Strategic delegation and voting rules

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Abstract

The selection of political representatives depends on the political system. Principals, such as voters or districts, may benefit by strategically electing representatives different from themselves. While a status-quo biased delegate may be a better negotiator, an enthusiastic representative has a better chance of being included in the majority coalition. A larger majority requirement leads to "conservative" delegation and hence a status quo bias; a poor minority protection does the opposite. Through strategic delegation, the political system also determines whether centralization or decentralization is beneficial.

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1. Introduction

Political decisions are made by delegates, not the citizens themselves. In most legislatures, every district is represented by a delegate who, on its behalf, negotiates and votes on whether certain policies should be approved. Each district may have an incentive to strategically elect a representative that is biased one way or the other. What determines the incentives to delegate strategically? Do they depend on the political system? Can institutions be designed to ensure "optimal" delegation?

Strategic delegation may be costly from a social point of view: If the delegates are "conservative" (status quo biased), they tend not to implement projects even if they are socially optimal. If, instead, the delegates are "progressive" (public-good lovers), they implement projects even if these are too costly. Strategic delegation may thus separate voters' preferences from those of the politicians. It is thus highly important to understand when and how voters strategically appoint representatives.

Unfortunately, there are contradictions in the literature on delegation. Starting with Schelling (1956), a large bargaining literature shows how principals delegate to status quo biased agents to gain "bargaining power". Such agents are less desperate in reaching an agreement and, therefore, able to negotiate a better deal.

On the other hand, a more recent literature in political economy argues that "voters attempt to increase the probability that their district is included in the winning coalition by choosing a representative who values public spending more" (Chari et al., 1997, p. 959). The majority coalition will typically consist of the winners, i.e., the representatives who are least costly to please (as in Ferejohn et al., 1987). And, being a member of the majority coalition is important, since this shares the surplus and expropriates the minority whose votes it does not need. To increase the "political power" (the probability of being a member of the majority coalition), districts should therefore delegate progressively — not conservatively.

This paper captures both the incentives to delegate conservatively (to gain bargaining power) and progressively (to gain political power). In equilibrium, the direction of delegation depends on which concern is stronger and this, it turns out, depends on the political system. In particular, if the majority requirement is small, however, the majority coalition expropriates a large minority, and divides the revenues on just a few majority members. Political power is then very beneficial, and districts delegate progressively, as argued by Chari, Jones and Marimon.
To return to the initial questions, strategic delegation does indeed depend on the political system; the voting rule in particular. But the strategic choice of delegate depends on several other parameters, as well. Some of these are details of the legislative game, such as the minority protection, the agenda-setting power, the majority coalition’s discipline and its stability. The characteristics of the political issue are also important, such as the heterogeneity, the expected value of the collective project, and its variance. In every case, the first-best can be achieved by carefully selecting the majority requirement.

Strategic delegation has also consequences for the optimal allocation of authority across alternative institutions. If the voting rules cannot be changed, they determine whether an issue ought to be centralized or decentralized to local governments.

The results are important for understanding empirical observations, since voting rules do differ across both countries and political chambers within the same country.1 The observation might also be important for the EU, applying various rules for different decisions and political chambers. While a serious test of the theory must await future research, the concluding section discusses some anecdotal evidence.

After a further discussion of the literature, the following section presents the simplest version of the model. Solving the game by backward induction, Section 4 shows how the districts have incentives to either delegate conservatively or progressively, depending on the political issue and the political system. The optimal majority rule balances the strategic concerns, and induces a first-best selection of projects. Section 5 applies the model to shed light on the trade-offs between centralization and decentralization, while Section 6 generalizes the legislative game by discussing the possibility to tax, minority rights and coalition stability. The final section concludes, while Appendix A contains all the proofs.

### 2. Related literature

As noticed above, there is a controversy in the literature on delegation. Starting with Schelling (1956), a large bargaining literature shows how principals delegate to status quo biased agents to gain “bargaining power”. Schelling’s argument is formalized by Jones (1989) and Segendorff (1998) in two-player games. Elsewhere (Harstad, 2008), I show how this argument hinges on the existence of side transfers, perhaps making transfers harmful. Milesi-Ferretti et al. (2002) compare majoritarian and proportional systems where three districts delegate to gain bargaining power. With one-dimensional policies, single-peaked preferences and without side payments, Klumpp (2007) shows that voters may delegate to status-quo biased representatives to make their acceptance sets smaller. An n-person bargaining game is studied by Brückner (2003); he finds that the bias may be mitigated by relaxing the unanimity requirement. Besley and Coate (2003) study strategic delegation in a context where two districts maximize joint utility. In a similar model, Dur and Roelfsema (2005) show that the direction of delegation may go either way, depending on the cost-sharing rules.

Much of the political economy literature goes the other direction, however, arguing that voters may want to delegate to (“progressive”) public good lovers since these are likely to be included in the winning coalition (Chari et al., 1997; Ferejohn et al., 1987); Austen-Smith and Banks (1988) and Baron and Diermeier (2001) show how voters consider the induced coalition-formation when electing representatives, although bargaining power is not considered. The trade-off between bargaining power and political power is apparent in the seminal contribution of Baron and Ferejohn (1989). In numerical examples, they show that a high probability of being recognized as the next agenda-setter makes the legislator less attractive as a coalition-partner. However, the trade-off is not explicitly discussed and they do not study strategic delegation. Recently, Christiansen (2009) does allow for both conservative and progressive delegation in the Baron–Ferejohn bargaining model. In equilibrium, however, delegation is never conservative since types are binary and deterministic in his model.

The model below compares centralization and decentralization. So do Besley and Coate (2003), and their “non-cooperative centralization” corresponds to my own definition of centralization. But they assume decentralization prevents any bargaining, and their “cooperative centralization” is a situation where the legislature simply maximizes the delegates’ total utilities. In contrast, this paper allows for cooperation even under decentralization. Gradstein (2004) studies how the threat of decentralization (or secession) affects delegation, while Lorz and Willmann (2008) let districts delegate before negotiating whether to centralize in the first place.

The emphasis on voting rules ties the paper to a large literature going back to Rousseau (1762), de Condorcet (1785), Wicksell (1896), Buchanan and Tullock (1962) and, more recently, Aghion and Bolton (2003).2 Wicksell, in particular, argued that unanimity were the appropriate requirement, since otherwise the majority would expropriate the minority.3 Without delegation, Wicksell would be right in my model. However, every district delegates conservatively if the majority requirement is large, and reluctant representatives implement too few projects. Taking this effect into account, the optimal majority requirement should be smaller.

In the model, each district trades off an incentive to gain bargaining power and the desire to be included in the majority coalition. This trade-off is similar to that in Harstad (2005), and the legislative games are quite similar in the two papers. However, Harstad (2005) ignores the upper boundary on taxes, and the analysis requires there to be small transaction costs related to the transfers. More substantially, Harstad (2005) studies optimal incentives to prepare for a collective project, ignoring the incentives to delegate strategically, emphasized in this paper.

### 3. The model

#### 3.1. Players and preferences

A set of districts, I, must agree to a policy, specifying whether a binary public project is to be implemented, in any case, how to allocate tax revenues. For individual j in district i ∈ I, the project has a realized gross value vi but a cost ci. These can be arbitrary distributed. Define the (net) value as vi = vi − ci, and district i’s average net value as \( \bar{v}_i \).

Each district i ∈ I elects a delegate i. As proven in Section 4.2, all voters in district i happen to agree on which i elected to, so a strong version of the Median Voter Theorem turns out to hold. The

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1. In the US, the majority requirement is effectively larger in the Senate than in the House, because of the possibility to filibuster. In Europe, the effective majority requirement varies across countries because of different explicit voting rules, but also because the number of parties, chambers and quorum requirements differ widely (Döring, 1995).

2. See also Mussner and Polborn (2004), who show how voters may prefer a super-majority rule as a way of delegating the pivotal role in the future, and Barbera and Jackson (2006), who explain how heterogeneity within countries determines their optimal voting weights. However, such heterogeneity is not important when side payments are available, as I assume.

