

Comments on "Time-varying Risk Premia and the Cost of Capital: An Alternative  
Implication of the Q Theory of Investment"

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## 1 Introduction

Martin Lettau and Sydney Ludvigson have written a series of papers exploring the ability of the consumption-wealth ratio to predict excess stock returns. The current paper takes this agenda another step by instead predicting real economic activity - specifically, investment in physical capital. The goal is an exciting and ambitious one: to connect the real and financial components of the macroeconomy. In the piece of the agenda undertaken here, the authors provide compelling evidence that the consumption-wealth ratio is a useful predictor of future investment growth.

In my comments, I first summarize the theoretical connection between the consumption-wealth ratio and future investment growth. This requires two basic equations. First, the intertemporal budget constraint for consumption links the consumption-wealth ratio to future returns. Second, the  $Q$ -theory of investment links asset returns to investment. Lettau and Ludvigson's earlier work examines the first alone, while this paper adds the second and tests the forecastability of investment using the consumption-wealth ratio. However, this second step also requires that the consumption-wealth ratio forecast changes in Tobin  $Q$ . This conceptual link is not tested in the paper, but is easily examined. I show in Section 3 that  $Q$  is indeed forecastable, consistent with the argument of the paper.

The more challenging question laid down by this line of research, however, is the source of forecastability. In most standard models without arbitrage, equity returns are not forecastable. Empirical evidence to the contrary has motivated a number of models that can generate this feature. While the evidence here consistent with forecastability of returns, it is silent on the source of this forecastability. However, the paper discusses the possibility that the forecasting power of the consumption-wealth ratio arises from changes in investors' required return to risk and hence, equity risk premia. Such a utility-based explanation might leave its footprints in consumption behavior, and indeed, the authors have examined whether the consumption-wealth ratio predicts future changes in consumption of nondurables and services. While they find no evidence of this, that result is not surprising. Alternatively, one might examine consumer durables expenditures for evidence of time varying risk premia. Since durables expenditures are physical capital investments made by households, they might be a more promising landscape in which to search for evidence of time varying risk aversion. I show below that there is only weak evidence relating the consumption-wealth ratio to future durable expenditures. The weakness of this finding may be because there are conflicting effects at work, but the evidence here is decidedly less compelling than for business investment.

Another implication explored by the authors is the presence of "investment lags". While this is not their main focus, the authors argue that a time lag in the dynamics of the investment response to changes in returns is evidence of an investment lag. I will argue below that such a time lag is not consistent with investment lags that involve time-to-build or other installation lags. These lags would quash rather than shift the investment

response. The reported data seem in fact more consistent with this implication of quasi-fixity of the capital stock in the short run.

## 2 Conceptually Linking the Consumption-Wealth Ratio and Investment Growth

Theoretically, there is a direct link between the consumption-wealth ratio and future investment. The paper develops this in the first section, so here I provide only a summary. Two equations provide the backbone of the empirical tests. The first is the intertemporal budget constraint. This links consumption and wealth with future returns to wealth, since the consumption expenditures are limited by the agent's wealth and the returns that can be achieved by investing. The second main equation comes from the  $Q$ -theory of investment, since this relates investment to asset prices and returns.

Beginning from the intertemporal budget constraint, the paper uses Campbell and Mankiw's expression for the log consumption-wealth ratio (equation (12) in the paper),

$$c_t - w_t = E_t \sum_{i=1}^{\infty} r_w^i (r_{w,t+i} - \Delta c_{t+i}) \quad (1)$$

This expression relates the consumption-wealth ratio to future returns and wealth and the future path of consumption. If consumption is expected to be flat,  $E_t \Delta c_{t+i} = 0$  for all  $i$ . In this case, current consumption is just an annuity payment from the stock of wealth, and

the consumption-wealth ratio is just the annuity factor,  $E_t \sum_{i=1}^{\infty} \mathbf{r}_w^i r_{w,t+i}$ . If the current consumption-wealth ratio exceeds this annuity factor, then future consumption must fall, so that  $E_t \sum_{i=1}^{\infty} \mathbf{r}_w^i \Delta c_{t+i} < 0$ . Similarly, if the current consumption-wealth ratio is less than the annuity factor, then future consumption may be increasing.

