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COMMENT ON "COMMITMENT VS. FLEXIBILITY"

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This comment corrects two results in the 2006 *Econometrica* paper by Amador, Werning, and Angeletos (AWA), that features a model in which individuals face a trade-off between flexibility and commitment. First, in contrast to Proposition 1 in AWA, we show that money-burning can be part of the ex ante optimal contract when there are two states. Second, in contrast to Proposition 2 in AWA, we show that money-burning can be imposed at the top (in the highest liquidity shock state), even when there is a continuum of states. We provide corrected versions of the above results.

KEYWORDS: Commitment, flexibility, self-control, money-burning.

1. INTRODUCTION

IN A PAPER IN THIS JOURNAL, Manuel Amador, Iván Werning, and George-Marios Angeletos (2006; from now on AWA) studied the optimal savings rule in a model where people are tempted to consume earlier, along the line of Strotz (1956), Phelps and Pollack (1968), and Laibson (1997, 1998), but full commitment is undesirable as it does not allow for incorporation of new information, such as taste shocks and income shocks.[2] They provided an optimal rule in two broad situations: if the shock variable can only take two values, and if the shock variable is continuous but a simple regularity condition on the density holds. An important feature of the optimum in the above characterization results is that there is no money-burning from the consumer's perspective: in every state, total consumption over time is equal to total endowment.[3]

We revisit the model of AWA and first analyze the case of two possible taste shocks. We show that money-burning may be used in equilibrium, imposed on the impatient type, so as to provide incentives for the more patient type not to imitate the impatient type. This is in contrast with Proposition 1 in AWA. The reason is that the arguments in AWA implicitly assume that the optimal contract involves allocating strictly positive amounts of the good to be consumed

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[2]See also Gul and Pesendorfer (2001) and Dekel, Lipman, and Rustichini (2001) for axiomatic foundations for preferences that imply temptation by present consumption and, relatedly, demand for commitment. For other papers studying optimal contracts with agents who suffer from self-control problems, see, for example, DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), Esteban and Miyagawa (2005), and Galperti (2012).

[3]Analogously, Athey, Bagwell, and Sanchirico (2004), Athey, Atkeson, and Kehoe (2005) showed, in various contract theory settings (that are technically connected to the original models they are primarily interested in), that money-burning is not part of the optimal contract. Ambrus and Egorov (2009), in a principal–agent setting different from the one in the current paper, characterized cases when money burning can be part of an optimal delegation scheme. See also Amador and Bagwell (2013).

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at both time periods, in every state. However, we show that there is an open set of parameter values for which the optimal contract involves zero consumption in the second time period in case of a negative liquidity shock in the first time period. In particular, this is the case when the probability of the impatient type is not too large, and when the negative liquidity shock is severe enough. For the same reason, Proposition 2 (as well as its corollary Proposition 6) in AWA is also incorrect, in claiming that with a continuum of types, money-burning is never imposed in the highest liquidity shock state.

2. THE MODEL

The setup reintroduces the model from AWA, and we preserve the notation. There are two periods and a single good. A consumer has a budget y and chooses his consumption in periods 1 and 2, c and k, respectively, so his budget set B is defined by $c \geq 0$, $k \geq 0$, $c + k \leq y$ (the interest rate is normalized to 0). The utility of *self-0* (the individual before the consumption periods) is given by

$$\theta U(c) + W(k),$$

where $U, W : \mathbb{R}^+ \to \mathbb{R}$ are two strictly increasing, strictly[4] concave, and continuously differentiable functions, and $\theta \in \Theta$ is a taste shock that is realized in period 1.

We assume that Θ is bounded and normalized so that $\mathbb{E}\theta = 1$. Denote the c.d.f. of θ by $F(\cdot)$ and the p.d.f. of θ by $f(\cdot)$. The utility of *self-1* is given by

$$\theta U(c) + \beta W(k),$$

where $0 < \beta \leq 1$ captures the degree of agreement between self-0 and self-1 (and $1 - \beta$ captures the strength of temptation toward earlier consumption). The goal is to characterize the optimal contract with self-0 as the principal and self-1 as the agent, that is, the consumption scheme that self-0 would choose from behind the veil of ignorance about the realization of the taste shock θ.

