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Market structure, innovation and vertical product differentiation

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Abstract

We reassess Arrow's (1962) [Economic Welfare and the Allocation of Resources for Invention, in NBER, The Rate and Direction of Innovative Activity (Princeton University Press, Princeton NJ)] results concerning the effect of market structure on the returns from process innovation. Here we consider product innovations that are *vertically differentiated* from older products, in the sense of Shaked and Sutton (1982) (Relaxing Price Competition through Product Differentiation, Review of Economic Studies 49, 3–13.), Shaked and Sutton (1983) (Natural Oligopolies, Econometrica 51, 1469–1484.). Competition and monopoly in the old product market provide *identical* returns to innovation when (i) the monopolist is protected from new product entry, and (ii) innovation is non-drastic, in the sense that the monopolist supplies positive quantities of both old and new products. If the monopolist can be threatened with entry, monopoly provides *strictly greater* incentives. Welfare may be greater under monopoly when innovation is valuable. Published by Elsevier Science B.V.

1. Introduction

In his seminal analysis of innovation, Arrow (1962) demonstrates that the returns generated by a process innovation are greater when the innovation is used in a competitive as opposed to a monopolized market, and hence competitive markets imply greater incentives to innovate. A key factor in the comparison of

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market structures is that under monopoly, part of the added returns from innovation serve simply to replace rents that the monopolist already earns prior to innovating. This *replacement effect* implies that the monopolist faces a higher opportunity cost of innovating, leading to a greater level of net returns under competition.[1]

In this paper, we ask whether this ranking of market structures extends to the case of *product innovations*. In particular, we consider product innovations that take the form of *vertically differentiated products*, in the sense of Shaked and Sutton (1982, 1983): the new product provides a greater level of new surplus to every consumer than does the old, if the two products are sold at a common price; but the old product remains more attractive to a subset of consumers if it is offered at a lower price. This gives rise to a natural notion of *non-drastic product innovation*, in which the existence of the old product alters the behavior of new-product sellers.

Given this notion of vertical product innovation, we ask whether returns from a new product are higher when it emerges from a competitive or a monopolized market for the old product. For a monopolized market, the monopolist can either be *protected*, in which case only the monopolist is able to market the innovation, or *threatened*, in which case any firm can market the innovation. We show that the comparison of market structures turns on two effects. Just as in Arrow's analysis, the replacement effect arises here, and it implies a higher opportunity cost of innovation under either protected or threatened monopoly, relative to competition. We identify an offsetting effect that is specific to product innovation: when the old product is competitively supplied, competition from firms producing the old product reduces the profits of the new-product supplier. This *product inertia effect* makes innovation relatively less attractive under competition, since the old-product monopolist internalizes this externality when it adopts the new product.

We demonstrate that, for the case of protected monopoly, the replacement and product inertia effects *exactly offset* when innovation is non-drastic in a vertically-differentiated market. This implies that competition and protected monopoly provide *identical* incentives for innovation. We also show that a threatened monopoly provides *strictly greater* incentives than does competition. These findings stand in sharp contrast to Arrow's results concerning process innovation.

Our analysis clarifies the conditions under which monopoly is socially preferable due to its effect in innovation. Since a protected monopoly provides no greater incentive to innovate than does competition, social surplus under competition must be greater. A threatened monopoly may, however, bring forth innovation that does not occur under competition, and threatened monopoly becomes superior when the value of innovation is great enough to outweigh the allocative efficiency

[1] See Tirole (1988), [chapter 10] for an explication of Arrow's result, along with further discussion of the replacement effect in related contexts.

of competition. These welfare rankings may be altered if total R&D spending is very sensitive to innovative rivalry, and in such cases product inertia may actually enhance welfare by *slowing* the pace of innovation, thereby reducing R&D costs. We illustrate the latter point below by means of a simple deterministic patent race model.

Our results are related to those of Gilbert and Newbery (1982), who also consider the returns to product innovation in a differentiated-products model. Restricting attention to the case of a threatened monopoly, Gilbert and Newbery show that the old-product monopolist has a greater return from preemptively adopting the new product than does an entrant, and therefore the monopoly will tend to persist. This finding, along with our own results, may be viewed as instances of a more general principle known as the *efficiency effect*: monopoly provides greater returns to innovation to the extent that it internalizes competitive externalities that dissipate post-innovation rents. In Gilbert and Newbery, potential new-product suppliers dissipate rents obtainable by the old-product monopolist, while in the present paper rivalry from old-product suppliers threatens the new-product monopolist. In either case, stronger innovation incentives are associated with monopolized market structures. The persistence of monopoly result hinges on the hypothesis that introducing the new product preempts all rivals, while product inertia continues to exert an effect under various conditions of rivalry in the innovative activity and structure of new-product supply, as we show below.[2]

A number of other papers have previously considered R&D rivalry from the point of view of vertical product differentiation: Beath et al. (1987); Dutta et al. (1993); Gruber (1992); Riordan and Salant (1994) and Shaked and Sutton (1990) study leader–follower relationships in models with two or three rivals; Chang (1995); Green and Scotchmer (1995); Scotchmer and Green (1990) and Van Dyck (1996) study patent policy in the context of vertically-differentiated innovations; and Motta (1992) and Rosenkranz (1995) considers cooperative R&D. In the present paper, we abstract from the structure of rivalry in the innovative activity in order to develop a more detailed analysis of product-market rivalry under market structures.

The paper is organized as follows. Section 2 outlines our basic model of product differentiation, and Section 3 compares the incentives to innovative under competitive and monopolistic structures of the old-product market. Section 4 compares the social surplus generated by competition and monopoly, and Section

[2] In other related work, Romano (1987) considers a version of Arrow's process-innovation model, and he shows that monopoly might provide greater incentives to innovate when the innovator can itself enter the product market. This finding and our results on threatened monopoly share a common basis: when a non-adopting monopolist can be threatened with new entry, its opportunity cost of innovating is reduced.

5 extends our analysis to allow for rivalry in the innovative activity, i.e., a patent race, as well as oligopolistic rivalry in the new-product market. Section 6 concludes the text. Proofs of propositions are given in Appendix A, and Appendix B further develops the equality-of-returns result by showing that either protected monopoly or competition may provide greater returns for appropriate perturbations of the utility functions.

2. A model of vertical product differentiation

Our analysis will build on the following model of vertical product differentiation, which is inspired by Shaked and Sutton (1982), (1983). There is a basic good that comes in old and new versions. We assume that production technology for the old version exhibits constant returns to scale, with C_O denoting its per-unit product cost. Variable production costs for the new product are constant at C_N (fixed R&D costs for the new product will be introduced in Sections 4 and 5). Further, there is a continuum of consumers with heterogeneous tastes, indexed by $\omega \in [0, 1]$, who are uniformly distributed on this interval with total mass one. Each consumer demands either zero units or one unit of the basic good, and either the old or the new product can be chosen. Let P_O and P_N be the prices of the old and new products. The net utilities for consumer ω purchasing the old and new products are given by $f_O(\omega) - P_O$ and $vf_N(\omega) - P_N$, where $v > 0$. The utility of not purchasing is zero.

We assume that $f_O(\omega)$ and $f_N(\omega)$ are twice continuously differentiable, and that $f_O(0) = f_N(0) = 0$. In addition we suppose $f'_O(\omega)$, $f'_N(\omega) > 0$ for all ω, so that higher ω indicates stronger preference for the basic good, and that $f_O(1) > C_O$, so that the old product is viable. Define $\underline{v} > 0$ by:

$$\underline{v}f_N(1) - C_N = f_O(1) - C_O$$

Assume further that $f'_O(\omega) < \underline{v}f'_N(\omega)$ for all ω; this implies that consumers $\omega > 0$ are willing to pay a greater amount for the new product, and also that the premium is greater for consumers with higher ω. Thus, for $v < \underline{v}$ the new product is not a viable competitor against the old, so we henceforth require $v \geqslant \underline{v}$. For technical purposes, we assume $\underline{v}f_N``(\omega) \leqslant f_O``(\omega) \leqslant 0$ for all ω (e.g., as a sufficient condition for concavity of profit functions). Finally, we impose the condition $f'_N(0) \leqslant C_O$, which is motivated below.

