

STRATEGIC ARGUMENTATION*

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Abstract

I analyze a game between an uninformed decision maker and a possibly biased expert. The expert receives a set of arguments, and each argument favors one of two alternatives. He can disclose each argument credibly, but cannot prove whether he has disclosed everything. In all equilibria, the biased expert sends messages containing arguments both for and against his preferred alternative. However, the decision maker is not influenced by the unfavorable arguments revealed by the biased expert. The latter is able to convince the decision maker to choose the biased expert's preferred alternative only if he reveals sufficiently many favorable arguments.

JEL Classification Numbers: D80, D82, D83

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1 Introduction

Consider a first-time camera buyer who is uninformed about its complexity; she does not know which and how many technical specifications are important for taking quality pictures. The salesperson knows all its relevant features and can credibly disclose each of them to the buyer. Disclosure is credible for many reasons; for example, the salesperson may be able to prove the zoom size by demonstrating it, the consumer may be able to test the features by using the camera, or liability laws may make it unprofitable for the salesperson to lie about any feature. Drawing upon everyday experience, one might expect that the salesperson will not fully inform the buyer. This observation is not in line with the unravelling result, which states that when the expert cannot lie, he or she reveals all information even if the parties have conflicting interests.¹ On the other hand, we often see, perhaps somewhat surprisingly, that the salesperson reveals features that seemingly go against his or her interest. In addition to revealing the favorable characteristics of the camera, the salesperson may also mention some unfavorable ones—for example, its short battery life.

There are many situations in which the expert does not disclose all information, but presents arguments which, if interpreted at face value, would go against what the expert is arguing for. For example, a financial adviser might receive a higher commission if investors choose a particular investment option following his advice, but he may nevertheless mention positive aspects of other investments as well. A doctor may receive a higher payment for providing a particular treatment, but may still disclose some drawbacks of this treatment. While trying to influence the reader, an author of an article may mention some arguments in favor of a competing view. The goal of this paper is to study whether these observations can be consistent with a game-theoretic model.

The model has the following structure. A decision maker (she) consults an expert (he) to help her choose between two alternatives. The expert observes a sequence of arguments, each

¹Since Grossman [12], Milgrom [22], Milgrom and Roberts [23], and Matthews and Postlewaite [21] first laid out the unravelling argument, subsequent research has focused on identifying situations in which this argument may fail. Viscusi [28] and Jovanovic [15] show that the expert reveals only favorable states when disclosure is costly. In Fishman and Hagerty [9] information transmission is hindered by the presence of decision makers who can verify the event of disclosure, but do not understand the disclosed information. Dye [8] and Shin [24, 25, 26] show that unravelling may fail if there is uncertainty about how well informed the expert is. In a dynamic setup, Grubb [13] shows that unravelling may be hindered further if senders want to build a reputation for being uninformed.

of which favors one alternative, and he can credibly disclose any argument. The expert may be an honest type who reveals all of the arguments, or he may try to convince the decision maker to choose a particular alternative. His type is his private information. The decision maker prefers the alternative which is favored by a sufficient fraction of arguments. She is uninformed, however, about the number of arguments and learns only about the arguments that the expert discloses.

First, the paper shows that, consistent with the camera example, full disclosure of information is not an equilibrium. Unravelling fails because the expert is unable to prove how many arguments exist. A biased expert has an incentive to conceal unfavorable information, and since the decision maker does not know how many arguments the expert has, she cannot force unravelling by forming a posterior unfavorable to the expert every time she receives few arguments.

Second, the paper sheds light on the type of information the expert reveals. The biased expert may reveal all arguments that are favorable to him, but in any equilibrium he may also provide some unfavorable ones. That is, in equilibrium a biased expert uses two-sided messages – messages containing arguments both for and against a given alternative – which is consistent with the motivating examples. The reason for this is as follows. If in equilibrium a biased expert always concealed all unfavorable arguments, the decision maker would discount such messages accordingly, but take all other messages at face value. In such a case, revealing at least one unfavorable argument would convince the decision maker that the expert is honest, and that the alternative he prefers is of high quality. Hence, the biased expert would have an incentive to deviate.

Third, the paper shows that, perhaps somewhat surprisingly, the decision maker is not influenced by the unfavorable arguments revealed by the biased expert. Given that a biased expert can conceal unfavorable arguments when his preferred alternative is not very attractive, if he reveals only a few unfavorable arguments, the decision maker must make the same choice independent of how many of them the expert reveals. If she did not, the biased expert would mislead her by revealing exactly the number of unfavorable arguments that would make her most likely to choose the expert's preferred alternative.

It is worth emphasizing that the use of two-sided messages in this model does not arise from dynamic considerations, such as reputation or career concerns. The rationale for their

use can be better understood if we think of an economy with infinitely many experts and decision makers who meet for one-shot interactions. Clearly, to avoid being manipulated, the decision makers must be skeptical whenever the experts provide arguments almost exclusively in favor of one alternative. Their beliefs, however, must be sustained by the behavior of the experts. If the experts stopped using two-sided messages, the decision makers would update their beliefs accordingly, and the experts would have a strict incentive to use them.

The paper also provides insight on when the expert is able to persuade the decision maker. If the biased expert has sufficiently many favorable arguments, he is able to convince the decision maker to choose the expert's preferred alternative even if it is inferior. However, if he has few favorable arguments, he may not be able to convince the decision maker to choose his preferred alternative even if that would be better for the decision maker.

This paper complements an extensive literature on communication and disclosure games.² The papers closest to mine, Shin [24] and Wolinsky [29], study games with a similar relationship between the payoff-relevant state and the set of available messages, and show that unravelling fails. Shin [24] analyzes a game in which the expert may have imprecise information about the state of nature and can reveal what he knows credibly, but he cannot prove how imprecise his information is. In Wolinsky [29], the states are ordered and the expert can prove only the lower bound on the state. In my model, the expert knows the state perfectly and can prove the type of each argument, but the rationale for the failure of unravelling is somewhat similar: the expert is unable to prove whether he has disclosed every piece of information. One important difference between the current paper and Shin [24] is that the latter does not allow for the uncertainty about the expert's preference. Without uncertainty, there exist many equilibria, and Shin [24] focuses only on the class of equilibria in which the expert reveals solely favorable information. In contrast to Shin [24], the current paper shows that allowing for uncertainty about the preferences of the expert makes the complete suppression of unfavorable information not an equilibrium.

A number of papers analyze communication games in which the preferences of the sender

²Among many others, Crawford and Sobel [5], Krishna and Morgan [19], Battaglini [1], and Chen [4] analyze communication situations in which talk is cheap. Kartik [16] and Kartik, Ottaviani, and Squintani [18] analyze communication games in which misrepresenting the state is costly. Verrecchia [27] is an extensive survey of the literature on disclosure games. From the modeling perspective, Glazer and Rubinstein [10, 11] also analyze a model in which information is a collection of arguments, but they focus on communication in cases in which there is a limit on the number of arguments that can be revealed.

are uncertain. In Wolinsky [29] the expert may prefer the lowest or the highest action. Callander and Wilkie [3] analyze an election game in which there is uncertainty about candidates' cost of lying. Kartik and McAfee [17] also analyze an election game but assume that some politicians are either honest or strategic in choosing their platform, and the voters care about the platform and the type of the politician. Like in the current work, in these papers the uncertainty about the expert's preferences results in the use of messages which, if taken at face value, would not be most favorable to the expert. Although the direct reasons for this result vary across these papers, the underlying principle is the same. If messages favorable to one type of the expert were taken at face value, this type would utilize only the most favorable messages, leaving the slightly less favorable ones to the other type. But if the other type is more truthful—either because of his preferences or his cost of lying—the decision maker would discount more heavily the most favorable messages, and as a result, the first type of the expert would prefer to tone down his message. In contrast to the current paper, however, the strategy of the expert in these papers is hard to map into the two-sided messages that are used by experts in the motivating examples of this paper.

