Political Economy of Redistribution*

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Abstract

It is often argued that additional constraints on redistribution such as granting veto power to more players in the society makes property better protected from expropriation. We use a legislative bargaining-type model to demonstrate that this intuition may be flawed. Increasing the number of veto players or raising the supermajority requirement for redistribution may reduce protection on the equilibrium path. The reason is the existence of two distinct mechanisms of property rights protection. One is formal constraints that allow individuals or groups to block any redistribution that is not in their favor. The other occurs in equilibrium where players without such powers protect each other from redistribution. Players without formal veto power anticipate that the expropriation of other similar players will ultimately hurt them and thus combine their influence to prevent redistributions. In a stable allocation, the society exhibits a "class" structure with class members having equal wealth, and strategically protecting each other from redistribution.

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1 Introduction

Economists have long viewed protection of property rights as a cornerstone of efficiency and economic development (e.g., Coase, 1937, Alchian, 1965, Hart and Moore, 1990). Yet whether property rights are effectively protected depends on the political economy of the respective society and its institutions. The idea that granting veto power to different actors in the society enhances protection dates back at least to the Roman republic (Polybius [2010], Machiavelli 1515[1984]) and, in modern times, to Montesquieu's *Spirit of the Laws* (1748[1989]) and the Federalist papers, the intellectual foundation of the United States Constitution. In essay No. 51, James Madison argued for the need to contrive the government "as that its several constituent parts may, by their mutual relations, be the means of keeping each other in their proper places." Riker (1987) concurs: "For those who believe, with Madison, that freedom depends on countering ambition with ambition, this constancy of federal conflict is a fundamental protection of freedom."

In modern political economy, an increased number of veto players has been associated with beneficial consequences. North and Weingast (1989) argued that the British parliament, empowered at the expense of the crown by the Glorious Revolution in 1688, provided "the credible commitment by the government to honour its financial agreement [that] was part of a larger commitment to secure private rights". Root (1989) demonstrated that this allowed British monarchs to have lower borrowing costs compared to the French kings. In Persson, Roland, and Tabellini (1997, 2000), separation of taxing and spending decisions within budgetary decision-makings improves the accountability of elected officials and limits rent-seeking by politicians. Keefer (2004) argues that "The absence of multiple veto players in countries often means that some groups in society are less represented than they otherwise would be."

We study political mechanisms that ensure protection of property against expropriation by a majority. In practice, institutions come in different forms such as the separation of powers between the legislative, executive, and judicial branches of government, multi-cameralism, federalism, super-majority requirements and other constitutional arrangements that effectively provide some players with veto power. Examples include a president with veto powers, a supreme court that can strike down a law as unconstitutional, or the Spartan *Gerousia*, the Council of Elders, that could veto motions passed by the *Apella*, the citizens' assembly (Plutarch [2010]). In a certain polity, it might be just individuals with guns who have effective veto power. Essentially, all these institutions allow individuals or collective actors to block any redistribution without their consent. If we interpret property rights as institutions that sustain allocations unless changed by the legislature, we can formally investigate the effect of veto power on the allocation of property.

In addition to property rights, formalized in constitutions or codes of law, i.e., game forms in a theoretical model, property rights might be protected as equilibrium outcomes of interaction of strategic economic agents. The property rights of an individual may be respected not because he is powerful enough to protect them on his own, i.e., has veto power, but because others find it in their respective interest to protect such rights. Specifically, members of a coalition, formed in equilibrium, have an incentive to oppose the expropriation of each other because they know that once a member of the group is expropriated, others will be next in the line for expropriation. As a result, property rights might be secure even in the absence of explicit veto power.

If property rights may emerge from strategic behavior of rational economic agents, such rights are necessarily dynamic in nature. A status quo allocation of assets stays in place for the next period, unless it is changed by the political decision mechanism in which case the newly chosen allocation becomes the status quo for the next period. This makes the legislative bargaining model, developed initially to study outcomes of parliament procedures (Baron and Ferejohn, 1989a, 1989b), the natural foundation of our analysis. Specifically, we use legislative bargaining with endogenous status quo, the literature that started in Baron (1996) and Kalandrakis (2004, 2007), and was developed further in Anesi and Seidmann (2014, 2015), Anesi and Duggan (2015, 2016), Baron and Bowen (2013), Bowen and Zahran (2012), Diermeier and Fong (2011, 2012), Duggan and Kalandrakis (2012), Kalandrakis (2010), Richter (2013), Vartiainen (2014), and Nunnari (2016). These papers differ in fine details of the setups—e.g., Baron and Bowen (2013) argue that it is important to assume that players vote against a proposal when indifferent, while Diermeier and Fong (2011) maintain the opposite assumption—yet the legislative-bargaining literature with endogenous quo provides us with the approach and instruments to study the political economy of redistribution and protection of property from expropriation.

We discuss the legislative bargaining literature in Section 5, after we describe our model and formulate and discuss the results. Though some of our results have clear parallels in the existing literature (e.g., the characterization of stable allocations as a von Neumann-Morgenstern stable set for a certain dominance relation between alternatives), most of our results are, to the best of our knowledge, novel. Our starting point is the results of Diermeier and Fong (2011) who discovered that with a sole agenda-setter, two other players could form a coalition to protect each other from expropriation by this agenda-setter, and characterized the stable allocation in a variant of this game. Example 1 demonstrates that more generally, this logic allows for formation of coalitions of more than two players.

Example 1 Consider five players who decide how to split 10 indivisible units of wealth, with the status quo being $(1, 2, 3, 4; 0)$. Player #5 is the sole veto player and proposer, any reallocation requires a majority of votes, and we assume that when players are indifferent, they support the proposer. In a standard legislative bargaining model, the game ends when a proposal is accepted. Then, player #5 would simply build a coalition to expropriate two players, say #3 and #4, and capture the surplus resulting in $(1,2,0,0;7)$. However, this logic does not hold in a dynamic model where the agreed upon allocation can be redistributed in the subsequent periods. That is, with the new status quo $(1,2,0,0;7)$, player #5 might propose to expropriate players #1 and #2 by moving to $(0,0,0,0;10)$, which is accepted in equilibrium. Anticipating this, players #1 and #2 should not agree to the first expropriation, thus becoming the effective guarantors of property rights of players #3 and #4. Starting with $(1,2,3,4;0)$, the ultimate equilibrium allocation will be $(3,3,3,0;1)$. In general, an allocation is stable if and only if there is a group of three non-veto players of equal wealth, and the remaining player has zero.

Critically, there is a single proposer in the Diermeier and Fong's model, so there is no possibility to isolate the impact of veto power from the impact of agenda-setting power; non-veto players have no chance to be agenda-setters, and their action space is very limited. With multiple veto players and multiple agenda-setters, not necessarily the veto players, we demonstrate that the endogenous veto groups have a certain "class structure": in a stable allocation, the non-veto players are subdivided into groups of equal size, with each of which individual players have the same amount of wealth, whereas the rest of the society is fully expropriated.

Example 2 Consider the economy as in Example 1, yet 4 votes, rather than 3, are required to change the status quo. Now, if the initial status quo is $(1,2,3,4;0)$, which is unstable, the ultimate stable allocation will be $(1,1,3,3;2)$, i.e. two endogenous veto groups will be formed. In general, with 5 players, 1 veto player and 4 votes required to change the status quo, all stable sets are of the form $(x_1,x_2,x_3,x_4;x_5)$ with $x_1=x_2$ and $x_3=x_4$ - the simplest example of a society exhibiting the "class" structure.

The number and size of these classes vary as a function of the number of veto players and the supermajority requirements. Perhaps paradoxically, adding additional exogenous protection (e.g., by increasing the number of veto players) may lead to the break-down of an equilibrium with stable property rights, as the newly empowered player (the one that was granted or has acquired veto power) now no longer has an incentive to protect the others. Thus, by adding additional hurdles to expropriation in the form of veto players or super-majority requirements (see Example 4 below), the protection of property rights may in fact be eroded. In other words, players' property may be well-protected in the absences of formal constraints, while strengthening formal constraints may result in expropriation. Our next example demonstrates this effect more formally.

Example 3 As in Example 1, there are 5 players and 3 votes are required to make a change, but now there are two veto players instead of one, #4 and #5. Allocations $(x_1,x_2,x_3;x_4,x_5)$, in which at least one of players #1, #2, #3 has zero wealth and at least one has a positive amount, are unstable as the two veto players will get the vote of one player with zero and redistribute the assets of the remaining two players. One can prove that an allocation is stable if and only if $x_1 = x_2 = x_3$. This means that if we start with $(3, 3, 3, 0; 1)$, which was stable with one veto player, becomes unstable if one more player, e.g., #1, becomes a veto player. The society will transition to another state, in which all 10 units are going to be split between the two veto players.

We see here an interesting phenomenon. The naive intuition would suggest that giving one extra player (player #1 in this example) veto power would make it more difficult for player #5 to expropriate the rest of the group. However, the introduction of a new veto player breaks the stable coalition of non-veto players, and makes #5 more powerful. Before the change, non-veto players sustained an equal allocation, precisely because they were more vulnerable individually. With only one veto player and an equal allocation for players #1, #2, and #3, the three non-veto players form an endogenous veto group, which block any transition that hurts the group as a whole (or even one of them). Here, an additional veto player makes expropriation more, not less, likely. Note that both the amount of wealth being redistributed and the number of players affected by expropriation are significant. The number of players who stand to lose is two, close to half the total number of players, and more than a half of the total wealth is redistributed through voting.

In addition to granting veto rights, changes to the decision-making rule (e.g., the degree of supermajority) can also have a profound, yet somewhat unexpected effect on protection of property. Higher supermajority rules are usually considered safeguards that make expropriation more difficult, as one would need to build a larger coalition. The next example shows that this intuition is flawed as well: in a dynamic environment, increasing the supermajority requirements may trigger further redistribution.

Example 4 As above, there are 5 players that make redistributive decisions by majority, and one of which (#5) has veto power. Allocation $(3, 3, 3, 0; 1)$ is stable. Now, instead of a change in the number of veto players, consider a change in the supermajority requirements. If a new rule requires 4 votes, rather than 3, the status quo allocation becomes unstable. Instead, $(3, 3, 0, 0; 4)$ becomes stable, and this move is supported by coalition of four players out of five. (The veto-player, #5, benefits from the move, #4 is indifferent as he gets 0 in both allocations, and #1-2 support this move as they realize that with the new supermajority requirement they form a group which is sufficient to protect its members against any expropriation.) Thus, an increase in supermajority may result in expropriation and redistribution.

As Example 4 demonstrates, raising the supermajority requirement does not necessarily strengthen property rights as some players are expropriated as a result. Proposition 6 below establishes that this phenomenon, as well the one discussed in Example 3, is generic: adding a veto player or raising the supermajority requirement almost always leads to a wave of redistribution. To provide a general characterization of politically stable allocations of wealth, we use the advances of legislative bargaining models with endogenous status quo. We characterize the Markov perfect equilibrium and the stable allocations as outcomes of a non-cooperative legislative bargaining game. Then, we use the characterization to obtain the comparative statics results described in Examples 3 and 4.

Technically, our model follows the literature on legislative bargaining started by Baron and Ferejohn (1989a), especially models focused on legislative bargaining with endogenous status quo. We review this literature in Section 5, in which we compare our assumptions and conclusions with those in the literature. Specifically, we discuss the following modeling choices that were made: continuous vs. discrete choice space, the degree of farsightedness, organization (agenda-setters, veto players, protocols, recognition, etc), voting, and the possibility of inefficient allocations and waste on the equilibrium path. In addition to the legislative bargaining approach to the role of veto players on policy stability that we discuss in Section 5, another approach has conceptualized veto players as constraints on majority rule in a social choice-theoretic environment, leading to the following stark prediction: "As the number of veto players of a political system increase, policy stability increases" (Tsebelis, 2002). In Diermeier and Myerson (1999), bicameralism increases the respective chambers' willingness to exercise veto power. In a rare departure, Gehlbach and Malesky (2010) provide a formal model where more numerous veto players might help a policy reform.

Generally, political economists identified three major sources of risks to private property. First, it is the expropriation by the powerful executive (the king in the times of Adam Smith or modern dictators). Second, expropriation by private agents who undermine property rights of each other (Hobbes, 1651[1991], Braguinsky and Myerson, 2007). Finally, redistribution through over-taxation (Persson and Tabellini, 2000) or an outright expropriation by the poor majority (e.g., Grossman, 1994, Acemoglu and Robinson, 2006). A large number of works explored the relationship between a strong executive and his subjects (see, e.g., Machiavelli, 1515[1984], ch. VII, on Alexander VI against the Orsini and Colonna clans in Rome; Greif, 2006, on the institute of podesteria in medieval Italian cities; Haber et al., 2003, on the 19^{th} century Mexican presidents; or Guriev and Sonin, 2009, on Russian oligarchs). Acemoglu, Robinson, and Verdier (2004) and Padro i Miquel (2006) build formal divide-and-rule theories of expropriation, in each of which a powerful executive exploited the existing cleavages for personal gain. In addition to the legislative bargaining literature, policy evolution with endogenous quo is studied, among others, in Dixit, Grossman, and Gul (2000), Hassler, Storesletten, Mora, and Zilibotti (2003), Dekel, Jackson, and Wolinsky (2009), Battaglini and Coate (2007, 2008), and Battaglini and Palfrey (2012).

The remainder of the paper is organized as follows. Section 2 introduces our general model. In Section 3, we establish the existence of (pure-strategy Markov perfect) equilibrium in a non-cooperative game and provide full characterization of stable wealth allocations. Section 4 focuses on the impact of changes in the number of veto players or supermajority requirements. In Section 5 we compare our modeling assumptions and results with those in the literature on legislative bargaining with endogenous status quo, while Section 6 concludes. The Online Appendix contains technical proofs and some additional examples and counterexamples.

