Module 1: Pricing Behavior

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Monopoly Pricing

- Consider a monopolist facing demand curve $D(p)$, where $D'(p) < 0$.
  - i.e., if the price is $p$, then demand for the good will be equal to $q = D(p)$.
  - Write $P(q)$ to denote the inverse demand function; i.e., $p = D^{-1}(q)$.

- The cost of producing $q$ units of the good is $c(q)$, where $c'(q) > 0$.
- The monopolist wants to choose the price to maximize his profit. So he solves:

$$\max_p \{ pD(p) - c(D(p)) \}$$

- First order condition:

$$\underbrace{D(p) + pD'(p)}_{\text{marginal revenue}} = \underbrace{c'(D(p))D'(p)}_{\text{marginal cost}}$$

$$\Rightarrow p - c'(D(p)) = \frac{-D(p)}{D'(p)}$$

$$\Rightarrow p - c'(D(p)) = \frac{1}{\epsilon}$$

(1)

where $\epsilon = -\frac{pD'(p)}{D(p)}$ denotes the demand elasticity at price $p$.

- Demand elasticity: % change in demand in response to a 1% price reduction.
- We usually denote this price $p^m$.
- Equation (1) tells us that the relative markup (i.e., the ratio between the profit margin and the price), also called the Lerner index, is inversely proportional to the demand elasticity.
Note: We assume that \( D(\cdot) \) and \( c(\cdot) \) are such that the monopolist’s objective function is concave in \( p \), so that the FOC is sufficient for a maximum.

- i.e., we assume that \( 2D'(p) + pD''(p) - c''(D(p)) [D'(p)]^2 \leq 0. \)

Cournot Competition

- Same setup as above with two changes:

  1. \( n \) instead of single firm compete in the market.
  2. Each firm \( i \) chooses a quantity \( q_i \) to produce, and the market price is determined by \( p = P(\sum_{i=1}^{n} q_i) \).

- Firm \( i \) chooses \( q_i \) by solving

\[
\max_{q_i} \{ q_i P(q_i + Q_{-i}) - c(q_i) \}
\]

- First order condition:

\[
P(Q) + qP'(Q) = c'(q)
\]

\[
\implies q = -\frac{P(Q) - c'(q)}{P'(Q)}
\]

Assuming symmetry (i.e., \( q = \frac{Q}{n} \)), we obtain (in equilibrium):

\[
\frac{Q}{n} = -\frac{P(Q) - c'(\frac{Q}{n})}{P'(Q)}
\]

\[
\implies \frac{P(Q) - c'(\frac{Q}{n})}{P(Q)} = -\frac{Q}{n} \frac{P'(Q)}{P(Q)}
\]

- Recall that \( P(Q) = D^{-1}(Q) \). Then \( P'(Q) = [D^{-1}(Q)]' = \frac{1}{D'(D^{-1}(Q))} = \frac{1}{D'(p)} \).

- Therefore, \( \frac{Q P'(Q)}{P(Q)} = \frac{D(p)}{p} = \frac{D(p)}{p D'(p)} \), and the equilibrium price satisfies

\[
\frac{p - c'(\frac{D(p)}{n})}{p} = \frac{1}{n\epsilon}
\]

where \( \epsilon = -\frac{p D'(p)}{D(p)} \)

- Remarks:
1. **Sanity check:** When \( n = 1 \), the price in (2) coincides with the monopoly price.

2. As \( n \) increases, the relative markup and the profit of each firm decreases. (In fact, the total profit of all firms decreases with \( n \).)

3. At the limit as \( n \to \infty \), the price equals marginal cost (**perfect competition**).

**Bertrand Competition**

- Two firms compete in a market.
- Each firm:
  - has constant marginal cost (so that \( c(q) = cq \)); and
  - faces market demand function \( q = D(p) \).
- Firm \( i \) sets a price \( p_i \) to maximize its equilibrium profit

\[
\Pi_i(p_i, p_j) = (p_i - c) D_i(p_i, p_j)
\]

where

\[
D_i(p_i, p_j) = \begin{cases} 
D(p_i) & \text{if } p_i < p_j \\
\frac{1}{2}D(p_i) & \text{if } p_i = p_j \\
0 & \text{if } p_i > p_j 
\end{cases}
\]

- Interpretation:
  - If a firm undercut the other firm’s price, then it captures the entire market.
  - If both firms set the same price, then each captures half of the market.

- **Claim:** The unique equilibrium of this game has both firms charging the competitive price: \( p_1^* = p_2^* = c \).

**Proof.**

- Suppose that \( p_1^* > p_2^* > c \).
  - Then firm 1 has no demand, and its profit is 0.
  - If instead firm 1 sets \( p_1^* = p_2^* - \epsilon > c \), then it obtains the entire demand \( D(p_2^* - \epsilon) \), and has a positive profit margin of \( p_2^* - \epsilon - c > 0 \).
  - Therefore, setting \( p_1^* \) cannot be optimal.
Now suppose that $p^*_1 = p^*_2 > c$.

- The profit of firm $i$ is $\frac{1}{2}D(p^*_i)(p^*_i - c) > 0$.
- If firm $i$ reduces its price to $p^*_i - \epsilon$, then its profit becomes $D(p^*_i - c)(p^*_i - \epsilon - c)$, which is greater for small $\epsilon$.
- Therefore, both firms setting some $p^* > c$ cannot be optimal either.

Lastly, suppose that $p^*_1 > p^*_2 = c$.

- Then firm 2, which makes no profit, could raise its price slightly, still supply all the demand, and make a positive profit - a contradiction.

Therefore, in the unique equilibrium, it must be that $p^*_1 = p^*_2 = c$.

Takeaway: Even with (only) two competing firms, firms price at marginal cost, and they do not make profits.

Note: Result extends to $n > 2$ competing firms.

This suggests that even a duopoly is enough to restore perfect competition.

We call this the Bertrand paradox. (Tough to believe!)

Solutions to the Bertrand Paradox:

   - Suppose that firms can product at most $\gamma$ units, where $D(c) > \gamma$.
   - Is $p^*_1 = p^*_2 = c$ still an equilibrium?
     - Suppose that firm 2 sets $p_2 > c$. Then firm 1 faces demand $D(c)$, but can only satisfy up to $\gamma$.
     - In this case firm 1 makes 0 profit, while firm 2 makes a positive profit. Therefore, this is not an equilibrium.
   - Characterizing the equilibrium of this game requires assumptions about how consumers are rationed.

2. Product differentiation.
○ Bertrand analysis assumes that the firms’ products are perfect substitutes.
○ This creates a pressure on price, which is relaxed when the products are not exactly identical.

3. Temporal dimension.

○ Bertrand analysis assumes that the firms set prices simultaneously.
○ In the real world, firms can observe their competitors’ prices and react.
○ Think of a dynamic environment where firms set $p_1 = p_2 > c$. Does any one firm have an incentive to set $p_i < p_j$?
  – Not clear! Must trade off the benefit of capturing all the market share “today”, and making no profits in the future.
  – This is called “tacit collusion”.

References