci & Rustichini (2006). Increasing α corresponds to increasing the agent's degree of ambiguity aversion. Note that Klibanoff et al. (2005) show that these preferences approach MaxMin-Expected-Utility preferences (Gilboa & Schmeidler 1989) if α approaches infinity. In our context, MaxMin-Expected-Utility-maximizers would evaluate every action according to the deviations in M that minimize the resulting expected utility.

For CAAA preferences it is often convenient to work with a transformed utility function, defined by $\tilde{U}(a, a', w) = -\frac{1}{\alpha}\log(-U(a, a', w))$. Such a function measures the prospects for the agents according to their utility equivalents: Given actions a and a', an agent is indifferent between a wage scheme w (resulting in utility $\tilde{U}(a, a', w)$) and a constant wage payment w^c that satisfies $u(w^c) = \tilde{U}(a, a', w)$. (Note that U does not have this property.) If we denote the average probability of output combination (q_i, q_j) following the action choices a and a' by $\bar{P}(q_i, q_j; a, a')$ (which actually simplifies to $\bar{P}(q_i, q_j; a, a') = \bar{p}_i^a \bar{p}_j^{a'} + \int_M \hat{p}_i^a \hat{p}_j^{a'} d\mu$), such a representation is given by

$$\begin{aligned}\tilde{U}(a, a', w) = &\sum_{i\in Q, j\in Q} \bar{P}(q_i, q_j; a, a') u(w(q_i, q_j)) - c^a \\ &-\frac{1}{\alpha}\log\int \exp\left[-\alpha\left(\sum_{i\in Q, j\in Q} (\bar{p}_i^a + \hat{p}_i^a)(\bar{p}_j^{a'} + \hat{p}_j^{a'}) - \bar{P}(q_i, q_j; a, a') u(w(q_i, q_j)\right)\right] d\mu.\end{aligned} \tag{2}$$

Here, only the second term depends on the agent's attitude towards ambiguity, and this term vanishes as the agent becomes ambiguity neutral (as α goes to zero), in which case the agent just uses the first term, the expected utility according to the average distribution of outcome vectors.

Finally, we turn to the principal's payoff. Assuming the firm is both ambiguity neutral and risk neutral (Klibanoff et al. (2005) show that ambiguity neutrality corresponds to ϕ affine), the firm's payoff if actions a and a' are chosen using payment scheme w is given by the function $\Pi : A \times A \times (\underline{w}, \infty)^{I\times I} \to \mathbb{R}$ defined by

$$\Pi(a, a', w) = \int\left[\sum_{i\in Q, j\in Q} (\bar{p}_i^a + \hat{p}_i^a)(\bar{p}_j^{a'} + \hat{p}_j^{a'})(q_i + q_j - w(q_i, q_j) - w(q_j, q_i))\right] d\mu$$

$$= \sum_i \bar{p}^a q_i + \sum_j \bar{p}^{a'} q_j - \sum_{i \in Q, j \in Q} \bar{P}(q_i, q_j; a, a') w(q_i, q_j) - \sum_{i \in Q, j \in Q} \bar{P}(q_i, q_j; a, a') w(q_j, q_i)$$

Thus the principal maximizes expected profits, where the expectation is taken with respect to the distribution given by $\bar{P}(q_i, q_j; a, a')$, which represents the average probability of the outcome combination $(q_i,\ q_j)$ when actions a and a' are taken. When looking for the optimal contract, we consider only deterministic wage schemes, since it can be verified that using a randomization device can never strictly improve the principal's profits.

2.1 The principal's problem

As the formulation of the principal's problem differs from, e.g., Mookherjee (1984) only in the way the uncertainty is modeled, we will be brief. It is generally useful to think about the principal's problem in two stages: First, the principal tries to find the cheapest way to implement each possible combination of effort levels. Second, she compares the implementation costs to the resulting expected output, and decides to implement the effort level that offers the largest spread between costs and benefits.

To solve the first part of the problem, we look for a Nash equilibrium solution. An agent accepts a wage contract and carries out the effort level the principal intends him to do, if it meets two constraints: The individual rationality constraint states that the the agent must be better off when she accepts the contract offered by the principal than with her outside option (which results in the utility level u_0), assuming that the other agent accepts the contract and chooses the intended effort level. The incentive constraint (IC) requires that the agent prefers choosing the action the principal intends to the other action, again assuming that the other agent does not deviate.[9]

As we consider only incentive schemes that treat the two agents in a symmetric way, it suffices to consider the incentives of agent one: they are the same for agent two. The solution to the first stage becomes:

$$C(a) = \min_w \sum_{i=1}^{I} \bar{p}^a q_i - \sum_{i \in Q, j \in Q} \bar{P}(q_i, q_j; a, a) w(q_i, q_j). \tag{3}$$

subject to the two constraints

$$\tilde{U}(a, a, w) \geq u_0 \tag{IR}$$

$$\tilde{U}(a, a, w) \geq \tilde{U}(a', a, w) \text{ for } a' \neq a, \tag{IC}$$

The second stage is to choose $a \in \{H, L\}$ to maximize

$$\Pi(a) = 2 \sum_{i=1}^{I} \bar{p}^a q_i - 2C(a).$$

To make the problem interesting, we assume that at least in a first best world it would be optimal to implement H for both agents. That is, $\mathrm{E}_{\bar{p}^H}[q] > \mathrm{E}_{\bar{p}^L}[q]$ and the difference in effort costs is sufficiently small. The problem of optimally implementing the high cost action is in fact the key part of our analysis, since the low cost action is most cheaply implemented using a constant

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[9] Note that there may be multiple equilibria. We focus on the incentive scheme that is best for the principal.

wage scheme. Consequently, profits are very closely linked to implementation costs: Profits can only increase if the implementation costs for the high cost action decrease. A decrease in these implementation costs guarantees an increase in profits if it is or at least becomes optimal to choose the high-cost action, and has no effect otherwise.

It will turn out to be convenient to state the problem in terms of the utility levels attributed to each output combination. This is possible since for any wage level w there is a unique corresponding utility level $u(w)$. Thus we denote by $h(u)$ the inverse utility function that satisfies $h(u(w)) = w$. Consequently, h is increasing and convex. Thus, instead of choosing a wage scheme, the principal can equivalently choose a scheme of utility levels. This approach is common in the agency literature.

3 The Optimal Incentive Contract

To discuss the optimal incentive scheme in the presence of ambiguity aversion, we proceed in the following way. For the case of risk neutrality, we can show that a particular kind of a tournament, which eliminates all ambiguity from the optimal wage scheme, is optimal. Moreover, it can achieve the first-best profit level and under a natural assumption about ambiguity, a contract can be optimal only if the total compensation paid to the agents does not depend on the output levels they achieve, as is typical for tournaments. The solution to the problem becomes very complex if also risk aversion is present. In the presence of risk aversion we will show that ambiguity aversion can significantly change the design of the optimal contract: Even in situations where the optimal incentive scheme cannot be tournament like under ambiguity neutrality, it may still be tournament like under ambiguity aversion.

To describe the set of optimal contracts, we introduce the following definition.

Definition 1. *A* strict tournament *is a wage scheme that increases iff the rank of the agent's output level increases, i.e.*

$$u_{ij} \equiv u(w(q_i, q_j)) = \begin{cases} u^W & \text{if } i > j \\ u^T & \text{if } i = j \\ u^L & \text{if } i < j \end{cases}$$

and $u^W > u^T > u^L$.

Thus, for any strict tournament, the agent's compensation depends only on the rank of their output. Under risk neutrality these compensation levels can be directly interpreted as wage levels (since u can be assumed to equal the identity function, scaling costs accordingly). When there are more than two output levels available, the principal might rank the agent using a coarser criterion than the set of all available output levels. For instance, if the revenue generated by each agent can be measured in dollars and cents, the principal could rank gents only based on the dollar-amount. We use the term coarse tournament to describe such tournaments.

Definition 2. *A* coarse tournament *is a tournament that ranks agents according to a connected partition of* Q, *i.e. there exists a weakly increasing function* $f : \{1, \dots I\} \to \{1, \dots I\}$ *such that*

$$u_{ij} \equiv u(w(q_i, q_j)) = \begin{cases} u^W & \text{if } f(i) > f(j) \\ u^T & \text{if } f(i) = f(j) \\ u^L & \text{if } f(i) < f(j) \end{cases}$$

and $u^W > u^T > u^L$*.*

Finally, we define a property that the optimal incentive contract will satisfy under risk neutrality:

Definition 3. *An incentive scheme is* constant-sum *if total compensation does not vary with output* ($u_{ij} + u_{ji} = u_{i'j'} + u_{j'i'}$) *for all* $i, i', j, j' \in \{0, \dots I\}$*).*

For risk neutrality, this implies that for constant-sum schemes, total wages do not change with the realized combination of output levels, only the division of wages among the agents may change. For tournaments, this property need not hold in general: While total wages do not change in a tournament as long as one of the agents achieves a strictly higher output level then the other one, total wages may change when a tie occurs. However, it is also possible to design a tournament so that the constant-sum property holds, by setting $u^T = (u^W + u^L)/2$. For the case of risk-neutral agents, this corresponds to dividing the prize for winning the tournament (the additional wage the winner gets over the loser) equally between both agents in case of a tie. Thus, the constant-sum property is satisfied by a very natural class of tournaments.

We can now describe the optimal contract for the case of risk neutrality:

Proposition 1. *Suppose agents are risk neutral but ambiguity averse. The set of optimal contracts includes a coarse tournament, and implementation costs are at the first-best level. Moreover, if ambiguity is rich enough (M is sufficiently diverse), all optimal schemes are constant-sum.*

Proof. See below for an outline, and the appendix for a full proof. ☐

Thus, the principal can always achieve the first-best level by choosing a (coarse) tournament. Moreover, whenever ambiguity is rich enough, only incentive schemes that have the constant-sum property are optimal for the principal. Thus, ambiguity aversion provides an argument why the principal would prefer a tournament over most other incentive schemes, including independent wage contracts, or even more complex contracts that combine features of tournaments with features of independent contracts. Note that no comparable agency model can provide such a justification for the use of tournaments: Without ambiguity aversion, tournaments are not optimal if risk aversion is introduced, but the optimal contract will use more information about the agents' own and relative performance than just their rank. For the case of risk neutrality however, many kind of incentive schemes are optimal. We formally describe in the appendix when the set M of possible output distributions is in fact sufficiently diverse to guarantee that constant-sum schemes are uniquely optimal. For the case of two outcomes, it is simply required that M contains at least three different beliefs about the outcome distribution of the high action.

We now give a brief outline of the proof, to explain the intuition behind the above result. First, any incentive scheme has to satisfy the IR constraint, which requires that

$$\sum_{i \in Q, j \in Q} \bar{P}(q_i, q_j; a, a') w(q_i, q_j) \geq \mathcal{A}^H + u_0 + c^H,$$

where $\mathcal{A}^H = \frac{1}{\alpha} \log \left(\int_M \exp[-\alpha \left(\sum_{i \in Q, j \in Q} \left[(\bar{p}_i^a + \hat{p}_i^a)(\bar{p}_j^{a'} + \hat{p}_j^{a'}) - \bar{P}(q_i, q_j; a, a') \right] w(q_i, q_j) \right)] d\mu \right)$

The principal's implementation costs correspond to the left hand side of this inequality. The (nonnegative) term $\mathcal{A}^H$ can be interpreted as an ambiguity premium, which the principal has to pay to compensate the agent for the ambiguity of the wage scheme. Thus implementation costs are at least $u_0 + c^H$. As we will show, for any constant-sum scheme, effort costs are purely risky. These schemes result in the same expected utility irrespective of which possible output distribution (in M) is considered. For tournaments, this property can be understood in a very intuitive way: If ties could not occur, the probability of winning the tournament must be identical for all agents (1/2 if there are two agents), irrespective of the actual distribution over output levels. Even if ties can occur, all agents will still have the same winning probability if ties are broken at random (but the principal may also choose a deterministic wage scheme that provides the same incentives to the agents).

Since for any purely risky wage schemes, $\mathcal{A}^H = 0$, ambiguity aversion does not change the IR constraint for constant-sum schemes. Thus, the first best implementation costs, $u_0 + c^H$, can be achieved whenever a constant-sum scheme satisfies the IC constraint. As we will demonstrate, it is in fact possible to do so using a coarse tournament that partitions the set of outcomes into two states. Thus, at least a coarse tournament is in the set of optimal contracts.

