Optimal Monitoring Design

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Motivation

- Performance monitoring is a crucial aspect of any incentive plan.
- Organizations devote substantial resources to identify or design good performance monitoring processes (Lazear, Gibbs, Murphy, Larcker,..)
- Contracting models typically assume exogenous performance measure.

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Optimal design of performance monitoring & wage scheme.

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In a Nutshell

- Standard principal-agent model under moral hazard
  (One-shot interaction, risk-averse agent, continuous effort, etc..)
- *Performance monitoring.*— The principal sequentially acquires costly i.i.d signals that are correlated with the agent's effort.
- Principal commits to:
  a. A path-contingent stopping rule for acquiring signals.
  b. A wage scheme, which conditions agent's wage on the acquired signals.

Main Result: Under certain conditions

i. Optimal information acquisition strategy is a two-threshold policy.

ii. *Single-bonus wage scheme* is optimal; *i.e.*, base wage + fixed bonus.

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Related Literature

- Informativeness Principle: Holmström (1979)
- Endogenous performance measures:
  - Dye (1986); Feltham & Xie (1994); Khalil and Lawarrée (1995)
  - Hoffman, Inderst, and Opp (2017); Li and Yang (2017)
- Simple contracts:
  - *Linear:* Holmström & Milgrom (1987); Edmans & Gabaix (2011); Carroll (2015); Barron, Georgiadis & Swinkels (2017)
  - *Single-Bonus:* Oyer (2000); Levin (2003); Palomino & Prat (2003); Herweg et al. (2010)
- Information design:
  - Aumann & Perles (1965); Kamenica & Gentzkow (2011)
  - Boleslavsky & Kim (2017)

Roadmap

Model

- *Players & timing:*
  1. Principal commits to information acquisition strategy & wage scheme.
  2. Agent chooses effort $a \geq 0$.
  3. Information acquisition strategy is implemented & payoffs are realized.
- *Information acquisition:* Principal observes the process

$$dX_t = a\,dt + dB_t \text{ , where } X_0 = 0\,,$$

  at cost 1 p.u of t, and chooses a *stopping time* $\tau(\omega)$.
- *Wage scheme:* $W(\omega_\tau) \geq \underline{w}$ (*the agent is cash constrained*)
- *Payoffs:*
  - *Agent:* $u(W) - c(a)$
  - *Principal:* $W + \tau$

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Principal's Problem and a 3-Step Reformulation

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- *Objective:* Motivate agent to choose *some* effort $a^* > 0$ at min. cost.

$$\inf_{W,\tau} \mathbb{E}_{a^*}\left[W\left(\omega_\tau\right)+\tau\right]$$

$$\text{s.t.} \quad a^* \in \arg\max_a \left\{\mathbb{E}_a\left[u\left(W\left(\omega_\tau\right)\right)\right]-c(a)\right\}$$

$$W\left(\omega_\tau\right) \geq \underline{w}$$

1. Replace (IC) with its first-order condition. Then will show that in an optimal contract, wages depend *only* on the *score* $s_\tau = \omega_\tau - a^*\tau$.
2. Express choice of stopping time as an information design problem. (Principal chooses zero-mean distribution s.t $s_\tau \sim F$ @ cost= $\mathbb{E}_F[s^2]$)
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Recall the Textbook Principal-Agent Model

- *Textbook model:* Effort generates a signal $x \sim G(\cdot|a)$. Principal solves

$$\inf_{w(\cdot)\geq \underline{w}} \int w(x) g(x|a^*) dx$$
$$\text{s.t. } \; a^* \in \arg\max_a \int u(w(x)) g(x|a) dx - c(a)$$

- *Standard approach:* Replace IC constraint with a *local* IC constraint,

$$\int u(w(x)) g_a(x|a^*) dx \geq c'(a^*),$$

and solve the Lagrangian

$$\inf_{w(\cdot)\geq \underline{w}} \int \left[w(x) - \lambda u(w(x)) \frac{g_a(x|a^*)}{g(x|a^*)} \right] g(x|a^*) dx + \lambda c'(a^*).$$

- An optimal contract depends *only* on the *score*, $g_a(x|a^*)/g(x|a^*)$.

