The Swing Voter's Curse*

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October 1995

Forthcoming *American Economic Review*

Abstract

We analyze two-candidate elections in which some voters are uncertain about the realization of a state variable that affects the utility of all voters.. We demonstrate the existence of a *swing voter's curse*: less informed indifferent voters strictly prefer to abstain rather than vote for either candidate even when voting is costless. The swing voter's curse leads to the equilibrium result that a substantial fraction of the electorate will abstain even though all abstainers strictly prefer voting for one candidate over voting for another.

In the 1994 State of Illinois elections there were 6,119,001 registered voters. Among those registered to vote only 3,106,566 voted in the gubernatorial race and only 2,144,200 voted on a proposed amendment to the state constitution.[1] There is nothing exceptional about the level of participation in the 1994 Illinois elections. As in most large elections in the United States, a substantial fraction of the registered electorate abstained from voting at all and of those who did vote a substantial fraction *rolled off*, i.e., did not vote on every item listed on the ballot.[2]

While abstention and roll-off are ubiquitous features of elections together they pose a challenge to positive political theory. One obvious explanation of abstention is costs to vote. However if voting is costly, since it is extremely unlikely that one person's vote changes the outcome, it is difficult to understand why so many people vote. Conversely, if voting is not costly, the problem is to explain why so many people abstain. This is "the paradox of not-voting".[3] The solution proposed by Anthony Downs (1957) and by William H. Riker and Peter C. Ordeshook (1968)[4] is that perhaps voting is costly for some citizens but not for others. This explanation for participation patterns runs into trouble however as an explanation for roll-off. Presumably most of the costs to vote are associated with getting to the polls. Roll-off occurs when voters who are already at the polls decide not to vote on a race or issue. One way that a cost theory of voting might explain roll-off is by ballot position. Voters get tired of voting and decline to vote on issues down the ballot. This explanation does not work for the example given above because in Illinois consititutional proposals appear *first* on the ballot.[5]

A useful theory of participation must explain not only abstention and roll-off but must also be consistent with the well known stylized fact that better educated and informed individuals are more likely to participate than the less well educated and informed.[6] In their seminal book, Who Votes, Raymond E. Wolfinger and Stephan J. Rosenstone (1980), using 1972 Bureau of Census data and controlling for a variety of demographic attributes including income, predict that every additional 4 years of schooling increases the liklihood of voting by between 4 and 13 percentage points (see table 2.4 page 26). We do not dispute the proposition that costs to vote influence participation. Our contribution here is to demonstrate that informational asymmetries may also influence both participation and vote choice independent of costs to vote and pivot probabilities. We show that less informed voters have an incentive to delegate their vote via abstention to more informed voters.

We use the insight underlying the "winner's curse"[7] in the theory of auctions to show that rational voters with private information may choose to abstain or even vote for a candidate that they consider inferior based on their private information alone. The paradigmatic example of the winner's curse is as follows. A group of bidders have private information about the value of an oil lease and each knows that other agents have private information as well.[8] If every bidder offers his expected evaluation determined from their private information the winning bidder has bid too much because, by virtue of winning, it follows that every other bidder's expected valuation is lower. Thus, the private information of the winning bidder is a biased estimate of the true value of the lease. The solution to the winner's curse is for every bidder to condition his offer not only on private information but also on what must be true about the world if his is the high bid and to bid less than they would if they were the only bidder.

There is an analog to the winner's curse in elections with asymmetric information: th*e swing voter's curse*. A swing voter is an agent whose vote determines the outcome of an election. Both in auctions and in elections an agent's action only matters in particular circumstances: when an agent is the high bidder in an auction or when an agent is a swing voter in an election. In either case, when some agents have private information that may be useful to an agent, the agent must condition his action not only on his information but also on what must be true about the world if the agent's action matters.

Consider the following example. There are two candidates, the status quo (candidate 0) and the alternative (candidate 1). Voters are uncertain about the cost of implementing the alternative. This cost is either high (state 0) or low (state 1). All voters prefer the status quo if the cost is high and the alternative if the cost is low. At least one of the voters is informed and knows the costs with certainty. However, voters do not know the exact number of informed voters in the electorate. All of the uninformed voters share a common knowledge prior that with .9 probability the cost is high and the status quo is the best candidate.

Suppose that all voters (informed and uninformed alike) vote only on the basis of their updated prior. All of the informed voters vote for the status quo if the cost is high and the alternative if the cost is low while all of the uninformed voters vote for the status quo in both states. The informed voters are behaving rationally while the uninformed are not. An uninformed voter is only pivotal if some voters have voted for the alternative. But this can only occur if the cost is low and the informed voters vote for the alternative. Therefore, an uninformed voter can affect the election outcome only if the cost is low. Consequently, an uninformed voter should vote for the alternative. On the other hand, it cannot be rational for all uninformed voters to vote for the alternative. In this case each uninformed voter would prefer to vote for the status quo. Thus it is not optimal for uninformed voters to vote only on the basis of their prior information.

In this example there is an easy solution for the uninformed voters: abstention. Abstention is an optimal strategy because it maximizes the probability that the informed voters decide the election. If all of the uninformed voters abstain it follows that there are only two conditions under which an uninformed voter might be pivotal: either there are no informed voters or there is exactly one informed voter. In our example we eliminated the first possibility because we assumed that there is always at least one informed voter. In the latter case the uninformed voter strictly prefers to abstain because the only way her vote effects the outcome is if she votes for the candidate not supported by the informed voter, i.e., the wrong candidate. Given the behavior by the other voters uninformed voters suffer the *swing voter's curse*: they are strictly better off abstaining than by voting for either candidate. This is true even though uninformed voters believe that the status quo is almost certainly the best candidate.

Our model formalizes and extends the above example to include voters with different preferences. We assume three kinds of voters: voters who prefer the status quo regardless of the state of the world (0-partisans), voters who prefer the alternative regardless of the state of the world (1-partisans) and independents. Independent voters sometimes prefer candidate 0 and sometimes prefer candidate 1 depending on the state of the world. All voters know the expected percentage of each type within the population but not the exact numbers. Finally, we assume that with positive probability any voter knows the true state of the world.

Asymmetric information fundamentally alters the calculus of voting. It may be rational for a voter in a two-candidate election to vote for the candidate he believes to be worse or to abstain even if voting is costless. Furthermore, our model predicts significant levels of abstention and participation. Our central results are as follows:

- If no agent uses a strictly dominated strategy then uninformed voters who are almost indifferent between voting for either of the two candidates suffer the *swing voter's curse* and are *strictly* better off by abstaining.
- For a wide range of parameters a significant fraction of the voters abstain in large elections.
- The asymptotic properties of the equilibria may be expressed in terms of the basic parameters of the model permitting a comparative statics analysis. Such an analysis demonstrates that an increase in the expected fraction of the electorate that is informed may lead to both a **lower** probability of being pivotal and **higher** participation.
- When voters behave strategically, large elections under private information almost always choose the same winner as would be chosen by a fully informed electorate.

This paper is in three sections. In the first section, we discuss the formal literature directly related to our model. In the second section we cover the model and results. The third section is a discussion of the results, their relationship to the empirical literature in American politics and some concluding remarks.

I. Related Literature

There is an extensive formal literature on participation and several recent surveys.[9] The effect of asymmetric information on the calculus of voting has not been analyzed in this literature. For example, Palfrey and Rosenthal (1985, p62) state that uncertainty over alternative outcomes "is of no consequence" in determining voting behavior; voters simply vote for the candidate associated with the most preferred expected outcome. We show that this is not the case if voters possess private information that might, if shared, cause other voters to change their preferences.