Finally, if ambiguity is rich enough, in the sense that the set of output distributions is sufficiently diverse, only constant-sum schemes lead to a purely risky wage contract. To understand why also other schemes might achieve the first best if agents would consider either very few or very similar output distributions, observe that according to μ, it might be possible, for instance, that a (non-trivial) event is unambiguous. That is, all probability distributions agree on the likelihood to observe a particular set of outcome distributions (containing more than one, but not all, output levels). Then, even an independent wage scheme may achieve the first best. To provide incentives, the principal could pay a bonus whenever this event occurs, and a base wage otherwise. Adding an output distribution that is sufficiently different from the others so that it assigns a different probability distribution to this event eliminates this possibility.

3.1 Two output levels

We now want to compare the case of ambiguity aversion under risk neutrality with the opposite case of risk aversion under ambiguity neutrality. To simplify the presentation, we restrict the model to the case of only two outcomes. Regarding the set of optimal contracts under ambiguity aversion, the case of two outcomes allows us to proof a more specific result about the uniqueness of the class of optimal schemes. We then show that the optimal contract may not even be tournament like under risk aversion. Finally, we use a numerical example to show how ambiguity aversion may affect the optimal contract if agents are both strictly risk averse and ambiguity averse. We begin by introducing a notation that is useful to deal specifically with the case of two outcomes.

When there are only two output levels, outcome level 1 can be interpreted as failure, F, and level 2 as success, S. We will identify any outcome distribution with its success probability, so that we will write $\bar{p}^H \equiv \bar{p}_2^H \in [0, 1]$, and $\bar{p}^L \equiv \bar{p}_2^L \in [0, 1]$. Similarly, any deviation $\hat{p}^a$ can be identified with the deviation around the success probability alone, so that we will write $\hat{p}^a = \hat{p}_2^a \in [-p^a, 1 - p^a]$.

For a symmetric wage scheme, it suffices to consider agent one.

To simplify the notation, we will denote $u^{SF} \equiv u(w(q_S, q_F)), u^{SS} \equiv u(w(q_S, q_S)), u^{FF} \equiv u(w(q_F, q_F)), u^{FS} \equiv u(w(q_F, q_S))$. It turns out useful to characterize the optimal incentive scheme using the following four variables. Let $\bar{u}$ denote the μ-average expected utility that the principal needs to provide to the agent, i.e.

$$\bar{u} \equiv \bar{p}^H \left((\bar{p}^H + \frac{\sigma_H^2}{\bar{p}^H}) u^{SS} + (1 - \bar{p}^H - \frac{\sigma_H^2}{\bar{p}^H}) u^{SF} \right) + (1 - \bar{p}^H) \left((\bar{p}^H - \frac{\sigma_H^2}{1 - \bar{p}^H}) u^{FS} + (1 - \bar{p}^H + \frac{\sigma_H^2}{1 - \bar{p}^H}) u^{FF} \right),$$

where $\sigma_H^2 \equiv \mathrm{E}_\mu[(\hat{p}^H)^2]$ (and for future use, also let $\rho_{HL} \equiv \mathrm{E}_\mu[\hat{p}^H \hat{p}^L]$). Now assume that both agents choose the high cost action. Let $u^{\Delta I}$ represent agent one's average incremental compensation for succeeding over failing (taking the expectation over the output of agent two and the possible success rates), i.e.

$$u^{\Delta I} \equiv (\bar{p}^H + \frac{\sigma_H^2}{\bar{p}^H}) u^{SS} + (1 - \bar{p}^H - \frac{\sigma_H^2}{\bar{p}^H}) u^{SF} - \left((\bar{p}^H - \frac{\sigma_H^2}{1 - \bar{p}^H}) u^{FS} + (1 - \bar{p}^H + \frac{\sigma_H^2}{1 - \bar{p}^H}) u^{FF} \right).$$

Finally let $u^{\Delta S} \equiv u^{SF} - u^{SS}$ be the change in agent one's compensation caused by a change of agent 2's output contribution, if agent one succeeds, and $u^{\Delta F} \equiv u^{FF} - u^{FS}$ be the equivalent change assuming agent one fails.

For the two outcome case, the distinction between a coarse tournament and a strict tournament becomes irrelevant, so that we define tournaments as follows.

Definition 4. *An incentive scheme is a tournament if the wage payment increases iff the rank of the agent's output contribution increases. In the case of only two outcomes, this condition is equivalent to* $u^{SF} > u^{SS} = u^{FF} > u^{FS}$.

The proposition below states that, as expected, under risk neutrality a certain kind of tournament can always implement the first-best also in the case of two outcomes. Moreover, all optimal incentive schemes will now be tournaments if two additional conditions hold. While it is still necessary that the agents perceives a rich enough pattern of ambiguity, all that is required in the two-outcome case is that agents think that at least three different probabilities possibly describe the success rate of the high cost action. It appears that this assumption is not overly restrictive. Second, we require that according to all probability distributions that the agent considers possible, the high cost action results in better odds than the low cost action, i.e. it is in a sense unambiguous that the high cost action has a higher success rate then the low cost action. Without this assumption an incentive scheme that has the constant-sum property, and is similar to a tournament, except that winning results in the lowest, while loosing results in the highest wage payment, could also be optimal.

Proposition 2. *Suppose the agent is risk neutral, but strictly ambiguity averse. Then the first-best can be achieved by a tournament which satisfies that* $u^{\Delta S} = u^{\Delta F} > 0$ *and* $u^{\Delta I} = (1 - \frac{\sigma_H^2}{\bar{p}^H(1-\bar{p}^H)}) u^{\Delta S}$. *If* M^H *has at least three elements, and* $\bar{p}^H + \inf(M^H) > \bar{p}^L + \sup(M^L)$, *all optimal incentive schemes are tournaments.*

We give a proof in the appendix. The main difference between this results and the general case is that only tournaments can be optimal if we assume that the high action is unambiguously better than the low action. To understand why this is true, observe that the only other kind of constant-sum scheme that could be designed in the case of two actions reward the winning agent less then the loser. Unless the agent assigns positive probability to any pair of success rates where the success rate generated by the low action is the higher one, such a scheme cannot provide any incentives to chose the high action.

In the presence of risk aversion, tournaments need no longer be optimal, even though they can eliminate all ambiguity. For the case of ambiguity neutrality, it has been argued that tournaments are actually almost never optimal (e.g. Mookherjee 1984, Holmstrom 1982). Even if they are not optimal, one could argue that they represent the main idea behind relative performance evaluation, where wages increase with the agent's own performance and decrease with the performance of other agents. Under ambiguity neutrality, however, Magill & Quinzii (2006) show that in a setting similar to ours (although with a continuum of actions), with uncertainty about a (common) outcome distribution, the optimal incentive scheme may not be tournament like at all under some conditions: An agent's wage may increase with the output of the other agents. They obtain this result in a setting where all agents are identical, but the stochastic link between effort and output may be affected by a random common shock, which allows for arbitrary changes to the outcome distributions (that are identical for both agents). Principal and agents agree on a probability distribution governing possible realizations of the common shock.[10] If a favorable realization of the common shock always goes together with circumstances in which effort is very productive, then the optimal incentive scheme will not be tournament like. We will argue now that this finding is not robust to ambiguity aversion. Before that, we explore when the optimal incentive scheme is tournament like under ambiguity neutrality in our setting.

As stated before, we call an incentive scheme tournament like, if increases in the output level of agent two never increase the wage for agent one (and vice versa), representing the idea that the agents compete with each other for monetary prizes (or promotions):

Definition 5. *An incentive scheme is* tournament like *if both $u^{\Delta S} \geq 0$ and $u^{\Delta F} \geq 0$.*

According to this definition, a tournament is always also a tournament like incentive scheme.

For ambiguity neutral agents the following proposition characterizes the set of principal agent problems (within the class of problems discussed in this chapter) in which the optimal incentive contract is tournament like.[11]

Proposition 3. *Assume strict risk aversion and ambiguity neutrality. In the optimal incentive scheme, $u^{\Delta S} \geq 0$ iff $\frac{\rho_{HL}}{\bar{p}^L} \geq \frac{\sigma_H^2}{\bar{p}^H}$ and $u^{\Delta F} \geq 0$ iff $\frac{\rho_{HL}}{1-\bar{p}^L} \leq \frac{\sigma_H^2}{1-\bar{p}^H}$. Thus the optimal incentive scheme is tournament like iff both conditions hold.*

Proof. Since the proof follows standard arguments, we only present an outline. Both $u^{\Delta S}$ and $u^{\Delta F}$ only change the incentive constraint, which simplifies to:

$$(\bar{p}^H - \bar{p}^L)u^{\Delta I} - \bar{p}^L\left(\frac{\sigma_H^2}{\bar{p}^H} - \frac{\rho_{HL}}{\bar{p}^L}\right)u^{\Delta S} + (1-\bar{p}^L)\left(\frac{\sigma_H^2}{1-\bar{p}^H} - \frac{\rho_{HL}}{1-\bar{p}^L}\right)u^{\Delta F} \geq c_H - c_L \tag{4}$$

To show sufficiency, suppose, for instance, that in the optimal incentive scheme $u^{\Delta S} < 0$ even though the condition presented in the proposition holds. But then consider another incentive scheme that differs only by setting $u^{\Delta S} = 0$. Then, the condition ensures that the incentive constraint still holds. The new incentive scheme is cheaper to implement, however, for the principal, since it provides

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[10] Thus they allow for the same kind of uncertainty about the output distributions as we do, while they assume that the agents are ambiguity neutral (or perceive all uncertainty to be objective.)

[11] Apart from our limitation to two outcomes, this results differs from Magill & Quinzii (2006) in that in they give a condition that guarantees an incentive scheme to be tournament like no matter which probabilities are attributed to the realization of the common shock. Moreover, their condition is only for a fixed output level of agent one.

the same expected utility to the agent in a less risky way (leading to lower implementation costs according to (12)). The other direction and the condition for $u^{\Delta F}$ can be shown using analogous arguments. $\square$

To understand this result, observe that choosing L over H has two effects: It changes the probability that the agent herself succeeds, so that her expected wage changes by $-(\bar{p}^H - \bar{p}^L)u^{\Delta I}$ (as reflected by the first term in the incentive constraint, equation (4)). Given she still succeeds, L changes the conditional probability that agent two fails by $(\frac{\sigma_H^2}{\bar{p}^H} - \frac{\rho_{HL}}{\bar{p}^L})$, which affects the expected wage the more agent one's payment following agent two's failure exceeds the payment following success, i.e. the larger $u^{\Delta S}$ (reflected by the second term in (4)). Similarly, choosing L changes agent two's failure rate conditional on agent one's failure by $-(\frac{\sigma_H^2}{1-\bar{p}^H} - \frac{\rho_{HL}}{1-\bar{p}^L})$ (compare the last term in (4)). Thus, $u^{\Delta S}$ and $u^{\Delta F}$ provide incentives according to the sign of the respective change in the conditional success probabilities.

We will discuss now two representative polar cases, in which intuitive arguments reveal whether the optimal incentive scheme is tournament like. These arguments are based on the well-known idea that an optimal incentive contract can be understood as a signal-extraction problem, where an outcome is rewarded the more, the stronger it signals that the agent has chosen the high cost action.

The first case corresponds to the following assumption:

Assumption 1. *For all pairs of deviations* $(\hat{p}^H, \hat{p}^L)$ *in the support of* μ, $\hat{p}^L = \hat{p}^H \equiv \hat{p}$.

Consequently, $\sigma_H^2 = \rho_{HL}$. In words, the decision maker contemplates that the probability distribution for H might exceed the average distribution by $\hat{p}$ precisely whenever the probability distribution for L exceeds the average distribution by the same amount $\hat{p}$. Hence, according to all (action-dependent) output distributions that the agents contemplate, effort changes the distributions to the same extent as it changes the average output distribution. Thus the effect of effort is unambiguous in this case.

The second case is given by:

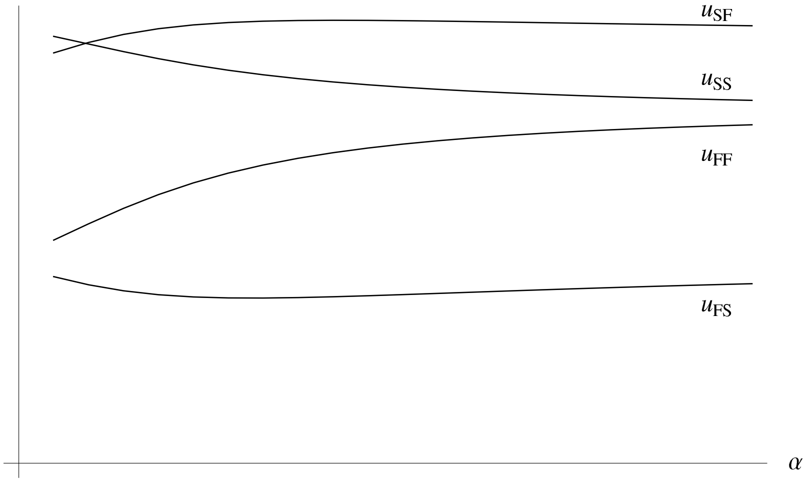
Assumption 2. *For all pairs of deviations* $(\hat{p}^H, \hat{p}^L)$ *in the support of* μ, $\hat{p}^H = -\hat{p}^L$.

