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Journal of MONETARY ECONOMICS

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Smoothing with liquid and illiquid assets ☆

Andrea L. Eisfeldt*

Department of Finance, Kellogg School of Management, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208, USA

Received 5 May 2006; received in revised form 30 May 2006; accepted 11 June 2006

Abstract

A quantitative examination of the demand for liquid assets arising from consumption smoothing motives reveals that such demand is very low. Consumers faced with income streams calibrated to match income and unemployment data and returns and transactions costs calibrated to match US Treasury Bill data almost exclusively buy and hold illiquid long term assets even though the return premium on long term assets is quite small. This is because, with standard preferences, savings are highly persistent even when risky income is not. In the calibrated model, the first order autocorrelation of savings is an order of magnitude larger than that of income.

JEL classification: E0; E2; G1; G2

Keywords: Liquidity; Portfolio choice; Consumption smoothing; Savings

1. Introduction

A quantitative examination of the demand for liquid assets arising from consumption smoothing motives reveals that such demand is very low. In a model where consumers receive risky income streams and can invest in liquid short term and illiquid long term

☆I thank John Cochrane, Cristina De Nardi, Lars Hansen, Adriano Rampini, Thomas Sargent, Noah Williams, the editor, Robert King, an anonymous referee, and seminar participants at the University of Chicago and the 2002 Society for Economic Dynamics Meetings for helpful comments. This paper is a revised and extended version of work awarded financial support by the John Lee Money and Banking Prize at the University of Chicago. All errors are mine.

*Tel.: +1 847 491 7843; fax: +1 847 491 5719.
E-mail address: a-eisfeldt@northwestern.edu.

0304-3932/$ - see front matter © 2006 Published by Elsevier B.V.
doi:10.1016/j.jmoneco.2006.06.004

assets to smooth consumption, such consumers almost exclusively buy and hold illiquid long term assets even if the return premium on long term assets is quite small. In an economy in which income streams are calibrated to match income and unemployment data and returns and transactions costs are calibrated to match US Treasury Bill data, the ratio of short term liquid to longer term illiquid assets held is 0.02. Empirically, this ratio is 1. Thus, in the model consumers choose to smooth consumption with far too few short term liquid assets to match the empirical maturity structure. This result is due to the fact that savings are highly persistent even when risky income is not. In the calibrated model, the first order autocorrelation of savings is an order of magnitude larger than that of income.

The focus of this study is the demand for short term, liquid assets relative to long term, illiquid assets. In practice, long term assets or investments are more productive but less flexible than short term investments, due to irreversibility. Financial intermediaries (and firms or governments themselves) convert long term illiquid investments into short term liquid claims. For example, bank deposits are backed by long term loans, and commercial paper is backed by long term investments by firms. It is often suggested that transforming long term illiquid investments into short term liquid claims creates assets which are more marketable to consumers.[1] However, the relative demand for short term liquid assets by consumers has not previously been quantitatively examined in a calibrated model.[2]

The following three main savings motives have been outlined in the consumption and savings literature:[3] consumption smoothing or precautionary saving, life cycle saving and saving for bequests, and saving for lumpy durables. This paper examines the demand for liquid assets generated in a calibrated model where savings are driven by consumption smoothing motives.[4] Simple, stylized versions of the second two savings motives are also examined.[5]

The model economy is populated by a continuum of risk averse consumers who face uninsurable individual income risk as in Bewley (1986). There is no aggregate risk in the economy. Hence the model focuses exclusively on maturity choice when long term assets are illiquid. Consumers have access to two assets with which to smooth their stochastic income, a one period riskless bond and a two period riskless bond. The two period bond can be liquidated early, but only at a cost.

To fix ideas and the calibration, returns and transactions costs are matched to US Treasury Bill data. The maturity structure, or the relative amounts of bonds of different maturities issued each period, and yield structure of government debt is readily characterized. Each model period is one quarter, so one and two period bonds correspond to three and six month bills. The government auctions three and six month treasury bills

[1]However, see Diamond and Rajan (2001), and Myers and Rajan (1998) for models where agency and commitment problems lead to liquid claims being backed by illiquid assets. See Cochrane (2001) for a fiscal theory of the maturity structure of government debt.

[2]A similar issue was studied by Aiyagari and Gertler (1991) who measured the equity premium implied by transactions costs in trading stocks.

[3]See, for example Deaton (1992), Carroll (1997), Attanasio (1999) and the references therein.

[4]See Kimball (1990) for necessary conditions for consumers to display "prudence" and to engage in precautionary savings. Carroll and Samwick (1998) report that between 32% and 50% of wealth held in their sample is due to precautionary savings motives.

[5]A complete description of these comparative statics appears in the unpublished appendix, Eisfeldt (2006). For a rigorous study of life cycle and precautionary savings in liquid assets, along with an exogenous accumulation of illiquid assets, see Gourinchas and Parker (2002). For a quantitative study of saving for bequests, see De Nardi (2004).