The motivating examples have a discrete message space, and this is what Shin [24, 25], and Wolinsky [29] assume. One technical contribution of this paper is to model the number of arguments as a continuous variable, which makes the model more tractable.

The paper is organized as follows. Section 2 presents the model. Section 3 shows that there are no equilibria with full or no information disclosure. Section 4 analyzes a version of the model in which the expert can be biased only in favor of one alternative. Section 5 extends the analysis to the case in which the expert can be biased in favor of any of the alternatives. Section 6 discusses the robustness of the model. Section 7 provides a summary and conclusions.

2 The Model

There are two players, the expert and the decision maker, who move sequentially.

The environment

There are two alternatives: *Right* and *Left*. A state is a tuple $(L, R) \in R_+^2 \equiv S$, where L represents the number of arguments in favor of *Left*, and R represents the number of arguments

in favor of *Right*.³ Let $f(L, R)$ be the prior probability density function over S , and $F(L, R)$ be the corresponding distribution function, which is common knowledge.

Note that the number of arguments is a continuous variable in this model. In my motivating example the number of arguments is discrete, but modeling L and R as continuous makes the model more tractable and leads to general results without putting much structure on $f(L, R)$.⁴ However, there are also situations in which the number of arguments could be reasonably viewed as a continuous variable. For example, consider an expert advising on the introduction of a new program. If the cost of the program is L , and the benefit is R , one can think of L and R as continuous variables. By providing examples of investments that need to be undertaken or groups that will benefit from the project, the expert can prove a lower bound on costs and benefits, but the decision maker can never be sure that there are no other costs or benefits that the expert does not reveal.⁵

The players

The expert observes the state and sends a *report* to the decision maker. A report is a tuple $(\lambda, \rho) \in R_+^2$, where λ is the number of arguments in favor of *Left* and ρ is the number of arguments in favor of *Right* that the expert reveals. A report (λ, ρ) is *feasible* in state (L, R) if $0 \leq \lambda \leq L$ and $0 \leq \rho \leq R$. In each state the expert can send any report from the feasible set at no cost, and cannot send any report from outside of the feasible set; that is, the expert can truthfully disclose any subset of the existing arguments, but cannot credibly convey that he has disclosed all of them. Let $V(L, R)$ denote the set of all reports that are feasible in state (L, R) . The expert is said to report *fully* if he reveals all of the arguments. Reports with either $\lambda = 0$ or $\rho = 0$ are called *one-sided messages*, and all other reports are called *two-sided messages*.

There are three types of experts: an honest expert, H ; a persuader toward *Right*, P_r ; and a persuader toward *Left*, P_l . The probability that the expert is of type $i \in \{P_l, P_r, H\} \equiv T$ is π_i . An honest expert is *non strategic* and reveals all of the arguments. The other types are strategic and differ only in their payoff function.

The decision maker chooses one of two alternatives, *Right* or *Left*, after observing the

³Since the set of arguments is uncountable, technically L and R represent a measure of arguments, but the word "number" reflects better the phenomena that the paper attempts to model. I will use this nomenclature throughout the paper.

⁴See Section 6.2 for some limitations of this assumption.

⁵I thank an anonymous referee for this example.

report of the expert. There is a continuum of types of the decision maker. A type is denoted by θ , is her private information, and is drawn from some commonly known distribution with full support on $[0, 1]$.

The game and the payoffs

The game proceeds as follows. First, nature determines the type of expert $i \in T$, the type of decision maker θ , and the state $(L, R) \in S$. The expert observes his type and the state (L, R) , and sends a report $(\lambda, \rho) \in V(L, R)$ to the decision maker. The decision maker observes her type and the report, and chooses one of the alternatives.

A persuader toward a given alternative receives a positive payoff if this alternative is chosen and 0 otherwise. If the underlying state is (L, R) , then the payoff of the decision maker of type θ from choosing *Right* is $U(\text{Right}|L, R) = \frac{R}{L+R} - \theta$, and her payoff from choosing *Left* is $U(\text{Left}|L, R) = \theta - \frac{R}{L+R}$.

The type θ describes an ex ante preference of the decision maker. For example, a consumer may have some intrinsic preference for Nikon cameras over Canon cameras, a shareholder may prefer stocks of environmentally friendly companies, or a voter may prefer a Republican candidate because of family tradition, all other things being equal. Let $q_R \equiv \frac{R}{R+L}$ denote the quality of *Right*, and assume that when $(L, R) = (0, 0)$, then $q_R = \frac{1}{2}$.⁶

Equilibrium concept

A strategy of a type i persuader, $M_i(\lambda, \rho|L, R)$, specifies for each (L, R) the probability distribution over the set of feasible reports; hence, $\int_{[0,L] \times [0,R]} dM_i(\lambda, \rho|L, R) = 1$ for all $(L, R) \in S$. Let $\Sigma_i(L, R)$ be the support of persuader i 's strategy. Let $b(i, L, R|\lambda, \rho)$ be the probabilistic belief of the decision maker over $T \times S$ given the report. In what follows, $E[\cdot]$ and $\Pr(\cdot)$ denote the expectation and the probabilities derived with respect to the prior, and $E_b[\cdot]$ and $\Pr_b(\cdot)$ denote the expectation and the probabilities derived with respect to $b(i, L, R|\lambda, \rho)$. Let $\eta(\lambda, \rho)$ be the equilibrium belief of the decision maker about q_R if she observes a report (λ, ρ) ; that is, $\eta(\lambda, \rho) \equiv E_b[q_R|\lambda, \rho]$. For short, I will call it a *belief* or *expected quality*.

The solution concept is perfect Bayesian equilibrium. Formally, a perfect Bayesian equilibrium (later referred to as simply an *equilibrium*) is characterized by measurable M_i for $i \in \{P_l, P_r\}$, $b(i, L, R|\lambda, \rho)$, $\eta(\lambda, \rho)$, and decision maker's mapping from $\eta(\lambda, \rho)$ to a choice of an alternative, such that:

⁶Since the state $(0, 0)$ will occur with probability 0, the quality assigned to it does not affect the analysis.

1. If (λ, ρ) is in the support of $M_{P_r}(\cdot|L, R)$, then

$$(\lambda, \rho) \in \arg \max_{(\lambda', \rho') \in V(L, R)} \eta(\lambda', \rho');$$

if (λ, ρ) is in the support of $M_{P_l}(\cdot|L, R)$, then

$$(\lambda, \rho) \in \arg \min_{(\lambda', \rho') \in V(L, R)} \eta(\lambda', \rho');$$

2. For any (λ, ρ) , the decision maker chooses *Right* if and only if $\eta(\lambda, \rho) \geq \theta$;⁷
3. $b(i, L, R|\lambda, \rho)$ and $\eta(\lambda, \rho)$ are derived using Bayes' rule from the strategy of each expert's type and the prior distribution over T and S , whenever possible.