Conceptually, the consumption-wealth ratio thus summarizes agents' views about future returns to wealth and planned consumption profiles.<sup>1</sup> In their earlier work, Lettau and Ludvigson use this expression directly. Since the consumption-wealth ratio should predict future returns to wealth, they develop an empirical proxy,  $\hat{c}\hat{y}$ , for the consumption-wealth ratio. This empirical proxy is used to predict returns to one component of wealth, the stock market.<sup>2</sup>

The link from the consumption-wealth ratio to real economic activity is also conceptually direct. Market efficiency implies that the expected required return to a firm's marginal unit of capital,  $r_K$ , equals the total expected return, or the sum of the expected capital gain and marginal profit derived from the marginal unit of capital. The value of the marginal unit of capital is marginal  $Q$ . Equating the gross required return and the expected return (capital gain plus marginal profit) to the marginal unit of capital yields

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<sup>1</sup> Campbell (1987) used this intuition by using a closely related measure, saving, to predict future labor income in his "saving for a rainy day" model.

<sup>2</sup> In principle, equation (1) relates the consumption-wealth ratio to future returns on total wealth. Lettau and Ludvigson predict one component of this, the returns to levered equity.

$$E_t(1 + r_{K,t}) = E_t \left[ \frac{Q_{t+1} + \frac{\partial p(K_t)}{\partial K_t} (1 - \mathbf{d})}{Q_t} \right] \quad (2)$$

where  $r_{K,t}$  is the required return to physical capital,  $Q_t$  is marginal  $Q$ ,  $p$  is cash flow, and  $\mathbf{d}$  is the depreciation rate of capital,  $K$ . The first term on the right hand side of equation (2) measures the expected capital gain on the marginal unit of capital, while the second term gives the expected net marginal cash flow. Together these give the total return to the marginal unit of capital, which must equal the required return on the left-hand side of the equation.

Lettau and Ludvigson then log-linearize this condition to obtain a linear expression for the required return to capital, which is equation (5) in the paper

$$E_t(r_{K,t+1}) \approx \mathbf{r}_q E_t \Delta q_{t+1} + (1 - \mathbf{r}_q) E_t(m_{t+1} - q_t) + \Phi_t \quad (3)$$

where  $\mathbf{r}_q$  is a constant,  $\Phi_t$  is a time-varying variance term, and  $m$  is the log of  $\frac{\partial p}{\partial K}(1 - \mathbf{d})$ .

I have written the return on the left hand side as  $r_{K,t+1}$  to emphasize that it is the required return on the firm's capital. In order to relate the consumption-wealth ratio to  $q$  (and subsequently to investment), the authors first equate  $r_{w,t}$  and  $r_{K,t}$ , the returns to wealth

and to physical capital, respectively.<sup>3</sup> With this assumption, equation (3) can be substituted into equation (1) to obtain

$$c_t - w_t = E_t \sum_{i=1}^{\infty} \mathbf{r}_w^i \left[ \mathbf{r}_q \Delta q_{t+1+i} + (1 - \mathbf{r}_q)(m_{t+1+i} - q_{t+i}) + \Phi_{t+i} - \Delta c_{t+i} \right] \quad (4)$$

This expression relates the current (log) consumption-wealth ratio not to future returns to wealth, but rather to future returns to physical capital, which are composed of capital gains,  $\Delta q$ , and marginal profits,  $m$ .

The final step connects the value of capital and investment in capital using  $Q$  theory, as suggested by the title of the paper. Following Abel (1983), the authors employ an adjustment cost function of the form  $g(I_t) = \mathbf{g}_t^b$ ,  $b > 1$ . Equating the marginal value of an additional unit of capital,  $Q$ , with its expected marginal cost yields  $Q_t = E_{t-1} \mathbf{b} \mathbf{g}_t^{b-1}$ . The paper again log-linearizes this expression to obtain

$$q_t = \ln(\mathbf{g}_t) + (\mathbf{b} - 1) E_{t-1} i_t + \frac{1}{2} (\mathbf{b} - 1)^2 \mathbf{s}^2(i_t) \quad (5)$$