The model we present here is similar to the model found in Timothy J. Feddersen and Wolfgang Pesendorfer (1994). In that model we demonstrate that elections fully aggregate private information for a broad class of environments. However, we do not consider abstention.

Our model is also similar in some respects to models developed by David Austen-Smith (1990) in a legislative setting and by Susanne Lohmann (1993a,b) in the context of participation in protest movements.[10] Austen-Smith showed that privately informed legislators may vote for an alternative he believes to be inferior even in a two-alternative election. Lohmann considers a model in which agents have private information about the state of the world and must decide to participate in a demonstration. A decision maker then observes the number of actions taken and determines the outcome. Our work extends Austen-Smith's insight by permitting abstention and differs from both Austen-Smith and Lohmann by considering the asymptotic properties of a model of elections with privately and asymmetrically informed voters.

Matsusaka (1992) develops a decision-theoretic informational approach to participation in which he argues that more informed voters get a higher expected benefit by voting for the candidate with the highest expected return than do less informed voters. His approach relies on the assumption that voting is costly. Voters in Matsusaka's model choose to acquire information at a cost and then choose if and for whom to vote. If voting is costless in Matsusaka's setting then all voters should vote. Our approach differs from Matsusaka's in that it is game-theoretic and uninformed voters may be strictly worse off by voting even if voting is costless.

II. Description of the Model

There are two states, state 0 and state 1, where $Z = \{0,1\}$ denotes the set of states. There are two candidates, candidate 0 and candidate 1. The set of candidates is $X = \{0,1\}$. There are three types of agents, where $T = \{0,1,i\}$ is the set of types. Type 0 and type 1 agents are partisans: irrespective of the state type 0 agents strictly prefer candidate 0 and type 1 agents strictly prefer candidate 1. Type *i* agents are independents: given a pair (*x*,*z*), $x \in X$ and $z \in Z$, the utility of a type *i* agent is

$$U(x,z) = \begin{cases} -1 & \text{if } x \neq z \\ 0 & \text{if } x = z \end{cases}. \tag{1}$$

Independent agents prefer candidate 0 in state 0 and candidate 1 in state 1.

At the beginning of the game nature chooses a state $z \in Z$. State 0 is chosen with probability α and state 1 is chosen with probability 1-α. Without loss of generality we assume that $\alpha \leq 1/2$. The parameter α is common knowledge and hence all agents believe that state 1 is at least as likely as state 0. Nature also chooses a set of agents by taking N+1 independent draws. We assume that there is uncertainty both about the total number of agents and the number of agents of each type. In each draw, nature selects an agent with probability $(1-p_\phi)$. If an agent is selected, then with probability $p_i/(1-p_\phi)$ she is of type *i*, with probability $p_0/(1-p_\phi)$ she is type 0, and with probability $p_1/(1-p_\phi)$ she is type 1. The probabilities $p = (p_i, p_0, p_1, p_\phi)$ are common knowledge.[11]

After the state and the set of agents have been chosen, every agent learns her type and receives a message $m \in M$, where $M = \{0, \alpha, 1\}$. Both her type and the message are private information. If an agent receives message *m* then the agent knows that the state is 0 with probability *m*. All agents who receive a message $m \in \{0,1\}$ are informed, i.e., they know the state with probability 1. Note that all informed agents receive the same message. The probability that an agent is informed is q. Agents who receive the message α learn nothing about the state beyond the common knowledge prior. We refer to these agents as uninformed.

III. Strategies and Equilibrium

Every agent chooses an action $s \in \{\phi,0,1\}$ where ϕ indicates abstention and 0 or 1 indicates her vote for candidate 0 or 1 respectively. The candidate that receives a majority of the votes cast will be elected. Whenever there is a tie, we assume that each candidate is chosen with equal probability.

A pure strategy for an agent is a map $s{:}T \times M \rightarrow \{\phi,0,1\}$. A mixed strategy is denoted by $\tau : T \times M \rightarrow [0,1]^3$, where τ_s is the probability of taking action *s*.

We analyze the symmetric Nash equilibria of this game, i.e., we assume that agents who are of the same type and receive the same message choose the same strategy. Note that the number of agents is uncertain and ranges from 0 to $N+1$. Therefore, there is a strictly positive probability that any agent is pivotal. It follows that all agents except the uninformed independent agents have a *strictly* dominant strategy.[12] Type-1 (type 0) agents always vote for candidate 1 (candidate 0) and all informed independent agents vote according to the signal they receive, that is if $m \in \{0,1\}$ then $s(i,m) = m$.

In equilibrium agents never use a strictly dominated strategy. Therefore we can simplify our notation and specify only the behavior of the uninformed independent agents (UIAs). We denote a mixed strategy profile by $\tau = (\tau_0, \tau_1, \tau_\phi) \in [0,1]^3$. Under profile τ all UIAs play according to the mixed strategy τ and all other agents choose their dominant strategies.

IV. Analysis

In order to facilitate the exposition of our results we introduce the following notation. For a given profile τ, define $\sigma_{zx}(\tau)$ to be the probability a random draw by nature results in a vote for candidate *x* if the state is *z*. The only agents who vote for *x* are *x*-partisans and independents. An informed independent agent votes for *x* only if $z=x$ while an UIA votes for *x* with probability τ_x

in both states. Therefore the probability that a draw by nature results in a vote for candidate *x* in state z is defined as follows:

(2) $$\sigma_{zx}(\tau) \equiv \begin{cases} p_x + p_i(1-q)\tau_x & \text{if } z \neq x \\ p_x + p_i(1-q)\tau_x + p_i q & \text{if } z = x \end{cases}$$

From the perspective of an UIA the probability that a draw by nature will result in a vote for candidate *x* in state *x*, $\sigma_{xx}(\tau)$, is the probability of a *correct* vote while $\sigma_{yx}(\tau)$, $y \neq x$, is the probability of a *mistaken* vote. Note that the probability of a draw resulting in a correct vote is always greater than the probability of a draw resulting in a mistaken vote.

Define $\sigma_{z\phi}(\tau)$ to be the probability that a random draw by nature does not result in a vote for either candidate in state *z*. This can happen either if no agent is drawn or if the agent who is drawn abstains. The only agents who might abstain are UIAs. Since both the probability that nature draws an agent and the strategy of an UIA do not depend on the state it follows that $\sigma_{z\phi}(\tau)$ is independent of the state. Thus

(3) $$\sigma_{0\phi}(\tau) = \sigma_{1\phi}(\tau) = \sigma_{\phi}(\tau) = p_i(1-q)\tau_{\phi} + p_{\phi}$$

In order to determine the best responses of UIAs we must specify the conditions in which an UIA's choice changes the outcome. There are three situations in which an agent may be pivotal:

- an equal number of other agents have voted for each candidate,
- candidate 1 receives one more vote than candidate 0,
- candidate 0 receives one more vote than candidate 1.

