The Seven Percent Illusion?

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Abstract

From 1995 to 1998, more than 90 percent of mid-range IPOs were charged underwriter fees, or spreads, of exactly seven percent, even though evidence suggests that economies of scale in the size of the offering should allow for lower fees. Chen and Ritter (2000) offer collusion as a possible explanation based on empirical analysis and observation. Alternatively, we investigate whether the spreads could be the result of non-cooperative behavior on the part of the underwriters.

To that end, we build a model that captures several important characteristics of competition in the IPO industry. Specifically, we examine a two-stage game where several underwriters compete in prices for a large number of heterogeneous entrepreneurs. Underwriters are differentiated by prestige, while entrepreneurs vary in terms of the value and quality of the firms they represent. Underwriters are assumed to have convex costs in the number of IPOs brought to market. Therefore, they find it unprofitable to underwrite low quality firms, so employ a screening technology to identify high quality firms.

In equilibrium, it becomes reasonable for underwriters to charge approximately the same spread to entrepreneurs of different valuations. As the ratio of the proportions of two different entrepreneurs converges to the ratio of their valuations, the difference between their spreads is made arbitrarily small. This condition implies that as the underlying distribution of entrepreneurs becomes increasingly skewed toward larger firms, prices for mid-range firms tighten, despite economies of scale. The intuition is that the shift in the distribution strengthens market segmentation along issue size and underwriter prestige, which forces less prestigious underwriters to react to increased competition, while more prestigious underwriters are able to exert increased market power. Thus, our model produces a pricing pattern that is consistent with observation, and does so in a non-cooperative environment.

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1 Introduction

Chen and Ritter (2000) document that from 1995 to 1998, more than 90 percent of mid-range domestic initial public offerings (IPOs) had underwriter fees, or spreads, of exactly seven percent.[1] This figure represents a significant increase from previous years. In 1985, only 19% of IPOs with proceeds greater than $20 million had seven percent spreads, but the number grew to 77% in 1998.[2] This is difficult to explain. Evidence suggests that underwriters enjoy economies of scale with respect to IPO size (Bhagat and Frost, 1986, Ritter, 1987, and Barry *et al*, 1991), which implies that underwriters could offer lower fees to larger IPOs. This unusual clustering has not gone unnoticed by the financial press (Kowalski, 1999, and Smith, 1999), regulatory authorities (Karmel, 2003), or economists.

The academic literature has yet to reach a consensus on the reasons behind this clustering. Explanations have typically centered around two contrasting mechanisms: some form of collusion to set spreads above competitive levels, or the natural outcome of competition. The former is endorsed by Chen and Ritter (2000), who argue that implicit, rather than explicit, collusion has been at work. Hansen (2001) supports the latter view that the seven percent spread arose naturally as the result of increased competition along non-price dimensions. One such alternative dimension is underwriter reputation, which is known to play a significant role in the underwriting process.[3] However, while there is a large empirical literature on IPOs and underwriter activity, little theoretical work has focused directly on the seven percent phenomenon. The question remains: Why has the seven percent spread become so popular?

Our goal is to further address this question by building a model that captures important characteristics of competition in the IPO industry. We know from Tirole (1988) that increasing marginal costs soften price competition, and use this basic result to drive our model. Specifically, we examine how price (the spread) is set in a game where several vertically differentiated underwriters compete for a large number of heterogeneous entrepreneurs. We explicitly incorporate several features of the underwriting industry, such as reputation acquisition and price competition, and study the link between them. Past models, such as Chen (2001) and Chemmanur and Fulghieri (1994), have only looked at these topics in isolation and frequently lack other important features of the industry. We will show that examining them together will give us a greater understanding of the relationship between reputation and price-setting in the underwriting industry, and provide insights into how the unusual pricing pattern of the last decade may have arisen from competition alone.

In equilibrium, we show that underwriters can charge approximately the same spread to entrepreneurs of different valuations. As the ratio of the proportions of two different entrepreneurs converges to the ratio of their valuations, the difference between their

[1] By their definition, mid-range IPOs are those with gross proceeds between $20 million and $80 million.

[2] However, the average gross spread has basically remained constant, hovering around seven percent.

[3] There is a large empirical literature that relates the reputation of underwriters to the values of securities they market. See Logue (1973), Beatty and Ritter (1986), Tinic (1988), Johnson and Miller (1988), and Carter and Manaster (1990).

spreads is made arbitrarily small. This condition implies that as the underlying distribution of entrepreneurs becomes increasingly skewed toward larger firms, prices for mid-range firms tighten, despite economies of scale. The intuition is that the shift in the distribution strengthens market segmentation along issue size and underwriter prestige, which forces less prestigious underwriters to react to increased competition, while more prestigious underwriters are able to exert increased market power. Thus, our model produces a pricing pattern that is consistent with observation, and does so in a non-cooperative environment.

Our initial model takes the form of a two-stage static game, where several underwriters compete in prices for a large number of entrepreneurs. Underwriters are differentiated by prestige (quality), while entrepreneurs vary in terms of the value of the firm they represent. The true value of a firm is unknown, but players observe a signal of its value. In addition to value, the quality of an entrepreneur's firm plays an important role. However, neither the entrepreneur nor the underwriter precisely knows the true quality of the firm at the beginning of the game, nor are they able to influence it. The quality of the firm is determined by an outside third party, independent of the underwriter involved. The true value of the firm depends probabilistically on the initial signal, the reputation of the underwriter that brings the firm public, and the quality of the firm itself. The future reputation of an underwriter is determined by the true value and true quality of the entrepreneurs it markets, so even though underwriters could profit in the short-run from issuing stock in bad firms, this will have a negative impact on their prestige, and thus, on their future profits. Consequently, underwriters attempt to screen low quality entrepreneurs by setting stringent evaluation standards, which determines the probability of marketing a bad entrepreneur.

Costs play an important role in the industry (Bhagat and Frost (1986), Ritter (1987), Barry *et al* (1991)), and they do so in our model as well. We assume that underwriters face convex costs, increasing both with the prestige of the underwriter and with the total volume of entrepreneurs underwritten. This formulation is intended to capture two key factors. First, large and prestigious underwriters have high fixed costs associated with maintaining ongoing operations.[4] Second, all underwriters face capacity constraints in terms of the total number of entrepreneurs they can bring public in a given period.[5] One important implication of these factors, observed in the industry, is that the marginal costs of underwriting prevents a single underwriter from capturing the entire IPO market.[6]

The rest of the document is structured as follows. Section 2 reviews the relevant literature and Section 3 describes the model. Section 4 presents results and Section 5 concludes. All proofs are located in the Appendix.

[4]Larger underwriters generally have employees devoted to underwriting while smaller underwriters may divert resources from other business segments. It is also costly for large underwriters to maintain a sufficient pool of primary market investors to absorb their offerings.

[5]It is reasonable to think that underwriters face capacity constraints due to limited personnel and analyst resources.

[6]See Table 4 of Corwin and Schultz (2003).

2 Related Literature

The literature has advanced two opposing explanations for the industry's convergence to the seven percent spread. One sees it as arising from collusion among underwriters, either explicitly or implicitly. The contrasting explanation is that the seven percent spread is the product of efficient competition, potentially along other (non-price) dimensions. To simplify the terminology, the term 'price' will refer to the spread from now on.

The collusive hypothesis. There has been some indirect evidence for the sorts of pressures that could give rise to explicit collusion. Prudential Securities Inc. was censured and fined \$100,000 by NASD Regulation Inc. in March 2000 for allegedly trying to pressure an issuer to boost the offering fee from six to seven percent. As a formality, Prudential paid the fine but neither admitted nor denied any wrongdoing in the case.[7] This could be taken as some evidence that non-competitive pricing exists, which may increase the likelihood of some form of collusion. However, it would be difficult for explicit collusion (a cartel) to operate in the underwriting industry for several reasons. First, the industry is not concentrated, which would make the logistics of a cartel exceedingly difficult. Second, there have been many new entrants, both small and large, over the last two decades. Most have been financial services firms with expertise in other areas, that happen to underwrite just several deals per year. However, some new entrants have become major players in the industry, such as Deutsche Bank Securities in the early 1990s. Third, the U.S. Anti-Trust Act strictly forbids the operation of cartels, and any firms involved would face severe legal action if caught. Even though the U.S. Department of Justice investigated the possibility of explicit collusion among underwriters, spurred by Chen and Ritter's (2000) findings, the investigation was dropped due to lack of evidence. Explicit collusion has also been empirically rejected by Hansen (2001). In summary, there is little reason to believe that seven percent spreads are the result of explicit collusion.

However, implicit collusion is more difficult to exclude. Observationally, implicit collusion and explicit collusion are nearly indistinguishable, since both produce prices above competitive levels. It is also quite difficult to differentiate unfettered competition from implicit collusion with a hidden guiding hand. Nevertheless, Chen and Ritter (2000) present evidence they felt support the implicit collusion hypothesis. They argue that most IPOs with proceeds greater than \$30 million had spreads above competitive levels. They cite evidence by Bhagat and Frost (1986), Booth and Smith (1986), and Bae and Levy (1990) on the significant economies of scale in issuing equity, on a per deal basis, and thus argue that profits from an \$80 million IPO must be substantially higher than one at \$20 million if both use a seven percent spread.[8] Chen and Ritter (2000) contend that certain features of the industry, such as the perceived importance of reputation, have allowed underwriters

[7] See PR Newswire Association, Inc., PR Newswire, March 9, 2000 and The New York Times, Late Edition - Final, March 10, 2000.

[8] Interestingly, Altınkılıç and Hansen (2000) find evidence supporting diseconomies of scale in secondary-equity offering (SEO) spreads. However, the same clustering phenomenon is not observed in SEO spreads, so the nature of costs between the two underwriting activities could be substantially different.

to keep spreads above competitive levels. They argue that underwriters have been acting strategically, but independently, to keep prices high, and that this serves as evidence for implicit collusion.

The only theoretical investigation of the implicit collusion hypothesis is Chen (2001), who applies Dutta and Madhavan's (1997) model of implicit collusion among NASD dealers to IPO underwriters. He derives conditions on the discount factor that would support implicit collusion as an equilibrium versus conditions resulting in competitive pricing. However, this model assumes the value of entrepreneurs is identical and known with certainty, and most of the results are based on the symmetric case where underwriters are not differentiated (i.e. they have an equal probability of winning an IPO). In his formulation, the cost of underwriting an individual issue exhibits constant-returns-to-scale with respect to issue size, which runs contrary to the widely accepted information cited by Chen and Ritter (2000) that costs have increasing-returns-to-scale This does not capture the importance of capacity constraints.[9]

Therefore, the hypothesis that explicit or implicit collusion is the cause of the seven percent clustering remains without compelling empirical or theoretical support.

The competitive hypothesis. Hansen (2001) argues that the seven percent spread is the efficient contract price brought about by competition among the underwriters along other non-price dimensions, such as underpricing and reputation. Yet the evidence he presents for competition is largely negative, excluding collusion as the cause. He does not test an explicit model of competition in the underwriting industry. Instead, to test for collusion, he examines several aspects of the market structure and whether underwriters earned unexpected surpluses from seven percent IPOs. He tests for competition by trying to establish that underwriters compete along other dimensions (such as underpricing) in seven percent IPOs, but that this competition is not evident in non-seven percent IPOs. However, as Hansen (2001) appreciates, it is intrinsically difficult to distinguish implicit collusion from efficient competition. Lacking appropriate data, he examines the spreads in the secondary equity offering market to see if there is a link between those charged in the IPO market. This forces him to assume a stronger form of implicit collusion, specifically one that jointly coordinates spreads in both markets. However, the test results are mixed, and even if this had not been the case, the tests would not have been able to identify implicit collusion in the IPO market alone.

Our Approach. These prior attempts have helped identify the factors that need to be taken into account for more satisfying tests and contrasts of the collusion and competition hypotheses. One prerequisite would simply be an explicit model of price competition. This is critical for both theoretical and empirical attempts to understand the observed pricing behavior. Such a model is developed here.

