Module 4: Moral Hazard - Linear Contracts

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- A principal employs an agent.
- Timing:

  1. The principal offers a linear contract of the form $w(q) = \alpha + \beta q$.
     - α is the salary, β is the bonus rate.
  2. The agent chooses whether the accept or reject the contract.
     - If the agent accepts it, then goto $t = 3$.
     - If the agent rejects it, then he receives his outside option U, the principal receives profit 0, and the game ends.
  3. The agent chooses action / effort $a \in A \equiv [0, \infty]$.
  4. Output $q = a + \varepsilon$ is realized, where $\varepsilon \sim \mathcal{N}(0, \sigma^2)$
  5. The principal pays the agent, and the parties' payoffs are realized.

- The principal is risk neutral. His profit function is

$$\mathbb{E}[q - w(q)]$$

- The agent is risk averse. His utility function is

$$U(w, a) = \mathbb{E}\left[-e^{-r(w(q)-c(a))}\right]$$

  with

$$c(a) = c\frac{a^2}{2}$$

- *Rationality assumptions:*

  1. Upon observing the contract $w(\cdot)$, the agent chooses his action to maximize his expected utility.

2. The principal, anticipating (1), chooses the contract $w(\cdot)$ to maximize his expected profit.

First Best

- *Benchmark:* Suppose the principal could choose the action a.
  - We call this benchmark the ***first best*** or the ***efficient outcome***.
  - Equivalent to say that the agent's action is *verifiable* or *contractible.*
- Principal solves:

$$\begin{aligned} \max_{a\,,\,w(q)} \quad & \mathbb{E}\left[a+\epsilon-w(q)\right] \\ \text{s.t.} \quad & \mathbb{E}\left[-e^{-r(w(q)-c(a))}\right] \geq U \quad \text{Individual Rationality (IR)} \end{aligned}$$

- Solution approach:
  - Jensen's inequality $\Longrightarrow \mathbb{E}_x\left[-e^{-rx}\right] \leq -e^{-r\mathbb{E}_x[x]}$
  - Because the principal chooses the action, optimal wage must be independent of q; *i.e.*, $w(q)=\alpha$
  - Because a higher $w(q)$ decreases the principal's profit and increases the agent's payoff, (IR) must bind. So:

$$\begin{aligned} -e^{-r(\alpha-c(a))} &= U \\ \Longrightarrow \alpha &= c(a)-\frac{\ln(-U)}{r} \end{aligned}$$

  - The last equation pins down the wage α as a function of the action a.
  - We now substitute into the objective function. We have:

$$\max_{a}\left[a-c\frac{a^2}{2}-\frac{\ln(-U)}{r}\right]$$

  - First order condition: $1-c\,a=0$
- Optimal solution:

$$a^*=\frac{1}{c} \quad \text{and hence} \quad w(q)=-\frac{\ln(-U)}{r}+\frac{1}{2c}$$

- *Notes:*
  - Intuitively, because the agent is risk averse and he does not choose the action, it is suboptimal to expose him to risk.
  - In general, (IR) will bind at the optimum. Otherwise, the principal is leaving money on the table.

Moral Hazard

- Now suppose that the principal cannot choose the agent's action.
- *Trade-offs:*
  1. Because the agent is risk averse and the principal is risk neutral, the principal wants to *insure* the agent.
  2. Because the principal cannot enforce a particular action, she must provide *incentives* to the agent.
- *Extreme cases:*
  - Full insurance (but no incentives): Pay a flat wage; *i.e.*, $w(q) = \alpha$.
  - Full incentives (but no insurance): Agents pays a flat fee and "buys" the output; *i.e.*, $w(q) = \alpha + q$.