2 Setup

Consider a set N of $n = |N|$ political agents who allocate a set of indivisible identical objects between themselves. In the beginning, there are b objects, and the set of feasible allocations is therefore

$$\mathbf{A} = \left\{ x \in (\mathbb{N} \cup \{0\})^n : \sum\nolimits_{i=1}^{n} x_i \leq b \right\}.$$

We use lower index x_i to denote the amount player i gets in allocation $x \in \mathbf{A}$ throughout the paper, and we denote the total number of objects in allocation x by $\|x\| = \sum_{i \in N} x_i$.

Time is discrete and indexed by $t > 0$, and the players have a common discount factor β. In each period t, the society inherits x^{t-1} from the previous period (x^0 is given exogenously) and determines x^t through an agenda-setting and voting procedure. A transition from x^{t-1} to some alternative $y \in \mathbf{A}$ is feasible if $\|y\| \leq \left\|x^{t-1}\right\|$; in other words we allow for the objects to be wasted, but not for the creation of new objects.[1] For a feasible alternative y to defeat the status quo x^{t-1} and become x^t, it needs to gain the support of a sufficiently large coalition of agents.

To define which coalitions are powerful enough to redistribute, we use the language of winning coalitions. Let $V \subset N$ be a non-empty set of veto players (denote $v = |V|$; without loss of generality, let us assume that V corresponds to the last v agents $n-v+1, \dots n$), and let $k \in [v, n]$ be a positive integer. A coalition X is *winning* if and only if (a) $V \subset X$ and (b) $|X| \geq k$. The set of winning coalitions is denoted by $\mathcal{W}$:[2]

$$\mathcal{W} = \left\{ X \in 2^N \setminus \{\varnothing\} : |X| \geq k \text{ and } V \subset X \right\}.$$

In this case, we say that the society is governed by a k-rule with veto players V, meaning that a transition is successful if it is supported by at least k players and no veto player opposes it. We will compare the results for different k and v. We maintain the assumption that there is at least one veto player—that V is non-empty—throughout the paper; this helps us capture various political

[1] An earlier version of the model required that there is no waste, so $\left\|x^t\right\| = \left\|x^0\right\| = b$ throughout the game, and the results were identical. It should be emphasized, however, that the possibility of waste can alter the set of outcomes in a legislative bargaining model (e.g., Richter, 2013). We are grateful to an anonymous referee for suggestion to explore the possibility of waste.

[2] Acemoglu, Egorov and Sonin (2012) consider an environment where political power may depend on the state, which in this model would correspond to the current allocation. In a legislative bargaining context, it is natural to assume that the set of all possible winning coalitions is the same in each state.

institutions, e.g., a supreme court, and it is also helpful in ruling out cycles. We do not require that $k > n/2$, so we allow for minority rules. For example, 1-rule with the set of veto players $\{i\}$ is a dictatorship of player i.

Our goal is to focus on redistribution from politically weak players to politically powerful ones, and especially on the limits to such redistribution. To avoid equilibria where non-veto players shuffle the units between themselves, we assume that there is a small transition cost $\delta > 0$, borne by all players. At the same time, we want veto players to retain the ability to transfer units to their group. We thus assume that for each unit transferred in period t to a veto player, there is a small transferable budget ε available to distribute among the agents; our interpretation of this budget is that politically powerful players are able to give minor gifts in order to break indifferences and overcome the (even smaller) transition costs.[3] A feasible proposal in period t is therefore a pair (y, ξ) such that $y \in \mathbf{A}$ that satisfies $\|y\| \leq \|x^{t-1}\|$ and $\xi_i \in \mathbb{R}^n$ satisfies $\xi_i \geq \delta$ for all $i \in N$ and $\|\xi_i\| \leq \max\left(\sum_{i \in V} y_i - \sum_{i \in V} x_i^{t-1}, 0\right)$. Throughout the paper, we assume $\delta < \frac{\varepsilon}{n} < \frac{1-\beta}{nb}$. (We will show that as $\varepsilon, \delta \to 0$, the equilibria converge to some equilibria of the game where $\varepsilon = \delta = 0$; thus, focusing on equilibria that may be approximated in this way may be thought of as equilibrium refinement that rules out uninteresting equilibria, specifically the ones that feature cycles.)

The timing of the game below uses the notion of a protocol (e.g., Acemoglu, Egorov, and Sonin, 2012, Ray and Vohra, 2015). By a protocol we mean any finite sequence of players (possibly with repetition); for existence results, however, we require it to end with a veto player.[4] We denote the set of protocols by Π, so

$$\Pi = \bigcup_{\eta=1}^{\infty} \{\pi \in N^{\eta} : \pi_{\eta} \in V\}.$$

The protocol to be used is realized in the beginning of each period, taken from a distribution $\mathcal{D}$ that has full support on Π (to save on notation, we assume that each veto player is equally likely to be last one, but this assumption does not affect our results). If the players fail to reach an agreement, the status quo prevails in the next period. Thus, in each period t, each agent i gets instantaneous utility $u_i^t = x_i^t + \left(\xi_i^t - \delta\right) \mathbf{I}_{x^t \neq x^{t-1}}$ and acts as to maximize his continuation utility

$$U_i^t = u_i^t + \mathbb{E} \sum_{j=1}^{\infty} \beta^j u_i^{t+j},$$

where the expectation is taken over the realizations of the protocols in the subsequent periods. We focus on the case where the players are sufficiently forward looking; specifically, we assume

[3] In most models of legislative bargaining, it is standard to assume that whenever an agent is indifferent, she agrees to the proposal (see Section 5). Naturally, otherwise the proposer would offer an arbitrarily small amount to an indifferent player. In our model, we assume indivisible units, but allow for these infinitesimal transfers to get the same natural property.

[4] Allowing non-veto players to propose may in some cases lead to non-existence of protocol-free equilibria as Example A2 in Appendix demonstrates.

$\beta > 1 - \frac{1}{b+2}$.[5] More precisely, the timing of the game in period $t \geq 1$ is the following.[6]

1. Protocol π^t is drawn from the set of possible protocols Π.

2. For $j = 1$, player π^t_j is recognized as an agenda-setter and proposes a feasible pair $\left(z^j, \chi^j\right)$, or passes.

3. If π^t_j passed, the game proceeds to step 5; otherwise, all players vote, sequentially, in the order given by protocol π^t, *yes* or *no*.

4. If the set of those who voted *yes*, Y^j, is a winning coalition, i.e. $Y^j \in \mathcal{W}$, then the new allocation is $x^t = z^j$, the transfers are $\xi^t = \chi^j$, and the game proceeds to stage 6. Otherwise, the game proceeds to the next stage.

5. If $j < \left|\pi^t\right|$, then the game moves to stage 2 with j increased by 1. Otherwise, the society keeps the status allocation $x^t = x^{t-1}$, and the game proceeds to the next stage.

6. Each player i receives an instantaneous payoff u^t_i.

The equilibrium concept we use is Markov Perfect equilibrium (MPE). In any such equilibrium σ, the transition mapping $\phi = \phi^\sigma : \mathbf{A} \times \Pi \to \mathbf{A}$, which maps the previous period's allocation and the protocol realization for the current period into the current period's allocation, is well-defined. In what follows, we focus on protocol-free equilibria (protocol-free MPE[7]), namely, σ such that $\phi^\sigma(x, \pi) = \phi^\sigma(x, \pi')$ for all $x \in \mathbf{A}$ and $\pi, \pi' \in \Pi$. We thus abuse notation and write $\phi = \phi^\sigma : \mathbf{A} \to \mathbf{A}$ to denote the transition mapping of such equilibria.

3 Analysis

Our strategy is as follows. We start by proving some basic results about equilibria of the non-cooperative game described above. Then, we characterize stable allocations, i.e. allocations with no redistribution, and demonstrate that the stable allocations correspond to equilibria of the non-cooperative game. We then proceed to studying comparative statics with respect to the number of veto players, different voting rules (majority requirements), and equilibrium paths that follow an exogenous shock to some players' wealth.

[5] This condition means that a player prefers $x + 1$ units tomorrow to x units today, for any $x \leq b + 1$. This assumption is relatively weak compared to some models of legislative bargaining that require β to approach 1 (see discussion in Section 5).

[6] There are many game forms that would yield identical results. For example, we could have each agenda-setter nominate an alternative and then proceed to choosing one that will be put for a vote against the status-quo. To simplify the exposition and proofs, we opted for a simpler game. An earlier version of the paper had a particular case of this setup with $\varepsilon = \delta = 0$, but made particular assumptions on breaking indifferences.

[7] See Examples A4 and A5 in the Appendix, where allowing for non-Markov strategies or dropping the requirement that transitions be the same for every protocol can lead to counterintuitive equilibria. Arguably, those equilibria are less robust, because they do not survive our refinements.

3.1 Non-cooperative Characterization

Consider a protocol-free MPE σ, and let $\phi = \phi_\sigma$ be the transition mapping that is generated by σ and is defined in the end of Section 2. (Using transition mappings, rather than individuals' agenda-setting and voting strategies, allows us to capture equilibrium paths in terms of allocations and transitions, i.e., in a more concise way). Iterating the mapping ϕ gives a sequence of mappings $\phi, \phi^2, \phi^3, \ldots : \mathbf{A} \to \mathbf{A}$, which must converge if ϕ is acyclic. (Mapping ϕ is *acyclic* if $x \neq \phi(x)$ implies $x \neq \phi^\tau(x)$ for any $\tau > 1$; we will show that every MPE satisfies this property.) Denote this limit by ϕ^∞, which is simply ϕ^τ for some τ as the set $\mathbf{A}$ is finite. We say that mapping ϕ is *one-step* if $\phi = \phi^\infty$ (this is equivalent to $\phi = \phi^2$), and we call an MPE σ *simple* if ϕ_σ is one-step. Given an MPE σ, we call allocation x *stable* if $\phi_\sigma(x) = x$. Naturally, ϕ_σ^∞ maps any allocation into a stable allocation.

Our first result deals with existence of an equilibrium and its basic properties.

Proposition 1 *Suppose $\beta > 1 - \frac{1}{b+2}$, $\varepsilon < \frac{1-\beta}{b}$, and $\delta < \frac{\varepsilon}{n}$. Then:*

1. *There exists a protocol-free Markov Perfect Equilibrium σ.*
2. *Every protocol-free MPE is acyclic.*
3. *Every protocol-free MPE is simple.*
4. *Every protocol-free MPE is efficient, in that it involves no waste (for any $x \in \mathbf{A}$, $\|\phi(x)\| = \|x\|$).*

These results are quite strong, and are made possible by the requirement that the equilibrium be protocol-free. For a fixed protocol, equilibria might involve multiple iterations before reaching a stable allocations (see Example A3 in the Appendix). However, these other equilibria critically depend on the protocol and are therefore fragile; in contrast, transition mappings supported by protocol-free MPE are robust (e.g., they would remain the protocols are taken from a different distribution, for example).

The proof of Proposition 1 is technically cumbersome and is relegated to the Appendix. However, the idea is quite simple. We construct a candidate transition mapping ϕ_σ that we want to be implemented in the equilibrium. If the society starts the period in state $x = x^{t-1}$ such that $\phi(x) = x$, we verify that it is a best response for the veto players to block any transitions except for those that are harmful to too many non-veto players, and thus x remains intact. If the society starts the period in state x such that $\phi(x) \neq x$, we verify that there is a feasible vector of transfers that may be redistributed from those who strictly benefit from such transition to those who are indifferent, and that the society would be able to agree on such vector over the course of the protocol. The second result, the acyclicity of MPE, relies on the presence of transaction costs, which rules out the possibility of non-veto players shuffling the objects around (example A1 in the Appendix illustrates cyclic equilibria that would exist in the absence of this assumption). To show that every protocol-free MPE is simple, we show that if there were an allocation from where the society would expect to reach a stable allocation in exactly two steps, then for a suitable protocol it would instead decide to skip the intermediate step and transit to the stable allocation immediately. Finally, given that every MPE is simple, the society may always allocate the objects that would otherwise be wasted to some veto player (e.g., the proposer) without facing adverse dynamic consequences ("the slippery slope"), which ensures that each transitions involves no waste and the allocations are efficient.

The following corollary highlights that the possibility of transfers may be viewed as an equilibrium refinement.

Corollary 1 *Suppose that for game Γ with parameter values $\beta, \varepsilon, \delta$ as in Proposition 1, $\phi = \phi_\sigma$ is the transition mapping that corresponds to a protocol-free MPE σ. Then consider game Γ' with the same $\beta' = \beta$, but $\varepsilon' = \delta' = 0$. Then there exists protocol-free MPE σ' with the same transition mapping $\phi_{\sigma'} = \phi$.*

Despite these characterization results stated in Proposition 1, the equilibrium transitions are not necessarily unique. The following Example 5 demonstrates this. Notice, however, that the set of stable allocations that may be reached with these equilibria is the same.

Example 5 Suppose there are $b = 3$ units of wealth, 4 agents, the required number of votes is $k = 3$, and the set of veto players is $V = \{\#4\}$. In this case, there is a simple equilibrium with transition mapping ϕ, under which allocations $(0,0,0;3)$, $(1,1,0;1)$, $(1,0,1;1)$ and $(0,1,1;1)$ are stable. Specifically, we have the following transitions: $\phi(2,1,0;0) = \phi(1,2,0;0) = (1,1,0;1)$; $\phi(0,2,1;0) = \phi(0,1,2;0) = (0,1,1;1)$; $\phi(2,0,1;0) = \phi(1,0,2;0) = \phi(1,1,1;0) = (1,0,1;1)$; and any allocation with $x_4 = 2$ has $\phi(x) = (0,0,0;3)$. However, another mapping ϕ' coinciding with ϕ except that $\phi'(1,1,1;0) = (1,1,0;1)$ may also be supported in equilibrium.