Our model draws upon several prior efforts. On the theoretical side, our model uses the

[9]Convex total costs are still consistent with the economies-of-scale view because the latter is with respect to the size of a single issue, while convex costs captures the notion that underwriters are capacity constrained in terms of the total number of IPOs they can handle.

framework of reputation acquisition in Chemmanur and Fulghieri (1994), but incorporates additional heterogeneity and price competition.[10] At a more general level, our efforts are related to the literature on vertical differentiation in product markets pioneered by Prescott and Vischer (1977) and Shaked and Sutton (1982).

3 The Model

Our static model has two stages and two kinds of agents, both of whom are risk neutral. There is a population of entrepreneurs who own private firms they wish to bring public using an underwriter.[11] Each firm represents the entrepreneur's claim to a single project with positive EDV. The goal of entrepreneurs is to maximize their net proceeds from bringing their firm public. This figure is determined by the gross valuation of the IPO after the stock has been sold in the primary-market, minus the percentage fee charged by the underwriter. This gross valuation is the true value of the firm. In our model, it is generated by a probability distribution that depends on specific traits of the entrepreneur's firm and the underwriter used in the offering.

Entrepreneurs are described by the pair (v, q^e), where v represents a *known* signal of the true value and q^e is the *unknown* quality of the firm. We assume that each value signal is drawn independently and identically from the density $f(v)$, where $v \in V \subset \mathbb{R}_+$. [12] For simplicity, we assume the quality of an entrepreneur's firm may only take on two values, 'good' ($q^e = G$) or 'bad' ($q^e = B$). While v and q^e share a common role as inputs into the distribution of the true value of the entrepreneur's firm, they represent two distinct concepts. Before going public, entrepreneurs solicit a group of underwriters who then compete to become the lead manager in the IPO. During this process, entrepreneurs convey some sense of their firm's value to the underwriters, which is a key factor in the underwriter's decision. We intend the value signal, v, to capture the initial value of the firm that is communicated during this underwriter selection process. The value signal represents an unbiased and unconditional estimate of the true value of the firm. However, the true quality of the entrepreneur's firm is revealed after the IPO, and investors in the secondary market take this into account when valuing the firm's shares. In a sense, q^e captures the difference in expectations held by the underwriter and investors in the secondary market about the valuation of the firm.

Let there be $J \geq 2$ underwriters, vertically differentiated by quality $q^u_j \in \mathbb{R}_+$, who compete in prices for the demand of the entrepreneurs. The particular values for underwriter quality are considered to be exogenous in the static model. High-quality underwriters have

[10] In their model, each underwriter evaluate entrepreneurs sequentially until one receives a good evaluation, at which point they accept the entrepreneur. However, they assume that investment banks perform their services in return for a fixed fee.

[11] For expository purposes, we will use the terms 'entrepreneurs' and 'firms' interchangeably.

[12] In the context of a standard model with vertical differentiation, the value signal represents an entrepreneur's taste for quality, though this taste parameter is usually unobserved.

an advantage, in that they make the true value of an entrepreneur larger, in a stochastic dominance sense, than will a low-quality underwriter. Thus, the quality of the underwriter chosen by an entrepreneur affects the probability distribution that determines the entrepreneur's true value. Underwriters would like to be able to discriminate between good and bad entrepreneurs since the latter yield lower profit, on average. Consequently, they evaluate each firm that approaches them before deciding whether to bring it public. Before conducting their evaluations, each underwriter must choose the stringency that they will apply to their evaluations. The chosen evaluation standard r_j determines an underwriter's ability to screen bad entrepreneurs. However, it is more costly for an underwriter to provide a more thorough evaluation of an entrepreneur.

To reduce notation, the 'e' and 'u' superscripts on quality q might be left out when it's clear which agent is being discussed.

3.1 Determining the True Value of an Entrepreneur

Denote the true value of an entrepreneur by v^*. As described earlier, the true value of an entrepreneur is a nonnegative random variable that depends on three factors:

1. the (known) value signal;
2. the (known) quality of the chosen underwriter;
3. the (unknown) quality of the entrepreneur's firm.

For expositional purposes, first consider only the effects of the value signal and underwriter prestige. Let the true value of an entrepreneur be described by the family of distributions:

$$\{\lambda(v^*|v, q^u) : v \in V, q^u \in \mathbb{R}_+\}$$

where the λ are stochastically increasing in both arguments. Without loss of generality, let $q_l^u < q_h^u$ and $v_l < v_h$. Then $E[v^*|v, q_l^u] < E[v^*|v, q_h^u]$ for all $v \in V$ and $E[v^*|v_l, q] < E[v^*|v_h, q]$ for all $q \in \mathbb{R}_+$. Both Johnson and Miller (1988) and Carter and Manaster (1988) provide evidence that, holding other variables constant, issuing firms raise higher proceeds with more reputable underwriters. This specification is intended to capture the added value that high-quality underwriters offer entrepreneurs. Conceptually, this added value could be monetary or in some other form. For example, a high-quality underwriter may contribute to an offering indirectly through other services, such as providing analyst recommendations and offering price support immediately following the IPO.[13] Both of these services are valuable from the perspective of the entrepreneur, but do not directly contribute to the proceeds received from the IPO.

Now consider the effects of firm quality on the true value. We assume the true value of a firm increases, on average, with quality. Then the distribution for the true value of an

[13] See Cliff and Denis (2003) and Aggarwal (2000) for more on these topics, respectively.

entrepreneur is fully specified by the set of distributions λ such that

$$\lambda = \left\{ \begin{array}{c} \lambda_G \\ \lambda_B \end{array} \right\} = \left\{ \begin{array}{c} \{\lambda_G(v^*|v, q^u)\} \text{ if } q^e = G \\ \{\lambda_B(v^*|v, q^u)\} \text{ if } q^e = B \end{array} \right\}$$

All the distributions in λ_G first-order stochastically dominate those in λ_B, such that for any given values of v and q^u, $E_B[v^*|v, q^u] < E_G[v^*|v, q^u]$.

Moreover, it we require that

$$E_B[v^*|v, q] < E[v^*|v] < E_G[v^*|v, q]$$

Hence, underwriter quality determines whether the expected value of the firm is above or below the unconditional value. This is the sense in which an underwriter 'adds value' to a particular IPO. We leave the mechanism that generates this outcome unspecified, since we are only concerned with the results. For example, a more reputable underwriter may be more affective at generating demand from its primary investor base and increasing the expected value of the IPO, or validation effects might lead investors to form higher expectations about the firm.

3.2 The Evaluation Process

The primary purpose of the evaluation process is to endogenize the acquisition of underwriter prestige. In contrast to many game-theoretic models, we do not allow underwriters to directly choose quality. Instead, they must accumulate 'reputation capital' over time by underwriting stock in good firms. Obviously, we can't model this complete process in a static model, but including an evaluation process forces underwriters to deal with part of the inherent trade-offs. Not only does underwriting a bad entrepreneur have a negative effect on an underwriter's reputation, but it also reduces their expected profits. Thus, even though underwriters are risk neutral, they still have incentives to screen bad entrepreneurs.

We now describe the evaluation process more formally. Let the prior probability of a firm being good, conditional on its value signal, be $\Pr(q^e = G|v) = h(v)$, where $h(v)$ is increasing in v and satisfies $0 < h(v) < 1$. This implies that firms with a larger value signal are more likely to be good. The reasoning is that larger IPOs are typically older and more established firms, and have relatively higher ex-ante probability of being good as compared to a smaller firm going public. Some evidence of this is found by taking post-IPO performance and volatility as measures of quality, and comparing these characteristics across small and large IPOs (see Carter and Manaster (1990) and Johnson and Miller (1988)).

Denote e_v as the outcome of an underwriter's evaluation for a particular entrepreneur. Following Chemmanur and Fulghieri (1994), we assume an entrepreneur receives either a 'good' evaluation ($e_v = G$) or a 'bad' evaluation ($e_v = B$). At the beginning of the game, each underwriter must choose the stringency of its evaluation process by choosing a particular level for its evaluation standard. For simplicity we assume that underwriters always receive a good evaluation for a good entrepreneur (i.e. when $q^e = G$). However, the evaluation standard that the underwriter chooses determines the probability of assigning a good evaluation to a bad entrepreneur. Thus, the underwriter must choose an $r_j \in [0, \bar{r}]$, $\bar{r} < 1$, which determines the conditional probability of receiving a good evaluation for a bad firm:

$$\Pr(e_v = G|q^e = G) = 1; \qquad \Pr(e_v = G|q^e = B) = 1 - r$$

A higher r is associated with a more stringent evaluation standard because it decreases the probability of a false positive. If the underwriter sets $r = 0$, it assigns a good evaluation for all entrepreneurs. Conversely, setting $r = \bar{r}$ corresponds to the most stringent standard possible, though there is still a positive probability that it could market a bad firm.[14] The evaluation standard essentially lets an underwriter control the probability of accepting a false-positive.

Thus, the probability of a firm being good, conditional on receiving a good evaluation and its size, is derived using Bayes' rule:

$$\Pr(q^e = G|e_v = G) = \frac{h(v)}{h(v) + (1-r)(1-h(v))} \tag{1}$$

Note that this expression makes sense even at extreme values for r. For instance, setting $r = 1$ implies that the underwriter could screen entrepreneurs perfectly, and (1) reduces to $\Pr(q^e = G|e_v = g, v) = 1$, for all v. On the other hand, if $r = 0$, the underwriter ignores the screening technology, and the prior becomes equivalent to the posterior probability: $\Pr(q^e = G|e_v = G, v) = \Pr(q^e = G|v) = h(v)$.

3.3 The Entrepreneur's Objective

Entrepreneurs seek to maximize their expected proceeds from the IPO. Their expected proceeds are their expected gross proceeds, minus the portion of the proceeds (the spread) paid to the underwriter.

Consider an arbitrary entrepreneur with value signal $v_i \in V$, who knows the quality of their own firm is q^e. Then the entrepreneur's objective is to choose the underwriter who maximizes his or her expected net proceeds, given the set of prices $\{p_{ij}\}_{j=1}^{J}$ offered by the underwriters to entrepreneurs with that value signal:

$$argmax_{j \in \{1,2,\ldots,J\}} (1 - p_{ij}) E[v_i^* | v_i, q_j^u, q_i^e] + \varepsilon_{ij} \tag{2}$$

Since entrepreneurs know the quality of the firm they represent, they are able to condition on this information. If $q^e = G$, then the expectation could be written as $E_G[v_i^*|v_i, q_j^u]$, and similarly for $q^e = B$.

The term ε_{ij} is a random variable that represents horizontal differentiation between underwriters, independent of quality. This might reflect idiosyncratic differences in the

[14] There are other potential formulations for the evaluation process, but this was chosen, for the moment, because of its simplicity. A more realistic mechanism could be used in the dynamic model.

tastes that entrepreneurs have for underwriters or simply random variation in expected proceeds.[15] We assume these idiosyncrasies are known to the entrepreneur but unobserved by the underwriter.

Since the $\{\varepsilon_{ij}\}$ are random, entrepreneur demand is probabilistic. We need to derive an expression for the portion of entrepreneurs with value signal v_i that choose some underwriter j. To obtain a closed-form for this probability, denoted as $\sigma(p_{ij}, v_i) \equiv \sigma(p_{ij}, p_{i-j}, q_j, q_{-j}, v_i)$, we assume the $\{\varepsilon_{ij}\}$ are distributed independently (over i and j) and identically according to a type I extreme value distribution.[16] Normalizing the value of the outside alternative to zero, the probability of an entrepreneur selecting a particular underwriter at a price p_{ij} and quality q_j, given the other underwriters' prices and qualities, follows a standard logit model:

$$\sigma(p_{ij}, v_i)=\frac{e^{(1-p_{ij})E[v_i^*|v_i,q_j^u,q_i^e]}}{1+\sum_{k=1}^{J} e^{(1-p_{ik})E[v_i^*|v_i,q_k^u,q_i^e]}} \tag{3}$$

This probability represents the market share of an underwriter from all entrepreneurs of a particular value signal. Note that the demand for an underwriter is always decreasing in price ($\frac{\partial \sigma(p_{ij},v_i)}{\partial p_{ij}} < 0$) and increasing in underwriter quality ($\frac{\partial \sigma(p_{ij},v_i)}{\partial q_j} > 0$).