Hereinafter, we find it convenient to characterize contracts in terms of utilities rather than allocations (each is a monotone transformation of the other). We let $C(u)$ and $K(w)$ be the inverse functions of $U(c)$ and $W(k)$, respectively, and we let set A be given by

$$A = \{(u, w) \in \mathbb{R}^2 : u \geq U(0), w \geq W(0), C(u) + K(w) \leq y\}.$$

Since $C(u)$ and $K(w)$ are convex functions, the set A is convex. Define function $z(\cdot)$ by

$$z(x) = W(y - C(x));$$

[4]Assuming strict concavity rules out linear utility functions, but simplifies characterization a great deal. Since any linear function may be approximated by strictly concave functions, the results may be extended to the case of linear functions as well.

then $z(\cdot)$ is decreasing and strictly concave. The set $\{u, w : w = z(u)\}$ is the frontier of the set A where there is no money-burning: $C(u)+K(w) = y$. Thus, self-0 solves

$$\max_{(u(\theta),w(\theta))_{\theta\in\Theta}} \int_{\theta\in\Theta} \big(\theta u(\theta) + w(\theta)\big)\, dF(\theta) \tag{1}$$

$$\text{subject to} \quad \big(u(\theta), w(\theta)\big) \in A \quad \text{for every } \theta \in \Theta, \tag{2}$$

$$\theta u(\theta) + \beta w(\theta) \geq \theta u\big(\theta'\big) + \beta w\big(\theta'\big) \quad \text{for every } \theta, \theta' \in \Theta. \tag{3}$$

Finally, let $(u^{\text{fb}}(\theta), w^{\text{fb}}(\theta)) = \arg\max_{(u,w)\in A}(\theta u + w)$ denote the first-best allocation.

3. TWO TYPES

Here we consider the case of two types, so $\Theta = \{\theta_l, \theta_h\}$ with $0 < \theta_l < \theta_h$ (and given the normalization $E\theta = 1$, $\theta_l < 1 < \theta_h$). This setup can be interpreted such that state θ_l represents "normal times," while state θ_h represents a negative liquidity shock, such as a job loss.

If we denote the probability that $\theta = \theta_l$ by μ, we must have

$$\mu\theta_l + (1-\mu)\theta_h = 1. \tag{4}$$

We are thus solving the problem

$$\max_{(u_l,w_l),(u_h,w_h)\in A} \big(\mu(\theta_l u_l + w_l) + (1-\mu)(\theta_h u_h + w_h)\big) \tag{5}$$

$$\text{subject to} \quad \theta_l u_l + \beta w_l \geq \theta_l u_h + \beta w_h, \tag{6}$$

$$\theta_h u_h + \beta w_h \geq \theta_h u_l + \beta w_l. \tag{7}$$

Throughout this section, we use subscripts l and h to denote the values at θ_l and θ_h, respectively, for example, $u_l \equiv u(\theta_l)$, etc.

AWA, as part of Proposition 1 in this paper, characterized the parameter regions in which (i) the optimal contract achieves the first-best; (ii) does not achieve the first-best but implies separation of the two types; and (iii) implies pooling of the two types. Parts of this proof relied on an argument that there is no money-burning in the optimal contract. We show that this need not hold without additional assumptions, and provide the complete proof of this result in the Appendix, even though Part 1 of Proposition 1 of AWA is correct as stated.

PROPOSITION 1: *Suppose* $\Theta = \{\theta_l, \theta_h\}$ *with* $\theta_l < \theta_h$. *Suppose that* $\theta_l < |\frac{dz}{du}|_{u=U(y)}|$ *and* $\theta_h > |\frac{dz}{du}|_{u=U(0)}|$.[5] *Then there exists* $\beta^* \in (\theta_l/\theta_h, 1)$ *such that, for* $\beta \in [\beta^*, 1]$, *the first-best allocation is implementable.*

If $\beta \leq \theta_l/\theta_h$, *then pooling is optimal, that is,* $u_h = u_l$ *and* $w_h = w_l$; *moreover, there is no money-burning in this case*: $w_l = z(u_l)$.

If, however, $\beta \in (\frac{\theta_l}{\theta_h}, \beta^*)$, *then separation is optimal, that is,* $u_h > u_l$ *and* $w_h < w_l$. *In this last case,* $w_l = z(u_l)$, *but both* $w_h = z(u_h)$ *and* $w_h < z(u_h)$ *are possible. In either case, the IC constraint of the low type* (6) *is binding and the IC constraint of the high type* (7) *is not.*

Proposition 1 of AWA also claims that money-burning is never part of the optimal contract, which, as we find, does not have to hold in general. Our next result below gives a necessary and sufficient condition for money-burning to be part of the optimal contract. The proof of Proposition 1 in AWA becomes invalid without further assumptions at the point where the authors write, "Then an increase in $c(\theta_h)$ and a decrease in $k(\theta_h)$ that holds $(\theta_l/\beta)U(c(\theta_h)) + U(k(\theta_h))$ unchanged...," which implicitly assumes that a decrease in $k_h = k(\theta_h)$ is possible. If $k_h = 0$, so type θ_h consumes only in period 1, then such a decrease is clearly impossible. We prove that this is the only possible case consistent with money-burning (i.e., money-burning implies $c_h < y$, $k_h = 0$). In fact, if $k_h > 0$ in the optimal contract, then the argument in AWA goes through, ruling out the possibility of money-burning.