3.2 Stable Allocations

Our next goal is to get a more precise characterization of equilibrium mappings and stable allocations. Let us define a binary relation $\rhd$ (interpreted as a dominance relation) on $\mathbf{A}$ as follows:

$$y \rhd x \Longleftrightarrow \|y\| \leq \|x\| \text{ and } \{i \in N : y_i \geq x_i\} \in \mathcal{W} \text{ and } y_j > x_j \text{ for some } j \in V.$$

Intuitively, allocation y dominates allocation x if transition from x to y is feasible and some powerful player prefers y to x strictly so as to be willing to make this motion, and also there is a winning coalition that (weakly) prefers x to y. Note that this does not imply that y will be proposed or supported in an actual voting against x because of further changes this move may lead to. Following the standard definition (von Neumann and Morgenstern, 1947; Greenberg, 1990), we call a set of states $\mathbf{S} \subset \mathbf{A}$ *von Neumann-Morgenstern- (vNM-)stable* if the following two conditions hold: (i) For no two states $x, y \in \mathbf{S}$ it holds that $y \rhd x$ (*internal stability*); and (ii) For each $x \notin \mathbf{S}$ there exists $y \in \mathbf{S}$ such that $y \rhd x$ (*external stability*).

The significance of this notion of vNM-stability is demonstrated by the following proposition.

Proposition 2 *For any protocol-free MPE σ, the set of stable allocations $\mathbf{S}_\sigma = \{x \in \mathbf{A} : \phi_\sigma(x) = x\}$ is a von Neumann-Morgenstern stable set for the dominance relation $\rhd$.*

This characterization is made possible by the requirement that the equilibrium be protocol-free. For a fixed protocol, there may exist other equilibria, where non-veto players propose alternatives that hurt themselves, but hurt other non-veto players even more, only to avoid other non-veto players do the opposite (see Example A3). However, these other equilibria critically depend on the protocol and are therefore fragile. In contrast, transition mappings supported by protocol-free MPE are robust (e.g., they would remain if the protocols are taken from a different distribution, for example).

Proposition 2 implies that the fixed points of transition mappings of non-cooperative equilibria described in Proposition 1 correspond to a von Neumann-Morgenstern stable set. Our next result states that such stable set is also unique; this implies, in particular, that for any two protocol-free MPE σ and σ', the set of stable allocations is identical. Consequently, even though equilibrium is not unique, we are able to study stable allocations irrespective of a particular equilibrium of the bargaining game.

Proposition 3 also gives a precise characterization of stable allocations. To formulate it, let us denote $m = n - v$, the number of non-veto players; $q = k - v$, the number of non-veto players that is required in any winning coalition; $d = m - q + 1 = n - k + 1$, the size of a minimal blocking coalition of non-veto players; and, finally, $r = \lfloor m/d \rfloor$, the maximum number of pairwise disjoint blocking coalition that non-veto players may be split into.

Proposition 3 1. *For the binary relation $\rhd$, a vNM-stable set exists and is unique.*

2. *Each element x of this set $\mathbf{S}$ has the following structure: the set of non-veto players $M = N \backslash V$ may be split into a disjoint union of r groups $G_1, \ldots, G_r$ of size d and one (perhaps empty) group G_0 of size $m - rd$, such that inside each group, the distribution of wealth is equal: $x_i = x_j = x_{G_k}$ whenever $i, j \in G_k$ for some $k \geq 1$, and $x_i = 0$ for any $i \in G_0$. In other words,*

$x \in \mathbf{S}$ if and only if the non-veto players can be permuted in such a way that

$$x = \left(\underbrace{\lambda_1, \ldots, \lambda_1}_{d\ times}, \underbrace{\lambda_2, \ldots, \lambda_2}_{d\ times}, \ldots, \underbrace{\lambda_r, \ldots, \lambda_r}_{d\ times}, \underbrace{0, \ldots, 0}_{m-rd\ times}; \underbrace{x_{m+1}, \ldots, x_n}_{veto\ players} \right).$$

for some $\lambda_1 \geq \cdots \geq \lambda_r \geq 0$ such that $d \sum_{j=1}^{r} \lambda_j + \sum_{l=1}^{n-m} x_{m+l} \leq b$.

The proof of this result is important for understanding the structure of endogenous veto groups, and we prove it in the text. We show that starting from any wealth allocation $x \in \mathbf{S}$, it is impossible to redistribute the units between agents without making at least d agents worse off, and thus no redistribution would gain support from a winning coalition. In contrast, starting from any allocation $x \notin \mathbf{S}$, such redistribution is possible. Furthermore, our proof will show that there is an equilibrium where in any transition, the set of individuals who are worse off is limited to the $d-1$ richest non-veto players.

Proof of Proposition 3. We will prove that set $\mathbf{S}$, as defined in Part 2, is vNM-stable, thus ensuring existence. To show internal stability, suppose that $x, y \in \mathbf{S}$ and $y \rhd x$, and let the r groups be $G_1, \ldots, G_r$ and $H_1, \ldots, H_r$, respectively. Without loss of generality, we can assume that each set of groups is ordered so that x_{G_j} and y_{H_j} are non-increasing in j for $1 \leq j \leq r$. Let us prove, by induction, that $x_{G_j} \leq y_{H_j}$ for all j.

The induction base is as follows. Suppose that the statement is false and $x_{G_1} > y_{H_1}$; then $x_{G_1} > y_s$ for all $s \in M$. This yields that for all agents $i \in G_1$, we have $x_i > y_i$. Since the total number of agents in G_1 is d, G_1 is a blocking coalition, and therefore it cannot be true that $y_j \geq x_j$ for a winning coalition, contradicting that $y \rhd x$.

For the induction step, suppose that $x_{G_l} \leq y_{H_l}$ for $1 \leq l < j$, and also assume, to obtain a contradiction, that $x_{G_j} > y_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > y_{H_s}$. Consequently, for agent $i \in \bigcup_{l=1}^{j} G_l$ to have $y_i \geq x_i$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)d = d$ agents in $\bigcup_{l=1}^{j} G_l \subset M$, it cannot be the case that $y_i \geq x_i$, which contradicts the assumption that $y \rhd x$. This establishes that $x_{G_j} \leq y_{H_j}$ for all j, and therefore $\sum_{i \in M} x_i \leq \sum_{i \in M} y_i$. But $y \rhd x$ would require that $x_i \leq y_i$ for all $i \in V$ with at least one inequality strict, which implies $\sum_{i \in N} x_i < \sum_{i \in N} y_i$, a contradiction to $\|y\| \leq \|x\|$. This proves internal stability of set $\mathbf{S}$.

Let us now show that the external stability condition holds. To do this, we take any $x \notin \mathbf{S}$ and will show that there is $y \in \mathbf{S}$ such that $y \rhd x$. Without loss of generality, we can assume that x_i is non-increasing for $1 \leq i \leq m$ (i.e., non-veto players are ordered from richest to poorest). Let us denote $G_j = \{(j-1)d + 1, \ldots, jd\}$ for $1 \leq j \leq r$ and $G_0 = M \setminus \left(\bigcup_{j=1}^{r} G_j \right)$. Since $x \notin \mathbf{S}$, it must be that either for some G_j, $1 \leq j \leq r$, the agents in G_j do not get the same allocation, or they do, but some individual $i \in G_0$ has $x_i > 0$. In the latter case, we define y by

$$y_i = \begin{cases} x_i & \text{if } i \le dr \text{ or } i > m+1; \\ 0 & \text{if } dr < i \le m; \\ x_i + \sum_{j \in G_0} x_j & \text{if } i = m+1 \end{cases}$$

(In other words, we take everything possessed by individuals in G_0 and distribute it among veto players, for example, by giving everything to one of them). Obviously, $y \in \mathbf{S}$ and $y \rhd x$.

If there exists a group G_j such that not all of its members have the same amount of wealth, let j be the smallest such number. For $i \in G_l$ with $l < j$, we let $y_i = x_i$. Take the first $d-1$ members of group G_j, $Z = \{(j-1)\,d+1, \ldots, jd-1\}$. Together, they possess $z = \sum_{i=(j-1)d+1}^{jd-1} x_i > (d-1)\,x_{jd}$ (the inequality is strict precisely because not all x_i in G_j are equal). Let us now take these z units and redistribute it among all the agents (perhaps including those in Z) in the following way. For each $s : j < s < r$, we let $y_{(s-1)d} = y_{(s-1)d+1} = \cdots = y_{sd-1} = x_{(s-1)d}$; this makes these d agents having the same amount of wealth and being weakly better off as the agent with number $(s-1)\,d$ was the richest among them.

Now, observe that in each group s, we spent at most $(d-1)\left(x_{(s-1)d} - x_{sd-1}\right) \le (d-1)\left(x_{(s-1)d} - x_{sd}\right)$. For $s = r$, we take d agents as follows: $D = \{(r-1)\,d, \ldots, m\} \cup Z'$, where $Z' \subset Z$ is a subset of the first $d - (m - (r-1)\,d + 1) = rd - m - 1$ agents needed to make D a collection of exactly d agents (notice that $Z' = \varnothing$ if $|G_0| = d-1$ and $Z' = Z$ if $G_0 = \varnothing$). For all $i \in D$, we let $y_i = x_{(r-1)d}$ (making all members of G_0 weakly better off and spending at most $(d-1)\,x_{(r-1)d}$ units) and we let $y_i = 0$ for each $i \in Z \setminus Z'$. We have thus defined y_i for all $i \in M$ and distributed

$$c \le (d-1)\left(x_{jd} - x_{(j+1)d} + \cdots + x_{(r-2)d} - x_{(r-1)d} + x_{(r-1)d}\right) = (d-1)\,x_{jd},$$

having $z - c > 0$ remaining in our disposal. As before, we let $y_{m+1} = x_{m+1} + z - c$ and $y_i = x_i$ for $i > m+1$. We have constructed $y \in \mathbf{S}$ such that $\|y\| = \|x\|$, $y_{m+1} > x_{m+1}$ and $\{i \in N : y_i < x_i\} \subset Z$. The latter, given $|Z| \le d-1$, implies $\{i \in N : y_i \ge x_i\} \in \mathcal{W}$, which means $y \rhd x$. This completes the proof of external stability, and thus $\mathbf{S}$ is vNM-stable.

Let us now show that $\mathbf{S}$ is a unique stable set defined by $\rhd$.[8] Suppose not, so there is $\mathbf{S}'$ that is also vNM-stable. Let us prove that $x \in \mathbf{S} \Leftrightarrow x \in \mathbf{S}'$ by induction on $\sum_{i \in M} x_i$. The induction base is trivial: if $x_i = 0$ for all $i \in M$, then $x \in \mathbf{S}$ by definition of $\mathbf{S}$. If $x \notin \mathbf{S}'$, then there must be some y such that $y \rhd x$. But for such y,

$$\sum_{i \in N} y_i \ge \sum_{i \in V} y_i > \sum_{i \in V} x_i = \sum_{i \in N} x_i,$$

which contradicts $\|y\| \le \|x\|$.

[8] An alternative (non-constructive) way to prove uniqueness would be to use a theorem by von Neumann and Morgenstern (1947) that states that if a dominance relation allows for no finite or infinite cycles, the stable set is unique.

The induction step is as follows. Suppose that for some x with $\sum_{i\in M} x_i = j > 0$, $x \in \mathbf{S}$ but $x \notin \mathbf{S}'$ (the vice-versa case is treated similarly). By external stability of $\mathbf{S}'$, $x \notin \mathbf{S}'$ implies that for some $y \in \mathbf{S}'$, $y \rhd x$, which in turn yields that $\sum_{i\in V} y_i > \sum_{i\in V} x_i$ and $\|y\| \leq \|x\|$. We have

$$\sum\nolimits_{i\in M} y_i = \|y\| - \sum\nolimits_{i\in V} y_i < \|x\| - \sum\nolimits_{i\in V} x_i = \sum\nolimits_{i\in M} x_i = j.$$

For y such that $\sum_{i\in M} y_i < j$ induction yields that $y \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}'$, and thus $y \in \mathbf{S}$. Consequently, there exists some $y \in \mathbf{S}$ such that $y \rhd x$, but this contradicts $x \in \mathbf{S}$. This contradiction establishes uniqueness of the stable set.■

Proposition 3 enables us to study the set of stable allocations $\mathbf{S}$ without reference to a particular equilibrium σ. The characterization obtained in this Proposition gives several important insights. First, the set of stable allocations (fixed points of any transition mapping under any equilibrium) does not depend on the mapping; it maps into itself when either the veto players V or the non-veto players $N \setminus V$ are reshuffled in any way. Second, the allocation of wealth among veto players does not have any effect on stability of allocations. Third, each stable allocation has a well-defined "class" structure: every non-veto player with a positive allocation is part of a group of size d (or a multiple of d) of equally-endowed individuals who have incentives to protect each other's interests.[9]

To demonstrate how this protection works, consider the following example.

