The Premium for Hedge Fund Lockups

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What excess return should a fund of funds expect for taking the risk of investing in a hedge fund with an extended lockup?

In this note we analyze a model for estimating the premium for long-term lockups. Because there is a persistence to the quality of hedge fund returns – good funds tend to continue to do well, and bad ones badly – a lockup deprives an investor of the opportunity to disinvest from bad funds and reinvest in good ones. The value of that foregone opportunity is the expected premium for submitting to the lockup.

We find the premium to be of order 1% for a two-year lockup over a one-year lockup. The increase in the premium for longer lockups does not increase in proportion to the length of the lockup, but approaches a constant value of about 3% per year.

Section 1. Introduction

As the capital invested in alternative strategies has increased, hedge funds have tended to incorporate increasingly longer lockups, the period during which an investor in the fund is unable to withdraw his capital.This illiquidity comes at a cost, the inability to invest one's money in other ways. What return premium should a fund of funds, faced with a variety of hedge fund alternatives, expect for giving up liquidity during the lockup period.

In this paper we group hedge funds into three categories: Good, Sick and Dead. Good funds perform acceptably to well, Sick funds underperform, and Dead funds have gone out of business owing to extreme underperformance relative to other funds in the same strategy. We then show that there is a persistence to hedge fund returns so that Sick/Good funds in a given year are likely to underperform/overperform in the following year too. We view the lockup and its illiquidity as an option foregone, the option to disinvest from Sick funds. One can estimate the value of the premium necessary to compensate a manager for the loss of this option.

Section 2. The Option to Disinvest

2.1 The Expected Premium

A typical fund of funds monitors its portfolio, deciding which additional funds to invest in, which to continue with and which to disinvest from.

Consider for simplicity a fund of funds invested in hedge fund A_1 with a specific strategy (equity long/short, for example) and a one-year lockup. Suppose that the portfolio manager of the fund of funds expects a one-year return μ_1 from investing in A_1. After one year the manager has a choice: stick with fund A_1 for another year, or liquidate the investment and reinvest in a different fund.

Suppose that you can identify Sick funds at the end of one year[1], and that Sick funds in one year tend to remain Sick or die in the following year. You can disinvest from them and so expect to earn a return $\mu_1 + \alpha$ in the second year, producing a total two-year return $2\mu_1 + \alpha$. Now consider a fund A_2 similar to A_1 except that it imposes a two-year lockup. Call its expected return μ_2. Because of the lockup, the expected return for two years is μ_2. In order for A_2 to be competitive with A_1, we require that $2\mu_2 = 2\mu_1 + \alpha$, so that the excess expected return for the two-year lockup is $\mu_1 + \alpha/2$.

In the following section we develop a simple model for the value α.

2.2 The Option Model

What would you do if you were a manager of a portfolio of hedge funds with a chance to re-evaluate your investment in A_1 at the end of one year? You would probably compare the return of A_1 to that of similar funds and, based on some criteria, decide whether the fund is Sick or Good. If the fund is judged to be Sick, you would disinvest; if it is Good, you would roll over your investment for one more year[2]. In what follows, we define Sick and Good in terms of the fund's historical returns, but the same method of finding the value of the option to disinvest would work for other definitions of Sick, where Sick means a fund whose future performance can be expected to be below average. Such a judgement could be based on qualitative considerations of the fund rather than strictly statistical ones applied to its returns.

Consider the distribution of returns for funds that have the same strategy as A_1 at the end of the first year, as illustrated schematically in Figure 1. Funds whose realized returns lies within the left-hand tail of the distribution, below level B, the fund will probably die, that is shut down, either due to the withdrawal of funds by disappointed investors or due to the fund managers throwing in the towel in the knowledge that they would be unlikely to ever rise about their high water mark again. Investors will therefore receive their share of the fund's NAV on liquidation, and have no option but to reinvest it in a different fund in the subsequent year.