3. Buchanan and Tullock (1962) argued that unanimity would imply too high “decision-making costs” and Aghion and Bolton (2003) suggested that the “winners” of a project may not have deep enough pockets to compensate the “losers”. The present paper does not include any of these features, giving Wicksell right — where it not for strategic delegation.
representative has a net value of the project equal to $v_i^d$, so the delegate's type, or bias, can be measured relative to the district average:

$$v_i^d = v_i + d_i.$$  

If $d_i > 0$, its delegate ("he") is "progressive" and generally has a higher value of the project than the average voter (she) in district $i$. If $d_i < 0$, its delegate is a status quo biased "conservative" who is less in favor of the project than she herself.

If we let $t_i$ represent the district-specific tax (or transfer), the utilities of $i$ and her representative becomes:

$$u_i = v_i - t_i.$$  

There is no need for asymmetric information in the model. However, the delegates may represent their districts for many projects and for a long time. Thus, at the delegation stage, there is uncertainty regarding the project that is going to be available, and the benefits are not yet known. In other words, after the delegation stage, the project's actual value is realized:

$$v_i = v_i^0 + \varepsilon_i - t_i.$$  

Moreover, and $\varepsilon_i$ represent some random local and global preference shocks, respectively. It is not important whether $i$ and her delegate are affected by the very same individual shock. The analysis below only uses the combined equation $\varepsilon_i = (\varepsilon_i + t_i)$, so it is possible to interpret $\varepsilon_i$ as the individual shock to $\varepsilon_i$'s value, or the uncertainty regarding $\varepsilon_i$'s preference.

The distribution of the initial net values, $v_i^d$, can be arbitrary. But to arrive at explicit solutions, let the $\varepsilon_i$ be independently drawn from a uniform distribution with mean zero and density $1/\theta$:

$$\varepsilon_i \sim U[-\theta, \theta].$$  

If $I$ is finite, the distribution of the $\varepsilon_i$ can take many forms, thus making the analysis quite complex. To simplify, I will assume that there is a large number of districts, such that $I$ can be approximated by a continuum, $I \in [0,1]$. Then, the distribution of the $\varepsilon_i$ is deterministic and uniform on $[-h/2, h/2]$. Consequently, $h$ would measure the heterogeneity in preferences across the delegates if $v_i^d + d_i$ should happen to be the same for all districts. I will order the delegates by decreasing value, such that $l_i < j_i$ if $v_i^d > v_j^d$. Variables without subscript denote the average, such that $v_i^d$ is the average (and the sum) of the $v_i^d$'s.

Parameter $\theta$ captures the uncertainty in the aggregate cost of the project. $\theta$ can be negative, of course, since the $v_i^d$'s already internalize the expected cost of the project. Let also $\theta$ be uniformly distributed, with $\sigma$ measuring the variance in the aggregate shock (to be precise, the variance of $\theta$ is $\sigma^2/12$):

$$\theta \sim U[-\sigma, \sigma].$$  

In the analysis, I refer to a one-dimensional policy space $(v_i \in \Theta^k, k = 1)$. But nothing prevents the model from capturing several dimensions $(k-1)$ if a district can choose a vector $d_i = (d_{i1}, ..., d_{ik})$, and if each dimension (or policy area) is voted over separately. This way, the model describes decision-making and strategic delegation on one dimension, and similar results hold independently for the other dimensions.4

3.2. The legislative game

After the representatives are appointed and the shocks realized, the legislative game begins (Fig. 1). Then, a majority coalition is formed and the coalition members negotiate a proposal. A clear separation of these two stages makes the mechanism of the model more transparent and easier to study.

First, the majority coalition is formed. A formateur (a political entrepreneur, president or initiator), randomly drawn among the delegates, selects a majority coalition $M \subset I_d$ to form the majority, where $I_d$ is the set of delegates. In equilibrium, the formateur will simply select the unique core of the game at this stage.

Second, the representatives in $M$ negotiate a policy proposal. The formateur makes the first offer, but if a proposal is rejected, every coalition member has the same chance of being recognized as the next proposer.5 Let $\delta \in [0,1)$ represent the common discount factor between offers. Each proposal specifies whether the project should be implemented and, in either case, how to distribute the transfers, with the constraint that the transfers must sum to zero and $t_i \leq T$. $T$ can be interpreted as some minority protection or as the tax paid by every district, if $t_i$ measures the net pay from district $i$ after the tax revenues are redistributed. To simplify, assume $T \geq \max_{i \in I_d} v_i$. Such that $t_i$ will not approve a proposal if the district is taxed maximally. Since a disagreement within the majority coalition can lead to a "government crisis", it may be reasonable to make:

**Assumption 1.** Every $i \in M$ must agree to the proposal.6

After a proposal has been accepted by all the members of the majority coalition, the proposal is submitted to the floor for a vote, and it is implemented if it is accepted by the required majority, $m$. To prevent the minority from bribing majority members, I also make:

**Assumption 2.** Before voting, minority members can negotiate with each others, but not with $M$-members.

If the proposal is not accepted, or if the majority members never agree, then the project is not implemented and no-one is taxed (or, equivalently, the tax revenues $T$ are repaid uniformly).

**Remark 1.** The model includes several special cases: If $\delta = 0$, for example, a delegate is randomly drawn to make a take-it-or-leave-it offer to the rest, and this proposal must be accepted by a fraction $m$. If $\delta \rightarrow 1$, the single role of the formateur is to create a coalition, and he has no more bargaining power than other coalition-members.

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4. With multiple dimensions, the model actually requires a majority coalition to form whenever a new dimension is voted over. If this is costly or cannot be done, Section 6.1 argues that the results still hold, qualitatively.

5. The reason why only coalition members can make proposals might be that they have committed to reject all other proposals, in line with Baron’s (1989) notion of “coalition discipline”. This assumption is relaxed in Section 6.2.

6. This assumption follows e.g. Persson et al. (2000) and Bennedsen and Feldmann (2002).
4. Politics, delegation and voting rules

Solving the game by backward induction, I start by discussing the outcome of the legislative game, taking the delegates’ bias as given.

4.1. The legislative outcome

Since the focus of the paper is on delegation, it is comforting that the legislative game has an intuitive outcome: the minority is taxed maximally, the majority coalition shares these revenues, they implement the project disregarding the minority’s cost, and the formateur selects a “least costly” minimum winning coalition.7 Before discussing its intuition, I first characterize the legislative outcome assuming that, at the bargaining stage, the outcome is a stationary sub-game perfect equilibrium in undominated strategies.

**Proposition 1.** Suppose \( \int_{i \in M} v_i^d di \geq 0 \) for some \( M \subset I_d \) s.t. \( |M| \geq m \). Then, the formateur:

(i) selects a minimum winning coalition, \(|M| = m\),
(ii) forms a coalition with the delegates that have the highest value of the project, \( M = \{i \mid v_i^d \geq v_m\} \), where \( v_m \) is the \((1-m)\)-fractile of the \( v_i^d \),
(iii) proposes the following transfers:

\[
t_i = T_v i \in I_d, M, \\
t_i = v_i^d - \frac{\delta}{m} \int_{j \in M} (1-m)T + \int_{j \in M} v_j^d di \forall i \in M.
\]

(iv) and that the project is implemented.

(v) This proposal is accepted by the majority coalition and in the final vote.

Part (i) states that, in this model, \( M \) is indeed a minimum winning coalition. A larger coalition is costlier since the surplus would have to be divided on a larger number of delegates (due to Assumption 1). A coalition smaller than \( m \) is also costlier, since then the minority can block the proposal, and they must thus be given a non-negative aggregate utility (by Assumption 2).