Example 6 There are $b = 12$ units, $n = 5$ individuals with one veto player (#5), and the supermajority of 4 is needed for a transition ($k = 4$). By Proposition 3, stable allocations have two groups of size two. Let ϕ be a transition mapping for some simple MPE σ, and let us start with stable allocation $x = (4, 4, 2, 2; 0)$. Suppose that we exogenously remove a unit from player #2 and give it to the veto player; i.e., consider $y = (4, 3, 2, 2; 1)$. Allocation y is unstable, and player #1 will necessarily be expropriated. However, the way redistribution may take place is not unique; for example, $\phi(y) = (3, 3, 2, 2; 2)$ is possible, but so is $\phi(y) = (2, 3, 3, 2; 2)$ or $\phi(y) = (2, 3, 2, 3; 2)$. Now suppose that one of the players possessing two units, say player #3, was expropriated, i.e., take $z = (4, 4, 1, 2; 1)$. Then it is possible that the other member, player #4, would be expropriated as well: $\phi(z) = (4, 4, 1, 1; 2)$. But it is also possible that one of the richer players may be expropriated instead: e.g., a transition to $\phi(z) = (4, 1, 1, 4; 2)$ would be supported by all players except #2.

Example 6 demonstrates that equilibrium protection that agents provide to each other extends beyond members of the same economic class. In the latter case, player #2 would oppose a move from $(4, 4, 2, 2; 0)$ to $(4, 4, 1, 2; 1)$ if in the subgame the next move is to $(4, 1, 1, 4; 2)$. This corresponds to a coalition among economic classes.

[9] It is permissible that two groups have equal allocations, $x_{G_j} = x_{G_k}$, or that members of some or all groups get zero. In particular, any allocation x where $x_i = 0$ for all $i \in M$ is in $\mathbf{S}$. Notice that if non-veto players get the same under two allocations x and y, so $x|_M = y|_M$, then $x \in \mathbf{S} \Leftrightarrow y \in \mathbf{S}$; moreover, this is true if $x_i = y_{\pi(i)}$ for all $i \in M$ and some permutation π on M.

We see that in general, an exogenous shock may lead to expropriation, on the subsequent equilibrium path, of players belonging to different wealth groups; the particular path depends on the equilibrium mapping, which is not unique. However, if we apply the refinement that only equilibria with a "minimal" (in terms of the number of units that need to be transferred) redistribution along the equilibrium path are allowed, then only the players with exactly the same wealth endowment would suffer from the redistribution that follows a shock. More importantly, Example 6 demonstrates the mechanism of mutual protection among players with the same wealth. If a non-veto player becomes poorer, at least $d-1$ other players would suffer in the subsequent redistribution. This makes them willing to oppose any redistribution from any of their members. Their number, if we include the initial expropriation target himself, is d, which is sufficient to block a transition. Thus, members of the same economic class have an incentive to act as a politically cohesive coalition, in which its members mutually protect each others' economic interests.

Proposition 3 also allows for the following simple corollary.

Corollary 2 *Suppose that in game Γ defined above, the set of stable allocations (in any protocol-free MPE) is $\mathbf{S}$. Take any integer $h>1$, and consider the set of allocations $\mathbf{A}^h$ given by*

$$\mathbf{A}^h = \left\{x \in \left(\mathbb{R}^+\right)^n : \|x\| \leq b \text{ and } \forall i \in N, hx_i \in \mathbb{Z}\right\}.$$

Suppose $\beta > 1 - \frac{1}{bh+2}$, $\varepsilon < \frac{1-\beta}{bh}$, $\delta < \frac{\varepsilon}{n}$. Then the set of stable allocations in the new game Γ^h (again, in any protocol-free MPE) $\mathbf{S}^h$ satisfies $\mathbf{S} \subset \mathbf{S}^h$.

In other words, taking a finer partition of units of redistributions (splitting each unit into h indivisible parts) preserves stable allocations. This result follows immediately from Proposition 3 Part 2. It effectively says that even though our results are obtained under the assumption of discrete number of indivisible units, they have a broader appeal: once dividing units into several parts is allowed, the stable allocations remain stable. This implies that the set $\mathbf{S}$ does not only describe stable outcomes for any appropriately refined equilibrium within the game, but is also a robust predictor of stable allocations if the minimal units are redefined, provided, of course, that players interact frequently enough.[10]

The next proposition generalizes Example 6 so that one can better understand the mechanics of mutual protection. It highlights that protection of a non-veto player is sustained, in equilibrium,

[10] Notice that since the sequence of stable sets satisfies $\mathbf{S} \subset \mathbf{S}^2 \subset \mathbf{S}^3 \subset \cdots$, their limit is a well-defined set $\mathbf{S}^\infty = \overline{\bigcup_{j>1} \mathbf{S}^j}$, where the bar denotes topological closure. This set has the following simple structure:

$$\mathbf{S}^\infty = \left\{x \in \Delta \mid \exists \rho \in S^n : x_{\rho(1)} = \cdots = x_{\rho(d)}, x_{\rho(d+1)} = \cdots = x_{\rho(2d)}, \ldots, x_{\rho((r-1)d+1)} = \cdots = x_{\rho(rd)}\right\},$$

where Δ is the $(N-1)$-dimensional unit simplex and $\rho \in S^n$ is any permutation. However, for these limit allocations to be approached in a noncooperative game that we study, one would have to take a sequence of discount factors β_j that tends to 1, so interactions should be more and more frequent. Intuitively, to study fine partitions of the state space, one would need finer partition of time intervals as well to prevent 'undercutting'. If this condition does not hold, veto players would be able to expropriate everything in the long run would apply (see, e.g., Nunnari, 2016).

by equally endowed or richer individuals, rather than by those who has less wealth. Proposition 4 is formulated as follows. We take some equilibrium characterized in Proposition 1, and consider a stable allocation. Then, we consider another, perturbed, allocation, in which one non-veto player has less wealth. We show that the resulting allocation is unstable, and compare the ultimate stable allocation with the initial one (before the perturbation).

Proposition 4 *Consider any MPE σ and let $\phi = \phi_\sigma$. Suppose that the voting rule is not unanimity ($k < n$), so $d > 1$. Take any stable allocation $x \in \mathbf{S}$, some non-veto player $i \in M$, and let new allocation $y \in \mathbf{A}$ be such that $y|_{M\setminus\{i\}} = x|_{M\setminus\{i\}}$ and $y_i < x_i$. Then:*

1. *Player i will never be as well off as before the shock, but he will not get any worse off: $y_i \leq [\phi(y)]_i < x_i$. Furthermore, the number of players who suffer as a result of a redistribution on the equilibrium path defined by σ is given by:*

$$\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| = d - 1;$$

2. *Suppose, in addition, that for any $k \in M$ with $x_k < x_i$, $x_k \leq y_i$, i.e., the shock did not make player i poorer than the players in the next wealth group. Then $[\phi(y)]_j < y_j$ implies $x_j \geq x_i$, i.e., members of poorer wealth groups do not suffer from redistribution.*

The essence of Proposition 4 is that following a negative (exogenous) shock to some player's wealth ($y_i < x_i$), at least $d-1$ other players are expropriated, and player i never fully recovers. If the shock is relatively minor so the ranking of player i with respect to other wealth groups did not change (weak inequalities are preserved),[11] then it must be equally endowed or richer people who suffer from subsequent redistribution. Thus, in the initial stable allocation x, they have incentives to protect i from the negative shock. This result may be extended to the case when a negative shock affects more than one (but less than d) non-veto players. The proof is straightforward when all the affected players belong to the same wealth group. However, this requirement is not necessary. If expropriated players belong to different groups, then the lower bound of the resulting wealth after redistribution is the amount of wealth that the poorest (post-shock) player possesses. In this case, the number of players who suffer as a result of the redistribution following the shock is still limited by $d-1$.

Our next step is to derive comparative statics with respect to different voting rules given by k and v.

[11] Note that this will always be the case if, e.g., $y_i = x_i - 1$.

4 Comparing Voting Rules

Suppose that we vary the supermajority requirement, k, and the number of veto players, v. The following result easily follows from the characterization in Proposition 3.

Proposition 5 *Fix the number of individuals n.*

1. *The size of each group G_j, $j \geq 1$, is decreasing as the majority requirement k increases. In particular, for $k = v + 1$, $d = n - v = m$, and thus all the non-veto players form a single group; for $k = n$ (unanimity rule), $d = 1$, and so each player can veto any change.*

2. *The number of groups is weakly increasing in k, from 1 when $k = v + 1$ to m when $k = n$ (from 0 when $k < v + 1$).*

3. *The size of each group G_j, $j \geq 1$ does not depend on the number of veto players, but as v increases, the number of groups weakly decreases, reaching zero for $v > n - d$.*

This result thus implies that the size of groups does not depend on the number of veto players, but only on the majority requirement as it determines the minimal size of blocking coalitions. As the majority requirement increases, groups become smaller. This has a very simple intuition: as redistribution becomes harder (it is necessary to get approval of more players), it takes fewer non-veto players to defend themselves; as such, smaller groups are sufficient. Conversely, the largest group (all non-veto players together) is formed when a single vote from a non-veto player is sufficient for veto players to accept a redistribution; in this case, non-veto players can only keep a positive payoff by holding equal amounts.

Now, consider the number of groups that (non-veto part of) the society is divided into. Intuitively, the number of groups corresponds to the maximum possible economic heterogeneity that a society can have in equilibrium. If we interpret the equally-endowed non-veto members of the society as economic classes (in the sense that members of the same class have similar possessions, whereas members of different classes have different fortunes, despite having the same political power), then the number of groups would correspond to the largest number of economic classes that the society can contain. With this interpretation, Proposition 4 implies that it is members of the same or richer economic classes that protect a non-veto player from expropriation. Notice that we cannot make predictions about the exact number of classes that would be realized. For example, for any parameters it is possible that all non-veto players possess zero and thus belong to the same class; similarly, Part 2 of Proposition 3 allows for classes that are larger than others and that span several groups G_j. Thus, societies with few groups are bound to be homogenous (among non-veto players), whereas societies with many veto groups have the ability to be heterogenous, at least with economic fortunes.

To better understand the determinants of the number of groups, consider the following. Take n large and v small (so that m is large enough to be interesting) and start with the smallest possible value of $k = v+1$. Then all the non-veto players possess the same allocation in any equilibrium. In other words, all players, except perhaps those explicitly endowed with veto power, must be equal. If we increase k, then two groups will form, one of which may possess a positive amount, while the rest possesses zero, which is clearly more heterogenous than for $k = v + 1$. If we increase k further beyond $v + (m+1)/2$, then both groups may possess positive amounts and a third group will form further, etc. In other words, as k increases, so does the number of groups, which implies that the society becomes less and less homogenous and can support more and more groups of smaller size. We see that in this model, heterogeneity of the society is directly linked to difficulty of expropriation, measured by the degree of majority needed for expropriation or, equivalently, by the minimal size of a coalition that is able to resist attempts to expropriate. If we interpret the equally-endowed groups as economic classes, then we have the following result: the more politically difficult it is to expropriate, the more fine is the class division of the society.

Corollary 3 *Suppose that $k = v+1$; as before, $d = n - v$. In this case, an allocation x is stable if $x_i = x_j$ for all non-veto players i and j, i.e., if all non-veto players hold the same amount. More generally, a single group of non-veto players with positive amount of wealth may be formed if and only if $k - v \equiv q \leq (m+1)/2$. In this case, some $n - k + 1$ non-veto players belong to the group and get the same amount, and the rest get zero.*

Proposition 5 dealt with comparing stable allocations for different k and v. We now study whether or not an allocation that was stable under some rules k and v remains stable if these rules change. For example, suppose that we make an extra individual a veto player (increase v), or increase the majority rule requirement (increase k). A naive intuition would say that in both these cases, individuals would not be worse off from better property rights protection. As the next proposition shows, in general, the opposite is likely to be true. Let $\mathbf{S}_{k,V}$ denote the set of stable allocations under the supermajority requirement k and the set of veto players V.

Proposition 6 *Suppose that allocation x is stable for k ($k < n$) and v ($x \in \mathbf{S}_{k,V}$). Then:*

1. *If we increase the number of veto players by granting an individual $i \notin V$ veto power so that the new veto set is $V \cup \{i\}$, then allocation $x \in \mathbf{S}_{k,V\cup\{i\}}$ if and only if $x_i = 0$;*
2. *Suppose $k + 1 < n$ and all groups G_j, $j \geq 0$, had different amounts of wealth under x: $x_{G_j} \neq x_{G_{j'}}$ for $j' \neq j$ (and $x|_M \neq 0$). If we increase the majority requirement from k to $k' = k+1$, and $k' < n$, then $x \notin \mathbf{S}_{k+1,V}$.*

The first part of this proposition suggests that adding a veto player makes an allocation unstable, and therefore will lead to a redistribution hurting some individual. There is only one exception to this rule: if the new veto player had nothing to begin with, then the allocation will remain stable. On the other hand, if the new veto player had a positive amount of the good, then, while he will be weakly better off from becoming a veto player, there will be at least one other non-veto player who will be worse off. Indeed, removing a member of one of the groups G_j without changing the required sizes of the groups *must* lead to redistribution. This logic would not apply if $V' = N$, when all players become veto players; however, the proposition is still true in this case because then i would have to be the last non-veto player, and under $k < n$ he would have to get $x_i = 0$ in a stable allocation x. Interestingly, removing a veto player i (making him non-veto) will also make x unstable as long as $x_i > 0$. This is, of course, less surprising, as this individual may be expected to be worse off.

The second part says that if all groups got different allocations (which is the typical case), then an increase in k would decrease the required group sizes, leading to redistribution. When some groups have equal amounts of wealth in a stable allocation, then allocation x may, in principle, remain stable. This is trivially true when all non-veto players get zero ($x_i = 0$ for all $i \notin V$), but, as the following Example 7 demonstrates, this is possible in other cases as well.

Example 7 Suppose $n = 7$, $V = \{\#7\}$, $b = 6$ and the supermajority requirement is $k = 5$. Then $x = (1, 1, 1, 1, 1, 1; 0)$ is a stable allocation, because $d = 3$ and the non-veto players form two groups of size three. If we increase k to $k' = 6$, then x remains stable, as then $d' = 2$ and x has three groups of size two.