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each week. Over the period from 1980 to 2002, the average ratio of three month bills to six month bills offered was just over one, implying that at any given issue date there are twice as many bills outstanding with three months to maturity as there are bills with six months to maturity.[6] Moreover, the yield curve describing these securities is rarely inverted, i.e. yields are usually increasing in maturity, and the average yield spread between three and six month bills was 0.244% annually over the period from 1952 to 1991.[7] This paper examines whether this yield spread, along with calibrated transactions costs, income processes and preferences imply a maturity structure that is close to the empirical one with equal amounts of one and two period bonds issued and purchased at each date.

In particular, consider the following set up where returns and transactions costs increase with maturity: the government faces transactions costs when issuing new debt. Consumers use government bonds to smooth idiosyncratic income shocks but face transactions costs when reselling long term debt prior to maturity.[8] Hence, the government finds it cheaper in terms of transactions costs to issue long term debt, but can offer a lower rate of return on short term debt since it offers consumers liquidity by maturing costlessly every period. The government trades off savings on issuing costs for long term debt against lower equilibrium returns on short term debt while consumers trade off higher returns on long term debt against the higher liquidity of short term claims. Given these tradeoffs between liquidity and returns, in such an environment one might observe the yield curve and maturity structure typical of US government securities.

First, the equilibrium maturity structure is computed in a model in which parameters are calibrated to their empirical counterparts. Comparative statics of how the relative equilibrium quantities of long and short term debt issued and held change with changes in preferences, changes to the transaction costs and return structure of bonds, and changes to the income process are also discussed. The question addressed in this paper is: do calibrated returns, transactions costs, and income processes along with standard consumer preferences imply an equilibrium maturity structure that is close to the empirical one? To match the stylized facts, the model should predict equal demand for newly issued one and two period bonds. It is actually quite difficult to get consumers to hold considerable amounts of short term bonds in equilibrium. Consumers with standard preferences faced with calibrated income streams have stable savings and hence take advantage of the higher return on illiquid assets while incurring few transactions costs due to early sale. One way of overturning this result is by introducing deterministic variation in income, although the variation needs to be quite large to generate the empirically observed maturity structure. This suggests that transactions demand as in monetary models, or saving for lumpy durables, may be important components of the demand for liquid assets.

Several interesting models of the effects of liquidity shocks on investor behavior, institutional structure, and asset prices have been developed in the literature. Two of the most prominent examples are Diamond and Dybvig (1983), who study the role of liquid assets in bank runs by consumers, and Holmström and Tirole (1996, 1998, 2001) who explore the role of liquid assets in buffering firms' liquidity shocks. Models constructed to

[6]See the US Treasury auction data at http://www.publicdebt.treas.gov. The mean ratio was 1.01 with a standard deviation of 0.06.

[7]See Campbell et al. (1997, p. 397 and 415).

[8]One could think of this cost as time spent finding a suitable buyer if their is no organized secondary market for treasury bills sold by individuals (as in the search model of Duffie et al., 2005) or as the commission on having the bill sold by an intermediary.

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study the effects of liquidity shocks often embed the demand for the liquid asset in the structure of the model (for example using a short time horizon or an overlapping generations framework) or assume a liquidity shock which is large enough to force the sale of the liquid asset. These are useful abstractions which have enabled important insights as to the effects of large liquidity shocks, taking as given relative holdings of liquid and illiquid assets.[9] This paper asks whether a standard consumption smoothing model can generate such holdings as a buffer against calibrated income shocks. Quantitatively measuring the sources of demand for liquid assets seems important for understanding the yield spread between liquid and illiquid assets and has important implications for policies aimed at providing liquidity.

Some of the results in this paper can be partially understood as a combination of the intuition from the theoretical literature on portfolio choice with transactions costs and on the magnitude of the equity premium.[10] However, there is quite a bit to learn from the explicit quantitative model. Constantinides (1986) shows that transactions costs have first order effects on quantities but second order effects on prices since investors avoid trading in illiquid assets. Idiosyncratic risk can increase the need for consumers to trade, enabling a greater effect of illiquidity on asset prices.[11] This study shows that with calibrated idiosyncratic risk it is difficult to generate a substantial demand for short term liquid claims even though consumers do avoid trading in illiquid assets. Thus, equilibrium quantities are not as sensitive to illiquidity as one might have thought, even if return premia are very small and long term assets are quite illiquid. Relatedly, it is well known that aggregate income streams exhibit too low a variance to explain the equity premium given standard preferences.[12] However, high levels of risk aversion can generate a large equity premium (although too volatile a risk free rate) while, in contrast, this paper shows that consumers' ability to smooth consumption with longer term illiquid claims in a long horizon model is insensitive to the level of risk aversion.

2. Model

There is a continuum of consumers with unit measure who face uninsurable idiosyncratic labor income shocks and invest in government issued one and two period bonds to smooth consumption. Each consumer has preferences given by

$$\mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{c(t)^{(1-\sigma)}}{1-\sigma}\right], \tag{1}$$

[9] See also Vayanos and Vila (1999) for a study of the liquidity premium in an overlapping generations life cycle model, and Huang (2003) for a model where agents face random liquidation shocks.