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1. We will define exactly what we mean by Sick Good and Dead shortly.
2. In this simple model we assume that the portfolio manager does not decide to change his allocation to the particular strategy in general, but only to particular hedge funds within that strategy

If fund A_1's return lies in the intermediate band above the extreme left tail – that is between B and U – we assume that the portfolio manager will choose to disinvest from A_1 in order to reinvest in a potentially better performing fund. This is where all the optionality of the portfolio manager lies.

Finally, if A_1's return lies to the right of both these bands – that is, above the level U, the portfolio manager will likely continue his investment in A_1 for one more year.

The actions a portfolio manager can take after one year of investment in A_1 are summarized in Table 1. We will discuss plausible values for B and U later.

Figure 1. The Distribution of One-Year Hedge Fund Returns

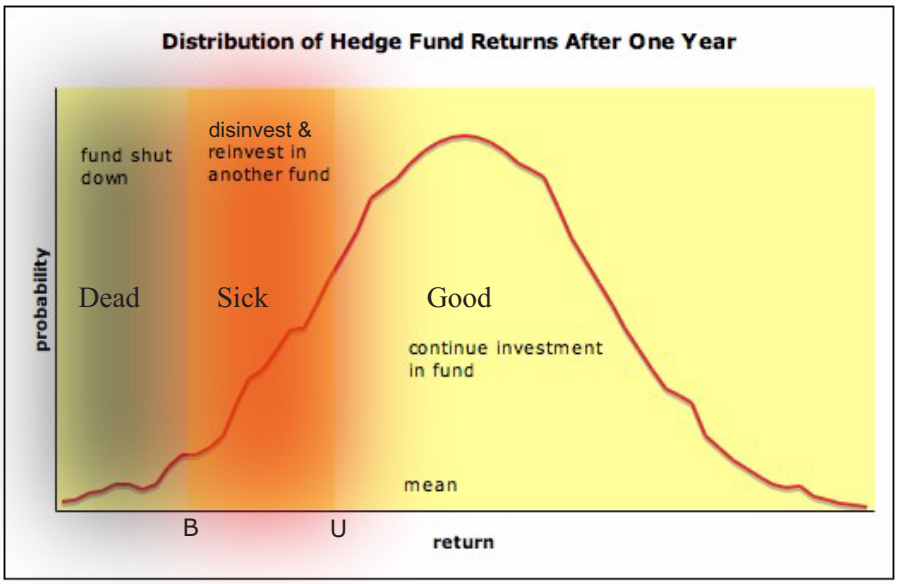


Table 1: Portfolio Manager's Choice After a One-Year Lockup in A_1

Return of A_1	Consequence	Action	Option
Dead: returns below B	Fund shuts down	reinvest returned money elsewhere	No
Sick: returns between B and U	Fund is sick but continues operating	Disinvest from A_1 and reinvest elsewhere	Yes
Good: returns above U	Fund continues operating	No need to reinvest	No

In this model, all the optionality involved in the one-year lockup lies in the portfolio manager's choice to disinvest from badly performing funds and then reinvest in a better performing funds. This option has value if the portfolio manager can reliably identify Sick funds whose future performance will be below average.