Part (iii) first states that all minority representatives are fully taxed and given nothing. This is not surprising, since the majority does not need their approval. These tax revenues, plus \( M \)'s aggregate value the project, are shared by the majority coalition. The second part of (iii) states that a representative favoring the project more, is taxed more. Intuitively, a delegate’s eagerness reduces his bargaining power, and he is held up by the other coalition members (and the formateur) unless he gives in by transferring some of his benefits to them. This transfer implies that a representative \( i \in M \) receives the utility

\[
u_i = \int_{j \in M} v_j^d di + (1-m)T, \tag{4.1}
\]

independent of \( v_i^d \) (as proven in Appendix A). Thus, every majority member receives a fraction of the coalition’s total value of the project and the taxes expropriated by the minority. This fraction is \( \delta \) (what is left after the formateur has expropriated his agenda-setting power).

4.2. Strategic delegation at the election stage

At the election stage, district \( i \) delegates by selecting \( d_i \). There are two reasons why \( i \) may delegate strategically by choosing \( d_i \neq 0 \).

On the one hand, a low \( d_i \) reduces the transfers to be paid by district \( i \), if \( I_d \) happens to be in the majority coalition. The reason for this is that a conservative delegate (small \( v_i^d \) raises \( i \)’s bargaining power (bp), since such a delegate is less eager to see the project implemented. This reduces the transfer \( i \) has to pay in equilibrium. From Proposition 1:

\[
t_i = v_i + d_i - \frac{\delta}{m} \left( T(1-m) + \int_{j \in M} v_j^d di \right) \quad \text{if } i \in M. \tag{bp}
\]

On the other hand, a high \( d_i \) makes it more likely that \( I_d \) becomes a member of the majority coalition, since this coalition consists of the most enthusiastic representatives. There will be some threshold \( v_m \) (the \((1-m)\)-fractile of the \( v_i^d \)) such that all representatives valuing the project more than \( v_m \) become coalition members, while those valuing the project less become minority members. Thus, a large \( d_i \) may increase district \( i \)’s political power (pp). For any given \( v_m \), \( i \)’s
representative becomes a majority member if \( i \)'s district-specific shock is such that:

\[
v_i^d \geq v_m \Rightarrow \epsilon_i \geq \epsilon_i^* \equiv 0 + v_m - v_i^0 - d_i.
\]

The two forces work in opposite directions. To increase the bargaining power, it is tempting to delegate conservatively, since such a delegate would be better able to receive compensation from the others. To increase the political power, however, it is wiser to delegate progressively. The choice of delegate must balance these concerns. Formally, voter \( j \) in district \( i \) prefers a representative that solves:

\[
\max_{d_i} \int_{-\sigma/2}^{-\alpha/2} \left( \hat{\epsilon}_i \left( v_j - T \right) \frac{d\hat{\epsilon}_i}{h} + \int_{-\alpha/2}^{-h/2} \left( v_j - \epsilon_i \right) \frac{d\epsilon_i}{h} \right) \frac{d\theta}{\sigma} \text{ s.t. (bp) and (pp). (4.2)}
\]

where \( \hat{\theta} \) is the highest \( \theta \) at which the project is implemented. Solving this problem gives:

**Proposition 2.**

(i) Every individual \( j \) in district \( i \) prefers a representative with bias given by

\[
d_i + v_i^0 = \left( d + v_i^0 \right) \frac{1 + \delta}{2} + T \left( 1 + \frac{\delta}{m} \right) - \frac{h}{4} \left[ 1 - \delta + m(1 + \delta) \right] - \frac{\sigma}{4} \left[ 1 - \delta \right].
\]

(ii) Thus, district \( i \) delegates conservatively \( \left( d_i < 0 \right) \) if \( v_i^0 \) is large while \( v_i^0 + d \) is small.

(iii) On average, districts delegate conservatively if \( m, h, v_i^0 \) and \( \sigma \) are large, while \( T \) and \( \delta \) are small: The equilibrium \( d \) is given by

\[
d(m) = -v_i^0 + 2T \left( 1 - \frac{\delta}{m - \delta} \right) - \frac{h}{2} \left[ 1 + m \left( 1 + \frac{\delta}{m - \delta} \right) \right] - \frac{\sigma}{2}.
\]

The equilibrium \( d_i \) depends on the value of political power, and this drives the comparative static of Proposition 2.

Part (i) shows that a strong version of the Median Voter Theorem holds in this model. Not only are preferences single-peaked in \( d_i \), but every voter in district \( i \) prefers the very same delegate, \( \hat{\epsilon}_i \). Since there are many districts, \( i \)'s delegate is not going to be pivotal for whether the project is implemented, and of importance is only whether the district's tax can be reduced. Obviously, every citizen in the district shares this objective.

Part (ii) states that district \( i \) delegates conservatively if its initial value of the project is large. In this case, there is a high probability of being included in the majority coalition and, instead of increasing this probability, it is relatively more important to appoint a reluctant representative, anticipating the bargaining game. The resulting probability must be the same for every representative. The equilibrium type \( d_i + v_i^0 \) is thus the same for all delegates, and they are distinguished only by the stochastic \( \epsilon_i \). If the districts, on average, are represented by delegates valuing the project a lot \( (d_i \) large), then Eq. (4.3) implies that \( i \) would also like to raise her \( d_i \). The reason is that when the other representatives are enthusiastic about the project, it is important to be a member of the majority coalition that shares all these values, and this probability increases in \( d_i \).

Part (iii) states several comparative statics that hold for \( d_i \) as well as for the average \( d \). Most importantly, if \( m \) increases, all districts delegate less progressively (or more conservatively). For large \( m \), it is not immensely important to become a member of the majority coalition, for two reasons: the minority \( (1 - m) \) which the majority can expropriate is then small and the total surplus is shared among more majority members. Thus, the gains from political power decrease in \( m \), as does the incentive to delegate progressively. The larger is the majority rule, the less progressively, or the more conservatively, does \( i \) delegate.\(^{11}\)

Since \( v_i^0 + d_i \) is, in equilibrium, the same for all districts, \( h \) does indeed measure the heterogeneity in preferences at the legislative stage. If \( h \) is large, the probability that \( \hat{\epsilon}_i \) becomes a majority member increases just a little, when \( d_i \) increases. Delegating progressively is then not a very effective way of gaining political power, and it is better to delegate conservatively to gain bargaining power instead.

If \( T \) is large, it is very important to become a member of the majority coalition, since the minority is taxed by a lot. Thus, the larger is \( T \), the more progressively the districts delegate.

However, if \( \delta \) is small, the formateur has a lot of agenda-setting power, and he leaves less surplus for the majority coalition. This makes it less beneficial to be a member of the majority coalition, and the equilibrium \( d \) decreases in order to gain bargaining power.

If \( v_i^0 \) and \( \sigma \), the aggregate uncertainty, are large, then the expected net value of the project is large — conditional on it being worthwhile to implement. Thus, the project becomes more beneficial, and this larger benefit is enjoyed whether or not \( \hat{\epsilon}_i \) should become a majority member. If he does, however, the formateur is expropriating some of the larger benefit by increasing \( t_i \). This cannot be done when the tax is already at its maximum. Hence, when \( v_i^0 \) or \( \sigma \) increases, being a majority member becomes relatively less important, and the \( d_i \)s decrease.

How does \( d \), derived here, affect the selection of projects, discussed in Section 4.1? Since the \( v_i^0 \)'s are uniformly distributed with mean \( v_i^0 + d(m) - \theta \) and density \( 1/h \),

\[
\int_{-\sigma/2}^{-\alpha/2} v_i^0 \frac{d\theta}{\sigma} \text{ s.t. (bp) and (pp). (4.2)}
\]

and the condition for when the project is implemented, \( \int_{-\sigma/2}^{-\alpha/2} v_i^0 \frac{d\theta}{\sigma} \geq 0 \), becomes \( \theta \leq \hat{\theta} \), where

\[
\hat{\theta} \equiv 2T \left( 1 + \frac{\delta}{m - \delta} \right) - \frac{h m}{2} \frac{\sigma}{1 - \delta}.
\]

**Corollary 2.** The project is implemented if and only if \( \theta \leq \hat{\theta} \), given by Eq. (4.5). Thus, fewer projects are implemented if \( m \) is large.