5 Discussion

Given our focus on strategic foundation of redistribution by voting, it is natural that we use the machinery of legislative bargaining with endogenous status quo that started with the work of Baron (1996) and Kalandrakis (2004). With respect to specific modeling assumptions, important contributions differentiate along the following lines. First, some models assume a continuous, while the other a discrete (e.g., finite) allocation space. Second, models differ with respect to the degree of farsightedness, which is, essentially, whether or not the results require bringing the discount factor close to 1. Third, there are multiple ways to set up the agenda-setting structure (protocols, recognition, etc). Fourth, voting rules might be different, ranging from minority quotes to the requirement of unanimity. Finally, some, but not all, models allow for waste of available resources. Each of these choices has been demonstrated to be consequential. In what follows, we compare our modeling choice with the ones made in the literature, and discuss how they affect the results.

In Baron and Ferejohn (1989a), with a closed amendment rule, any proposer makes a policy proposal that fully expropriates all voters who are not members of the minimal winning coalition. Bernheim, Rangel, and Rayo (2006), in a distributive model of legislative bargaining with the possibility of a finite round of reconsideration and a pre-determined sequence of proposers, arrive to a similar conclusion: the last proposer is able to implement his ideal policy. Eraslan (2002) demonstrated that, even with unequal recognition probabilities and discount factors, there is a single vector of expected utilities common to all stationary subgame perfect equilibria. (See a comprehensive recent survey of the related literature in Eraslan and McLennan, 2013.)

Baron (1996) initiated the study of legislative bargaining with endogenous default.[12] Kalandrakis (2004) analyzed distributive politics in a dynamic legislative bargaining with three players, random recognition, and an evolving status quo in a continuous policy space, and constructed a class of mixed strategy stationary equilibria, in which after some initial phase the current agenda setter captures all the benefits. He assumed risk-neutral players, statically efficient (no waste) allocations, and continuous action space. Kalandrakis (2007) extended this example to allow for a larger number of players. In a general setup, Kalandrakis (2010) constructed a particular mixed-strategy MPE, where for any initial status quo, the outcomes converge to an ergodic set with stochastically rotating dictator allocations.

Anesi and Seidmann (2015) demonstrated that almost any allocation is possible in a pure strategy MPE when the space is continuous, and legislators have heterogeneous discount factors and recognition probabilities. They assume that when indifferent, a player votes for status quo if and only if she belongs to winning coalition under status quo and the status quo is in the supported set. They found that, unlike in Baron and Ferejohn (1989a) and Eraslan (2002), equilibrium coalitions are not necessarily minimal, and there might be waste and inefficiency along the equilibrium path. Anesi and Duggan (2015) extends the Anesi and Seidmann (2015) construction to the spatial setting.

Notably, Anesi and Seidmann (2015) use a constructive approach in describing equilibria. (Duggan and Kalandrakis, 2012, use a fixed point argument to prove existence of pure strategy MPEs in games with any quota in which preferences and the default are subject to stochastic shocks.) Similarly, Diermeier and Fong (2011, 2012) describe an algorithm that constructs the stable set in their model. The existence of the recursive algorithm follows from the fact that there is a suitable dominance relationship defining the von-Neumann-Morgenstern stable set (von Neumann and Morgenstern, 1947). The advantage of focusing on the vNM stable set is that it allows to work with a much simpler static object, instead of an equilibrium in a strategic voting game. The first non-cooperative foundation of von Neumann-Morgenstern stable set in a voting context is constructed

[12] There is an important parallel in the coalition formation literature. See, e.g., Seidmann and Winter (1998) on the impact of the possibility of renegotiation on the structure of the ultimate coalition, and Hyndman and Ray (2007) on equilibria in games with possible binding constraints.

by Anesi (2010). (See Anesi, 2006, for a brief survey of the preceding literature.) The vNM stable set depends, critically, on the specific dominance relationship defined over the set of states: in Anesi (2010), the dominance relation is defined as follows: $x \succ y$ if and only if there is a winning coalition S such that $x_i > y_i$ for any $i \in S$. Acemoglu, Egorov, and Sonin (2012) prove a similar result with a slightly weaker restrictions on the set of winning coalitions (sets of winning coalitions are not assumed to be the same in different states). In the current paper, we use a similar approach with a different dominance relation as we need to resolve possible indifferences for non-veto players. Anesi and Duggan (2016) obtained further results, demonstrating, among other things, that if there is a veto player with positive recognition probability and players are sufficiently patient, then starting from any allocation, the equilibrium path leads to a unique absorbing point.

In Diermeier and Fong (2011, 2012), there is only one strategic player. The other $N-1$ players merely grant their approval for a move to another allocation, rather than choose strategies in a non-cooperative game. Thus, our results not only extend Diermeier and Fong (2011, 2012) to the case of more than one veto player, they also considerably strengthen and refine them. Although Diermeier and Fong (2011) established existence of stable states, they characterized them only for the case of three players; in particular, they did not observe the existence of 'economic classes' structure of groups of players that have incentives to protect each other, nor did they compare different voting rules. Finally, Diermeier and Fong (2011, 2012) do not consider veto players (as they have the solo proposer who is, effectively, a veto player); in contrast, our paper emphasizes the role of veto players in such game.

Nunnari (2016) considers a legislative-bargaining model with a veto player and random recognition of the proposer. He demonstrates that irrespective of the initial status-quo, discount factors, and recognition probabilities the allocation eventually converges to one in which the veto player gets all the surplus. (Still, in contrast with our comparative static results, in his three-player model, an additional veto player does not affect the equilibrium payoff of the non-veto player.) In our model, we allow for random protocols, but require equilibria to be protocol-free. Baron and Bowen (2013) characterize a certain set of equilibria that sustain policies that cannot be defeated by another policy in the set of sustained policies, and cannot be defeated by another policy outside of the sustained set. While the bargaining model is different, the structure of equilibrium allocations in Baron and Bowen (2013) resembles that of our paper (see also Richter, 2013).

The basic reasoning underlying the stability of certain allocations in this paper (as well as in Diermeier and Fong, 2011, 2012) is similar to that of Acemoglu, Egorov, and Sonin (2012), where stability of a certain political state may be supported by instability of the states where it can be immediately moved and that are preferred by a winning coalition. However, at the level of generality in Acemoglu, Egorov, and Sonin (2012), no characterization or even meaningful comparative statics is possible. The second critical difference is that the former paper does not allow indifferences, while the current one depends critically on assumption about voters' behavior in such situations. (Other papers that achieve characterization results in models with endogenous status quo, but only in the case where indifferences are largely ruled out, are Anesi, 2010, Diermeier and Fong, 2012, and Acemoglu, Egorov, and Sonin, 2008.)

When indifferences are present because of the nature of the model, the literature is split on the approach to resolution of indifferences. Kalandrakis (2004), Diermeier and Fong (2011), Anesi and Duggan (2015) assume that a player supports a proposal when indifferent between the proposal and status-quo. In contrast, Baron and Bowen (2013) argue that it is important to assume that players vote against a proposal when indifferent. Anesi and Seidmann (2015) assume that players are supportive of the proposal, when indifferent, depending on the point on the equilibrium path. We assume that transitions to veto players unlock an arbitrarily small budget that may be used to resolve indifferences. The fact that the results hold for any size of this additional budget provided that it small enough, points to robustness of our equilibria.

Richter (2013) demonstrates that allowing waste in a dynamic legislative bargaining model can allow the universal allocation to be supported. (Concavity of the utility function is important as in such case an equal division maximizes the sum of players' utilities.) Bowen and Zahran (2012) do not allow for waste. While Baron and Ferejohn (1989a) allow for waste, this does not change the equilibrium outcomes as the last voting stage gives the proposer a lot of bargaining power. In our paper, waste is allowed, yet does not happen in equilibrium allocations; this is not surprising given that all allocations are reached in a one-step transition.

Finally, the literature is split between papers that assume a continuous (divide-a-dollar) policy space and a discrete (e.g., finite) one. Baron and Ferejohn (1989a), Kalandrakis (2004), Baron and Bowen (2013), Richter (2013), Anesi and Seidmann (2014), Nunnari (2016), among others, assume that the policy space is continuous. Anesi (2010), Diermeier and Fong (2011, 2012), Anesi and Duggan (2016) assume that the space is discrete. On one hand, our assumption of a finite space considerably simplifies the analysis: in fact, the use of the von-Neumann-Morgenstern stable set in all voting models that we are aware of requires discrete space. On the other hand, the limit set of our equilibrium allocations when the size of the unit approaches zero has the same 'class' structure as the set of stable sets in Proposition 3, suggesting further robustness of this result.

6 Conclusion

The modern literature often considers constitutional constraints and other formal institutions as instruments of property rights protection. Allston and Mueller (2008) proclaim: "A set of universally shared beliefs in a system of checks and balances is what separates populist democracies from democracies with respect for the rule of law." The relationship between veto power given to different government bodies, supermajority requirements, or additional checks and balances and better protection seems so obvious that there is little left to explain. Yet, from a political economy perspective, property rights systems should be understood as equilibrium outcomes rather than exogenous fixed constraints. Legislators or, more generally, any political actors cannot commit to entitlements, prerogatives, and rights. Rather, any allocation must be maintained in equilibrium. By varying characteristics of the political institutions (here modeled as veto power and supermajority requirements in a legislative-bargaining model), one can assess the consequences for economic institutions.

The legislative-bargaining approach suggests that a dynamic perspective may lead to a more subtle understanding of the effects of veto players and supermajority rules. In a dynamic environment, they lead to emergence of endogenous veto groups: groups of players that sustain a stable allocation in equilibrium. The society has a "class structure": any non-veto player with a positive wealth is part of a group of equally-endowed individuals who have incentives to protect each other's interests. The effect of exogenous constraints on endogenous veto groups is complex. One the one hand, endogenous veto groups may protect each other in equilibrium even in the absence of formal veto rights. One the other hand, adding more veto players may lead to more instability and policy change if such additions upset dynamic equilibria where players were mutually protecting each other.

Models of legislative bargaining with endogenous status quo seem to be a natural and very fruitful approach to study the political economy of property rights protection. Our results point to the importance of looking beyond formally defined property rights, and more, generally, beyond formal institutions. While formal institutions provide better incentives for investment and production, the incentives provided by informal equilibrium institutions are substantial as well. Thus, a change in formal institutions might strengthen protection of property rights of designated players, yet have negative consequences for protection of property rights of the others, and, as a result, a negative overall effect.

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Online Appendix

A1 Proofs

We start with a few auxiliary Lemmas that help us prove Proposition 1. In what follows, we let ζ_i^t denote transfers less transition costs, if any, obtained by player i in period t.

Lemma A1 *Any protocol-free MPE σ is acyclic.*

Proof of Lemma A1. Let $\phi = \phi_\sigma$ be the equilibrium transition mapping generated by equilibrium σ. Suppose that there is a cycle starting from x: $\phi(x) \neq x$, but $\phi^l(x) = x$ for some $l > 1$; without loss of generality, let l be the minimal such value, i.e., the length of the cycle. Let us first show that for every $i \in V$, $\left[\phi^j(x)\right]_i = x_i$ for all j. Suppose not, then without loss of generality we may assume to have chosen x such that $x_i \geq \left[\phi^j(x)\right]_i$ for all j (so i gets his maximum allocation along the cycle), and moreover that $[\phi(x)]_i < x_i$. Then, in the period that started with $x^{t-1} = x$ and where in equilibrium, transition to $\phi_\sigma(x)$ is made, the continuation utility of player i satisfies (after taking the expectation over possible realizations of the protocols)

$$U_i^t \leq [\phi_\sigma(x)]_i + \zeta + \beta\left(\left[\phi_\sigma^2(x)\right]_i + \zeta\right) + \cdots + \beta^{l-1}\left(\left[\phi_\sigma^l(x)\right]_i + \zeta\right) + \beta^l U_i^t,$$

where $\zeta \in [0, b\varepsilon]$ is the maximum possible value of ζ_i^t over different periods. We thus have

$$\begin{aligned} U_i^t &\leq \frac{[\phi_\sigma(x)]_i + \zeta + \beta\left(\left[\phi_\sigma^2(x)\right]_i + \zeta\right) + \cdots + \beta^{l-1}\left(\left[\phi_\sigma^l(x)\right]_i + \zeta\right)}{1-\beta^l} \\ &\leq \frac{(x_i - 1) + \zeta + \beta(x_i + \zeta) + \cdots + \beta^{l-1}(x_i + \zeta)}{1-\beta^l} \\ &= \frac{x_i + \zeta}{1-\beta} - \frac{1}{1-\beta^l} < \frac{x_i + \zeta}{1-\beta} - 1. \end{aligned}$$

At the same time, if player i always vetoes all proposals in all subsequent periods, his continuation utility would equal $\tilde{U}_i^t = \frac{x_i}{1-\beta}$. Since $\frac{\zeta}{1-\beta} < \frac{b\varepsilon}{1-\beta} < 1$, we have $U_i^t < \tilde{U}_i^t$, which implies that player i has a profitable deviation. Hence, it must be that $\left[\phi_\sigma^j(x)\right]_i = x_i$ for all $j \geq 1$ and for all $i \in V$.