Part 1 is equivalent to each persuader maximizing his expected utility given the behavior and the beliefs of the decision maker. Since each persuader wants to convince the largest set of types of the decision maker to choose his preferred alternative, maximizing his payoff is equivalent to either maximizing (in case of P_r) or minimizing (in case of P_l) the belief of the decision maker. Part 2 says that the decision maker maximizes her expected utility given her beliefs.

One technical comment is in place. The honest expert uses all possible reports; therefore, there are no off-equilibrium beliefs. However, given that there is a continuum of possible reports, in equilibrium some of them might be sent with probability 0, in which case $b(i, L, R|\lambda, \rho)$ and $\eta(\lambda, \rho)$ are not pinned down by the Bayes' rule. However, $b(i, L, R|\lambda, \rho)$ and $\eta(\lambda, \rho)$ must satisfy Bayes' rule on any set of a strictly positive measure. Hence, some statements in the propositions do not necessarily apply to some sets of reports of measure zero, but when confusion is unlikely to occur, I omit this qualification from the main text in the interest of simplicity and space.

The parameter θ affects the choice of the decision maker, but does not affect how she forms the beliefs. Since part 1 of the definition of the equilibrium says that the expert sends a report that maximizes the belief of the decision maker, the behavior of the persuader does not depend on the distribution of θ as long as it has full support.

⁷For convenience, I assume that when indifferent, the decision maker chooses *Right*.

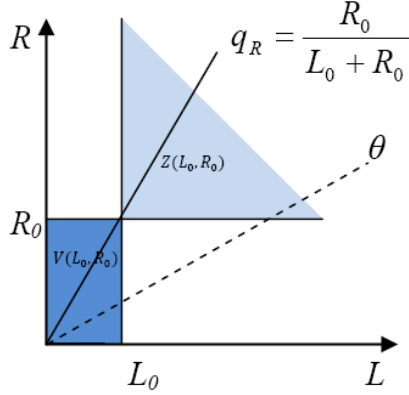


Figure 1: This figure represents the state space and the report space with a representative element (L_0, R_0) . $V(L_0, R_0)$ is the set of feasible reports in state (L_0, R_0) , and $Z(L_0, R_0)$ is the set of states of nature in which report (L_0, R_0) is feasible.

Graphic representation

Figure 1 represents the state space and the report space of this model. Define $Z(\lambda, \rho) = \{(L, R) : L \geq \lambda \text{ and } R \geq \rho\}$. If the state of nature is (L_0, R_0) , then the shaded region $V(L_0, R_0)$ is the set of feasible reports. The shaded region $Z(L_0, R_0)$ is the set of states of nature that allow the expert to send a report $(\lambda, \rho) = (L_0, R_0)$. All points situated on the same ray going through $(0, 0)$ —except $(0, 0)$ —represent the same quality of *Right*. For example, the dashed ray represents the states with $q_R = \theta$, and the solid ray represents the states with $q_R = \frac{R_0}{L_0 + R_0}$. Since $\frac{R_0}{L_0 + R_0} > \theta$, in state (L_0, R_0) the decision maker of type θ prefers *Right*.

Assumptions on $f(L, R)$

Assume that $f(L, R)$ is continuous with full support on S . Let $f^L(L|R)$ denote the conditional density of L given R , and $f^R(R|L)$ the conditional density of R given L . Let $F^L(L|R)$ and $F^R(R|L)$ be the corresponding distribution functions. I assume that for any $R_1 < R_2$, $F^L(L|R_1)$ first-order stochastically dominates $F^L(L|R_2)$. Similarly, for any $L_1 < L_2$, $F^R(R|L_1)$ first-order stochastically dominates $F^R(R|L_2)$. That is,

$$\frac{d(1 - F^L(L|R))}{dR} \leq 0, \quad \frac{d(1 - F^R(R|L))}{dL} \leq 0. \quad (1)$$

In words, condition (1) says that the probability that the number of arguments against a given alternative is above a certain threshold is not increasing in the number of arguments in favor.

This rules out situations in which "good news is bad news," that is, in which a higher number of arguments favoring a given alternative makes this alternative less likely to be attractive.

3 Preliminaries

It is easy to see that there is no equilibrium with full information disclosure. By definition, in such an equilibrium the experts' reports would be taken at *face value*; i.e., $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$, but then each persuader would prefer to conceal all unfavorable arguments, thereby convincing the decision maker to choose the persuader's preferred alternative.⁸ The failure of unraveling in this model contrasts with Milgrom [22]. Milgrom [22] analyzes a persuasion game in which a biased expert can prove which state has occurred. In both models, a biased expert wants to conceal information if this results in a higher belief about the quality of his preferred alternative. Rationality of the decision maker, however, implies that the expert cannot manipulate the decision maker in all states. For example, if in some states he convinces the decision maker that *Right* is better than it really is, then in other states he must induce the decision maker to believe that *Right* is worse than it really is. But this cannot be an equilibrium in Milgrom [22]: in the latter case the persuader toward *Right*, for example, would simply prove the true state. Hence, in Milgrom [22] information must be fully revealed. In the current model, the expert cannot prove the true state, and the same logic therefore fails.

In Lemma 2 in the appendix, I prove that there is also no equilibrium in which the decision maker's belief is independent of the expert's report. The basic intuition for this is as follows. Condition (1) implies that as the number of arguments in favor of one alternative increases, the expected quality of this alternative increases as well. This means that independent of the persuaders' strategy, when the expert reveals many arguments with the overwhelming majority favoring one alternative, the probability that the expert is honest and its quality is high outweighs the probability that the expert is a persuader and its quality is low. Hence, the decision maker must expect the quality of this alternative to be high. This means that in states with a sufficiently high number of arguments in favor of one alternative the expert must induce a belief about its quality that is higher than the prior. Hence, the decision maker cannot hold the same belief independent of the report.

⁸Strictly speaking, this argument applies to the truthful equilibrium, but if there exists any fully revealing equilibrium, there must exist also a truthful one.

4 One-sided Bias

In this section, I consider a simpler case in which the expert is either honest or biased toward *Right* ($\pi_{P_l} = 0$), and both types appear with strictly positive probability. This case conveys the main intuitions in a simpler setting. On the application front, it also better describes situations in which the decision maker knows which alternative the persuader favors.

First, I show that all equilibria of this model share two features: the persuader uses two-sided messages with positive probability; and the decision maker ignores the unfavorable arguments that the persuader reveals. The equilibria may differ, however, in whether the persuader reveals all favorable arguments. Next, I show that there exist equilibria in which the persuader reveals all favorable arguments, and all such equilibria are outcome equivalent. Moreover, the persuader reveals all favorable arguments in equilibrium if and only if the belief of the decision maker is a continuous function of the expert's report. However, there are also equilibria in which the persuader conceals some favorable arguments, and at the end of this section I discuss why such equilibria must be characterized by a discontinuous belief function. I provide an example of such an equilibrium in the appendix.

