Module 7: Debt Contracts & Credit Rationing

Information Economics (Ec 515) · George Georgiadis

Two Applications of the principal-agent model to credit markets

- An entrepreneur (E - borrower) has a project.
  - Project requires investment $I > 0$.
  - Entrepreneur has assets $A \in [0, I)$.
  - Requires to borrow $I - A$ from a Lender (L).
- If undertaken, project either succeeds and yields profits $\pi = R > 0$, or it fails and yields $\pi = 0$.
- Both E and L are risk-neutral.
- E privately chooses effort $e \in \{e_H, e_L\}$
  - Assume $c(e_H) = B > 0$ and $c(e_L) = 0$.
  - Let $p(e)$ be the probability that project succeeds, where $\Delta = p(e_H) - p(e_L) > 0$.
- Assumptions: The project has
  - positive NPV if E works: $p(e_H) R - I - B > 0$
  - negative NPV if E shirks: $p(e_L) R - I < 0$
- L offers E a contract to lend him $I - A$:
  - Contract specifies repayment z from E to L, as a function of the realized profits.
  - There is a competitive lending market, so lender earns zero expected profits.
- Assume E is protected by limited liability, so $z \leq \pi$.
  - If $\pi = 0$, then repayment is zero $\Longrightarrow$ both E and L get zero profits.

- If $\pi = R$, then repayment is $z \in [0, R] \implies$ E gets $R - z$ and L gets z.

- If E puts high effort:
  - Lender's expected profits are: $p(e_H) z - (I - A)$.
  - Entrepreneur's expected profits are: $p(e_H)(R - z) - A - B$.

- If Entrepreneur puts low effort:
  - L's expected profits are $p(e_L) z - (I - A)$.
  - E's expected profits are $p(e_L)(R - z) - A$.

- Recall that project has positive NPV only if E puts effort:
  - Suppose L offers a contract that induces E to put low effort. Then:

$$\underbrace{[p(e_L) z - (I - A)]}_{\text{Profits to Lender}} + \underbrace{[p(e_L)(R - z) - A]}_{\text{Profits to Entrepreneur}} < 0.$$

  - No loan that induces E to put low effort will ever be given out - such a loan would give a negative payoff either to E or to L.

- Suppose that L offers a contract that induces E to put high effort.
  - If E puts high effort, L's expected profits are $p(e_H) z - (I - A)$.
  - Perfect competition among lenders implies that

$$z = \frac{I - A}{p(e_H)}$$

- L must provide incentives for E to put high effort.
  - Incentive compatibility constraint:

$$\begin{aligned} p(e_H)(R - z) - B - A &\geq p(e_L)(R - z) - A \\ \implies \Delta (R - z) &\geq B \\ \implies R - \frac{B}{\Delta} &\geq z \end{aligned}$$

- These two equations imply that

$$
\begin{aligned}
R - \frac{B}{\Delta} &\geq \frac{I - A}{p\left(e_H\right)} \\
\Longrightarrow A &\geq I - p\left(e_H\right)\left(R - \frac{B}{\Delta}\right) = \bar{A}.
\end{aligned}
$$

  - E will only get financing if $A \geq \overline{A}$.
  - To provide incentives, E must have a high stake in the project (*i.e.*, enough "skin in the game").
  - If the principal cannot provide incentives, then he will not finance the project.

- *Case 1:* $A \geq \overline{A}$

  - E will get financing, and his repayment scheme is $z = \frac{I-A}{p(e_H)}$.
  - L earns zero profits (competitive lending market).
  - E's stake in the firm:

$$
R - z = R - \frac{I - A}{p\left(e_H\right)} \geq R - \frac{I - \overline{A}}{p\left(e_H\right)} = \frac{B}{\Delta}.
$$

  - E has incentives to put effort.

- *Case 2:* $A < \bar{A}$

  - E must borrow a large amount, and hence repay a large amount to L.
  - This reduces his stake in the project, so he doesn't have incentives to put effort.
  - There is no loan agreement that induces effort and allows L to recover the investment.
  - There is credit rationing!

- Determinants of credit rationing:

  - Level of assets that E owns A.
  - How costly it is to provide incentives: how large B is relative to Δ.
  - How costly the investment is (*i.e.*, how large I is).

- Crucial constraint for these results: limited liability constraint.

- Recall that in the general principal-agent problem, we could implement the optimal solution when the agent was risk-neutral.
  - In that case, the optimal contract was to "sell the firm" to the agent.
  - But this doesn't satisfy limited liability!
- In this problem, credit rationing wouldn't matter without limited liability.
  - If we drop the limited liability constraint, we are assuming that E has enough money to fund the project herself!