For any agent the probabilities of each of these events given state *z*, *N* other possible agents and strategy profile τ are as follow. The probability an equal number of other agents have voted for each candidate, i.e., a tie is:

(4) $$\pi_t(z,\tau) = \sum_{j=0}^{\frac{N}{2}} \frac{N!}{j!\,j!(N-2j)!} \sigma_{\phi}(\tau)^{N-2j} (\sigma_{z0}(\tau)\sigma_{z1}(\tau))^{j}\ .$$

The probability that candidate *x* receives exactly one less vote than candidate y (the probability that candidate *x* is down by 1 vote) is:

(5) $$\pi_x(z,\tau)=\sum_{j=0}^{\frac{N}{2}-1}\frac{N!}{(j+1)!\,j!(N-2j-1)!}\sigma_\phi(\tau)^{N-2j-1}\sigma_{zy}(\tau)(\sigma_{zx}(\tau)\sigma_{zy}(\tau))^j\ .$$

By $Eu(x,\tau)$ we denote the expected payoff to an UIA of taking action *x* when the strategy profile used by all other agents is τ. To determine a best response by an UIA it is only necessary to consider the expected utility differences between every pair of strategies. The expected utility differentials are given below as a function of *N* and τ:

(6) $$Eu(1,\tau)-Eu(\phi,\tau)=\frac{1}{2}\Big[(1-\alpha)[\pi_t(1,\tau)+\pi_1(1,\tau)]-\alpha[\pi_t(0,\tau)+\pi_1(0,\tau)]\Big]$$

(7) $$Eu(0,\tau)-Eu(\phi,\tau)=\frac{1}{2}\Big[\alpha[\pi_t(0,\tau)+\pi_0(0,\tau^N)]-(1-\alpha)[\pi_t(1,\tau)+\pi_0(1,\tau)]\Big]$$

(8) $$Eu(1,\tau)-Eu(0,\tau)=(1-\alpha)[\pi_t(1,\tau)+\tfrac{1}{2}(\pi_1(1,\tau)+\pi_0(1,\tau))]-\alpha[\pi_t(0,\tau)+\tfrac{1}{2}(\pi_1(0,\tau)+\pi_0(0,\tau))]$$

Proposition 1 states that an UIA *strictly* prefers to abstain whenever he is indifferent between voting for candidate 1 and voting for candidate 0 and no agent uses a *strictly* dominated strategy. This is the *swing voter's curse*. It is often thought that strategic voting requires complicated mental gymnastics. The following proposition provides advice that is easy for the uninformed indifferent voter to swallow: *abstain*. All proofs are in the appendix.

Proposition 1 *Let* $p_\phi > 0$ *,* $q > 0$ *,* $N \geq 2$ *and N even. For any symmetric strategy profile* τ *in which no agent plays a strictly dominated strategy,* $Eu(1,\tau) = Eu(0,\tau)$ *implies* $Eu(1,\tau) < Eu(\phi,\tau)$.

To provide an intuition for Proposition 1 recall that the *correct* candidate for the UIA is the candidate favored by the informed independent voters. If an UIA is indifferent between voting for candidate 0 and candidate 1 from equations (6) and (7) it follows that the utility difference between voting for candidate 1 and abstaining is

(9) $$Eu(1,\tau) - Eu(\phi,\tau) = \tfrac{1}{4}\left[(1-\alpha)[\pi_1(1,\tau) - \pi_0(1,\tau)] + \alpha[\pi_0(0,\tau) - \pi_1(0,\tau)]\right]$$

The rhs of equation (9) is the weighted sum of the differences between the probability of creating a tie by voting for the correct candidate and the probability of creating a tie by voting for the incorrect candidate in each state. Each of these differences is a negative number since in each state it is more likely that the incorrect candidate is behind by one vote than that the correct candidate is behind by one vote. This is because all informed independents vote for the correct candidate in each state. [13]

Proposition 1 demonstrates that there are a wide variety of settings in which UIAs have an incentive to abstain. In particular it implies that there can be no mixed strategy equilibrium in which UIAs mix between voting for candidate 0 and voting for candidate 1. The only possible equilibria in our model are either pure strategy equilibria or mixed strategy equilibria in which UIAs mix between abstention and voting for a single candidate.

V. Voting and Participation in Large Elections

In the previous section we demonstrated the existence of the swing voter's curse. We now use the result to provide a theory of participation in large elections. We define a sequence of games with *N*+1 potential voters indexed by *N* and a sequence of strategy profiles for each game as $\{\tau^N\}_{N=0}^{\infty}$. First we show that avoiding the swing voter's curse can lead to large scale abstention by the UIAs in large elections. This abstention is unrelated to the fact that the probability of being pivotal is very small in large elections (recall that voting is costless). Next, we show that equilibrium voting behavior virtually guarantees that the winning candidate will be the same as the candidate that would win if voters had perfect information.

In order to prove our central results on voter behavior and information aggregation we require the following lemma. Suppose that the probability that a random draw by nature results in a vote for candidate 1 in state 0 is larger than the probability that a random draw results in a vote for candidate 0 in state 1. In other words, it is more likely that a random draw leads to a mistaken vote in state 0. *Lemma* 1 states that in this case all UIAs will prefer to vote for candidate 0 if the electorate is large. Conversely, if it is more likely that a random draw leads to a mistaken vote in state 1 then all UIAs will prefer to vote for candidate 1 if the electorate is large.

Lemma 1 *Suppose* $p_i q > 0$ *and* $0 < \alpha < 1$. *Consider a sequence of voting games and strategy profiles* $\{\tau^N\}_{N=0}^{\infty}$. *Then:*

A. *if there exists an ε>0 such that* $\sigma_{xy}(\tau^N) - \sigma_{yx}(\tau^N) > \varepsilon$ *for any* $N \geq 0$ *and* $x \neq y$ *then there exists an* $\overline{N}$ *such that for any N>* $\overline{N}$ $Eu(x,\tau^N) > Eu(\phi,\tau^N) > Eu(y,\tau^N)$;

B. *if for all* $N \geq 0$ *there are two actions s,s' with* $s \neq s'$ *such that* $Eu(s,\tau^N) = Eu(s',\tau^N)$ *then for any ε>0 there is an* $\overline{N}$ *such that for N>* $\overline{N}$ $|\sigma_{01}(\tau^N) - \sigma_{10}(\tau^N)| < \varepsilon$.

The intuition behind Lemma 1.A can be summarized as follows: if the probability a random draw results in a mistaken vote in state 1 is larger than the probability of a mistaken vote in state 0, i.e., $\sigma_{10}(\tau^N) - \sigma_{01}(\tau^N) > \varepsilon$, then the conditional probability that the world is in state 1 given the agent is pivotal goes to 1 as the size of the electorate, *N*, increases. This follows from the fact that an agent is only pivotal if enough agents make a mistake to compensate for the votes of the informed independent agents. If the probability of a mistake is higher in state 1 than state 0 then an UIA is much more likely to be pivotal in state 1 than in state 0 and he strictly prefers to vote for candidate 1 rather than abstain and would rather abstain than vote for candidate 0. Lemma 1.B follows as a corollary of part A.

UIAs do not know the state with certainty and therefore are unsure of the candidate that they prefer to win. On the other hand UIAs would always prefer that informed independent agents decide the election. The effect of equilibrium behavior of the UIAs is to maximize the probability that the informed independent agents determine the winner. UIAs vote to compensate for the partisans and having achieved that compensation they abstain.

