Module 3: Natural Monopolies

Market Organization & Public Policy (Ec 731) · George Georgiadis

Market Entry and Monopoly

- Consider the following two period game:
  - In $t = 1$, a “large” number of identical firms sequentially decide whether to pay an entry fee $F > 0$ to enter the market.
  - In $t = 2$, the firms that entered, engage in Cournot competition.

- Assume that
  - each firm has product cost $c(q) = cq$; and
  - the inverse demand function $P(Q) = \alpha - \beta Q$.

- From the previous section, we know that if $n$ firms have entered, in $t = 1$, each will set quantity
  \[
  q = -\frac{P(Q) - c'(q)}{P'(Q)} = \frac{\alpha - \beta Q - c}{\beta}
  \]
  and using that $Q = nq$ yields that $q = \frac{\alpha - c}{\beta(n+1)}$.

- This corresponds to price $P(Q) = \frac{\alpha + cn}{n+1}$.
  - Observe that it decreases in $n$, and converges to $c$ as $n \to \infty$.

- Each firm’s profit is then
  \[
  \pi_n = q(\alpha - \beta nq) - cq = \frac{1}{\beta} \left( \frac{\alpha - c}{n+1} \right)^2
  \]
  - Observe that $n\pi_n$ decreases (monotonically) with $n$.

- Suppose $n$ firms have already entered the market. Will the next firm choose to enter or not?
- Yes, if \( \pi_{n+1} \geq F \)
- No, if \( \pi_{n+1} < F \)

Therefore, the equilibrium number of firms that will enter the market (denote \( n^* \)) is the largest \( n \) such that \( \pi_n \geq F \), or equivalently

\[
n^* = \left[ \frac{\alpha - c}{\sqrt{\beta F}} - 1 \right]
\]

- If \( F > \frac{(\alpha - c)^2}{9\beta} \), then \( n^* = 1 \), and we have a monopoly.
- As \( F \to 0 \) (i.e., as entry costs vanish), \( n^* \to \infty \) (perfect competition).

**Questions:**
- What if firms engage in Bertrand competition?
- What if firms decide whether to enter simultaneously?

**Monopoly Regulation**

- How can a regulator restore the social optimum?

- Suppose that a regulator taxes monopoly output at rate \( t \).
  - i.e., if the monopolist sets price \( p \), then consumers must pay \( p + t \).

- The monopolist chooses \( p \) by solving

\[
\max_p \{ pD(p + t) - c(D(p + t)) \}
\]

- First order condition:

\[
D(p + t) + D'(p + t) (p - c') = 0
\]

\[
\implies [D(p + t) - tD'(p + t)] + D'(p + t) (p + t - c') = 0
\]

- To restore the social optimum, the price faced by consumers (i.e., \( p + t \)) must equal marginal cost \( c' \).
Therefore, we must set \( t = \frac{D(p+t)}{D'(p+t)} \).

- Denoting the competitive price by \( p_c \), we can re-write \( t = -\frac{p_c}{\epsilon} \), where \( \epsilon = -\frac{p_c D'(p_c)}{D(p_c)} \).

Observe that because \( D' < 0, t < 0 \); i.e., the regulator must subsidize the monopolist. (Somewhat paradoxical!)

**Intuition:**

- The problem with monopoly pricing is that it induces consumers to consume too little.
- In order to achieve efficiency, we must induce them to consume more, which requires to subsidize the good.

**Problems:** Determining the proper subsidy requires that the regulator knows (i) the demand elasticity of the monopolist, and (ii) his entire cost curve.

- Demand information can be obtained through sampling, but this is potentially expensive and inaccurate if the monopolist supplies only a few customers.
- Cost information is harder to extract, because the monopolist will be reluctant to release accurate estimates of its cost structure.

### Regulating a Monopolist with Unknown Costs

**Baron and Myerson (Ecta, 1982)**

- Setting where the firm has a privately known cost parameter; i.e., its cost is \( c(q, \vartheta) \), where \( c_q > 0 \) and \( c_\vartheta > 0 \).
- Regulator can choose (i) the price \( p \) that the firm can charge, and (ii) a subsidy \( s \) to be paid to the firm.

  - Solution Approach: the firm is asked to report \( \hat{\vartheta} \), and receives \( p(\hat{\vartheta}) \) and \( s(\hat{\vartheta}) \).

- Application of the revelation principle.

**Laffont and Tirole (JPE, 1986)**

- Argue that accounting costs are usually observable to the regulator.
- Study a problem with both moral hazard and adverse selection.
Setup

- Natural monopolist has exogenous cost parameter $\theta \in \{\theta_L, \theta_H\}$. (Define $\Delta \theta = \theta_H - \theta_L > 0$.)
  - Assume that $\theta$ is private information of the monopolist.
  - The regulator has beliefs over $\theta$: $Pr \{ \theta = \theta_L \} = \beta$.

- Production cost: $c = \theta - e$, where $e$ stands for “effort” (e.g., investment in cost reduction).
  - Effort has cost $\psi(e) = \frac{e^2}{2}$.
  - Assume that $c$ is contractible.

- The objective of the regulator is to choose the smallest payment $P = c + s$ such that the firm produces the good.