As a prelude to the next result, the following figures illustrate the two types of separating contracts that are possible in optimum. Note that if the IC constraint is binding for the low type, then the line connecting (u_l, w_l) and (u_h, w_h) has to have a slope of $-\theta_l/\beta$. Below, we refer to this line as the IC_l line. Figure 1 (left) depicts the case where, at the optimum, the IC_l line intersects set A twice at the Pareto frontier. This corresponds to a separating equilibrium with no money-burning, as in AWA. Figure 1 (right) depicts a different case, when, at the optimum, the IC_l line crosses the horizontal boundary of set A (on the $w = W(0)$ line), implying that there is money-burning in equilibrium. Below, we show that both of these cases can indeed occur at the optimum.

To give a precise characterization of when money-burning is part of a separating optimal contract, we need to introduce some further notation. Proposition 1 implies that the IC constraint of type θ_l is binding; let us denote, for any $\kappa \in \mathbb{R}$,

$$\lambda^\kappa = \left\{(u, w) \in A : u + \frac{\beta}{\theta_l} w = \kappa\right\}. \tag{8}$$

[5]This requirement ensures that the first-best contract is not pooling, and should have been included in Proposition 1 of AWA as well. If $\theta_l \geq |\frac{dz}{du}|_{u=U(y)}|$, then the optimal contract is $c_l^{\text{fb}} = c_h^{\text{fb}} = y$, $k_l^{\text{fb}} = k_h^{\text{fb}} = 0$, and if $\theta_h \leq |\frac{dz}{du}|_{u=U(0)}|$, then the optimal contract is $c_l^{\text{fb}} = c_h^{\text{fb}} = 0$, $k_l^{\text{fb}} = k_h^{\text{fb}} = y$. In either of these cases, the first-best is implementable for all β.

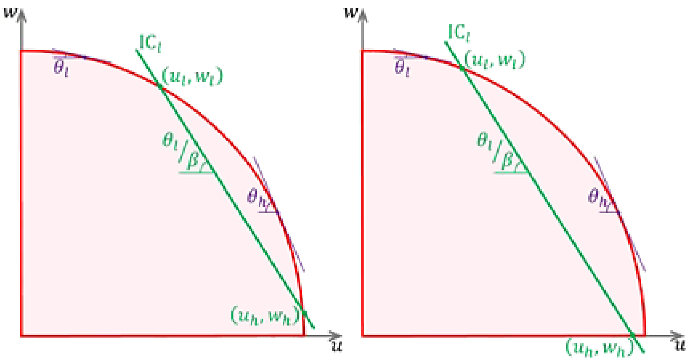

FIGURE 1.—Optimal contracts without and with money-burning.

For any κ, the above set of points is either a line segment, a point, or the empty set, although for simplicity we just refer to it as the IC$_l$ line. Whenever $\lambda^\kappa \neq \emptyset$, let $\lambda_l^\kappa = (u_l^\kappa, w_l^\kappa)$ and $\lambda_h^\kappa = (u_h^\kappa, w_h^\kappa)$ be the points of λ^κ that minimize and maximize u, respectively. Fixing $\kappa = u_l + \frac{\beta}{\theta_l} w_l = u_h + \frac{\beta}{\theta_l} w_h$, we observe that $(u_l, w_l) = \lambda_l^\kappa$ and $(u_h, w_h) = \lambda_h^\kappa$ (if it were not the case, then moving (u_l, w_l) north-west along the IC$_l$ line would not violate (6) or (7) and would increase (5), as $\theta_l < \frac{\theta_l}{\beta}$, and moving (u_h, w_h) along the same line would have the same effect as $\theta_h > \frac{\theta_l}{\beta}$). Let us now take a particular value of κ,

$$\kappa_0 = U(y) + \frac{\beta}{\theta_l} W(0); \tag{9}$$

then κ_0 is finite if $W(0) \neq -\infty$ and $\kappa_0 = -\infty$ otherwise. In the case κ_0 is finite, notice that $\lambda_h^{\kappa_0} = (U(y), W(0))$ by definition. The leftmost point of intersection of λ^{κ_0} with A, $\lambda_l^{\kappa_0}$, plays a critical role in the following formulation, and we let $u_0 \equiv u_l^{\kappa_0}$.

