Problem Set 4

Information Economics (Ec 515)   ·   George Georgiadis

Due in class or by e-mail to quel@bu.edu at 12:30, Monday, December 8

Problem 1. Screening

A monopolist can produce a good in different qualities. The cost of producing a unit of quality s is $\frac{s^2}{2}$. Consumers buy at most one unit and have utility function

$$u(s|\theta) = \begin{cases} \theta s & \text{if they consume one unit of quality } s \\ 0 & \text{if they do not consume} \end{cases}$$

The monopolist decides on the quality (or qualities) it is going to produce and price. Consumers observe qualities and prices and decide which quality to buy if at all.

1. Characterize the first-best solution.

2. Suppose that the seller cannot observe θ, and suppose that

    $$\theta = \begin{cases} \theta_H & \text{with probability } 1 - \beta \\ \theta_L & \text{with probability } \beta \end{cases}$$

    with $\theta_H > \theta_L > 0$. Characterize the second-best solution and consumers' informational rent.

3. Suppose now that θ is uniformly distributed on the interval $[0,1]$. Characterize the second-best optimal quality-pricing schedule.

Problem 2. Screening #2

John has a grape farm and sells grapes to a liquor company. The relationship between John and the liquor company is mutually exclusive: there is no outside options for both. The liquor company produces wines, and its profit function is

$$\pi(x, a) = 2a^2 - (a + 3 - x)^2,$$

where x is the amount of grapes supplied by John, and a is the type of the liquor company, which is not known to John. We assume that $\Pr\{a = 1\} = \Pr\{a = 4\} = 0.5$. John offers a contract, which is represented by a function $T : \mathbb{R} \to \mathbb{R}$ where $T(x)$ is the money paid by the liquor company when John supplies x amount of grapes. T can be negative; John subsidizes the liquor company. We assume that there is no cost to produce grapes. Thus, John wants to maximize the expectation of $T(x)$.

1. Suppose that a is known to John. What is the optimal contract?

2. Suppose that a is not known to John. What is the optimal contract when John wants to supply grapes to both types?

3. Suppose that a is not known. What is the optimal contract when John wants to accommodate the high type?

4. Compare John's profits in parts 2 and 3. Discuss.

Problem 3. Auctions

A decision where to locate a hazardous waste dump is taken through an auction between n towns in a given country. Call d_i town i's disutility from taking on the dump. Assume the d_i's are uniformly and independently distributed on $[0, 1]$. Call T_i the transfer the town requests from taking the hazardous waste dump. The lowest bidder gets the dump and receives its requested transfer, which is paid equally from the other towns. Compute the symmetric equilibrium of this auction. Is this an efficient allocation mechanism? Discuss.