Proposition 2 describes the case where the expected fraction of UIAs is too small to compensate for the partisan advantage enjoyed by candidates 0 and 1 respectively. For example, if the probability a draw results in a mistake is higher in state 1 than in state 0 independent of the strategy of UIA's all UIAs vote for candidate 1. Proposition 2 is an immediate consequence of Lemma 1.A

Proposition 2 *Suppose* $q > 0$, $p_i(1-q) < |p_0 - p_1|$ *and* $p_\phi > 0$. *Let* $\{\tau^N\}_{N=0}^{\infty}$ *be a sequence of equilibria.*

(i) *If* $p_i(1-q) < p_0 - p_1$ *then* $\lim_{N\to\infty} \tau_1^N = 1$, *i.e., all UIAs vote for candidate 1.*

(ii) *If* $p_i(1-q) < p_1 - p_0$ *then* $\lim_{N\to\infty} \tau_0^N = 1$, *i.e., all UIAs vote for candidate 0.*

In Proposition 3 the expected fraction of UIAs is large enough to fully offset the bias introduced by partisans. In this case there are no pure strategy equilibria. In this case UIAs mix between abstention and voting and exactly compensate for the differences in partisan support.

Proposition 3 *Suppose* $q > 0$, $p_i(1-q) \geq |p_0 - p_1|$ *and* $p_\phi > 0$. *Let* $\{\tau^N\}_{N=0}^{\infty}$ *be a sequence of equilibria.*

(i) *If* $p_i(1-q) \geq p_0 - p_1 > 0$ *then UIAs mix between voting for candidate 1 and abstaining*; $\lim \tau_1^N = \frac{p_0 - p_1}{p_i(1-q)}$ *and* $\lim \tau_\phi^N = 1 - \frac{p_0 - p_1}{p_i(1-q)}$.

(ii) *If* $p_i(1-q) \geq p_1 - p_0 > 0$ *then UIAs mix between voting for candidate 0 and abstaining*; $\lim \tau_0^N = \frac{p_1 - p_0}{p_i(1-q)}$ *and* $\lim \tau_\phi^N = 1 - \frac{p_1 - p_0}{p_i(1-q)}$.

(iii) *If* $p_0 - p_1 = 0$ *then UIAs abstain;* $\lim \tau_\phi^N = 1$.

Proposition 3 is an immediate consequence of Proposition 1 and Lemma 1. For simplicity, consider the case where $p_i(1-q) > |p_0 - p_1|$, i.e., where the expected fraction of UIA's is larger than the expected difference in partisan support. By Proposition 1 there are no equilibria in which UIAs mix between voting for each candidate. On the other hand if all UIA's vote for one of the two candidates, for example, candidate 1, then it is more likely to draw a mistaken vote in state 0 and hence by Lemma 1 UIA's have a strict preference to vote for candidate 0. Thus there cannot be a pure strategy equilibrium. But then it must be the case that UIAs mix between abstention and voting for one of the candidates so as to exactly compensate for the differences in partisan support. Only then is the probability of a mistaken vote equal in both states and hence voters can be indifferent between voting for one of the two candidates and abstaining.

Propositions 2 and 3 also demonstrate that equilibrium voting behavior is much different than the voting behavior predicted by standard voting models. In Proposition 2, even though there is no abstention, all of the UIA's may be voting for the candidate that on the basis of their prior information alone they believe is likely to be the incorrect candidate.

Finally it should be emphasized that the result that a positive fraction of the electorate abstains in equilibrium can be generalized to the case where independents have different preferences.[14] Similarly, the fact that informed independents are *perfectly* informed about the state simplifies the analysis but is not crucial for the results.[15]

VI. Information Aggregation

In Proposition 4 we show that the winning candidate is almost surely the same as the candidate that would win if the electorate were fully informed. We say that the election mechanism *fully aggregates information* when the electoral outcome is the same under private information as it would be under perfect information. Consider the case in which the independent agents may be expected to decide the election, i.e., the case where $|p_0 - p_1| < p_i$. In this case the election fully aggregates information, if the right choice from the point of view of the independent agents is made, i.e., if candidate 0 is chosen in state 0 and candidate 1 is chosen in state 1. The following result shows that the probability that an election fully aggregates information goes to one as the size of the electorate increases.

Proposition 4 *Suppose* $p_\phi > 0$ *and* $q > 0$ *and* $p_i \neq |p_0 - p_1|$*. Then for every* ε *there exists an* $\overline{N}$ *such that for N>* $\overline{N}$ *the probability that in equilibrium the election fully aggregates information is greater than* $1-\varepsilon$.

Proposition 4 relies on the fact that as the size of the electorate gets large the expected vote share for each candidate converges to the actual vote share. If the expected fraction of UIAs is larger than the difference between the fractions of partisans (e.g., $p_i > |p_0 - p_1|$) then given the characteristics of the equilibrium strategies described in Propositions 2 and 3 the expected vote share for the correct candidate from the perspective of the independents is always larger than the expected vote share for the incorrect candidate. It follows from the law of large numbers that the probability the correct candidate receives the most votes goes to one as *N* gets large. On the other hand if $p_i < |p_0 - p_1|$ one set of partisans is expected to constitute a majority and, since all of the partisans vote for their favored candidate, by the law of large numbers the probability that candidate wins goes to one [16].

The information aggregation result given here should be compared to the result that would occur if all agents voted naively[17] on the basis of their private information. For example, in the completely symmetric case (v) for small q and because $\alpha < 1/2$ the expected vote share for candidate 1 is always greater than the expected vote share for candidate 0. In this case, candidate 1 would always win and α (if N is large) would be the probability the election results in an outcome not preferred by a fully informed majority. Thus strategic behavior improves the information aggregation properties of the electoral mechanism.

VII Empirical Predictions

A. Comparative Statics

The expected fraction of agents who abstain is equal to the probability that a randomly chosen agent is an uninformed independent times the probability that an uninformed independent abstains. Thus, in a large electorate the fraction of agents who abstain is well approximated by:

(10) $$\tau_\phi p_i (1-q)$$

Consider the case where $p_i(1-q) > |p_0 - p_1|$, i.e., the expected difference in partisan support is smaller than the expected fraction of UIA's . For a large electorate Proposition 3 implies that the fraction of UIA's who abstain is well approximated by the equation:

(11) $$\tau_\phi = 1 - \frac{|p_0 - p_1|}{p_i(1-q)}.$$

Thus the fraction of voters who abstain may be written in terms of the model parameters:

(12) $$p_i(1-q) - |p_0 - p_1|.$$

Holding constant the difference in the expected fractions of type-0 and type-1 partisans, abstention is increasing in the percentage of independents (p_i). The increased abstention is due to two factors. First, as the percentage of independents increases it follows that there is an increase in the percentage of UIAs who abstain (τ_ϕ). Second, the percentage of uninformed independents also increases. Similarly, abstention decreases as the expected percentage of informed voters (q) increases.

UIAs play a mixed strategy of abstention and voting for the candidate with the lower expected fraction of partisan support when $p_i(1-q) > |p_0 - p_1|$.[18] Thus, an increase in the expected fraction of informed voters results in an increased probability that the uninformed independents will vote for the candidate with the lower partisan support.

Without changing either the probability of being informed or the probability that a voter is independent, an increase in the expected difference of partisan support ($|p_0 - p_1|$) results in a decrease in abstention.