PROPOSITION 2: *Suppose $\frac{\theta_l}{\theta_h} < \beta < \beta^*$, so the optimal contract is separating. Money-burning will be used as part of the optimal contract if and only if* (i) $u_0 > U(0)$, *where u_0 is defined as $u_l^{\kappa_0}$ for $\kappa_0 = U(y) + \frac{\beta}{\theta_l} W(0)$, and* (ii) *the following inequality holds*:

$$\mu\left((1-\beta)\Big/\left(\frac{1}{\left|\frac{dz}{du}\Big|_{u=u_0}\right|} - \frac{\beta}{\theta_l}\right)\right) > 1. \tag{10}$$

The formal proof is in the Appendix. We want to point out that imposing a condition that $W'(0) = \infty$ (commonly referred to as the Inada condition) does not rule out the possibility that the optimal contract involves money-burning and zero second-period consumption in the high state. The intuition is that the IC constraint for the low type is binding. Therefore, while marginally increasing second-period consumption in the high state, starting from 0, increases the consumer's expected utility at an infinite rate, it also increases the temptation of the low type to pretend to be a high type, thus tightening the IC constraint and decreasing utility in the low state at an infinite rate.

EXAMPLE 1: Suppose $U(c)=\sqrt{c}$, $W(k)=\sqrt{k}$.[6] In this case, $z(u)=\sqrt{y-u^2}$ and $u_0=\frac{(\theta_l/\beta)^2-1}{(\theta_l/\beta)^2+1}\sqrt{y}$, $\frac{dz}{du}(u_0)=-\frac{u_0}{\sqrt{y-u_0^2}}=-\frac{1}{2}(\frac{\theta_l}{\beta}-\frac{\beta}{\theta_l})$, and the condition (10) becomes

$$\mu(1-\beta)\frac{\theta_l}{\beta}\frac{(\theta_l/\beta)^2-1}{(\theta_l/\beta)^2+1}>1.$$

Now, if we take $\theta_l=\frac{1}{10}$, $\theta_h=10$, $\mu=\frac{10}{11}$, $\beta=\frac{1}{20}$, the left-hand side equals $\frac{57}{55}>1$. One can check that the optimal contract is $c_l=\frac{121}{346}$, $k_l=\frac{225}{346}$, $c_h=\frac{1369}{1384}$, $k_h=0$, and indeed involves money-burning. (The optimal contract with the constraint that money-burning is not allowed would be $c_l=\frac{9}{25}$, $k_l=\frac{16}{25}$, $c_h=1$, $k_h=0$, and the ex ante expected utilities in the two contracts are $\frac{3257}{220\sqrt{346}}=0.795897$ and $\frac{87}{110}=0.790909$, respectively, with the difference of 0.005.) In a working paper version, we provided examples with different (power) utility functions, where the use of money-burning increased the gain in ex ante welfare by more than 36%.

Example 1, which shows that money-burning is possible, is not atypical. In particular, this has nothing to do with the choice of utility functions: as long as the utility functions in both periods are the same, one can find an open set of parameter values (relative to the possible set of parameter values defined in the model) for which having money-burning is optimal, that is, (10) is satisfied. We formalize this result in the Supplemental Material (Ambrus and Egorov (2013)).

4. CONTINUUM OF TYPES

Let us restrict attention to the case where the support of θ is a compact segment $\Theta=[\underline{\theta},\overline{\theta}]$, and that $f(\theta)$ is positive on Θ. Denote

$$G(\theta)=F(\theta)+\theta(1-\beta)f(\theta), \tag{11}$$

[6]We thank an anonymous referee for a suggestion that made the example simpler.

and let θ_p be the lowest $\theta \in \Theta$ such that

$$\int_{\hat{\theta}}^{\overline{\theta}} \big(1 - G(\tilde{\theta})\big)\, d\tilde{\theta} \leq 0 \quad \text{for all } \hat{\theta} \geq \theta_p.$$

Since $F(\overline{\theta}) = 1$ and $f(\overline{\theta}) > 0$, we must have $\theta_p < \overline{\theta}$. The following proposition proves that there is "bunching at the top," that is, all types $\theta > \theta_p$ get the same allocation.

PROPOSITION 3: *An optimal allocation* $\{(u(\theta), w(\theta))\}_{\theta\in\Theta}$ *satisfies* $u(\theta) = u(\theta_p)$ *and* $w(\theta) = w(\theta_p)$ *for* $\theta \geq \theta_p$. *Both* $w(\theta) = z(u(\theta))$ *and* $w(\theta) < z(u(\theta))$ *are possible for* $\theta \geq \theta_p$.

This proposition corrects Proposition 2 in AWA. Like AWA, we claim that the types $[\theta_p, \overline{\theta}]$ are pooled. Unlike AWA, we do not claim that the budget constraint holds with equality for these types and there is no money-burning at the top. On the contrary, we show that it is possible that types $[\theta_p, \overline{\theta}]$ will have to burn money. The difference in the conclusions again arises because of the possibility that the optimal contract does not specify an interior consumption plan. In particular, in the proof of Proposition 2, AWA suggested that if θ_p is interior (i.e., $\theta_p \in (\underline{\theta}, \overline{\theta})$), then $u(\theta_p)$ can be increased in a way that the IC constraint is preserved and the objective function does not decrease. However, preserving the IC constraint for type θ_p necessarily implies that $w(\theta_p)$ must be decreased, which is impossible if $w(\theta_p) = 0$. As in the case with two types, therefore, we only can have money-burning at the top if $w(\theta) = 0$ for high types.