Optimally, the project should be implemented if \( \theta \leq \hat{\theta} = v_i^0 \). But in equilibrium, \( \partial \theta / \partial m < 0 \) and there may be a status-quo bias when \( m \) is large. This is not so because too few projects are implemented from the delegates’ point of view: Corollary 1 states that the selection is optimal from the delegates point of view if \( m \sim 1 \). And, if \( d = 0 \) and \( m \sim 1 \), too many projects are implemented, not too few. However, if \( m \) is large, all districts delegate conservatively in equilibrium, and reluctant delegates are less willing to implement projects.

4.3. The optimal voting rule

Considering the expected utilities, the transfers are irrelevant and of importance is only whether projects are implemented too often or too seldom. The selection of project depends on the majority requirement for two reasons. On the one hand, for \( d \) given, the majority coalition implements more projects if \( m \) is small since it can then ignore a larger majority, possibly hurt by the project. In isolation, this argument implies that the agents implement too many projects, relative to what is optimal from the agents’ point of view. This is the argument by Wicksell (1896), captured by Corollary 1. Corollary 2, on the other hand, shows that if \( m \) increases, all districts delegate to conservative representatives, and these are less willing to implement the public project, compared to what

\(^{11}\) There is a third reason for this: The benefit of delegating conservatively is large if \( \hat{\epsilon}_i \) is very likely to become a majority member (because only then can his bargaining power be exploited), and this probability increases in \( m \).
is optimal from the districts’ point of view. By carefully selecting \( m \), these two effects cancel. This has two consequences.

First, if the parameters change such that one effect dominates the other, \( m \) must adjust accordingly. A larger \( T \), for example, increases the incentive to delegate progressively and, to restore an optimal selection of project, \( m \) must increase. For similar reasons, \( m \) should increase in \( \phi \) but decrease in \( h, v^0 \) and \( \sigma \).

Second, since too many projects are always implemented from the representatives’ point of view when \( m < 1 \), the selection of projects can be first-best for \( m = 1 \) only if the average voters have higher valuations of the project that their agents. Hence, \( d < 0 \) at the optimal \( m \) (Fig. 2).

### Proposition 3.

Assume \( h > 2T - \left( \theta^0 + \frac{\sigma}{2} \right) \mid (1 - \phi) \).

(i) The optimal majority requirement \( m^\ast \) increases in \( T \) and \( \phi \) but decreases in \( h, v^0 \) and \( \sigma \):

\[
2T \left( 1 + \frac{\phi}{m - \phi} - hm^\ast \right) = \left( v^0 + \frac{\sigma}{2} \right) \mid (1 - \phi).
\]

(ii) At \( m = m^\ast, d < 0 \).

The proof follows simply by equalizing \( \hat{m} \) in Eq. (4.5) to the optimal threshold for \( \theta \), which is \( \hat{v}^0 \).

By interpreting \( \phi \) and \( T \), as well as \( m \), as institutional variables, there is a plane of combinations that induces the first-best selection of project. One can allow \( m \) to increase, for example, if \( T \) simultaneously decreases. Thus, the two means of protecting the minority \( (m \uparrow \text{or} T \downarrow) \) are substitutes. More power to the formateur \( (\phi \uparrow) \) reduces the value of being in the majority coalition, and \( m \) should decrease to compensate for this. Finally, \( \phi \) and \( T \) are substitutes: if the formateur gets less power \( (\phi \downarrow) \), districts delegate progressively and too many projects are implemented, unless \( T \) decreases (or \( m \) increases).

### 5. Centralization or decentralization?

The model above is simple and can be employed, for example, to compare decentralization and centralization. This choice affects the selection of projects directly, for \( \hat{d} \) given, but it also affects the identity of the representatives that are elected. Clearly, both effects must be taken into account.

While the regime above is first best whenever Eq. (4.6) holds, it is often difficult to change the institutional parameters \( m, T \) or \( \phi \). For example, most legislatures rely extensively on the simple majority rule, even though this is unlikely to be recommended by the theory above. If the legislature uses a voting rule \( m \neq m^\ast \), one can easily calculate the social loss, \( L^\ast \), compared to the first-best:14

\[
L^m = \frac{1}{2\sigma} \left( \hat{v} - v^0 \right)^2 = \frac{1}{2\sigma} \left[ 2T \left( 1 + \frac{\phi}{m - \phi} - hm^\ast \right) - \frac{\sigma}{2} \mid (1 - \phi) \right] ^2.
\]

Instead of modifying \( m, T \) or \( \phi \), an alternative approach is to take these institutional parameters as given, and instead look for the appropriate level of government. Referring to the above regime as “majority rule”, this section makes comparisons to both further “centralization” and “decentralization”.

#### 5.1. Decentralization

By “decentralization”, I refer to a context where the local governments have the authority, such that (i) no district can be forced to participate, and (ii) the representatives are domestic politicians, not only representatives in a national legislature. The first feature implies that unanimity is required when decentralized districts consider to implement a collective project. The second feature suggests that it may be costly for a district to delegate “strategically” under decentralization, since such a politician has domestic power as well. This is the case for the European Council, for example, where the representatives are, first of all, ministers (or heads of states) in their home countries.

If district \( i \) elects a minister that is biased towards liberalization in agriculture, for example, then he might be so for local decisions as well as for international projects. This creates distortions when the representative has local power. Suppose district \( i \)'s value of a typical local liberalization project is given by some parameter \( \theta_i \)

\[
\theta_i = U \left[ a_i - \frac{c_i}{2} a_i + \frac{c_i}{2} \right],
\]

and that \( i \)'s representative is decisive on a number of \( n_i \) such issues.

The cost of selecting a delegate with the bias \( d_i \) (such that his value of the project is \( \theta_i + d_i \)) is then16

\[
c_i d_i^2 / 2, \text{ where } c_i = n_i / a_i.
\]

Delegating to a very progressive minister (\( d_i > 0 \)) is costly since he will liberalize too much also locally. Delegating to a conservative

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12 If \( h \geq 2T - \left( \theta^0 + \frac{\sigma}{2} \right) \mid (1 - \phi) \) does not hold, too many projects are implemented for any \( m < 1 \), even if \( m \rightarrow 1 \). But as proven in the next section, too few projects are implemented if \( m = 1 \), so the first best can then not be achieved by any fixed \( m \). However, one can show that the first best is possible if, at the election stage, \( m = 1 \) with probability \( p \) and \( m = 1 - \epsilon, \epsilon > 0 \), with probability \( 1 - p \), where \( p \) must be given by

\[
\frac{1}{1 + \frac{2p}{\epsilon - p}} = 2T - \left( \theta^0 + \frac{\sigma}{2} \right) \mid (1 - \phi).
\]

13 The effect of the project’s value \( \hat{v}^0 \) is the opposite of its effect in Harstad (2005).

14 This can be seen since, if the threshold is \( \hat{d} \neq \hat{d}^\ast = \hat{v}^0 \), the social loss is

\[
\int_{-\sigma/\hat{v}^0}^{\sigma/\hat{v}^0} (\hat{v}^0 - \theta) \hat{d} \frac{d\theta}{\sigma} - \int_{-\sigma/\hat{v}^0}^{\sigma/\hat{v}^0} (\hat{v}^0 - \theta) \hat{d} \frac{d\theta}{\sigma} = \frac{(\hat{v}^0 - \hat{v})^2}{2\sigma}.
\]

15 Throughout the paper, I ignore the possibility that a sub-group of districts could implement the project without requiring everyone to do so. Relaxing this assumption leads to a much richer model, left for future research.

16 To see this, notice that the delegate implements a local project if \( \theta_i + d_i \geq 0 \). District \( i \)'s expected utility of the local policies becomes:

\[
n_i = \frac{\theta_i + d_i}{\theta_i + d_i} k_i = \frac{\theta_i + d_i}{\theta_i + d_i} x_i \cdot \frac{n_i}{\theta_i}, \text{ where } c_i = n_i / \theta_i.
\]

and \( k^\ast = n_i(a_i + c_i/2)^2 / 2a_i \) is constant.
This is the case, for example, for $T$ large, $\sigma$ small, because then districts elect unbiased local decision-makers. The regimes in this section are thus comparable even though local decisions are not discussed for centralization or majority rule.