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$$\inf_{W,\tau} \mathbb{E}_{a^*}\left[W(\omega_\tau) + \tau\right]$$
$$\text{s.t.} \quad a^* \in \arg\max_a \left\{\mathbb{E}_a\left[u\left(W(\omega_\tau)\right)\right] - c(a)\right\}$$
$$W(\omega_\tau) \geq \underline{w}$$

- Fix $\tau \equiv t$ & let the contract condition wages *only* on X_τ.
- This is a special case of the textbook model where $X_\tau \sim N(a^*\tau, \tau)$ and

$$g(X_\tau|a) = \frac{1}{\sqrt{2\pi\tau}} e^{-(X_\tau - a\tau)^2/2\tau}.$$

- Therefore, the *score*

$$\frac{g_a(X_\tau|a^*)}{g(X_\tau|a^*)} = X_\tau - a^*\tau$$

  is a sufficient statistic for the optimal wage scheme.
- **Question:** Can principal benefit from information about path of X?

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Girsanov's Theorem

- Incentive compatibility constraint:

$$a^* \in \arg\max_a \{\mathbb{E}_a [u(W(\omega_\tau))] - c(a)\}$$

- Using Girsanov's Theorem, agent's expected utility can be written as

$$\mathbb{E}_a [u(W(\omega_\tau))] = \mathbb{E}_{a^*} \left[u(W(\omega_\tau))\, e^{(a-a^*)B_\tau - \frac{1}{2}(a-a^*)^2 \tau} \right]$$

- Differentiating wrt a and evaluating at $a = a^*$ yields relaxed IC constr.

$$\mathbb{E}_{a^*} [u(W(\omega_\tau)) \underbrace{B_\tau}_{'score'\ s_\tau = X_\tau - a^* \tau}] \geq c'(a^*) \qquad \text{(IC-FOC)}$$

Lemma 1.

- Consider the relaxed problem where (IC) is replaced by (IC-FOC).
- In an optimal contract, the wage depends *only* on the score s_τ.

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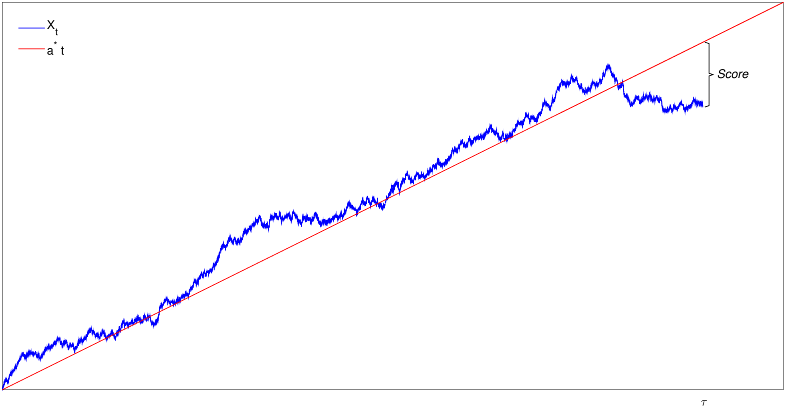
The Score

1. Recall that $dX_t = adt + dB_t$
2. Notice that in equilibrium, $a = a^*$, and hence, $ds_t = dB_t$



Information Design

- Each stopping time generates a *zero-mean* distribution over scores.*

Lemma 2.

- Consider a stopping time τ such that $\mathbb{E}_{a^*}[\tau] < \infty$.
- Then $s_\tau \sim F_\tau$, where

$$F_\tau \in \mathcal{F} = \left\{ F \in \Delta(\mathbb{R}) \, : \, \mathbb{E}_F[s] = 0, \, \mathbb{E}_F[s^2] < \infty \right\}$$

- The reverse is also true.

Lemma 3. (Root, 1969 and Rost, 1976)

i. For any $F \in \mathcal{F}$, there exists a τ such that $s_\tau \sim F$ and $\mathbb{E}_{a^*}[\tau] = \mathbb{E}_F[s^2]$.

ii. Any τ' such that $s_{\tau'} \sim F$ satisfies $\mathbb{E}_{a^*}[\tau'] \geq \mathbb{E}_{a^*}[\tau]$.