Here, whenever the success rate for the high cost action differs from its average by some $\hat{p}$, the success rate for L differs from its average by $-\hat{p}$. Consequently, deviations from the average output distribution for the high cost action H are perfectly negatively correlated with deviations following L, so that $\rho_{HL} = -\sigma_H^2$. If $\hat{p} > 0$, the productivity of effort is above average since the two success rates differ by $\bar{p}^H - \bar{p}^L + 2\hat{p}$. However, the more $\hat{p}$ decreases, effort becomes less productive and thus ambiguity about the success probabilities can be completely identified with ambiguity about the effectiveness of effort. In that sense, *only* the effect of effort is ambiguous.

Corollary 1. *Assume strict risk aversion and ambiguity neutrality. The optimal incentive scheme is always tournament like under assumption 1 (i.e. when the effectiveness of effort is unambiguous), while it is never tournament like if* $\rho_{HL} \leq 0$ *(so in particular, under assumption 2).*

If the effect of effort is unambiguous (assumption 1), in an expected utility context, success of agent two signals that the success rate is in fact above average, irrespective of which action agent

Figure 1: Optimal Incentive Contract Depending on Ambiguity Aversion



one chose. While the absolute change in agent one's success rate between H and L remains constant, the relative difference decreases and thus agent two's success makes agent one's success a weaker signal that she chose H.

Now consider the case where only the effect of effort is unambiguous (assumption 2). Again, success of agent two could be interpreted as indicating an above-average success rate (as she succeeds more likely under high success rates). But now, this also implies that effort was more productive than usual. This intuitively suggests that the success of agent one should be rewarded more if agent two succeeds, resulting in a non-tournament-like incentive scheme. This intuition generalizes to the case where $\rho_{HL} \leq 0$, since even if nothing is learned about the effectiveness of the low cost action ($\rho_{HL} = 0$) agent two's success still signals that the high cost action is more productive than on average, indicating that the productivity of effort is above average.

These two case describes (abstracting from the differences between the two models) comparable circumstances for which Magill & Quinzii (2006) show that the optimal incentive schemes cannot be tournament like. Especially the second case illustrates that the optimal incentive scheme might be very different between ambiguity averse and risk neutral (in which case it needs to be a tournament) and ambiguity neutral and risk averse agents (in which case it will not even be tournament like).

Finally, note that also when agents are both risk averse and ambiguity averse, assuming that only the effect of effort is unambiguous (assumption 2) is not a sufficient condition for the optimal contract not to be tournament like. This is illustrated in Figure 3. It presents the optimal solution to a set of principal agent problems which differ only by the agents' ambiguity aversion (obtained using numerical methods). It belongs to the case where only effort is ambiguous ($\rho_{HL} = -\sigma_H^2$). As the figure shows (in agreement with Proposition 3), under ambiguity neutrality the optimal scheme is not tournament like: The payment of an agent who succeeds is higher if also the other agent succeeds. However, as ambiguity aversion increases, agent one's compensation (measured in utility terms), given success, varies less whether agent two fails or not, up to the point where the optimal incentive scheme becomes tournament like.

As a direction for future research this example suggests that it might be worthwhile to investigate whether, (keeping fixed the agent's risk aversion) the optimal incentive scheme always becomes tournament like for some (finite) level of ambiguity aversion.

4 The Effect of Ambiguity and Ambiguity Aversion on Popular Contracts

As we argued in the introduction, it seems that in many applied contexts principals select either tournaments (for instance via promotions) or independent contracts to provide incentives to the agents. As they are not optimal in general, it might be the case that practical constraints prevent the principal from implementing the optimal contract, which may be quite complex. The principal might be forced to choose among sufficiently simple contracts, such as tournaments and independent schemes. (However, note that we have provided another justification for the use of tournaments: Under risk neutrality and ambiguity aversion essentially only tournaments are optimal. Moreover, for risk averse agents independent schemes are optimal in the absence of ambiguity.) Thus we want to investigate whether, keeping everything else equal, introducing ambiguity and aversion makes it more likely that the principal prefers a tournament over independent schemes.[12] To do so, we analyze how the profits of the principal change for both tournaments and independent incentive schemes.

In general, the effects of introducing ambiguity and ambiguity aversion may depend on very specific details about the problem. However, we will show that for tournaments that have the constant-sum property the effects of ambiguity and ambiguity aversion can be determined quite generally. We will then argue why the principal might do even better by using a tournament that does not have the constant-sum property. We will illustrate the issues that can arise in this case using a simplified version of our model, which allows for only two outcomes.

4.1 Constant-sum tournaments

We begin with the case of introducing ambiguity, assuming agents are ambiguity neutral. While it is true in a single-agent model that the introduction of ambiguity without ambiguity aversion does not change anything about the problem as long as the average output distributions remain unchanged, this is no longer the case for the case of two agents. Therefore, we will show first how the introduction of ambiguity alone changes the principal's profits, and then show how profits change if the agents also become ambiguity averse.

We will show first that ambiguity (in the absence of ambiguity aversion) leaves the profits of the principal unchanged for some important cases, which we will identify using the following assumptions. In doing so, we do not attempt to give a complete characterization of these cases (which is provided in the appendix), but we focus on three representative cases.

Assumption 3. *The* effect of effort is unambiguous *if* $\forall(\hat{p}^H, \hat{p}^L) \in M$, $\hat{p}^H = \hat{p}^L$

In this case, for every member of M, the probability distribution that is attributed to the high action fully determines the probability distribution of the low action, since both output distributions

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[12] In fact, also the optimal independent contract might still be complex, so one could consider certain classes of independent contracts instead – by imposing restrictions such as monotonicity or linearity – but we expect that this would not change our results in any essential way.

always differ from their average to the same extent. This case generalizes assumption 1 to the case of two outcomes.

Assumption 4. Only the effect of effort is ambiguous *if* $\forall(\hat{p}^H, \hat{p}^L) \in M$, $\hat{p}^H = -\hat{p}^L$

Equivalently, this case could be defined by requiring that $.5(\bar{p}^H + \hat{p}^H) + .5(\bar{p}^L + \hat{p}^L)$ does not vary between the elements in M. Thus, the average of the high and low output distribution is constant (hence unambiguous), while the effect of effort (the difference between the two output distributions) is not. In this sense, only the effect of effort is ambiguous. Note that this case generalizes assumption 2 to the case of more than two outcomes.

Assumption 5. *Ambiguity is uncorrelated between actions if* $\forall i, j \in I$, $E_\mu(\hat{p}_i^H \hat{p}_j^L) = 0$.

Here, we assume that the output distribution of the high action is, according to μ, uncorrelated with the output distribution of the low action, in the sense that the probability of any output level under H is uncorrelated with the probability of any output level under L.

Finally, we also consider a special form of constant-sum tournaments, where the prize for winning the tournament, resulting in an incremental compensation $u^W - u^L$, is awarded randomly to one of the two agents in case of a tie. So from the agents' point of view, $u^T = .5(u^W + u^L)$, as in a constant-sum scheme. However, the (expected) wage paid the principal pays for a tie becomes $.5h(u^W) + .5h(u^L)$ (instead of $h(.5u^W + .5.u^L)$ as in deterministic constant-sum schemes). We call such tournaments 1-prize tournaments, as they arise naturally in a situation where the principal can only award a single indivisible prize (like a promotion). Constant-sum tournaments utilize instead two different prizes (in addition to a baseline compensation): Both winning the tournament and ties are rewarded, but to a different extent. Note that in the case of infinitely many outcomes with atomless output distributions all tournaments are equivalent to 1-prize tournaments, since ties do not occur with positive probability.

Proposition 4. *Assume either assumption 1, 2, or 3 holds. For ambiguity neutral agents, introducing ambiguity leaves profits that can be achieved using a 1-prize tournament constant, while profits that can be achieved using a CS-tournament will (weakly) increase.*

Proof. See Appendix. □

This result shows that for 1-prize tournaments, ambiguity alone may not have any effect on implementation costs for some cases. The effect may become positive when constant-sum tournaments are considered (but this can never be the case if there are infinitely many outcomes, as in this case 1-prize tournaments and constant-sum tournaments are equivalent). While a full proof is in the appendix, observe that for any constant-sum tournament, as well as 1-prize tournaments, ambiguity never changes the agent's expected payoff resulting from the high action. For the low action, this remains true for instance if one of the three assumptions hold. Thus, the set of implementable constant-sum tournaments (or 1-prize tournaments), remains unchanged. For 1-prize tournaments, also the principal's implementation costs remain unchanged. However, for constant-sum tournaments, introducing ambiguity means that ties occur more often. In a constant-sum scheme, agents exactly get the average compensation whenever a tie occurs. Thus, if ties occur more often, any constant-sum tournament provides the same average utility in a less risky way to the agent. Thus, implementation costs decrease.

The effect of ambiguity aversion on constant-sum tournaments is more straightforward, as the following proposition indicates.

Proposition 5. *Increasing ambiguity aversion can never decrease the profits that the principal can achieve if a constant-sum tournament is used.*

Proof. We will argue that the set of implementable tournaments cannot shrink, but may grow. (The result follows since ambiguity aversion changes nothing about implementation costs for a given tournament.) As we have argued in the proof of proposition 1, the agent's utility does not change with increases in ambiguity aversion when she chooses the high action, as constant-sum tournaments are purely risky. Thus, the set of constant-sum tournaments which satisfy the IR constraint remains unchanged. However, the low cost action may become less attractive (but never more attractive): The last term in equation (2) represents the effect of ambiguity aversion. As we have argued earlier, this term will weakly increase with increases in ambiguity aversion (but remains constant if also the low action makes wages purely risky). In fact, it is easy to construct an example where the wage scheme following a deviation is not purely risky, even if e.g. assumption 3 holds. As only the low cost action can become less attractive, any constant-sum tournament that satisfies both constraints under ambiguity neutrality still does under ambiguity aversion (and some additional constant-sum tournament do), and hence the set of all such schemes which implement the high action pair cannot shrink. ☐

In short, constant-sum tournaments eliminate all ambiguity from the agent's wage contract on equilibrium. However, deviating to the low cost action may still result in ambiguous wage scheme. This might allow the principal to choose an incentive scheme that is less risky. Thus, for constant-sum tournaments, ambiguity aversion will often increase the principal's profit, while it can never decrease them. This provides tournaments with an advantage over other schemes.[13]

As we will show below, the effect of ambiguity aversion is quite different for independent schemes.

4.2 Independent wage schemes

Before we compare tournaments to independent wage schemes, we briefly summarize the main properties of independent wage schemes under ambiguity aversion. Such wage schemes are characterized by the restriction that for all $q_j, q_i, q_k \in Q$, $w(q_i, q_j) = w(q_i, q_k)$. They fit directly into the framework discussed in Kellner (2008), as the problem can be studied for each agent separately. Introducing ambiguity without ambiguity aversion neither changes anything about set of feasible independent contracts, nor about the implementation costs. To describe the effect of introducing ambiguity aversion, we can adapt Proposition 14 in Kellner (2008) to obtain the following result:

Proposition 6. *Suppose either assumption 3 holds or assumption 4 holds and ambiguity is symmetric ($\forall a \in A, \mu(p^a) = \mu(-p^a)$). Then it follows that an increase in ambiguity aversion cannot increase the profits of the principal if the optimal independent incentive scheme is used.*

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[13] Note that the preceding two propositions also allow us to address the effect of introducing ambiguity for ambiguity averse agents: In the absence of ambiguity, ambiguity aversion is irrelevant. Hence, introducing ambiguity for ambiguity averse agents has the same effect as first introducing ambiguity for an ambiguity neutral agent, followed by introducing ambiguity aversion. Thus, for the cases we addressed in propositions 4 and 5, the combined effect will be positive.

Thus, in these two cases, ambiguity aversion can only be positive for the principal, if she implements a constant-sum tournament.

In other cases (for instance, when assumption 5 holds), the effect of ambiguity aversion partly depends on which of the two actions results in a more ambiguous wage scheme. While Kellner (2008) discusses this issue in greater detail, consider the two polar cases where only output distributions generated by one of the two actions are uncertain. If it is only the high action, ambiguity aversion can never increase the principal's profits. If it is only the low action, ambiguity aversion can never decrease them. But note that even if they are equally ambiguous, the effect of ambiguity aversion will be negative.