Since unanimity is required, everyone is a member of the majority coalition, and there is no point of delegating progressively. To increase the bargaining power, therefore, $d_i$ is going to be negative in equilibrium. And, $d_i$ will be more negative if the cost of delegation, $c_i$, is small and if the project has a high chance of being implemented ($v^0 + d + \sigma/2$ large), since then costly delegation is likely to pay off. Consequently, the project will be implemented too seldom in equilibrium, and the social loss is particularly large if $v^0 + \sigma/2$ is large, while the $c_i$s are small.

To let $\delta$ be a characteristic of the majority rule regime, assume now that the discount factor approaches one, such that no district has agenda-setting power under decentralization.

**Proposition 4. Under decentralization,**

(i) more conservative representatives pay less transfers:

$$t_i = (d_i + v_i^0) - (d + v^0) + \epsilon_i.$$  

(ii) **District $i$ delegates more conservatively if $c_i$ is small while $d$ is large:**

$$d_i = -\frac{v^0 + d + \sigma/2}{c_i}.$$  

(iii) **All districts delegate conservatively, and more so if the $c_i$s are small while $v^0$ and $\sigma$ are large:**

$$d = -\frac{v^0 + \sigma/2}{c + 1} < 0.$$  

(iv) **Projects are implemented too seldom, particularly if the $c_i$s are small while $\sigma$ is large, and the social loss is given by $L^D$:**

$$L^D = \left(\frac{1}{2\alpha} + \frac{\sigma}{2}(\frac{v^0 + \sigma/2}{c + 1})^2\right).$$  

If the $c_i$s are large, delegation is very costly under decentralization, and the representatives are almost identical to their principals. Then, $d_i \approx 0$ and **Corollary 3** suggests that $m = 1$ is optimal, exactly as decentralization requires.

**Corollary 3.** If $c_i \rightarrow \infty \forall i \in I$, decentralization (requiring $m = 1$) becomes first-best.

But in general, both decentralization and majority rule tend to be inefficient and the best choice depends on the parameters. For example, if $T$ and $\delta$ are large, while $h$, $v^0$, and $\sigma$ are small, the optimal voting rule $m^*$ is going to be large and probably larger than $m$, when $m$ is fixed. The social loss under majority rule, $L^m$, is then large, and decentralization is better.

**Proposition 5. Decentralization is better than majority rule ($L^D < L^m$) if**

$$(1 + \alpha)\left(\frac{v^0 + \sigma/2}{c + 1}\right)^2 < 2T\left(\frac{1 + \delta}{1 - h}\right)\frac{hm}{1 - h} - \frac{\sigma}{2}(v^0)^2.$$  

This is the case, for example, for $T$ large, $\delta$ large, $c_i$ large, $h$ small, $v^0$ small and $\sigma$ small.

5.2. **Centralization**

Let “centralization” mean that one representative, from one of the districts, takes the decision on the behalf of everyone. Since no consent is necessary, the other districts may require to be compensated in advance for the high taxes this “president” is going to set or, equivalently, abandon the possibility to set taxes. Thus, let $T = 0$ under centralization. To be consistent, suppose this president is appointed, or elected, at the “delegation stage”, i.e., before the shocks are realized. If the president is from district $i$, and he has the bias $d_i$, then he implements the decision if $\theta \leq \theta_i \equiv v^0 + d_i + \epsilon_i$, and the social loss is

$$\frac{1}{2\alpha}(v^0 - \theta_i)^2 = \frac{1}{2\alpha}(v^0 + \epsilon_i + d_i - v_0)^2.$$  

This loss depends on $\epsilon_i$, the shock in district $i$. Clearly, the expectation of this loss is minimized if $v^0 + d_i = v_0$: that is, if the president has the average value of the project, at least before the shocks are realized. Assume this to be the case.\(^\dagger\) The loss from centralization is then:

$$L^C = \int_{-\h/2}^{\h/2} \frac{\epsilon^2_i dc_i}{2\alpha h} = \frac{h^2}{24\alpha}.$$  

**Corollary 4.** If $h \rightarrow 0$, centralization becomes first best.

By comparing with the two alternative regimes, we immediately get:

**Proposition 6. Centralization is better than majority rule ($L^C < L^m$) if**

$$\frac{h^2}{12} < \left[2T\left(\frac{1 + \delta}{1 - h}\right) - \frac{hm}{1 - h} - \frac{\sigma}{2}(v^0)^2\right].$$  

This is the case, for example, for $T$ large, $\delta$ large, $h$ small, $v^0$ small, and $\sigma$ small.

**Proposition 7. Centralization is better than decentralization ($L^C < L^D$) if**

$$\frac{h}{\sqrt{12}} < \left(\frac{v^0 + \sigma/2}{c + 1}\right)\sqrt{1 + \alpha \sigma}.$$  

This is the case for $h$ small, $v^0$ large, $\sigma$ large and if the $c_i$s are small.

5.3. **The best regime**

Sections 5.1 and 5.2 compare “decentralization”, “centralization” and majority rule. While majority rule is first-best if $m = m^*$, the more $m$ deviates from the optimal rule, the less efficient it is. Thus, each of Eqs. (5.1) and (5.2) gives two conditions for when majority rule is best, one for $m < m^*$, and another for $m > m^*$. Together with Eq. (5.3), we get five conditions describing the best of these three regimes.  

Fig. 3 illustrates these conditions. Majority rule is best in area $a$, centralization is best in the areas marked with $C$, while decentralization is best in the areas marked with $D$. In fact, decentralization is first best along the first axis ($v^0 = -\sigma/2$), centralization is first best along the second axis ($h = 0$), while majority rule is first-best along the dotted line (then, $m = m^*$). Below the dotted line, $m < m^*$ and majority rule performs worse if $h$ and $v^0$ are large, because then districts elect representatives that are too progressive. The opposite is the case above the dotted line: There, $m > m^*$ and majority rule performs worse if $h$ and $v^0$ are large, because then districts appoint representatives

\(^\dagger\) This assumption holds if (i) the distribution of the voters’ $v_i$s is symmetric or (ii) with Coasian bargaining over the choice of “president” at the delegation stage. If these assumptions did not hold, the results would be qualitatively similar, but centralization would be less efficient than predicted above.
that are too conservative. Compared to centralization, decentralization is naturally better if the heterogeneity is large, and it is also better if $v^D$ is large, since then the districts, under decentralization, delegate to very conservative representatives.\(^{18}\)

6. Extensions and generalizations

The legislative game in Sections 3 and 4 is simple and it can easily be extended. It builds on three strong assumptions: (i) The composition of the majority coalition $M$ only depends on the project-values, (ii) only $M$-members can make proposals, and (iii) side transfers are always possible. These assumptions are relaxed, one by one, in the following three subsections. Each extension is discussed in isolation, although it is straightforward to combine them.\(^{19}\)

6.1. Coalition stability

In the model above, the majority coalition consists of the representatives with the highest valuation of the project. This arises as an equilibrium phenomenon and it is often simply assumed elsewhere in the literature.\(^{20}\) In reality, however, there may be other reasons for selecting coalition members, not only their valuation of the project.

Suppose that, with probability $s$, the coalition is formed independently of the $v^D$s, and every representative has then the same probability $|M| = m$ of being included in $M$. This may be reasonable, for example, if the policy space is multi-dimensional and deciding on one dimension (or political issue) is not worth the formation of a new coalition. Alternatively, some earlier coalition may already exist and this may be stable with probability $s$.

If $s$ is large, it is unlikely that a progressive delegate is helpful in gaining political power, and the districts prefer instead to delegate conservatively since this, at least, increases their bargaining power. Thus, if $s$ increases, $d_i$ decreases, unless $m$ decreases. In Fig. 2, increasing $s$ would imply a downward shift in the steepest curve.

At the same time, the socially optimal level of $d$ increases in $s$. At the optimal voting rule, $d = 0$, just as before. This implies that when the coalition is random, and not only including the winners, then the project is implemented too seldom. When $s$ increases, this occurs too often and to mitigate this inefficiency, the optimal $d$ should increase towards zero. This can be done by reducing $m$.