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Problem Reformulation

- We can reformulate the principal problem as

$$\inf_{\widetilde{W}(\cdot)\,,\,F\in\mathcal{F}} \int \left[\widetilde{W}(s) + s^2\right] dF(s)$$

$$\text{s.t.} \int su(\widetilde{W}(s))dF(s) \geq c'(a^*) \tag{IC}$$

$$\widetilde{W}(s) \geq \underline{w} \text{ for all } s \tag{LL}$$

- We will solve this problem in two stages:
  1. Characterize optimal wage scheme for *given* F. Denote objective $\Pi(F)$.
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Optimal Wage Scheme for any given $F \in \mathcal{F}$

- Write the Lagrangian:

$$L(\lambda, F) = \inf_{\widetilde{W}(\cdot) \geq \underline{w}} \int \left[\widetilde{W}(s) - \lambda s u(\widetilde{W}(s)) + s^2 + \lambda c'(a^*) \right] dF(s)$$

- Define, for every s, the wage scheme

$$w(\lambda, s) = \begin{cases} \underline{w} & \text{if } s \leq s_*(\lambda) \\ u'^{-1}\left(1/\lambda s\right) & \text{if } s > s_*(\lambda), \end{cases}$$

where $s_*(\lambda) = 1/\lambda u'(\underline{w})$. (This minimizes the term in brackets $\forall s$.)

Lemma 4.

i. Strong duality holds; *i.e.*, $\sup_{\lambda \geq 0} L(\lambda, F) = \Pi(F)$.

ii. An optimal wage scheme exists iff $\exists \widehat{\lambda} > 0$ such that $L(\widehat{\lambda}, F) = \Pi(F)$. In this case, (IC) binds and $\{w(\widehat{\lambda}, s\}$ is uniquely optimal.

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i. Strong duality holds; *i.e.*, $\sup_{\lambda \geq 0} L(\lambda, F) = \Pi(F)$.

ii. An optimal wage scheme exists iff $\exists \widehat{\lambda} > 0$ such that $L(\widehat{\lambda}, F) = \Pi(F)$. In this case, (IC) binds and $\left\{ w(\widehat{\lambda}, s \right\}$ is uniquely optimal.

Minimax Problem

- Strong duality ⇒ we can write the principal's problem as

$$\inf_F \sup_\lambda L(\lambda, F)$$

- Unfortunately, we cannot solve this problem... ☹
- However, we can solve

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Solution Approach

- Consider the following (auxiliary) zero-sum game:
  - Principal chooses $F \in \mathcal{F}$ to minimize $L(\lambda, F)$
  - *Nature* chooses $\lambda \geq 0$ to maximize $L(\lambda, F)$

Lemma 5. (Von Neumann, 1928)

If $\{\lambda^*, F^*\}$ is an equilibrium in the zero-sum game, then

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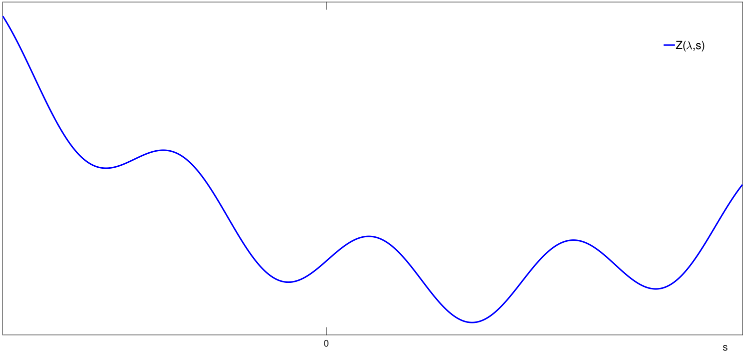
Two-Point Distribution

- The principal best-responds to any $\lambda \geq 0$ by solving

$$\inf_{F \in \mathcal{F}} L(\lambda, F) = \inf_{F \in \mathcal{F}} \mathbb{E}_F \underbrace{\left[w(\lambda, s) - \lambda s u(w(\lambda, s)) + s^2 + \lambda c'(a^*) \right]}_{\triangleq Z(\lambda, s)}$$