Lastly, we note that Proposition 6 in AWA, which generalizes Proposition 2 there, is also incorrect in claiming the absence of money-burning, for the same reason as Proposition 2. As the arguments are analogous to the ones regarding Proposition 2, we omit the details here.

APPENDIX

PROOF OF PROPOSITION 1: First note that the first-best allocation is implementable if $\beta \geq \beta^*$, where

$$\beta^* = \theta_l \frac{u_h^{\text{fb}} - u_l^{\text{fb}}}{w_l^{\text{fb}} - w_h^{\text{fb}}},$$

and moreover $\beta^* > \frac{\theta_l}{\theta_h}$. This is correctly proven in AWA.

From now on, consider the case $\beta < \beta^*$. Adding the incentive constraints (6) and (7) implies $\theta_h(u_h - u_l) \geq \theta_l(u_h - u_l)$, which implies $u_h \geq u_l$. Trivially, if (6) holds with equality, then (7) holds as well. Let us prove that (6) binds (so we can forget about (7)) and that $(u_l, w_l) \in \partial A$ and $(u_h, w_h) \in \partial A$.

To see that $(u_l, w_l) \in \partial A$, assume the contrary. If $(u_l, w_l) \notin \partial A$, then we can use reasoning analogous to AWA: we can lower u_l and raise w_l slightly while holding $\theta_l u_l + \beta w_l$ unchanged, so that the modified contract is still in A; this would not change (6), will relax (7), and will increase the objective function (5), which contradict optimality of the initial contract.

To prove that $(u_h, w_h) \in \partial A$, suppose that $(u_h, w_h) \notin \partial A$, and consider the following three cases separately. If $\beta > \frac{\theta_l}{\theta_h}$, then a slight increase in u_h and a corresponding decrease in w_h that holds $\theta_l u_h + \beta w_h$ unchanged will not change (6), will relax (7), and will increase the objective function (5). If $\beta < \frac{\theta_l}{\theta_h}$, then a slight decrease in u_h and a corresponding increase in w_h will do the same. Finally, if $\beta = \frac{\theta_l}{\theta_h}$, then moving (u_h, w_h) to ∂A while preserving $\theta_l u_h + \beta w_h$ will not violate any constraint and will preserve the objective function, so without loss of generality we may assume that $(u_h, w_h) \in \partial A$ in the optimal contract in this case as well.

Let us now prove that (6) holds with equality in the optimal contract. Denote $\partial_f A = \{(u, w) \in \partial A : C(u) + K(w) = y\}$, $\partial_c A = \{(u, w) \in \partial A : C(u) = 0\}$, and $\partial_k A = \{(u, w) \in \partial A : K(u) = 0\}$. Note that $\partial A = \partial_f A \cup \partial_c A \cup \partial_k A$.

Suppose, to obtain a contradiction, that (6) is not binding; this already implies that the optimal contract is separating. We must have $(u_h, w_h) \in \partial_f A$, for otherwise we would be able to increase u_h slightly without violating either of the constraints and increasing the objective function. Second, we must have $(u_l, w_l) \in \partial_f A$. Indeed, suppose not; then either $(u_l, w_l) \in \partial_c A$ or $(u_l, w_l) \in \partial_k A$. Notice that (7) must bind, for if (7) did not bind, we could increase c_l to increase the objective function. Now, if $(u_l, w_l) \in \partial_k A$, then we must have $w_l \le w_h$ (w_l is the lowest possible), we also have $u_l \le u_h$, and if the contract is separating, one of the inequalities is strict, but then (7) cannot be binding. The remaining case is $(u_l, w_l) \in \partial_c A \setminus \partial_f A$. Since (7) binds, we must have $|\frac{dz}{du}|_{u=u_h}| > \frac{\theta_h}{\beta}$. But then slightly increasing w_l, coupled with moving (u_h, w_h) along $\partial_f A$ so as to preserve (7), would unambiguously increase the objective function. This means that if (6) is not binding, then $(u_l, w_l) \in \partial_f A$, $(u_h, w_h) \in \partial_f A$, and also $u_l < u_h$ (otherwise the contract would be pooling, not separating). Again, suppose first that (7) binds; then $u_l < u_h$ means that (u_h, w_h) is the rightmost point of intersection of the line corresponding to (7) and $\partial_f A$, and so $|\frac{dz}{du}|_{u=u_h}| > \frac{\theta_h}{\beta}$; in this case, moving (u_h, w_h) slightly in the direction of $(u_h^{\text{fb}}, w_h^{\text{fb}})$ would relax (7) and increase the objective function. The last possibility is that (7) does not bind. Then we could move either (u_h, w_h) slightly in the direction of $(u_h^{\text{fb}}, w_h^{\text{fb}})$ or (u_l, w_l) slightly in the direction of $(u_l^{\text{fb}}, w_l^{\text{fb}})$ so as to increase the objective function without violating any of the non-binding constraints. The only case where such deviation would not be possible is where $(u_h, w_h) = (u_h^{\text{fb}}, w_h^{\text{fb}})$ and $(u_l, w_l) = (u_l^{\text{fb}}, w_l^{\text{fb}})$. But this is not an incentive compatible contract if $\beta < \beta^*$ by the definition of β^*. This contradiction proves that (6) binds.