Motivating Debt Contracts

- *Debt contract:* First $D of profits go to investors.

Model:

- Risk-neutral entrepreneur seeks funding from risk-neutral investor
- Output $q \sim f(q \mid a)$ satisfies MLR
- Investor puts in funds I
- Entrepreneur makes a TIOLI offer to repay $r(q) \in [0, q]$ in state q.
- Entrepreneur's utility: $w(q) - c(a)$, where $w(q) = q - r(q)$.

- Entrepreneur's Problem:

$$\begin{aligned} \max_{r(q),\, a} \quad & \mathbb{E}\left[q - r(q) \mid a\right] - c(a) & \\ \text{s.t.} \quad & \mathbb{E}\left[r(q) \mid a\right] \geq I & \text{(IR)} \\ & a \in \arg\max_{a'} \mathbb{E}\left[q - r(q) \mid a'\right] - c(a') & \text{(IC)} \\ & 0 \leq r(q) \leq q & \text{(feasibility)} \end{aligned}$$

- Straightforward that IR should bind.

- Ignore (feasibility) and write the Lagrangian:

$$\begin{aligned} L &= \int_{\mathbb{R}} [q - r(q)]\, dF(q \mid a) - c(a) + \lambda \left[\int_{\mathbb{R}} r(q)\, dF(q \mid a) - I \right] \\ &\quad + \mu \left\{ \int_{\mathbb{R}} [q - r(q)] \frac{f_a(q \mid a)}{f(q \mid a)} dF(q \mid a) - c'(a) \right\} \\ &= \int_{\mathbb{R}} q \left[1 + \mu \frac{f_a(q \mid a)}{f(q \mid a)} \right] dF(q \mid a) + \int_{\mathbb{R}} r(q) \left[-1 + \lambda - \mu \frac{f_a(q \mid a)}{f(q \mid a)} \right] dF(q \mid a) - \lambda I - \mu c'(a) \end{aligned}$$

  - Second line follows from FOC approach.

- Take FOC with respect to r:

$$\frac{dL}{dr} = -1 + \lambda - \mu \frac{f_a(q \mid a)}{f(q \mid a)}$$

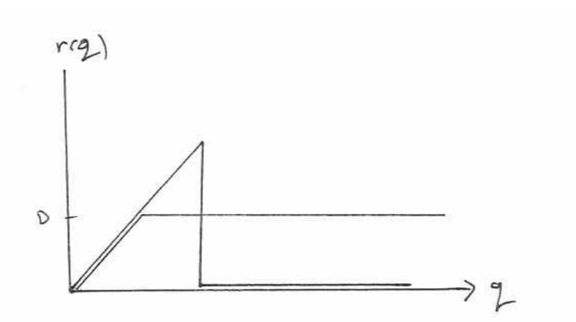
  - r does not appear anywhere $\Longrightarrow$ solution will be "bang-bang".

- Optimal contract:

$$r(q) = \begin{cases} q & \text{if } \lambda \geq 1 + \mu \frac{f_a(q \mid a)}{f(q \mid a)} \\ 0 & \text{otherwise.} \end{cases}$$

  - Optimal λ and μ will be such that (IR) binds.
  - MLR $\Longrightarrow$ $\frac{f_a(q \mid a)}{f(q \mid a)}$ increases in q. Therefore (assuming $\mu > 0$), $\exists q^*$ such that $r(q) = q$ for $q \leq q^*$.

    (Can show that $\mu > 0$ using a similar approach as in standard principal-agent problem.)



- *Intuition:*

  - Incentive problem: induce the agent to exert high effort.
  - Must be rewarded when q is large.

- The entrepreneur's reward $= q - r(q)$.

◦ *Problem:* Because $r(q)$ decreases in q,

1. the entrepreneur can borrow money (without the investor knowing), reduce payment, and repay the borrowed money later ; and
2. the investor has incentives to sabotage the project if q is "large".

◦ *Solution:* Add the constraint $r'(q) \geq 0$.

◦ Then the optimal contract becomes a debt contract:

$$r(q) = \begin{cases} q & \text{if } q \leq D \\ D & \text{otherwise.} \end{cases}$$

◦ D is chosen such that the investor's IR constraint binds:

$$\mathbb{E}[r(q) \mid a^*] = I$$

where $a^* \in \arg\max_{a'} \mathbb{E}[q - r(q) \mid a'] - c(a')$.

References

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