4.1 The properties of the equilibria

Proposition 1 describes the properties that are shared by all equilibria.

Proposition 1 *In each equilibrium, for any ρ there exists $\lambda_\rho > 0$ (possibly different across equilibria) such that for almost all (λ, ρ) with $\lambda \in [0, \lambda_\rho)$:*

- i. the decision maker's belief $\eta(\lambda, \rho)$ is constant in λ ;*
- ii. $\eta(\lambda, \rho)$ is weakly increasing in ρ ;*
- iii. there exists (L, R) such that $(\lambda, \rho) \in \Sigma_{P_r}(L, R)$.*

Proof. All proofs are in appendix B. ■

Proposition 1 contains two main findings of this paper. First, the belief of the decision maker is not strictly decreasing in the number of unfavorable arguments that the expert reveals (part *i*). For a given ρ , the decision maker chooses the same alternative when the expert reveals no unfavorable arguments as she does when he reveals any number smaller

than λ_ρ . Put informally, if the number of the revealed unfavorable arguments is small, the decision maker ignores them and bases her decision on the favorable arguments only. Second, in all equilibria the persuader uses two-sided messages with positive probability (part *iii*).

The intuition for Proposition 1 is as follows. First, if there are some reports that are not used in equilibrium by the persuader, they must be taken at face value; that is, $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$. Fix ρ and assume—contrary to what Proposition 1 says—that for any $\lambda_\rho > 0$ the belief is not constant in $\lambda < \lambda_\rho$. Since the persuader sends only reports that result in the highest belief, many reports with small λ are sent only by the honest expert.⁹ Hence, each (λ, ρ) with $\lambda < \lambda_\rho$, is either taken at face value, or generates the highest belief, which by definition is higher than the beliefs attached to nearby reports that are taken at face value. This means that for all small λ , $\eta(\lambda, \rho) \geq \frac{\rho}{\lambda + \rho}$. But then there exist λ for which the belief of the decision maker is arbitrarily close to 1. This implies that for all states with at least ρ arguments in favor of *Right*, the persuader can convince almost any type of the decision maker to choose *Right*. But this cannot happen because in equilibrium the persuader cannot consistently fool a rational decision maker.

The belief of the decision maker cannot be decreasing in ρ (part *ii*). If sending more favorable arguments generates a lower belief than sending fewer of them, then only the honest expert reveals more favorable arguments. But if at the same time the honest expert reveals no unfavorable arguments, the decision maker must believe that $q_R = 1$. This is, however, a contradiction.

The behavior of the persuader described in part *iii* is a direct consequence of the decision maker's belief. Since it does not hurt the persuader to reveal some of the unfavorable arguments, he uses two-sided messages, justifying in this way the decision maker's beliefs.

While all equilibria share the properties described in Proposition 1, they may differ in how many favorable arguments the persuader reveals. Proposition 2, however, states that there exist equilibria in which the persuader reveals all favorable arguments.

Proposition 2 *There exist equilibria in which the persuader reveals all favorable arguments.*

All equilibria in which the persuader reveals all favorable arguments are characterized by the

⁹To be precise, the report that generates the highest belief for a given ρ may not be feasible in some states, but this can only happen in states with small L . Therefore the belief of the decision maker after those reports must be close to 1 anyway, and the same argument holds.

same (up to zero measure perturbations) belief function. In all these equilibria,

i. for each ρ , λ_ρ is defined by

$$\frac{\rho}{\lambda_\rho + \rho} = \frac{\pi_H \Pr(L \leq \lambda_\rho | R = \rho) E[q_R | R = \rho, L \leq \lambda_\rho]}{\pi_H \Pr(L \leq \lambda_\rho | R = \rho) + \pi_{P_r}} + \frac{\pi_{P_r} E[q_R | R = \rho]}{\pi_H \Pr(L \leq \lambda_\rho | R = \rho) + \pi_{P_r}} ; \quad (2)$$

ii. $\eta(\lambda, \rho) = \frac{\rho}{\lambda_\rho + \rho}$ if $\lambda \leq \lambda_\rho$, and $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$ if $\lambda > \lambda_\rho$;

iii. the persuader sends only reports (λ, ρ) such that $\lambda \leq \min\{L, \lambda_\rho\}$.

Associated with each strategy profile is a distribution function that assigns to each quadruplet $(i, \theta; L, R,)$ the probability that *Right* is chosen if the type of the expert is i , the type of the decision maker is θ , and the state is (L, R) . Two strategy profiles are said to be outcome equivalent if their associated distribution functions are equal almost everywhere. Since the belief function is the same (almost everywhere) across all equilibria, and in each state the persuader induces the highest feasible belief, we can state the following corollary of Proposition 2.

Corollary 1 *All equilibria in which the persuader reveals all favorable arguments are outcome equivalent.*

Figure 2 represents all of these equilibria for a fixed π_{P_r}, π_H and $f(L, R)$. For each R , the boundary of the shaded area is determined by λ_ρ as defined by Eq. (2) with $\rho = R$. The unshaded region is the set of reports that are used in equilibrium only by the honest expert. The shaded region is the set of reports used in equilibrium also by the persuader; that is, it includes all reports that do not allow the decision maker to identify the type of expert. In some state (L, R_0) , for example, the persuader reveals R_0 and sends some $\lambda \leq \min\{L, \lambda_{R_0}\}$, thereby generating a belief equal to $\frac{R_0}{\lambda_{R_0} + R_0}$.

The intuition behind Proposition 2 is as follows. Fix $R = \rho$, and let $\eta_\rho = \max_\lambda \eta(\lambda, \rho)$.¹⁰ That is, η_ρ is the highest belief that the persuader can induce if he reveals all favorable arguments. The persuader will never reveal $\lambda > \frac{1 - \eta_\rho}{\eta_\rho} \rho$ because this would prove that the

¹⁰The proof of Proposition 2 shows the existence of the equilibrium formally. This implies that $\max_\lambda \eta(\lambda, \rho)$ exists; otherwise the solution to the problem of the persuader would not exist. The intuition for the existence of the equilibrium is not very insightful and is omitted.

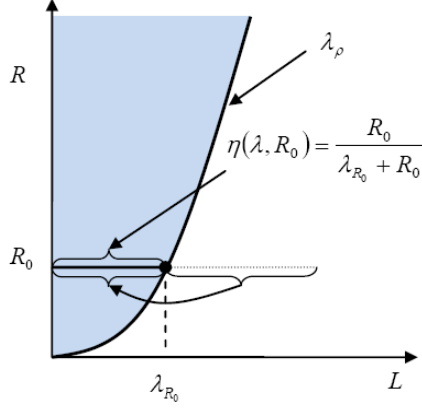


Figure 2: The details of the equilibria in which the persuader reveals all favorable arguments. The shaded region represents all reports that the persuader uses in these equilibria.

quality of *Right* is lower than η_ρ , and clearly, this is not in the persuader's interest. This means that reports with $\lambda > \frac{1-\eta_\rho}{\eta_\rho} \rho$ are sent only by the honest expert, and as such, they must be taken at face value. On the other hand, for all reports with $\lambda < \frac{1-\eta_\rho}{\eta_\rho} \rho$, the decision maker must hold $\eta(\lambda, \rho) = \eta_\rho$. If she did not, then by the definition of η_ρ those reports would generate a belief lower than η_ρ . As such, they would be sent only by the honest expert, and hence generate $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho} > \frac{1-\eta_\rho}{\eta_\rho} \frac{\rho}{\rho + \rho} = \eta_\rho$, which is a contradiction. By denoting $\lambda_\rho \equiv \frac{1-\eta_\rho}{\eta_\rho} \rho$, the argument above implies that the persuader sends only reports with $\lambda \leq \lambda_\rho$, $\eta(\lambda, \rho) = \eta_\rho = \frac{\rho}{\rho + \lambda_\rho}$ for all $\lambda \leq \lambda_\rho$, and $\eta(\lambda, \rho) = \frac{\rho}{\rho + \lambda}$ for all $\lambda > \lambda_\rho$, as stated in parts *ii* and *iii* of Proposition 2.