Finally, let M denote the total size of the market for entrepreneurs. This is the aggregate number of entrepreneurs seeking financing across all valuations.

3.4 The Underwriter's Objective

The objective of an underwriter is to maximize the expected value of its profits. It does this first by choosing an evaluation standard and then setting prices. The particular value of the evaluation standard will determine the portion of entrepreneur demand that an underwriter chooses to accept, for each value signal. First, we explain how market share in (3) is mapped into the actual number of deals accepted by an underwriter. Then, we derive the underwriter's objective function for a given value signal, before generalizing it to a distribution of entrepreneurs.

An underwriter may or may not underwrite all the entrepreneurs that desire its services. Of the total number of entrepreneurs with signal v_i that approach underwriter j, $\sigma(p_{ij}, v_i)$, we assume the underwriter accepts only those that received a good evaluation. The probability of an entrepreneur receiving a good evaluation is

$$\gamma(v, r) \equiv \Pr(e_v = G|v) = h(v) + (1 - r)(1 - h(v))$$

So the total demand for an underwriter may be expressed as

$$d(p_{ij}, v_i, r) = \gamma(v_i, r_j)\sigma(p_{ij}, v_i)$$

[15]For example, many underwriters specialize in certain industries, so an entrepreneur may be more likely to select an underwriter in a particular field.

[16]See de Palma, Ginsburgh, Papageorgiou, and Thisse (1985) and Anderson and de Palma (1992) for theoretical work on using the logit as a model of horizontal product differentiation.

While this assumption might seem strong, it's possible to specify the appropriate conditions that would lead to this behavior in equilibrium. All that's necessary is that the marginal benefit of underwriting a bad entrepreneur must be less than the marginal cost. Such a condition could be constructed by placing restrictions on the total cost function, but this would not add any qualitatively interesting implications.

It is important to incorporate the results of the evaluations into the demand function for several reasons. First, if underwriters could set evaluation standards independently of demand, the value of r would not impact underwriters' profits. As the underwriter sets a more stringent evaluation standard (r increases), fewer entrepreneurs receive good evaluations ($\gamma(v, r)$ decreases), so total demand must decrease as well ($d(p, v, r)$ decreases). Second, in a dynamic setting, this endogenizes reputation acquisition by linking an underwriter's future expected profits to its quality, and vice versa.[17]

Chemmanur and Fulghieri (1994) show that, in equilibrium, underwriters in their model only accept entrepreneurs that receive good evaluations. While we make this assumption more for the sake of simplicity, a similar result could be obtained in our model. If a bad entrepreneur decreased an underwriter's reputation sufficiently, it would outweigh the benefits of having accepted the entrepreneur in the first place. This assumption is made implicitly by Chemmanur and Fulghieri (1994) by their particular choice of parametric forms.

This setup is similar to the one in Chemmanur and Fulghieri (1994), but with two important modifications. First, underwriters simultaneously evaluate all entrepreneurs as opposed to a sequential evaluation process. Second, entrepreneur demand is random in our model.[18]

There is no role in the static model for a link between market share and future underwriter quality. However, in a dynamic model, the prestige of an underwriter could be determined by the quality of entrepreneurs they brought to market in the previous period. The empirical literature has produced a number of different measures of underwriter reputation, but it is still difficult to quantify precisely (see Carter and Dark (1992) for a thorough comparison). A common proxy, used in much of the applied industrial organization literature, is market share. This measure is popular because firm-level data are frequently available and capture the basic notion of product rank accurately in many industries.

Underwriters face symmetric information with respect to value signals, but asymmet-

[17] There is a potential issue that arises in a multi-period game, but not in the static model that we have been discussing. The profit from underwriting a bad entrepreneur could be greater than the subsequent loss in profits resulting from a lower reputation. This prompts the question: Why wouldn't an underwriter accept some bad entrepreneurs if it was more profitable? Though the evaluation standard set by an underwriter affects its current demand and profits, there are no reputation effects. In the present model, underwriters do not care about their future reputation. That being the case, underwriters could accept bad entrepreneurs if they produced more profits than good entrepreneurs. However, we feel it is reasonable to rule out this type of behavior in the static model.

[18] An implicit assumption of this specification is that demand varies linearly with the evaluation standard. More realistic function forms could be used, but should yield qualitatively similar results.

ric information with respect to firm quality. In some sense, this situation is similar to an affiliated value auction, since each underwriter observes a common signal v and independent private signals $\{e_v(r_j)\}_{j=1}^J$. However, there are several important differences. First, underwriters are asymmetric with respect to quality. Second, the private signals are not identically distributed because underwriters set independent evaluation standards. Third, the value of an entrepreneur to a specific underwriter is independent of other underwriters' private signals (each underwriter receives their own e_v). Denote R_{ij} as the revenue obtained by underwriter j from entrepreneurs of type v_i. Thus, for $k \neq j$,

$$e_v(r_j) > e_v(r_k) \not\Rightarrow E[R_{ij}|e_v(r_j)] > E[R_{ij}|e_v(r_j), e_v(r_j) > e_v(r_k)]$$

In other words, our model does not support the conditions necessary to guarantee a winner's curse.[19]

With demand fully specified, let $\hat{r}_j$ be some evaluation standard chosen by the underwriter in the first stage of the game. As described earlier, the quality of an entrepreneur is unknown to the underwriter, but they receive a noisy signal based on their evaluations. Though underwriters accept all that receive good evaluations, a portion of these will be low quality entrepreneurs. Since the quality of an entrepreneur affects the true valuations, the objective function for an underwriter for a single value signal v_i is:

$$\max_{p_{ij} \in [0,1]} Md(p_{ij}, v_i, \hat{r}_j) p_{ij} E[v_i^*|v_i, q_j^u] - C(Md(p_{ij}, v_i, \hat{r}_j), q_j^u, \hat{r}_j)$$

where the total cost function C is increasing in all its arguments and strictly convex. This yields a simple one-dimensional maximization problem over a concave objective function, so a unique optimum is guaranteed to exist. The expected value of the entrepreneur's firm, conditional on their value signal and underwriter quality, may be expanded to

$$\begin{aligned} E[v_i^*|v_i, q_j^u] &= \Pr(q^e = G|e_v = G) E_G[v_i^*|v_i, q_j^u] + Pr(q^e = B|e_v = G) E_B[v_i^*|v_i, q_j^u] \\ &= \frac{h(v) E_G[v_i^*|v_i, q_j^u] + (1 - \hat{r}_j)(1 - h(v)) E_B[v_i^*|v_i, q_j^u]}{h(v) + (1 - \hat{r}_j)(1 - h(v))} \end{aligned}$$

This expression demonstrates the importance of providing the underwriters with a mechanism to screen potential entrepreneurs. It's clear that an underwriter would prefer to maximize the first term in the numerator due to the stochastic dominance conditions. The evaluation standard allows the underwriter to control their exposure to potentially low-quality (low-profit) entrepreneurs. Moreover, the evaluation standard softens price competition for the underwriters by acting as an additional 'degree of freedom': instead of lowering prices, an underwriter could lower their evaluation standard.

Until now, we have only considered the case of a single entrepreneur. The extension to a distribution of entrepreneurs is straightforward, but first a comment on notation is required. So far, we have only discussed the prices charged by underwriters in the context

[19] For example, see Krishna, 2002, p. 83-85.

of a particular value signal. This must be generalized to a continuum of value signals, which implies that underwriters actually choose a pricing function $p_j : V \to [0,1]$. Let Ω be the space of such functions.

Denote

$$\Psi_j = M \int_V d(p_j(v), v, \hat{r}_j) dF(v)$$

as the total *volume* of firms accepted by an underwriter. The continuous form of the underwriter's objective function is found by integrating over the distribution of entrepreneurs and maximizing with respect to the function $p_j(v)$. The general form is:

$$\max_{p_j \in \Omega} M \int_V d(p_j(v), v, \hat{r}_j) p_j(v) E[v^* | v, q_j^u] dF(v) - C(\Psi_j, q_j^u, \hat{r}_j) \tag{4}$$

Unfortunately, a closed-form expression for this integral does not exist due to the arbitrary functional form of $F(v)$. Additionally, the problem cannot be decomposed into a set of one-dimensional problems because each pricing decision is linked to the others through the convex total cost function. This difficulty represents the fundamental trade-off faced by underwriters. By competing more heavily for entrepreneurs in one market (i.e. a particular value signal), capacity constraints reduce their potential to compete in other markets. In the next section, we see how these factors may allow underwriters both to charge higher prices than would otherwise be expected, and to charge the same price for different entrepreneurs.

3.5 Game Summary

Though the game is essentially comprised of two stages, the following is the specific order of events:

1. Underwriters choose values for r_j given their current level of quality q_j^u (considered exogenous in the static model).
2. The value signals for each entrepreneur are realized and observed by all.
3. Underwriters compete in prices for the demand of the entrepreneurs.[20]
4. The random horizontal taste of each entrepreneur, ε_{ij}, is realized, and they make their choice of underwriter.
5. Given the demand of the entrepreneurs, underwriters select the portion of this demand to underwrite.
6. The true values for v and q^e are realized and profits for the stage game are determined.

In a dynamic model, in the next period, the prestige of the underwriters would be updated based on the proportion of good firms they brought public in this period.

[20] When approached by entrepreneurs, underwriters only specify a spread, and not a potential valuation. Thus, as Chen and Ritter (2000) assert, it's valid to examine competition based solely on the spread offered by the underwriters.

4 Equilibrium

Given our model, defining an equilibrium entails finding a set of evaluation standards and pricing functions that are best-responses to each other. Unfortunately, examining uniqueness in a function space is difficult, so we make the following simplifying assumptions:

Assumption 1 *The distribution of entrepreneurs is given by a discrete distribution that takes on values* $\{v_1, \ldots, v_I\}$ *with associated probabilities* $\{f(v_1), \ldots, f(v_I)\}$.

Assumption 1 lets us consider a discrete version of the objective function in (4). We believe the discretized version serves as a good approximation to the true pricing function. Further, computational evidence, not reported here, suggests that it is possible to solve the model for reasonably large values of I and J, and get results that seem consistent with a more coarse discretization.

Assumption 2 *The total cost function in (4) can be decomposed additively into two components*

(i). $k(q, r)$ *– a fixed costs that is linear and increasing in both arguments, so that* $\frac{\partial k(q,r)}{\partial q} > 0$ *and* $\frac{\partial k(q,r)}{\partial r} > 0$*;*

(ii). $c(q, \Psi)$ *– a strictly convex cost that depends on the total volume of entrepreneurs that an underwriter accepts.*

Let $\beta > 0$ *be a scaling parameter. Thus, the total cost function may be written as*

$$C(\Psi, q, r) = k(q, r) + \beta c(q, \Psi)$$

The two cost components in Assumption 2 are intended to capture the fixed costs and capacity constraints, in terms of the number of deals accepted, faced by underwriters in providing their services.[21]

Let $p_{ij} \equiv p_j(v_i)$. After incorporating Assumptions 1 and 2, the underwriter's maximization problem is:

$$\pi_j = \max_{\{p_{ij}\}_{i=1}^{I}} M \sum_{i=1}^{I} d(p_{ij}, v_i, \hat{r}_j) p_{ij} E[v_i^* | v_i, q_j^u] f(v_i) - \left[k(q_j^u, \hat{r}_j) + \beta c(q_j^u, \Psi_j) \right] \tag{5}$$

We analyze the sub-game perfect Nash equilibrium of the one-shot game in which underwriters first choose an evaluation standard and then choose a set of prices. Given an

[21] While we have assumed a simple cost structure for the evaluation standard, alternative specifications should yield qualitatively similar results. For example, more prestigious underwriters could enjoy evaluation cost efficiencies due to their experience at evaluating entrepreneurs. However, such a formulation will only shift the set of prices up for high quality underwriters, and will not have any significant impact on the other dynamics of the model.

evaluation standard $\hat{r}$ selected in the first stage, the corresponding price sub-game is solved by $\{p^*_{1,j}, ..., p^*_{I,J}\}_{j=1}^{J}$ such that equation (5) is maximized for each underwriter and pricing function. Given the optimal pricing functions, an equilibrium for the first stage is a set of evaluation standards $\{r^*_j\}_{j=1}^{J}$ that maximize equation (5). Thus, a sub-game perfect Nash equilibrium for the two-stage game consists of the set of pairs $\{r^*_j, p^*_{1,j}, ..., p^*_{I,J}\}_{j=1}^{J}$.

