Two Asset Portfolios

Two assets have the following expected returns and standard deviations:

<table>
<thead>
<tr>
<th></th>
<th>µ</th>
<th>σ</th>
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<tbody>
<tr>
<td>Asset 1</td>
<td>.15</td>
<td>.20</td>
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<tr>
<td>Asset 2</td>
<td>.25</td>
<td>.50</td>
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a) What is the expected return of a portfolio with a share "w" in Asset 1?

b) What is the variance of returns of a portfolio with a share "w" in Asset 1 if the correlation between the two assets is ρ?

c) If ρ = .5, will Asset 2 be part of the minimum variance portfolio that could be formed with these two assets? Would your answer change with different values of ρ?

Solution:

a) \[ E(r) = w(0.15) + (1-w)(0.25) \]

b) \[ \text{Var}(r) = w^2(0.04) + (1-w)^2(0.25) + 2w(1-w)ρ(0.2)(0.5) \]

c) The minimum variance portfolio will typically include some of the riskier asset due to diversification and/or hedging. When the assets are not highly correlated, then positive amounts of both assets are held for diversification. When the assets are highly correlated, then lower portfolio risk can be obtained by taking a short position in one as insurance, or a hedge, against movements in the other.

Borrowing and Lending Rates

a) Draw the efficient frontier describing the optimal combinations of risky assets in terms of the portfolio expected return (μ) and standard deviation (σ).

b) Draw the capital market line (CML) for a given riskless rate, rf. In what range of the CML does the investor have a long position in the riskless asset? - a short position?

c) Suppose that the investor now has to pay a percentage of the returns as a fee in order to take a short position. Thus, with a long position, the investor still earns rf. However, with a short position, the investor has to pay rf ×(1+τ). Draw the CML for a long position and the CML for a short position.

d) Using the situation you've diagrammed in part (c), does the market (risky) portfolio held by the investor depend on whether he has a long or short position in the riskless asset?

e) Using the situation you've diagrammed in part (c), if an investor wanted to take on more risk than that implied by the market portfolio and the riskless (lending, or long) rate, what portfolio should he choose - to achieve low levels of additional risk? - for high levels of additional risk?
The upper (flatter) bold line is the CML for short positions. The lower (steeper) bold line is the CML for long positions.

d) If the investor is long the riskless asset, then he holds the market portfolio given at point "p". If the investor is short the riskless asset, then he holds the market portfolio given at point "q".

e) When there is a premium for taking a short position (or borrowing), then to move to the right of point "p" and take on more risk, the investor first chooses a riskier portfolio of risky assets (between points "p" and "q"). Then, to take on more risk than the standard deviation of the "q" portfolio, the investor borrows at rate \( r_f(1+\tau) \) and invests in the "q" portfolio (he leverages up).