Consider the case $\frac{\theta_l}{\theta_h} < \beta < \beta^*$. Let us prove that the contract is separating. Indeed, if it were pooling, then, first of all, $(u_l, w_l) = (u_h, w_h) \in \partial_f A$. If this contract is λ_h^κ (but not λ_l^κ) for $\kappa = u_l + \frac{\beta}{\theta_l} w_l$, then we can lower u_l and raise w_l slightly while holding $\theta_l u_l + \beta w_l$ unchanged; this would not change (6), will relax (7), and will increase the objective function (5). If this contract corresponds to λ_l^κ (but not λ_h^κ), then we can raise u_h and lower w_h slightly while holding $\theta_l u_h + \beta w_h$ unchanged with similar effects. The remaining case is where $\lambda_l^\kappa = \lambda_h^\kappa$; this means that $|\frac{dz}{du}|_{u=u_h}| = \frac{\theta_l}{\beta}$, and then moving (u_l, w_l) in the direction of $(u_l^{\text{fb}}, w_l^{\text{fb}})$ and moving (u_h, w_h) in the direction of $(u_h^{\text{fb}}, w_h^{\text{fb}})$ in a way that (6) continues to bind will relax (7) and will increase the objective function. Consequently, the optimal contract is separating. This implies $u_l < u_h$, and thus (7) does not bind. From this, one can easily prove that $(u_l, w_l) \in \partial_f A$ (otherwise, slightly increasing $\theta_l u_l + \beta w_l$ would create an incentive compatible contract which yields a higher ex ante payoff) and, moreover, $|\frac{dz}{du}|_{u=u_l}| \in [\theta_l, \frac{\theta_l}{\beta}]$ (in particular, $u_l \in [u_l^{\text{fb}}, u_h^{\text{fb}}]$). Indeed, if $|\frac{dz}{du}|_{u=u_l}| < \theta_l$, then moving (u_l, w_l) in the direction of $(u_l^{\text{fb}}, w_l^{\text{fb}})$ would increase the ex ante payoff, and $|\frac{dz}{du}|_{u=u_l}| > \frac{\theta_l}{\beta}$ makes $(u_l, w_l) \in \partial_f A$ and (6) binding incompatible with $u_h > u_l$. As for (u_h, w_h), we can rule out $(u_h, w_h) \in \partial_c A$ (as then $u_l < u_h$ is impossible), but as we show, both $(u_h, w_h) \in \partial_f A$ and $(u_h, w_h) \in \partial_k A$ are possible.

Now consider the case $\beta < \frac{\theta_l}{\theta_h}$. Let us prove that the contract is pooling. If it were separating, then we can lower u_h and raise w_h slightly while holding $\theta_l u_h + \beta w_h$ unchanged (the fact that $(u_l, w_l) \in A$ ensures that such deviation results in a contract within A, but it also preserves (6), (7) and increases the ex ante payoff (5)). Hence, the contract is pooling. This means that $(u_l, w_l) = (u_h, w_h) \in \partial_f A$ and also $u_l \in [u_l^{\text{fb}}, u_h^{\text{fb}}]$, for otherwise, moving the pooled contract along $\partial_f A$ in the direction of the first-best contract would increase the ex ante payoff.

We thus showed that the contract is separating if $\frac{\theta_l}{\theta_h} < \beta < \beta^*$, pooling if $\beta < \frac{\theta_l}{\theta_h}$, and money-burning is possible only in the separating case and for type θ_h only. The possibility of money-burning for type θ_h is established by Example 1; the construction of an example without money-burning at optimum is trivial. This completes the proof. *Q.E.D.*

PROOF OF PROPOSITION 2: Take $\beta \in (\frac{\theta_l}{\theta_h}, \beta^*)$. From the proof of Proposition 1, if money-burning is part of the optimal contract, then $(u_h, w_h) \in \partial_k A \setminus \partial_f A$, so $k_h = 0$, $c_h < y$.