Section 3. The Persistence of Hedge Fund Returns

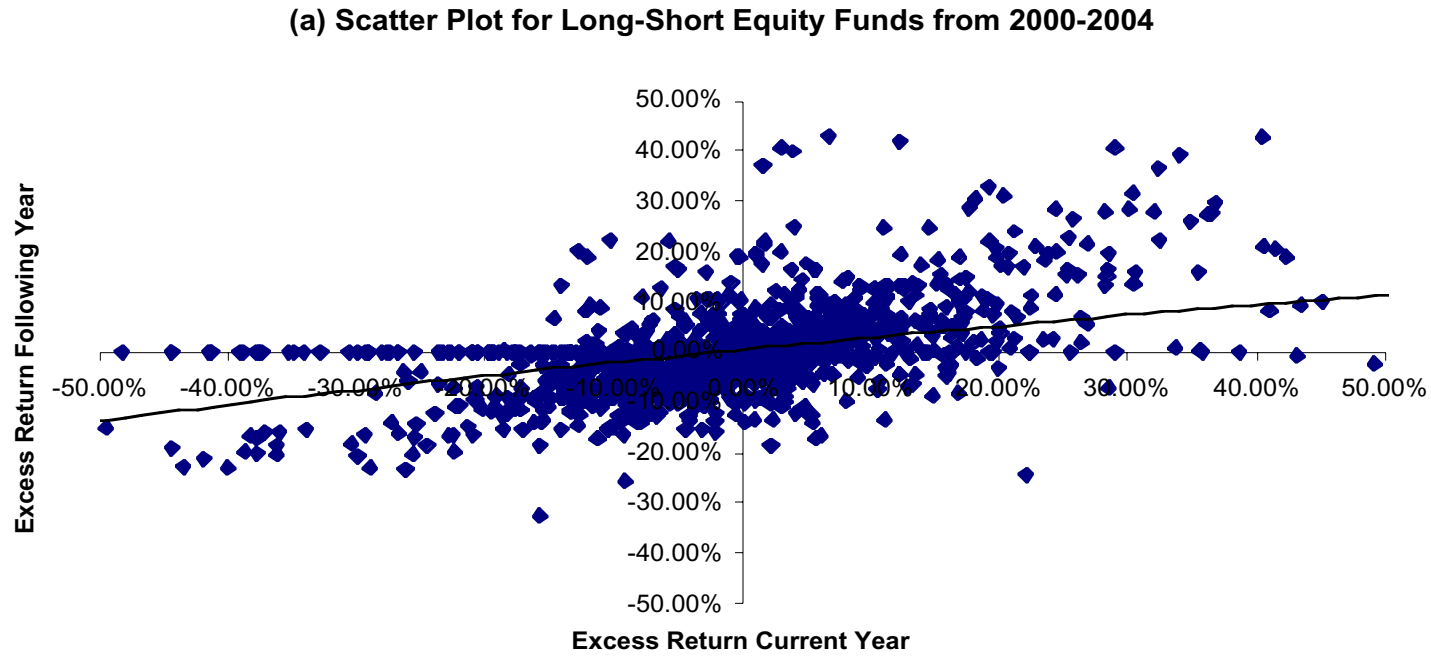
In this section we present qualitative evidence for the persistence of hedge fund performance[3].The return data is taken from the TASS database.