4.3 Comparing tournaments with independent wage schemes

We can now compare the effect of introducing ambiguity and ambiguity aversion, based on the results presented earlier in this section. Consider first the case of introducing ambiguity for ambiguity neutral agents. Under assumptions 3, 4 or 5, profits remain unchanged for both one-prize tournaments and independent schemes, so that ambiguity alone has no effect at all. If constant-sum tournaments are considered, even ambiguity alone will favor tournaments over independent schemes, as profits can only increase for the former, while they will remain constant for the latter.

For the case of ambiguity neutrality, our findings are related to Green & Stokey (1983), but there are important differences. Their proposition 2 studies, in a context with a continuum of outcomes and additive effort, the effects of introducing an additive common shock to the agent's output level in a tournament between identical agents. As this common shock becomes sufficiently diffuse, a tournament eventually does better than an independent incentive scheme. Thus, in a model with infinitely many outcomes additive common shocks favor tournaments over independent schemes.

Such a common shock could be interpreted as ambiguity about the mean of the outcome distribution, and in this sense, also Green & Stokey (1983) suggest that ambiguity favors tournaments. Note, however, that we have argued that for the case of infinitely many outcomes (where tournaments can use only one prize to provide incentives), in many cases ambiguity has no effect on the principal's profits. In fact, also a common shock leaves profits that can be achieved with a tournament unchanged. However, a common shock does not preserve the average output distribution. Specifically, the average output distribution becomes more risky. Thus, it becomes less attractive to choose an independent wage scheme, and this is the only reason why tournaments become relatively more attractive.

We showed instead that ambiguity may have a positive effect, even if the average output distribution is unchanged, as ambiguity may make tournaments more profitable. However, for some cases, ambiguity alone may not have an effect on both kind of schemes.

Also, Green & Stokey (1983) do not allow for ambiguity aversion. But even when ambiguity alone does not favor tournaments under ambiguity neutrality, introducing ambiguity aversion will often still do.

This can be most clearly seen for the case of one-prize tournaments in which either assumption 3 or assumption 4 hold. Comparing propositions 4 and 6 we see that in this case ambiguity has no effect on either tournaments or independent schemes, if agents are ambiguity neutral. However, leaving the expected utility framework by allowing for ambiguity aversion makes independent schemes less attractive, while the profits that can be achieved with a 1-prize tournament can never decrease. In fact, ambiguity aversion favors constant-sum tournaments over independent schemes beyond these two cases. This is the case whenever the high action is at least as ambiguous as the low action.

4.4 The optimal tournament

We now turn to studying the properties of the optimal tournament. As we have argued earlier, the optimal tournament differs from a constant-sum tournament only in the way ties are treated. As, in general, the occurrence of ties becomes less likely the more output levels are observable, one can expect the difference between constant-sum tournaments and more general tournaments to matter most if there are only few output levels. As a simplifying assumption we assume that there are only two output levels, interpreted as success and failure. In this section, we want to illustrate first why the optimal tournament need not have the constant-sum property. Then we show, making further assumptions when necessary, how the optimal tournament looks like, and how the the effects of ambiguity and ambiguity aversion might differ from the case of constant-sum schemes.

To understand the design of the optimal contract, it is useful to illustrate how ambiguity about the output distribution translates to ambiguity about the three pay-off relevant events (winning, losing and ties), given the agents' action choice, and thus, ambiguity about the wage scheme. As we did in section 3.1, we identify every probability distribution with the probability for the better outcome (the success rate), and we assume for simplicity that assumption 1 holds, so that $\sigma_H^2 = \rho_{HL}$. We begin with the case where both agents choose the high cost action. For every possible deviation $\hat{p}$ with corresponding success probability $\bar{p}^H + \hat{p}$, if both agents choose the high cost action the probability of winning, losing and ties is given by

$$\begin{aligned} P(\text{win}|\text{both choose H}) &= \bar{p}^H(1-\bar{p}^H) + (1-2\bar{p}^H)\hat{p} - \hat{p}^2 \\ P(\text{tie}|\text{both choose H}) &= 1 - 2\left(\bar{p}^H(1-\bar{p}^H) + (1-2\bar{p}^H)\hat{p} - \hat{p}^2\right) \\ P(\text{lose}|\text{both choose H}) &= \bar{p}^H(1-\bar{p}^H) + (1-2\bar{p}^H)\hat{p} - \hat{p}^2. \end{aligned}$$

To see how these probabilities vary with deviations around the average success probability, it suffices to focus on the probability of winning the tournament: The probability of losing the tournament is always exactly the same, and ties can be understood as the complement of winning and losing. As the likelihood of any deviation $\hat{p}$ is determined by the zero-mean measure μ, the average probability of winning (resp. losing) the tournament, denoted by $\bar{p}^W$ (resp. $\bar{p}^{LT}$) is in turn given by $\bar{p}^W = \bar{p}^H(1-\bar{p}^H) - \sigma_H^2 = \bar{p}^{LT}$.

Thus, if a success probability differs from the average success probability by $\hat{p}$, the corresponding probability of winning the tournament will differ by $(1-2\bar{p}^H)\hat{p}-\hat{p}^2+\sigma_H^2$ from the average probability of winning the tournament. To understand this expression, observe first that, for instance, a positive deviation $\hat{p}$ from the average success probability increases on the one hand the probability that agent one produces the high output, but on the other hand it decreases the probability that agent two fails to do so. In isolation, an increase in agent one's success probability by $\hat{p}$ would increase the winning probability by $\hat{p}(1-\bar{p}^H)$. A decrease in the other agent's failure rate by $\hat{p}$ decreases the winning probability by $-\bar{p}^H(\hat{p})$. The two effects work in opposite direction, and the net effect is positive if the first effect dominates, which is true iff $\bar{p}^H < (1-\bar{p}^H)$, or $\bar{p}^H < 1/2$. Note however that the first effect becomes smaller, and the second effect more negative since both probabilities change at the same time, which is reflected by the term $-(\hat{p})^2$. Finally, the average probability of winning will typically not equal the winning probability corresponding to the average success probability - they differ by σ_H^2.

Similarly, if an agent contemplates to deviate to the low cost action (under the equilibrium assumption that the other agent sticks to the high cost action), a success probability of $\bar{p}^H + \hat{p}$ would result in the following probabilities for the outcomes of the tournament

$$\begin{aligned}
P(\text{win}|\text{one defects to L }) &= \bar{p}^L(1-\bar{p}^H) + (1-\bar{p}^H-\bar{p}^L)\hat{p} - \hat{p}^2 \\
P(\text{tie}|\text{one defect to L}) &= 1 - (\bar{p}^H + \bar{p}^L - 2\bar{p}^H\bar{p}^L + 2(1-\bar{p}^H-\bar{p}^L)\hat{p} + 2\hat{p}^2) \\
P(\text{lose}|\text{one defects to L}) &= \bar{p}^H(1-\bar{p}^L) + (1-\bar{p}^H-\bar{p}^L)\hat{p} - \hat{p}^2.
\end{aligned}$$

The key difference to the high cost action is that the impact of the change of agent two's success probability on agent one's winning probability is reduced, since it is now less likely that agent one actually is successful herself. Consequently, the average probability of winning the tournament now has decreased to $\bar{p}^L(1-\bar{p}^H) - \sigma_H^2 = \bar{p}^W - (1-\bar{p}^H)(\bar{p}^H - \bar{p}^L)$ while the average probability of losing has increased to $\bar{p}^H(1-\bar{p}^L) - \sigma_H^2 = \bar{p}^{LT} + \bar{p}^H(\bar{p}^H - \bar{p}^L)$. However, the winning probability still differs from its average, given a deviation $\hat{p}$, to the same extent as the probability of losing, and this difference now amounts to $(1-\bar{p}^H-\bar{p}^L)\hat{p} - \hat{p}^2 + \sigma_H^2$.

Having identified the possible probabilities for the payoff-relevant events, we can now compute the utility associated with each action under a given tournament. We used to describe general wage contracts via the extent to which they provide incentives based on an agent's own outcome and how much these are adjusted based on relative performance. Since a tournament requires that $u^{SS} = u^{FF}$, the principal cannot choose individual incentives independent from relative performance adjustments.[14] The following notation turns out more useful to describe tournaments. First, let the number $u^{WL} = u^W - u^L = u^{\Delta F} + u^{\Delta S}$ represent the incentives for winning the tournament relative to losing it. Second, the number $u^{\Delta T}$ is defined by $u^T - .5(u^W + u^L) = .5(u^{\Delta F} - u^{\Delta S})$. It reflects, measured in utility terms, how much the compensation for ties exceeds the average compensation for winning and losing. Finally, as before, $\bar{u} = u^T + \bar{p}^W(u^W - u^T) - \bar{p}^W(u^L - u^T) = u^T - 2\bar{p}^W u^{\Delta T}$ is the expected utility level according to the average probability distribution over the output combinations if H is chosen by both agents.

To find the best tournament, the principal's profit maximization problem becomes to minimize implementation costs

$$\begin{aligned}
C^T(H) = \min_{u^{\Delta T}} \Bigg(& \bar{p}^W h\left(\bar{u}(u^{\Delta T}) - (1-2\bar{p}^W)u^{\Delta T} + \frac{u^{WL}(u^{\Delta T})}{2}\right) \\
& + \bar{p}^W h\left(\bar{u}(u^{\Delta T}) - (1-2\bar{p}^W)u^{\Delta T} - \frac{u^{WL}(u^{\Delta T})}{2}\right) \\
& + (1-2\bar{p}^W) h\left(\bar{u}(u^{\Delta T}) + 2\bar{p}^W u^{\Delta T}\right)\Bigg),
\end{aligned} \tag{5}$$

subject to the constraints

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[14] This follows since for a tournament, $u^{\Delta I} = (1-\bar{p}^H - \frac{\sigma_H^2}{\bar{p}^H})u^{\Delta S} + (\bar{p}^H - \frac{\sigma_H^2}{1-\bar{p}^H})u^{FS}$

$$\begin{aligned}\bar{u}(u^{\Delta T}) &= u_0 + c_H + \mathcal{A}^H(u^{\Delta T}) \text{ (from IR)}\\ u^{WL}(u^{\Delta T}) &= 2(1-2\bar{p}^H)u^{\Delta T} + 2\frac{c^H - c^L + \mathcal{A}^H(u^{\Delta T}) - \mathcal{A}^L(u^{\Delta T})}{\bar{p}^H - \bar{p}^L} \text{ (from IC)}\end{aligned} \tag{6}$$

where $\mathcal{A}^H(u^{\Delta T}) = \frac{1}{\alpha}\log\left(\int(\exp(2\alpha[(1-2\bar{p}^H)\hat{p} - (\hat{p}^2 - \sigma_H^2)]u^{\Delta T})d\mu\right)$ represents the ambiguity premium required for H, and $\mathcal{A}^L(u^{\Delta T}) = \frac{1}{\alpha}\log\left(\int(\exp(2\alpha[(1-\bar{p}^H - \bar{p}^L)\hat{p} - (\hat{p}^2 - \sigma_H^2)]u^{\Delta T})d\mu\right)$ is the equivalent for the low cost action.

Comparing with the case of more general wage schemes, observe first that the incentive constraint can still be interpreted as stating that the average expected utility, after adding an ambiguity premium, must exceeds the value of the outside option by more than the effort costs. The ambiguity premium however now only depends on one characteristic of the optimal contract alone, $u^{\Delta T}$, as it should be clear from our discussion of the likelihood of the payoff-relevant events. For the incentive constraint, observe first that, naturally, the larger the spread between winning and loosing the tournament, u^{WL}, the larger are the incentives for the agent to choose H. However, also $u^{\Delta T}$ may provide incentives to the agents: If the other agent succeeds (which happens on average with probability $\bar{p}^H$), choosing the low cost action reduces the probability of a tie (at the expense of losing) by $\bar{p}^H - \bar{p}^L$ while, with average probability $1 - \bar{p}^H$, if the other agent fails, agent ones chances for winning $\bar{p}^H - \bar{p}^L$ are reduced at the expense of obtaining a tie. Thus, $u^T - u^L$ matters more than $u^W - u^L$ if $\bar{p}^H > 1/2$, which implies that a positive $u^{\Delta T}$ provides incentives iff the sign of $-(1-2\bar{p}^H)$ is positive.

Note that the constraint set may be empty, if the range of u is small, in which case it is optimal to implement the low cost action.[15] Otherwise, it can be verified that this problem in fact has a solution.

Since $\mathcal{A}^H(0) = \mathcal{A}^L(0) = 0$, we can obtain the following observation.

Observation 1. *If the range of u is large enough, the principal can always design a tournament that eliminates all ambiguity from the resulting equilibrium wage scheme. This tournament, which we henceforth call the ambiguity free tournament, is given by $u^{\Delta T} = 0$, $u^{WL} = 2\frac{c^H - c^L}{\bar{p}^H - \bar{p}^L}$ and $\bar{u} = u_0 + c_H$.*

For this tournament to exist, it suffices that $(u_o + c^H \pm \frac{c^H - c^L}{\bar{p}^H - \bar{p}^L})$ is in the range of u, which is always true if c^H and c^L are close enough.