Proposition 8.

(i) The equilibrium $d$ decreases in $s$: 
$$d = d(m) - \frac{smq}{1-s},$$ where 
$$q = \frac{v^0 + d + \sigma/2}{v^0 + d + \sigma/2 + h(1-m)/2}$$

(ii) The socially optimal $d$ increases in $s$: 
$$d^* = -\frac{h(1-m)(1-s)}{2}.$$

(iii) For both reasons, the optimal $m$ decreases in $s$.

The function $d(m)$ is the same as before (Eq. (4.4)), and $q$ is the probability that the project is implemented if the majority coalition is independent of the $v^D$s compared to when it is not. The comparative static with respect to the other parameters turns out to be just the same as before.

A large $s$ is, in practice, associated with less important political issues, for which it is not worth to form a new coalition. This suggests a smaller majority requirement for such issues, to prevent a too conservative delegation on these dimensions. In other words, more important issues should require a larger majority. Similarly, if majority coalitions are quite "stable" for a particular political system, then it should use smaller majority requirements to discourage districts from appointing too conservative delegates.

6.2. Minority proposal power

To emphasize the importance of being in the majority coalition, the analysis above assumed that only majority members could make proposals. This section, in contrasts, assumes that after every rejected offer, there is a probability $\epsilon$ that a minority-member can make the next offer while, with probability $1 - \epsilon$, a majority member makes the next offer. As before, I assume that every member of each coalition has the same chance of being recognized as the next proposer, and everyone in the majority coalition must approve the proposal, even if it is suggested by a minority-member. To simplify, let there be only two stages, such that if the formateur’s initial offer is rejected, the next proposer makes a final take-it-or-leave-it offer.

The second proposer will always ensure that the (other) $M$-members only receive utility $0$, just enough to make them approve the project. In the first stage, a majority member approves a proposal if 
$$u^0_i \geq \frac{b(1-\epsilon)}{m} \left[ \int_{j \in M} v^D_j + (1-m)T \right].$$

and in equilibrium, the formateur will propose exactly this, while the minority is taxed by $T$, just as before.

The minority’s proposal power clearly diminishes the value of being in the majority coalition. In fact, increasing $\epsilon$ has the same effect

\(^{18}\) Another way around the cost of strategic delegation may be to decide upon the issue at the beginning of the game. If the districts commit to always do the project, no matter $\theta$, the social loss is:

$$\frac{v^0}{|M|} \left( v^0 - \frac{f_0}{\alpha} \right) - \int_{-\alpha/2}^{\alpha/2} \frac{\sigma}{\alpha} \left( v^0 - \frac{f_0}{\alpha} \right) = \frac{1}{20} (\alpha/2 - v^0)^2.$$

if $\alpha/2 \geq v^0$, while it is 0 otherwise. Similarly, by committing to never consider the project, the expected social loss is:

$$\frac{v^0}{|M|} \left( v^0 - \frac{f_0}{\alpha} \right) = \frac{1}{20} (\alpha/2 + v^0)^2,$$

if $\alpha/2 \geq -v^0$, while it is 0 otherwise. Of these alternatives, it is clearly better to do the project if $v^0 > 0$. Compared to the other regimes above, it is better to decide in advance when the project is either almost for sure, or almost for sure not.

\(^{19}\) Another strong assumption is that the majority coalition is formed prior to the bargaining. If everyone could propose and no coalition was formed in advance, the equilibrium would be in mixed strategies, $M \subset M_i$ would be random, and so would the $\theta$-threshold. When $\theta$ is in the range of the $\theta$-thresholds, the project is undertaken with some probability only, and the probability that $\epsilon \in M$ would depend on the random $\theta$-threshold. For these reasons, such a model becomes too complicated to solve.

\(^{20}\) See, for example, Aghion and Bolton (2003).
as decreasing $\delta$; the formateur asks for larger taxes from the majority-coalition. The minority is taxed maximally, just as before. Equilibrium delegation (Eq. (4.4)), and the optimal majority rule (Eq. (4.6)), continue to hold if just $\delta$ is replaced by $\delta(1 - \epsilon)$.

**Proposition 9.** If the minority has a larger chance of making a proposal $(\epsilon \uparrow)$, $d$ declines and $m^*$ should be smaller.

If the minority is protected more on one dimension (a larger $\epsilon$), they should be less protected by the majority requirement. Otherwise, the incentive to become a member of the majority coalition would be weak, districts would delegate conservatively, and too few projects would be implemented. The proof is similar to those above, and thus omitted.

6.3. The possibility to transfer and tax

This subsection relaxes the assumption that transfers and taxes are possible whether or not the project is implemented. One interpretation of the taxes and transfers is that they could be alterations of the project being implemented: By modifying the collective project, by making exceptions and reallocations, it is often possible to transform a collective project into one that satisfies particular interests. This may be possible even if explicit taxes are not. With this interpretation, $T$ may measure the extent to which it is possible to modify a project (by transferring its benefits), for a particular political issue. But with this interpretation, the “taxes” are possible only if the project is, in fact, implemented. If the project is not implemented, it is not possible to “tax” the minority in this way. This changes the model above. If $M$ can tax the minority if and only if the project is implemented, it is implemented whenever

$$0 \leq \hat{\theta} + T(1 - m)/m,$$

where $\hat{\theta}$ is the threshold (Eq. (4.5)) derived above. Thus, the project is implemented more often than in the model above, since this is the only way in which the majority can expropriate the minority. The equilibrium delegation becomes

$$d = d(m) + T(1 - m)/m,$$

(6.1)

where $d(m)$ is given by Eq. (4.4) as before. So, in this case, each district appoints a delegate that is more progressive, compared to the case above. The reason is that when the $\theta$-threshold increases, projects are, on average, costlier, and this has the same effect as when $v^0$ decreases: $d_i$ increases. So, in this case, the delegates are more progressive, and any set of delegates would implement the project more often. Clearly, then, the majority requirement should increase, relative to the situation above.

**Proposition 10.** Assume the minority can be taxed if and only if a project is implemented. Equilibrium delegation is given by Eq. (6.1), and the optimal majority requirement satisfies:

$$2T/m^*hm^* = \left( v^0 + \frac{\alpha^2}{2}(1 - \hat{\theta}) \right).$$

The proof is similar to those above, and thus omitted.

7. Concluding remarks

If voters elect representatives strategically, such as to gain political or bargaining power, they may implement decisions that are suboptimal for the electorate as a whole. This paper shows how districts delegate conservatively or progressively depending on the political system in general and the voting rule in particular: If the majority requirement is large, the districts appoint more status quo biased representatives. The direction and magnitude of strategic delegation also depend on the characteristics of the relevant policy and the political system, such as the project’s value, its variance, the heterogeneity, the minority protection, the agenda-setting power and the coalition’s stability. But in each case, the selection of policies is first best if carefully selecting the voting rule. The model is applied to compare decentralization and centralization, taking strategic delegation into account.

An empirical test of the theory is clearly beyond the scope of this paper. But a future test is possible, since the model has several sharp predictions. For example, if one legislative chamber has a larger majority requirement than another, the prediction is that the first set of legislators should be more status-quo biased. For environmental issues, for example, a status-quo bias would typically mean a hesitation to pass environmental regulation. Consistent with the theory, the League of Conservation Voters typically gives representatives in the US Senate (where the possibility to filibuster implies a 60% majority rule) a lower average score than the representatives in the House (where the majority rule is 50%).