- If a BR exists, it is either a *two-point*, or a *degenerate distribution*.*

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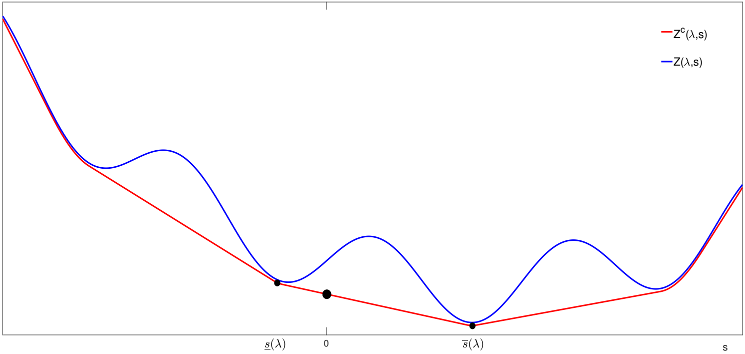

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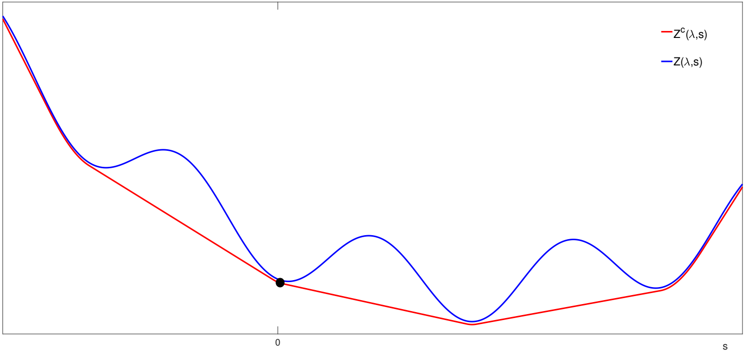

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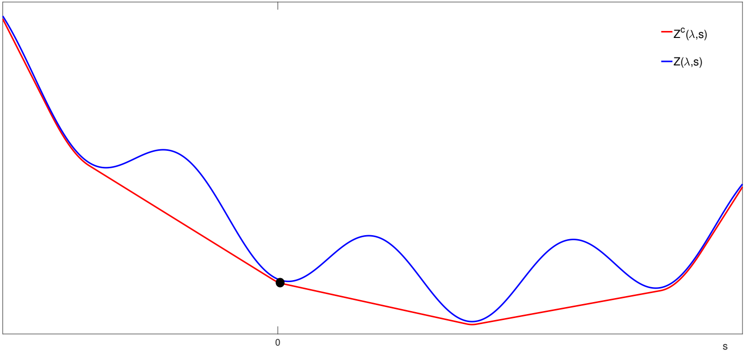

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Main Result

Theorem 1.

- Assume $\rho(z) = u\left(u'^{-1}(1/z)\right)$ is strictly concave and $\lim_{z\to\infty} \rho'(z) = 0$.
- There exists a unique equilibrium $\{\lambda^*, F^*\}$ in the zero-sum game, in which $\text{supp}\{F^*\} = \{\underline{s}, \overline{s}\}$ for some $\underline{s} < 0 < \overline{s}$.

- *Implication.*— There is a unique contract $\{\tau^*, W^*\}$ which solves the original problem. In this contract, the stopping rule

$$\tau^* = \min\left\{t > 0 : X_t = a^* t + \underline{s} \quad \text{or} \quad X_t = a^* t + \overline{s}\right\},$$

and the wage scheme

$$W^*(\omega_{\tau^*}) = \begin{cases} \underline{w} & \text{if } \omega_{\tau^*} = a^* t + \underline{s} \\ w(\lambda^*, \overline{s}) & \text{if } \omega_{\tau^*} = a^* t + \overline{s} \end{cases}$$

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- Conditions are satisfied by many common utility functions; *e.g.*,
  - *CRRA*: $u(w) = w^{1-\gamma}$, where $\gamma > 1/2$ (coefficient of RRA)
  - *CARA*: $u(w) = 1 - e^{\alpha w}$
  - *Logarithmic*: $u(w) = \log(\alpha w + \beta)$
  - *HARA*: $u(w) = \frac{\gamma}{1-\gamma}\left(\frac{\alpha w}{\gamma} + \beta\right)^{1-\gamma}$, where $\gamma > 1/2$