Solution Approach

- First, solve the agent's maximization problem for arbitrary $w(q)$:

$$\begin{aligned}
\max_a U &= \max_a \mathbb{E}\left\{-e^{-r[w(q)-c(a)]}\right\} \\
&= \max_a \mathbb{E}\left\{-e^{-r\left[\alpha+\beta(a+\varepsilon)-c\frac{a^2}{2}\right]}\right\} \\
&= \max_a \left\{-e^{-r\left[\alpha+\beta a-c\frac{a^2}{2}\right]})\mathbb{E}\left[e^{-r\beta\varepsilon}\right]\right\} \\
&= \max_a \left\{-e^{-r\left[\alpha+\beta a-c\frac{a^2}{2}\right]})e^{\frac{1}{2}r^2\beta^2\sigma^2}\right\} \\
&= \max_a \left\{-e^{-r(\alpha+\beta a-c\frac{a^2}{2}-\frac{1}{2}r\beta^2\sigma^2)}\right\}
\end{aligned}$$

- Therefore, the agent's problem reduces to

$$\max_{a} \left\{ \alpha + \beta a - c\frac{a^2}{2} - \frac{1}{2} r \beta^2 \sigma^2 \right\}$$

- The first-order condition for the agent's optimal effort choice is:

$$a(\beta) = \frac{\beta}{c}$$

- Unless $\beta \geq 1$, in equilibrium, effort is less than first best.
- The principal will then maximize

$$\begin{aligned} \max_{a,\alpha,\beta} \quad & \mathbb{E}\left[a + \epsilon - \alpha - \beta (a + \epsilon)\right] = (1 - \beta) a - \alpha \\ \text{s.t.} \quad & a = \frac{\beta}{c} \\ & \alpha + \frac{\beta^2}{2} \left(\frac{1}{c} - r\sigma^2 \right) \geq \frac{\overline{u}}{r} \end{aligned}$$

- First equation is the incentive compatibility constraint (IC) and the second is the individual rationality (IR) with $\overline{u} = \ln\left(-\overline{U}\right)$.
- The principal will choose $\alpha = \frac{\overline{u}}{r} - \frac{\beta^2}{2}\left(\frac{1}{c} - r\sigma^2\right)$ (s.t. IR binds).
- Substituting into the principal's objective function:

$$\max_{\beta} \left\{ \frac{(1-\beta)\beta}{c} + \frac{\beta^2}{2}\left(\frac{1}{c} - r\sigma^2\right) - \frac{\overline{u}}{r} \right\}$$

- Solution:

$$\beta^* = \frac{1}{1 + rc\sigma^2} \tag{1}$$

  and

$$\alpha^* = \frac{\overline{u}}{r} - \frac{1 - rc\sigma^2}{2c^2\left(1 + rc\sigma^2\right)^2},$$

- Because negative salaries are allowed, the IR constraint is binding.
- The equilibrium level of effort is

$$a^* = \frac{1}{c\left(1 + rc\sigma^2\right)}$$

which is always lower than the first-best level of effort, $a^{fb} = \frac{1}{c}$.

Comparative Statics

$$\beta^* = \frac{1}{1 + rc\sigma^2}$$

- Incentives are *lower powered* ; *i.e.*, β^* is lower when:
  - the agent is more risk-averse; *i.e.*, if r is larger
  - effort is more costly; *i.e.*, if c is larger
  - there is greater uncertainty; *i.e.*, if σ^2 is larger.

- Is a linear contract optimal (among all possible contracts)?
  - NO!
  - Mirrlees's "shoot-the-agent" contract is optimal here:

$$q^*(x) = \begin{cases} w_H & \text{if } x \geq q_0 \\ w_L & \text{otherwise} \end{cases}$$

    where $w_H > w_L$.
  - By choosing w_H, w_L and q_0 appropriately, it is possible to implement first best (approximately).
    * Agent receives w_H almost surely, yet has incentives from fear of w_L.
  - But this result depends *crucially* on the assumption $\epsilon \sim N(0, \sigma^2)$.

- What to make of linear contracts
  - Even if linear contracts are not optimal here, they are attractive for their simplicity and for being easy to characterize and interpret.
  - Nonlinear models are often very sensitive to the particular assumptions of the model (*e.g.*, the distribution function of ϵ).

- Nonlinear contracts are also prone to "gaming".
  - Consider Mirrlees' "shoot-the-agent" contract in a dynamic world.
  - After output has reached q_0, the agent has no incentive to exert effort.

References

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