We can also make predictions about changes in the expected margin of victory (MV). The margin of victory is the percentage difference in votes between the winning and the losing candidate. If $p_i(1-q) > |p_0 - p_1|$ then in large elections MV is the percentage of informed independents ($p_i q$) divided by the expected fraction of active agents who vote. Therefore, MV is well approximated by the equation:

(13) $$MV = \frac{p_i\, q}{1 - p_i\,(1-q) + |p_0 - p_1|}.$$

Thus, our model predicts that MV increases with an increase in the percentage of independents and with an increase in the probability of being informed.[19]

In contrast to the predictions of standard models of participation (Riker and Ordeshook 1968) in our model there is no causal relationship between pivot probabilities and abstention. Changes in pivot probabilities due to dramatic changes in population size do not change the patterns of abstention and voting in our model. When we combine the comparative static results on abstention with those on the expected margin of victory we see that an increase in the expected fraction of informed voters, q, will result in both *higher margins of victory* and *lower levels of abstention.* Thus, abstention may actually increase as the probability of being pivotal increases.[20]

On the other hand, an increase in the percentage of independents gives the same comparative statics as the standard model: an increase in abstention and a decrease in pivot probabilities.

One parameter in our model that does not play a critical role in either the decision to participate or vote choice is the common knowledge prior belief (α) concerning the state of the world--and therefore which candidate is ex ante believed to be the best candidate by the UIAs. This can be seen by examining the strategy profiles specified in Propositions 2 and 3 and noting that the parameter α does not appear. If the population is sufficiently large, a small change in α does not cause large changes in the voting strategies. However, for fixed population sizes and α sufficiently close to zero a small change in α may have significant effects on the voting strategies. The intuition here is that if the common knowledge prior is strong enough then the information gained by being pivotal will be unable to overcome it.

B. Example

We provide the following example not as a test of our model but for the purposes of illustrating the inner workings of our model under fairly reasonable assumptions. Let $p_0 = 0.36$ and $p_1 = 0.28$ and $p_i = 0.36$.[21] If the margin of victory is 5% then from (13) we must have

(14)
$$MV = \frac{p_i\, q}{1 - p_i\,(1-q) + |p_0 - p_1|} = \frac{.36q}{1 - .36(1-q) + .08} = .05$$

This implies that $q = .10$, i.e., that 10% of the independent voters are informed. Now the fraction of voters expected to abstain follows from (12) and is

(15)
$$p_i\,(1-q) - |p_0 - p_1| = .36 \cdot .9 - .08 \approx .24$$

In other words, for this choice of parameters 24% of the voters abstain. Since all of the partisans vote this implies that the fraction of independents who abstain is approximately $0.24 / 0.36 = 2/3$ and hence turnout among independents is approximately 33%.[22]

VIII. Conclusion

Private information and common values together can radically alter the calculus of voting due to the swing voter's curse. Under private information and common values less informed voters may have an incentive to abstain even though voting is costless and they have a strict ex ante preference between the two candidates; those voters who do vote may not vote for their ex ante preferred candidate; finally, strategic voting and abstention may lead to an informationally superior election outcome.

Our informational explanation for turnout constrasts with earlier models that focus on the costs and benefits of voting. Cost-benefit models can only give a partial answer to the question of why people vote because they cannot adequately explain roll-off. The phenomenon of roll-off is particularly difficult to explain for models that rely on costs and changes in pivot probabilities. Voters who roll-off are already at the voting booth and generally forego voting in down-the-ballot elections in which they are more likely to be pivotal.[23] Furthermore, our model gives results that are consistent with patterns of participation observed by Wolfinger and Rosenstone (1980) who note that the single best predictor of participation is education level. In our model it is always true that informed voters are more likely to vote than uninformed voters. Empirical work has also demonstrated that independent voters are much more likely to abstain than partisans.

If costs to vote are introduced the effect will be to eliminate participation altogether following the argument of Palfrey and Rosenthal (1985) because the probability of being pivotal goes to zero as the population size gets large. If we follow the literature and add a benefit from voting then the prediction would be that those with positive costs to vote would never vote while those with negative costs to vote will always vote. If the benefit from voting is obtained simply by showing up at the ballot booth our model can still be used to explain roll-off. In addition, the comparative statics results relating information and participation would still hold.

One possible extension is to endogenize information acquisition. A seemingly natural approach would be to permit voters to acquire information at some cost. However, because of very small pivot probabilities only voters with nearly zero information costs would acquire information. A more interesting approach would consider the role of elites as information providers. One question in this context is, who gets informed and what consequences does that have on patterns of participation and election outcomes. In addition, the comparative static result showing margin of victory increasing in information (see equation 13) might be challenged under such a scenario. If the election were not expected to be close elites would have a lower incentive to provide information.

Finally, since election results reveal information, victorious candidates may have an incentive to utilize this information when choosing policies after the election. If winning candidates are responsive in this fashion voters may influence election outcomes even when they are not pivotal: votes may be used to signal private information to the winning candidate. This may explain, for example, support for minor parties in plurality rule elections. Analyzing information aggregation in multi-candidate elections also suggests a new criterion for comparing alternative voting systems such as proportional representation and plurality rule.

XI. **Appendix**

Proposition 1 *Let* $p_\phi > 0$, $q > 0$, $N \geq 2$ *and N even. For any symmetric strategy profile* τ *in which no agent plays a strictly dominated strategy,* $Eu(1,\tau) = Eu(0,\tau)$ *implies* $Eu(1,\tau) < Eu(\phi,\tau)$.

Proof: Given $Eu(1,\tau) = Eu(0,\tau)$ it follows from (8) that

(16)
$$(1-\alpha)\pi_t(1,\tau) + \alpha\pi_t(0,\tau) = \tfrac{1}{2}\big[\alpha[\pi_1(0,\tau) + \pi_0(0,\tau)] - (1-\alpha)[\pi_1(1,\tau) + \pi_0(1,\tau)]\big].$$

It follows from (6) that

(17)
$$(1-\alpha)\pi_t(1,\tau) + \alpha\pi_t(0,\tau) = 2[Eu(1,\tau) - Eu(\phi,\tau)] - \big[(1-\alpha)\pi_1(1,\tau)] - \alpha\pi_1(0,\tau)\big].$$

Combining these two expressions we get:

(18)
$$4[Eu(1,\tau) - Eu(\phi,\tau)] = (1-\alpha)[\pi_1(1,\tau) - \pi_0(1,\tau)] + \alpha[\pi_0(0,\tau) - \pi_1(0,\tau)].$$

Thus it is sufficient to show that

(i) $\pi_1(1,\tau) - \pi_0(1,\tau) < 0$ and

(ii) $\pi_0(0,\tau) - \pi_1(0,\tau) < 0$

To see (i) note that

(19)
$$\pi_1(1,\tau) - \pi_0(1,\tau) = (\sigma_{10}(\tau) - \sigma_{11}(\tau))\sum_{j=0}^{\frac{N}{2}-1} \frac{N!}{(j+1)!\,j!(N-2j-1)!}\sigma_\phi(\tau)^{N-2j-1}(\sigma_{10}(\tau)\sigma_{11}(\tau))^j .$$

Since $\sigma_\phi(\tau) = p_\phi + (1-q)p_i\tau_\phi$ and $\sigma_{11}(\tau) = p_i q + \sigma_{10}(\tau)$ (i) follows from $p_\phi > 0$, $q > 0$ and $N \geq 2$. A similar argument is used for (ii). □

To prove Proposition 2 we require the following two Lemmas. Lemma 0 is technical fact.