Now, it remains to show how η_ρ , and therefore λ_ρ , are determined. Since the belief of the decision maker is constant for all $\lambda \leq \lambda_\rho$, it is equal to the belief she would form if she knew that the expert sent fewer than λ_ρ unfavorable arguments, without knowing their exact number. In that case she would know that either she is facing a persuader, in which case L can be arbitrarily large and the expected quality of *Right* is $E[q_R | R = \rho]$, or she is facing the honest expert, in which case $L \leq \lambda_\rho$ and the expected quality of *Right* is $E[q_R | R = \rho, L \leq \lambda_\rho]$. Therefore, the belief she holds is the weighted average of those two, as expressed by the right-hand side of Eq. (2). More precisely, this belief is equal to the probability that the expert is a persuader conditional on $R = \rho$ and $\lambda \leq \lambda_\rho$, times $E[q_R | R = \rho]$, plus the probability that the expert is honest conditional on $R = \rho$ and $\lambda \leq \lambda_\rho$, times $E[q_R | R = \rho, L \leq \lambda_\rho]$. Eq. (2)

equates this belief to $\eta_\rho = \frac{\rho}{\lambda_\rho + \rho}$.

The uniqueness of the belief function follows directly from the uniqueness of λ_ρ , and the latter can be understood as follows. Consider λ_ρ satisfying Eq. (2), and recall that in this equilibrium $\eta(\lambda, \rho) = \frac{\rho}{\lambda_\rho + \rho}$ for all $\lambda \leq \lambda_\rho$. Assume that there exists another $\lambda'_\rho > \lambda_\rho$ that solves Eq. (2). In all states in which λ s smaller than λ_ρ are sent in the equilibrium with λ_ρ , λ s smaller than λ_ρ are sent in the equilibrium with λ'_ρ . However, λ s smaller than λ_ρ are additionally sent when the expert is honest and $L \in (\lambda_\rho, \lambda'_\rho)$. But when $L \in (\lambda_\rho, \lambda'_\rho)$, $\frac{R}{L+R} \in \left(\frac{\rho}{\lambda'_\rho + \rho}, \frac{\rho}{\lambda_\rho + \rho}\right)$. Since in the equilibrium with λ_ρ the reports with $\lambda \leq \lambda_\rho$ generate belief equal to $\frac{\rho}{\lambda_\rho + \rho}$, the last observation implies that in the equilibrium with λ'_ρ the decision maker must hold a belief which is a convex combination of $\frac{\rho}{\lambda_\rho + \rho}$ and $\frac{\rho}{\lambda'_\rho + \rho}$. This in turn implies that this belief is strictly higher than $\frac{\rho}{\lambda'_\rho + \rho}$, which contradicts that λ'_ρ solves Eq. (2). In the appendix, I show formally that condition (1) guarantees that $\frac{\rho}{\rho + \lambda_\rho}$ is strictly increasing in ρ . The latter assures that the persuader indeed has an incentive to reveal all favorable arguments.

Proposition 2 is silent on exactly which $\lambda \leq \lambda_\rho$ the persuader sends in each state because there are many strategies that can support the equilibria outlined in this proposition. The equilibrium strategy must, however, justify a belief that is constant in λ for each ρ and $\lambda \leq \lambda_\rho$. In other words, the strategy must justify a belief that satisfies

$$\frac{\rho}{\lambda_\rho + \rho} = \Pr_b(H|\lambda, \rho) \frac{\rho}{\lambda + \rho} + (1 - \Pr_b(H|\lambda, \rho)) E_b \left[\frac{\rho}{L + \rho} | \lambda, \rho, P_r \right], \quad (3)$$

where $\Pr_b(H|\lambda, \rho)$ is the equilibrium belief of the decision maker that the expert is H conditional on observing (λ, ρ) . If the expert is honest, then a smaller number of arguments in favor of *Left* implies a higher quality of *Right*. The strategy of the persuader must offset this effect. Assuming that Eq. (3) is differentiable with respect to λ , we obtain

$$\frac{d\Pr_b(H|\lambda, \rho)}{d\lambda} \left(\frac{\rho}{\lambda + \rho} - E_b \left[\frac{\rho}{L + \rho} | \lambda, \rho, P_r \right] \right) = \Pr_b(H|\lambda, \rho) \frac{\rho}{(\lambda + \rho)^2} - (1 - \Pr_b(H|\lambda, \rho)) \frac{dE_b \left[\frac{\rho}{L + \rho} | \lambda, \rho, P_r \right]}{d\lambda}$$

The expression in brackets on the left-hand side is positive because the persuader can send λ only if $L \geq \lambda$. This implies that in equilibrium either $\frac{d\Pr_b(H|\lambda, \rho)}{d\lambda} > 0$ or $\frac{dE_b \left[\frac{\rho}{L + \rho} | \lambda, \rho, P_r \right]}{d\lambda} > 0$, or both. That is, as the number of the unfavorable arguments that the expert reveals increases, either the decision maker's posterior belief that the expert is honest increases, or the expected

quality of *Right* conditional on the expert being a persuader increases, or both. The first effect implies that two-sided messages might be more credible. The second effect implies that the less favorable the state is, the more likely the persuader is to disclose few unfavorable arguments. Although there is no guarantee that revealing unfavorable information increases the credibility of the expert in any particular equilibrium, there always exist equilibria in which this is true. In one such equilibrium, for each $\rho = R$ the persuader reports fully if $L \leq \lambda_\rho$; otherwise, he randomizes over $\lambda \in [0, \lambda_\rho]$ using some probability density function $m_{P_r}^\rho(\lambda)$. The details of this function are outlined in the proof of Proposition 2, but the crucial observation is that $m_{P_r}^\rho(\lambda)$ must be decreasing in λ , which means that the persuader is more likely to reveal fewer unfavorable arguments.

It turns out that the equilibria from Proposition 2 share another property.

Lemma 1 *In equilibrium, the persuader always reveals all favorable arguments if and only if the belief function $\eta(\lambda, \rho)$ is continuous (up to zero measure perturbations) in reports. In those equilibria the belief function is strictly increasing in ρ .*

The continuity of $\eta(\lambda, \rho)$ in λ is immediate from part *ii* of Proposition 2. Since $f(L, R)$ is continuous, the unique solution to Eq. (2) is continuous in ρ ; therefore, $\eta(\lambda, \rho)$ is also continuous in ρ .