4.1 Results

To determine the optimal set of prices, an underwriter must solve a system of non-linear equations given by the first-order conditions (FOCs) of π_j with respect to each p_{ij}. That is, we find $\frac{\partial \pi_j}{\partial p_{ij}}$:

$$ME[v^*_i|v_i, q^u_j]\left(\frac{\partial d(p_{ij}, v_i, \hat{r}_j)}{\partial p_{ij}}p_{ij} + d(p_{ij}, v_i, \hat{r}_j)\right) - \beta M \frac{\partial c(\hat{r}_j, \Psi_j)}{\partial \Psi_j(p_{ij}, v_i, \hat{r}_j)} \frac{\partial \Psi_j(p_{ij}, v_i, \hat{r}_j)}{\partial p_{ij}} = 0 \tag{6}$$

for $i = 1, \ldots, I$. Re-arranging and simplifying yields an implicit solution for the optimal set of prices for a given underwriter:

$$p^*_{ij} = \frac{1}{E[v^*_i|v_i, q^u_j]}\left(\beta \frac{\partial c(\hat{r}_j, \Psi_j)}{\partial \Psi(p^*_{ij}, v_i, \hat{r}_j)} f(v_i) + \frac{1}{1 - \sigma(p^*_{ij})}\right) \qquad \text{for } i = 1, \ldots, I \tag{7}$$

The price charged by an underwriter reflects the sum of the total marginal cost for a particular signal and its relative market share, scaled by the expected true value of the firm. This form is more general than those usually encountered in the literature with vertical differentiation and logit demand specifications. The key difference is that the marginal cost term, $\frac{\partial c(\hat{r}_j, \Psi_j)}{\partial \Psi(p_{ij}, v_i, \hat{r}_j)}$, depends explicitly on all underwriter prices and qualities through Ψ_j.

The expression in (7) rules out negative price solutions. Values above one are invalid since our model considers price as a percentage of gross proceeds. However, both corner price solutions ($p_{ij} = 1$) and interior solutions ($0 < p_{ij} < 1$) are still possible depending on the total marginal cost term. To restrict our attention to interior solutions, we make the following additional assumption concerning the bounds on the convex cost coefficient.

Assumption 3 *Consider any set of value signals* $\mathbf{v} = \{v_1, \ldots, v_I\}$ *with associated probabilities* $\mathbf{f} = \{f(v_1), \ldots, f(v_I)\}$, *where* $\sum f(v_i) = 1$, *and some underwriter qualities* $\mathbf{q} = \{q_1, \ldots, q^u_j\}$. *Then the convex cost coefficient,* $\beta = \beta(\mathbf{q}, \mathbf{v}, \mathbf{f})$, *must satisfy:*

$$\beta > \frac{M}{c(q^u_j, \Psi_j)} \sum_{i=1}^{I} p_{ij} E[v^*_i|v_i, q^u_j] f(v_i) = \underline{\beta}$$

The assumption states that the coefficient on the convex cost function must be large enough so that for any set of prices $\{p_{ij}\}_{i=1}^{I}$, it is impossible (i.e. unprofitable) for the underwriter to achieve a monopoly and carry the entire market. This assumption effectively rules out unrealistic equilibria.[22] For example, if β is too low, capacity constraints are no longer an issue for underwriters. Consequently, the high-quality underwriter charges uniform prices across all value signals and captures most of the market. As the degree of vertical differentiation increases, the high-quality underwriter can effectively drive the low-quality underwriter out of the market completely and continue to raise its uniform price. Such pricing behavior is not observed, so it seems reasonable to bound β above values that lead such equilibria. Conversely, if β is too high, then the price equilibrium may exceed one. Since price is interpreted as a percentage, values above one are invalid. Such price equilibria are the result of the extreme value distribution assumptions on ε_{ij}, which have infinite support, so an underwriter's demand will always be strictly greater than zero for finite values of the price. We choose to ignore such equilibria.

Proposition 1 *An interior pure-strategy sub-game perfect Nash equilibrium, composed of the pairs* $\{r_j^*, (p_{1j}^*, ..., p_{Ij}^*)\}_{j=1}^J$, *that solves the system in (7), exists and is unique.*

Note that the equilibrium is unique in both outcomes and strategies. This is in contrast to many vertical differentiation models where players choose qualities and then prices, and the equilibrium is only unique in outcomes.

Lemma 1 *For any given value signal, high-quality underwriters charge higher prices than low-quality underwriters.*

Lemma 1 is a standard result in models of vertical differentiation. In general, consumers are willing to pay more for quality, and as a result, firms that offer a higher-quality product are able to sustain higher prices. Due to the relationship between underwriter quality and expected value of an entrepreneur's firm, high-quality underwriters gain pricing power as the value signal increases.

Lemma 2 *Consider two underwriters with qualities* $q_l^u < q_h^u$ *and equilibrium prices* $p_l^* < p_h^*$. *Then* $\frac{\partial d(p_l^*, q_l^u, v)}{dv} < 0$ *and* $\frac{\partial d(p_h^*, q_h^u, v)}{dv} > 0$.

Note that Lemma 2 only holds for equilibrium prices. The intuition is that, as the size of the entrepreneur's firm increases, the vertical differentiation among the underwriters overshadows the effects of the horizontal differentiation. In short, high-quality underwriters have more of an advantage when competing for larger entrepreneurs.

Lemma 3 *Consider two underwriters with* $q_l^u < q_h^u$. *Then there always exists a* β^* *such that for all* $\beta > \beta^*$, *the high-quality underwriter obtains lower profits than the low-quality underwriter.*

Lemma 3 shows that if costs are sufficiently high, it may not be optimal to be the high-quality underwriter.

[22] Note that we are able to ignore fixed costs since they do not affect equilibrium prices.

Proposition 2 *Consider two underwriters with* $q_l^u < q_h^u$*. Then for any value signals* $v_1 < v_2$*, there exists a* β^* *such that for all* $\beta > \beta^*$*, the following statements hold:*

(i). The low-quality underwriter has a larger equilibrium market share for the smaller entrepreneurs: $\sigma(p_{1l}^*) > \sigma(p_{1h}^*)$*.*

(ii). The high-quality underwriter has a larger equilibrium market share for the larger entrepreneurs: $\sigma(p_{2l}^*) < \sigma(p_{2h}^*)$*.*

Building off Lemma 2, this result supports the empirical observation that the market for entrepreneurs is segmented by issue size.[23] As a group, the 'bulge bracket' underwriters (a common term for the most prestigious underwriters) essentially control the market for large IPOs. However, the number of small IPOs a bulge bracket underwriter markets is significantly lower by proportion, because of the substantial fixed costs they face. While only a handful of underwriters compete in the market for the large IPOs, there are many other underwriters who compete for much smaller issues. Many of these less established underwriters acknowledge that they can not compete with the most prestigious underwriters, either in reputation capital or sheer resources. In a sense, the market segmentation implies that each group of underwriters is able to maintain local monopolies: The high quality underwriters have little incentive to compete in the low end market for entrepreneurs, and low quality underwriters are simply unable to compete on the high end. An important feature to note is that the market structure within each local monopoly is different, with many more underwriters competing on the low end compared to the relatively select group that comprises the bulge bracket. This point will be important after the next proposition.

Proposition 3 *Consider a given underwriter with equilibrium prices* p_l^* *and* p_h^* *for two value signals* $v_l < v_h$*. Then for any* $\delta > 0$*,* $\exists \beta^*, \varepsilon > 0$*, such that if*

$$\left| \frac{f(v_h)}{f(v_l)} - \frac{v_h}{v_l} \right| \leq \delta$$

then $\forall \beta \in \beta(\mathbf{q}, \mathbf{v}, \mathbf{f})$*, with* $\beta > \beta^*$*,*

$$|p_l^* - p_h^*| \leq \varepsilon \tag{8}$$

This proposition states our key result, namely that it's possible for an underwriter to charge the same price for two entrepreneurs of different valuations. This also shows how the underlying distribution of entrepreneurs plays an important role in the price competition between underwriters.[24] If the distribution of entrepreneurs becomes skewed toward larger issues (as it did during the late 1990s), the price for smaller IPOs decreases while the

[23] See Table 2 of Hansen (2001) for a breakdown of IPOs from 1980 to 2000 by offering size and underwriter prestige.

[24] As noted earlier, an underwriter may charge similar prices for two different entrepreneurs if costs (β) are sufficiently low. However, for values of $\beta < \underline{\beta}$, this results in uniform pricing across all value signals, as opposed to a subset, which is not realistic.

price for larger IPOs increases.[25] If the distribution becomes sufficiently skewed, it may counteract the economies of scale, and the price charged for small and large issues could be similar.

This result may seem counter-intuitive, because one might expect that increasing the relative number of large entrepreneurs should intensify the competition for them, and thus, prices should decrease. Instead, prices for large IPOs rise because of the market segmentation and capacity constraints. The high quality underwriters act as local monopolists and exert their market power, since it is difficult for less prestigious underwriters to compete in the high end market. At the same time, skewing the distribution towards larger values places less relative mass on smaller entrepreneurs. Consequently, competition for small IPOs increases and their price decreases. Combining these two effects – prices for large IPOs rising and prices for small IPOs falling – it is possible that entrepreneurs within some range of values may be charged similar prices by an underwriter. Thus, our model of competition is able to reproduce a pricing pattern that is consistent with the price clustering observed in the real world.

While this proposition provides conditions under which the prices charged by a single underwriter may be close for different entrepreneurs, it does not compare prices across underwriters. According to Lemma 1, low quality underwriters always charge lower prices than high-quality underwriters. Ideally, we would also like to derive conditions that both make the prices for a given underwriter similar and the difference in prices between underwriters small.

If underwriters were less vertically differentiated, their equilibrium prices would be closer. However, it is not necessary for underwriters to actually become less vertically differentiated. All that is required is that the value entrepreneurs assign to quality decreases. An implication in the real world is that as entrepreneurs' willingness-to-pay for reputable underwriters decreases, then prices between low- and high-quality underwriters should converge, as each begins to compete along dimensions other than price and prestige. One possible path to this outcome would be for high-quality underwriters to shift competition away from reputation toward other dimensions, such as analyst coverage and underpricing. Carter, Dark, and Singh (1998), among others, find that companies taken public in the 1980s by highly reputable underwriters were associated with less underpricing. During this period, the seven percent IPO was less prominent. Interestingly, as seven percent IPOs gained popularity, Beatty and Welch (1996) find that this relationship reversed in the first half of the 1990s, with high quality underwriters associated with more, as opposed to less, underpricing.

An example of the conditions necessary for Proposition 3 are illustrated in Figure 4.1. The figure plots equilibrium prices for a large set of value signals with two underwriters of different quality. The underlying distribution of the values was calibrated to approximate the actual distribution of IPOs in the late 1990s. The following parameter values were used

[25] For instance, the number of IPOs between \$120 million and \$250 million rose by 422%, while the entire IPO market only doubled in size.

to generate Figure 1: $\bar{r}$, the maximum allowable evaluation standard, is set to 0.99; β is set to 16. In terms of underwriter qualities, we plot the non-differentiated case in which $q_l^u = q_h^u = 1$ and a differentiated case in which $q_l^u = 1$ and $q_h^u = 1.2$. The set of value signals and its distribution are calibrated according to data taken from Corwin and Schultz (2003).