Since each veto player gets x_i in each period, the equilibrium payoff of each player must equal $U_i^t = \frac{x_i - \delta}{1-\beta}$. However, player i can always guarantee himself $\tilde{U}_i^t = \frac{x_i}{1-\beta}$, by vetoing all proposals. Therefore, he has a profitable deviation, which is impossible in equilibrium. This contradiction completes the proof. ■

Lemma A2 *Consider a one-step mapping ϕ, which is independent of protocols, and suppose that the current period is t and the current allocation is $x = x^{t-1}$. Suppose that some player i has*

$[\phi(y)]_i > [\phi(z)]_i$ *for some* $y, z \in \mathbf{A}$. *Then player* i *prefers transition to* y *to transition to* z, *in other words (expectations are with respect to realization of protocols),*

$$y_i + \mathbb{E}\zeta_i^t + \sum_{\tau=1}^{\infty} \beta^\tau \left([\phi(y)]_i + \mathbb{E}\zeta_i^{t+\tau}\right) > z_i + \mathbb{E}\tilde{\zeta}_i^t + \sum_{\tau=1}^{\infty} \beta^\tau \left([\phi(z)]_i + \mathbb{E}\tilde{\zeta}_i^{t+\tau}\right), \tag{A1}$$

where ζ *and* $\tilde{\zeta}$ *reflect the transfers on path that follow acceptance of* y *and* z, *respectively. Furthermore, the same is true if* $[\phi(y)]_i = [\phi(z)]_i$, *but* $y_i > z_i$.

Proof of Lemma A2. Suppose $[\phi(y)]_i > [\phi(z)]_i$, but the inequality (A1) does not hold. Since $\xi_i^{t+\tau}, \tilde{\xi}_i^{t+\tau} \in [0, b\varepsilon]$ for any $\tau \geq 0$, this must imply

$$y_i + \sum_{\tau=1}^{\infty} \beta^\tau [\phi(y)]_i \leq z_i + \sum_{\tau=1}^{\infty} \beta^\tau [\phi(z)]_i + \frac{b\varepsilon}{1-\beta}. \tag{A2}$$

Since $[\phi(y)]_i > [\phi(z)]_i$ implies $[\phi(y)]_i - [\phi(z)]_i \geq 1$, this implies

$$y_i + \frac{\beta}{1-\beta} \leq z_i + \frac{b\varepsilon}{1-\beta}.$$

Given that $z_i - y_i \leq b$, this implies $\frac{\beta - b\varepsilon}{1-\beta} \leq b$, which, since we assumed $b\varepsilon < 1-\beta$, implies $\frac{2\beta-1}{1-\beta} \leq b$, which is equivalent to $\beta \leq 1 - \frac{1}{b+2}$, a contradiction. This proves the first part of the Lemma.

Now suppose that $[\phi(y)]_i = [\phi(z)]_i$, but $y_i > z_i$. As before, assume not, in which case (A2) would hold. Now, given that $y_i - z_i \geq 1$, (A2) would imply $1 \leq \frac{b\varepsilon}{1-\beta}$, which contradicts our assumption that $b\varepsilon < 1-\beta$. This contradiction completes the proof. ■

Lemma A3 *Suppose that in protocol-free MPE* σ, $x \in \mathbf{A}$ *is such that* $x \neq \phi_\sigma(x) = \phi_\sigma^2(x)$. *Then* $\phi_\sigma(x) \rhd x$.

Proof of Lemma A3. Denote $y = \phi_\sigma(x)$. Let us first prove that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$. Suppose, to obtain a contradiction, that this is not the case. Take some veto player l and consider protocol π where only player l proposes, and only once (so $\pi = (l)$). Under this protocol, alternative y must be proposed, and subsequently supported at the voting stage by a winning coalition of players. Now consider any agent i such that $y_i < x_i$, which implies $x_i - y_i \geq 1$. If y_i is accepted, agent i gets continuation utility (assuming the current period is t) that satisfies

$$U_i^t \leq y_i + b\varepsilon + \beta(y_i + b\varepsilon) + \cdots = \frac{y_i + b\varepsilon}{1-\beta}.$$

If, however, y_i is rejected, then the continuation utility satisfies

$$\tilde{U}_i^t \geq x_i + \beta y_i + \beta^2 y_i + \cdots = x_i + \frac{\beta}{1-\beta} y_i.$$

Since $b\varepsilon < 1-\beta$, we have

$$\begin{aligned} U_i^t - \tilde{U}_i^t &\leq \frac{y_i + b\varepsilon}{1-\beta} - \left(x_i + \frac{\beta}{1-\beta} y_i\right) \\ &= y_i - x_i + \frac{b\varepsilon}{1-\beta} \leq \frac{b\varepsilon}{1-\beta} - 1 < 0. \end{aligned}$$

Therefore, such player i prefers the alternative y to fail at the voting stage. This implies that $U_i^t - \tilde{U}_i^t \geq 0$ is possible only if $y_i \geq x_i$, and by assertion, the set of such players does not form a winning coalition, which means that y cannot be accepted at this voting stage. This contradicts that σ is equilibrium, which proves that $\{i \in N : y_i \geq x_i\} \in \mathcal{W}$.

It remains to prove that for some $i \in V$, $y_i > x_i$ and that $\|\phi_\sigma(x)\| \leq \|x\|$. Both results immediately follow from that transition to $\phi_\sigma(x)$ is feasible and is not blocked by any veto player because of transition cost. Now, by definition of the binary relation $\rhd$, we have $\phi_\sigma(x) \rhd x$, which completes the proof. ■

Lemma A4 *Every protocol-free equilibrium is simple, i.e., for every $x \in \mathbf{A}$, $\phi_\sigma^j(x) = \phi_\sigma(x)$ for all $j \geq 1$.*

Proof of Lemma A4. Suppose that there is a protocol-free equilibrium σ that is not simple, which means that there is $x \in \mathbf{A}$ such that $\phi_\sigma^2(x) \neq \phi_\sigma(x)$. By Lemma A1, σ is acyclic, and therefore the path starting from x: $\phi_\sigma(x), \phi_\sigma^2(x), \ldots$ stabilizes after no more than $|\mathbf{A}|$ iterations, and thus its limit $\phi_\sigma^\infty(x) = \phi_\sigma^{|\mathbf{A}|}(x)$ is well-defined. Denote the set of all such $x \in \mathbf{A}$ by Y, so

$$Y = \left\{x \in \mathbf{A} : \phi_\sigma^2(x) \neq \phi_\sigma(x)\right\} \neq \varnothing.$$

Take allocation $y \in Y$ such that $\phi_\sigma^\infty(y) = \phi_\sigma^2(y)$ (notice that such y exists: indeed, if we take any $x \in Y$ and the minimal number such that $\phi_\sigma^\infty(x) = \phi_\sigma^j(x)$ is $j > 2$, then we can take $y = \phi_\sigma^{j-2}(x)$). Notice that we must have $\sum_{i \in V} \left[\phi_\sigma^2(y)\right]_i > \sum_{i \in V} [\phi_\sigma(y)]_i$, for otherwise the transition from $\phi_\sigma(y)$ to $\phi_\sigma^2(y)$ would be blocked by some veto player due to the cost of transition.

Consider veto player l for whom $\left[\phi_\sigma^2(y)\right]_l > [\phi_\sigma(y)]_l$. Suppose that in period t where the status quo is y, protocol $\pi = (l)$ is realized. Since σ is protocol-free, this must imply that player l proposes alternative allocation $\phi_\sigma(y)$ and some feasible transfers ξ, and this proposal is subsequently accepted. Now suppose that protocol $\pi' = (l, l)$ is realized, and suppose that the game reached the second stage of the protocol. This subgame is isomorphic to one where protocol π has just been realized; consequently, in equilibrium, it must be that $\phi_\sigma(y)$ is proposed and accepted.

Let us prove that if in the second stage, the society decides to move to $\phi_\sigma(y)$, then in the first stage player l would be better off proposing $\phi_\sigma^2(y)$ and some feasible vector of transfers $\tilde{\xi}$, which would be accepted. Notice that in the following period, a transition from $\phi_\sigma(y)$ to $\phi_\sigma^2(y)$ would take place, which means that each player would receive a certain vector of transfers $\tilde{\zeta}$ (we must have . These vectors of transfers satisfy

$$\begin{aligned}\|\xi\|+\|\zeta\| &\leq \varepsilon\left(\max\left(\sum_{i\in V}\left[\phi_\sigma(y)\right]_i-\sum_{i\in V}y_i,0\right)+\max\left(\sum_{i\in V}\left[\phi_\sigma^2(y)\right]_i-\sum_{i\in V}\left[\phi_\sigma(y)\right]_i,0\right)\right)\\ &\leq \varepsilon\left(\sum_{i\in V}\left[\phi_\sigma^2(y)\right]_i-\sum_{i\in V}y_i,0\right),\end{aligned}$$

consequently, $\xi+\zeta$ is a feasible vector. Furthermore, since player l would propose ξ at the last stage, he must get a transfer strictly larger than the cost of transition, as in equilibrium he only keeps the other players indifferent, so $\xi_i>\delta$. Thus, there exists a small value $\alpha>0$ such that if we define χ by $\chi_i=\xi_i+\zeta_i+\alpha$ for $i\neq l$ and $\chi_l=\xi_l+\zeta_l-(n-1)\alpha$, we would still get a feasible vector of transfers.

Now, consider any $i\in N\setminus\{l\}$ such that $\left[\phi_\sigma^2(y)\right]_i\geq\sum_{i\in V}\left[\phi_\sigma(y)\right]_i$. Since $(\xi_i-\delta)+\beta(\zeta_i-\delta)>\xi_i+\zeta_i-\delta$, this player is better off if $\phi_\sigma^2(y)$ is accepted at the first stage. For player l, the same is true, because $\left[\phi_\sigma^2(y)\right]_l>\left[\phi_\sigma(y)\right]_l$. Since the transition from to $\phi_\sigma^2(y)$ would happen in a period starting with $\phi_\sigma(y)$, Lemma A3 implies $\phi_\sigma^2(y)\rhd\phi_\sigma(y)$, but this implies that the set of players who are better off if $\phi_\sigma^2(y)$ is accepted at the first stage is a winning coalition. This means that $\phi_\sigma^2(y)$ would be accepted if proposed, which implies that player l has a profitable deviation. This is a contradiction that completes the proof. ■

Lemma A5 *If σ is a simple protocol-free MPE, then for all $x\in\mathbf{A}$ either $\phi_\sigma(x)=x$ or $\phi_\sigma(x)\rhd x$.*

Proof of Lemma A5. By Lemma A1, σ is acyclic, and by Lemma A4 it is simple. Then for any $x\in\mathbf{A}$, we must have $\phi_\sigma^2(x)=\phi_\sigma(x)$. Now if $\phi_\sigma(x)=x$, the result is automatically true, and if $\phi_\sigma(x)\neq x$ then it follows immediately from Lemma A3. ■

Lemma A6 *Suppose that protocol-free MPE σ is played, and suppose that in period t, $x^{t-1}=x$. Then if there exists $y\in\mathbf{A}$ such that $\phi_\sigma(y)=y$ and $y\rhd x$, then x cannot be stable: $\phi_\sigma(x)\neq x$.*

Proof of Lemma A6. Suppose, to obtain a contradiction, that $\phi_\sigma(x)=x$. Let l be a veto player such that $y_l>x_l$ (such l exists as $y\rhd x$). Consider protocol $\pi=(l)$ (or any protocol ending with l). In equilibrium, each player i gets $U_i^t=\frac{x_i}{1-\beta}$; they would get the same amount if the proposal is made and is rejected.

If player l makes proposal (y,ξ), where $\xi_i=\frac{(\|y\|-\|x\|)\varepsilon}{n}$. Since $\|y\|-\|x\|\geq1$ and $\delta<\frac{\varepsilon}{n}$, we have $\xi_i>\delta$ for all $i\in N$, so ξ_i is a feasible transfer. This means that each player i for which $y_i\geq x_i$ would get $\frac{y_i}{1-\beta}+\xi_i-\delta$ if the proposal is accepted, which exceeds U_i^t that he would get if the proposal is rejected. Since $y\rhd x$, such players form a winning coalition, which implies that the proposal (y, ξ) would be accepted if made. Then player l has a profitable deviation, which is impossible. This contradiction completes the proof. ■

Proof of Proposition 2. Part 1. Take any simple protocol-free MPE σ, and let $\mathbf{S}_\sigma = \{x \in \mathbf{A} : \phi_\sigma(x) = x\}$. By Lemma A1, it is nonempty. Let us prove that it satisfies internal stability. Suppose that for some $x, y \in \mathbf{S}_\sigma$, we have $y \triangleright x$. Then by Lemma A6, $\phi_\sigma(y) = y$ implies $\phi_\sigma(x) \neq x$, which contradicts that $x \in \mathbf{S}_\sigma$. This contradiction proves that $\mathbf{S}_\sigma$ satisfies internal stability.