Recall that such a tournament was the optimal incentive scheme for risk-neutrality. With risk aversion, it need not be the best tournament. However, it will be relatively attractive if ambiguity concerns are large in relation to risk aversion. We illustrate using a numerical example. Suppose that $\bar{p}^H = .7$, $\bar{p}^L = .6$, $u_0 = 0$, $c^H = 10$, $c^L = 9.5$. Let $M = \{0, -.15, .15\}$ and μ assigns equal probability to all elements of M. It follows that $\bar{p}^W = .145$ We begin with the case of ambiguity neutrality. In this case, the tournament defined by $u^{\Delta T} = 0$ requires that $u^{WL}(u^{\Delta T}) = 10$ and $\bar{u}(u^{\Delta T}) = 10$. Thus, $u^T(u^{\Delta T}) = 10$, $u^L(u^{\Delta T}) = 5$ and $u^W(u^{\Delta T}) = 15$. If $u(w) = \sqrt{w}$ (for positive w), so that $h(u) = u^2$, the implementation costs will equal 214.5. The principal can reduce the costs

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[15] Recall that we have implicitly assumed that costs are measured in utility terms. While it would be possible to rescale the utility function so that it still represents the same preferences, this would make it necessary to rescale the costs assigned to actions as well. Thus, more precisely, non-emptiness requires the range of u to be small in comparison to the difference in the cost of the two actions.

by choosing the incentive scheme identified by $u^{\Delta T'} = 2.80$, so that $u^{WL}(u^{\Delta T'})$ reduces to 6.64 while $\bar{u}(u^{\Delta T'})$ stays at 10. This implies that $u^{T}(u^{\Delta T'})$ increases to 10.81, $u^{W}(u^{\Delta T'})$ decreases to 11.32 and $u^{L}(u^{\Delta T'})$ decreases to 4.69. The resulting implementation costs decrease to 209.621 (and this is in fact the best the principal can achieve), which is intuitive since the primed incentive scheme provides the agent with the same expected utility (under the average probability distribution), but varies less between the three possible outcomes of the tournament.

Now allow for the presence of ambiguity aversion (we assume $\alpha = 30$). The tournament given by $u^{\Delta T} = 0$ is the ambiguity-free tournament, so it will continue to implement the high cost action at a cost of 214.5. The tournament previously associated with $u^{\Delta T'} = 2.8$ fails to satisfy the IR constraint if ambiguity aversion is present. The principal needs to increase $\bar{u}(u^{\Delta T'})$ to 10.426, reflecting the ambiguity premium that needs to be paid to the agent (and in this particular example also u^{WL} needs to increase), so that the tournament results in $u^{W}(u^{\Delta T'}) = 12.59$, $u^{L}(u^{\Delta T'}) = 4.27$ and $u^{T}(u^{\Delta T'}) = 11.23$. This increases implementation costs to 230.67, showing that $u^{\Delta T'} = 2.8$ no longer results in an attractive tournament when compared to the ambiguity-free tournament.

This example illustrates that ambiguity aversion may affect different tournaments in a different way, so that in choosing the best tournament it will be one goal of the principal to reduce the ambiguity induced by the equilibrium wage scheme, which can be done by moving in the direction of the ambiguity-free tournament. We have also seen already that this goal might conflict with another objective of the principal: To reduce the riskiness of the equilibrium wage scheme (according to the average wage distribution). Moreover we will discuss later that choosing a tournament that increases ambiguity about the equilibrium wage scheme might imply that the wage scheme resulting from deviating to the low cost action incurs even higher ambiguity costs. This might make it possible to reduce the riskiness of the equilibrium wage scheme, adding a further consideration to the design of the optimal tournament.

We will now describe the optimal tournament, which results from balancing these trade-offs.

4.4.1 Ambiguity neutrality

We begin with the absence of ambiguity aversion, while allowing for ambiguity. As we have implicitly dealt with the case of risk neutrality in the section on optimal incentive schemes, we assume that agents are strictly risk averse. We find the cheapest tournament that still implements the high cost action for an agent who perceives subjective uncertainty about the success probabilities, while treating it identical to objective uncertainty. We continue to focus on the case of unambiguous effort ($\sigma_H^2 = \rho_{HL}$). In the absence of ambiguity aversion, the two constraints that any optimal tournament has to satisfy, simplify to:

$$\bar{u}(u_\Delta) = u_0 + c^H \qquad \text{(IR)}$$

$$u^{WL}(u_\Delta) = 2\frac{c^H - c^L}{\bar{p}^H - \bar{p}^L} - 2(1 - 2\bar{p}^H)u^{\Delta T} \qquad \text{(IC)}$$

It becomes easy to see that if $\bar{p}^H = 1/2$ the incentive constraint determines that $u^{WL}(u^{\Delta T})$ does not vary with $u^{\Delta T}$. If $\bar{p}^H > 1/2$, $u^{\Delta T}$ and u^{WL} are substitutes in providing incentives to the agent, and else, u^{WL} and $-u^{\Delta T}$ become substitutes. Finding the optimal tournament amounts to choosing the right balance between u^{WL} and $u^{\Delta T}$ in order to minimize the riskiness of the optimal tournament.

Note that the constraint set does not vary with changes in ambiguity (when tournaments are described using $u^{\Delta T}$, u^{WL} and $\bar{u}$). However, the resulting wage payment does, since u^W, u^T and u^L depend on σ_H^2. Also, ambiguity changes the objective function in the cost-minimization problem (since $\bar{p}^W$ changes in equation (5)).

For a general utility function it can be quite complicated to describe the optimal tournament even in the absence of ambiguity aversion. As it is not unusual in the agency literature and in particular the literature on tournaments, we focus on a specific utility function to obtain a more tractable solution.

Assumption 6. *Let $u(w) = 2\sqrt{w}$, which implies that $h(u) = .5u^2$.*

In this case, the objective function becomes

$$C(u^{\Delta T}) = .5\bar{u}(u^{\Delta T})^2 + \bar{p}^W(1-2\bar{p}^W)u^{\Delta T2} + \bar{p}^W(u^{WL}(u^{\Delta T})/2)^2. \tag{7}$$

The benefit of choosing this functional form is that u^{WL} and $u^{\Delta T}$ enter the objective function separately, so that we can decompose it into three parts: First, $.5\bar{u}(u^{\Delta T})^2$ reflects the costs of providing the average utility level, if this was possible in an risk-free way. In general, the wage scheme will be risky however, and the principal also needs to compensate the agent for this riskiness. These risk costs are determined by the second and third term. They increase separately with $|u^{\Delta T}|$ and $|u^{WL}(u^{\Delta T})|$. In designing the optimal incentive scheme the principal needs to keep these risk costs as low as possible by finding the right balance between the two sources of risk, $u^{\Delta T}$ and u^{WL}. Note that one could argue that this intuition remains to a large extent valid for other utility functions, but additional considerations may arise since the effect that $u^{\Delta T}$ and u^{WL} has on the costs of a wage scheme can no longer be considered in isolation.

Assuming an interior solution, the first order condition to the problem of optimally implementing the high cost action determines its solution (since the problem is convex if viewed as a function of $u^{\Delta T}$ alone):

$$u^{WL}(u^{\Delta T}) = \frac{\frac{1}{2} - \bar{p}^W}{\frac{1}{2} - \bar{p}^H} u^{\Delta T}$$

Plugging in the constraints, the optimal tournament becomes

$$\bar{u}^* = u_0 + c_H$$

$$u^{\Delta T*} = (-.5)\frac{c_H - c_L}{\bar{p}^H - \bar{p}^L}\frac{(1-2\bar{p}^H)}{1 - 3\bar{p}^H + 3(\bar{p}^H)^2 + \sigma_H^2}$$

$$u^{WL*} = \frac{c_H - c_L}{\bar{p}^H - \bar{p}^L}\left(2 - \frac{(1-2\bar{p}^H)^2}{1 - 3\bar{p}^H + 3(\bar{p}^H)^2 + \sigma_H^2}\right)$$

Thus, we can observe that the optimal tournament has the following properties: Unsurprisingly, the agent who wins the tournament receives a higher wage than the agent who loses the tournament. Also, measured in utility terms, the remuneration the agent receives for a tie is higher than the average utility she receives otherwise iff $\bar{p}^H > 1/2$. This seems intuitive since whether $\bar{p}^H > 1/2$

determines whether the high cost action reduces the probability of observing a tie. Note that these properties are true for all levels of ambiguity, and even in the absence of ambiguity.

Ambiguity affects the design of the optimal contract in two ways: On the one hand, $u^{\Delta T^*}$ decreases in its absolute value (but without changing its sign), since σ_H^2 increases. That means that the optimal contract relies to a lesser extent on the occurrence of ties to provide incentives to the agents. On the other hand, the utility difference between winning and losing (as measured by u^{WL^*}) increases with ambiguity. This finding is consistent with the fact that under the presence of uncertainty about the probability distribution, an agent achieving the high outcome more strongly indicates that the agent has actually chosen the high cost action if she is the only one to do so. Analogously, the failure to achieve the better outcome is a stronger signal that the low cost action was chosen, if the other agent succeeds.

Note that in the case of only two outcomes, it is easily possible to define and study gradual increases in ambiguity. The average probability distribution for both actions as well as any possible deviation can in this case be identified with a unique real number, the success probability. Deviations around the average success probability can then viewed as a real valued random variable whose distribution is given by μ, and it is possible to state the following definition.

Definition 6. *Suppose that two ambiguous success rates are described by* $(\bar{p}, M, \mu)$ *and* $(\bar{p}', M', \mu')$. *The second success rate is the more ambiguous one if* μ' *is a mean preserving spread of* μ.

If a quadratic utility function is assumed, it turns out that more ambiguity actually decreases implementation costs.

Proposition 7. *Under assumptions 1 and 6, if the agent is ambiguity neutral, increases in ambiguity (keeping the average success rates fixed) reduce the implementation costs if the optimal tournament is chosen.*

Proof. See Appendix. ☐

Since ambiguity aversion does not change the benefits from implementing the high cost action, the implications for the principal's profits follow immediately: If it is (or becomes) optimal to implement the high cost action, profits increase with ambiguity if a tournament is chosen. Else, profits remain the same, which is always true for the independent wage scheme (under assumption 1).

4.4.2 Ambiguity aversion

We now want to describe the optimal tournament in the presence of both risk and ambiguity aversion. We focus on two main goals. First, in case the principal decides to implement a tournament, we want to show how ambiguity aversion changes the design of the optimal tournament. In the two-outcome case, this mainly means whether she will reward winning the tournament (over a tie) more than she will punish loosing. Second, the discussion of the risk-neutral case might lead to the impression that under the present of ambiguity aversion, it is the goal of the principal to eliminate ambiguity about the wage scheme. As this might increase the riskiness, risk aversion would only limit the extent to which this is done. We will show that this is not necessarily true. In some situations, the principal might chose a tournament that entails a more ambiguous wage scheme after the introduction of ambiguity aversion. For simplicity, we continue to focus on the case where effort is unambiguous.

Recall that the optimal tournament is the solution to the minimization problem given in equation (5). Before we turn to the solution of this problem note that ambiguity aversion only changes the constraints for this problem (equation (6)), but not the objective function. In particular, it is straightforward to see that the individual rationality constraint becomes (weakly) harder to satisfy, since in any tournament the principal needs to compensate the agent for the ambiguity in the optimal wage scheme, to the extent given by the ambiguity premium $\mathcal{A}(u^{\Delta T})$.

The incentive constraint, however, is either weakened or tightened. In fact, introducing ambiguity aversion relaxes the incentive constraint if $\mathcal{A}^H(u^{\Delta T}) < \mathcal{A}^L(u^{\Delta T})$. It turns out that under a natural assumption about ambiguity, whether a tournament makes the distribution of the wages resulting from the high cost action become more ambiguous depends only on the average success rates:

Observation 2. *Suppose ambiguity is symmetric, i.e.* $\mu(\hat{p}) = \mu(-\hat{p})$*. The introduction of ambiguity aversion tightens the incentive constraint if and only if* $\bar{p}^H > 1/2$ *and* $\bar{p}^H > 2/3 - \bar{p}^L/3$*.*

See the appendix for a proof. To understand this observation it is useful to recall our earlier discussion how ambiguity about the success probability translates to ambiguity about the winning probability. A (positive, e.g.) deviation $\hat{p}$ has two opposing consequences for the corresponding winning probability: It increases the probability that agent one succeeds (which makes her win if agent two fails, which on average occurs with probability $(1 - \bar{p}^H)$) but decreases the probability that agent two fails (which makes agent one win if she succeeds, which happens on average with probability $\bar{p}^H$ or $\bar{p}^L$ respectively). If $\bar{p}^H > 1/2$, and H is chosen, the second effect will dominate, since $\bar{p}^H > 1 - \bar{p}^H$. With the low cost action, agent 1 succeeds less often, so that the decreasing failure rate of agent two becomes less relevant. Thus the second effect becomes less relevant, so that net effect decreases (for large levels of $\bar{p}^L$) or even changes its sign. In this case, the net effect remains smaller until $\bar{p}^H > 2/3 - \bar{p}^L/3$. It turns out that this difference has important implications for the optimal incentive schemes, so that the two possible cases need to be treated separately.