Lemma 0 *Let* $(a_N, b_N, c_N)_{N=1}^{\infty}$ *be a sequence that satisfies* $(a_N, b_N, c_N) \in [0,1]^3$, $a_N < b_N - \delta$ *and* $\delta < c_N$ *, for all N and for some* $\delta > 0$. *Then for i=0,1*

$$\frac{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{\,N-2j-i} a_N^{\,j}}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{\,N-2j-i} b_N^{\,j}} \to 0 \text{ as } N \to \infty$$

Proof: The proof is in two steps. First, since $0 < a_N < b_N - \delta < 1$ choose k so that $(a_N / b_N)^k < \varepsilon$ for any N. Choose L such that $1/L < \varepsilon$. Given k, L and $c_N > \delta$ for any N, we can choose $N > 2L(k+1) + 2i$ large enough so that

$F(N, j) \equiv \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{\,N-2j-i} b_N^{\,j}$ is increasing in j for $j<(k+1)L$. To see that we can choose such an N note that $F(N, j) < F(N, j+1)$ if

(20)

$$\frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N^{\,N-2j-i} b_N^{\,j} < \frac{N!}{(j+i+1)!(j+1)!(N-2j-i-2)!} c_N^{\,N-2j-i-2} b_N^{\,j+1}$$

but now by canceling terms we get

(21) $$\frac{c_N^{\,2}}{b_N}(j+i+1)(j+1) < (N-2j-i)(N-2j-i).$$

Since $b_N > \delta$, $c_N \le 1$ and $j<(k+1)L$ it follows that

(22) $$\frac{c_N^{\,2}}{b_N}(j+i+1)(j+1) < \frac{1}{\delta}((k+1)L+i+1)((k+1)L+1)$$

and

(23) $$(N-2(k+1)L-i)(N-2(k+1)L-i) < (N-2j-i)(N-2j-i)$$

Now we can choose N so that

(24) $$\frac{1}{\delta}((k+1)L+i+1)((k+1)L+1) < (N-2(k+1)L-i)(N-2(k+1)L-i)$$

so $F(N, j)$ is increasing for $j<(k+1)L$.

Step 2. Now we split the equation in the Lemma into two parts both of which are shown to be less than ε.

(25)
$$\frac{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} a_N{}^j}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} b_N{}^j} =$$

$$\frac{\sum_{j=0}^{k} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} a_N{}^j + \sum_{j=k+1}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} a_N{}^j}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} b_N{}^j}.$$

We now show that the first term is less than ε.

Note that

$$\frac{\sum_{j=0}^{k} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} a_N{}^j}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} b_N{}^j} <$$

(26)
$$\frac{\sum_{j=0}^{k} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} b_N{}^j}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\,j!(N-2j-i)!} c_N{}^{N-2j-1} b_N{}^j} =$$

$$\frac{\sum_{j=0}^{k} F(N,j)}{\sum_{j=0}^{\frac{N}{2}-i} F(N,j)} < \frac{\sum_{j=0}^{k} F(N,j)}{L\sum_{j=0}^{k} F(N,j)} = 1/L < \varepsilon.$$

(This is the case since $F(N,j)$ is increasing for $j<L(k+1)$ and $N/2-i>L(k+1)$.)

Finally, we show that the second term is less than ε.

Note that

(27)
$$\frac{\sum_{j=k+1}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-1} a_N{}^{j}}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-1} b_N{}^{j}} =$$

$$(a_N / b_N)^k \frac{\sum_{j=k+1}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-i} b_N{}^{k} a_N{}^{j-k}}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-i} b_N{}^{j}} .$$

Now from $b_N > a_N$ it follows that $b_N{}^k a_N{}^{j-k} < b_N{}^j$ for j>k and therefore

(28)
$$(a_N / b_N)^k \frac{\sum_{j=k+1}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-i} b_N{}^{k} a_N{}^{j-k}}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-i} b_N{}^{j}} <$$

$$(a_N / b_N)^k \frac{\sum_{j=k+1}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-i} b_N{}^{j}}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!\, j!(N-2j-i)!} c_N{}^{N-2j-i} b_N{}^{j}} < (a_N / b_N)^k < \varepsilon .$$

□

Lemma 1 *Suppose* $p_i q > 0$ *and* $0 < \alpha < 1$. *Consider a sequence of voting games and strategy profiles* $\{\tau^N\}_{N=0}^{\infty}$. *Then:*

A. *if there exists an ε>0 such that* $\sigma_{xy}(\tau^N) - \sigma_{yx}(\tau^N) > \varepsilon$ *for any* $N \geq 0$ *and* $x \neq y$ *then there exists an* $\overline{N}$ *such that for any N>* $\overline{N}$ $Eu(x,\tau^N) > Eu(\phi,\tau^N) > Eu(y,\tau^N)$ *;*

B. *if for all* $N \geq 0$ *there are two actions s,s' with* $s \neq s'$ *such that* $Eu(s,\tau^N) = Eu(s',\tau^N)$ *then for any ε>0 there is an* $\overline{N}$ *such that for N>* $\overline{N}$ $|\sigma_{01}(\tau^N) - \sigma_{10}(\tau^N)| < \varepsilon$ *.*

Proof: Condition B follows as a corollary of A so we only need to show A. Suppose that there exists an ε>0 such that $\sigma_{01}(\tau^N) - \sigma_{10}(\tau^N) > \varepsilon$ for any $N \ge 0$. Since $p_i q > 0$ and $\sigma_{xx}(\tau^N) = p_i q + \sigma_{yx}(\tau^N)$ for $x \ne y$ we can state the following facts:

There exists an $\eta > 0$ such that for all $N \ge 0$, $\sigma_{01}(\tau^N) > \eta$,

$\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) - \sigma_{11}(\tau^N)\sigma_{10}(\tau^N) > \eta$ and $\sigma_{\phi}(\tau^N) > \eta$. (This follows since $p_{\phi} > 0$ and $\sigma_{\phi}(\tau^N) = p_{\phi} + p_i(1-q)\tau_{\phi}^N$). Furthermore $1 > \sigma_{00}(\tau^N) > \sigma_{10}(\tau^N)$ for all $N \ge 0$.

From Equation (1) it follows that $Eu(\phi,\tau^N) > Eu(1,\tau^N)$ if and only if: $(1-\alpha)\pi_t(1,\tau^N) - \alpha\pi_t(0,\tau^N) + (1-\alpha)\pi_1(1,\tau^N) - \alpha\pi_1(0,\tau^N) < 0$.

From Equation (2) it follows that $Eu(0,\tau^N) > Eu(\phi,\tau^N)$ if and only if:

$(1-\alpha)\pi_t(1,\tau^N) - \alpha\pi_t(0,\tau^N) + (1-\alpha)\pi_0(1,\tau^N) - \alpha\pi_0(0,\tau^N) < 0$

Therefore $Eu(\phi,\tau^N) > Eu(1,\tau^N)$ and $Eu(0,\tau^N) > Eu(\phi,\tau^N)$ if the following three conditions hold:

(i) $(1-\alpha)\pi_t(1,\tau^N) - \alpha\pi_t(0,\tau^N) < 0$,

(ii) $(1-\alpha)\pi_1(1,\tau^N) - \alpha\pi_1(0,\tau^N) < 0$

(iii) $(1-\alpha)\pi_0(1,\tau^N) - \alpha\pi_0(0,\tau^N) < 0$.

Note that $0 < \alpha < 1$. Lemma 0 and the fact that $\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) - \sigma_{11}(\tau^N)\sigma_{10}(\tau^N) > \eta$, $\sigma_{\phi}(\tau^N) > \eta$ imply that

$$(29)\qquad \frac{\pi_t(0,\tau^N)}{\pi_t(1,\tau^N)} = \frac{\alpha\sum_{j=0}^{\frac{N}{2}} \frac{N!}{j!\,j!(N-2j)!}\sigma_{\phi}(\tau^N)^{N-2j}(\sigma_{00}(\tau^N)\sigma_{01}(\tau^N))^j}{(1-\alpha)\sum_{j=0}^{\frac{N}{2}} \frac{N!}{j!\,j!(N-2j)!}\sigma_{\phi}(\tau^N)^{N-2j}(\sigma_{10}(\tau^N)\sigma_{11}(\tau^N))^j} \to \infty$$

as $N \to \infty$.