Interpretation of Two-point Distribution

- Given $\underline{s} < 0 < \overline{s}$, the principal uses the stopping rule

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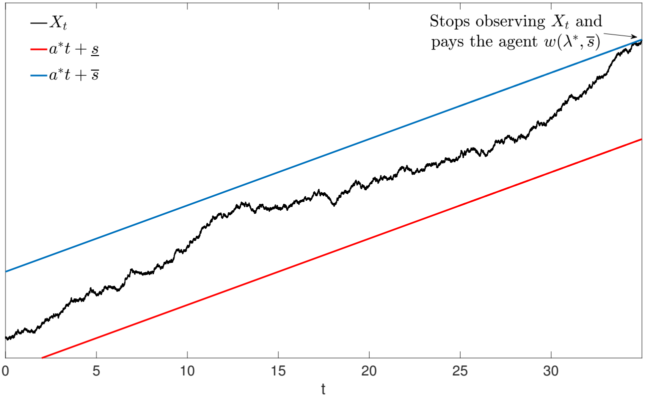

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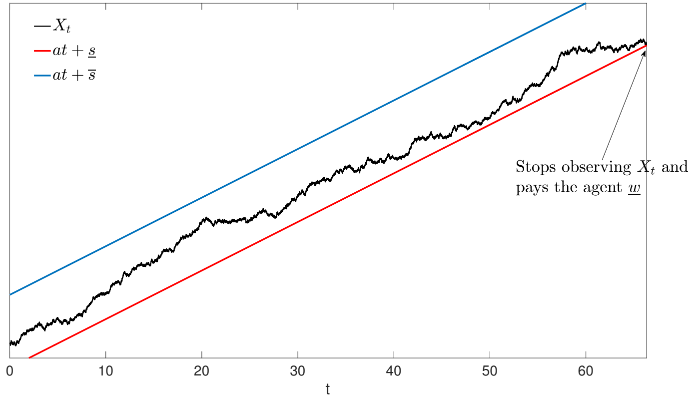

Sketch of the Proof 1/3: Nature's Best Response

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- Nature best-responds to $F \in \mathcal{F}$ by solving $\sup_{\lambda \geq 0} \{\mathbb{E}_F[Z(\lambda, s)\}$.
- This is equivalent to choosing $\lambda \geq 0$ such that (IC) binds:

$$\int su(w(\lambda, s))\, dF(s) = c'(a^*).$$

- If (IC) binds for some $\lambda < \infty$, then this is the unique best response.
- Otherwise, nature *does not* have a best response.

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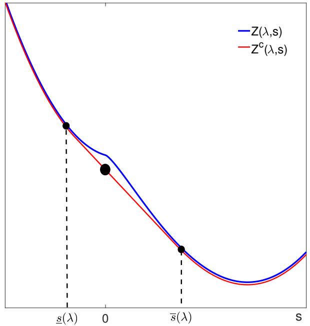
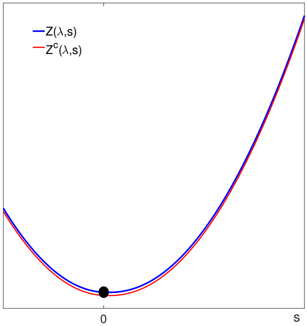
Sketch of the Proof 2/3: Principal's Best Response

- Assumptions that ρ is strictly concave and $\lim_{z\to\infty}\rho'(z) = 0$

  $\Rightarrow Z(\lambda,\cdot)$ is *convex-concave-convex* if $\lambda > \lambda_c$, and convex otherwise.