22 In the European Union, the Commission and the Parliament apply simple majority rules, while the Council typically requires super-majorities or unanimity. Finally, Proposition 3 predicts that if the president becomes

21 A comparison could also be made to the case where transfers are unfeasible even when the project is implemented. District $i$ would then like to see the project implemented if $v_i \geq \hat{\theta}$, while it is, in equilibrium, implemented if a fraction $m$ of the representatives votes for it. Clearly, a district would never benefit from choosing $d_i \neq 0$. If $v_i = v_i = \delta$, the project would be approved if

$$v_i - h(m - 1/2) \geq 0.$$

The social loss, compared to the social optimum, is:

$$\int_0^{1/2} \left( v_i - h(m - 1/2) \right) dv = \frac{h^2(m - 1/2)^2}{2a^2}.$$  

The optimal majority requirement is clearly $m = 1/2$, but otherwise this social loss can be compared to the regimes in Section 5 to determine when an m-rule without transfers is a better idea. By comparing to centralization, for example, an m-rule without transfers is better if

$$m < 1/2 + \sqrt{1/12} \approx 0.79.$$  

The model in Harstad (2008) builds on this framework, studying when allowing transfers improves efficiency. In that model, however, only unanimity rule $(m = 1)$ is considered, there are no individual shocks, and there is a finite number of principals.

22 The scores are available at http://lcv.org/scorecard/past-scorecards/. The averages are compared for the years 1981–2004, and the score is higher for the House than the Senate in 18/24 of these years. I am grateful to Yosh Halberstam for pointing this out.

23 For the current rules, see Hix (2005).

24 “For some commentators and practitioners, the Council is the blockage to European political integration, always looking to put obstacles in the way of bright ideas from the Commission or the EP” (Hayes-Renshaw and Wallace, 1997, p. 2). Also for environmental policies, Weale (2002, p. 210) observes that “the Parliament has the general reputation of having a policy position that is more pro-environmental than the Council of Ministers”. Section 5.1 discusses the modified predictions when taking into account that it is costlier to distort the Council-members’ preferences, since they are, first of all, domestic ministers.
more powerful, the majority requirement should decrease. Both features are, indeed, combined in the Treaty of Lisbon.

The analysis raises a host of questions that can be analyzed in future research. How do other aspects of the political system affect the legislators’ type? What if these types have other dimensions than the status quo bias emphasized here?

More widely, delegation is often implemented by institutional rules, not necessarily by selecting representatives. For example, Haller and Holden (1997) suggest that groups may require a local super-majority to ratify collective projects. This, in effect, delegates the ratification decision from the median voter to a more reluctant citizen, increasing the group’s bargaining power. Such delegation is, in this paper, argued to be desirable when the federal majority rule is large. Combined, the prediction is a positive correlation between the majority requirements at the federal (or the international) and the local level. Thus, one set of institutions may be strategically designed in the response to another set of institutions. This opens up a large set of questions that should be investigated in future research.

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Appendix A. Proofs

Proof of Proposition 1. The intuition for the proof is clearer if we assume a finite number $n$ of legislators, and let $n \to \infty$.25

Let $\pi_i^p$ (respectively, $\pi_i$) represent $i$‘s equilibrium utility, conditional on being (respectively, not being) the proposer. If one proposal is rejected, each $i \in M$ is the next proposer with probability $1/|M|$, where $|M|$ is the number of representatives in $M$.

Suppose, first, that $|M|/n \geq m$, and let $i \equiv p \in M$ be the current proposer. Conditional on the proposal being accepted, $p$ would like to maximize

$$\pi_i^p = \lambda v_i^p + \sum_{i \in M \setminus \{p\}} t_i + \sum_{i \in \{M \setminus \|\}} t_i,$$

(8.1)

where $\lambda = 1$ if the project is proposed, and $\lambda = 0$ otherwise. Since every coalition-member must approve the proposal, and $|M|/n \geq m$, no votes from the minority are necessary. Hence $p$ sets $t_i = TV i \in M \setminus \{p\}$. Coalition-member $i$ approves if

$$\lambda v_i - t_i \geq \delta \left(1 - \frac{1}{|M|}\right) \pi_i + \frac{\lambda}{|M|} \pi_i^p \forall i \in M.$$

(8.2)

By substituting in Eq. (8.1), any proposer maximizes Eq. (8.1) by making Eq. (8.2) bind and by setting $\lambda = 1$ if and only if $\sum_{i \in M} v_i \geq 0$. Suppose, from now on, that this is the case. Rewriting $\pi_i^p$ gives

$$\pi_i^p = v_i^p + \sum_{i \in M \setminus \{p\}} \left(\lambda v_i - \delta \right) + \left(1 - \frac{1}{|M|}\right) \pi_i + \frac{1}{|M|} \pi_i^p + (n - |M|) T.$$

(8.3)

Since Eq. (8.2) binds in equilibrium, the left-hand side of Eq. (8.2) must equal $\pi_i$. Solving for $\pi_i$ gives

$$\pi_i = \frac{\delta m^p}{|M| - \delta |M| - \delta |M|}.$$

(8.4)

where we can substitute in Eq. (8.3) to get:

$$\pi_i^p = \sum_{i \in M} v_i^p - \sum_{i \in M \setminus \{p\}} \pi_i^p + \left(1 - \frac{1}{|M|}\right) \left(1 - \frac{\delta}{|M| - \delta |M| - \delta |M|}\right) + \frac{1}{|M|} \pi_i^p + (n - |M|) T,$$

or

$$\pi_i^p = \sum_{i \in M} v_i^p - \sum_{i \in M \setminus \{p\}} \pi_i^p + \left(1 - \frac{\delta}{|M| - \delta |M| - \delta |M|}\right) + (n - |M|) T,$$

(8.5)

implying that $\pi_i^p = \pi_i^p \forall i \in M$, since the right-hand side is independent of $p$. Solving Eq. (8.5) for $\pi_i^p$ gives

$$\pi_i^p = \sum_{i \in M} v_i^p + (n - |M|) T\left(1 - \delta (1 - 1/|M|)\right).$$

(8.6)

Clearly, $\pi_i^p$ is maximized by selecting the largest $v_i$’s as $M$-members. Moreover, since $v_i^p - T > 0$, $\pi_i^p$ is maximized by reducing $|M|$. Since $|M| \geq m$, $|M| = m$ is optimal. Substituting in Eq. (8.4) gives (after dividing numerator and denominator by $n$):

$$\pi_i = \frac{\delta v_i^d m (1 - m - T)}{m},$$

where $v_i^d = \sum_{i \in \{M \setminus \{p\}\}} v_i^d/|M|$ is the average $v_i$ in $M$. With a continuum of delegates, $v_i^d = \int v_i^d d i$ implying Eq. (4.1).

Now, suppose $|M|/n = m < m$. Then, it is not enough that the majority coalition approves the proposal, other votes are needed in addition. But, by Assumption 2, the minority may collude and vote against the proposal if they, collectively, lose from the project. Thus, for a proposal to be accepted, $\sum_{i \in \{M \setminus \{p\}\}} v_i^d - t_i \geq 0$ when $m < m$. Instead of Eq. (8.6), $\pi_i$ becomes

$$\pi_i^p (m) = \sum_{i \in M} v_i^p + \sum_{i \in \{M \setminus \{p\}\}} v_i (1 - \delta) (1/\min(m)) = (1 - \delta) (1 - 1/\min(m)) \sum_{i \in \{M \setminus \{p\}\}} v_i^d.$$

In contrast, let $\pi_i^p = \pi_i^p (m)$ be given by Eq. (8.6) if $|M|/n = m$. By comparison, we find that for any $m < m$,

$$\lim_{n \to \infty} \left(\pi_i^p (m) - \pi_i^p (m)\right) = \sum_{i \in \{M \setminus \{p\}\}} (T - v_i^d) (1 - \delta) > 0,$$

where $M$ is the coalition when $|M|/n = m$. Thus, a coalition smaller than $m$ is never optimal. QED