Therefore condition *(i)* is satisfied for sufficiently large *N*.

Similarly, Lemma 0 and the fact that $\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) - \sigma_{11}(\tau^N)\sigma_{10}(\tau^N) > \eta$, $\sigma_{\phi}(\tau^N) > \eta$, $\sigma_{00}(\tau^N) > \eta$, $\sigma_{01}(\tau^N) > \eta$ imply that

(30)

$$\frac{\pi_1(0,\tau^N)}{\pi_1(1,\tau^N)} = \frac{\alpha\sigma_{00}(\tau^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!\,j!(N-2j-1)!}\sigma_\phi(\tau^N)^{N-2j-1}(\sigma_{00}(\tau^N)\sigma_{01}(\tau^N))^j}{(1-\alpha)\sigma_{10}(\tau^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!\,j!(N-2j-1)!}\sigma_\phi(\tau^N)^{N-2j-1}(\sigma_{10}(\tau^N)\sigma_{11}(\tau^N))^j} \to \infty$$

as $N \to \infty$ and

(31)

$$\frac{\pi_0(0,\tau^N)}{\pi_0(1,\tau^N)} = \frac{\alpha\sigma_{01}(\tau^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!\,j!(N-2j-1)!}\sigma_\phi(\tau^N)^{N-2j-1}(\sigma_{00}(\tau^N)\sigma_{01}(\tau^N))^j}{(1-\alpha)\sigma_{00}(\tau^N)\sum_{j=0}^{\frac{N}{2}}\frac{N!}{(j+1)!\,j!(N-2j-1)!}\sigma_\phi(\tau^N)^{N-2j-1}(\sigma_{10}(\tau^N)\sigma_{11}(\tau^N))^j} \to \infty$$

as $N \to \infty$.

Hence conditions *(ii)* and *(iii)* are also satisfied.

Using an analogous argument we can show that if there exists an ε>0 such that $\sigma_{01}(\tau^N) - \sigma_{10}(\tau^N) > \varepsilon$ for any $N \geq 0$ then $Eu(1,\tau^N) > Eu(\phi,\tau^N) > Eu(0,\tau^N)$. □

Proposition 2 *Suppose* $q > 0$, $p_i(1-q) < |p_0 - p_1|$ *and* $p_\phi > 0$. *Let* $\{\tau^N\}_{N=0}^{\infty}$ *be a sequence of equilibria.*

(i) *If* $p_i(1-q) < p_0 - p_1$ *then* $\lim_{N\to\infty} \tau_1^N = 1$, *i.e., all UIAs vote for candidate 1.*

(ii) *If* $p_i(1-q) < p_1 - p_0$ *then* $\lim_{N\to\infty} \tau_0^N = 1$, *i.e., all UIAs vote for candidate 0.*

Proof: Note that $\sigma_{00}(\tau^N) = p_i q + \sigma_{10}(\tau^N)$, $\sigma_{11}(\tau^N) = p_i q + \sigma_{01}(\tau^N)$, $\sigma_{zx}(\tau^N) = p_i(1-q)\tau_x^N + p_x$ for $z \neq x$. In case (i) it follows from $p_i(1-q) < p_0 - p_1$ and $\tau_1^N \leq 1$ that $\sigma_{10}(\tau^N) + \delta > \sigma_{01}(\tau^N)$ for any τ^N and some $\delta > 0$. Therefore $\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) < \sigma_{11}(\tau^N)\sigma_{10}(\tau^N) - \delta'$ for some $\delta' > 0$ where δ' is independent of *N*.

The result follows directly from Lemma 1.A. The argument for case (ii) is analogous.

Proposition 3 *Suppose* $q > 0$, $p_i(1-q) \geq |p_0 - p_1|$ *and* $p_\phi > 0$. *Let* $\{\tau^N\}_{N=0}^{\infty}$ *be a sequence of equilibria.*

(i) *If* $p_i(1-q) \geq p_0 - p_1 > 0$ *then UIAs mix between voting for candidate 1 and abstaining*; $\lim \tau_1^N = \frac{p_0 - p_1}{p_i(1-q)}$ *and* $\lim \tau_\phi^N = 1 - \frac{p_0 - p_1}{p_i(1-q)}$.

(ii) *If* $p_i(1-q) \geq p_1 - p_0 > 0$ *then UIAs mix between voting for candidate 0 and abstaining*; $\lim \tau_0^N = \frac{p_1 - p_0}{p_i(1-q)}$ *and* $\lim \tau_\phi^N = 1 - \frac{p_1 - p_0}{p_i(1-q)}$.

(iii) *If* $p_0 - p_1 = 0$ *then UIAs abstain;* $\lim \tau_\phi^N = 1$.

Proof: *Cases (i) and (ii) with strict inequality:* First, we show that for large *N* there are no pure strategy equilibria. We describe the argument only for case (i). An analogous argument with all inequalities reversed holds for case (ii). Suppose $\tau_1^N = 0$. By the same argument as in case (i), Proposition 2, $\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) < \sigma_{11}(\tau^N)\sigma_{10}(\tau^N) - \delta'$ for some $\delta' > 0$. From Lemma 1.A all UIAs strictly prefer to vote for candidate 1 if *N* is large. It follows that $\tau_1^N > 0$ and, by Lemma 2, $\tau_0^N = 0$. Suppose $\tau_1^N = 1$. A simple calculation shows that this implies that $\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) - \delta' < \sigma_{11}(\tau^N)\sigma_{10}(\tau^N)$ for some $\delta' > 0$. By Lemma 1.A all UIAs strictly prefer to vote for candidate 0. Since there is always a mixed strategy equilibrium, in any equilibrium agents mix between abstention and voting for candidate 1. Now the result follows from Lemma 1.B.

Cases (i) and (ii) with equality: follows from the upper hemi continuity of the equilibrium correspondence in (p_0, p_1).

Case (iii): Suppose $\tau_\phi^N = 0$. Then it follows from Lemma 2 that, for large *N*, $\tau_1^N = 1$ or $\tau_0^N = 1$. But $\tau_0^N = 1$ implies that $\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) < \sigma_{11}(\tau^N)\sigma_{10}(\tau^N) - \delta'$ for some $\delta' > 0$, and by Lemma 1.A every UIA prefers to vote for candidate 1. Similarly, $\varsigma_1^N = 1$ implies that $\sigma_{00}(\tau^N)\sigma_{01}(\tau^N) - \delta' > \sigma_{11}(\tau^N)\sigma_{10}(\tau^N)$ for some $\delta' > 0$, and by Lemma 1.A every UIA prefers to vote for candidate 0. Thus for large *N*, and for any voting equilibrium it must be true that $\tau_\phi^N > 0$. The result now follows from Lemma 1.B. ☐

Proposition 4 *Suppose* $p_\phi > 0$ *and* $q > 0$ *and* $p_i \neq |p_0 - p_1|$. *Then for every* ε *there exists an* $\overline{N}$ *such that for N>* $\overline{N}$ *the probability that in equilibrium the election fully aggregates information is greater than* $1-\varepsilon$.