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<sup>3</sup> This substitution equates the return on wealth and a different sub-component than the authors examined in their work on equity returns and in Table 1. In this case, the returns to physical capital are substituted for the returns to total wealth.

where  $i_t \propto \ln(I_t)$ , and  $s^2(i_t)$  is the conditional variance of log investment. Differencing equation (5) yields an expression for  $Dq_t$ , which can be substituted into equation (4) to yield

$$c_t - w_t = E_t \sum_{i=1}^{\infty} r_w^i \left[ r_q (b-1) \Delta i_{t+1+i} + (1 - r_q) (m_{t+1+i} - q_{t+i}) + \Phi_{t+i} - \Delta c_{t+i} \right] \quad (6)$$

Equation (6) relates the current (log) consumption-wealth ratio to future changes in investment, which will be the empirical substance of the paper.

This expression builds on two pieces of economic theory. The first is the intertemporal budget constraint for consumption, and the second is the  $Q$  theory of investment. The intertemporal budget constraint relates the consumption-wealth ratio to future returns. The second links asset returns to investment. In their earlier work, the authors have examined the first piece. Here they add the second.

### 3 Is Tobin's $Q$ forecastable?

Before the expression for optimal investment was substituted above, equation (4) linked the consumption wealth ratio to future growth in  $q$ . While the paper moves directly to examining equation (6) by looking at investment growth, this link is only present conceptually if equation (4) holds and changes in  $q$  are forecastable.

Since the authors graciously provided me their data, the ability of  $c\hat{a}y$  to forecast future growth in (log) Tobin's Q is easily tested. The results of this test are reported in Table 1; this table follows the format of Lettau and Ludvigson's (LL) tables by reporting the regression coefficient with the standard error in parentheses below, followed by the  $R^2$  of the regression. The first two rows of the table reproduce data from LL Tables 1 and 2, respectively, for comparison. The third row of the table reports the results of forecasting future growth in log Tobin's Q at the reported quarterly horizons. The results suggest that the ability of  $c\hat{a}y$  to predict future growth in Tobin's Q is very similar to its ability to predict stock prices. This is reassuring given the close relationship between Tobin's Q and equity values, as well as the important role played by Tobin's Q in the conceptual development of the subsequent investment regressions.

(Table 1 about here)

A potentially related effect is shown in the last row of Table 1. This row reports the ability of  $c\hat{a}y$  to predict future equity issues at long horizons, which is substantial. These data suggest another potential link from forecastable equity returns to corporate behavior, via corporate financing and capital structure.

#### **4 What is the source of $c\hat{a}y$ 's forecasting power?**

Recall that in the theoretical development of the connection between  $c\hat{a}y$  and investment, there were two main equations. The first was the intertemporal budget

constraint for consumption, and the second was the  $Q$  theory of investment. Neither of these involves utility maximization; indeed utility is entirely absent from this conceptual framework. This structure is silent on the source of return predictability; it only requires that agents respect the intertemporal budget constraint. The source of return predictability is a challenging question, and the authors discuss the possibility that the forecasting ability of  $c\hat{y}$  derives from its ability to capture movements in risk premia.

Where might one look for evidence of changes in risk premia? Changes in risk premia might affect consumption growth; however, in the introduction of the paper, the authors say, "movements in the consumption-wealth ratio ... bear virtually no relation to contemporaneous or future consumption growth." This is perhaps not surprising, given the well-known and theoretically well-motivated smoothness of consumption. Moreover, evidence suggests that consumption is close to a random walk, so changes should be difficult to predict. However, other components of consumption are not so smooth, and in fact, are conceptually and empirically more similar to investment. In particular, one might find evidence of time varying risk premia in consumer durables expenditures.

Table 2 reports long horizon regressions using  $c\hat{y}$  to forecast growth in consumer durables expenditures.<sup>4</sup> The table has the same format as Table 1 and the LL tables, reporting the regression coefficients, standard errors, and  $R^2$  at each horizon. The results suggest a much weaker relationship between  $c\hat{y}$  and future residential and durables investment than is found for business investment. Residential investment is the more

reliably predicted of the two durables series, though the  $R^2$  peaks at shorter horizons (.08 at four quarters) than business investment (.18 at 12 quarters in LL Table 2). Consumer durables are even less reliably predicted; the maximum  $R^2$  is only .03.