Proof of Proposition 2. Notice that $t_i$ is a function of $i$’s shock, $t_i (\epsilon_i)$. For any $v_i$, the first-order condition of Eq. (4.2) is:

$$\frac{\partial}{\partial \epsilon_i} \left( \frac{1}{h} (T - t_i (\epsilon_i)) + \int_{\epsilon_i} (1 - d \epsilon_i) \frac{d \theta}{\theta} \right) = 0 \Rightarrow$$

$$\left( \frac{2}{h} (T - t_i (\epsilon_i)) + \int_{\epsilon_i} (1 - d \epsilon_i) \frac{d \theta}{\theta} \right) \times \frac{d \theta}{\theta} = 0 \Rightarrow$$

$$\hat{\delta} (T - t_i (\epsilon_i)) + \left( \frac{h}{2} - \delta \right) \epsilon_i = 0 \Rightarrow$$

$$\epsilon_i = \frac{h}{2} - \delta \Rightarrow$$

$$t_i (\epsilon_i) = T \hat{\delta} \epsilon_i.$$

(8.7)
→d_i + v_i^\theta E_\theta \left( T + \frac{\delta}{m}(T(1-m) + \int_{i \in M} v_i^\theta di) + \theta \cdot h /2 \right). \tag{8.8}

where E_\theta is taking the expectation over \theta conditional on \theta \leq \bar{\theta}. The derivative of Eq. (8.7) w.r.t. d_i is \int_{\sigma > 0} \frac{\partial}{\partial \sigma} \delta d_i \cdot d_i / \sigma, so the second-order conditions are fulfilled. Thus, all v_i^\theta = d_i + v_i^\theta + \epsilon_i = \theta = \delta + v_i^\theta + \epsilon_i - \theta are uniformly distributed on \{\theta + \theta - h/2, \theta + \theta - \theta + h/2\}. Since I have already ordered the derivatives by decreasing value, such that k_i < j if v_i > v_j, v_i^\theta = \theta + \theta - h/2 - h. v_m is the (1 - m)-fractile of the v_i^\theta's, and it becomes

\nu_m = d + \theta + h/2 - h \cdot m - \theta, and

\int_{i \in M} v_i^\theta di / m = d + \theta + h/2 - h \cdot m /2 - \theta.

The project is implemented whenever \theta \leq \bar{\theta}, where

\hat{\theta} = d + \theta + h/2 - h \cdot m /2, and

E_\theta = E(\theta | \theta \leq \bar{\theta}) = \frac{(d + \theta + h/2 - h \cdot m /2 - \theta)}{2} \tag{8.9}

Substituted in Eq. (8.8) gives:

\begin{align*}
d_i + v_i^\theta & = T + \frac{\delta}{m} \cdot (T(1-m) + \bar{\theta}(d + \theta + h/2 - h \cdot m/2) + E_\theta \cdot h/2 - \bar{\theta} \\
& = T(1 + \bar{\theta} / m - \bar{\theta} - \bar{\theta}) + (d + \theta) + \frac{1}{2}(1 - \bar{\theta}) - \frac{h}{4}(1 - \bar{\theta}) - \frac{m}{2}(1 + \bar{\theta}) - \bar{\theta} \frac{1}{4}.
\end{align*}

Solving for d + \theta = d_i + v_i^\theta \forall i gives Eq. (4.4), and substituting d into Eq. (8.9) gives Eq. (4.5). That d increases in \hat{\theta} can best be seen from Eq. (8.8). QED

Proof of Proposition 4. Part (i) follows from (bp). Each district chooses d_i in order to

\begin{align*}
\max_{d_i} \bar{\theta}_i & = \int_{\alpha / 2 \leq \bar{\theta} \leq 1 / 2} \int_{\alpha / 2 \leq \bar{\theta} \leq 1 / 2} (v_i^\theta + \epsilon_i - \bar{\theta} - \bar{\theta} \frac{d}{h \cdot \alpha}) - c_i d_i^2 / 2 s.t. (bp) and m = 1 and \bar{\theta} \rightarrow 1.
\end{align*}

This gives the first-order condition:

\begin{align*}
- c_i d_i = \bar{\theta}_i + \alpha / 2,
\end{align*}

where the project is implemented if

\begin{align*}
v_i^\theta + d \geq \delta \geq \bar{\theta}_i \equiv v_i^\theta + d \rightarrow - d_i = \frac{v_i^\theta + d + \alpha / 2}{c_i}.
\end{align*}

\begin{align*}
d = \frac{v_i^\theta + d + \alpha / 2}{\bar{c}_i} \rightarrow - d = \frac{v_i^\theta + d + \alpha / 2}{\bar{c}_i + 1}.
\end{align*}

\begin{align*}
\hat{\theta}_i \equiv v_i^\theta + d \geq \frac{v_i^\theta + \alpha / 2 / c_i + 1}{c_i + 1} = \frac{v_i^\theta - \alpha / 2 + c_i d_i^2 / 2}{c_i + 1},
\end{align*}

so the social loss, compared to the first-best, is:

\begin{align*}
\frac{1}{2 \gamma^2} \left( \hat{\bar{\theta}}_i - v_i^\theta \right)^2 + c_i d_i^2 / 2 = \left( \frac{v_i^\theta + \alpha / 2 / c_i + 1}{c_i + 1} \right)^2 + 2 \frac{\left( v_i^\theta + \alpha / 2 \right) d_i}{c_i + 1} = L^i. \tag{8.10}
\end{align*}

QED

Proof of Proposition 8. When M is random, so is \int_M v_i^\theta d_i. But when each i \in J is a member of M with the same probability, m, and these are i.i.d., then, by (a slight abuse of) the law of large numbers, \int_M v_i^\theta d_i = v_i^\theta + d - \theta, and the project is implemented if \theta \leq \bar{\theta} \equiv v_i^\theta + d. The principal’s problem becomes:

\begin{align*}
d_{m} \left( 1 - s \right) \int_{\alpha / 2 \leq \bar{\theta} \leq 1 / 2} \left( \frac{\epsilon_i}{h /2} \left( v_i^\theta - T \cdot \frac{d}{h} \right) - h /2 \right) \bar{\theta} d\bar{\theta} + \text{sm}E_\theta \left( \frac{d}{h} \bar{\theta} d\bar{\theta} \right) \text{ s.t. (bp) and (pp).}
\end{align*}

and it can be solved the same way as in Section 4.2. The first-order condition becomes:

\begin{align*}
d = d(\theta) - \text{sm} \frac{h}{1 - s} \quad \text{where } q = \frac{v_i^\theta + d + \alpha / 2}{v_i^\theta + d + \alpha / 2 + h(1 - m) / 2}. \tag{8.11}
\end{align*}

d(\theta) is the same function as before (Eq. (4.4)), and q is the probability that the project is implemented if the majority coalition is independent of the v_i^\theta's compared to when it is not. By introspection, the comparative static for d is just the same as before.

Take d, for a moment, as given. The social loss, compared to the first-best, can be written as:

\begin{align*}
\int_{\alpha / 2 \leq \bar{\theta} \leq 1 / 2} \left( \frac{v_i^\theta - \bar{\theta}}{2} \right)^2 + \left( \hat{\bar{\theta}}_i - v_i^\theta \right)^2 = \frac{d^2}{2 \gamma^2} + \left( 1 - s \right) \left( \frac{d + h(1 - m) / 2}{2} \right)^2.
\end{align*}

Minimizing this w.r.t d gives the optimal d^*:

\begin{align*}
d^* = -h(1 - m) / (1 - s) / 2.
\end{align*}

Combined with Eq. (8.11), the optimal m must ensure that d^* = d, implying

\begin{align*}
-v_i^\theta + 2 \frac{h}{2} \left( 1 + m / -h \right) \left( \frac{1 + m / -h}{1 - m / -h} \right) \frac{\text{sm} \frac{h}{1 - s}}{2} = -h(1 - m) / (1 - s) / 2
\end{align*}

\begin{align*}
-v_i^\theta + 2 \frac{h}{2} \left( 1 + m / -h \right) \left( \frac{1 + m / -h}{1 - m / -h} \right) \frac{\text{sm} \frac{h}{1 - s}}{2} = -h(1 - m) / (1 - s) / 2
\end{align*}

\begin{align*}
-v_i^\theta + 2 \frac{h}{2} \left( 1 + m / -h \right) \left( \frac{1 + m / -h}{1 - m / -h} \right) \frac{\text{sm} \frac{h}{1 - s}}{2} = -h(1 - m) / (1 - s) / 2
\end{align*}

By introspection, the comparative static is the same as before. QED

References