Proof: Proposition 4 is a straightforward consequence of Propositions 2 and 3. If $p_i > |p_0 - p_1|$ then Propositions 2 and 3 imply that if the state is 1 then the probability of any agent voting for candidate 1 is larger than the probability that any agent chooses candidate 0 by at least $\min\{qp_i, p_i - |p_0 - p_1|\} > 0$. Conversely, if the state is 0 then the probability that any agent chooses candidate 0 is larger than the probability that any agent chooses candidate 1 by at least $\min\{qp_i, p_i - |p_0 - p_1|\} > 0$. By the law of large numbers it then follows that the probability that candidate 1 wins in state 1 and candidate 0 wins in state 0 goes to one as $N \to \infty$. This is the same outcome that would occur if voters were fully informed. If $p_i < |p_0 - p_1|$ then the expected vote share for the candidate with the greatest expected support is always greater than 50% regardless of the state. By the law of large numbers it follows that this candidate will win goes to 1 as *N* goes to infinity. □

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[***]Thanks to Daniel Diermeier, Roger Myerson, Susanne Lohmann, John Matsusaka and seminar participants at Northwestern University, Washington University, Princeton University, California Institute of Technology, the University of Southern California Business School and Johns Hopkins University. We are responsible for all remaining errors. Timothy Feddersen thanks the Department of Political Science at Northwestern University for supporting this research while he was a member of that department. Mailing addresses: Timothy Feddersen, Kellogg Graduate School of Business, Department of Managerial Economics and Decision Sciences, Northwestern University, 2001 Sheridan Road, Evanston, ILL 60208. Wolfgang Pesendorfer, Department of Economics, Northwestern University, 2001 Sheridan Road, Evanston, ILL 60208.

[1]Source: The State Board of Elections. *State of Illinois Official Vote Cast at the General Election on November 8, 1994*. Springfield, 1994.

[2] Roll-off is the “tendency of the electorate to vote for ‘prestige’ offices but not for lower offices on the same ballot”. Walter Dean Burnham (1965, p.9).

[3]Thomas R. Palfrey and Howard Rosenthal (1985).

[3]More recent models of rational participation in the decision theoretic vein include: John A. Ferejohn and Morris P. Fiorina (1974); Rebecca B. Morton (1991); Carole J. Uhlaner (1989); John G. Matsusaka (1992). Game-theoretic models by John O. Ledyard (1983), Palfrey and Rosenthal (1983, 1985) and Timothy J. Feddersen (1992, 1993) demonstrate that significant levels of participation may be rationalizable even if voting is costly for some equilibria. However, Palfrey and Rosenthal (1985) demonstrate that these game-theoretic explanations of costly participation are not robust to the introduction of reasonable uncertainty. They show that, if there is sufficient uncertainty about preferences and about participation costs of voters, participation by those with strictly positive costs to vote will go to zero as the size of the population gets large.

[5]See Gary W. Cox and Michael C. Munger (1990) for a discussion of the literature on ballot position. A recent study (William K. Hall and Larry T. Aspin 1987) on roll-off in judicial retention elections found very little impact of ballot position.

[6]See Raymond E. Wolfinger and Stephan J. Rosenstone 1980; John H. Aldrich 1993; and Matsusaka (1992)

[7] See Paul Milgrom and Robert Weber (1982).

[8]We might imagine that each bidder has privately commissioned a study of the property being leased in an attempt to determine the amount of oil that may be productively exploited.

[9] See Aldrich (1993) or Matsusaka (1992).

[10]There is also a related social choice literature on Condorcet's Jury Theorem that examines majority rule elections as information aggregation devices. See for example, Krishna Ladha 1992; Peyton Young 1988; and Norman Schofield 1972. In an related paper Alvin K. Klevorick, Michael Rothschild and Cristopher Winship (1985) show that if each juror only considers her private information then the majority rule outcome is inefficient. See also David Austen-Smith and Jeffrey Banks (1994).

[11]Thus, the actual number of voters n is uncertain and follows a binomial distribution with parameters $(N+1, 1-p_\phi)$. Similarly, the number of type j voters, j=0,1,i, follows a binomial distribution with parameters $(N+1, p_j)$.

[12]In standard two-candidate elections with no common values voters have a weakly dominant strategy to vote for the candidate that they believe to be best rather than either abstain or vote for the other candidate. That is not the case in our model where UIAs do not always support the candidate that they ex ante believe to be the best. Note also that in the standard model voters have a **strictly** dominant strategy to support the candidate they believe to be the best if there is some uncertainty about the population size.

[13] Note that a similar conclusion would hold in a model for which independents had different preferences. Suppose that a voter type y prefers candidate 1 to candidate 0 if the probability of state 1 is greater than y (we are grateful to Ariel Rubinstein for suggesting this simple generalization of our model). In this case an analog of Proposition 1 can be proven: suppose type y^* is indifferent between voting for candidate 0 and candidate 1. Then there exists a strictly positive interval of types around y^* such that each type in that interval strictly prefers to abstain over voting for either candidate. Moreover, the length of this interval is bounded uniformly for all N. Details are available from the authors upon request.

[14]The crucial step in the argument that a positive fraction of voters abstains is a modified version of Proposition 1 which as argued in footnote 13 holds even in the generalized case with preference diversity.

[15] As long as the informed independents get a signal that is strictly informative Proposition 1 still holds. If informed independents receive a noisy but informative signal then this implies that they do not have a strictly dominant strategy anymore. Thus we would have to analyze also the decision problem of the informed independents. This would complicate the analysis without adding any new insight. In particular, it does not affect the conclusion that a strictly positive fraction of voters abstains in equilibrium.

[16] In the limiting case where $p_i = |p_0 - p_1|$ the probability that the correct candidate is chosen from the point of view of a majority may stay bounded away from 1. In this case there is a state such that (for large *N*) exactly half of the electorate prefers candidate 0 and half of the electorate prefers candidate 1. Clearly this is a knife-edge case.

[17]We say an agent behaves naively by voting for the candidate he believes best on the basis of him private information alone. We call this naive rather than sincere as has been proposed by Austen-Smith and Banks (1994) because sincere implies that voters would all prefer that the candidate that they vote for win. This is not necessarily the case when voters do not know the state.

[18]If the expected difference in partisan support is large enough then our model predicts no abstention by active agents.

[19]Note that this is the case as long as the percentage of independents is less than one.

[20]Pivot probabilities unambiguously decline when the expected margin of victory increases and the size of the electorate increases.

[21]Bruce E. Keith, et al., (1992, p.14) report that in 1990 36% of US voters considered themselves to be “strong” or “weak” Democrats and 28% consider themselves to be “strong” or “weak” Republicans.

[22] Keith, et al., (1992 p14, Table 1.1) state that the average self reported turnout in midterm elections 1990 was 53% among strong and weak Republicans, 55% among strong and weak Democrats and 37% among Independents (which includes independent Democrats, independent Republicans and Independents).

[23]Mark W. Crain, et al., (1987) demonstrate that there is variability in voting on House and Senate races on the same ballot. They find support for the hypothesis that the variability in voting may be explained as a

function of the closeness of the election: the closer the election the higher the participation. However, given that voters are already in the ballot booth it is hard to see why cost should be a factor in deciding not to vote. Other studies have contested the linkage between closeness and participation. See for example recent work by Matsusaka (1992 and 1993), and Cox and Munger (1989). Crain et al did not control for the information in each election.