Fig. 1 illustrates these demand conditions for prices $P_N > P_O > 0$. As shown, consumers with $\omega_N < \omega \leqslant 1$ maximize utility by purchasing the new product, those with $\omega_O < \omega < \omega_N$ purchase the old product, and those with $0 \leqslant \omega < \omega_O$ choose to make no purchase. The marginal consumers ω_O and ω_N are determined by:

$$f_O(\omega_O) - P_O = 0$$

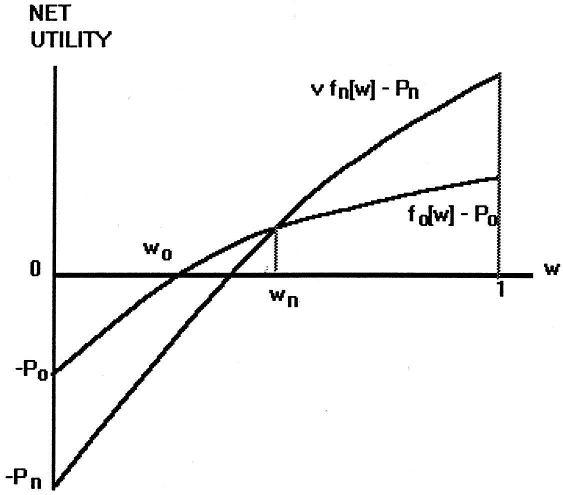

Fig. 1. Net utility from product O and product N under prices P_O and P_N.

$$f_O(\omega_N) - P_O = vf_N(\omega_N) - P_N$$

Manipulating these equations and setting $Q_O \equiv \omega_N - \omega_O$ and $Q_N \equiv 1 - \omega_N$ gives the inverse demand functions:

$$P_O = f_O(1 - Q_O - Q_N) \tag{1a}$$

$$P_N = vf_N(1 - Q_N) - f_O(1 - Q_N) + f_O(1 - Q_O - Q_N) \tag{1b}$$

Our analysis will focus primarily on the case in which the new-product market is monopolized, e.g., due to a patent.[3] Either monopoly or competition will prevail in the old-product market. This leads to three possible configurations of post-innovation market structures:

1. *Joint Monopoly*. Both the old and new products are monopolized by a single firm, which is the joint monopolist.

2. *Differentiated Duopoly*. The old and new products are monopolized by different firms, who consequently sell vertically differentiated products. For differentiated duopoly we will assume that the firms play a Nash equilibrium in quantity choices. For the fringe, we take the limiting case of quantity competition as the number of rivals becomes infinite; thus, the old-product price will be equal to C_O.

3. *Dominant–Fringe Structure*. The old-product market is competitive, and the

[3] Post-innovation market structures in which the new product need not be monopolized will be considered in Section 5.2.

new-product market is monopolized. Thus, the new-product monopolist is a dominant firm facing a competitive fringe that sells a vertically differentiated product. In this case, we assume that the dominant firm has a first-mover advantage in choosing quantities.

We will say that the new product represents a *drastic innovation* if the presence of the old product has no effect on the maximized profits of the new product monopolist, i.e., the latter may simply act as if the old product did not exist. This obviously occurs when the new product's quality is much higher than that of the old product, without a substantially higher cost. If the presence of the old product constrains the new-product monopolist, then innovation is *non-drastic*.[4] Regions of drastic and non-drastic innovation under the three market structures are characterized in the following proposition, whose proof is given in Appendix A:

Proposition 1. *There exist v^A and v^B, satisfying $\underline{v} < v^A < v^B$, such that (i) under joint monopoly, innovation is non-drastic if $\underline{v} \leq v < v^A$, and drastic if $v \geq v^A$; and (ii) under dominant–fringe and differentiated-duopoly structures, innovation is non-drastic if $\underline{v} \leq v < v^B$, and drastic if $v \geq v^B$.*

Observe that under joint monopoly, innovation is non-drastic for a strictly smaller range of v than under dominant–fringe and differentiated-duopoly structures. In proving the proposition, we invoke the condition $f_N(0) \leq C_0$ to rule out the possibility that innovation is non-drastic for all v.[5]

3. Returns to innovation

We now turn to the central question of our paper: what market structure generates the greatest returns to innovation? More specifically, how does the structure of product-market rivalry impact on innovation incentives? We measure

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[4] Our notion of drastic innovation is analogous to Bain (1949) concept of blockaded entry, where the old-product suppliers are thought of as entrant firms. Arrow (1962) defines a process innovation to be drastic if and only if the monopoly price under the new technology lies below the competitive price under the old technology; hence the profits of a monopolist operating the new technology are unaffected by the existence of the old. Our notion of drastic innovation extends Arrow's basic idea to the case of product innovation.

[5] To see how the latter may happen, consider the case of joint monopoly and suppose the monopolist chooses $Q_O = 0$. The elasticity of demand for the new product at $Q_O = 0$ is, using (1b), $\varepsilon(Q_N) = f_N(1 - Q_N)/f_N(1 - Q_N)Q_N$. Note that demand elasticity is independent of v and strictly decreasing in Q_N. Profit maximizing choice of Q_N given $Q_O = 0$ requires $\varepsilon(Q_N) > 1$. Further, let Q'_N denote the smallest level of Q_N such that the marginal profitability of the old product at $Q_O = 0$ is nonpositive. As long as $\varepsilon(Q'_N) \leq 1$, it is never optimal for the monopolist to choose $Q_O = 0$, no matter how large v is. The condition $f'_N(0) \leq C_O$ implies $\varepsilon_N(Q'_N) > 1$, however, and innovation becomes drastic once v reaches the critical level v^A. Similar comments apply with respect to the other market structures.

the returns to innovating in terms of the post-innovation profits obtainable by a new-product monopolist, i.e., returns are given by the value of a patent for the new product, gross of R&D costs. Following Arrow, we frame the question in terms of a rhetorically convenient conceptual experiment, in which the innovation is controlled by a patent-holding inventor, whose returns are generated by licensing revenue. Thus the question becomes, what market structure allows the inventor to obtain the greatest possible revenue?

Consider first the case of competition in the old-product market. Here the inventor may license to as many firms as desired, and he can appropriate any profits generated by the new product by means of licensing fees. Denote by Π^{C} the profits earned by the new-product monopolist under dominant–fringe structure; thus Π^{C} gives the maximum licensing revenue of the inventor, which the inventor may obtain by licensing to a single firm that becomes the new-product monopolist.

Now suppose there is a monopoly in the old-product market. In this case, there are two possible situations. First, the monopoly power of the old-product monopolist might extend as well to the new-product market, so that the inventor cannot license to any firm other than the old-product monopolist. This may occur when the monopolist controls essential production or distribution facilities or brand names, or when the monopolist holds complementary patents. We call this the situation of *protected monopoly*. Alternatively, the inventor might be free to license to any firm, either the old-product monopolist or a new entrant; we call this the case of *threatened monopoly*.

In either situation, by adopting the new product the old-product monopolist earns the joint monopoly profits, denoted Π^{M}, less the licensing fee. Under protected monopoly, adoption will be chosen only if these postadoption profits exceed what the monopolist would have earned by simply keeping its old-product monopoly; profits in the latter case are written Π^{OM}. Thus, $\Pi^{M}-\Pi^{OM}$ gives the maximum licensing revenue available to the inventor under protected monopoly.

Competitive market structure provides greater returns to the inventor than does protected monopoly if $\Pi^{C}>\Pi^{M}-\Pi^{OM}$, which can be reexpressed as:

$$\Pi^{OM} > \Pi^{M} - \Pi^{C}$$

On the left-hand side we have the difference between firms' opportunity costs of adoption under protected monopoly and competition, which is certain to be strictly positive. This is the *replacement effect* discussed by Arrow, and it may be seen that the effect tends toward giving competition the advantage as far as innovation incentives.

On the right-hand side we have the difference between the total benefits of adoption under competition and monopoly, and this will be strictly positive as long as the new product does not completely displace the old. This is a version of the efficiency effect: competition from the old product, which takes the form of a competitive fringe of old-product suppliers, limits the rents that can be extracted from the new product. The old product acts as a drag on the incentives to adopt the new; hence, to highlight the vertical-differentiation context, we call this the *product inertia effect*. Note that this effect tends toward providing greater incentives under monopoly.

Since the replacement and product inertia effects cut in opposite directions, it is not immediately clear which market structure provides greater incentives for innovation. In fact, we can show that when protected monopoly leads to non-drastic innovation, i.e., when a joint monopolist supplies positive quantities of both old and new products, the two effects *exactly offset*, and protected monopoly and competition provide precisely the same returns to innovation. This surprising result hinges on the relationships between pre- and post-innovation quantities. Let Q_O^M and Q_N^M denote the quantities supplies under joint monopoly, Q_N^C the new-product supply under dominant–fringe structure, and Q_O^{OM} the monopoly supply of the old product when the new product has not been adopted. In Appendix A we prove the following:

Lemma. *If $\underline{v} \leq v \leq v^A$, then: (a) $Q_O^{OM} = Q_O^M + Q_N^M$, i.e., the protected monopolist's total output is the same whether or not it adopts the new product; and (b) $Q_N^M = Q_N^C$, i.e., output of the new product under joint monopoly is identical to output under dominant–fringe structure.*

The Lemma relates important restrictions on the equilibrium quantities when innovation is non-drastic under joint monopoly. To develop some feel for these restrictions, let us express the profit function of the joint monopolist in terms of the total output $Q \equiv Q_O + Q_N$ and the new-product output Q_N; using (1a) and (1b), we have:

$$[f_O(1-Q)-C_O][Q-Q_N]+[vf_N(1-Q_N)-f_O(1-Q_N)+f_O(1-Q)-C_N]Q_N$$
$$=[f_O(1-Q)-C_O]Q+[vf_N(1-Q_N)-f_O(1-Q_N)+C_O-C_N]Q_N \quad (2)$$

Observe that on the right-hand side of (2), profits have been decomposed into two components, with one component depending only on Q and the other depending only on Q_N. Part (a) follows from the fact that the Q component has exactly the same form as the profit function of a non-adopting monopolist. Essentially, the total quantity produced in the non-drastic case hinges on the marginal old-product purchaser, and under our demand structure this purchaser's behavior is the same whether or not the monopolist adopts the new product.