[10] See also the related literature on the portfolio choice between stocks and bonds in the presence of uninsurable idiosyncratic labor income risk and borrowing constraints, for example, Guiso et al. (2002) and the references therein, Gomes and Michaelides (2005), Polkovnichenko (2006), and Haliassos and Michaelides (2003).

[11] Aiyagari and Gertler (1991) generate half of the observed equity premium by including transactions costs in a model with uninsurable idiosyncratic income risk, and Heaton and Lucas (1996) extend this result by including aggregate as well as idiosyncratic risk, generating a larger equity premium.

[12] See Hansen and Singleton (1982) for estimates of risk aversion based on the empirical risk premium, and Mehra and Prescott (1985) for a quantitative examination of the equity premium given aggregate consumption growth data.

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where $c(t)$ is the consumer's consumption at time t and $\sigma > 0$. Each period consumers receive idiosyncratic labor income $y(t)$. Income follows a Markov chain with transition probabilities given by

$$\Pi_y(y, y') = \Pr(y(t+1) = y' | y(t) = y). \tag{2}$$

Consumers decide how much to consume and how much one and two period debt to invest in by trading off the liquidity of short term (one period) debt against the higher return on long term (two period) debt. Denote investment in new two period bonds as $i_2(t)$ and investment in new one period bonds or disinvestment in current holdings of mid-term two period bonds as $i_1(t)$. Then, $i_1(t)$, along with current holdings of mid-term two period bonds, determines the quantity of bonds maturing tomorrow held. The quantity of mid-term two period bonds held at time t which will mature in the following period is denoted $B_2(t)$, and $B_1(t)$ denotes bonds held at time t which mature in the current period. The laws of motion for mid-term and maturing bond holdings are then

$$B_2(t+1) = i_2(t), \tag{3}$$

$$B_1(t+1) = B_2(t) + i_1(t). \tag{4}$$

Note that there are two ways to invest in bonds maturing at t (i.e. $B_1(t)$), namely, through $i_2(t-2)$ and $i_1(t-1)$.

The following borrowing constraints are imposed:[13]

$$i_1(t) \geqslant -B_2(t) \quad \forall t, \tag{5}$$

$$i_2(t) \geqslant 0 \quad \forall t. \tag{6}$$

Consumers can only disinvest in bonds maturing in the following period up to the amount they currently own, and cannot borrow.

Consumers face the following period by period budget constraint:

$$c(t) + \frac{i_1(t)}{1+r_1} + \frac{i_2(t)}{(1+r_2)^2} \leqslant y(t) - \tau + B_1(t) - \frac{\gamma}{1+r_1}\min\{i_1(t), 0\}, \tag{7}$$

where τ is a lump sum tax, γ is the proportional transactions cost of selling long term debt mid-term, r_1 is the net return on one period bonds as well as the net one period rate of return on resold two period bonds, and r_2 is the net one period rate of return on two period bonds held to maturity. Consumers must be indifferent between buying mid-term two period bonds and buying one period bonds in order for both to be purchased. Thus the return on new one period bonds and resold two period bonds must be equal and it is not necessary to distinguish between different types of maturing bonds. Rather, purchases of either of these types of bonds are denoted $i_1(t)$. Notice also that since consumers cannot short one period bonds, $\min\{i_1(t), 0\}$ represents mid-term sales of two period bonds. In words, the budget constraint states that a consumer's consumption, plus additional investment or disinvestment in bonds which mature tomorrow, plus investment in two period bonds, must be less than or equal to the consumer's idiosyncratic income realization, less tax payments, plus income from maturing bonds, less transactions costs

[13]Relaxing the borrowing constraints would make the liquid asset less desirable since the consumer could smooth adverse shocks by borrowing as well as dissaving.

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incurred from selling two period bonds early. In equilibrium this constraint will hold with equality.

The government is passive in this economy, and simply spends its income from taxes and proceeds from bond sales less interest payments on goods not valued by consumers. The government issues debt and collects taxes to fund government spending and faces the following per capita per period budget constraint:[14]

$$g(t) + B_1(t) = \frac{B_2(t+1)}{(1+r_2)^2}(1-\theta) + \frac{B_1(t+1) - B_2(t)}{(1+r_1)}(1-\theta) + \tau, \tag{8}$$

where $g(t)$ is government spending at t, τ is the per period per capita tax and θ is the proportional issuing cost for one and two period debt. Note that in the stationary equilibrium, government spending is constant. It is slightly smaller than the constant lump sum per capita tax due to the issuing costs.

Two conditions restricting returns relative to transactions costs are necessary to get an interior solution with both long and short term debt issued and held. Condition 1 comes from the government budget constraint. For the government to be indifferent between offering long and short term debt the 'cost of funds' including transactions costs and interest paid must be equal, i.e.,

$$\frac{(1-\theta)}{(1+r_2)^2} = \frac{(1-\theta)^2}{(1+r_1)^2}$$

which implies:

Condition 1. $\theta = 1 - (1+r_1)^2/(1+r_2)^2$.