1. Principal's best-response, $F_\lambda = \arg\sup_{F\in\mathcal{F}}\{\mathbb{E}_F[Z(\lambda,s)]\}$, is *unique*.
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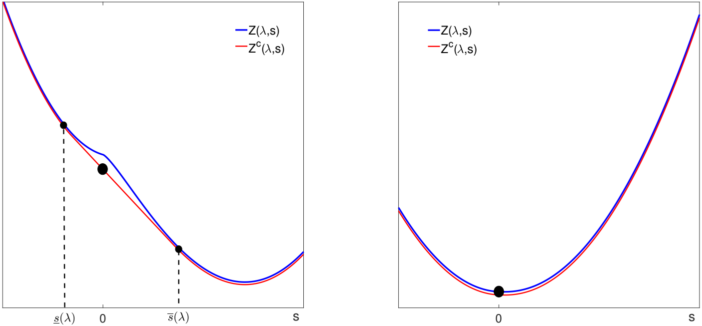

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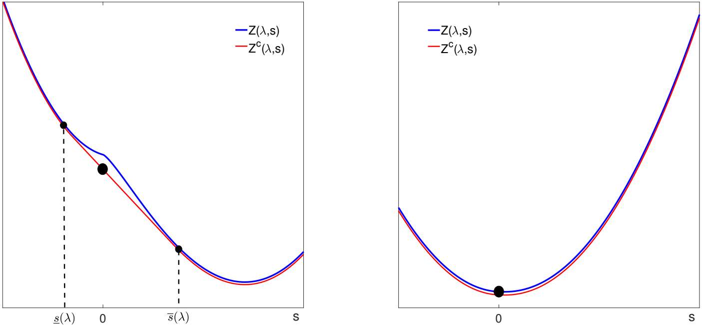

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- Recall λ is a best response to F if & only if IC binds at $\{\lambda, F\}$; *i.e.*,

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and F_λ denotes the principal's best response to λ.

Lemmas 10-11: There exists a λ^* such that

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Attaining Efficiency

Construction

- *First-best:* Principal pays $\underline{w}$ and chooses $F(s) = \mathbb{I}_{s \geq 0}$ at cost $= \underline{w}$.

Theorem 2.

- Suppose that there exists some $\zeta > 1$ such that

$$\lim_{w \to \infty} \frac{[u'(w)]^3}{u''(w)} [u(w)]^{-\frac{\zeta - 1}{\zeta}} = -\infty .$$

- For every $\epsilon > 0$, there exists a single-bonus wage scheme, and a two-point distribution satisfying (IC) and (LL) such that $\Pi^* \leq \underline{w} + \epsilon$.

- Condition satisfied if, for example, $u(w) = w^{1-\gamma}$ and $\gamma < 1/2$ (*CRRA*)

- Optimal contract pays $\underline{w}$, plus a *large* bonus with *small* probability. *i.e.*, inverse of a *Mirrlees shoot-the-agent* contract.

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- Fix any two-point distribution $F \in \mathcal{F}$ and wage scheme $\{\widetilde{W}(s)\}$.
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- Noting that $ds_t = (a - a^*)dt + dB_t$, we have

$$p(a) := \Pr\{s = \overline{s} | a\} = \frac{e^{-2(a-a^*)\overline{s}} - 1}{e^{-2(a-a^*)\underline{s}} - e^{-2(a-a^*)\overline{s}}}$$

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Validation of the First-Order Approach (Cont'd)

- Thus, the global IC constraint can be written as:

$$a^* \in \arg\max_{a \geq 0} \left\{ u\left(\widetilde{W}(\bar{s})\right) + p(a)\frac{c'(a^*)}{p'(a^*)} - c(a) \right\}.$$

- FOA is valid if maximand is single-peaked at a^*.

Proposition:

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Validation of the First-Order Approach (Cont'd)