Part (b) is a bit more subtle. Note first that under dominant–fringe structure, the competitive fringe imposes an ‘‘implicit tax’’ on the new-product supplier; using (1a), (1b) and $P_O = C_O$, we may express the new-product price as:

$$P_N = vf_N(1-Q_N) - \{f_O(1-Q_N) - C_O\} \quad (3)$$

The implicit tax is given by the term in braces. Similarly, the existence of the old product reduces the new-product price under joint monopoly, but the effect is more complex since the monopolist earns profits from both products. Note however that in the Q_N component of the right-hand side of (2), the effective new-product price is exactly the same as (3). Since the implicit tax is the same in each case, the incentives to produce the new product are also the same.

The equality of implicit tax rates is explained by two effects. First, the existence of the old product under joint monopoly directly reduces P_N by an amount $f_O(1-Q_N)-P_O$, as can be seen in (1b). When innovation is non-drastic, however, the joint monopolist reduces its sales of the old product at a one-to-one rate when it sells additional units of the new; this imposes an added tax of P_O-C_O, which is the profit margin on the old product. The sum of these effects yields an implicit tax rate that is identical to the level under dominant–fringe structure.

We now establish $\Pi^C=\Pi^M-\Pi^{OM}$. Using the Lemma and (2), maximized profits under joint monopoly may be written:

$$\Pi^M=[f(1-Q^{OM})-C_O]Q^{OM}$$
$$+[vf_N(1-Q_N^C)-f_O(1-Q_N^C)+C_O-C_N]Q_N^C=\Pi^{OM}+\Pi^C$$

which establishes the result. In essence, the implicit tax rates under dominant–fringe and joint monopoly structures serve as per-unit measures of the product inertia and replacement effects, respectively. Equality of the tax rates directly reflects equality of the two effects.

For innovations of greater value, in particular those that are drastic under joint monopoly but not under dominant–fringe structure, the product inertia effect is partially attenuated by the fact that $Q_N^C>Q_M^N$ when $v^A<v<v^B$. As a consequence, competition provides strictly greater returns than does protected monopoly. The arguments supporting this result are given in Appendix A, and here we summarize with:

Proposition 2. *(a) If $\underline{v}\leqslant v\leqslant v^A$, then $\Pi^C=\Pi^M-\Pi^{OM}$, and competition and protected monopoly provide the same returns to innovation; (b) If $v>v^A$, then $\Pi^C>\Pi^M-\Pi^{OM}$, and competition provides strictly greater returns.*

The equality of returns shown in part (a) of Proposition 2 establishes that competition and protected monopoly can provide equal incentives to innovate under reasonable market conditions. It is appropriate to ask, however, whether the equality is part of a larger weak inequality over a broader class of demand conditions, or whether equality itself holds up for broader conditions. This question is addressed in Appendix B, where it is shown that by appropriately perturbing the utility functions in the $\underline{v}<v\leqslant v^A$ case, either competition or protected monopoly may provide strictly greater returns; thus equality should be interpreted to mean that returns will be very close over a large range of demand conditions, but that it is possible for either market structure to provide greater returns.

Now consider the case of threatened monopoly, in which the inventor can license the innovation to any firm. As with competition, the inventor does best by licensing to whatever number of firms maximizes industry profits, and the latter is accomplished by selling the rights to a single firm. If this firm is a new entrant, then by adopting the innovation it earns the new-product monopolist's Nash equilibrium profit, given by Π^{ND}, less the licensing fee, as adoption leads to a differentiated duopoly that pits the entrant against the old-product monopolist. The opportunity cost of adoption for the new entrant is the normal profit rate, i.e., zero.

If the inventor sells to the old-product monopolist, then the latter's profits upon adoption are Π^{M} less the licensing fee. The monopolist's opportunity cost is the Nash equilibrium profit of the old-product monopolist, written Π^{OD}, since the inventor will sell to a new entrant if the monopolist declines to adopt. Thus $\Pi^{\mathrm{M}} - \Pi^{\mathrm{OD}}$ is the maximum revenue that the inventor can gain by licensing to the old-product monopolist. Now, as long as $\underline{v} \leqslant v < v^{\mathrm{B}}$ we have $\Pi^{\mathrm{M}} > \Pi^{\mathrm{OD}} + \Pi^{\mathrm{ND}}$, i.e., joint monopoly yields strictly greater industry profits than does differentiated duopoly, and it is apparent that the inventor's best policy is to license to the old-product monopolist (here we have an instance of Gilbert and Newbery (1982) ''persistence of monopoly'' result).

In view of Proposition 2, it is simple to compare competition and threatened monopoly for the non-drastic case: we have $\Pi^{\mathrm{OD}} < \Pi^{\mathrm{OM}}$, since the entrant cuts into the old-product monopolist's market, and thus $\Pi^{\mathrm{M}} - \Pi^{\mathrm{OD}} > \Pi^{\mathrm{M}} - \Pi^{\mathrm{OM}} = \Pi^{\mathrm{C}}$ when innovation is non-drastic under joint monopoly. In this case, the replacement effect is mitigated by the threat of entry. The resulting dominance of the product inertia effect means that threatened monopoly provides *strictly greater* returns to innovation than does competition. In Appendix A we extend this conclusion to the interval $(v^{\mathrm{A}}, v^{\mathrm{B}})$, on which innovation is non-drastic under dominant–fringe structure but not under joint monopoly. This establishes:

Proposition 3. *(a) If $\underline{v} \leqslant v < v^{\mathrm{B}}$, then $\Pi^{M} - \Pi^{OD} > \Pi^{C}$, and threatened monopoly provides strictly greater returns to innovation than does competition; (b) If $v \geqslant v^{B}$, then $\Pi^{M} - \Pi^{OD} = \Pi^{C}$, and the two market structures provide the same returns.*

It follows that, for product innovations that are vertically differentiated from established products, Arrow's comparison of market structures is reversed when (i) the innovation is non-drastic, and (ii) the old-product monopolist can be threatened by a new-product entrant.

We conclude this section by considering one final situation, in which the old-product market is monopolized, but the inventor can license the new product only to a new entrant, e.g., because institutional constraints prohibit the old-product monopolist from adopting the new product. We call this the case of *excluded monopoly*. Since the inventor cannot license to the old-product monopolist, his maximum licensing revenue becomes Π^{ND}, and since $\Pi^{\mathrm{ND}} < \Pi^{\mathrm{M}} - \Pi^{\mathrm{OD}}$ for $\underline{v} \leqslant v < v^{\mathrm{B}}$ it follows that excluded monopoly provides strictly lower returns to innovation than does threatened monopoly. In Appendix A we demonstrate $\Pi^{C} < \Pi^{ND}$ for these levels of v, and this gives:

Proposition 4. *(a) If $\underline{v} \leq v < v^{B}$, then $\Pi^{M} - \Pi^{OD} > \Pi^{ND} > \Pi^{C}$, and excluded monopoly provides returns to innovation that are strictly between those of threatened monopoly and competition; (b) If $v \geq v^{B}$, then $\Pi^{M} - \Pi^{OD} = \Pi^{ND} = \Pi^{C}$, and all of the market structures provide the same returns.*

The superiority of excluded monopoly over competition in the non-drastic case illustrates how the product inertia effect is sensitive to the intensity of competition from the old-product market. Differentiated duopoly conveys greater market power in the new-product market than does dominant–fringe structure, and as a consequence innovation under excluded monopoly generates more rents for the inventor to extract. This suggests a more general conclusion: if innovation is non-drastic, then returns to innovation rise as the old-product market becomes more concentrated. In this way, the efficiency effect, as reflected here by product inertia, gives a direct link between monopoly power and innovation incentives.

4. Social welfare

In this section, we consider normative aspects of vertically-differentiated product innovation. While competition in the old-product market yields better allocative efficiency for a given set of products, monopoly may give rise to innovation that would not occur under competition. Monopoly may then be superior on balance if the innovation is of sufficient value. Let S^{C}_{NA} denote the social surplus obtaining from competition in the old-product market when the new product is not adopted, and let S^{M}_{A} denote the surplus associated with an old-product monopoly that adopts the new product. This trade-off between market structures is made explicit in the following proposition, which is proved in Appendix A:

Proposition 5. *There exists v^*, satisfying $\underline{v} < v^* < v^{B}$, such that $S^{C}_{NA} > S^{M}_{A}$ if $v < v^*$, $S^{C}_{NA} = S^{M}_{A}$ if $v = v^*$, and $S^{C}_{NA} < S^{M}_{A}$ if $v > v^*$.*

According to this proposition, there exists a level v^* such that for v above this level, monopoly with adoption provides strictly greater social surplus than competition without adoption. It follows that normative comparison of the market structures depends on the incentives that are provided to adopt the new product, together with the value of the innovation.