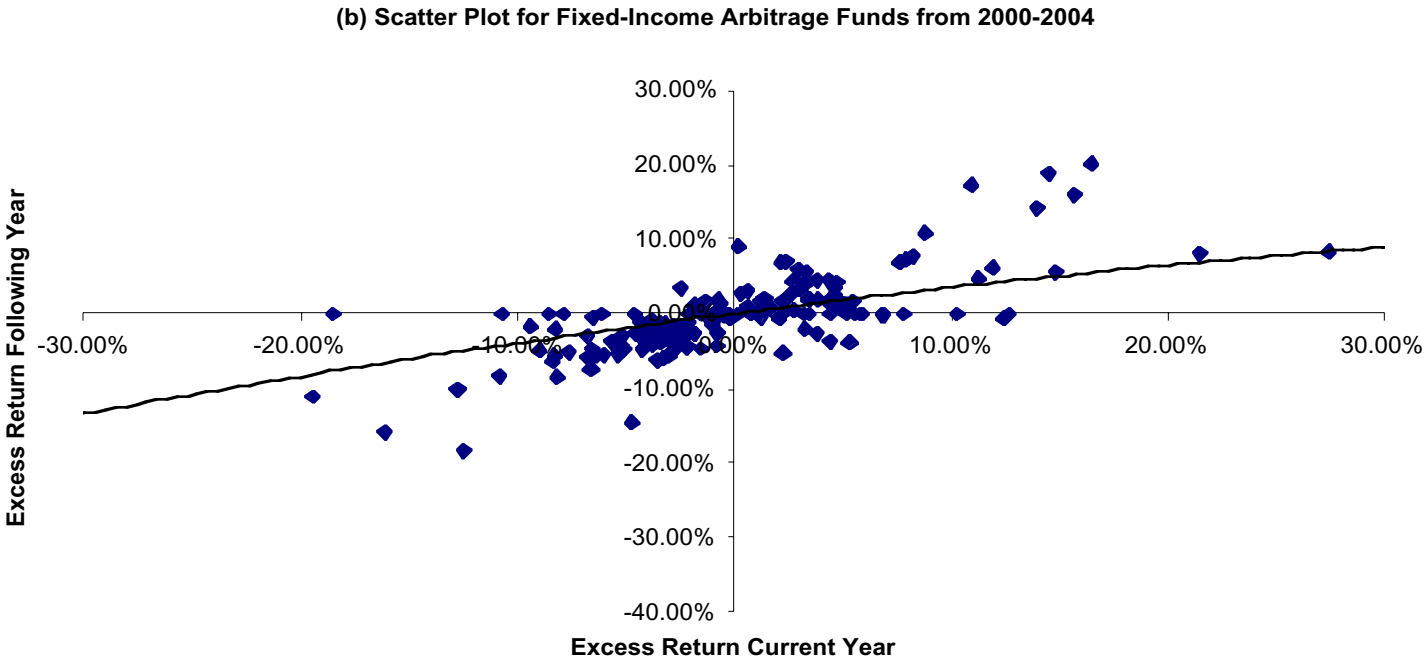
Figure 2(a) displays the relation between the successive annual returns of Long/Short Equity hedge funds from 2000 - 2004. You can see that funds with high/low returns in a given year tended to have high/low returns in the subsequent year. There is a tendency for the quality of returns to persist. Figure 2(b) shows the same effect for Fixed-Income Arbitrage funds.

A similar trend is noticeable for many other hedge fund strategies. Though some strategies show less persistence than others, and though persistence varies from year to year, we conclude that, overall, better-than-average funds are likely to do better, and worse-than-average funds are likely to do worse.

3. For a thorough analysis of persistence in hedge fund returns see Jagannathan, Malakhov and Novikov, *Do Hot Hands Persist Among Hedge Fund Managers? An Empirical Evaluation,* NBER Working Paper No. 12015, February 2006, and the papers cited therein.

Figure 2. (a) The relation between the returns of Long/Short Equity Funds from year to year. (b) The relation between the returns of Fixed-Income Arbitrage funds from year to year.





Section 4. A Model of Hedge Fund Behavior

In this section we describe a model for hedge fund evolution that can be used to quantify the qualities Good, Sick and Bad. We can then use this model to estimate the excess return expected for a longer lockup.

We begin by stating the following stylized facts about hedge funds.

1. Individual hedge fund volatilities tend to be low compared to those of stocks, and vary by strategy. We assume a volatility of 7% for individual funds in Long/Short Equity, 10% for Macro and 5% for Multi-Strategy and Event Funds.

2. Hedge funds seem to have a half-life of about five years, corresponding to a decay rate of about 13%. However, not all of the funds that vanish each year die owing to lying on the left tail of the return distribution. Some funds continue to do business but become closed to new investors and stop reporting. Others simply go out of business due to a series of mediocre returns that provide them with no incentive fee. The number of funds that enter the realm of the Dead each year due to annual returns that lie in the far left tail appears to be in the neighborhood of 3%.

3. The levels B and U in Table 1 that determine a fund's death or sickness are estimated as follows. We assume B and U are determined by the relative performance of a fund compared to the performance of other funds in the same strategy in the same year, so that B and U are levels of relative return. We will take the level B to correspond to approximately 20 percentage points below the mean of the returns of similar funds; that is, we assume that if a fund's return in a given year lies more than 20 percentage points below the mean of its cohort, the fund will die and return each investor's share of the final NAV on liquidation. Similarly, we take the level U to correspond to a return of about 10 percentage points below the mean. Funds whose relative returns lie between 10 and 20 percentage points below the mean are Sick, and we assume the portfolio manager will disinvest and reinvest in a similar fund.