The solution to the optimal tournament is, even in our simplified model with only two outcomes, in general quite difficult to describe. Thus, as in the case of ambiguity neutrality, we specify again a particular utility function, stated in assumption 6. Then, the first order condition for a minimum of the objective function defined in equation (7) becomes:

$$2\bar{p}^W(1-2\bar{p}^W)u^{\Delta T} + \left((1-2\bar{p}^H) + \frac{\mathcal{A}^{H'}(u_\Delta) - \bar{p}^W \mathcal{A}^{L'}(u_\Delta)}{\bar{p}^H - \bar{p}^L} \right) u^{WL}(u_\Delta) + \mathcal{A}^{H'}(u^{\Delta T})\bar{u}(u^{\Delta T}) = 0$$

We see that varying $u^{\Delta T}$ has several different effects. As in the absence of ambiguity aversion, the first term, $2\bar{p}^W(1-2\bar{p}^W)u^{\Delta T}$ stands for the direct effect of changing the (relative) remuneration for ties on the riskiness in the optimal wage scheme. The term pre-multiplying $u^{WL}(u_\Delta)$ represents the effect that changing $u^{\Delta T}$ has on the necessary spread in the payment for winning and losing. Ambiguity aversion changes the size of this effect depending on which is the more ambiguous action. Finally, ambiguity aversion adds an entirely new effect: The more $u^{\Delta T}$ differs from zero, the more the principal has to compensate the agent for ambiguity in the equilibrium wage scheme. This effect is captured by the term $\mathcal{A}^{H'}(u^{\Delta T})\bar{u}(u^{\Delta T})$.

Unfortunately, the objective function need not remain convex, and thus the first order condition may have solutions which do not identify the optimal tournament.

To cope with this problem, we make a further simplifying assumption: We compare the case of no ambiguity aversion only to the limiting case of infinite ambiguity aversion, which corresponds to max-min expected utility preferences (compare proposition (3) in Klibanoff et al. (2005)). In this case, an action is only evaluated using the deviation $\hat{p} \in M$ that minimizes expected utility, or in terms of the ambiguity premia,

$\mathcal{A}^H(u^{\Delta T}) = \max_{\hat{p} \in M}[(1-2\bar{p}^H)\hat{p} - (\hat{p}^2 - \sigma^2_H)]u^{\Delta T}$ and $\mathcal{A}^L(u^{\Delta T}) = \max_{\hat{p} \in M}(1-\bar{p}^H-\bar{p}^L)\hat{p} - (\hat{p}^2 - \sigma^2_H)]u^{\Delta T}$

We begin with the case where ambiguity aversion tightens both constraints. (Also in the case of infinite ambiguity aversion, this case is given by $\bar{p}^H > 1/2$ and $\bar{p}^H > 2/3 - \bar{p}^L/3$). It turns out that in this case the optimal tournament is the ambiguity free tournament for a large (non-generic) class of problems: It suffices that u_0, the outside option, is large enough. If the ambiguity free tournament is not optimal, ties result in a higher utility than the average of winning or losing. However, this difference is smaller than in the absence of ambiguity aversion. If $u^{\Delta T^{**}}$ characterizes the optimal tournament for ambiguity averse agents and $u^{\Delta T^*}$ characterizes the optimal tournament in an otherwise identical problem without ambiguity aversion, we can state this result formally as:

Proposition 8. *Suppose ambiguity aversion is infinite, M is a closed interval centred at 0, $\bar{p}^H > 1/2$ and $\bar{p}^H \geq 2/3 - \bar{p}^L/3$ and $u(w) = 2\sqrt{w}$. Then $0 \leq u^{\Delta T^{**}} < u^{\Delta T^*}$. Moreover, the ambiguity-free tournament is the best tournament if u_0 is large enough (or, alternatively, $c^H - c^L$ is small enough).*

A proof is given in the appendix. We show that for negative $u^{\Delta T}$, the objective function is always decreasing. For positive $u^{\Delta T}$, the objective function is always increasing as soon as u_0 is large enough. Else, the objective function is convex in that region so that the FOC determines its minimum.

To understand this result intuitively, note that our assumption about u entails decreasing absolute risk aversion. Eliminating ambiguity (by setting $u^{\Delta T} = 0$) comes at the cost of increasing the riskiness of the wage scheme. But as the agent becomes wealthy enough (high u_o), the increase in risk matters less, so that it may become optimal to remove all ambiguity from the wage scheme.

If risk aversion stays important enough (if u_0 is small), the goal of reducing ambiguity still makes the principal choose a smaller level of $u^{\Delta T}$, without eliminating all ambiguity.

Figure (2) represents graphically a typical example for the case where the high cost action results in a more ambiguous wage scheme (here, we choose $\bar{p}^H = .8$ and $\bar{p}^L = .6$). In the figure, and ambiguity attitude α ranges from 0 to 35.[16] Part a) illustrates how the compensation from the agent for each of the three outcomes "win", "loose" and "tie" changes in the optimal tournament (measured in utility terms). We see that the spread in the compensation between winning and losing increases, but the compensation for ties moves towards the average compensation in these two outcomes, reducing ambiguity induced by the equilibrium wage scheme.

Part b) shows that in this example profits decrease with ambiguity aversion also for the optimal tournament, but to a far less extent than with an independent wage scheme. As a benchmark we have also included the profits from the optimal incentive scheme (which does not impose any

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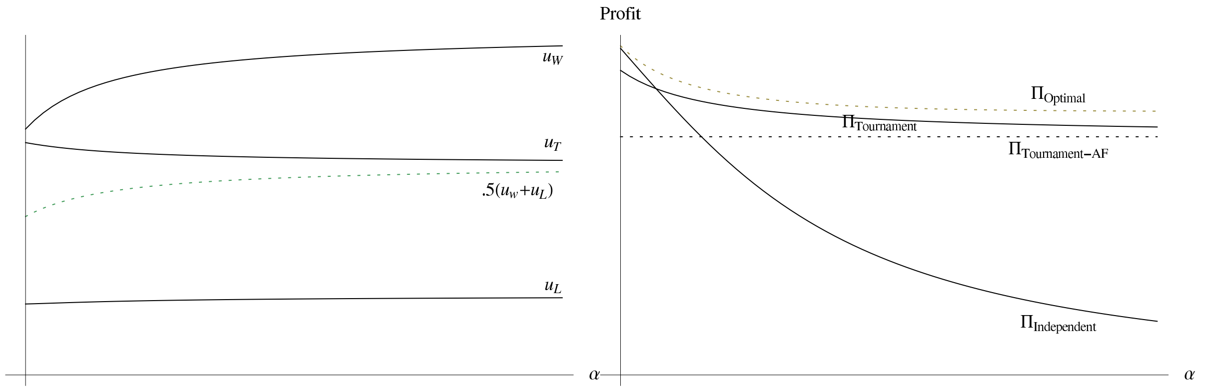
[16] The costs of the two actions were assumed to be $c^H = 4$ and $c^L = 3.5$, $M = \{.15, 0, .15\}$ all members of M are considered equally likely, . q^F and q^S are chosen such that the expected benefits from the high cost action are 40.



(a) Compensation in the optimal tournament (measured in utility terms)

(b) Profit Comparison

Figure 2: Case 1 (High-Cost induces more ambiguous wage scheme)

restriction on the class of possible contracts). For large levels of ambiguity aversion, the best tournament is much closer to the optimal wage scheme than the best independent wage scheme.

Regarding the implementation costs, the fact that ambiguity aversion tightens both constraints results in the following observation (independent of any assumptions about risk, ambiguity or ambiguity attitude.)

Observation 3. *Suppose $\bar{p}^H > 1/2$ and $\bar{p}^H > 2/3 - \bar{p}^H/3$. As ambiguity aversion is introduced, profits can never increase for the optimal tournament (while they will typically decrease), while profits for the ambiguity-free tournament remain constant.*

The situation changes considerably if we consider the case where $p_H \leq 1/2$ or $\bar{p}^H < 2/3 - \bar{p}^L/3$ which makes choosing the low cost action increase the ambiguity inherent in the optimal wage scheme. Proceeding as above, it can still be established that the ambiguity free tournament is optimal for large enough u_0. However, $u^{\Delta T}$ will sometimes increase and sometimes decrease if infinite ambiguity version is introduced. It is even possible to construct examples where $u^{\Delta T}$ changes its sign. The shape of the optimal tournament is much harder to predict in this case, since ambiguity aversion has a negative effect (the increase ambiguity premium needed to satisfy the IR constraint), but it relaxes the incentive constraint. Thus, it might sometimes be optimal to choose an incentive scheme that induces a considerable amount of ambiguity in equilibrium, since a deviation to the low cost action might entail even higher ambiguity costs for the agent. The following example illustrates.

Figure (3) represents the case where choosing the high cost action results in a less ambiguous wage scheme in the optimal tournament than deviating to the low cost action (in this example, $\bar{p}^H = .4 < .5$). In the figure, ambiguity aversion (α) increases from 0 to 35.[17] We see from part a) that, at least for low values of ambiguity aversion, the spread between winning and losing decreases, but the difference between the average compensation for these two outcomes and the compensation for ties (given by $|u^{\Delta T}|$) increases. The latter implies that, at least for low levels of ambiguity aversion, the

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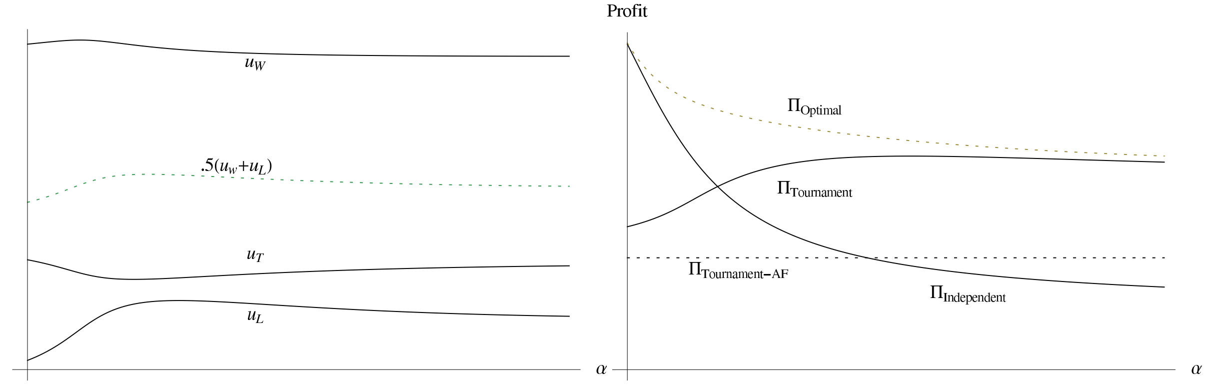
[17] The costs of the two actions were assumed to be $c^H = 2$ and $c^L = 1.5$, $M = \{.15, 0, .15\}$ all members of M are considered equally likely and q^F and q^S are chosen such that the expected benefits from the high cost action are 15.



(a) Compensation in the optimal tournament (measured in utility terms)

(b) Profit Comparison

Figure 3: Case 2 (High cost action induces less ambiguous wage scheme)

principal should devise a wage scheme that actually induces more ambiguity as ambiguity aversion increases. Part b) reveals that the introduction of ambiguity aversion nevertheless increases the profits of the best tournament, which is possible since the wage scheme becomes less risky. However, this is not always true for marginal increases of ambiguity aversion. The profits resulting from the best independent wage scheme decrease. In this particular example the profits of the tournament in fact exceed the best independent wage scheme after ambiguity aversion exceeds some limit. As a benchmark we have also included the profits from the optimal incentive scheme (without putting any restriction on the class of possible contracts). Again, for large levels of ambiguity aversion, the best tournament is much closer to the optimal wage scheme than the best independent wage scheme.