Intuitively, when the persuader reveals all favorable arguments, two reports that differ infinitesimally in ρ carry similar information: the number of favorable arguments is similar, and since $f(L, R)$ is continuous, the expected number of unfavorable arguments is similar as well. Hence, the decision maker must form a similar belief, and the belief function must be continuous in ρ . In equilibria in which the persuader conceals some favorable arguments, ρ and ρ' that are close to each other may carry very different information about the number of arguments. More precisely, assume that ρ is infinitesimally higher than ρ' , and the persuader always reveals all favorable arguments if $R = \rho$, but sometimes conceals some of them if $R = \rho'$.¹¹ The persuader is willing to conceal favorable arguments when $R = \rho'$ only if the belief generated by ρ' is the same as a belief generated by some $\rho'' < \rho'$.¹² But the belief can

¹¹That for any ρ' such ρ exists is guaranteed by condition (1). High enough R implies that the expected q_R is arbitrarily close to 1; hence, independent of the strategy of P_r , the presence of the honest expert guarantees that upon observing high enough ρ , the decision maker must hold a belief arbitrarily close to 1. This in turn gives P_r the incentive to reveal all favorable arguments.

¹²Recall that Proposition 1 has already established that $\eta(\lambda, \rho)$ is weakly increasing.

be the same for ρ' and ρ'' only if the persuader conceals favorable arguments in states in which q_R is relatively high, thereby decreasing the belief of the decision maker when she observes ρ' and increasing it when she observes ρ'' . This implies that upon receiving a report with ρ' , the decision maker knows that q_R is likely to be low—otherwise the persuader would have sent ρ'' —and as a result she must form a discretely lower belief than upon receiving a report with ρ . Hence, her belief is discontinuous. In appendix A, I illustrate this intuition by presenting an example of an equilibrium in which the persuader conceals some favorable information.

Note that the discontinuous beliefs cannot be excluded by standard refinement concepts, mainly because most of the refinements put restrictions on off-equilibria beliefs, while in this model all reports are on an equilibrium path. However, since most of the qualitative features are shared by all equilibria, for the remainder of this paper I focus solely on the continuous equilibria to simplify the analysis.¹³

I conclude this section by describing the choice of the decision maker in the continuous equilibria. Since experts of different types use different strategies, whether the decision maker chooses the correct alternative depends on the state and the type of the expert that she actually faces. Figure 3 depicts the choices of a decision maker with preference parameter θ as a function of the state when the expert is biased (the left panel) and when the expert is honest (the right panel). Note that R_θ in Figure 3 is such that $\eta(\lambda_{R_\theta}, R_\theta) = \frac{R_\theta}{\lambda_{R_\theta} + R_\theta} = \theta$; that is, R_θ is the number of arguments in favor of *Right* that the persuader must reveal to make the decision maker θ indifferent between the alternatives.

The left panel shows that when the expert happens to be biased, the decision maker is susceptible to two types of errors. First, if there are few arguments supporting *Right* (fewer than R_θ in Figure 3), a persuader will be unable to convince the decision maker to choose *Right* even if $q_R > \theta$. The states in which this happens are represented by the striped area. Second, if there are many arguments in favor of *Right*, the persuader will succeed in convincing the decision maker to choose *Right* even if $q_R < \theta$. In the states in which this happens—represented by the dotted area—persuasion is successful.

The right panel shows that if the expert is honest, the decision maker will make only one type of errors: choosing *Left* when the state is in the striped area.

¹³The previous version of this paper contained a proposition that if we perturb the game by adding a small fixed cost of concealing information, its unique equilibrium converges to a continuous equilibrium. Results are available upon request.

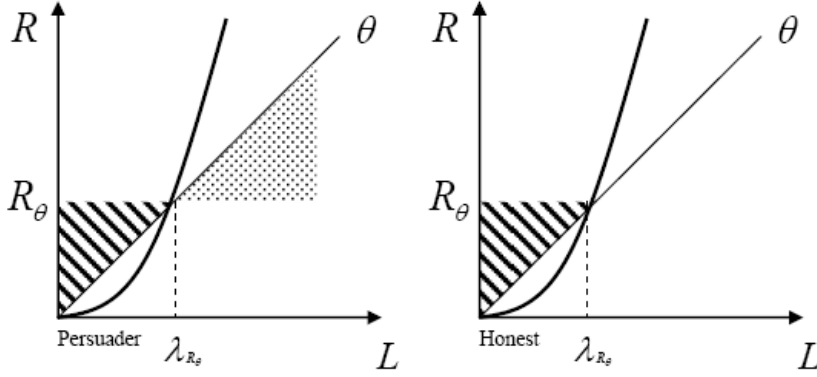


Figure 3: The behavior of the decision maker with the preference parameter θ . The left panel represents the choice of the decision maker as a function of the state when she faces the persuader, and the right panel represents the same when the decision maker faces the honest expert.

4.2 Comparative statics

Main assumptions of the model are that the decision maker is uncertain about the type of expert and about the number of arguments. In this section, I analyze how the equilibria change when we vary the probability of facing the honest expert and the prior distribution of arguments.

4.2.1 Varying the probability of facing the honest expert

Proposition 3 describes what happens with the continuous equilibria when one varies the probability of facing the honest expert.

Proposition 3 *In any continuous equilibrium, for any ρ ,*

- i.* $\frac{d\lambda_\rho}{d\pi_H} < 0$, and $\frac{d\eta(\lambda_\rho, \rho)}{d\pi_H} > 0$;
- ii.* $\lim_{\pi_H \rightarrow 1} \lambda_\rho = 0$, and $\lim_{\pi_H \rightarrow 1} \eta(\lambda_\rho, \rho) = 1$;
- iii.* $\lim_{\pi_H \rightarrow 0} \lambda_\rho = \frac{1 - E\left[\frac{\rho}{L+\rho} | R=\rho\right]}{E\left[\frac{\rho}{L+\rho} | R=\rho\right]} \rho$, and $\lim_{\pi_H \rightarrow 0} \eta(\lambda_\rho, \rho) = E\left[\frac{\rho}{L+\rho} | R = \rho\right]$.

Proposition 3 says that as the probability of facing the honest expert increases, the largest number of unfavorable arguments that the persuader reveals decreases, and the belief that he induces in any state increases (part *i*). When the expert is likely to be honest, one-sided reports

are likely to signal a high q_R , and this must be reflected in the decision maker's beliefs. The persuader will therefore not reveal too many unfavorable arguments, as this would necessarily make the decision maker revise her belief downward.

As $\pi_H \rightarrow 0$, the belief induced by P_r in any state (L, R) converges to the expected quality of *Right* conditional on R , and the highest number of unfavorable arguments that the persuader reveals converges to the highest number that does not prove this belief wrong (part *iii*). Since by Proposition 1 for any $\pi_H > 0$, all $\lambda \leq \lambda_\rho$ must lie in the strategy of P_r , Proposition 3 implies that as P_r becomes more likely, he reveals a higher number of unfavorable arguments. Proposition 3 does not say anything, however, about the probability with which P_r does so, because this varies across equilibria.¹⁴ At the minimum, the strategy of P_r has to offset the behavior of the honest expert, therefore the smallest probability with which P_r sends two-sided messages is π_H . However, there are equilibria in which P_r sends two-sided messages with probability 1.