   Alternatively, one could also interpret B and U as percentile levels. With this view, funds whose relative returns lie below the B^{th} percentile will shut down; portfolio managers will withdraw their investment in funds whose relative returns lie between the B^{th} and U^{th} percentile. Reasonable value for B and U in this framework are 25 and 35 respectively.

4. The NAV of a fund evolves smoothly from month to month most of the time, with occasional sharp downward moves. A fund dies through a sequence of several sharp downward monthly moves in its NAV.

We incorporate these facts into our model as follows. The low volatility of hedge funds implies that hedge funds cannot die at a rate of about 3% if their value evolves by smooth diffusion. We therefore assume that hedge fund values undergo a jump-diffusion process. The diffusion process in our model is a geometric Brownian motion with expected return μ and a low volatility σ; the jump is a Poisson process with a jump probability per year of λ, and a jump magnitude that follows a lognormal distribution with a mean of – J% with a standard deviation of 2 percentage points. In order to account for the serial correlation of jumps that characterizes Sick and dying funds, we assume that once a fund has suffered one downward jump, it is more likely to become Sick, so that the probability of a jump must then increase, typically to a rate of order 5λ. In order to calibrate to a fund death rate of about 3%, we find $\lambda \sim 1$ and $J \sim 4$. Since jumps are the predominant cause of both sickness and death, this calibration to the death rate also deter-

mines the probability of a fund's return lying between B and U at the end of a given year, that is, of being Sick.

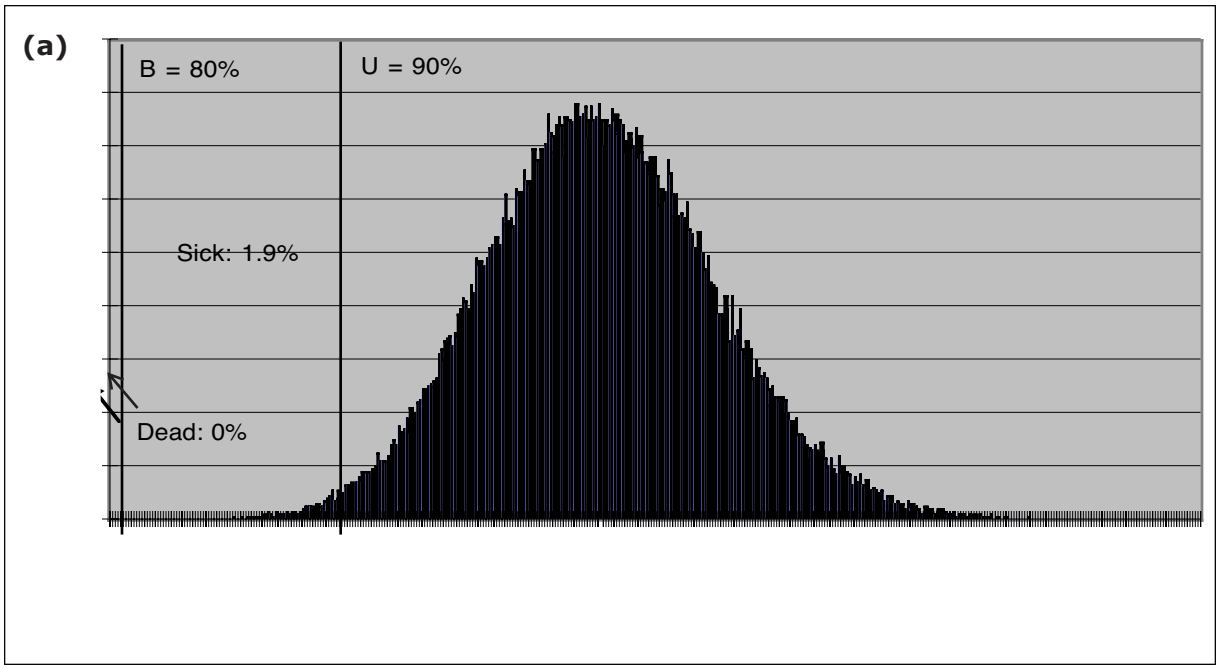
We assume that the portfolio manager of a fund of funds will withdraw his money from Sick funds because of the likelihood that they will remain Sick or die. We assume that the manager will invest the funds withdrawn in a similar Good fund that, because of persistence, can be expected to generate a higher expected return than a Sick fund.