(Table 2 about here)

These regressions ignore other sources of variation in the forecasting relationship. Specifically, equation (6) suggests that the consumption-wealth ratio should predict future investment, controlling for the expected future path of  $m$ , or marginal profitability. That is, if future investment is predictable, it should be related to  $c\hat{y}$  even after controlling for profits. However, the predictability of durables expenditure found in Table 2 essentially disappears in the last row of the table, when we control for initial income.

## 5 Investment Lags

While unrelated to their main line of research, the authors also examine "investment lags". They argue, following Lamont (2000), that these lags are responsible for the delayed timing observed in their investment data. Specifically, they argue that when long run returns rise, the contemporaneous return falls. Without any "investment lag", this "sign flip" or reversal would be mirrored in investment growth, leading to an immediate decrease in investment growth, followed by a long run increase.

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<sup>4</sup> The data are real Personal Consumption Expenditures on durable goods and real Residential Investment, both in chained 1996 dollars, seasonally adjusted annual rates, from the Bureau of Economic Analysis.

In their data, this sign reversal occurs immediately (as predicted) in forecasted returns (LL Table 1, reproduced in Table 1 here), so that the effect of  $c\hat{a}y$  on future returns has the same sign from the first quarter onward. However, in the investment regressions the sign reversal is delayed until the fourth quarter. This is argued to be evidence of a four-quarter investment lag.

However, if by "investment lags" one means time-to-build or "delivery lags, planning lags, construction lags" (Section 4.2) as described in the paper, optimal investment would not be simply delayed, or temporally shifted. Instead, the firm chooses optimal investment knowing that there will be a delay. For example, if there is a four-quarter investment lag, the firm chooses future investment today based on today's expectation of returns to capital and  $Q$  four quarters hence. It would not be optimal to choose investment based on today's returns and  $Q$  and then face a four-quarter delay. Investment lags that delay installation would negate any immediate effect of returns and  $Q$  on current investment. Thus, lags of this sort *quash* the sign reversal, rather than delaying it. Indeed, this seems more consistent with the data reported in Tables 2 through 4 and Table 6, where the sign reversal is only significantly different from zero in two of the eight regressions involving  $c\hat{a}y$ . Instead, the data seem to provide evidence of a muted short-run response and therefore short-run quasi-fixity of the capital stock.<sup>5</sup>

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<sup>5</sup> An "expenditure lag", rather than an investment lag, might appear to temporally shift investment relative to returns and  $Q$ . With such a lag, investment would be installed immediately, but the expenditure would be recorded at some future date and measured by the econometrician as occurring then. This would

## 6 Concluding Remarks

Lettau and Ludvigson provide compelling evidence that a proxy for the consumption-wealth ratio forecasts future investment growth at the two to four year horizon. They extensively "stress test" this result, showing that it holds, with only small variations, in equipment and structures separately, and controlling for other variables often used to explain investment, such as lagged investment, profits, Tobin's (average)  $Q$ , and GDP. While these variables explain a larger share of the variation in investment growth than the financial variables examined in LL Table 2, they do not eliminate the marginal forecasting power of  $c\hat{a}y$ .

The conceptual link between  $c\hat{a}y$  and investment growth developed here builds from the intertemporal budget constraint for consumption and the  $Q$  theory of investment. Section 3 of this comment shows that the intermediate link from the intertemporal budget constraint to investment returns and  $Q$  is a solid basis for the subsequent investment regressions. That is,  $c\hat{a}y$  is hypothesized to predict investment because it predicts the return to capital; the results in Table 2 suggest that indeed  $c\hat{a}y$  does predict future changes in  $Q$ .