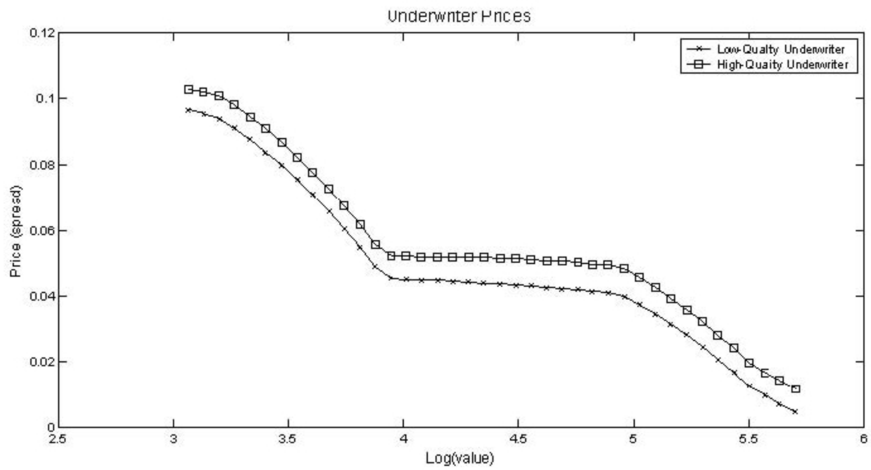


Figure 1: Equilibrium prices for IPOs with two vertically differentiated underwriters.

Proposition 4 *Fix an* $\epsilon > 0$ *and let* $w_i = f(v_i)$ *for an arbitrary* i. *Next, define*

$$B_w(M) = \{(w_l, w_h) : \beta \in \beta(M, \mathbf{q}, \mathbf{v}, \mathbf{f}), |p_\ell^*(M) - p_h^*(M)| \leq \epsilon\}$$

where $p_\ell^*(M), p_h^*(M)$ *denote the equilibrium prices as implicit functions of market size. Let* $\{M_n\}_{n=1}^{\infty}$ *be a bounded monotonically increasing sequence such that* $\lim_{n\to\infty} M_n = \bar{M}$. *Then for any* $1 < n < m$,

$$B_w(M_n) \subset B_w(M_m)$$

and $\lim_{n\to\infty} B_w(M_n) = \mathbf{B}_w$.

The thrust of this proposition is that as the number of firms issuing equity increases, the probability that two underwriters will charge the same fees for low and high value IPOs also increases. Increasing the market size strengthens the degree of local market power within each segment of underwriters, which is precisely what occurred in the industry.

Proposition 5 *The equilibrium evaluation standard* r_j^* *is strictly increasing in* β, *regardless of underwriter quality.*

Assuming that most of the costs associated with performing evaluations are fixed, as underwriting bad entrepreneurs becomes more costly, underwriters are inclined to screen entrepreneurs more effectively. Essentially, the marginal cost of raising the evaluation standard is less than the marginal cost of underwriting bad entrepreneurs. One could imagine that a similar result would hold in a dynamic model, because the additional effect on reputation of accepting bad entrepreneurs would further lower expected future profits.

Proposition 6 *Consider two underwriters with $q_l^u < q_h^u$. Then the equilibrium evaluation standards exhibit the following properties:*

(i). *The high-quality underwriter. If underwriters are not too differentiated ($q_h^u - q_l^u \leq \varepsilon_h$), then r_j^* increases as q_h^u increases. Conversely, if underwriters are sufficiently differentiated ($q_h^u - q_l^u > \varepsilon_h$), then r_j^* decreases as q_h^u increases.*

(ii). *The low-quality underwriter.*

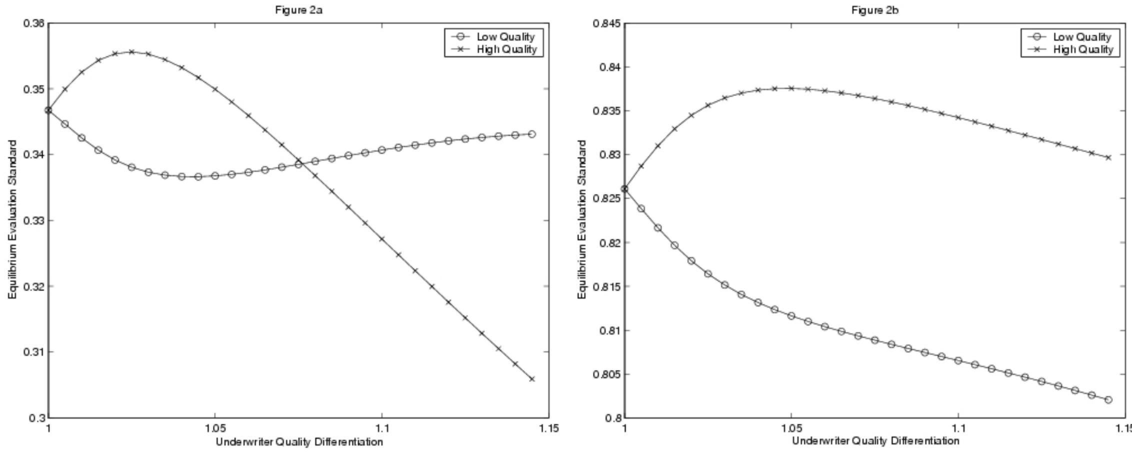
(Low costs) *If costs are sufficiently low, and if underwriters are not too differentiated ($q_h^u - q_l^u \leq \varepsilon_l$), then r_j^* decreases as q_h^u increases. Conversely, if underwriter are sufficiently differentiated ($q_h^u - q_l^u > \varepsilon_l$), then r_j^* increases as q_h^u increases.*

(High costs) *If costs are sufficiently high, then r_j^* decreases as q_h^u increases.*

The results for the high-quality underwriter are similar to those obtained for the high-cost underwriter in Chemmanur and Fulghieri (1994), though we are able to provide an analysis of the dynamics of the low-quality underwriter's evaluation standard as well. Setting a stricter evaluation both reduces an underwriters demand and is costly. However, if underwriters are not too differentiated, then it remains profitable for the high-quality underwriter to increase its evaluation standard up until a critical value of quality q_h^*. The benefits of being the high-quality underwriter are less than those from maintaining a stricter standard when $q_h < q_h^*$. When $q_h > q_h^*$, the underwriter is able to rely more on its reputation to generate additional profits and it can relax its evaluation standard. In other words, the underwriter has established a sufficient reputation such that it can withstand the effects of underwriting some bad entrepreneurs.

The situation is more complicated for the low-quality underwriter. When costs are low, its evaluation standard almost follows the inverse behavior of the high-quality underwriter. However, when costs (β) are sufficiently high, its evaluation standard always decreases as the underwriter become more differentiated. To compete with the high-quality underwriter in a costly environment, the low-quality underwriter must accept as many entrepreneurs as possible, and essentially ignore the effects a bad entrepreneur will have on its own reputation.

Figures 2a and 2b illustrate the dynamics of both underwriters' evaluation standards under low costs and high costs. Each figure plots the variation in the low- and high-quality underwriters' equilibrium evaluation standards w.r.t. changes in the degree of vertical differentiation. As before, the quality of one underwriter is fixed at $q_l^u = 1$ while the other underwriter's quality starts $q_h^u = 1$ and increases. The set of possible values signals is $\{10, 30, 70, 150\}$ with associated probabilities $\{0.25, 0.3, 0.35, 0.1\}$; $\bar{r}$, the maximum allowable evaluation standard is set to 0.99. The results above differ only in the particular value for the convex cost coefficient, where Figure 2a sets $\beta = 8$, while Figure 2b sets $\beta = 21$.



(a) Low Costs

(b) High Costs

Figure 2: Equilibrium evaluation standards varying the degree of underwriter quality differentiation. Figure 2a depicts the low cost case, while Figure 2b considers high costs.

5 Conclusion and Future Work

This paper presents a model of the underwriting industry with vertical differentiation and endogenous reputation acquisition. We show that the model captures several important characteristics of the industry, and more importantly, provide theoretical evidence that the seven percent IPO phenomenon may be the result of differentiated firms acting competitively in interacting local monopolies. Market segmentation and a skewed entrepreneur distribution can cause underwriters to charge similar prices for entrepreneurs of different size, even in the presence of economies of scale.

One difficulty in extending the present model to multiple periods is the lack of closed-form expressions for the players' decision variables. It might be possible to simplify the demand side, while retaining the concept of a distribution of entrepreneurs, and derive results in a two-period model. Such an extension would create the key feedback between an underwriter's current decision variables and their future reputation.

Later, we plan to extend the model to a dynamic multi-agent game along the lines of Ericson and Pakes (1995). Extending our model would be an important step because the issues we wish to examine are inherently dynamic: reputation is acquired and varies over time; prices today depend on expectations about tomorrow; future demand is uncertain. A dynamic model would allow us to explore these topics more fully and give us a better opportunity to look at policy and welfare questions. Once this has been accomplished, we hope this paper will serve as an important example of an application of this dynamic approach to the finance literature. These methods have become popular in the applied industrial organization literature because they allow the researcher to move away from stylized models and provide greater opportunities for generating testable hypotheses.[26]

Future work could also be directed at conducting econometric evaluations of some of the hypotheses put forth here or to estimate any of the model's parameters. Is there a link between the changes in the distribution of entrepreneurs and observed spreads? Is it possible to measure underwriter capacity constraints to estimate the affect it may have on softening price competition? When deciding whether or not to underwrite a firm, does an underwriter consider capacity constraints or 'reputation constraints' more? One observation, made by Anand & Galetovic (2000), is that the market structure of the underwriting industry has been remarkably stable through time. It might be interesting to see if a computational model exhibited similar market structure properties.

[26]See Fershtman and Pakes (2000), de Roos (2002), and Benkard (2003) for examples of applications of the Ericson and Pakes (1995) framework.

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A Appendix

Proof of Proposition 1. First we find the equilibrium in the pricing sub-game, and then given the optimal prices, solve the first-stage of the game for the optimal evaluation standards.

The pricing sub-game. The basic principle behind the proof in the pricing sub-game is a monotonicity argument. However, it will facilitate exposition if we present the proof in the case that $f(v)$ takes on a point-mass at some value $v \in V$. We'll then extend the proof to the case that $f(v)$ takes on an arbitrary discrete distribution.