To build the intuition for this, it is worth discussing the case in which $\pi_H = 0$. When $\pi_H = 0$, there exist equilibria—among others—in which the persuader reveals all favorable arguments and suppresses all unfavorable ones. In such equilibria, upon seeing only favorable arguments, the decision maker must believe that $\eta(0, \rho) = E\left[\frac{\rho}{L+\rho} | R = \rho\right]$. Since $\pi_H = 0$, all other reports are off the equilibrium path; hence, they only have to assure that the suppression of unfavorable arguments is optimal; i.e., $\eta(\lambda, \rho) \leq E\left[\frac{\rho}{L+\rho} | R = \rho\right]$. In particular, $\eta(\lambda, \rho) = E\left[\frac{\rho}{L+\rho} | R = \rho\right]$ for $\lambda \leq \lambda_\rho$ and $\eta(\lambda, \rho) = \frac{\rho}{\lambda+\rho}$ for $\lambda > \lambda_\rho$ is a part of some equilibrium in which the persuader suppresses the unfavorable arguments completely. Under this belief function, however, the persuader is indifferent between doing so and revealing some unfavorable arguments. Consider a strategy of P_r in which he reports fully when $L \leq \lambda_\rho$, and when $L > \lambda_\rho$, he randomizes over all $\lambda \leq \lambda_\rho$ with some density function that is decreasing in λ . By the same argument as provided in the discussion of Proposition 2, such a strategy would support the constant belief of the decision maker. Therefore, even when $\pi_H = 0$, there exist equilibria in which the persuader uses two-sided messages with probability 1.

¹⁴Recall from the discussion of Proposition 2 that there are many strategies of P_r that support the equilibrium belief function.

4.2.2 Varying the familiarity of the problem

In this section I look at what happens when the uncertainty about the number of arguments decreases. To this aim, it is more convenient to work directly with the distribution of q_R and the total number of arguments $N \equiv L + R$ instead of $f(L, R)$.

Changing the distribution of the arguments affects the utility of the decision maker for two reasons. First, it affects the expected quality of *Right*. Second, it affects the uncertainty faced by the decision maker, which in turn may determine how much information is revealed. Since in this section I am interested in the impact of the latter, I assume that q_R is independent of N . Let q_R be uniformly distributed on $[0, 1]$, and let $G(N; z)$ be the distribution of N parametrized by z , with $g(N; z)$ being the corresponding probability density function.¹⁵ Let $G(N; z)$ be the family of distributions with the following properties:

1. the expected value of N is independent of z , $\bar{N} \equiv \int_0^\infty N dG(N; z)$;
2. for all z and for all N , we have $G_z(N; z) < 0$ if $N < \bar{N}$, and $G_z(N; z) > 0$ if $N > \bar{N}$;
3. as $z \rightarrow \infty$, $G(\cdot)$ becomes degenerate at \bar{N} .

These conditions are equivalent to the single crossing property of Diamond and Stiglitz [7] with the additional requirement that the crossing of $G(N; z)$ happens at \bar{N} . Any two distribution functions from a family satisfying the above conditions differ by a mean preserving spread.¹⁶

Proposition 4 *For every preference type of the decision maker θ , if $z_1 > z_2$, then the decision maker's utility is higher for $G(N; z_1)$ than for $G(N; z_2)$. As $z \rightarrow \infty$, there is full revelation of information.*

¹⁵These assumptions mean that (q_R, N) are jointly distributed on $[0, 1] \times (0, \infty)$ with the probability density function $g(N; z)$. Consider the mapping $(q_R, N) \rightarrow \left(\frac{R}{L+R}, L+R\right)$. Using equation 20.20 from Billingsley [2], we obtain that the distribution of (L, R) must satisfy $f(L, R) = \frac{g(L+R; z)}{L+R}$. For a particular $z < \infty$, this does not restrict the correlation between L and R . For example, if $g(N; z) = e^{-N} N$ then $f(L, R) = e^{-(L+R)}$, which implies that L and R are independent. However, below I state the conditions on $G(N; z)$ that require that in the limit as $z \rightarrow \infty$, $R = \bar{N} - L$; hence, for z large enough L and R become negatively correlated.

¹⁶For $\bar{N} = 1$, an example of such family is $G(N; z) = \begin{cases} \frac{1}{2} \left(\frac{z}{(z-1)(1-N)+1} - 1 \right) \frac{1}{z-1} & \text{if } N \leq 1 \\ 1 - \frac{1}{2} e^{\frac{z}{\gamma}(1-N^\gamma)} & \text{if } N > 1 \end{cases}$, with $z \in (1, \infty)$ and γ defined implicitly by $\int_1^\infty N^\gamma e^{\frac{z}{\gamma}(1-N^\gamma)} dN = \frac{z-2}{(z-1)z} + \frac{1}{(z-1)^2} \ln(z)$. The proof of this is in appendix B.

Proposition 4 says that the lower the uncertainty about N is, the better off the decision maker is. When the decision maker knows more about how many arguments are available to the expert, she can more easily infer his information: when she receives a report, she can estimate rather precisely how many arguments have been concealed from her. When the uncertainty disappears, we are back to the world of Milgrom [22].

With the distribution of q_R held constant, the distribution of N reflects the decision maker's uncertainty about the choice problem. It describes how the total number of arguments varies from situation to situation for the same decision problem. For example, in each political campaign a different number of issues may be important, and this could be represented in the model by a relatively dispersed prior belief over N . Other choice problems are likely to be characterized by roughly the same number of arguments every time the decision maker faces them, such as choosing an investment option or buying a car; this would be captured by a distribution of N that is concentrated around the mean. Alternatively, the prior distribution of N may describe the decision maker's knowledge about the problem. An investor with a dispersed distribution of N knows little about the nature of the problem, while an experienced or educated investor is likely to have a relatively good idea about N . Proposition 4 implies that the decision maker is better off if she faces familiar problems for which she has a pretty good estimate of the number of relevant arguments.

5 Two-sided Bias

In this section, I consider the situation in which the expert can be biased toward either alternative. The expert can be P_r , P_l , or H with positive probabilities π_{P_r} , π_{P_l} , and π_H , respectively.

The previous section shows that when the expert is either honest or biased toward *Right*, the decision maker knows which arguments may be favorable to the expert and bases her posterior on how many of these arguments the expert reveals. When the expert can be biased in either direction, the decision maker cannot use the same logic; therefore, we can expect that much less information will be revealed. Section 5.1, shows that this is only partially true.