To illustrate the normative comparison of market structures, let us consider the very simple situation in which the inventor must incur a fixed R&D cost of $F > 0$

in order to bring forth the new product. This can be thought of as a case with no rivalry in R&D, where only one inventor has the knowledge to see the innovation; we consider alternate possibilities in Section 5.1. In the present instance, adoption of the new product occurs if and only if the returns from adoption exceed *F*. It is clear from Proposition 2 that competition provides strictly greater social welfare than protected monopoly, since $\Pi^{C} > \Pi^{M} - \Pi^{OM}$ implies that adoption occurs under competition whenever it occurs under protected monopoly. Threatened monopoly may provide greater welfare than competition, however, since we have $\Pi^{M} - \Pi^{OD} > \Pi^{C}$ for $\underline{v} < v < v^{B}$.

The latter comparison is illustrated in Fig. 2, which depicts $\Pi^{M} - \Pi^{OD}$ and Π^{C} as functions of v for a given specification of the functions $f_{O}(\omega)$ and $f_{N}(\omega)$.[6] For $F \leqslant \Pi^{C}$, adoption occurs under both threatened monopoly and competition, while for $F > \Pi^{M} - \Pi^{OD}$ adoption occurs under neither; in these regions competition is superior based on static allocative efficiency. In the area between the curves,

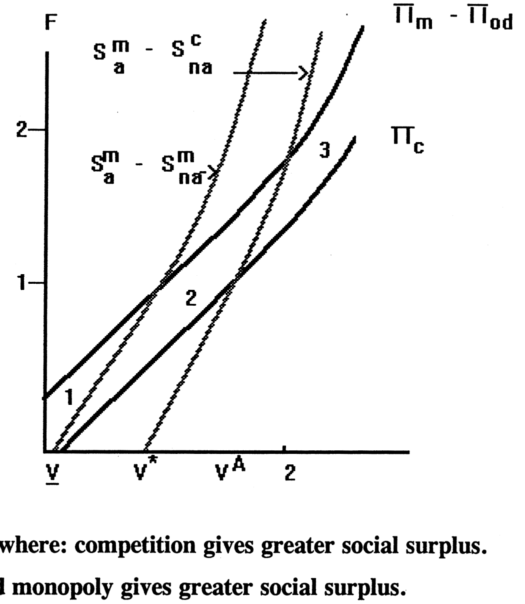

Fig. 2. Social Surplus with fixed R&D cost F. Regions 1, 2 and elsewhere: competition gives greater social surplus. Regions 3: threatened monopoly gives greater social surplus.

[6] In particular, we specify $f_{O}(\omega) = f_{N}(\omega) = \omega$. Fig. 2 is calculated for the values $C_{O} = 4/7$ and $C_{N} = 6/7$; this gives $\underline{v} = 9/7$, $v^{A} = 1.5$ and $v^{B} = 6$. The propositions hold for this example despite the violation of the assumption $f'_{N}(0) \leqslant C_{O}$; for $f_{O}(\omega) = f_{N}(\omega) = \omega$ the assumption may be replaced by $C_{O} > 1/2$.

labelled Regions 1, 2 and 3, adoption occurs under threatened monopoly but not under competition. Note further that S^{M}_{NA} denotes surplus under monopoly in the old-product market when the new product is not adopted, and the social surplus measures are gross of R&D expenditures. In Regions 1 and 2 we have $S^{M}_{A} - F < S^{C}_{NA}$, so competition remains superior. In Region 3, however, the value of innovation is great enough to outweigh the efficiency advantage of competition, and the inequality is reversed. In this region, it is threatened monopoly that provides the greater social welfare.

Interestingly, in Region 1 social welfare under monopoly would be greater if the monopolist did not adopt the innovation. This exemplifies the familiar proposition that rivalry may lead to socially excessive innovation, where in this instance it is product market rivalry, rather than rivalry to acquire the patent, that leads to excessive adoption. Finally, excluded monopoly and competition may be compared by noting that the curve Π^{ND} lies between $\Pi^{M} - \Pi^{OD}$ and Π^{C}, so that the areas corresponding to Regions 1, 2 and 3 become smaller.

5. Extensions

5.1. Rivalry in the innovative activity

In the preceding analysis we have taken the point of view of an inventor whose innovation decision is essentially static, and who faces no rivals in the innovative activity. Actual R&D activity, however, has important dynamic aspects and frequently involves intense rivalry among potential discoverers of the innovation. Further, rivalry typically affects the amount of R&D expenditures that are undertaken, and thereby exerts an added effect on net social surplus. In this section, we will introduce rivalry in the innovative activity that determines the timing of innovation together with the resources devoted to R&D. The main new finding is that, in situations where delaying innovation generates large reductions in R&D costs, product inertia may become favorable for welfare, due to its tendency to slow innovation and thereby to mitigate against excessive R&D expenditures.

To avoid unnecessary complications, we will follow Gilbert and Newbery and restrict attention to the simple deterministic patent-race model developed by Barzel (1968). There is a pool of potential innovators, any of whom can discover the innovation at time T by paying an R&D cost of $F(T)$, to be incurred at the instant of discovery. Time is continuous, and $F(T)$ is positive and strictly decreasing in T; the assumption of declining R&D costs can be justified in terms of complementary discoveries in other sectors, or diseconomies stemming from compression of research activity. When one innovator makes the discovery at T, he obtains a patent on the product, and his subsequent profits are determined as above.

Suppose first that the pool of potential innovators includes any agent that desires to make the R&D investment, so that R&D activity is disintegrated from the production process for the new product; we call this *disintegrated R&D*. Innovation occurs as soon as R&D costs exactly dissipate all rents available from the patent. In the case of competition, for example, the equilibrium discovery time is determined by:

$$\frac{\Pi^{C}}{r} - F(T) = 0 \tag{4}$$

where r is the rate of discount, and we have assumed for simplicity that the flow of profits from the new product continues unchanged for all time. The discovery time is determined similarly for protected and threatened monopoly.

Let us now consider social surplus in patent race equilibria for the specification of Fig. 2, where we also set $F(T)=F(0)e^{-dT}$. The parameter $d>0$ indicates the rate of which R&D costs decline over time. Further, we put $F(0)=(\Pi^{M}-\Pi^{OD})/r$, i.e., time zero is taken to be the instant at which adoption occurs under threatened monopoly; measuring discounted social surplus at $t=0$ then serves to maximize the relative advantage of threatened monopoly versus competition.[7]

The results are summarized in Fig. 3. For $v<v^{B}$, adoption occurs sooner under threatened monopoly than under competition. But threatened monopoly gives greater equilibrium social surplus than competition only in Region 1, where d is small and the value of innovation is relatively high. The small d case approximates the static situation of Section 4; in particular, as d approaches zero, the patent race outcomes converge to the points on the curve $\Pi^{M}-\Pi^{OD}$ in Fig. 2, and it follows from above that threatened monopoly is superior if v is sufficiently large. For larger d, delayed adoption leads to greater marginal reductions in R&D costs, and as a consequence the slower adoption makes competition more attractive. Here product inertia becomes desirable precisely because it slows innovation.

A similar effect arises when competition and protected monopoly are compared. Adoption occurs no later under competition, but in Region 4 social surplus is greater under protected monopoly due to the savings in R&D costs results from slower adoption. Now it is the replacement effect that becomes relatively favorable for welfare. Interestingly, the relative advantage of protected monopoly increases as v rises, since an increase in v serves to widen the gap between adoption times. Finally, threatened monopoly is superior to protected monopoly only in Regions 1 and 2, where again the slower adoption under protected monopoly becomes attractive when d is large.

Let us now consider the alternative possibility that R&D is directly related to

[7] Measuring social surplus at other times will increase the weight placed on times at which adoption has occurred under neither or both threatened monopoly and competition, and at such times threatened monopoly is relatively less attractive. Thus our results give an upper bound on the attractiveness of threatened monopoly.