We would like to show that a unique Nash equilibrium exists for some number of underwriters J and a single entrepreneur signal $v \in V$.[27] Since the objective function is quasi-concave, it suffices to find the solution to the FOCs, which will correspond to the Nash equilibrium. Denoting the objective (profit) function as π_j, the FOC with respect to p_j is

$$\frac{\partial \pi_j}{\partial p_j} : ME[v^*|v, q_j^u] \left(\frac{\partial d(p_i, v, \hat{r}_j)}{\partial p_j} p_j + d(p_j, v, \hat{r}_j) \right) - M \frac{\partial c(q, \Psi_j)}{\partial \Psi_j(p_j, v, \hat{r}_j)} \frac{\partial \Psi_j(p_j, v, \hat{r}_j)}{\partial p_j} = 0 \tag{9}$$

In the point-mass case, $\Psi_j = MD_j$. Therefore, this expression may be simplified by using the following equalities and notational abbreviations. Letting $v_j^* \equiv E[v^*|v, q_j^u]$, $\sigma_j \equiv \sigma(p_j(v), v)$, and $\Lambda(p_j) \equiv \Lambda(\sigma(p_j), q_j^u, \hat{r}_j, M) \equiv \frac{\partial c(q_j^u, \Psi_j)}{\partial \Psi_j(p_j, v, \hat{r}_j)}$, we have $\frac{\partial d(p_j, v, \hat{r}_j)}{\partial p_j} = \gamma(\hat{r}_j) \frac{\partial \sigma(p_j)}{\partial p_j}$ and $\frac{\partial \sigma(p_j)}{\partial p_j} = v_j^* \sigma(p_j)(\sigma(p_j) - 1)$. These expressions allow us to rewrite equation (9) as

$$v_j^* \gamma(v, r) \left[v_j^* \sigma_j (\sigma_j - 1) p_j + \sigma_j \right] = \beta \Lambda(p_j) \gamma(\hat{r}_j) v_j^* (\sigma_j (\sigma_j - 1))$$

$$p_j^* = \frac{1}{v_j^*} \left(\beta \Lambda(p_j) + \frac{1}{1 - \sigma_j(p_j^*)} \right) \text{ for } j = 1, \ldots, J \tag{10}$$

The equilibrium set of prices $\{p_j^*\}_{j=1}^J$ is implicitly given by the solution to the system of nonlinear equations in (10). To show the equilibrium is unique, we must show that the system of equations in (10) has a unique solution. Set $x_j \equiv (1 - p_j^*) v_j^*$ for $j = 1, \ldots, J$. This allows us to write the logit probabilities in (3) as

$$\sigma(x_j) = \frac{\exp(x_j)}{1 + \sum_k \exp(x_k)} \tag{11}$$

We must also rewrite $\Lambda(p_j) = \frac{\partial c(\Psi, \hat{r}_j)}{\partial \Psi(p_j, v, \hat{r}_j)}$ to explicitly account for the fact that it is also a function of $\sigma(x_j)$, through $d(p_j, v, \hat{r}_j)$. Using (11), we rewrite (10) as

$$v_j^* - x_j = \beta \Lambda(\sigma(x_j), q_j^u, \hat{r}_j) + \frac{1}{1 - \frac{\exp(x_j)}{1 + \sum_k \exp(x_k)}} \tag{12}$$

Clearly, a solution to the system defined in (10) has a unique solution if and only if (12) has a unique solution in x_j. To prove this statement, we need to break apart the term $\sigma(x_j)$, which we'll rewrite as $\tilde{\sigma}(x_j, Q) = \exp(x_j)/(1 + Q)$, where $Q > 0$ is a given parameter such that $\exp(x_j) - 1 < Q$. This implies that Equation (12) can be restated as

$$v_j^* - x_j = \beta \Lambda(\tilde{\sigma}(x_j, Q), v, \hat{r}_j) + \frac{1}{1 - \tilde{\sigma}(x_j, Q)} \tag{13}$$

[27] The proof in the point-mass case is along the lines of one presented in Section 7.10.1 of Anderson, de Palma, and Thisse (1992), but with a more general cost structure.

Note that $\lim_{Q\to\infty} \tilde{\sigma}(x_j, Q) = 0$. Since $\Lambda(\sigma(x_j), v, \hat{r}_j)$ is increasing in $\sigma(x_j)$, then $\lim_{Q\to\infty} \Lambda(\tilde{\sigma}(x_j, Q), v, \hat{r}_j) = 0$. Suppose that (13) has a single solution $x_j^*(Q)$ for each $Q > 0$. We can draw several implications immediately from (13):

1. $x_j^*(Q) < v_j^* - 1$ for all $Q > 0$, since $\tilde{\sigma}(x_j, Q) > 0\ \forall Q > 0$ and $x_j < \ln(Q + 1)$;
2. $x_j^*(Q)$ is an increasing function of Q;
3. $\lim_{Q\to\infty} x_j^*(Q) = v_j^* - 1$;
4. $\lim_{Q\to-\infty} x_j^*(Q) = -\infty$;

Recall that $\sigma(p_j)$ is a decreasing, convex function of p_j. Since x_j is a decreasing function of p_j, $\tilde{\sigma}(x_j, Q)$ must be an increasing function of x_j, so the second term on the r.h.s. of (13) must be convex and increasing in x_j. From Assumption 2 on c, we know that $\Lambda(\tilde{\sigma}(x_j, Q), v, \hat{r}_j)$ must also be a convex and increasing function of x_j. Therefore, for any given $Q > 0$, (13) has a unique solution $x_j^*(Q)$.

We would like to show that (14) has a unique solution Q^*, which implies that the corresponding solution of (13) is also a unique solution of (12). Define, $\gamma(x_1^*(Q), \ldots, x_J^*(Q)) = \sum \exp(x_j^*(Q))$. Thus, we wish to show that

$$\gamma(x_1^*(Q), \ldots, x_J^*(Q)) = Q \tag{14}$$

has a unique solution $Q = Q^*$. Consider the following ratio

$$\frac{\gamma}{Q} = \frac{\sum \exp(x_j^*(Q))}{Q} \tag{15}$$

Re-arranging (13) yields $\frac{\exp(x_j^*(Q))}{Q} = 1 - \frac{1}{v_j^* - x_j^*(Q) - \beta\Lambda(\tilde{\sigma}(x_j^*(Q), Q), v, \hat{r}_j)}$, and plugging this expression into (15) gives us

$$\frac{\gamma}{Q} = J - \sum \frac{1}{v_j^* - x_j^*(Q) - \beta\Lambda(\tilde{\sigma}(x_j^*(Q), Q), v, \hat{r}_j)} \tag{16}$$

Properties 1–4 above and equation (16) imply that:

1. γ/Q is a (monotonically) decreasing function of Q;
2. $\lim_{Q\to\infty} \gamma/Q = 0$;
3. $\lim_{Q\to-\infty} \gamma/Q = J \geq 2$.

It follows from the last implication that there exists a Q small enough such that $\gamma/Q > 1$. Taken together, these three properties, and the fact that γ/Q is a continuous function of Q, imply the existence of a unique value Q^* such that $\gamma/Q^* = 1$. Thus, Q^* is the unique solution to (14), and hence, a unique solution exists to (12) as well.

Now that the point-mass case has been covered, we extend the proof above to consider an arbitrary discrete distribution $f(v)$ that takes on values $\{v_1, \ldots, v_I\}$ with associated probabilities $\{f(v_1), \ldots, f(v_I)\}$, where $\sum f(v_i) = 1$. The notation will be more compact than presented above, so the reader should refer to the point-mass case for details.

For the sake of concreteness, let the convex cost function take on the function form $c(q_j^u, \Psi_j) = \beta(q_j^u \Psi_j)^2$ and denote $Z_{ij} = 2\beta\left(q_j^u \gamma(\hat{r}_j)\right)^2 f(v_i) > 0$. Now consider the set of FOCs for a given underwriter, for $i = 1, \ldots, I$:

$$\frac{\partial \pi_{ij}}{\partial p_{ij}} : E[v_i^* | v_i, q_j^u] \left(\frac{\partial d(p_{ij}, v_i, \hat{r}_j)}{\partial p_{ij}} p_{ij} + d(p_{ij}, v_i, \hat{r}_j) \right) - Z_{ij} \left[\sum_{k=1}^{I} \sigma(p_{kj}) w_k \right] \frac{\partial \sigma(p_{ij})}{\partial p_{ij}} = 0$$

After breaking apart the summation term, solving for the equilibrium prices yields:

$$p_{ij}^* = \frac{1}{v_j^*}\left(Z_{ij}\sum_{k\neq i}^{I}\sigma(p_{kj}^*)w_k + Z_{ij}\sigma(p_{ij}^*)f(v_i) + \frac{1}{1-\sigma(p_{ij}^*)}\right) \text{ for } i = 1,\ldots,I \tag{17}$$

This expression may be rewritten in a similar manner to (13) above

$$v_j^* - x_{ij} = Z_{ij}\sum_{k\neq i}^{I}\tilde{\sigma}(x_{kj},Q_k)w_k + Z_{ij}\tilde{\sigma}(x_{ij},Q_i)f(v_i) + \frac{1}{1-\tilde{\sigma}_j(x_{ij},Q_i)} \text{ for } i = 1,\ldots,I \tag{18}$$

for some set of $\{Q_1, Q_2, \ldots, Q_I\}$ such that $Q_i > 0$ and $e^{x_{ij}} - 1 < Q_i$, $\forall i,j$. Note that the only substantial difference between the FOC in the point-mass case, given by (13), and the FOC in the general discrete case, given by (18), is the addition of the summation term $\sum_{k\neq i}^{I}\tilde{\sigma}(x_{ij},Q_i)w_k$. However, this term only depends on prices other than that of the current entrepreneur i. The constants Z_{ij} and $f(v_i)$ do not depend on prices.

Let $x_{-ij} = \{x_{kj}\}_{k\neq i}^{I}$ and $Q_{-i} = \{Q_k\}_{k\neq i}^{I}$. Let $x_{ij}^*(Q_{ij}, x_{-ij}, Q_{i-j})$ be a specific solution to (18), which depends on the values of x_{-ij} and Q_{-i} through the additional summation term. The affects of the capacity constraint become clear. If the underwriter has accepted a large volume of entrepreneurs, the summation term will be high, and this will force the equilibrium price for this value signal to increase. The underwriter must charge more because his marginal costs are higher, so convex costs have successfully softened price competition. Using the same arguments in the point-mass case, it is clear that for a particular value signal, a unique solution $x_{ij}^*(Q_{ij}, x_{-ij}, Q_{i-j})$ exists for all j, given values of x_{-ij}^* and Q_{-i}.

Define $x_i^*(Q) = [x_{i1}^*(Q_i, x_{-ij}, Q_{-i}), \ldots, x_{iJ}^*(Q_i, x_{-ij}, Q_{-i})]$ and

$$\gamma(x_i^*(Q)) = \sum_{j=1}^{J}\exp(x_{ij}^*(Q_i, x_{-ij}, Q_{-i}))$$

As before, we need to show that there exists a unique set $Q^* = \{Q_i^*\}_{i=1}^{I}$ that solves the set of FOCs for all value signals and underwriters. Such a solution would also solve the system of equations $\gamma(x_i^*(Q)) = Q_i$, for $i = 1, ..., I$. This function can be rewritten in similar form to (16) as

$$\frac{\gamma_i}{Q_i} = J - \sum_{j=1}^{J}\frac{1}{v_{ij}^* - x_{ij}^*(Q_i, x_{-ij}, Q_{-i}) - Z_{ij}\tilde{\sigma}(p_{ij}, Q_{ij})f(v_i) - Z_{ij}\sum_{k\neq i}^{I}\tilde{\sigma}(p_{kj}, Q_i)w_k} \tag{19}$$

for $i = 1,\ldots,I$

Unfortunately, the mean-value theorem and monotonicity argument used in the point-mass case does not apply here since we are trying to solve a system of nonlinear equations. One way that we can show that the system above has a unique solution is to prove that the Jacobian of the system possesses a general property of matrices called diagonal dominance.

Definition 1 *(diagonal dominance) Let J be an $M \times M$ matrix and $a_{m,n}$ an arbitrary element. Then J is diagonally dominant if and only if it satisfies both of the following conditions:*

$$\text{(i) } |a_{m,m}| > \sum_{k\neq m}^{M}|a_{k,m}| \qquad \text{(ii) } |a_{m,m}| > \sum_{k\neq m}^{M}|a_{m,k}|$$

for all $m = 1, ..., M$.