Let us now show that $\mathbf{S}_\sigma$ satisfies external stability. Take $x \notin \mathbf{S}_\sigma$; then by Lemma A5, $\phi_\sigma(x) \triangleright x$. Since σ is simple, $\phi_\sigma(x) \in \mathbf{S}_\sigma$, which shows that there exists $y \in \mathbf{S}_\sigma$ such that $y \triangleright x$. This proves that $\mathbf{S}_\sigma$ satisfies external stability. This proves that $\mathbf{S}_\sigma$ is von Neumann-Morgenstern stable set. ■

Lemma A7 *If σ is a protocol-free MPE, then $\|\phi_\sigma(x)\| = \|x\|$ for all $x \in \mathbf{A}$.*

Proof of Lemma A7. Suppose not, then there exists $x \in \mathbf{A}$ for which $\|\phi_\sigma(x)\| < \|x\|$. Since σ is simple by Lemma A4, we have $\phi_\sigma(x) \in \mathbf{S}$. Take some veto player l and consider the protocol $\pi = (l)$; at this stage, player l must propose $\phi_\sigma(x)$ and it must be accepted. Notice, however, that player l may propose allocation y that has $y_l = [\phi_\sigma(x)]_l + 1$ and $y_i = [\phi_\sigma(x)]_i$ for all $i \neq l$, and split the extra ε of available transfers equally among players. By Proposition 3, such allocation y is stable as well, consequently, all players would be strictly better off from proposal y (with the corresponding transfers) than the equilibrium proposal $\phi_\sigma(x)$. Thus, if a winning coalition was weakly better off from supporting $\phi_\sigma(x)$, it is strictly better off supporting y. Thus, player l has a profitable deviation at the proposing stage, which is a contradiction that completes the proof. ■

Proof of Proposition 1. Part 1. Consider the unique von Neumann-Morgenstern stable set for dominance relation $\triangleright$, $\mathbf{S}$ (its existence and uniqueness follow from Proposition 3 proven in the main text. Take any mapping ϕ such that $\phi(x) = x$ for any $x \in \mathbf{S}$ and for any $x \notin \mathbf{S}$, $\phi(x) \in \mathbf{S}$ and $\phi(x) \triangleright x$ (existence of such mapping follows from external stability of mapping $\mathbf{S}$ implying that for any $\mathbf{S}$ we can pick such $\phi(x) \in \mathbf{S}$) and, moreover, $\|\phi(x)\| = \|x\|$ (existence of such x follows from Proposition 3 as well, as otherwise one can add $\|x\| - \|\phi(x)\|$ units to some veto player and get an allocation in $\mathbf{S}$ with the required property). Let us prove the following (stronger) result: there is protocol-free MPE σ such that $\phi_\sigma = \phi$ (notice that σ will in this case be simple, because $\phi^2 = \phi$.)

We construct equilibrium σ using the following steps. For each possible status quo $x \in \mathbf{A}$ and each protocol $\pi \in \Pi$, we define transfers that each player is supposed to get in that period. We use allocations and these transfers utilities to define continuation utilities. After that, we use these continuation utilities to define strategies players would use for each $x \in \mathbf{A}$ and each $\pi \in \Pi$. We then check that under these strategies, players indeed get the transfers that we defined, and no player has a one-shot deviation. This would prove that σ is MPE, which would be protocol-free by construction.

If $x \notin \mathbf{S}$, then let $V_x = \{i \in V : [\phi(x)]_i = x_i\}$ and let $v_x = |V_x|$. Furthermore, let $Z = \sum_{i \in V} [\phi(x)]_i - \sum_{i \in V} x_i > 0$. Define transfers $\xi_i(x, \pi)$ as follows:

$$\xi_i(x,\pi) = \begin{cases} \delta & \text{if } i \notin V_x \cup \{l\}; \\ \delta + \beta \frac{Z\varepsilon - n\delta}{(1-\beta)v + \beta v_x} & \text{if } i \in V_x \setminus \{l\}; \\ Z\varepsilon - \sum_{j \neq l} \xi_i(x,\pi) & \text{if } i = l. \end{cases} \tag{A3}$$

Given these transfers, the continuation utilities (at the beginning of the period, before protocol is realized), is given by

$$V_i(x) = [\phi(x)]_i + \sum_{\pi \in \Pi} \xi_i(x,\pi) + \frac{\beta}{1-\beta} [\phi(x)]_i . \tag{A4}$$

Let us now define strategies as follows. Suppose that in period t, the current status quo is $x = x^{t-1}$ and protocol π was realized. To define strategies, consider the game with timing from Section 2, but define payoffs in case transition to some $y \in \mathbf{A}$ and set of transfers ξ is decided upon given by

$$U_i(y,\xi) = y_i + \xi_i + \beta V_i(y)$$

(in other words, consider the game truncated at the end of the period, i.e., a finite game, but with payoffs coinciding with continuation payoffs of the original game).

Define strategies by proceeding by backward induction, with a few exceptions. In the last stage, the proposer $\pi_{|\pi|}$ proposes to transfer to $\phi(x)$ (or to stay, if $\phi(x) = x$), and offers transfers $\xi_i(x,\pi)$. We require that all players who are at least indifferent vote for the proposal to pass. If any other proposal is made, as well as in all previous stages, we require that players play any strategies consistent with backward induction, except that we require that players vote 'no' when indifferent.

Let us show that the players have no incentive to deviate for any strategy that we defined. One-shot deviation principle applies, and one can immediately check that after all transitions were declined, the strategies we defined form an equilibrium in the subgame, with each player i getting, in expectation, $\check{\xi}_i$. Now consider the two cases, $\phi(x) = x$ and $\phi(x) \neq x$, separately.

First, consider the case $\phi(x) \neq x$. Let us check that at the last stage, it is a best response for any player i with $[\phi(x)]_i \geq x_i$ to accept, which would imply that this proposal is indeed accepted. Indeed, both accepting and rejecting results in getting the same allocation $[\phi(x)]_i$ in two periods, thus, if for some player i, $[\phi(x)]_i > x_i$, then by Lemma A2 he is strictly better off if $\phi(x)$ is accepted. Consider a player i with $[\phi(x)]_i = x_i$. If $i \notin V$, then he gets transfer $\xi_i(x,\pi) = \delta$ if $\phi(x)$ is accepted, which is just enough to compensate for the transition cost, but he gets the same in the following period if the proposal is rejected, which implies that he is indifferent, so supporting $\phi(x)$ is a best response. If $i \in V_x \setminus \{l\}$, then he gets the transfer $\xi_i(x,t)$ if the alternative is accepted, and it makes him exactly indifferent between accepting and rejecting. Finally, if $i \notin V_x$ or $i = l$, the player is strictly willing to accept. Thus, for all veto players, it a best response to support the alternative. Since $\phi(x) \rhd x$, the set of players with $[\phi(x)]_i \geq x_i$ is a winning coalition. Finally, $\|\phi(x)\| = \|x\|$, so the transition is feasible. Consequently, there are best responses that result in $\phi(x)$ being accepted.

Taking one step back, let us verify that it is a best response for player $l = \pi_{|\pi|}$ to propose $\phi(x)$. First, since he prefers $\phi(x)$ to be accepted rather than rejected, he would only propose an alternative y if this alternative will be accepted at the voting stage. Suppose there is such alternative; it suffices to prove that proposing it does not make the player l better off. By Lemma A2, if $[\phi(y)]_i < [\phi(x)]_i$ for some player i, then this player would be better off if y is rejected. Consequently, for y to be accepted in equilibrium, it is necessary that $[\phi(y)]_i \geq [\phi(x)]_i$ for a winning coalition of players, in particular, for all veto players $i \in V$.

Let us prove that $[\phi(y)]_i = [\phi(x)]_i$ for all $i \in V$; to do so, suppose it is not the case, meaning that for some $j \in V$, the strict inequality $[\phi(y)]_j > [\phi(x)]_j$ holds. In addition, notice that $\|y\| \leq \|x\|$ since transition to y is feasible, but $\|\phi(y)\| \leq \|y\|$ (because transition to $\phi(y)$ would be feasible) and $\|x\| = \|\phi(x)\|$ (by assumption that transition to $\phi(x)$ does not result in waste). This implies $\|\phi(y)\| \leq \|\phi(x)\|$ which, together with $\{i \in N : [\phi(y)]_i \geq [\phi(x)]_i\} \in \mathcal{W}$ and $[\phi(y)]_j > [\phi(x)]_j$ imply $\phi(y) \rhd \phi(x)$. Since $\phi(x), \phi(y) \in \mathbf{S}$, this contradicts internal stability of $\mathbf{S}$, which proves that $[\phi(y)]_i = [\phi(x)]_i$ for all $i \in V$.

Notice that for the proposer, player $l = \pi_{|\pi|}$, to prefer transition to y to transition to $\phi(x)$, it must be that $y_l = [\phi(y)]_l = [\phi(x)]_l$, for otherwise we would get a contradiction with Lemma A2. Consider two possibilities. If $\phi(y) = y$, then for player l to be better off, he needs to get a larger transfer $\chi_l > \xi_l(x, \pi)$. However, since all other veto players in V_x were indifferent between accepting their transfer $\xi_i(x, \pi)$ and rejecting, they need to get at least this transfer as well; since other players need to get $\chi_i \geq \delta$ as well, such deviation cannot be profitable. If, however, $\phi(y) \neq y$, then $\phi(y)$ will be reached in the following period. Notice that for each $i \in V$ it must be that $y_i \geq x_i$, for otherwise this player would block the transition. This means, in particular, that for players in V_x, $x_i = y_i = \phi(x_i) = \phi(y_i)$ holds, and they therefore need discounted transfer $\chi_i^t + \mathbb{E}\chi_i^{t+1} \geq \xi_i(x, \pi)$ in order to be willing to accept. However, since the transfers available over the two periods are capped at $Z\varepsilon$, player l cannot be better off from such deviation. Therefore, proposing $\phi(x)$ at the last stage is a best response.

We now prove that for any proposal z made at the previous stage by player $\pi_{|\pi|-1}$, the set of players who strictly prefer transition to z does not form a winning coalition. Indeed, suppose that it is; then by Lemma A2 it must be that for all $i \in V$, $[\phi(z)]_i = z_i = y_i$, for otherwise we would have $\phi(z) \rhd y$, which would contradict internal stability of $\mathbf{S}$. This implies that $z = \phi(z)$, for otherwise transition from z to $\phi(z)$ would be impossible; furthermore, the set of transfers χ proposed at this stage must coincide with $\xi_i(x, \pi)$. If so, if some player $i \notin V$ strictly prefers transition to z, this implies that $z_i > y_i$ for such player. However, this would contradict the characterization results from Proposition 3. This shows that it is a best response for at least $n-k+1$ players to vote against proposal z, which implies that there is an equilibrium in this subgame where it is not accepted. Proceeding by backward induction, we can conclude that there is an equilibrium in this finite game where no proposal is accepted until the last stage, where y is accepted.

Now consider the game with $x \in \mathbf{S}$. We allow any strategies, but require that players vote against the proposal when indifferent. Now, again by backward induction, we can conclude that if a winning coalition strictly prefers to accept some proposal z, then either $\phi(z) \rhd x$, which contradicts internal stability of $\mathbf{S}$, or $[\phi(z)]_i = z_i = x_i$ for all $i \in V$, in which case veto players are actually worse off because of transition costs. Thus, there is an equilibrium in the finite game where no proposal is accepted, so x remains stable.

Lastly, it is straightforward to check that if the strategies are played, then in every period, transfers are indeed given by $\xi(x, \pi)$ defined above, and thus continuation utilities are indeed at the beginning of period with x as the status quo are $V(x)$. This means that if these strategies are played in the original game Γ, no player has a one-shot deviation. Since by construction the strategies are Markovian and transitions do not depend on the realization of the protocol, then σ is a protocol-free MPE. Moreover, it is simple and efficient by construction, which completes the proof of existence of such equilibrium.

Part 2. Follows from Lemma A1.

Part 3. Follows from Lemma A4.

Part 4. Follows from Lemma A7.

Proof of Proposition 4. Part 1. Lemma A5 implies that $\phi(y) \rhd y$; in particular, for each $j \in V$, $[\phi(y)]_j \geq y_j$ and for at least one of them the inequality is strict. Suppose, to obtain a contradiction, that $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| < d-1$; then $\left|\left\{j \in M : [\phi(y)]_j < x_j\right\}\right| < d$. But we also have that for each $j \in V$, $[\phi(y)]_j \geq x_j$, with at least inequality strict. This means $\phi(y) \rhd x$, which is impossible, given that $x, \phi(y) \in \mathbf{S}$. Now suppose, to obtain a contradiction, that $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| > d-1$. But then for at least d agents $[\phi(y)]_j < y_j$, which contradicts $\phi(y) \rhd y$. This contradiction proves that $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| = d-1$. It remains to prove that $y_i \leq [\phi(y)]_i < x_i$. Suppose not, i.e., either $[\phi(y)]_i < y_i$ or $[\phi(y)]_i \geq x_i$. In the first case, we would have that at least d agents have $[\phi(y)]_j < y_j$, contradicting $\phi(y) \rhd y$. In the second case, $[\phi(y)]_i \geq x_i$, coupled with the already established $\left|\left\{j \in M \setminus \{i\} : [\phi(y)]_j < y_j\right\}\right| = d-1$, would mean $\left|\left\{j \in M : [\phi(y)]_j < x_j\right\}\right| = d-1$, and therefore $\phi(y) \rhd x$. This is impossible, and this contradiction completes the proof.

Part 2. This proof is similar to the proof of internal stability in the proof of Proposition 3. Denote $\phi(y) = z$; then $z \rhd y$ and $x, z \in \mathbf{S}$. We know that x and z have the group structure by Part

2 of Proposition 3; then let the r groups be $G_1, \ldots, G_r$ for x and $H_1, \ldots, H_r$ for z, respectively. Without loss of generality, we can assume that each set of groups are ordered so that x_{G_j} and z_{H_j} are non-increasing in j for $1 \leq j \leq r$. Suppose, to obtain a contradiction, that for some agent $i' \in M$ with $x_{i'} \leq y_i < x_i$, $z_{i'} < y_{i'}$. In that case, among the set $\{j \in M : x_j \geq x_i\}$ there are at most $d-1$ agents with $z_j < y_j$; similarly, among the set $\{j \in M : x_j < x_i\}$ there are at most $d-1$ agents with $z_j < y_j$.

