The Retail Planning Problem Under Demand Uncertainty

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Introduction

- Many retail store chains carry private label products.

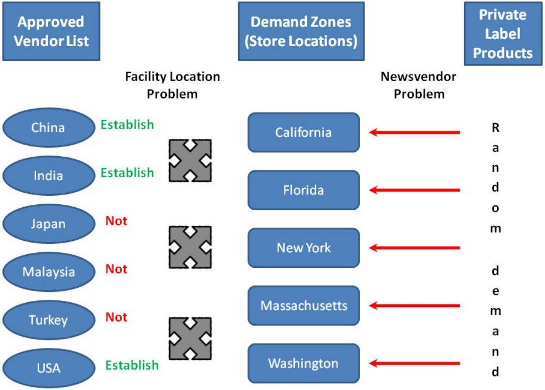
Examples:

- *Macy's* carries Alfani, Club Room, and others.
- *GAP* and *Zara*: Exclusively private labels.

- In addition to deciding inventory levels, the retailer must

  1. Choose suppliers / establish production facilities.
  2. Make production and distribution decisions.

Illustration of the Retail Planning Problem



We develop a framework to address the retailer's supplier choice, as well as her production, distribution and inventory decisions.

Related Literature

- Facility Location under Uncertain Demand:
    - Reviews by Aikens (1985), Snyder (2006), and Shen (2007).
- Integrated Supply Chain Models:
    - Balachandran and Jain (1976).
    - Le Blanc (1977).
    - Daskin et al. (2002).
    - Shen et al. (2003).
- Retailing: Fisher and Raman (2010).

Model Formulation

i : suppliers || j : stores || k : products

$$Z_P = \min \left\{ \sum_i f_i z_i + \sum_{i,j,k} c_{ijk} x_{ijk} + \sum_{i,k} d_{ik} w_{ik} + \sum_{i,j} e_{ij} v_{ij} + \sum_{j,k} S_{jk}\left(y_{jk}\right) \right\}$$

$$\sum_i x_{ijk} = y_{jk} \quad \text{for all } j \text{ and } k$$

$$L_i z_i \le \sum_j \sum_k \alpha_{ijk} x_{ijk} \le U_i z_i \quad \text{for all } i$$

$$\sum_j \alpha_{ijk} x_{ijk} \le U_i w_{ik} \text{ and } \sum_k \alpha_{ijk} x_{ijk} \le U_i v_{ij} \quad \text{for all } i, j, \text{ and } k$$

$$x_{ijk} \ge 0 \,,\; y_{jk} \ge 0 \,,\; w_{ik} \in \{0,1\} \,,\; v_{ij} \in \{0,1\} \,,\; z_i \in \{0,1\} \text{ for all } i, j, \text{ and } k$$

- $S_{jk}(y)$: Inventory cost (newsvendor model).
  - *Fashion industry*: short product lifecycles relative to lead times.

A Basic Result and a Roadmap

- The RPP is strongly NP-hard.
  - Reduction to CPLP (Cornuejols et. al. (1991)).
  - Large-sized instances unlikely to be solvable to optimality.

How to proceed?

- Construct heuristics to obtain a feasible solution.
- Obtain a lower bound on Z_P using a Lagrangean relaxation.
- Evaluate how close the feasible solution is to the lower bound.
  - CVX Heuristic: Average suboptimality gap $= 3.4\%$.
- Analyze the computational results to draw insights.

Convex Programming Heuristic

1: Solves a convex programming relaxation.
- w_{ik}'s, v_{ij}'s and the *unfixed* z_i's relaxed to lie in $[0,1]$.

2: Permanently fix any $z_i \in \{0,1\}$.

3: Temporarily fixes largest fractional z_i to 1.
- Solves remaining problem and rounds to 1 fract. w_{ik} and v_{ij}.

Temporarily fixes smallest fractional z_i to 0.
- Solves remaining problem and rounds to 1 fract. w_{ik} and v_{ij}.

4: Permanently fix the z_i that yielded lowest total cost.
- Return to 1 until all z_i's have been fixed.

- *LP-based version*: Uses *Lagrangean* inventory levels to solve a sequence of linear programs.

Lagrangean Relaxation

- Relax $\sum_i x_{ijk} = y_{jk}$. Decomposes problem into:
  - I Facility Location Subproblems:

$$L_i^{milp}(\lambda) = \min \left\{ f_i z_i + \sum_k \left[d_{ik} w_{ik} + \sum_j (c_{ijk} - \lambda_{jk}) x_{ijk} \right] \right\}$$

  - $J \times K$ Inventory Subproblems:

$$L_{jk}^{cvx}(\lambda) = \min \{ \lambda_{jk} y_{jk} + S_{jk}(y_{jk}) \}$$

- $L(\lambda) = \sum_i L_i^{milp}(\lambda) + \sum_j \sum_k L_{jk}^{cvx}(\lambda)$
  - For any $\lambda \in \mathbb{R}^{J \times K}$, $L(\lambda)$ is a lower bound for Z_P.