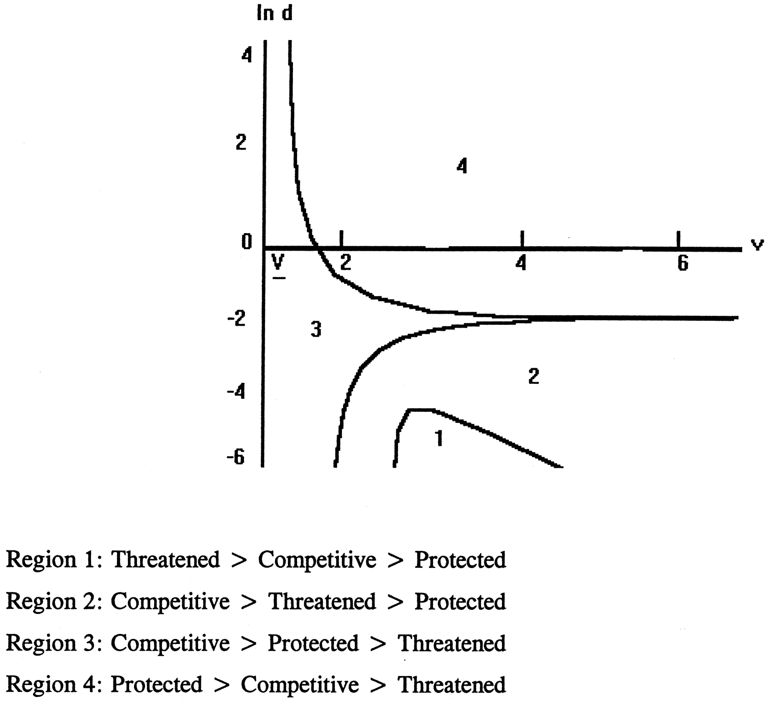

Fig. 3. Social Surplus in patent race equilibria, case of disintegrated R&D. Region 1: Threatened > Competitive > Protected. Region 2: Competitive > Threatened > Protected. Region 3: Competitive > Protected > Threatened. Region 4: Protected > Competitive > Threatened.

the production process, so that the producing firm must pay its own R&D cost rather than licensing from an outsider; this is called *integrated R&D.* The distinction between integrated and disintegrated R&D is immaterial under competition, as any firm can produce the new product; and (4) continues to give the equilibrium discovery time. Protected monopoly is affected, however, since integrated R&D gives the old-product producer a monopoly over the innovative activity. Now the firm chooses its discovery time to maximize discounted profits net of R&D costs, given by:

$$e^{-rT}\left[\frac{\Pi^{\mathrm{M}}-\Pi^{\mathrm{OM}}}{r}-F(T)\right]$$

The first-order condition for maximization with respect to T is:

$$\frac{\Pi^{\mathrm{M}}-\Pi^{\mathrm{OM}}}{r}-F(T)+\frac{F'(T)}{r}=0 \tag{5}$$

Since $F'<0$, it follows that discovery occurs strictly later when R&D is integrated. Threatened monopoly is similarly affected, except that new entrants can earn Π^{ND} by innovating, and this places an upper bound on how long the threatened monopolist can delay innovation.[8]

Fig. 4 summarizes equilibrium social surplus in the integrated R&D case, for the same specification of the model as in Fig. 3. For low values of *d*, the reductions in R&D costs from delay are small relative to the surplus that consumers earn from the new product, so that delay is excessive under protected and threatened monopoly. Thus Region 1, on which threatened monopoly is superior to competition, is smaller here than in the disintegrated R&D case of Fig. 3. For large *d*, in contrast, delay leads to large cost reductions, and protected and threatened monopoly become relatively more attractive; note that Region 4, on

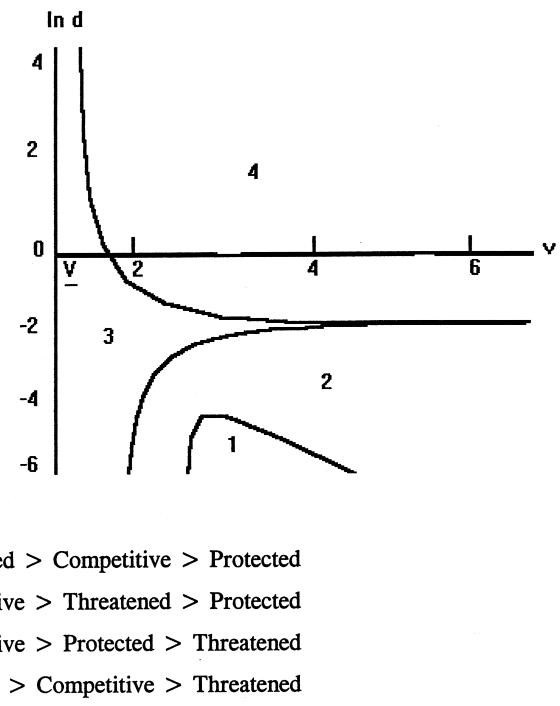

Fig. 4. Social Surplus in patent race equilibria, case of integrated R&D. Region 1: Threatened> Competitive>Protected. Region 2: Competitive>Threatened>Protected. Region 3: Competitive> Protected>Threatened. Region 4: Protected>Competitive>Threatened.

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[8] Since new entrants fully dissipate rents, the entry time for new entrants is determined by (4) with Π^{ND} replacing Π^{C}. The threatened monopolist is thus constrained to innovate no later than the new-entrant entry time, which may be sooner than the unconstrained time determined by (5). It can be shown analytically that this constraint becomes tighter as *d* rises.

which protected monopoly dominates competition, becomes much larger under integrated R&D due to the added delay under protected monopoly.

5.2. Rivalry in the new-product market

Thus far we have assumed that the new product is controlled exclusively by a patent-holding inventor. In many cases, however, patent protection is not available to developers of new products, and there arises rivalry between firms that market different variants of the new technology. In this section, we show that product inertia continues to play a fundamental role when there is the potential for rivalry in the new-product market.

We now suppose that two firms produce the old product. Each has the option of adopting the new product at a cost of F. We consider a two-stage game between these firms: in the first stage, the firms choose simultaneously whether or not to market the new product. In the second stage, the firms choose quantities to produce, where the new product can be produced only if the firm had decided to market it in the first stage, while the old product can be produced irrespective of the stage-one decision. Demand and costs take the same form as above. We study subgame–perfect equilibria of this game, in which the quantity decisions give Nash equilibria of the second stage for every possible adoption profile in the first stage.[9]

Fig. 5 summarizes the pure-strategy adoption equilibria in terms of F and v, for the specification considered in Fig. 2.[10] In Regions 1a–d, neither firm adopts due to the high adoption costs. In Region 2 only one firm adopts, and in Regions 3a–b both firms adopt. Under threatened monopoly, in contrast, there is no adoption in Regions 1a and 1b, while adoption occurs in the remaining regions. Thus Region 1c and 1d are associated with outcomes in which adoption fails to occur in the duopoly case, even though a threatened monopolist would adopt. Here product inertia derives from the rival producer of the old product: although either firm could adopt and acquire a monopoly position in the new-product market, the presence of the rival reduces the returns to adoption and makes it unattractive relative to threatened monopoly.

Let us now compare the adoption equilibria to the adoption profiles that would maximize social surplus. Social surplus is calculated under the assumption that outputs are determined by Cournot quantity-setting, i.e., imperfect competition in the quantity stage is taken as a constraint. In Regions 1a and 1c of Fig. 5,

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[9] Shaked and Sutton (1990) consider a reduced-form adoption game having a similar decision-theoretic structure; here we derive the payoffs from a specific model of differentiated-products duopoly.

[10] Duopoly equilibria are unique on the interiors of Regions 1 and 3, in which neither adopt or both adopt. In Region 2, there exist a pair of asymmetric pure-strategy equilibria in which only one firm adopts, as well as a single symmetric mixed-strategy equilibrium.

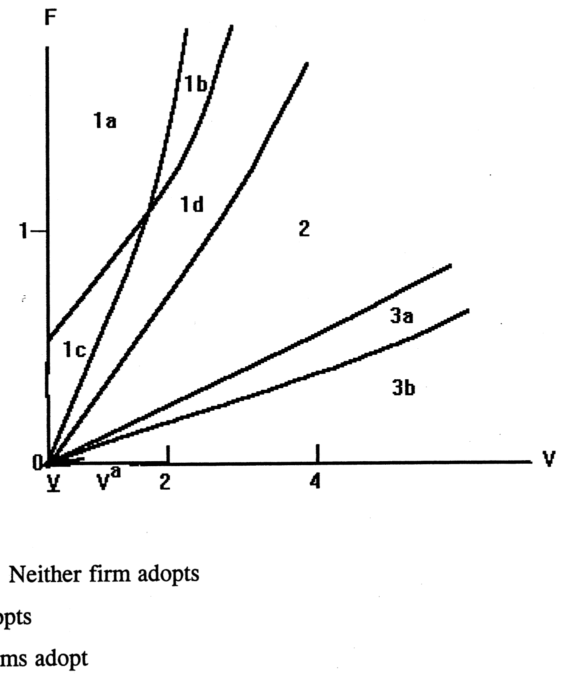

Fig. 5. Duopoly adoption equilibria. Regions 1a, 1b, 1c, 1d: Neither firm adopts. Region 2: One firm adopts. Region 3a, 3b: Both firms adopt.

non-adoption maximizes social surplus, while in Regions 1b, 1d, 2 and 3a the highest social surplus is associated with adoption by only one firm. Adoption by both firms is optimal only in Region 3b. It follows that adoption is suboptimally low in Regions 1b and 1d, as a consequence of product inertia together with incomplete appropriability of consumer surplus, while adoption is suboptimally high in Region 3a. In the latter region, which is associated with relatively high v for given F, most of the gains in consumer surplus are generated by the initial adoption, and the cost of the second adoption outweighs the gain in surplus from having lower concentration in the new-product market.