Let Φ be the $I \times I$ Jacobian matrix of partial derivatives of (19) with $\phi_{i,h} \equiv \frac{\partial(\gamma_i/Q_i)}{\partial q_h^u}$ being an arbitrary element. Consider the expression for an element on the diagonal:

$$\phi_{i,i} = \sum_{j=1}^{J} \frac{\frac{\partial x^*_{i,j}(Q_i,x_{-i,j},Q_{-i})}{\partial Q_i} + \frac{\partial Z_{i,j}\tilde{\sigma}(p_{i,j},Q_i)f(v_i)}{\partial Q_i}}{(v^*_{i,j} - x^*_{i,j}(Q_i,x_{-i,j},Q_{-i}) - Z_{i,j}\tilde{\sigma}(p_{i,j},Q_{i,j})f(v_i) - Z_{i,j}\sum_{k\neq i}^{I}\tilde{\sigma}(p_{k,j},Q_k)w_k)^2} \tag{20}$$

Now consider an off-diagonal element ($i \neq h$):

$$\phi_{i,h} = \sum_{j=1}^{J} \frac{\frac{\partial x^*_{i,j}(Q_i,x_{-i,j},Q_{-i})}{\partial q_h^u} + \frac{\partial Z_{i,j}\tilde{\sigma}(p_{h,j},Q_{h,j})w_h}{\partial q_h^u}}{(v^*_{i,j} - x^*_{i,j}(Q_i,x_{-i,j},Q_{-i}) - Z_{i,j}\tilde{\sigma}(p_{i,j},Q_{i,j})f(v_i) - Z_{i,j}\sum_{k\neq i}^{I}\tilde{\sigma}(p_{k,j},Q_k)w_k)^2} \tag{21}$$

The only difference between these two expressions are in the numerators. Using (18) and the definition of $\tilde{\sigma}(x_j, Q)$, we find that

$$\frac{\partial x^*_{i,j}(Q_i,x_{-i,j},Q_{-i})}{\partial Q_i} = -\frac{\partial Z_{ij}\tilde{\sigma}(p_{ij},Q_{ij})f(v_i)}{\partial Q_i} + \frac{1}{(1-\tilde{\sigma}_j(x_{ij},Q_i))^2}\frac{x_{i,j}}{(1+Q_i)^2} \tag{22}$$

$$\frac{\partial x^*_{i,j}(Q_i,x_{-i,j},Q_{-i})}{\partial q_h^u} = -\frac{\partial Z_{i,j}\tilde{\sigma}(p_{h,j},Q_{h,j})w_h}{\partial q_h^u} \tag{23}$$

Thus, equation (23) implies that the numerator in (21) is zero, so every off-diagonal element in Φ is zero. To show that Φ is diagonally dominant, we just must show that the diagonal elements are non-zero, which simultaneously satisfies the row and column dominance conditions. After substituting in equation (22), we can rewrite (20) as:

$$\phi_{i,i} = \sum_{j=1}^{J} \frac{\frac{1}{(1-\tilde{\sigma}_j(x_{ij},Q_i))^2}\frac{x_{i,j}}{(1+Q_i)^2}}{(v^*_{i,j} - x^*_{i,j}(Q_i,x_{-i,j},Q_{-i}) - Z_{i,j}\tilde{\sigma}(p_{i,j},Q_{i,j})f(v_i) - Z_{i,j}\sum_{k\neq i}^{I}\tilde{\sigma}(p_{k,j},Q_k)w_k)^2} > 0$$

The inequality must hold since $0 < \tilde{\sigma}_j < 1$ and $Q_i > 0$ together imply that the numerator is positive and obviously the same is true for the denominator. This implies that the Jacobian of the set of first-order conditions satisifies diagonal dominance. Thus, there must exist a unique set Q^* that solves the set of equations and guarantees a unique pricing equilibrium exists.

The evaluation sub-game. Given the equilibrium prices above, we can now solve the first-stage of the game for the optimal evaluation standard. Define the total revenue for an underwriter as $R_j(r,p;q,v) = \sum_{i=1}^{I} d(p_{ij}, v_i, \hat{r}_j)p_{ij}E[v_i^*|v_i, q_j^u]f(v_i)$. Differentiating the profit function w.r.t. r_j yields:

$$\frac{\partial \pi_j}{\partial r_j} : \frac{\partial R_j(r_j,p_j;q_j^u,v)}{\partial r_j} - \frac{\partial k(r_j,q_j^u)}{\partial r_j} - \beta\frac{\partial c(q,\Psi_j)}{\partial \Psi_j(p_j,v,r_j)}\frac{\partial \Psi_j(p_j,v,r_j)}{\partial r_j} = 0$$

Substituting in terms and re-arranging gives an explicit expression for an underwriter's optimal choice of r_j

$$\frac{\partial R_j(r_j,p_j;q_j^u,v)}{\partial r_j} - \frac{\partial k(r_j,q_j^u)}{\partial r_j} = \beta q_j^2\left(\sum_{i=1}^{I}\gamma(r_j,v_i)\sigma(p_{ij})f(v_i)\right)\left(\sum_{i=1}^{I}(h(v_i)-1)\sigma(p_{ij})f(v_i)\right)$$

$$\frac{\frac{\partial R_j(r_j,p_j;q_j^u,v)}{\partial r_j} - \frac{\partial k(r_j,q_j^u)}{\partial r_j}}{\beta q_j^2\sum_{i=1}^{I}(h(v_i)-1)\sigma(p_{ij})f(v_i)} = \sum_{i=1}^{I}\sigma(p_{ij})f(v_i) + (h(v_i)-1)\sigma(p_{ij})f(v_i)r_j$$

Finally, solving for r_j gives us an explicit expression for the optimal evaluation standard:

$$\frac{\sum_{i=1}^{I}(h(v_i)-1)\sigma(p_{ij})p_{ij}E[v_i^*|v_i,q_j^u]f(v_i)-\frac{\partial k(r_j,q_j^u)}{\partial r_j}}{\beta q_j^2\left(\sum_{i=1}^{I}(h(v_i)-1)\sigma(p_{ij})f(v_i)\right)^2}-\frac{\sum_{i=1}^{I}\sigma(p_{ij})f(v_i)}{\sum_{i=1}^{I}(h(v_i)-1)\sigma(p_{ij})f(v_i)}=r_j^* \qquad (24)$$

■

Proof of Lemma 1.

Consider an arbitrary entrepreneur signal v and two underwriters with profits denoted by $\pi_j(p)$ and $\pi_k(p)$, and without loss of generality let $q_j^u < q_k$. The proof follows by showing that we can always construct a price $\hat{p} > p$ such that the high-quality underwriter obtains higher profits at this new price, $\pi_k(\hat{p}) > \pi_k(p)$, and thus would never charge the a price that is equal to or less than the price charged by the low-quality underwriter. We can show that such a $\hat{p}$ exists by looking at the difference obtained from charging either price:

$$\begin{aligned}\pi_k(\hat{p}_k)-\pi_k(p_k) &= d(\hat{p}_k)\hat{p}_k v^* - \beta q_k^2 d(\hat{p}_k)^2 - d(p_k)p_k v^* + \beta q_k^2 d(p_k)^2\\ &= \beta q_k^2[d(p_k)^2-d(\hat{p}_k)^2]+v^*[d(\hat{p}_k)\hat{p}_k-d(p_k)p_k]\end{aligned}$$

Using our assumption concerning the lowerbound on β, we can rewrite the above as

$$\begin{aligned}\beta q_k^2[d(p_k)^2-d(\hat{p}_k)^2]+v^*[d(\hat{p}_k)\hat{p}_k-d(p_k)p_k] &> v^*\left[\frac{q_k^2[d(p_k)^2-d(\hat{p}_k)^2]}{q_k^2 d(p_k)^2}+[d(\hat{p}_k)\hat{p}_k-d(p_k)p_k]\right]\\ &= v^*\left[1-\frac{d(\hat{p}_k)^2}{d(p_k)^2}+d(\hat{p}_k)\hat{p}_k-d(p_k)p_k\right] \qquad (25)\end{aligned}$$

If we can show that (25) is greater than zero, we are done. For this to be true, it must be that

$$1 > \frac{d(\hat{p}_k)^2}{d(p_k)^2}+(d(\hat{p}_k)\hat{p}_k-d(p_k)p_k) \qquad (26)$$

Since $\frac{\partial d(p)}{\partial p} < 0$ is always true (i.e. market share is declining in price), we know the first term on the r.h.s. of (26) must be less than 1. And since $p_k < \hat{p}_k \leq 1$, it follows that $\frac{d(\hat{p}_k)^2}{d(p_k)^2}+(d(\hat{p}_k)\hat{p}_k-d(p_k)p_k) < \frac{d(\hat{p}_k)^2}{d(p_k)^2}+d(\hat{p}_k)-d(p_k) < 1$. Thus, $\pi_k(\hat{p}_k)-\pi_k(p_k) > 0$, and the high-quality underwriter always obtain higher profits by raising its price above the low-quality underwriter's, so in equilibrium, the high-quality underwriter charges higher prices. ■

Proof of Lemma 2. To prove this result, we need to derive $\frac{\partial d(p^*,v)}{dv}$ for the low- and high-quality underwriter. Totally differentiating this expression yields

$$\begin{aligned}\frac{\partial d(p^*,q,v)}{dv} &= \frac{\partial d(p^*,q,v)}{\partial p^*}\frac{\partial p^*}{\partial v}+\frac{\partial d(p^*,q,v)}{\partial q}\frac{\partial q}{\partial v}+\frac{\partial d(p^*,q,v)}{\partial v}\\ &= \frac{\partial d(p^*,q,v)}{\partial p^*}\frac{\partial p^*}{\partial v}+\frac{\partial d(p^*,q,v)}{\partial v}\end{aligned}$$

Consider the case of the high-quality underwriter. We know that $\frac{\partial d(p^*,q,v)}{\partial p^*} < 0$ due to the properties of the logit demand system. Additionally, $\frac{\partial p^*}{\partial v} < 0$ for all reasonable values of v. For values of v close to zero, this expression can actually be positive due to the implicit assumption of a unit variance on the ϵ_{ij}. To simplify, for any variance, we can always pick some $V \subset \mathbb{R}_+$ such that $\inf(V) > V^*$ is sufficiently bounded from below. Intuitively, this makes sense, since price should decline with the magnitude of the value signals. Indeed, this is observed in the data. A similar condition is required for the third term, which could be negative for extreme values of v. Otherwise, $\frac{\partial d(p^*,q,v)}{\partial v} > 0$, and so we can conclude that the entire expression satisfies $\frac{\partial d(p_h^*,q_h^u,v)}{dv} > 0$. A similar argument applies for the low quality underwriter. ■

Proof of Lemma 3. We would like to show for equilibrium prices $p_l^* < p_h^*$, where this inequality follows from Lemma 1, that there exists a β^* that such for all $\beta > \beta^*$, $\pi_l(p_l^*) > \pi_h(p_h^*)$. We know that $c(q, \Psi)$ is increasing in both arguments, so if both firms had identical revenues, the high-quality underwriter's costs would be higher. Let $R_h \equiv \sum_i d(p_{iH}, \hat{q}_h, v_i) p_{iH} v_{iH}^*$ be the revenues obtained by the high-quality underwriter and R_l be defined similarly. Consider the cases when the high-quality underwriter has higher profits at these prices and demands, so that $\pi_l(p_l^*) < \pi_h(p_h^*)$. This implies that either of the following is true

$$R_h > R_l \text{ and } C_h < C_l + (R_h - R_l) \tag{27}$$

or

$$R_h < R_l \text{ and } C_h + (R_h - R_l) < C_l \tag{28}$$

Let $R_\Delta = (R_h - R_l)$ and $C_\Delta = (C_h - C_l)$. Consider the first case in which the high-quality underwriter has higher revenues and higher costs than the low quality underwriter. Obviously, since revenues do not depend on the cost parameter, we only need to look at the function $N(\beta) = \frac{R_\Delta}{C_\Delta} > 1$. Since the high-quality underwriter has greater marginal costs due to its reputation, they are also more sensitive to changes in the cost coefficient, such that $\frac{\partial C_h}{\partial \beta} > \frac{\partial C_l}{\partial \beta}$. Therefore, $N(\beta)$ is a decreasing (continuous) function of β, which implies that there exists a β_1 such that $N(\beta) < 1$ and the condition in (27) is violated and the low-quality underwriter obtains higher profits. Similarly, we may redefine $N(\beta) = \frac{R_\Delta}{C_\Delta} < 1$ to satisfy the second condition, and find another β_2 such that $N(\beta) > 1$ and the condition in (28) is violated and the low-quality underwriter obtains higher profits. Taking $\beta^* = \max\{\beta_1, \beta_2\}$, it follows that for any $\beta > \beta^*$ the low-quality underwriter obtains higher profits. ■

Proof of Proposition 2. Consider the case when $\beta = 0$. Then neither underwriter faces any capacity costs and their is intense price competition (due to the absence of increasing marginal costs), which allows the high-quality underwriter to continually increase their market share as their quality increases. Slowly increasing marginal costs will soften price competition since underwriting demand becomes costly. Since the marginal cost of demand increases quadratically with underwriter quality, at some point, it will become unprofitable for the high-quality underwriter to accept more entrepreneurs. At this point, say β^*, the high-quality underwriter faces a trade-off between competing for the small entrepreneurs versus the larger ones. Thus, the high-quality underwriter will relax price competition for the value signals that minimize profits lost from doing so. This takes place for the smallest value signals, since it is just as costly to underwriter large entrepreneurs as it is to underwrite small entrepreneurs. When $\beta > \beta^*$, the low-quality underwriter gains market share for the small value signals, and at some point, $\sigma(p_{1L}^*) > \sigma(p_{1H}^*)$. The reverse argument shows that the opposite must be true for the large entrepreneurs, $\sigma(p_{2L}^*) < \sigma(p_{2H}^*)$. ■

Proof of Proposition 3. We would like to derive the conditions under which the equilibrium prices for two different value signals, given implicitly by the solution to (17), become close, holding the value signals constant.