- The payoff of the firm is: $P - c - \psi(e) = P - (\theta - e) - \frac{e^2}{2}$ (if it chooses to produce).

First Best

- Suppose that the regulator knows $\theta$.

- The regulator’s problem then is:

\[
\min_{P, e} P \\
\text{s.t.} \quad P - (\theta - e) - \frac{e^2}{2} \geq 0
\]

- This problem has solution: $e^* = 1$ and $P^* = \theta + \frac{1}{2}$.
  - Let $s = P - c = P - \theta + e$ denote the subsidy. Observe that $s^* = \frac{3}{2}$ (independent of $\theta$).
  - $P(\theta = \theta_H) > P(\theta = \theta_L)$: If the regulator does not know $\theta$, then the firm would like “convince” the regulator that $\theta = \theta_H$ to elicit a larger payment.
Adverse Selection

- The regulator would like to design a “menu” of contracts \( \{ s_L, c_L \} \) and \( \{ s_H, c_H \} \) such that a firm with cost parameter \( \theta_i \) will choose contract \( \{ s_i, c_i \} \) and exert effort \( e_i = \theta_i - c_i \).
  - Implemented using a price \( P_i = s_i + c_i \).
  - Then the firm’s payoff is \( P_i - c_i (e_i) - \frac{e_i^2}{2} = s_i - \frac{e_i^2}{2} \).

- The regulator solves the following problem:

\[
\begin{align*}
\min_{s_L, e_L, s_H, e_H} & \quad \beta (s_L - e_L) + (1 - \beta) (s_H - e_H) + [\beta \theta_L + (1 - \beta) \theta_H] \\
\text{s.t.} & \quad s_L - \frac{e_L^2}{2} \geq 0 \quad (IR_L) \\
& \quad s_H - \frac{e_H^2}{2} \geq 0 \quad (IR_H) \\
& \quad s_L - \frac{e_L^2}{2} \geq s_H - \frac{(e_H - \Delta \theta)^2}{2} \quad (IC_L) \\
& \quad s_H - \frac{e_H^2}{2} \geq s_L - \frac{(e_L + \Delta \theta)^2}{2} \quad (IC_H)
\end{align*}
\]

- The first two inequalities are participation constraints.
- The next two are incentive constraints: if a firm with \( \theta_L \) reports \( \theta_H \), then it must exert effort \( e = \theta_H - c_L = \Delta \theta - e_L \).

- First best has the same effort level and the same subsidy for both types (\( \theta_L \) and \( \theta_H \)), but a higher actual cost for \( \theta = \theta_H \).
  - Incentive problem arises, because the efficient type (i.e., \( \tilde{\theta} = \theta_L \)) wants to mimic the inefficient type to collect the same subsidy while expending only effort \( e^* - \Delta \theta \), and achieving actual cost \( c_H \).

- **Claim:** The \( (IR_L) \) and \( (IC_H) \) are obsolete, while \( (IR_H) \) and \( (IC_L) \) bind.
  - \( s_H - \frac{e_H^2}{2} \geq 0 \Rightarrow s_H - \frac{(e_H - \Delta \theta)^2}{2} \geq 0 \Rightarrow s_L - \frac{e_L^2}{2} \geq 0 \), so \( (IR_L) \) is obsolete.
  - Add \( (IC_H) \) and \( (IC_L) \) to find \( e_H \leq e_L + \Delta \theta \). So \( s_H - \frac{e_H^2}{2} \geq s_L - \frac{e_L^2}{2} \geq s_L - \frac{(e_L + \Delta \theta)^2}{2} \), so \( (IC_H) \) is obsolete.
  - Suppose \( (IR_H) \) is slack. Then decrease \( s_H \) to increase the regulator’s payoff until it binds. Note also that this relaxes \( (IC_L) \).
– Suppose \((IC_L)\) is slack. Then decrease \(s_L\) to increase the regulator’s payoff until it binds.

○ Therefore:

\[
s_H - \frac{e_H^2}{2} = 0 \quad \text{and} \quad s_L - \frac{e_L^2}{2} = s_H - \frac{(e_H - \Delta \theta)^2}{2}
\]

(1)

○ Re-writing the objective function using these equalities yields

\[
\min \left\{ \beta \left( \frac{e_L^2}{2} - e_L + \frac{e_H^2}{2} - \frac{(e_H - \Delta \theta)^2}{2} \right) + (1 - \beta) \left( \frac{e_H^2}{2} - e_H \right) \right\}
\]

○ First-order conditions:

– \(e_L = 1 = e^*\)

– \(e_H = 1 - \frac{\beta}{1 - \beta} \Delta \theta < e^*\)

– Note: Assume that \(\frac{\beta}{1 - \beta} \Delta \theta < 1\).

○ We can now solve for the subsidies \(s_L\) and \(s_H\) using (1).

○ Intuition:

– The “low” type would like to imitate the “high” type, but not vice verse.

– So for IC, mechanism gives inefficient incentives to the “high” type and lower his payoff to make it undesirable to the “low” type to imitate him.

– Give efficient incentives to the “low” type.

\textbf{References}