5 Conclusions and Robustness

We have argued that in many cases ambiguity aversion provides a stronger justification why a principal might set up a tournament as a way to provide incentives. In the existing literature, which stays within the expected-utility framework, tournaments are either not optimal (under risk aversion), or many possible incentive schemes are optimal (under risk neutrality). Under ambiguity aversion however, if agents are risk neutral, tournaments are always optimal, and most other schemes are not. Even if agents are risk averse, both ambiguity and ambiguity aversion will often make it relatively more attractive for the principal to choose certain tournaments over independent contracts. We conclude that, if agents are in fact ambiguity averse, one could expect that a principal chooses tournaments or schemes that are very similar to tournaments if the distribution of output levels is very ambiguous. In applications, this might more likely be true for white color workers, as opposed to blue collar workers who routinely face the same task. Indeed, it has often been argued that tournament like incentives (for instance via promotions) seem to play a more important role for white collar workers.

Note that we considered a particular class of agency problems to obtain these results. We will now address to which extent our findings rely on our assumptions about the nature of the problem.

5.1 Robustness

To show how much our results depend on our assumptions, we focus mostly on the question whether the principal can still design a tournament which is purely risky. In this case, tournaments would do well at least under risk neutrality.

As we have already discussed, we assumed that there are only two agents only for notational convenience: Also if there are many agents, it is possible to design a purely risky wage scheme. If all agents exert the high effort, the principal could still break ties in a way that all agents have the same probability of winning the tournament, irrespective of the actual probability distribution.

5.1.1 Non-Uniform abilities

We will now address to what extent our results remain valid in situations where there is a difference in abilities between the two agents. First, note that if we model changes in abilities as changes in the effort costs, our results will not be altered in important ways. It will still be possible to design an ambiguity free tournament. Of course, a symmetric wage scheme will not be optimal in this case, as the agent with higher effort costs needs to be compensated more to choose the high action. The situation is different if the two agents differ in the actions that they have available, that is if the high action of agent one results in different output distributions than the high action of agent two.

Even then it can be sometimes possible that a tournament results in an ambiguity free wage scheme. We focus on the case where this difference is perceived not to be ambiguous. That is, the probability distribution over output levels for the high-ability agent differs from its average always to the same extent as the probability distribution of the low-ability worker. Consider a 1-prize tournament (where ties are broken at random). We begin with the case of two outcomes. If we reinterpret $\bar{p}^L$ as the success rate attributed to the agent with the lower ability when she chooses the high action, it follows that her winning probability is still given by $P(\text{prize}|\text{low ability}) = .5 - .5(\bar{p}^H - \bar{p}^L)$ for all possible success rates. Hence, equilibrium wages are purely risky. Thus, in this case, tournaments remain a good reaction to ambiguity, at least if risk aversion is sufficiently small.

The situation becomes slightly more complicated if there are more than three outcomes. In this case, for any $\hat{p} \in M^H$ the probability to win for the agent with the low ability, assuming both agents choose the high action, becomes

$$P(\text{prize}|\text{low ability}) = 0.5 + 0.5 \sum_{i=2}^{I} \left(\bar{p}_i^L \left(\sum_{j=2}^{i-1} \bar{p}_j^H - \sum_{j=1+i}^{I} \bar{p}_j^H \right) + \left(\bar{p}_i^L - \bar{p}_i^H \right) \left(1 + \sum_{j=2}^{i-1} \hat{p}_j - \sum_{j=1+i}^{I} \hat{p}_j \right) \right). \tag{8}$$

In most cases, this expression is not constant as it depends on the deviations $\hat{p}_j$ around the average probability of the outcomes. Thus, a purely risky tournament need not exist. Through the last term, ambiguity enters with the difference in abilities. While future work might explore this point in greater detail, it seems that the resulting level of ambiguity may often be smaller than the ambiguity costs inherent in an independent wage scheme. Moreover, we could still explore the possibility to use handicaps to eliminate more ambiguity from the tournament.

Our results however may change quite considerably, if differences in abilities are perceived to be ambiguous. Any incentive scheme that uses relative performance measures is likely to be vulnerable to such kind of ambiguity. Similarly, if the agents do not face exactly the same task, or if they do not operate in the same environment, the agent may consider it to be ambiguous how their own outcome distribution compares to their competitors'. For instance, agent one may consider a scenario where agent two's outcome distribution is more favorable than the average, while she herself faces a below-average outcome distribution (and vice versa). But even in these cases, tournaments can still do better than independent schemes, as also independent schemes are exposed to ambiguity in this case.

5.1.2 Infinitely many outcomes

The case where there are infinitely many outcomes deserves special attention, as ties need not occur. If there are infinitely many outcomes, and all entertained outcome distributions are non-atomic, the introduction of ambiguity and any increase in ambiguity aversion can never decrease profits when the optimal tournament is used. If there are no ties, all tournaments that implement the high cost action are one-prize tournaments (and there is only one tournament that satisfies both constraints with equality), and thus our discussion of one-prize tournaments becomes a discussion of the optimal tournament. Thus, the following proposition can be derived based on arguments presented in section 4:

Proposition 9. *Suppose there are infinitely many outcomes and* $\bar{p}^H + \hat{p}$ *is atomless for all* $\hat{p} \in M^H$*. Then, implementation costs for the optimal tournament will (weakly) decrease, as ambiguity aversion or ambiguity is introduced.*

Regarding the comparison with independent contracts, also in this case, ambiguity aversion makes independent schemes worse off, whenever the high action is at least as ambiguous as the low action. Thus, in this case, it follows quite generally that ambiguity aversion favors the use of tournaments over independent wage schemes.

6 Appendix

Proposition 1

Since under risk neutrality, u can be assumed to be the identity function, the individual rationality constrain becomes

$$\sum_{i \in Q, j \in Q} \bar{P}(q_i, q_j; H, H') w(q_i, q_j) \geq \mathcal{A}^H + u_0 + c^H,$$

where $\mathcal{A}^H = \frac{1}{\alpha} \log \left(\int \exp \left[-\alpha \left(\sum_{i \in Q, j \in Q} (\bar{p}_i^a + \hat{p}_i^a)(\bar{p}_j^{a'} + \hat{p}_j^{a'}) - \bar{P}(q_i, q_j; a, a') u(w(q_i, q_j)) \right) \right] \right)$.

The left hand side equals the principal's implementation costs. Since the expectation (over μ) of $\sum_{i \in Q, j \in Q} [(\bar{p}_i^H + \hat{p}_i^H)(\bar{p}_j^h + \hat{p}_j^h) - \bar{P}(q_i, q_j; H, H)] u_{ij} = 0$, and $\exp(0) = 1$, according to Jensen's inequality the integral evaluates to a number larger than 1, so that $\mathcal{A}^H \geq 0$.

Thus implementation costs are at least $u_0 + c^H$, and they reach this lower bound whenever $\mathcal{A}^H = 0$. We now describe necessary and sufficient conditions which guarantee that $\mathcal{A}^H = 0$. For this purpose, we can assume that $u_{11} = 0$, since $\mathcal{A}^H$ is unchanged whenever a constant is added to all compensation levels. We focus on the case where M is finite. Due to Jensen's inequality, $\mathcal{A}^H = 0$

implies that $\sum_{i\in Q, j\in Q}[(\bar{p}_i^H + \hat{p}_i^H)(\bar{p}_j^H + \hat{p}_j^H) - \bar{P}(q_i, q_j; a, a')]u_{ij}$ must be constant over all elements in M. As $\bar{p}^H$ and $\bar{P}(q_i, q_j; a, a'))$ do not vary between elements in M^H, the condition becomes that $\sum_{i\in Q, j\in Q}(\hat{p}_j^H \bar{p}_i^H + \hat{p}_i^H \bar{p}_j^H + \hat{p}_i^H \bar{p}_j^H)u_{ij}$ is constant, and since $0 \in M^H$, this constant needs to equal zero. Rearranging this term, the condition becomes

$$\begin{aligned}
&\sum_{i\geq 2} \sum_{1<j<i} \left(\bar{p}_i^H \hat{p}_j^H + \bar{p}_j^H \hat{p}_i^H + \hat{p}_i^H \hat{p}_j^H\right)\left(u_{ij} + u_{ji} - (u_{1j} + u_{j1}) - (u_{1i} + u_{i1})\right) \\
&+ \sum_{i\geq 2} \left(2\bar{p}_i^H \hat{p}_i^H + (\hat{p}_i^H)^2\right)\left(u_{ii} - (u_{1i} + u_{i1})\right) \\
&+ \sum_{i\geq 2} \hat{p}_i^H \left(u_{1i} + u_{i1}\right)
\end{aligned} \tag{9}$$

A scheme with constant-sum property satisfies that $u_{ij} + u_{ji} = u_{11} + u_{11} = 0$, where the last equality results from our normalization. Thus, it follows that $\mathcal{A}^H = 0$ in such cases. To obtain an example where only constant-sum schemes can satisfy this equation suppose that M^H consists of at least the following elements: Assume that, for every pair (i, j) such that $1 \leq j < I$, M^H contains an element $\hat{p}^i$ which attributes 0 to all output levels except that it assigns the deviation ϵ to output level i and $-\epsilon$ to output level j. Also suppose that for every such pair M^H also contains $-\hat{p}^{ij}$. Comparing how the above sum evaluates for $\hat{p}^{i1}$ and $-\hat{p}^{i1}$, it follows that $u_{ii} = 0$. Consequently, comparing $\hat{p}^{i1}$ and $\hat{p}^{ij}$ it follows that $u_{ij} + u_{ji} - u_{1j} + u_{j1}$ must be zero, and then, considering $-\hat{p}^{i1}$ it follows that u_{i1+1i} =0. The only schemes that satisfies these conditions for all i and j are constant-sum schemes. [18]

Thus, if beliefs are diverse enough only constant-sum schemes can implement the first best. As we mentioned in the text, an example how this condition could fail is the case where a non-trivial event is unambiguous, i.e. $\sum_{i\in Q'} \hat{p}_i = 0$, where $Q' \subset Q$, and Q' is neither a singleton nor equal to Q. Since an independent scheme satisfies $u_{ij} = u_{i1}$ for all $1 \leq j \leq I$ (so that $u_{1j} = u_{11} = 0$), it can be easily verified that $u_{1j} = u_{11} = 0$. Thus, all but the last line of equation 9 vanish, and also the last line can be made to equal 0 by having u_{i1} constant on Q' and on $Q \setminus Q'$.

Now, we show that a course constant-sum tournament always provides the necessary incentives.

Since we assume that H is better for the principal, $\sum_i \bar{p}_i^H q_i > \sum_i \bar{p}_i^L q_i$. Thus, there must exists a $k > 0$ such that $\sum_{i>k} \bar{p}_i^H > \sum_{i>k} \bar{p}_i^L$ (otherwise, $\bar{p}^L$ would first-order stochastically dominate $\bar{p}^H$). Now suppose that the principal designs a coarse tournament that considers only whether or not each agents' output exceeds q_k or not (i.e the function f, used in the definition of a coarse tournament, is given by $f(i) = k$ for $i \leq k$and 0 otherwise). Then, the problem is essentially equivalent to the case of two outcomes discussed below, and in this case it is indeed possible to find a tournament that implements H.

Proposition 2

Proof. For the case of two outcomes, the IR constraint specializes to

---

[18] A possibility to verify for any given set of beliefs whether or not this condition holds, is the following. For any member $\hat{p}^H$ of M^H, define the vector $v(\hat{p}^H)$ whose first I-1 elements are of given by $\hat{p}^i$ (for $1 < i \leq I$), followed by I-1 entries of the form $\left(2\bar{p}_i^H \hat{p}_i^H + (\hat{p}_i^H)^2\right)$, while the remaining entries are $(\left(\bar{p}_i^H \hat{p}_j^H + \bar{p}_j^H \hat{p}_i^H + \hat{p}_i^H \hat{p}_j^H\right)$ for any combination of i and j that satisfies $1 < j < i$. Each such vector has $I(I+1)/2 - 1$ entries. Then, $\mathcal{A}^H = 0$ for no contract who is not a constant-sum scheme whenever the matrix composed of all vectors $v(\hat{p}^H)$ for $p \in M^H$ is has rank $I(I+1)/2 - 1$.

$$\bar{u} - \mathcal{A}^H \geq u_0 + c_H \qquad \text{(IR)} \tag{10}$$

where $\mathcal{A}^H = \frac{1}{\alpha} \log \left(\int \exp \left(-\alpha \left(\hat{p} u^{\Delta I} + (\bar{p}^H + \hat{p})(\frac{\sigma_H^2}{\bar{p}^H} - \hat{p}) u^{\Delta S} + (1 - \bar{p}^H - \hat{p})(-\frac{\sigma_H^2}{1-\bar{p}^H} - \hat{p}) u^{\Delta F} \right) \right) d\mu \right)$.