We can now proceed by induction, similarly to the proof of Proposition 3, and show that $x_{G_j} \leq z_{H_j}$ for all j. Base: suppose not, then $x_{G_1} > z_{H_1}$; then $x_{G_1} > z_s$ for all $s \in M$. But this mean that for all agents $l \in G_1$ have $x_l > z_l$; since their total number is d, we get a contradiction. Step: suppose $x_{G_l} \leq z_{H_l}$ for $1 \leq l < j$, and suppose, to obtain a contradiction, that $x_{G_j} > z_{H_j}$. Given the ordering of groups, this means that for any l, s such that $1 \leq l \leq j$ and $j \leq s \leq r$, $x_{G_l} > z_{H_s}$. Consequently, for a agent $i'' \in \bigcup_{l=1}^{j} G_l$ to have $z_{i''} \geq x_{i''}$, he must belong to $\bigcup_{s=1}^{j-1} H_s$. This implies that for at least $jd - (j-1)\,d = d$ agents in $\bigcup_{l=1}^{j} G_l \subset M$, $z_{i''} \geq x_{i''}$ does not hold (denote this set by D. If that is true, it must be that $\bigcup_{l=1}^{j} G_l$ includes all the agents in D, including agents i and i' found earlier, and in particular, $x_{G_j} \leq y_i < x_i$. But on the other hand, these d agents are not in $\bigcup_{s=1}^{j-1} H_s$. In particular, this implies that for any $i'' \in D$, $z_{i''} < x_{G_j}$, but $x_{i'} \geq x_{G_j}$, which means $z_i < x_{i'}$. But $z_i \geq y_i$ by Part 1 of this Proposition, so $y_i < x_{i'}$. But this contradicts with the way we chose i' to satisfy $x_{i'} \leq y_i < x_i$. This proves that such i' cannot exist, and thus the $d-1$ agents other than i who are made worse off satisfy $x_j \geq x_i$. ■

Proof of Proposition 5. This result immediately follows from the formulas $m = n - v$, $d = n - k + 1$, $r = \lfloor m/d \rfloor$ and from Proposition 3. ■

Proof of Proposition 6. Part 1. If $k < n$, then $d > 1$. An allocation x is stable only if $|\{j \in M : x_j > 0\}|$ is divisible by d. If x is stable and some agent i with $x_i > 0$ is made a veto agent, then the set $|\{j \in M' : x_j > 0\}| = |\{j \in M : x_j > 0\}| - 1$ and is not divisible by d, thus x becomes unstable. At the same time, if $x_i = 0$, then the group structure for all groups with a positive amount is preserved; thus x remains a stable allocation.

Part 2. In this case, the size of each group in x is $d > 2$, and every positive amount is possessed by either none or d non-veto players. If k increases by 1, d decreases by 2. Then allocation x becomes unstable, except for the case $x|_M = 0$. ■

A2 Examples

In the examples below, we do not explicitly consider costs of transition and transfers explicitly, as they would complicate the exposition. Unless specified otherwise, each of the examples below may be modified to accommodate.

Example A1 ***(If costs of transition are assumed to be*** 0***)*** Suppose $n = 3$, $v = 1$, $k = 2$, so there are three players, one of them veto player, and the rule is simple majority rule. Assume for simplicity that there is only one unit that initially belongs to a non-veto player (say, player #1), so the initial allocation is $(1, 0; 0)$. Then there would be an equilibrium where the veto player (proposing last) would propose to move the unit from player #1 to player #2 if it belongs to player #1, and then propose to move it the other way around if it belongs to player #2. Such proposal would then be supported by the veto player and the other player who receives the unit.

To complete the description of strategies, we can also assume that any proposal made at a protocol stage before the last one, except for the proposal to transfer the good to the veto player, would be vetoed by the veto player (he is indifferent anyway). On the other hand, if a proposal to transfer the unit to the veto player is ever made, the two non-veto players vote against this proposal. They both have incentives to do so, because the equilibrium play give them the unit in possession every other period, which is better than having the unit taken away.

Thus, without transaction costs, it is possible to have cyclic equilibria, which do not seem particularly natural.

Example A2 ***(Example where non-veto player proposes last)*** Suppose $n = 11$, $v = 1$, $k = 9$, so there are eleven players, one of them veto player, and the rule requires agreement of nine players. The size of a minimal blocking coalition is then three. In this case, in any protocol-free MPE (where the last proposal is done by veto player), allocation $(3, 3, 3, 2, 2, 2, 1, 1, 1, 10; 0)$ is unstable, and in any equilibrium, it results in a transfer to an allocation where all players except for player #10 (the one possessing 10 units in the beginning of the game) are better off. To simplify the following argument, let us focus on the equilibrium where an immediate transition to $(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$ takes place.

Consider, however, what would happen if a protocol has a non-veto player propose last. Specifically, suppose the protocol has two players: first the veto player (player #11) proposes, and then the non-veto player #6 proposes. Consider the last stage and suppose that player #6, instead of proposing to move to $(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$ or to stay in the current allocation $(3, 3, 3, 2, 2, 2, 1, 1, 1, 10; 0)$, instead proposes to transfer to allocation $(3, 3, 1, 2, 2, 3, 1, 1, 2, 4; 6)$; in other words, in addition to moving some units to the veto player, he also proposes to take two units from player #3 and takes one himself and gives the other one to player #9 in order to 'buy' his vote. This proposal leads to a stable allocation, and it makes only two players (player #3 and player #10) worse off. It therefore would be accepted; the veto player would agree, because it gives him six of the units right away, and he would be able to get the other four the following period. (Notice that player #4 might prefer not to get more units for himself in the short run, out of fear that having four or more units in the next period would make him a candidate for complete expropriation.)

Taking one step back, consider the stage where the veto player makes the proposal. He would use the opportunity to get the ten units belonging to him immediately (which hurts player #10). However, he would not be able to make the society move to $(3, 3, 3, 2, 2, 2, 1, 1, 1, 0; 10)$, which they are supposed to do in equilibrium, because doing so would make players #6 and #9, in addition to #10, worse off, and thus such proposal would not gather nine votes needed to pass. This means that by allowing non-veto players to propose, in some examples we would lose existence of protocol-free MPE.

This example relies on the fact that non-veto players are not indifferent between different stable allocations, and would want to make the society reallocate the units in their favor. As the results in this paper show, these moves cannot happen in protocol-free equilibria studied in the paper. Consequently, we do not view such possibility to be natural or robust, and impose the assumption that non-veto players cannot be last ones in a protocol to avoid such issues and obtain existence of protocol-free equilibria.

Example A3 ***(Example with fixed protocol)*** Suppose $n = 3$, $v = 1$, $k = 2$, so there are three players, one of them veto player, and the rule is simple majority. Consider the allocation $(1, 1; 0)$, where the veto player possesses nothing initially. In a protocol-free equilibrium, this allocation would be stable.

Consider a game where the protocol is fixed at $\pi = (1, 3)$ in each period (we can allow the second player to propose in-between the other two and get the same result). We claim that the following transitions are possible in an equilibrium. Player #1 is recognized first, and he proposes to move to $(1, 0; 1)$, which is supported by him and the veto player, and in the following period the veto player gets all the surplus, as usual. If the proposal by player #1 is rejected, however, then player #3 is recognized and proposes to move to $(0, 1; 1)$, and this proposal is supported by himself and player #2. Thus, in equilibrium, the society moves from $(1, 1; 0)$ to $(1, 0; 1)$, and then to $(0, 0; 2)$.

The reason for why this example works is the following. Player #1 knows that if he does not promise the veto player a transfer of one unit, then he would lose his possession immediately (later the same period), whereas delivering the unit to the veto player allows him to postpone for another period. The veto player knows that he cannot take both units at once (as players #1 and #2 would like to stick to them for another period); however, if he allows player #2 to keep his unit, the latter would not mind participating in expropriation of player #1, because in either case he keeps his unit for the current period and loses it in the following one, along the equilibrium path. Furthermore, if these strategies are played, preserving the status quo $(1, 1; 0)$ is not an option. Thus, there is an equilibrium where non-veto players participate in expropriation of each other.

Notice that this transition (from $(1, 1; 0)$ to $(1, 0; 1)$) cannot arise in a protocol-free equilibrium for the following reason. Suppose the protocol only involves the veto player. In such equilibrium, he needs to propose to transit to $(1, 0; 1)$. But player #2 will oppose it for obvious reasons, and player #1 would know that if he agrees, then he keeps his unit for one extra period (the current one), but if he rejects, then in protocol-free MPE he faces the same transition to $(1, 0; 1)$ the following period, and thus he would be able to keep the unit for two extra periods, which he obviously prefers. Consequently, such transition would be impossible in this protocol, which contributes to the idea that such transitions are not particularly robust.

Example A4 ***(Example of equilibrium that is not Markov perfect)*** Suppose $n = 3$, $v = 1$, $k = 2$, so there are three players, one of them veto player, and the rule is simple majority. Consider the allocation $(1, 1; 0)$, where the veto player possesses nothing initially.

Suppose that the veto player is always the proposer, so the protocol is $\pi = (3)$. Then the following transitions may be supported in equilibrium. As long as the allocation is $(1, 1; 0)$, the veto player proposes to move to $(1, 0; 1)$ if the period is odd and to move to $(0, 1; 1)$ if the period is even, and the proposal is supported by him and by the non-veto player who keeps the unit (player #1 in odd periods and player #2 in even periods). Once this transition has taken place, in the following period, the veto player gets everything, thus moving to $(0, 0; 2)$.

The rationale for non-veto players to support such proposals is that they get to keep their unit for exactly one extra period, regardless of the outcome of the voting. Thus, they are indifferent in such situations, in which case the veto player is able to allocate a small transfer to break this indifference. As a result, there is a SPE where the society moves to $(1, 0; 1)$ and then to $(0, 0; 2)$; it is supported by the threat of a move to $(0, 1; 1)$ (and then again to $(0, 0; 2)$) if this proposal is rejected.

Two comments are warranted. First, this SPE does not require knowledge of all history, in particular, players' proposals and votes. It only requires that the veto player acts differently in odd and even periods. In particular, this is a dynamic equilibrium (DE) in the sense of Anesi and Seidmann (2015), as if the players are allowed to condition their moves on the past history of alternatives, they of course can make use of the length of this history. Second, such transitions are impossible in a protocol-free equilibrium. Indeed, the proposal to move to $(1, 0; 1)$ made by the veto player would not be accepted if player 1 knew that the veto player would make this very proposal again in the following period, rather than proposing $(0, 1; 1)$.

Example A5 ***(Example with random recognition of players but without protocol-free requirement)*** Suppose $n = 5$, $v = 2$, $k = 3$, so there are five players, two of them veto players, and the rule is simple majority. Consider the allocation $(1, 1, 1; 0, 0)$, where the veto players possess nothing initially. In a protocol-free equilibrium, this allocation would be stable.

Consider a game, where in each period, one player is recognized as the proposer. Furthermore, assume for simplicity that only veto players may be recognized, and each of them is recognized with probability 0.5. Then the following strategies would form a MPE. Suppose that player #4, if he is the agenda-setter, proposes to move to $(2, 0, 0; 1, 0)$, and this proposal is supported by the two veto players and player #1. Similarly, if player #5 gets a chance to propose, he proposes to move to $(0, 2, 0; 0, 1)$, which is supported by the two veto players and player #2. If either of the proposals is accepted, then in the following period the society moves to $(0, 0, 0; 2, 1)$, where the veto players possess everything.

To understand why player #1 supports the transition to $(2, 0, 0; 1, 0)$, notice that in this case, he gets payoff 2 in the current period and 0 thereafter. If he rejects, then he keeps 1 in the current period, but in the next period he faces a lottery between 2 and 0, and gets 0 thereafter. His expected continuation payoff is therefore $1 + \beta\frac{2+0}{2} = 1 + \beta < 2$. Consequently, he prefers to agree on the transition to $(2, 0, 0; 1, 0)$. For the same reason, player #2 would support the transition to $(0, 2, 0; 0, 1)$. Notice that neither of the veto player can do better by choosing some other proposal, and therefore these transitions are possible in equilibrium.

Notice that if we impose the requirement that equilibria be protocol-free, which in this case would mean that the transition is the same regardless of the player that gets to make the proposal, such equilibrium will be ruled out. Thus, the requirement that equilibria do not depend on the protocol is important for our results, but also these equilibria may be considered more robust than the one in this example.

A3 Characterization for $n = 3, 4, 5$

The following tables contain a summary of stable allocations if the number of players is small ($n = 3, 4, 5$). The nontrivial cases, where non-veto players form groups and protect each other, are highlighted.

Number of veto players (v)

Voting rule (k)

$\boldsymbol{n = 3}$	$v = 1$	$v = 2$	$v = 3$
$k = 1$	Non-veto players possess nothing		
$k = 2$	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k = 3$	All allocations are stable	All allocations are stable	All allocations are stable

Number of veto players (v)

Voting rule (k)

$\boldsymbol{n = 4}$	$v = 1$	$v = 2$	$v = 3$	$v = 4$
$k = 1$	Non-veto players possess nothing			
$k = 2$	**Non-veto players have equal amounts**	Non-veto players possess nothing		
$k = 3$	**Two non-veto players have same amounts, third has nothing**	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k = 4$	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable

Number of veto players (v)

Voting rule (k)

$\boldsymbol{n = 5}$	$v = 1$	$v = 2$	$v = 3$	$v = 4$	$v = 5$
$k = 1$	Non-veto players possess nothing				
$k = 2$	**Non-veto players have equal amounts**	Non-veto players possess nothing			
$k = 3$	**Three non-veto players have same amounts, fourth has nothing**	**Non-veto players have equal amounts**	Non-veto players possess nothing		
$k = 4$	**Non-veto players form either two classes of two or one class of four**	**Two non-veto players have same amounts, third has nothing**	**Non-veto players have equal amounts**	Non-veto player possesses nothing	
$k = 5$	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable	All allocations are stable