The deeper question underlying this research agenda, however, is the source of return predictability. While the theoretical development and tests employed here are formally silent on this point, the discussion and results might provide some clues. Predictability of

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temporally shift the entire investment series. This expenditure lag would generate a "sign flip" at the fourth

returns is consistent with time variation in risk premia, and the authors argue that  $c\hat{a}y$  may be measuring these changes in agents' required risk premia. Since current  $c\hat{a}y$  forecasts changes in investment, these data are consistent with a significant response from the corporate sector to changes in risk premia. Less supportive, however, is the evidence in Table 2 of this note, suggesting a much weaker response of households' investment in consumer durables. If  $c\hat{a}y$  is measuring changes in agents' risk premia, it would be very convincing to see these changes reflected in their own investments, in addition to those made indirectly via the corporate sector. While it may be that the effects are masked by measurement or offsetting conceptual effects, the results for consumer investment are considerably weaker than those for business investment.

By linking forecastability of equity returns to forecastability of business investment, these results move forward the enterprise of linking financial and macroeconomic variables. The empirical work here connects the consumption-wealth ratio, asset returns, and investment. The fascinating question for future work is whether this connection is evidence for a statistical model with reliable forecasting properties, or whether it goes further toward an asset-pricing model - linking utility and asset returns.

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quarter if capital were installed immediately, but the expenditures were not recorded for a year.

## References

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Table 1. Forecasting growth in Tobin's Q using  $\hat{c}ay$

| $\hat{c}ay$ forecasts:                        | Forecast Horizon, H      |                           |                         |                         |                         |                         |
|---|--------------------------|---------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
|   | 1                        | 2                         | 4                       | 8                       | 12                      | 16                      |
| Stock returns<br>(LL Table 1)                 | 1.77<br>(3.08)<br>0.07   | 3.34<br>(2.98)<br>0.10    | 5.95<br>(2.85)<br>0.17  | 9.65<br>(2.67)<br>0.24  | 11.10<br>(3.85)<br>0.26 | 11.02<br>(2.33)<br>0.22 |
| Growth in total<br>investment<br>(LL Table 2) | -0.19<br>(-1.44)<br>0.00 | -0.05<br>(-0.24)<br>-0.01 | 0.89<br>(2.15)<br>0.02  | 3.54<br>(4.11)<br>0.13  | 4.83<br>(4.33)<br>0.18  | 4.86<br>(3.86)<br>0.16  |
| Growth in<br>Average Q                        | 3.31<br>(2.10)<br>0.05   | 6.43<br>(3.35)<br>0.09    | 10.21<br>(3.40)<br>0.15 | 15.62<br>(3.89)<br>0.22 | 20.21<br>(7.74)<br>0.29 | 18.77<br>(8.25)<br>0.25 |
| Growth in<br>Equity Issues                    | 6.62<br>(0.97)<br>0.01   | 11.15<br>(1.94)<br>0.02   | 17.31<br>(2.85)<br>0.05 | 34.70<br>(4.89)<br>0.14 | 35.52<br>(4.46)<br>0.16 | 33.97<br>(5.35)<br>0.17 |

Table 2. Forecasting growth in durables and residential investment using *cây*

| cây forecasts<br>growth in:                                  | 1   | 2                        | 4                        | 8                        | 12                       | 16                       |
|--|---|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|  | Consumption<br>(non-durables &<br>services) | -0.05<br>(-1.80)<br>0.02 | -0.09<br>(-1.81)<br>0.02 | -0.10<br>(-1.12)<br>0.01 | -0.10<br>(-0.71)<br>0.00 | -0.18<br>(-1.22)<br>0.01 |
| Residential<br>Investment                                    | 0.57<br>(2.57)<br>0.02                      | 1.46<br>(3.61)<br>0.04   | 3.40<br>(4.45)<br>0.08   | 3.89<br>(3.08)<br>0.05   | 2.27<br>(1.61)<br>0.01   | 0.01<br>(0.01)<br>0.00   |
| Durables<br>Investment                                       | 0.03<br>(0.20)<br>0.00                      | 0.36<br>(1.31)<br>0.01   | 1.02<br>(2.06)<br>0.03   | 1.48<br>(2.05)<br>0.03   | 1.79<br>(2.79)<br>0.02   | 1.31<br>(1.67)<br>0.01   |
| Durables<br>Investment,<br>controlling for<br>initial income | 0.00<br>(0.00)                              | 0.27<br>(1.04)           | 0.79<br>(1.72)           | 0.80<br>(1.12)           | 0.56<br>(0.70)           | -0.21<br>(-0.22)         |