6. Conclusion

Our analysis demonstrates that, for the case of a product innovation that is vertically differentiated from old products, monopoly in the old-product market may provide greater incentives for innovation than does competition. The key effect is product inertia, which is a variant of the efficiency effect wherein competition from the old product reduces the rents available from the new. When innovation is non-drastic under protected monopoly, our model demonstrates that the incentives to innovate under competition and protected monopoly are identical. Further, incentives are strictly greater under monopoly when the monopolist can be threatened by entry. These findings constitute a reversal of Arrow (1962) conclusions with respect to process innovations. Threatened monopoly may provide greater social welfare than competition when the innovation is sufficiently valuable to offset the static allocative efficiency of competition, and realized R&D costs are not excessively sensitive to rivalry in the innovative activity.

Stated more briefly, monopoly threatened with entry gives greater social welfare when innovations are valuable and adoption costs are high, while competition is best when adoption costs are low. This welfare analysis suggests a potentially useful classification of industries by scope for innovation. Highly innovative sectors in which entirely new markets emerge from rapid technological progress (computers, communication equipment, biotech) may most often provide greater welfare when markets are monopolized. In sectors where new technologies are less fundamental and innovation takes the form of nonprice competition via small product improvements (food and household items, apparel), welfare may tend to be higher under competition, since the gains from innovation are small. As pointed out above, this classification may be reversed if excessive R&D expenditures become the dominant consideration.

We count ourselves among the growing number of researchers who have found that models of vertically-differentiated markets open up fruitful avenues for developing new insights about innovation. A number of further questions seem very well-suited to this approach: How important is product inertia for the returns to low-quality, low-cost innovations (e.g., the replacement of fresh vegetables by cheaper frozen vegetables)? How does product inertia affect firms that market a portfolio of old and new products, or who spread incremental improvements across a product line? Does product inertia exert a greater effect when there is a large number of old-product firms, or is there a nonmotonic relation between concentration and innovation incentives? These and related questions about the relationship between market structure and innovation arise naturally once innovation is formulated in terms of vertical product differentiation.

Acknowledgements

This paper is a revised and extended version of Greenstein and Ramey (1988). We thank seminar participants at the 1989 AEA Winter Meetings, the European Association for Research in Industrial Organization, Stanford University, the University of Illinois and UCSD, as well as an anonymous review.

Appendix A

Proofs of Propositions and Lemma

Proof of Proposition 1. The profits of a joint monopolist may be written, using (1a–b):

$$\Pi = [f_0(1 - Q_O - C_O]Q_O + [vf_N(1 - Q_N) - f_O(1 - Q_O) + f_O(1 - Q_O - Q_N) - C_N]Q_N \quad \text{(A1)}$$

Under our assumptions, (A1) is a strictly concave function; let Q_O^M and Q_N^M denote the unique maximizers of (A1). We have $Q_N^M > 0$ as a consequence of $v > \underline{v}$. The remaining possibilities are $Q_O^M > 0$, which constitutes non-drastic innovation, and $Q_O^M = 0$, which is drastic innovation. Q_O^M and Q_N^M in the non-drastic case are characterized by:

$$-f'_O(1 - Q_O^M - Q_N^M)[Q_O^M + Q_N^M] + f_O(1 - Q_O^M - Q_N^M) - C_0 = 0 \quad \text{(A2a)}$$

$$-f'_O(1 - Q_O^M - Q_N^M)[Q_O^M + Q_N^M] - [vf'_N(1 - Q_N^M) - f'_O(1 - Q_N^M)]Q_N^M + vf_N(1 - Q_N^M) - f_O(1 - Q_N^M) + f_O(1 - Q_O^M - Q_N^M) - C_N = 0 \quad \text{(A2b)}$$

and in the drastic case by:

$$-f'_O(1 - Q_N^M)Q_N^M + f_O(1 - Q_N^M) - C_O \leq 0 \quad \text{(A3a)}$$

$$-vf'_N(1 - Q_N^M)Q_N^M + vf_N(1 - Q_N^M) - C_N = 0 \quad \text{(A3b)}$$

Let Q'_N be the level of Q_N that gives quality in (A3a). Since $Q'_N < 1$ necessarily, we may choose v sufficiently close to $\underline{v}$ to give:

$$vf_N(1 - Q'_N) - C_N - f_O(1 - Q'_N) + C_O < 0$$

For $Q_N^M \geq Q'_N$ we may invoke $vf'_N(\omega) > vf'_O(\omega)$ to obtain:

$$vf_N(1 - Q_N^M) - C_N - f_O(1 - Q_N^M) + C_O < 0$$

Thus if Q_N^M satisfied (A3b) we would have:

$$0 = -vf'_N(1 - Q_N^M)Q_N^M + vf_N(1 - Q_N^M) - C_N < -f'_O(1 - Q_N^M)Q_N^M + f_O(1 - Q_N^M) - C_O$$

in violation of (A3a); thus, innovation must be non-drastic for v sufficiently close to $\underline{v}$.

Observe next that our assumptions imply, for all $\omega > 0$:

$$f_O(\omega) - f_N(\omega) - f'_O(\omega)[1 - \omega] + f'_N(\omega)[1 - \omega] < f'_N(0) \leq C_O$$

Rearranging gives:

$$-f'_O(\omega)[1-\omega]+f_O(\omega)-C_O < -f'_N(\omega)[1-\omega]+f_N(\omega)$$

and at $\omega=1-Q'_N$ we have:

$$0=-f'_O(1-Q'_N)Q'_N+f_O(1-Q'_N)-C_O < -f'_N(1-Q'_N)Q'_N+f_N(1-Q'_N)$$

It follows that for sufficiently large v:

$$-f'_N(1-Q'_N)Q'_N+vf_N(1-Q'_N)-C_N>0$$

and (A3b) will be satisfied by $Q_N^M>Q'_N$, which implies satisfaction of (A3a). Thus innovation is drastic for sufficiently large v. Note finally that the Q_N^M satisfying (A3b) is strictly increasing in v, which implies that once innovation is drastic at a given v, it continues to be drastic for all larger v. It follows that v^A is given by the value of v such that (A3a) holds with equality.

Next, let Q_O^C denote the fringe supply, and let Q_N^L be defined by:

$$f_O(1-Q_N^L)-C_O=0 \tag{A4}$$

If the new-product monopolist chooses $Q_N<Q_N^L$, then Q_O^C is determined by the requirement that price equal marginal cost in the old-product market:

$$f_O(1-Q_O^C-Q_N)-C_O=0$$

while if $Q_N\geqslant Q_N^L$ we have $Q_O^C=0$; thus Q_N^L is the limit quantity. If $Q_N\leqslant Q_N^L$, then the monopolist's profits are:

$$\Pi=[vf_N(1-Q_N)-f_O(1-Q_N)+C_O-C_N]Q_N$$

which is a strictly concave function. Let Q_N^C denote its maximizer on $[0, Q_N^L]$; we have $\underline{Q}_N^C>0$ as a consequence of $v>\underline{v}$. If $\underline{Q}_N^C<Q_N^L$, then $\underline{Q}_N^C$ is characterized by:

$$-[vf'_N(1-\underline{Q}_N^C)-f'_O(1-\underline{Q}_N^C)]\underline{Q}_N^C+vf_N(1-\underline{Q}_N^C)-f_O(1-\underline{Q}_N^C)+C_O-C_N$$
$$=0 \tag{A5}$$

If instead we have $Q_N\geqslant Q_N^L$, then the monopolist's profit function is:

$$\Pi=[vf_N(1-Q_N)-C_N]Q_N$$

which again is strictly concave. Let $\bar{Q}_N^C$ denote the maximizer of this function over *all* quantities, including $Q_N<Q_N^L$; thus $\bar{Q}_N^C$ is defined by (A3b), with $\bar{Q}_N^C$ replacing Q_N^M. It follows that innovation is drastic as long as $\bar{Q}_N^C\geqslant Q_N^L$, since the new-product monopolist may simply implement the unconstrained profit maximum by choosing $Q_N^C=\underline{Q}_N^C$, while if $\bar{Q}_N^C<Q_N^L$ the presence of the competitive fringe affects the monopolist's profits, so that innovation is non-drastic. In the latter case, the monopolist's optimal choice is $Q_N^C=\underline{Q}_N^C$.

The earlier arguments are easily modified to establish that innovation is non-drastic for v sufficiently close to $\underline{v}$ (replace Q'_N with Q_N^L). Further, the condition $f'_N(0) \leq C_O$ implies

$$-f'_N(1-Q_N^L)Q_N^L + f_N(1-Q_N^L) > 0 \tag{A6}$$

so that $\bar{Q}_N^C > Q_N^L$ for sufficiently large v. Since $\bar{Q}_N^C$ is strictly increasing in v, it follows that a $v^B > \underline{v}$ exists having the desired property; in particular, $v = v^B$ solves (A3b) with Q_N^L replacing Q_N^M. Finally, at $v = v^A$ we have $Q_N^M = \bar{Q}_N^C$ and thus $\bar{Q}_N^C$ satisfies (A3a) with equality; this implies $\bar{Q}_N^C < Q_N^L$, so that we must have $v^B > v^A$.