Another restriction is necessary for short term debt to be held in positive quantities. The reasoning is as follows. Two period debt is always preferred over one period debt for moving resources two periods forward as long as $r_2 > r_1$. The transactions cost γ must be high enough so that consumers do not also prefer to use two period debt to move resources one period forward. A consumer can buy one unit of two period debt this period for $1/(1+r_2)^2$ and sell it next period for $(1-\gamma)/(1+r_1)$. Investing this same amount in short term debt yields $(1+r_1)/(1+r_2)^2$. But then if $(1-\gamma) > (1+r_1)/(1+r_2)^2$ consumers will always choose two period bonds over one period bonds. Combining this with Condition 1 yields:

Condition 2. $\gamma > 1 - (1-\theta)/(1+r_1)^2$.

The consumers' problem is formulated recursively in order to study the stationary equilibrium of the above economy. Denote the individual state vector $z = (B_1, B_2, y)$. The Bellman equation characterizing a consumer's optimization problem is

$$v(z) = \max_{i_1, i_2}\{u(c) + \beta \mathrm{E}[v(z')|z]\}, \tag{9}$$

subject to (5)–(7) given (2)–(4). The solution to this program is characterized by the investment policy:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \phi(z). \tag{10}$$

[14]Here I am abusing notation a bit since the government budget constraint holds in per capita terms.

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An economy $\mathscr{E}$ is described by the risk aversion parameter, discount factor, idiosyncratic income process, returns on one and two period bonds, transaction cost for selling two period bonds early, government issuing costs, and per capita lump sum taxes. Thus, an economy $\mathscr{E}$ is defined as: $\mathscr{E} \equiv \{\sigma, \beta, y, r_1, r_2, \gamma, \theta, \tau\}$. A stationary equilibrium can be defined as follows:

Definition 1. A *stationary equilibrium* for an economy $\mathscr{E}$ consists of an investment policy function $\phi(z)$, a probability distribution $\pi(z)$ and positive real numbers for one and two period bond holdings b_1 and b_2, the quantity of bonds sold mid-term $b_{2,\text{sold}}$, and per capita government spending g, such that:

1. The transactions cost and return parameters (θ, γ, r_1, r_2) satisfy Conditions 1 and 2.
2. The policy function $\phi(z)$ solves the consumer's optimization problem (9).
3. The probability distribution $\pi(z)$ is the stationary distribution associated with ($\phi(z)$, Π_y), i.e. it is consistent with consumer optimization and the exogenous Markov process for income.
4. The average values of liquid one period bonds and illiquid two period bonds issued and purchased (b_1, b_2) and illiquid bonds sold mid-term ($b_{2,\text{sold}}$) are implied by the average consumer behavior and satisfy bond market clearing as well as the government's budget constraint.
5. Per capita government spending (g) is positive (τ is large enough to cover issuing costs).

The consumer's optimization problem is solved using discrete state dynamic programming.[15] The solution yields the investment policy function which along with the equilibrium distribution across income and bond holdings states can be used to compute the maturity structure of bonds issued and purchased in the model economy. Per capita two period bond purchases, b_2, are computed as

$$b_2 = \sum_z \pi(z) i_{2,z}. \tag{11}$$

The per capita quantity of two period bonds which are sold mid-term, $b_{2,\text{sold}}$, is computed as the per capita disinvestment in bonds maturing in the following period:

$$b_{2,\text{sold}} = \sum_z \pi(z) \min\{i_{1,z}, 0\}. \tag{12}$$

It can be trivially shown that no consumer would simultaneously sell mid-term two period debt and buy one period debt. The quantity sold is thus a probability weighted sum over investment and disinvestment in bonds maturing next period. Per capita purchases of new one period debt, b_1, are computed as

$$b_1 = \sum_z \pi(z) \max\{i_{1,z}, 0\}. \tag{13}$$

To satisfy bond market clearing it is necessary that $b_1 \geqslant b_{2,\text{sold}}$. Finally, consumption is implied by the consumer budget constraint (7) given the investment policy function (10). If the government's budget constraint (8) is satisfied as well then by Walras Law market clearing is satisfied.

[15] See Lungqvist and Sargent (2000).

3. Results

The model is calibrated to quarterly US Treasury Bill return data and income and unemployment data, and uses standard values from the literature for the subjective discount factor and for risk aversion. Returns for one and two period bonds are matched to data on three and six month US Treasury Bills. The return on one period debt is set to zero[16] and the value for r_2 is set to match the annual yield spread between three and six month treasury bills of 0.244% reported in Campbell et al. (1997, p. 415) for the period 1952–1991. This translates to a quarterly spread of 0.061%, or approximately six basis points, thus $r_2 = 0.061\%$. The calibration uses the yield spread since there is no aggregate risk in the model, however for comparison the associated return spread over the same period was 0.071%.