The incentive constraint becomes

$$\begin{aligned}(\bar{p}^H - \bar{p}^L) u^{\Delta I} - \bar{p}^L (\frac{\sigma_H^2}{\bar{p}^H} - \frac{\rho_{HL}}{\bar{p}^L}) u^{\Delta S} + (1 - \bar{p}^L)(\frac{\sigma_H^2}{1 - \bar{p}^H} - \frac{\rho_{HL}}{1 - \bar{p}^L}) u^{\Delta F} \\ -(\mathcal{A}^H - \mathcal{A}^L) \geq c_H - c_L. \qquad \text{(IC)}\end{aligned} \tag{11}$$

with $\rho_{HL} = \mathrm{E}_\mu [\hat{p}^H \hat{p}^L]$ and

$\mathcal{A}^L = \frac{1}{\alpha} \log \left(\int e^{\left(-\alpha \left(\hat{p}^L u^{\Delta I} + (\bar{p}^L + \hat{p}^L)(\frac{\sigma_H^2}{\bar{p}^H} - \hat{p}^H) u^{\Delta S} + (1 - \bar{p}^L - \hat{p}^L)(-\frac{\sigma_H^2}{1-\bar{p}^H} - \hat{p}^H) u^{\Delta F} \right) \right)} d\mu \right)$

Finally, note that the principal's implementation costs are given by:

$$\begin{aligned}(\bar{p}^H(1-\bar{p}^H) - \sigma_H^2) h \left(\bar{u} + (1 - \bar{p}^H) u^{\Delta I} + (1 - \bar{p}^H)(\bar{p}^H + \frac{\sigma_H^2}{\bar{p}^H}) \right) \\ +((\bar{p}^H)^2 + \sigma_H^2) h \left(\bar{u} + (1 - \bar{p}^H) u^{\Delta I} - \bar{p}^H (1 - \bar{p}^H - \frac{\sigma_H^2}{\bar{p}^H}) \right) \\ +((1 - \bar{p}^H)^2 + \sigma_H^2) h \left(\bar{u} - \bar{p}^H u^{\Delta I} + (1 - \bar{p}^H)(\bar{p}^H - \frac{\sigma_H^2}{1 - \bar{p}^H}) \right) \\ +(\bar{p}^H(1 - \bar{p}^H) - \sigma_H^2) h \left(\bar{u} - \bar{p}^H u^{\Delta I} - \bar{p}^H (1 - \bar{p}^H + \frac{\sigma_H^2}{1 - \bar{p}^H}) \right)\end{aligned} \tag{12}$$

Since h can be assumed to equal the identity function, the principal's implementation costs (according to equation (12)) equal $\bar{u} = u_0 + c_h + \mathcal{A}^H$, and thus they (weakly) exceed $u_0 + c_h$. They equal $u_0 + c_h$ iff $\mathcal{A}^H = 0$. Since the agent is strictly ambiguity averse, this is true iff the term in the integral defining $\mathcal{A}^H$ does not vary with $\hat{p}$ (almost everywhere if μ is not discrete). This is the case iff $\hat{p}(u^{\Delta I} - (\bar{p}^H - \frac{\sigma_H^2}{\bar{p}^H}) u^{\Delta S} + (1 - \bar{p}^H - \frac{\sigma_H^2}{1-\bar{p}^H}) u^{\Delta F}) - (\hat{p}^2 - \sigma_H^2)(u^{\Delta S} - u^{\Delta F}) = 0$ for almost all $\hat{p} \in M$.

If there are more than three elements in M (the support of μ), $\hat{p}^2 - \sigma_H^2$ has at least two distinct values. Thus it is necessary that both $u^{\Delta I} - (\bar{p}^H - \frac{\sigma_H^2}{\bar{p}^H}) u^{\Delta S} + (1 - \bar{p}^H - \frac{\sigma_H^2}{1-\bar{p}^H}) u^{\Delta F} = 0$ and $u^{\Delta S} - u^{\Delta F} = 0$. Given the second condition, the first condition becomes $u^{\Delta I} = (1 - \frac{\sigma_H^2}{\bar{p}^H(1-\bar{p}^H)}) u^{\Delta S}$, which can be verified to equal $u^{SS} = u^{FF}$.

Now assume that these two constraints are satisfied and in addition $\bar{u} = u_0 + c_H$. There exists a positive $u^{\Delta S}$ such that both constraints are satisfied: The IC simplifies to $(\bar{p}^H - \bar{p}^L) u^{\Delta S} + A^L \geq c_H - c_L$. Since $\mathcal{A}^L \geq 0$, the IC holds whenever $u^{\Delta S}$ is large enough. Hence, a tournament that satisfies the two conditions achieves the first best. For the other direction, it remains to show that there exists no non-positive $u^{\Delta S}$ that satisfies the two conditions along with the IC and the IR. After some reformulation, $\mathcal{A}^L = \frac{1}{\alpha} \log \left(\int \exp \left(-\alpha \left(\hat{p}^L u^{\Delta S} - \hat{p}^H u^{\Delta S} \right) \right) d\mu \right)$. Then the assumption that H results unambiguously in a higher success rate than L guarantees that $\mathcal{A}^L < |(\bar{p}^H - \bar{p}^L) u^{\Delta S}|$ so that no negative $u^{\Delta S}$ can satisfy the incentive constraint. Thus no other incentive scheme achieves the first best. $\square$

Proposition 4

Proof. As argued in the main text, we only need to show that the deviation to the low action does not change the agent's expected utility, if any of the three assumptions hold. Under ambiguity neutrality, if the agent deviates to the low action, her utility becomes (rearranging equation 2) $U(L,H) = \sum_I \sum_J \bar{p}_i^L \bar{p}_j^H u_{ij} + \sum_I \sum_J E_\mu[\hat{p}_i^L \hat{p}_j^H] u_{ij} - c^L$

The first and last term do not vary with mean-preserving ambiguity. Assuming, wlog, that the tournament pays 1 to the winner, -1 to the loser and 0 otherwise, the second term becomes $\sum_i \sum_{j<i} E_\mu[\hat{p}_i^L \hat{p}_j^H - \hat{p}_i^H \hat{p}_j^L]$. Since under assumption 3 and 4 $\hat{p}_i^L \hat{p}_j^H = \hat{p}_i^H \hat{p}_j^L$ while under assumption 5, $E_\mu[\hat{p}_i^L \hat{p}_j^H] = 0$, this second term is always 0 as well, even if ambiguity is introduced.

□

Proposition 6

Proof. Some algebraic manipulations reveal that the difference in implementation costs is given by

$$-\frac{\sigma_H^2 \left(2\sigma_H^2(1-3\bar{p}^W) + (1-3\bar{p}^W)^2 + (\bar{p}^W)^2\right)}{2(1-3\bar{p}^W)(1-3\bar{p}^W)}$$

where here $\bar{p}^W = \bar{p}^H(1-\bar{p}^H)$ referring to the winning probability without ambiguity. This term is clearly negative, and the benefits from the high cost action remain unchanged. □

Observation 3

Proof. (sketch) By taking the first two derivative of $\mathcal{A}^H$ w.r.t $1-\bar{p}^H-\bar{p}^L$ we show that this term is at a local minimum at 0. Also, in this variable $\mathcal{A}^H$ is symmetric if ambiguity is symmetric. Thus ambiguity in H is lower if $1-2\bar{p}^H$ is closer to zero then $1-\bar{p}^H-\bar{p}^L$ □

Proposition 8

Proof. Let $f(u^{\Delta T}) = 1 - 2\bar{p}^H - \frac{\mathcal{A}^{H\prime}(u^{\Delta T}) - \mathcal{A}^{L\prime}(u^{\Delta T})}{\bar{p}^H - \bar{p}^L}$. The first derivative of the objective function (5) can be rearranged to

$$\begin{gathered}\left(\mathcal{A}^{H\prime}(u^{\Delta T})^2 + \bar{p}^W\left(\left(2(1-2\bar{p}^W) + (f(u^{\Delta T}))^2\right)\right)u^{\Delta T}\right. \\ +2\bar{p}^W \frac{(c^H - c^L) f(u^{\Delta T})}{\bar{p}^H - \bar{p}^L} + \mathcal{A}^{H\prime}(u^{\Delta T})(u_0 + c_H).\end{gathered}$$

Observe that $f(u^{\Delta T})$ and $\mathcal{A}^{H\prime}(u^{\Delta T})$ depends only on the sign of $u^{\Delta T}$, and all terms premultiplying $u^{\Delta T}$ are positive, so that the objective function is convex if limited to either positive or negative values of $u^{\Delta T}$. We will now show that given the assumptions in the proposition, the objective function is decreasing in $u^{\Delta T}$ whenever $u^{\Delta T} < 0$. To do so it suffices to show that $f(u^{\Delta T})$ and $\mathcal{A}^{H\prime}(u^{\Delta T})$ are negative for any negative value of $u^{\Delta T}$. Now we need to identify which element of M is used to compute $\mathcal{A}^{H\prime}(u^{\Delta T})$ and $\mathcal{A}^{L\prime}(u^{\Delta T})$. For negative $u^{\Delta T}$ the problem of finding the maximizer in M is convex for both actions, resulting in a corner solution. As we assume M is centred around 0, it is easy to verify that the largest element of M (not the smallest), is selected. We label this element $\hat{p}^*$. Then $\mathcal{A}^H(u^{\Delta T}) = 2((1-2\bar{p}^H)\hat{p}^* - (\hat{p}^*)^2 + \sigma_H^2)]u^{\Delta T}$ and $\mathcal{A}^L(u^{\Delta T}) = 2((1-\bar{p}^H-\bar{p}^L)\hat{p}^* - (\hat{p}^*)^2 + \sigma_H^2)]u^{\Delta T}$. Consequently, $\mathcal{A}^{H\prime}(u^{\Delta T}) < 0$ and $\mathcal{A}^{H\prime}(u^{\Delta T}) - \mathcal{A}^{L\prime}(u^{\Delta T}) = -2\hat{p}^*$. Thus $f(u^{\Delta T}) < 0$ as well.

Now suppose $u^{\Delta T} > 0$. Now, the problem of identifying the worst deviation in M (in order to compute $\mathcal{A}^H(u^{\Delta T})$) is concave. If $\hat{p}^{**}$ denotes the smallest deviation in M, the worst deviation in M is given by by $\max\{\hat{p}^{**}, .5(1-2\bar{p}^H\}$, if agent one chooses the high cost action. Thus, $\mathcal{A}^{H\prime}(u^{\Delta T}) > 0$. For the low cost action, if $1-\bar{p}^H-\bar{p}^L \geq 0$, the worst possible is given by $\max\{\hat{p}^{**}, .5(1-\bar{p}^H-\bar{p}^L\}$, and if $1-\bar{p}^H-\bar{p}^L > 0$, the maximizer is $\min\{\hat{p}^{**}, 5.(1-\bar{p}^H-\bar{p}^L\}$. In any case, it follows either from $\bar{p}^H > 2/3 - pl/3$ or from assuming that M is centred at 0, that $\mathcal{A}^H(u^{\Delta T}) - \mathcal{A}^L(u^{\Delta T}) > 0$. However, $f(u^{\Delta T})$ remains negative - based on our assumptions, it can be verified that it cannot exceed $f(u^{\Delta T}) \leq -.5(\bar{p}^H - \bar{p}^L)$. From considering the first derivation, we see that the objective function remains increasing for all $u^{\Delta T}$ close to zero if $2\bar{p}^W \frac{(c^H-c^L)f(u^{\Delta T})}{\bar{p}^H-\bar{p}^L} + \mathcal{A}^{H\prime}(u^{\Delta T})(u_0+c_H)$, which is true if either u_0 becomes large enough or $c^H - c^L$ becomes small enough.

Otherwise the FOC indeed identifies a minimum at

$$u^{\Delta T**} = -\frac{2\bar{p}^W \frac{(c^H-c^L)f(u^{\Delta T})}{\bar{p}^H-\bar{p}^L} + \mathcal{A}^{H\prime}(u^{\Delta T})(u_0+c_H)}{\mathcal{A}^{H\prime}(u^{\Delta T})^2 + \bar{p}^W((2(1-2\bar{p}^W) + (f(u^{\Delta T}))^2)} < \frac{(c^H-c^L)}{\bar{p}^H-\bar{p}^L} \frac{-f(u^{\Delta T})}{(1-2\bar{p}^W) + .5(f(u^{\Delta T}))^2}.$$

Now define $g(x) = \frac{(c^H-c^L)}{\bar{p}^H-\bar{p}^L} \frac{x}{(1-2\bar{p}^W)+.5x^2}$. It is easy to verify that $g(-f(u^{\Delta T})) = u^{\Delta T**}$ and $g(-(1-2\bar{p}^H) = u^{\Delta T*}$. Also, $-(1-2\bar{p}^H) > f(u^{\Delta T})$ (since $\mathcal{A}^H(u^{\Delta T}) - \mathcal{A}^L(u^{\Delta T}) > 0$) given the assumptions of our case. Since g is decreasing for all $x \in [0,1)$, $u^{\Delta T**} < u^{\Delta T*}$ as desired.

□

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