Under differentiated duopoly, the profit functions for the old and new monopolists respectively are:

$$\Pi_O = [f_O(1-Q_O-Q_N) - C_O]Q_0$$

$$\Pi_N = [vf_N(1-Q_N) - f_O(1-Q_N) + f_O(1-Q_O-Q_N) - C_N]Q_N$$

Let the Nash equilibrium quantities be denoted Q_O^D and Q_N^D. As above, $v > \underline{v}$ assures $Q_N^D > 0$. Under our assumptions, there exist downward-sloping and continuous reaction functions in the space of quantity pairs, and moreover these reaction functions have one and only one intersection; thus Q_O^D and Q_N^D are uniquely defined.

Innovation is drastic in this case if and only if $Q_O^D = 0$. Comparing (A4) with the first-order condition of the old-product duopolist, it follows that innovation is drastic under differentiated duopoly if and only if $Q_N^D \geq Q_N^L$, and in the latter instance we have $Q_N^D = Q_N^C$ since the first-order condition of the new-product duopolist coincides with (A3b). Thus, innovation is drastic under differentiated duopoly precisely when it is drastic under dominant–fringe structure. *Q.E.D.*

Proof of Lemma. (a) The profit function of an old-profit monopolist that does not adopt the new product are:

$$\Pi = [f_O(1-Q_O) - C_O]Q_O$$

and the unique maximizer Q_O^{OM} is defined by:

$$f'_O(1-Q_O^{OM})Q_O^{OM} - f_O(1-Q_O^{OM}) - C_O = 0 \tag{A7}$$

Since $\underline{v} < v \leq v^A$, Q_O^M and Q_N^M are defined by (A2a–b), and substituting $Q_O^M + Q_N^M = Q_O^{OM}$ into (A2a) gives (A7).

(b) Subtracting (A2a) from (A2b) gives a condition equivalent to (A5), with Q_N^M replacing $\underline{Q}_N^C$. Thus the result holds if the dominant firm does not choose the limit pricing strategy for $\underline{v} < v \leq v^A$. Now, for $v = v^A$ we may use (A3a) with equality and (A4) to obtain $\underline{Q}_N^c < Q_N^L$, while subtracting (A3a) with equality from (A3b) gives (A5) with $\bar{Q}_N^M$ replacing $\underline{Q}_N^C$; since (A3b) also determines $\bar{Q}_N^C$, this establishes $\bar{Q}_N^C < Q_N^L$ for $v=v^A$. It is also true that $\underline{Q}_N^C$ is strictly increasing in v at $v=v^A$, for differentiation of (A5) gives:

$$\text{sign}\left[\frac{\partial \underline{Q}_N^C}{\partial v}\right] = \text{sign}[-f'_N(1-\underline{Q}_N^C)\underline{Q}_N^C + f_N(1-\underline{Q}_N^C)] > 0 \tag{A8}$$

where the inequality follows from (A3b) and $\underline{Q}_N^C = Q_N^M$. Moreover, the second term in (A8) remains positive for smaller {{UNDERLINE Q}_{func {N}}×{func {C}}}, so that $\underline{Q}_N^C$ continues to be an increasing function of v at lower levels of v; thus $\underline{Q}_N^C < \bar{Q}_N^L$ for all $\underline{v} < v < v^A$. Similarly, we may use (A3b) to show $\bar{Q}_C^L < Q_N^L$ for all $\underline{v} < v < v^A$. *Q.E.D.*

Proof of Proposition 2. It remains to consider the case of $v>v^A$. Note first that $Q_N^M<Q_N^L$ for $v^A<v<v^B$: this may be seen by subtracting (A3a) with equality from (A3b) and comparing with (A5), when $Q_N^C<Q_N^L$; and by comparing (A3b) at $v=v^A$ with (A3b) at $v=v^B$ and Q_N^L replacing Q_N^M, using (A6), when $Q_N^C=Q_N^L$. Further, $Q_N^C=Q_N^M$ for $v\geq v^B$. Making the dependence on v explicit, we have:

$$\begin{aligned}\Pi^C(v) - (\Pi^M(v) - \Pi^{OM}) &= \Pi^C(v^A) - (\Pi^M(v^A) - \Pi^{OM}) \\ &\quad + \int_{v^A}^{v} \left[\frac{\partial \Pi^C(t)}{dt} - \frac{\partial \Pi^M(t)}{dt}\right] dt \\ &= \int_{v^A}^{\min\{v,v^B\}} [f_N(1-Q_N^C)Q_N^C \\ &\quad - f_N(1-Q_N^M)Q_N^M]\, dt > 0\end{aligned}$$

using $Q_N^M<Q_N^C\leq Q_N^L$ over the relevant interval, along with $f_N(1-Q_N)Q_N$ strictly increasing in Q_N for $Q_N\leq Q_N^L$, which follows from (A6). *Q.E.D.*

Proof of Proposition 3. Here we provide a sketch of the proof; a more detailed proof is available from the authors upon request. First, it can be shown that $v^A<v<v^B$ implies two facts: (i) $Q_O^D<Q_N^C$ and (ii) $\text{sign}[Q_N^C-Q_N^D]=\text{sign}[Q_N^D-Q_O^D]$. Fixing v and parameterizing by C_O, it can be shown that there exist C_O^A and C_O^B, with $\underline{C}_O < C_O^A < C_O^B$, such that innovation is non-drastic under joint monopoly if and only if $\underline{C}_O < C_O < C_O^A$, and non-drastic under dominant–fringe structure and differentiated duopoly if and only if $\underline{C}_O < C_O < C_O^B$. Consider the region $C_O^A< C_O<C_O^B$ (corresponding to $v^A<v<v^B$). To ease notation, we will use a lower bar to indicate that a function is evaluated at $1-Q_O^D-Q_N^D$, and an upper bar to indicate evaluation at $1-Q_N^D$. Making explicit the dependence on C_O, we may write:

$$\begin{aligned}\Pi^{M} - \Pi^{OD}(C_O) - \Pi^{C}(C_O) &= \{\Pi^{M} - \Pi^{OD}(C_O^B) - \Pi^{C}(C_O^B)\} \\ &\quad + \int_{C_O}^{C_O^B} \left[-\underline{f}'_O Q_O^D \frac{\partial Q_N^D}{\partial C_O} + Q_N^D \right] dC_O \\ &= \int_{C_O}^{C_O^B} \frac{1}{\Delta} (-\underline{f}'_O Q_O^D[-\underline{f}_O‘ Q_N^D + \underline{f}'_O] + \Delta Q_N^C)\, dC_O\end{aligned} \tag{A9}$$

where Π^{M} is independent of C_O due to $C_O > C_O^A$; the term in braces vanishes since innovation is drastic under all three market structures for $C_O \geqslant C_O^B$; and Δ is given by:

$$\begin{aligned}\Delta \equiv\ & -\underline{f}_O‘ \underline{f}'_O[Q_O^D + Q_N^D] + 3[\underline{f}'_O]^2 + [\underline{f}_O‘ Q_O^D - 2\underline{f}'_O] \cdot \{v\overline{f}_N‘ - \overline{f}_O‘]Q_N^D \\ & - 2[v\overline{f}'_N - \overline{f}'_O)\} > 0\end{aligned}$$

In the last integrand in (A9), the numerator may be written:

$$\begin{aligned}& \underline{f}_O‘ \underline{f}'_O Q_O^D Q_N^D - [\underline{f}'_O]^2 Q_O^D - \underline{f}_O‘ \underline{f}'_O[Q_O^D + Q_N^D]Q_N^C + 3[\underline{f}'_O]^2 Q_N^C + \{\underline{f}_O‘ Q_O^D - 2\underline{f}'_O\} \cdot \\ & \{[v\overline{f}_N‘ - \overline{f}_O‘]Q_N^D - 2[v\overline{f}'_N - \overline{f}'_O]\}Q_N^C\end{aligned} \tag{A10}$$

The last term in (A10), which includes the product of expressions in braces, is strictly positive, and the remaining terms may be written:

$$\underline{f}_O‘ \underline{f}'_O Q_O^D[Q_N^D - Q_N^C] + [\underline{f}'_O]^2[3Q_N^C - Q_O^D] - \underline{f}_O‘ \underline{f}'_O Q_N^D Q_N^D \tag{A11}$$

The third term in (A11) is evidently positive. Using fact (i), the second term is strictly positive. If $Q_N^D \geqslant Q_N^C$, then we have $Q_O^D \geqslant Q_N^D$, by fact (ii), but these inequalities contradict $Q_N^C > Q_O^D$; thus $Q_N^D < Q_N^C$ and the first term is positive. *Q.E.D.*