The value for the government's issuing cost, θ, is implied by Condition 1 given bond returns. The issuing cost θ then determines the lower bound on the selling cost for mid-term two period debt, γ, by Condition 2. Condition 1 implies $\theta = 0.12\%$; hence $\gamma > 0.12\%$. The baseline selling cost is set to $\gamma = 0.2\%$, or twenty basis points. This cost is conservatively large. For example, the bid ask spread on actively traded bills is about two basis points.[17]

Government spending per capita is implied by the values chosen for taxes and issuing costs along with the solution to the consumers' maximization problem. Taxes are set at a level which ensures that the government budget constraint is satisfied in the stationary equilibrium (i.e., issuing costs are covered and government spending is positive.) Taxes are set to $\tau = 0.01$ which constitutes one percent of average income and implies positive government spending in all parameterizations studied.

The Markov chain constructed in Aiyagari and Gertler (1991) to be consistent with the US average duration of unemployment and the unemployment rate in the US of 1.5 quarters and 5%, respectively, is used for the baseline income process. Average income is set to one with a standard deviation of about 30% per quarter or 15% per year, which is consistent with estimates from the PSID in Kydland (1984). There are three states for income: unemployment, low income employment and high income employment. As in Diaz-Gimenez and Prescott (1989) income while unemployed is one third of mean income when employed, which matches the ratio of manufacturing to minimum wages of three to one. The state vector for income in unemployment, low income employment and high income employment is $y = [0.31\ 0.7254\ 1.347]$. The two employment states have symmetric transition probabilities and the unemployment state is less persistent than either employment state. Transition probabilities for income are given in Table 1.

Utility function parameters are set to standard values from the literature. The subjective discount factor β is set to $0.96^{0.25}$ or 0.96 annually, and risk aversion is set to 2. Finally, the maximum values of the discretized state vectors were chosen so as not to be binding in the stationary equilibrium and the minima were set to zero to impose the borrowing constraints. Increments are equal to 15% of average quarterly income.

In contrast to the empirical maturity structure, the relative quantities of one and two period bonds issued and purchased implied by the model using parameters calibrated to match their empirical counterparts consist predominantly of illiquid two period debt.

[16]See Labadie (1989).
[17]See Cook and LaRoche (1993).

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Table 1
Model results: baseline calibration

Parameters		Results	
γ	0.2%	b_1	0.0195
r_2	0.061%	b_2	0.8462
β	$0.96^{0.25}$	$b_{2,\text{sold}}$	0.0024
σ	2	$b_1 + b_2$	0.8657
B	[0:0.15:3.15]	$\frac{b_1}{b_2}$	0.0230
y	[0.31 0.7254 1.347]	$\frac{b_{2,\text{sold}}}{b_2}$	0.28%
Π_y	$\begin{bmatrix} 0.34 & 0.33 & 0.33 \\ 0.035 & 0.4825 & 0.4825 \\ 0.035 & 0.4825 & 0.4825 \end{bmatrix}$		

The results of the model based on the baseline calibration are presented in Table 1. Empirically, the ratio of three month bills to six month bills issued and purchased is about one. The implications of the model are quite different. The ratio of three month bills to six month bills issued and purchased is only about 0.02, almost two orders of magnitude smaller than the empirical ratio. It is also notable how few bonds are sold mid-term (0.28%). The intuition for this result can be explored in two ways. First, note that the persistence of consumers' savings is striking. Both the small fraction of three month bonds issued and purchased in equilibrium and the small amount of six month bonds sold mid-term are due to the fact that consumers' savings are highly persistent. Second, consumer's investment policies reveal that investors typically save in staggered two period bonds. They invest in one period bonds only if they hold more debt which matures this period relative to the following period, and sell two period bonds mid term only if they hold more debt which matures in the next period relative to this period.

For a parsimonious illustration of consumers' highly persistent savings, it is useful to study savings patterns in an analogous model with one period debt only. Panel A of Fig. 1 plots bond holdings at $t+1$ as a function of bond holdings at t and the 45° line and illustrates the high autocorrelation in bond holdings in such a model. Points along the diagonal indicate unchanged net bond holdings. Points below the diagonal indicate net decreases in bond holdings and points above the diagonal indicate net purchases. The persistence in savings obtains despite the fact that the baseline income process exhibits very weak autocorrelation and has a fairly high variance. The variance of the income process is 0.1101 on a quarterly basis which implies a standard deviation of just over 30% per quarter or 15% annually. The autocorrelation functions for income and bond holdings in the one period bond economy are graphed in Panel B of Fig. 1. The first order autocorrelation for the baseline income process is only 0.0725 and as seen in the graph it decays quickly. Conversely, the first order autocorrelation for savings is 0.9330 and decays slowly. Thus, the persistence in savings is more than an order of magnitude larger than the persistence in income.

Clearly, consumers rarely change their bond position by very much. Thus, it is not surprising that when two period bonds with a higher return than one period bonds are introduced consumers are able to exploit the additional yield while avoiding selling costs.