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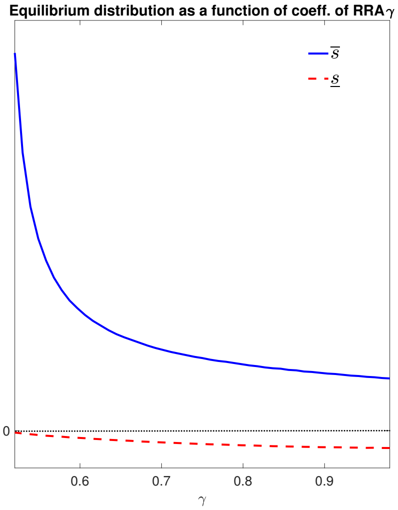
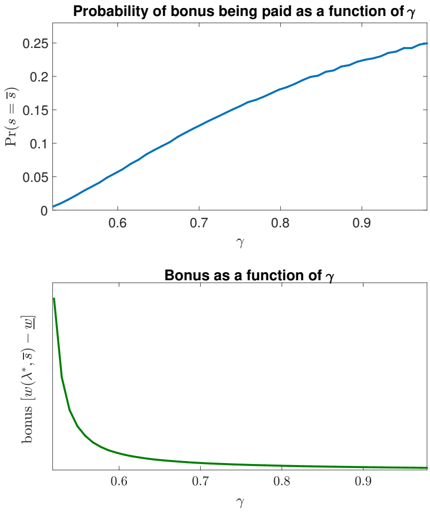
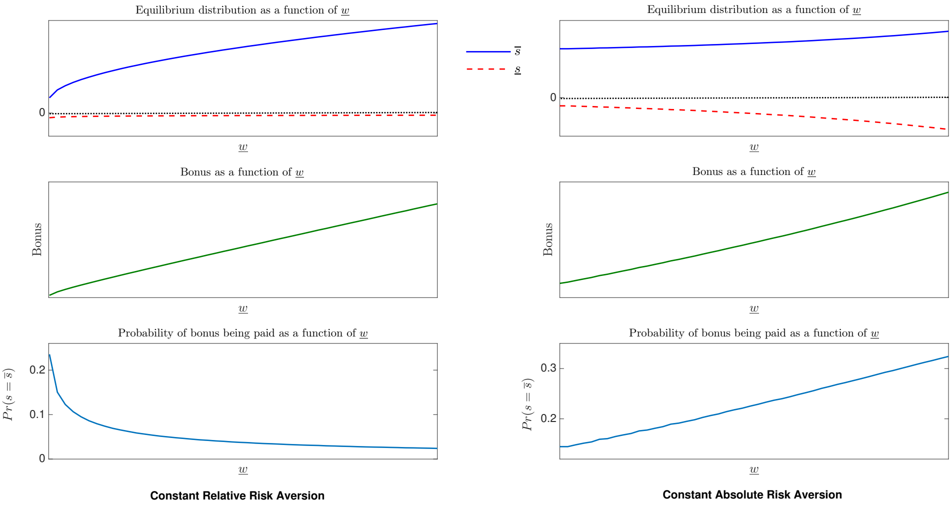
Comparative Statics: Varying the (constant) coeff. of RRA





Comparative Statics: Varying the minimum wage



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- Results hold if principal must satisfy (IR) constraint or effort is binary.
- Main theorem holds also if the principal can choose $F \in \mathcal{F}$ at cost $\mathbb{E}_F[\varphi(s)]$ for some convex function such that $\varphi'''(s) \geq 0$ for all $s > 0$.
- What if signals are discrete; *i.e.*, each signal $X_i \sim G(\cdot|a)$?
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- Flexible framework for analyzing design of performance measures.
- Under certain conditions, optimal contract pays 2 wage levels.
  - *Ideal* performance measure is binary — model highlights trade-off.
  - Rationale for commonly observed *single-bonus contracts.*
- *Next steps:*
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  - How to think about performance measure design more broadly?
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Attaining Efficiency: Construction

Return

- Pick a $\zeta > 1$ such that

$$\lim_{w\to\infty} \frac{[u'(w)]^3}{u''(w)} [u(w)]^{-\frac{\zeta-1}{\zeta}} = -\infty \qquad (*)$$

- Define the sequence of two-point distributions & wage schemes

$$F_n(s) = \begin{cases} 0 & \text{if } s < -n^{-\zeta} \\ \frac{n}{n+n^{-\zeta}} & \text{if } s \in [-n^{-\zeta}, n) \\ 1 & \text{if } s \geq n \end{cases} \quad \text{and} \quad w_n(s) = \begin{cases} \underline{w} & \text{if } s = -n^{-\zeta} \\ \overline{w}_n & \text{if } s = n, \end{cases}$$

where $\overline{w}_n$ is chosen such that (IC) binds.

- As $n \to \infty$, the principal's expected cost

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