Proof of Proposition 4. Here we provide a sketch of the proof; a more detailed proof is available from the authors upon request. We may write:

$$\Pi^{ND}(v) - \Pi^{C}(v) = \int_{\underline{v}}^{v} \left\{ \overline{f}_N Q_N^D - \underline{f}_O Q_N^D \frac{\partial Q_O^D}{\partial v} - f_N(1 - Q_N^C)Q_N^C \right\} dv \tag{A12}$$

using $\Pi^{ND}(\underline{v}) = \Pi^{C}(\underline{v}) = 0$. Since Q_O^D is strictly decreasing in v, the second term within the braces is strictly positive. Further, it was established above that $f_N(1 - Q_N)Q_N$ is strictly increasing in Q_N for $Q_N \leqslant Q_N^L$, so $\Pi^{ND}(v) > \Pi^{C}(v)$ if $Q_N^D \geqslant Q_N^C$. Otherwise, parameterizing by C_O, we have

$$\Pi^{\mathrm{ND}}(c_{\mathrm{O}}) - \Pi^{\mathrm{C}}(C_{\mathrm{O}}) = -\int_{C_{\mathrm{O}}}^{C^{\mathrm{B}}} \frac{1}{\Delta}\{-\underline{f}'_{\mathrm{O}} Q_{\mathrm{N}}^{\mathrm{D}}([v\bar{f}''_{\mathrm{N}} - \bar{f}''_{\mathrm{O}} + \underline{f}''_{\mathrm{O}}]Q_{\mathrm{N}}^{\mathrm{D}} - 2[v\bar{f}'_{\mathrm{N}} - \bar{f}'_{\mathrm{O}} + \underline{f}'_{\mathrm{O}}]) - \Delta Q_{\mathrm{N}}^{\mathrm{C}}\}\, \mathrm{d}C_0$$

using $\Pi^{\mathrm{ND}}(C_{\mathrm{O}}^{\mathrm{B}}) - \Pi^{\mathrm{C}}(C_{\mathrm{O}}^{\mathrm{B}}) = 0$, where Δ is defined in the proof of Proposition 3. The expression in braces may be written:

$$\{-\underline{f}''_{\mathrm{O}}\underline{f}'_{\mathrm{O}}Q_{\mathrm{N}}^{\mathrm{D}} + 2[\underline{f}'_{\mathrm{O}}]^2 - \underline{f}'_{\mathrm{O}}([v\bar{f}''_{\mathrm{N}} - \bar{f}''_{\mathrm{O}}]Q_{\mathrm{N}}^{\mathrm{D}} - 2[v\bar{f}'_{\mathrm{N}} - \bar{f}'_{\mathrm{O}}])\}\cdot[Q_{\mathrm{N}}^{\mathrm{D}} - Q_{\mathrm{N}}^{\mathrm{C}}]$$
$$-\{-\bar{f}''_{\mathrm{O}}\bar{f}'_{\mathrm{O}}Q_{\mathrm{O}}^{\mathrm{D}}[\underline{f}'_{\mathrm{O}}]^2 + [\underline{f}''_{\mathrm{O}}Q_{\mathrm{O}}^{\mathrm{D}} - \underline{f}'_{\mathrm{O}}]\cdot([v\bar{f}''_{\mathrm{N}} - \bar{f}''_{\mathrm{O}}]Q_{\mathrm{N}}^{\mathrm{D}} - 2[v\bar{f}'_{\mathrm{N}} - \bar{f}'_{\mathrm{O}}])\}Q_{\mathrm{N}}^{\mathrm{C}}$$

which is strictly negative as long as $Q_{\mathrm{N}}^{\mathrm{D}} < Q_{\mathrm{N}}^{\mathrm{C}}$. *Q.E.D.*

Proof of Proposition 5. Social surplus when the new product has been adopted by a joint monopolist is:

$$S_{\mathrm{A}}^{\mathrm{M}} = \max\{Q_{\mathrm{O}}^{\mathrm{OM}} - Q_{\mathrm{N}}^{\mathrm{M}}, 0\} \int_{1-Q_{\mathrm{O}}^{\mathrm{OM}}}^{1-Q_{\mathrm{N}}^{\mathrm{M}}} (f_{\mathrm{O}}(\omega) - C_{\mathrm{O}})\, \mathrm{d}\omega + \int_{1-Q_{\mathrm{N}}^{\mathrm{M}}}^{1} (vf_{\mathrm{N}}(\omega) - C_{\mathrm{N}})\, \mathrm{d}\omega$$

It can be shown using (A2a–b) that $Q_{\mathrm{N}}^{\mathrm{M}}$ is strictly increasing in v for $\underline{v} < v \leq v^{\mathrm{A}}$, while we have already noted that $Q_{\mathrm{N}}^{\mathrm{M}}$ is strictly increasing in v for $v > v^{\mathrm{A}}$; thus $S_{\mathrm{A}}^{\mathrm{M}}$ is strictly increasing in v. Social surplus under competition in the old-product market when the new product has not been adopted is given by:

$$S_{\mathrm{NA}}^{\mathrm{C}} = \int_{1-Q_{\mathrm{N}}^{\mathrm{L}}}^{1} (f_{\mathrm{O}}(\omega) - C_{\mathrm{O}})\, \mathrm{d}\omega$$

where $Q_{\mathrm{N}}^{\mathrm{L}}$ is given by (A4). Now, $Q_{\mathrm{N}}^{\mathrm{M}} \to 0$ as $v \to \underline{v}$, while $Q_{\mathrm{N}}^{\mathrm{L}} > Q_{\mathrm{O}}^{\mathrm{OM}}$ follows from (A4) and (A7); thus $S_{\mathrm{NA}}^{\mathrm{C}} > S_{\mathrm{A}}^{\mathrm{M}}$ for v close to $\underline{v}$. At $v = v^{\mathrm{B}}$, in contrast, we have $Q_{\mathrm{N}}^{\mathrm{M}} = Q_{\mathrm{N}}^{\mathrm{L}}$, and moreover (A3a–b) imply:

$$vf_{\mathrm{N}}(1 - Q_{\mathrm{N}}^{\mathrm{L}}) - C_{\mathrm{N}} > f_{\mathrm{O}}(1 - Q_{\mathrm{N}}^{\mathrm{L}}) - C_{\mathrm{O}}$$

Thus, $S_{\mathrm{A}}^{\mathrm{M}} > S_{\mathrm{NA}}^{\mathrm{C}}$ for $v = v^{\mathrm{B}}$. It follows that $S_{\mathrm{A}}^{\mathrm{M}} = S_{\mathrm{NA}}^{\mathrm{C}}$ for a unique v^* satisfying $\underline{v} < v^* < v^{\mathrm{B}}$, with $S_{\mathrm{A}}^{\mathrm{M}} < S_{\mathrm{NA}}^{\mathrm{C}}$ for $v < v^*$ and $S_{\mathrm{A}}^{\mathrm{M}} > S_{\mathrm{NA}}^{\mathrm{C}}$ for $v > v^*$. *Q.E.D.*

Appendix B

Comparison of Returns for Perturbed Utility Functions

First, for given Q_{O} and Q_{N}, the utility of consumer ω_{N} will be denoted by:

$$U_{\mathrm{N}} = f_{\mathrm{O}}(1 - Q_{\mathrm{N}}) - f_{\mathrm{O}}(1 - Q_{\mathrm{O}} - Q_{\mathrm{N}})$$

Since U_N is strictly increasing in Q_O, we can recast the joint monopolist's profit maximization problem in terms of Q_N and U_N, and there will be unique maximizers Q_N^M and U_N^M. In the dominant–fringe case, $f_O(1-Q_O-Q_N)=C_O$ and U_N is increasing in Q_N, so we can think of the new-product monopolist as choosing a profit-maximizing level U_N^C. It is easy to see that $U_N^C > U_N^M$ when $v < v^A$.

Now consider perturbations of the utility function that have the form $g(vf_N(\omega) - P_N)$, where $g' > 0$ and $g(U) \geqslant U$ for all U. The utility of the old product is held fixed, which ensures that Π^{OM} is unaffected by the perturbation. Now fix a constant X with $U_N^M < U_N^C$, and suppose $g(U) > U$ if and only if $U < X$. If the joint monopolist chooses Q_N^M and U_N^M in the same way as before the perturbation, then P_O is unchanged if the perturbation is slight, while P_N becomes strictly greater; thus the maximized profit level is strictly greater following the perturbation. Under dominant–fringe structure, in contrast, choosing U_N^C gives the same level of profits, since utility is unaffected for $U \geqslant X$, while offering utility levels $U < X$ will continue to be unattractive if the perturbation is sufficiently slight; thus Π^C is unaffected. It follows that we have $\Pi^M - \Pi^{OM} > \Pi^C$ following the perturbation, and returns are strictly greater under protected monopoly. By a symmetric argument, it follows that competition will provide strictly greater returns following perturbations such that $g(U) > U$ if and only if $U > X$.

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