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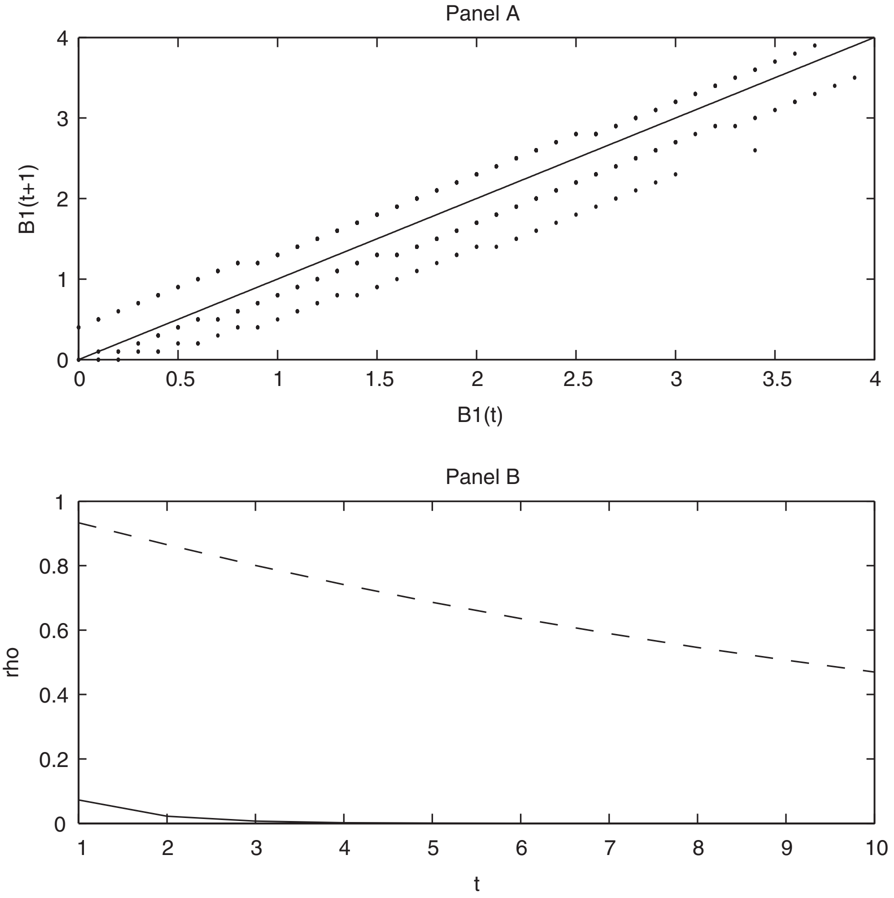

Fig. 1. Persistence in savings: model with one period bonds only. Panel A: $B_1(t+1)$ as a function of $B_1(t)$ in a simulation of an economy with one period bonds only. Panel B: Autocorrelation function for income (solid) and savings (dashed).

Savings rates are similar in the model with only one period bonds compared to the model with both one and two period bonds. Total bond holdings are 1.6739 in the one period bond model, compared to 1.7119 in the model with one and two period bonds. However, in the model with one and two period bonds about half of total savings is in two period bonds maturing in the following period while the other half is in two period bonds maturing in two periods, i.e. saving is in staggered two period bonds.

The investment policies for one and two period bonds also provide intuition for the very low demand for liquid one period bonds. The policy functions documenting which consumers sell long term bonds mid-term and which consumers buy short term bonds are of particular interest for determining which consumers demand liquid bonds. Panel A on the left-hand side of Fig. 2 plots $B_1(t+1)$ as a function of $B_2(t)$ for the three income levels, unemployment, low employment and high employment. Any points below the diagonal indicate mid-term sales. One period bond purchases are indicated by points above the diagonal. Consumers sell mid-term two period bonds when they are in the unemployment

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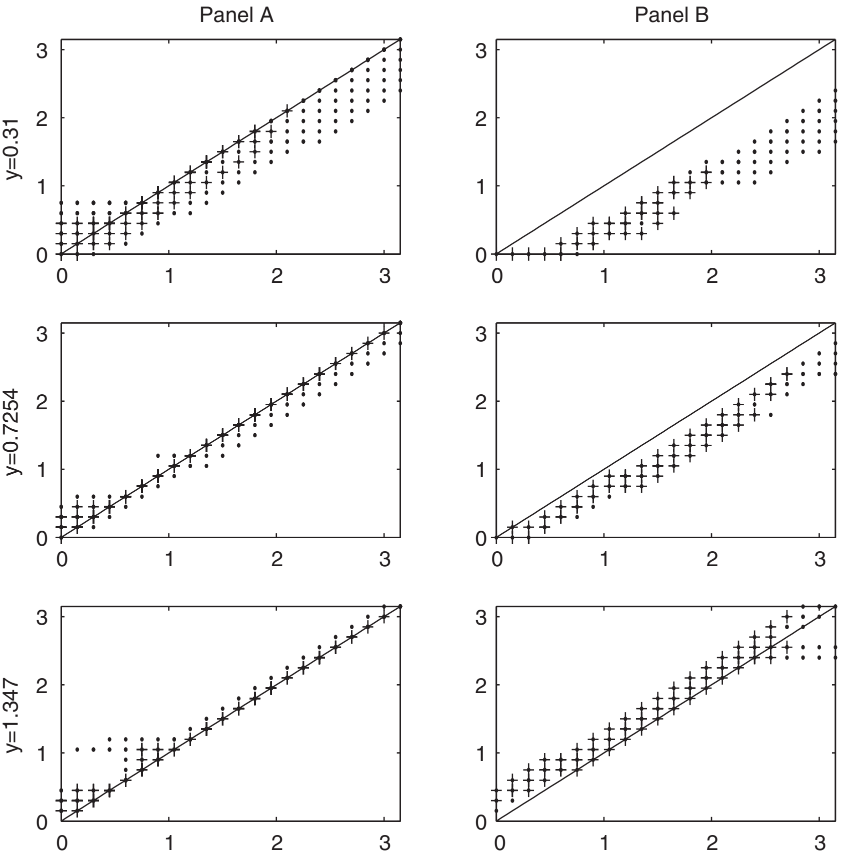