Also, by Proposition 1, we know that (6) is binding. Consequently, if (u_l, w_l, u_h, w_h) is the optimal contract, then $u_h + \frac{\beta}{\theta_l} w_h = u_l + \frac{\beta}{\theta_l} w_l$, which we denote by κ. This means that $(u_l, w_l), (u_h, w_h) \in \lambda^\kappa$. Moreover, from the proof of Proposition 1, we know that $(u_l, w_l) = \lambda_l^\kappa$, $(u_h, w_h) = \lambda_h^\kappa$. This proves that the optimal contract solves the following problem (formulated in terms of κ, which remains the only degree of freedom):

$$\max_{\kappa:\lambda^\kappa\neq\emptyset}\bigl(\mu\bigl(\theta_l u_l^\kappa + w_l^\kappa\bigr) + (1-\mu)\bigl(\theta_h u_h^\kappa + w_h^\kappa\bigr)\bigr). \tag{12}$$

Indeed, the constraints (6) and (7) would then hold automatically: The IC constraint of type θ_l (6) would hold as equality because (u_l^κ, w_l^κ) and (u_h^κ, w_h^κ) lie on the same λ^κ, and the IC constraint of type θ_h (7) would follow from the fact that (6) holds with equality and $u_h^\kappa \geq u_l^\kappa$. Moreover, again from the proof of Proposition 1, we have $(u_l, w_l) \in \partial_f A$, and also $u_l \geq u_l^{\text{fb}}$, so it suffices to optimize over $\kappa \geq u_l^{\text{fb}} + \frac{\beta}{\theta_l} w_l^{\text{fb}}$ only.

Let us first establish that (12) is strictly concave in κ. Take two values of κ, κ_1 and κ_2, and denote the value of the maximand in (12) by $v(\kappa_1)$ and $v(\kappa_2)$, respectively. Now take any $\delta \in (0, 1)$. Given the linearity of the objective function (12) and the constraints (6) and (7), the contract given by $u_l' = \delta u_l^{\kappa_1} + (1-\delta)u_l^{\kappa_2}$, $u_h' = \delta u_h^{\kappa_1} + (1-\delta)u_h^{\kappa_2}$, $w_l' = \delta w_l^{\kappa_1} + (1-\delta)w_l^{\kappa_2}$, $w_h' = \delta w_h^{\kappa_1} + (1-\delta)w_h^{\kappa_2}$ satisfies the constraints and yields the value of (12) v' equal to $\delta v(\kappa_1) + (1-\delta)v(\kappa_2)$; moreover, it lies in A due to convexity of A. Since we proved that we can only improve by moving (u_l, w_l) to the upper-left and (u_h, w_h) to the lower-right, we get that $v(\delta\kappa_1 + (1-\delta)\kappa_2) > \delta v(\kappa_1) + (1-\delta)v(\kappa_2)$ (to see that the inequality is strict, notice that at least (u_l', w_l') necessarily lies in the interior of A). Hence we established that (12) is strictly concave in κ.

We now see that money-burning is optimal if and only if (12) increases if we decrease κ a little bit from the value $\kappa_0 = U(y) + \frac{\beta}{\theta_l} W(0)$. If $u(0) = U(0)$, then doing so decreases the value of the objective function, because both the low type and the high type will get a smaller payoff. Now consider two cases. Suppose first that $\kappa_0 > u_l^{\text{fb}} + \frac{\beta}{\theta_l} w_l^{\text{fb}}$; then the formula (10) is derived in the main text. If $\kappa_0 \leq u_l^{\text{fb}} + \frac{\beta}{\theta_l} w_l^{\text{fb}}$, then $|\frac{dz}{du}|_{u=u_0}| \leq \theta_l$ as $u_0 \leq u_l^{\text{fb}}$. But then the right-hand side of (10) does not exceed $\mu\theta_l < 1$, so the formula is correct in this case as well. *Q.E.D.*

PROOF OF PROPOSITION 3: The proof that $u(\theta) = u(\theta_p)$ for $\theta \geq \theta_p$ in AWA is correct, and is omitted here. Trivially, we must have $w(\theta) = w(\theta_p)$ for $\theta \geq \theta_p$ as well (otherwise, only the contracts with the highest w will be chosen). This proves the first part of the proposition.