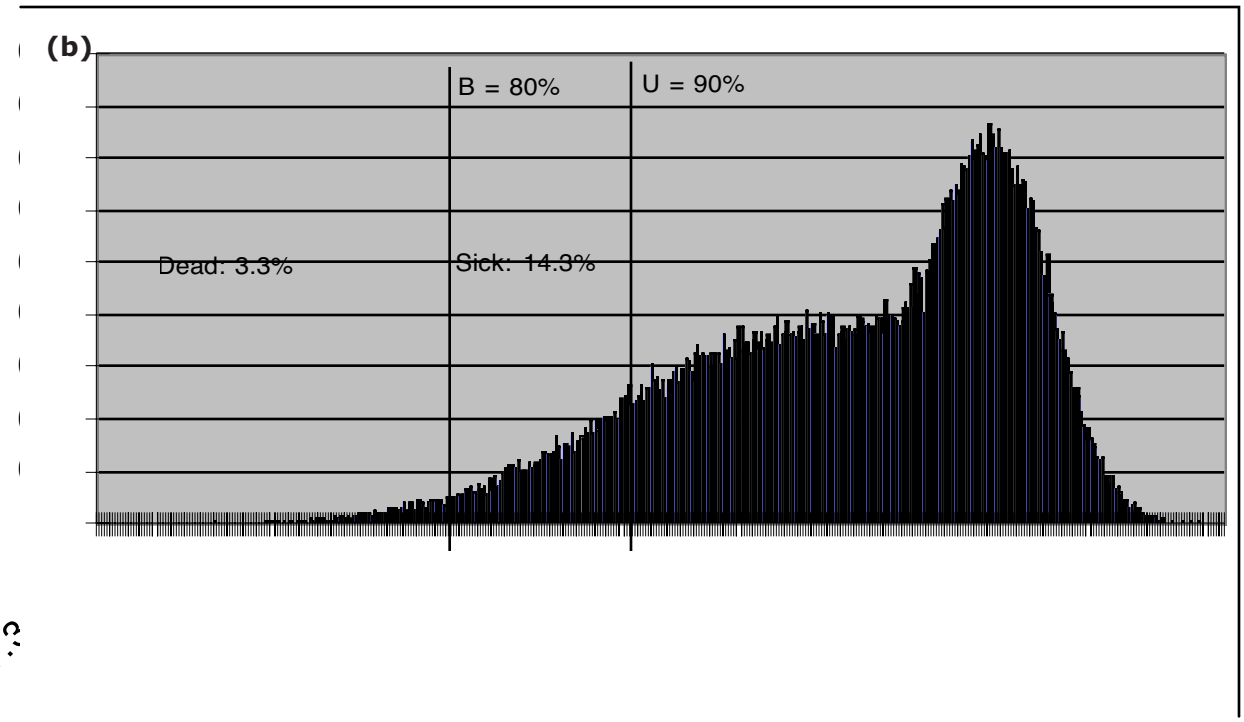
Figure 3 displays plots of the distributions of hedge fund relative values and returns after one year in this model. In Figure 3a the hedge fund diffuses with a drift of 8% and a volatility of 5% with no jumps. No funds fall below B = 80% of the expected value, and hence this version of the model cannot match the 3% rate at which funds appear to die. Figure 3b displays a similar distribution when the jump rate after an initial jump is chosen to be 5 jumps per year. Then the proportion of funds that fall below B = 80% is 3.3%, roughly matching the observed rate of death of hedge funds. In this case 14.3% of funds are Sick after one year, with returns lying between U and B.