Fig. 2. Model with one and two period bonds: investment policy functions for low, medium, and high income realizations. Panel A: $B_1(t+1)$ as a function of $B_2(t)$. Panel B: $B_2(t+1)$ as a function of $B_1(t)$. '+' indicates points with significant mass in the stationary equilibrium.

state and when they are in the low employment state and hold relatively many two period bonds that mature tomorrow. Conversely, consumers buy one period bonds when they are in the high employment state and when they have few two period bonds which mature tomorrow.

Most of the mass in the stationary equilibrium is attributable to points along the diagonal, as indicated by the '+' marks. These points exhibit a high level of savings persistence with no mid-term sales or one period bond purchases. Accordingly, consumers smooth their consumption mainly with illiquid two period debt. In fact consumers smooth their consumption about as well as in the model with one period debt only. The mean level of consumption in the one period bond model is 1.0022 with a variance of 0.0083 compared to a mean of 1.0035 with a variance of 0.0090 in the one and two period bond model. Consumers accept a slightly higher variance of consumption in return for a slightly higher mean when long term claims are available at a higher rate of return.

In addition to savings being highly persistent, almost all of the action in net savings or dissavings is in changes in two period bond holdings. Panel B on the right-hand side of

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Fig. 2 plots $B_2(t+1)$ as a function of $B_1(t)$ for the three income levels. Since Panel A of Fig. 2 shows that consumers rarely purchase one period debt, one can attribute the difference between what maturing debt they hold today and what new two period bonds they purchase as savings or dissavings in terms of two period debt. As expected, net savings are increasing in income. Unemployed consumers dissave the most and consumers with high employment have the largest net increases in savings.

Clearly, for the baseline calibration, most consumers hold portfolios associated with investment policy functions which indicate neither sales of mid-term two period bonds, nor purchases of one period bonds. In this sense, consumers avoid transactions costs, as in Constantinides (1986). However, they are able to do this while trading, and holding, very few one period liquid bonds.

The following findings summarize the analysis of the baseline calibration: consumers hold far too little liquid short term bonds relative to illiquid long term bonds to match the empirical maturity structure. Savings patterns in the model are very persistent, despite the low autocorrelation of the income process. Consequently, consumers are able to smooth their consumption with illiquid two period debt as well as they do in a model with only liquid one period debt. In the model with an infinite horizon, they can stagger their long term bonds to smooth consumption without selling bonds mid-term. Consumers sell two period debt mid-term when they have low income shocks and relatively more debt maturing today than tomorrow and buy one period debt when they have relatively more debt maturing tomorrow than today.

Reasonable modifications to preferences, transactions costs and returns, and the income process do not substantially alter these results.[18] Coefficients of relative risk aversion of up to one hundred yield similar low fractions of one period bonds held. De-linking the elasticity of substitution and the coefficient of relative risk aversion following Epstein and Zin (1989) and Weil (1989) illustrates the key preference parameter governing the level of savings is the elasticity of substitution, however, low elasticities lead to high levels of savings in illiquid longer term debt. Conversely, higher discount rates, or probabilistic death, lead to lower savings but again savings are in longer term debt. The results are also insensitive to increases in transactions costs and decreases in the return differential; a minimal differential is enough to lead consumers to save in longer term debt given the persistence in savings. Introducing three period debt leads consumers to replace two period debt with three period debt. Introducing a zero income state as in Gourinchas and Parker (2002) increases savings dramatically, but again these savings are achieved via two period bonds. Similarly, persistent shocks as in Storesletten et al. (2004) cannot be smoothed away and hence do not increase one period bond holdings.