Next we show, by example, that $w(\theta_p) < z(u(\theta_p))$ is possible, so money-burning for high types is possible. Our strategy is to build on Example 1, approximate it with a continuous distribution, and show that, for sufficiently close approximations, the optimal contract must have money-burning. Take $U(c) = \sqrt{c}$, $W(k) = \sqrt{k}$, $y = 1$ (then $z(u) = \sqrt{1-u^2}$), $\beta = \frac{1}{20}$. Take $\varepsilon \in (0, \frac{1}{10})$, and let F_ε be the atomless distribution with finite support given by the following p.d.f.:

$$f_\varepsilon(\theta) = \begin{cases} 0, & \text{if } \theta < \dfrac{1}{10} - \varepsilon, \\ \dfrac{\dfrac{10}{11} - \dfrac{\varepsilon}{2}}{\varepsilon}, & \text{if } \dfrac{1}{10} - \varepsilon \le \theta < \dfrac{1}{10}, \\ \dfrac{\varepsilon}{10 - \dfrac{1}{10}}, & \text{if } \dfrac{1}{10} \le \theta < 10, \\ \dfrac{\dfrac{1}{11} - \dfrac{\varepsilon}{2}}{\varepsilon}, & \text{if } 10 \le \theta < 10 + \varepsilon, \\ 0, & \text{if } 10 + \varepsilon \le \theta. \end{cases}$$

We compute $\theta_p = \theta_p(\varepsilon)$ and show that, for every $\varepsilon \in (0, \frac{1}{10})$, $\theta_p < \frac{1}{2}$. Hence, for any such ε, all agents with $\theta \ge \frac{1}{2}$ receive the same allocation in the optimal contract. Denote the ex ante payoffs in Example 1 (for $y = 1$) in the optimal contract by V and in the optimal contract subject to no money-burning constraint by $\tilde{V}$. We prove in the Supplemental Material that, if we take the optimal contract from 1 and offer it to the agents in this example, the ex ante payoff will be close to V for ε sufficiently small (formally, we show that $\liminf_{\varepsilon\to 0} V_\varepsilon \ge V$). On the other hand, we show there that if we restrict attention to contracts without money-burning, we cannot improve much more over $\tilde{V}$ (formally, $\limsup_{\varepsilon\to 0} \tilde{V}_\varepsilon \le \tilde{V}$). But we know that $V > \tilde{V}$, and therefore, for ε close to 0, $V_\varepsilon > \tilde{V}_\varepsilon$. This shows that, for some $\varepsilon > 0$, the optimal contract involves money-burning for all agents with $\theta > \theta_p(\varepsilon)$, and the mass of these agents is positive (at least $\frac{1}{11}$). This argument shows that $w(\theta_p) < z(u(\theta_p))$ is possible. *Q.E.D.*

REFERENCES

AMADOR, M., AND K. BAGWELL (2013): "The Theory of Optimal Delegation With an Application to Tariff Caps," *Econometrica*, 81 (4), 1541–1599. [2113]

AMADOR, M., I. WERNING, AND G.-M. ANGELETOS (2006): "Commitment vs. Flexibility," *Econometrica*, 74 (2), 365–396. [2113]

AMBRUS, A., AND G. EGOROV (2009): "Delegation and Nonmonetary Incentives," Report, Harvard University. [2113]

——— (2013): "Supplement to 'Comment on "Commitment vs. Flexibility"'," *Econometrica Supplemental Material*, 81, http://www.econometricsociety.org/ecta/supmat/10739_proofs.pdf. [2118]

ATHEY, S., A. ATKESON, AND P. KEHOE (2005): "The Optimal Degree of Discretion in Monetary Policy," *Econometrica*, 73, 1431–1475. [2113]

ATHEY, S., K. BAGWELL, AND C. SANCHIRICO (2004): "Collusion and Price Rigidity," *Review of Economic Studies*, 71, 317–349. [2113]

DEKEL, E., B. LIPMAN, AND A. RUSTICHINI (2001): "Representing Preferences With a Unique Subjective State Space," *Econometrica*, 69, 891–934. [2113]

DELLAVIGNA, S., AND U. MALMENDIER (2004): "Contract Design and Self-Control: Theory and Evidence," *Quarterly Journal of Economics*, 119, 353–402. [2113]

ELIAZ, K., AND R. SPIEGLER (2006): "Contracting With Diversely Naive Agents," *Review of Economic Studies*, 73, 689–714. [2113]

ESTEBAN, S., AND E. MIYAGAWA (2005): "Optimal Menu of Menus With Self-Control Preferences," Report, Universitat Autonoma de Barcelona. [2113]

GALPERTI, S. (2012): "Commitment, Flexibility, and Optimal Screening of Time Inconsistency," Report, Northwestern University. [2113]

GUL, F., AND W. PESENDORFER (2001): "Temptation and Self-Control," *Econometrica*, 69, 1403–1435. [2113]

LAIBSON, D. (1997): "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 112, 443–477. [2113]

——— (1998): "Life-Cycle Consumption and Hyperbolic Discount Functions," *European Economic Review*, 42, 861–871. [2113]

PHELPS, E., AND R. POLLACK (1968): "On Second Best National Savings and Game-Equilibrium Growth," *Review of Economic Studies*, 35, 185–199. [2113]

STROTZ, R. (1956): "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, 23, 165–180. [2113]

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