One can repeat the analysis for a jump rate after of 2.5 jumps per year after the initial jump. We then characterize Sick funds by U=95% and B = 85%, a less forgiving disinvestment criterion. Now the proportion of funds that die is 3.9% and the proportion that are Sick is 21.3%, providing a substantially greater option value for withdrawal and reinvestment in other Good funds.

To summarize, in this jump-diffusion model of hedge fund evolution, when we match the roughly 3% death rate of hedge funds by means of serially correlated jumps of magnitude –4%, then approximately 10% – 20% of funds are candidates for disinvestment by virtue of their annual performance. These numbers are consistent with the experience of fund of funds managers, who indicate that they disinvest from about 10% of their funds annually.

Figure 3. (a) Distribution of fund relative values after one year for an initial fund value of 100, volatility of 5%, and no jumps. U = 90% and B = 80%. (b) Distribution of fund relative values after one year for an initial fund value of 100, volatility of 5%, one jump per year and a Sick fund rate of 5 jumps per year. U = 90% and B = 80%.





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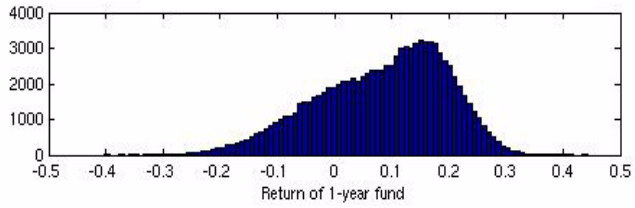
Section 5. The Premium for a Lockup

Figure 4a displays the distribution of relative annual returns after two years for a fund with rolling one-year lockups, assuming evolution as specified in Figure 3b. After one year 14% of funds are Sick and their NAV is reinvested in Good funds. Figure 4b displays the two-year distribution of funds with a two-year lockup in which no disinvestment occurs at the end of the first year. The distribution of returns for the funds invested in the two-year lockup has a lower left tail owing to the underperformance of Sick funds in the second year.The mean two-year return in Figure 4a is 1.7% greater than that of Figure 4b. We must therefore expect an annual excess return of at least 0.85% over a one-year lockup in order to find the two alternatives equally attractive.

Figure 4. (a) The distribution of returns relative to the mean after rolling one-year lockups for the fund of Figure 3 b. After one year the money invested in both Sick and Dead funds has been withdrawn and reinvested in Good funds. (b) The distribution of two-year relative returns for similar funds with a two-year lockup. The mean two-year return of rolling one-year lockups for two years is 1.7% greater than the mean return of a two-year lockup.

(a) Two rolling one-year lockups



(b) One two-year lockup.

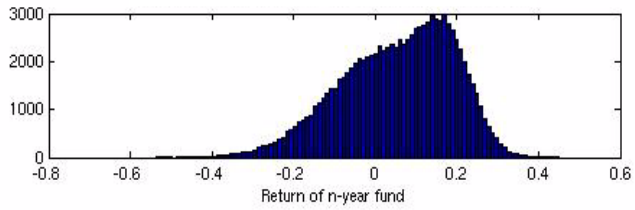


Table 2 displays the expected excess annual returns for an N-year lockup over the average annual return of rolling one-year lockups for N years, assuming that both investments must provide the same expected return. Note that the excess return increases monotonically wit the length of the lockup, though the rate of growth slows.

In the next section we provide a more intuitive explanation of this pattern.

Table 2: The Jump-Diffusion Model of Hedge Fund Evolution. Excess Return for Longer Lockups in when Compared with Rolling One-Year Lockups for the Funds Described in Figure 4b.