Transactions demand, or saving for lumpy durables, may be important for understanding the demand for short term, liquid bonds. To illustrate this, deterministic income fluctuations, or "pay cycles" are introduced into the income process. The income process with fluctuations is constructed by adding cycles to a two state income process which matches the unemployment rate and duration. Consumers in a particular income state fluctuate deterministically between 'bonus periods' and 'bill periods'. For one particular income state, say employment, $y_s = \bar{y}_s + pay$ in even periods (for example) and $y_s = \bar{y}_s - pay$ in odd periods, where $\bar{y}_s$ denotes average income in that particular income state. One can liken this process to alternating 'bonus' pay checks and bills. This income process

[18]The complete description of these comparative statics appears in the unpublished appendix, Eisfeldt (2006).

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Table 2
Income process comparative statics

Pay cycles		
$\bar{\Pi}_y = \begin{bmatrix} 0 & 0.3400 & 0 & 0.6600 \\ 0.3400 & 0 & 0.6600 & 0 \\ 0 & 0.0350 & 0 & 0.9650 \\ 0.0350 & 0 & 0.9650 & 0 \end{bmatrix}$	$y = [0.3109 - 0.3 \quad 0.3109 + 0.3 \quad 1.0363 - 0.3 \quad 1.0363 + 0.3]$	$y = [0.3109 - 0.3 \quad 0.3109 + 0.3 \quad 1.0363 - 0.9 \quad 1.0363 + 0.9]$
b_1	0.1479	0.4332
b_2	0.6191	0.4926
$b_1 + b_2$	0.7670	0.9259
$\frac{b_1}{b_2}$	0.2389	0.8794
$b_{2,\text{sold}}$	0.0013	0

combines that of Aiyagari and Gertler (1991) with the endowment process in Townsend (1980) in which there are even consumers who are endowed with one unit in even periods and zero units in odd periods and odd consumers who are endowed with one unit in odd periods and zero units in even periods.[19]

As before, average income is one and income while unemployed is 30% of employment income. Thus, $y = [0.3109 - pay_l \; 0.3109 + pay_l \; 1.0363 - pay_h \; 1.0363 + pay_h]$ and the transition probabilities are given in Table 2. Given this additional "transactions" motive to hold one period bonds, relative equilibrium quantities in the model with a cyclical income process are closer to the empirical maturity structure. The left two columns of Table 2 show the results of the model with cyclical income. The first column gives results for a model where the *pay* amount is equal in the unemployment and employment states, and the second column gives results for an income process where the *pay* amount in the unemployment state is one third of that in the employment state, corresponding to the relationship between average unemployment and employment income. If income is cyclical enough, the model can generate a realistic ratio of one to two period bonds. However, the cyclical income process is obviously highly negatively autocorrelated, and the cyclical income process which comes closest to matching the empirical ratio of one to two period bonds has an 89% standard deviation per quarter. But, the results of this section are suggestive. Holding liquid assets as alternatives to cash for use in transactions for lumpy durables may be a quantitatively important motive for holding liquid assets.

4. Conclusions

Empirically, the majority of outstanding government securities will mature in under one year, and the ratio of three to six month bills issued and purchased is about 1. In contrast, very few one period bonds are held by consumers who save to smooth stochastic income when faced with calibrated incomes, returns and transactions costs. Highly persistent

[19]In related work, Shen and Starr (1998) employ a turnpike model based on Townsend (1980) to interpret the term premium in interest rates as attributable to the bid ask spread on treasury bills. Shen and Starr do not calibrate their turnpike model, nor report results on equilibrium quantities.

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savings drive these results. Calibrated preferences imply that savings are highly persistent even when consumers' income processes are not. To simultaneously explain the equilibrium prices and quantities of liquid assets it will be necessary to break this persistence in savings. This paper characterizes the response of bond holdings to changes in preference parameters, transactions costs and returns, maximum available maturity and the income process. The fraction of savings comprised by short term bonds is insensitive to these changes since these modifications do not effectively reduce savings persistence. Cyclical income streams reduce savings persistence but at the cost of introducing high income variance and negative autocorrelation.

Quantifying the demand for liquid assets arising from different motives is important for understanding portfolio choice and liquidity needs, for understanding the behavior of liquidity premia over time, and for policy questions regarding liquidity provision. If investors indeed hold liquid assets primarily to smooth consumption, one would expect liquidity premia to be high when income or consumption is expected to be particularly low, and liquidity provision should be targeted at consumers. If liquid assets are instead primarily held to take advantage of new investment opportunities, then liquidity premia might instead be high when outside funding is expected to be particularly costly, and liquidity provision should be targeted at firms.

The consumption and savings literature emphasizes three main motives for saving, namely, consumption smoothing or precautionary motives, life cycle motives and the motive for intergenerational transfers, and saving for lumpy durables. The model considered here incorporates the consumption smoothing motive explicitly but abstracts from the second two motives. However, since life cycle motives and intergenerational transfers would have opposing effects on the demand for liquid assets it is unlikely that considering them would change the results here, in particular for the relatively small differences in maturities considered in this paper. Fruitful extensions might include calibrating the "transactions" demand for liquid assets, and incorporating the demand for liquid assets by firms.

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at 10.1016/j.jmoneco.2005.06.004.

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