Some Analytical Results

Proposition 1

Lagrangean Relaxation solved in closed form for any given λ.

- Lagrangean bound $= \max_\lambda \{L(\lambda)\}$.
- $L(\lambda)$ in closed form $\Rightarrow$ Can solve max. problem *directly*.

Lemma 1

Lagrangean problem does not possess the integrality property.

- Lagrangean lower bound $\geq$ convex relaxation lower bound.

Proposition 2

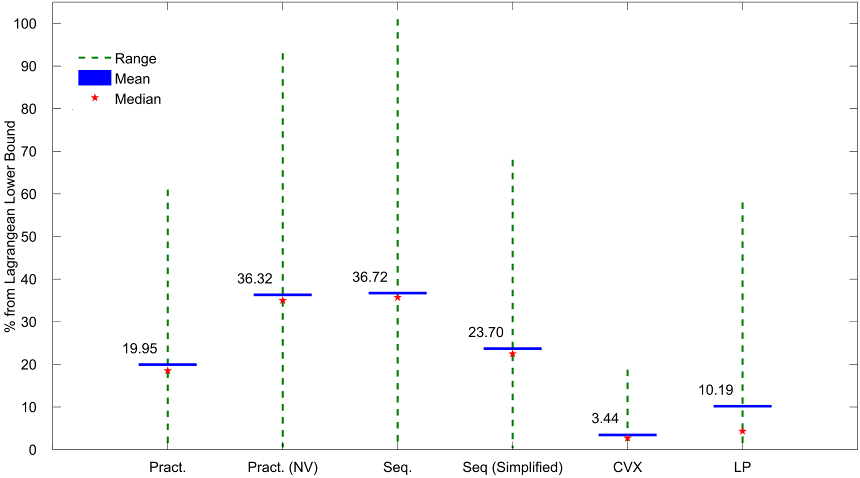
- Conditions so that λ^*_{jk} can be characterized analytically.

Computational Experiments

- 500 randomly generated problem instances.
  - $5-20$ candidate facility locations.
  - $10-40$ stores.
  - $1-25$ products.

- We evaluate:
  - Objective functions of CVX heuristic, and LP variation;
  - Lagrangean lower bound; and
  - Objective functions of two benchmark heuristics:
    1. Practitioner / Greedy heuristic.
    2. Sequential heuristic.

Suboptimality Gap



An Insight regarding Inventory Decisions

- Inventory levels in CVX heuristic $<$ newsvendor levels.

Why?

CVX solution accounts for effect of inventory to upstream SC costs.

Intuitively: a higher inventory level increases

(i) production and distribution costs; and

(ii) costs associated establishing production capacity.

When these costs are accounted for, lower inventory is preferable.

Take-away

When managing the entire SC, a lower fill rate may be preferable.

Analyzing the Computational Results

- How does
  - a. the computational time ;
  - b. the gap between CVX and the best benchmark heuristic ;
  - c. the suboptimality gap ; and
  - d. the total expected cost of CVX heuristic

  depend on the size and the cost parameters of the problem.

Finding 1: Computational Time.

- Depends primarily on problem size (i.e., I, J and K).
- Linear regression yields $R^2 = 0.64$.
  - Scales up approximately linearly in problem size.

Finding 2: CVX heuristic is robust to changes in parameters.

- All regressors and their std. errors are close to 0.

Gleaning Insights from the Computational Results (Cont'd)

Finding 3: Performance Advantage of CVX heuristic is Robust.

- Performance advantage increases in problem size.
- Insensitive to the cost parameters.

Finding 4: Total Expected Cost vs. Problem Parameters.

- Increases in the problem size and the mean demand.
- *Key influencing factors:*
  1. Inventory underage and overage costs; and
  2. Marginal production and distribution costs.

- Emphasizes value of improved demand forecast.
- Supplier capacity and fixed costs have a secondary effect.

Summary

- Integrated SC problem: Retailer chooses suppliers, and determines production, distribution and inventory planning.
  - Use Lagrangean relaxation to obtain a lower bound.
  - Develop heuristics to obtain feasible solutions.
- Computational experiments.
  - Solutions are close to optimal (within 3.4% on average).
  - Suboptimality gap is robust to problem size and parameters.
  - Computational time scales up $\sim$ linearly in problem size.
- Insights:
  1. Lower fill rate may be preferable when managing the entire SC.
  2. Inventory costs are key drivers of total expected SC costs.
     - Fixed costs and supplier capacity have a secondary effect.

Future Research

- Embed this problem in a dynamic environment.
  - Allow for replenishing of inventory.
- Incorporate multiple echelons in the SC.
  - e.g., wholesalers, distribution centers, etc.
- Explicitly model economies of scale.