Lockup (years)	Excess Annual Return (%)
2	0.9
3	1.3
4	1.6
5	1.9
7	2.2
10	3.0

Section 6. A Simple Markov Model for Hedge Fund Evolution

One can understand the results of the previous section by modeling the evolution of hedge funds as a simple Markov process.

Suppose that a hedge fund at the end of any given year can be in one of three states: G for Good, S for Sick and D for Dead. We define Sick funds as funds that can be identified as likely to underperform. Then, each year, Good funds can become Sick, whereas Sick funds can recover, remain Sick or die. Table 3 shows an assumed set of transition probabilities between states: a Good fund has a 10% probability of becoming Sick in the following year; a Sick fund has a 50% probability of recovering and becoming Good again, a 20% probability of remaining Sick, and a 30% probability of becoming Dead.

Table 3: The Transition Matrix for Hedge Funds

	G	S	D
G	0.9	0.1	
S	0.5	0.2	0.3
D			

Table 4 displays the evolution of an ensemble of 10,000 hedge funds over five years, assuming that all hedge funds are initially Good. Notice that within a few years the rate of dying converges to 3.5% and the rate of becoming Sick to 12.6%.

Table 4: Annual Evolution of Hedge Funds

Year	Good Funds	Sick Funds	No. of Funds that Died During Previous Year	Rate of Dying	Rate of Becoming Sick
0	10000				
1	9000	1000	0	0.0%	10.0%
2	8600	1100	300	3.0%	12.2%
3	8290	1080	330	3.4%	12.6%
4	8001	1045	324	3.5%	12.6%
5	7723	1009	314	3.5%	12.6%

Suppose a fund of funds manager invests in funds that have rolling one-year lockups. Each year he can reassess his investments and divest from Sick funds, thereby reinvesting 12.6% of his portfolio into Good funds. Now suppose that because of the demonstrated persistence of returns, the 12.6% of the portfolio invested in Good funds outperform the Sick funds by 15 percentage points. Then, in each year after the

first, the rolling portfolio can be expected to outperform the locked-up portfolio by $12.6\% \times 15 \text{points} \approx 1.9\%$.

The second column in Table 5 shows the excess annual return necessary to compensate for the value of the option to reinvest in Good Funds. At the end of the second year the cumulative value is 1.9%, or 0.85% per year. At the end of the third year the loss is (3.8 / 3)%, or 1.3% per year. Asymptotically, as the length of lockup becomes large the cumulative loss per year approaches 1.9%. The monotonic growth of the excess expected annual return with the length of the lockup in Table 5 matches the behavior of the excess returns of Table 2.

Given the persistence of above average and below average returns, a fund of funds manager can do even better by divesting from Sick funds at the end of each year and reinvesting the 12.6% of his portfolio into Very Good funds, i.e. funds that consistently tend to produce above average returns. Suppose that Very Good funds outperform Sick funds by 25 percentage points rather than 15 points, The resultant excess returns are 1.67 times higher, as displayed in the third column of Table 5.

Table 5: Excess Expected Return for an N-year Lockup in a Markov Model.

Lockup (years)	Excess Annual Return from Reinvesting in Good funds (%)	Excess Annual Return from Reinvesting in Very Good funds (%)
2	0.9	1.5
3	1.3	2,2
4	1.4	2.4
5	1.5	2.5
7	1.6	2.7
10	1.7	2.8

Conclusion

There is a demonstrable persistence to the returns of hedge funds. Good funds tend to continue to outperform the mean return in their strategy cohort from year to year, and bad funds continue to underperform. Therefore, fund of funds managers can improve their performance by identifying and disinvesting from Sick funds each year. As a result, the excess expected annual return for a fund with a two-year lockup is of order 1% more than the expected annual return for a similar fund with rolling one-year lockups. The increase in expected annual return is monotonic in the length of the lockup, but increases at a less than proportional rate to the length of the lockup.

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