Examining the difference between the equilibrium price for two different signals, we get

$$\frac{1}{v_j^*}\left(MZ_{ij}\sum_{k\neq i}^{I}\sigma(p_{kj}^*)f(v_k)+MZ_{ij}\sigma(p_{ij}^*)f(v_i)+\frac{1}{1-\sigma(p_{ij}^*)}\right)-\frac{1}{v_j^*}\left(MZ_{lj}\sum_{k\neq l}^{I}\sigma(p_{kj}^*)f(v_k)+MZ_{lj}\sigma(p_{lj}^*)f(v_l)+\frac{1}{1-\sigma(p_{lj}^*)}\right)$$

To simplify matters, consider only two value signals, v_l and v_h which allows us to rewrite the above as

$$\frac{\left(MZ_l(\sigma(p_h)f(v_h)+\sigma(p_l)f(v_l))+\frac{1}{1-\sigma_l(p_l)}\right)}{v_l^*}-\frac{\left(MZ_h(\sigma(p_l)f(v_l)+\sigma(p_h)f(v_h))+\frac{1}{1-\sigma_h(p_h)}\right)}{v_h^*} \tag{29}$$

As the difference between these two terms converges to zero, the prices p_l and p_h must converge. Suppose that $p_l = p_h$: we want to determine the conditions under which this is true.

Denote $p_l = p_h = p$ and $\alpha = v_h^*/v_l^*$, where $\alpha > 1$ since $v_H > v_L$. Also, let $\Delta_\sigma(\alpha) = \frac{\alpha}{1-\sigma_l(p_l)} - \frac{1}{1-\sigma_h(p_h)}$ and note that $Z_l, Z_h, \sigma(p) > 0$ for all parameter values. Finally, using these abbreviations, we can rewrite (29) as

$$M\left[\alpha Z_l(\sigma(p)f(v_h)+\sigma(p)f(v_l)) - Z_h(\sigma(p)f(v_l)+\sigma(p)f(_h))\right]+\Delta_\sigma(\alpha) \tag{30}$$

Next, let $\left\{\frac{f_n(v_H)}{f_n(v_L)}\right\}_{n=1}^{\infty}$ be a sequence such that $\frac{f_n(v_H)}{f_n(v_L)} \nearrow \frac{v_H}{v_L}$ as $n\to\infty$. Of course, $f(v)\in V$, where V is a compact subset of $\mathbb{R}_+$, so we must be able to construct such a sequence. Since v_L and v_H are fixed, we require convergence from below to mimic conditions in actual data. In the past, the distribution of IPO valuations in real terms was skewed to the lower end, and over time this distribution shifted out. This sequence is intended to simulate this process.

Substituting in the expression for Z_{ij}, arbitrary terms from the sequence above, and re-arranging yields

$$\left|\left[\sigma_h(p)f_n(v_h)(\alpha f_n(v_l)-f_n(v_h))+\sigma_l(p)f_n(v_l)(\alpha f_n(v_l)-f_n(v_h))\right]+\frac{\Delta_\sigma(\alpha)}{2\beta\left(q\gamma(\hat{r})\right)^2}\right|$$

Note that when $\alpha f_n(v_l) - f_n(v_h)$ is zero, the first two terms are zero. This term will converge to zero as $n\to\infty$. This implies that for every $\varepsilon>0$, $\exists N_1\in\mathbb{Z}$ such that $\forall n\geq N_1$,

$$\left|M\alpha(\sigma(p)f(w_h)+\sigma(p)f(v_l)) - (\sigma(p)f(v_l)+\sigma(p)f(v_h))\right| \leq \frac{\varepsilon}{2} \tag{31}$$

Additionally, since the second term continuous in β, we can easily construct an increasing sequence $\{\beta_m\}_{m=1}^{\infty}$ such that for every $\varepsilon>0$, $\exists N_2\in\mathbb{Z}$, where $\forall m\geq N_2$,

$$\left|\frac{\Delta_\sigma(\alpha)}{2\beta_m(q\gamma\hat{r})^2}\right| \leq \frac{\varepsilon}{2} \tag{32}$$

Put $N^* = \max\{N_1, N_2\}$ and this implies that $\forall n, m \geq N^*$,

$$\begin{aligned}&\left|\left[\sigma_h(p)f_n(v_h)(\alpha f_n(v_l)-f_n(v_h))+\sigma_l(p)f_n(v_l)(\alpha f_n(v_l)-f_n(v_h))\right]+\frac{\Delta_\sigma(\alpha)}{2\beta_m(q\gamma\hat{r})^2}\right|\\ &\leq \frac{\varepsilon}{2}+\frac{\varepsilon}{2}\\ &\leq \varepsilon\end{aligned}$$

■

Proof of Proposition 4. This proof builds off the previous one. To simplify notation, let $w_i = f(v_i)$. Fix some particular $\varepsilon > 0$. Then from the previous proof, we require the value density to have positive support on all points, this equivalent to requiring that $\exists \underline{w}_l, \underline{w}_h > 0$ such that for all $w_l > \underline{w}_l$ and $w_h > \underline{w}_h$, the expression above is satisfied. Also, there must exist upper bounds $\bar{w}_l$ and $\bar{w}_h$ such that for all $w_l < \bar{w}_l$ and $w_h < \bar{w}_h$ the expression is satisfied.

Substituting the expression defined earlier for Z_{ij}, in the proof of Proposition 1, yields

$$\begin{aligned} &\left|2\beta\left(q\gamma(\hat{r})\right)^2\left[\alpha\sigma_h(p)w_h^* w_l^* + \alpha\sigma_l(p)w_l^* - \sigma_l(p)w_h^* w_l^* - \sigma_h(p)w_h^*\right] + \Delta_\sigma(\alpha)\right| \\ &= \left|\left[\sigma_h(p)w_h^*(\alpha w_l^* - w_h^*) + \sigma_l(p)w_l^*(\alpha w_l^* - w_h^*)\right] + \frac{\Delta_\sigma(\alpha)}{2\beta\left(q\gamma(\hat{r})\right)^2}\right| \end{aligned} \tag{33}$$

Thus, the first term is minimized if $\alpha w_l^* = w_h^*$. Since $\beta \in R_+$, the second term is a continuous function of β, so $\forall \varepsilon > 0$, $\exists \beta^* > \underline{\beta}$, such that for all $\beta > \beta^*$, $\frac{|\Delta_\sigma(\alpha)|}{2\beta(q\gamma(\hat{r}))^2} \leq \varepsilon$. So if costs are sufficiently high and the weights are set according to $\alpha w_l^* = w_h^*$, then $|p_l^* - p_h^*| \leq \varepsilon$. If $\alpha w_l^* \neq w_h^*$, then the condition may still be satisfied by choosing a larger value of $\beta^{**} > \beta^*$, so that (33) will hold for all $\beta > \beta^{**}$. More generally, for any given β, such that $\frac{\Delta_\sigma(\alpha)}{2\beta(q\gamma(\hat{r}))^2} \leq c$, $\exists w_l^{**}, w_h^{**} \in (0,1)$such that for all $w_l^* < w_l^{**}$ and $w_h^* > w_h^{**}$, we must have $[\sigma_h(p)w_h^*(\alpha w_l^* - w_h^*) + \sigma_l(p)w_l^*(\alpha w_l^* - w_h^*)] \leq \varepsilon - c$. (To be completed)

Proof of Proposition 5. This result follows immediately from the expression for the equilibrium value of r_j^* in (24). Since $0 < h(v) < 1$, the numerator in the first term is negative and the denominator is positive, so the first term is always negative. The second term is also a negative fraction, which cancels with the negative sign in front of it. As $\beta \to \infty$, the first term converges upwards to 0, so r_j^* increases. ■

Proof of Proposition 6. We could differentiate (24) w.r.t. r_j^*, however, it is easier to use implicit differentiation and examine $\frac{\partial r_j^*}{\partial q_j^u} = -\frac{\partial \pi_j / \partial q_j^u}{\partial \pi_j / \partial r_j}$. The sign of the numerator can be determined casually by examining the profit function. We know that $d(p_{ij}, v_i, r_j)$and $E[v_i^* | v_i, q_j^u]$are always increasing in quality. Since demand increases, the costs for an underwriter must increase as well, but only proportional to the demand increase, while revenue increases proportionally to the demand and the higher true values. Thus, $\frac{\partial \pi_j}{\partial q_j^u} > 0$, for all values of β, reflecting the gains associated with being a higher quality underwriter. We use this result to prove each point in the proposition.

The rest of the results will rely on the following expression:

$$\begin{aligned} \frac{\partial \pi_j}{\partial r_j} &: \sum_{i=1}^{I}(h(v_i) - 1)\sigma(p_{ij})p_{ij}E[v_i^* | v_i, q_j]f(v_i) - \frac{\partial k(r_j, q_j)}{\partial r_j} \\ &\quad -\beta q_j^2 \sum_{i=1}^{I} d(p_{ij}, q_j, r_j)f(v_i) \sum_{i=1}^{I}(h(v_i) - 1)\sigma(p_{ij})f(v_i) \end{aligned} \tag{34}$$

When underwriters are undifferentiated, they choose the same evaluation standards, the same prices, and obtain identical market shares and profits. Now consider the case increasing one underwriters quality by some small amount. The first term in (34) is negative, and essentially represents the total revenue with each value signal being weighed by the probability of the entrepreneur being bad. If we assume that the fixed costs are additive in each argument, then the second term is zero. Since $0 < h(v), \gamma(q, r), \sigma(p_j) < 1$implies that

$$\sum_{i=1}^{I} \gamma(r_j, v_i)\sigma(p_{ij})f(v_i) < I$$

and

$$\left|\sum_{i=1}^{I}(1-h(v_i))\sigma(p_{ij})f(v_i)\right| < I$$

then we must be able to find some q_h^* such that $\forall q_h \in \{q_h : q_l < q_h < q_h^*\}$, the third term in (34) is less than the first term in absolute value, which implies the whole expression is negative. Thus, $\frac{\partial \pi_j}{\partial r_j} < 0$ and $\frac{\partial r_j^*}{\partial q_j^u} > 0$ for small enough values of q_h^u. Now, as q_h^u increases, revenues grow linearly while costs increase quadratically. Thus, for all $q_h > q_h^*$, $\frac{\partial \pi_j}{\partial r_j} > 0$and $\frac{\partial r_j^*}{\partial q_j^u} < 0$. This result is true for all β.

A similar argument holds for the cast of hte low-quality underwriter in part (ii).