5.1 The properties of the equilibria

Define λ_ρ using Eq. (2) from section 4, and define ρ_λ analogously by

$$\frac{\rho_\lambda}{\lambda + \rho_\lambda} = \frac{\pi_H \Pr(R \leq \rho_\lambda | L = \lambda) E[q_R | L = \lambda, R \leq \rho_\lambda]}{\pi_H \Pr(R \leq \rho_\lambda | L = \lambda) + \pi_{P_l}} + \frac{\pi_{P_l} E[q_R | L = \lambda]}{\pi_H \Pr(R \leq \rho_\lambda | L = \lambda) + \pi_{P_l}}. \quad (4)$$

For any \bar{L} and \bar{R} , let $\lambda_{\bar{R}} \equiv \lambda_\rho$ for $\rho = \bar{R}$ and $\rho_{\bar{L}} \equiv \rho_\lambda$ for $\lambda = \bar{L}$. Define

$$\begin{aligned} \Psi(\bar{L}, \bar{R}) \equiv & \{(\lambda, \rho) : (\lambda, \rho) \in [0, \bar{L}] \times [0, \bar{R}] \\ & \cup [\min\{\bar{L}, \frac{\lambda_{\bar{R}}}{\bar{R}} \rho\}, \max\{\bar{L}, \frac{\lambda_{\bar{R}}}{\bar{R}} \rho\}] \times [\min\{\rho_{\bar{L}}, \bar{R}\}, \max\{\rho_{\bar{L}}, \bar{R}\}]\}. \end{aligned} \quad (5)$$

Finally, let \bar{L} and \bar{R} solve the following system of Eqs.:

$$\frac{\bar{R}}{\lambda_{\bar{R}} + \bar{R}} = \frac{\rho_{\bar{L}}}{L + \rho_L}, \quad (6)$$

$$\frac{\bar{R}}{\lambda_{\bar{R}} + \bar{R}} = \frac{\pi_{P_r} \Pr(R \leq \bar{R}) E[q_R | R \leq \bar{R}] + \pi_{P_l} \Pr(L \leq \bar{L}) E[q_R | L \leq \bar{L}] + \pi_H \Pr((L, R) \in \Psi(\bar{L}, \bar{R})) E[q_R | (L, R) \in \Psi(\bar{L}, \bar{R})]}{\pi_{P_r} \Pr(R \leq \bar{R}) + \pi_{P_l} \Pr(L \leq \bar{L}) + \pi_H \Pr((L, R) \in \Psi(\bar{L}, \bar{R}))} \quad (7)$$

Proposition 5 characterizes all equilibria in which the belief function is continuous.

Proposition 5 *Consider $\bar{L}, \bar{R}, \lambda_\rho, \rho_\lambda$, and $\Psi(\bar{L}, \bar{R})$ as defined above. There exists a unique belief function that is continuous and a part of equilibrium. This belief function satisfies*

- i. for all $\rho > \bar{R}$ and $\lambda \leq \lambda_\rho$, $\eta(\lambda, \rho) = \frac{\rho}{\lambda_\rho + \rho}$; for all $\lambda > \bar{L}$ and $\rho \leq \rho_\lambda$, $\eta(\lambda, \rho) = \frac{\rho_\lambda}{\lambda + \rho_\lambda}$;*
- ii. for all $(\lambda, \rho) \in \Psi(\bar{L}, \bar{R})$, $\eta(\lambda, \rho) = \frac{\bar{R}}{\lambda_{\bar{R}} + \bar{R}}$;*
- iii. for all other (λ, ρ) , $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$.*

In any equilibrium with a continuous belief function the strategies of the persuaders satisfy

- iv. P_r sends $\rho = R$ and $\lambda \leq \min\{\lambda_\rho, L\}$ for all (L, R) with $R > \bar{R}$; for almost all (λ, ρ) such that $\rho \geq \bar{R}$ and $\lambda \leq \lambda_\rho$ there exist (L, R) such that $(\lambda, \rho) \in \Sigma_{P_r}(L, R)$; P_r sends reports from $\Psi(\bar{L}, \bar{R})$ for all (L, R) with $R < \bar{R}$;*
- v. P_l sends $\lambda = L$ and $\rho \leq \min\{\rho_\lambda, R\}$ for all (L, R) with $L > \bar{L}$; for almost all (λ, ρ) such that $\lambda \geq \bar{L}$ and $\rho \leq \rho_\lambda$ there exist (L, R) such that $(\lambda, \rho) \in \Sigma_{P_l}(L, R)$; P_l sends reports from $\Psi(\bar{L}, \bar{R})$ for all (L, R) with $L < \bar{L}$.*

I will call all reports in $\Psi(\bar{L}, \bar{R})$ *short*. Proposition 5 says that, as in the one-sided case, persuaders do not suppress the unfavorable arguments completely (parts *iv* and *v*). Moreover, when a persuader has sufficiently many favorable arguments, he reveals all of them, separating himself from the persuader of the other type (parts *iv* and *v*). In such a case, the decision maker updates her belief using only arguments that favor that persuader (part *i*). The decision maker, however, treats all reports with few arguments of both types in the same way (part *ii*).

Let me provide the intuition for Proposition 5, using an example with a symmetric $f(L, R)$ and $\pi_{P_l} = \pi_{P_r}$. Note that the prior belief of the decision maker in this example is equal to $\frac{1}{2}$. Figure 4 represents an equilibrium for this example.¹⁷ A persuader who reveals many arguments in favor of *Right* proves that the quality of *Right* is likely to be high. Given this, P_l will never reveal many arguments in favor of *Right*. As a result, when observing high ρ , the decision maker knows that she faces either the honest expert or the persuader toward *Right*; hence, she faces the same problem as in the one-sided case. That is, in equilibrium for any ρ large enough, there must exist $\lambda_\rho > 0$ such that $\eta(\lambda, \rho)$ is constant for all $\lambda \leq \lambda_\rho$, and in an equilibrium with a continuous belief function, λ_ρ is derived in the same way as in the one-sided case. A similar logic applies to the reports with large λ , and the corresponding ρ_λ must solve Eq. (4). Figure 4 depicts such λ_ρ and ρ_λ functions. The shaded region represents reports used only by the honest expert and the persuader toward *Right*, and the striped area represents reports used only by the honest expert and the persuader toward *Left*. For example, as illustrated in Figure 4, in state (L_0, R_0) , P_r sends $\rho = R_0$ and some $\lambda \leq \lambda_{R_0}$, and P_l sends $\lambda = L_0$ and some $\rho \leq \rho_{L_0}$. Upon seeing (λ, ρ) such that $\rho = R_0$ and $\lambda \leq \lambda_{R_0}$, the decision maker believes $\eta(\lambda, \rho) = \frac{R_0}{\lambda_{R_0} + R_0}$; upon seeing (λ, ρ) such that $\lambda = L_0$ and $\rho \leq \rho_{L_0}$, the decision maker believes $\eta(\lambda, \rho) = \frac{\rho_{L_0}}{L_0 + \rho_{L_0}}$.

However, the same logic does not work in states with few arguments of one type. Figure 4 shows that λ_ρ and ρ_λ cross for $\rho = \bar{R}$ and $\lambda = \bar{L}$. This means that if the persuaders separated themselves in all states, then after observing any report with few arguments the decision maker would hold a lower posterior if she believed that the report was sent by P_r (in which case she would believe $\frac{\rho}{\lambda_\rho + \rho}$) than if she believed that the report was sent by P_l (in which case she would believe $\frac{\rho_\lambda}{\lambda + \rho_\lambda}$). In this case, each persuader would have an incentive to

¹⁷The figure is drawn for $f(L, R) = \frac{1}{(1+L)^2} \frac{1}{(1+R)^2}$ and $\frac{\pi_{P_r}}{1-P_l} = 0.9$. In this example $\bar{L} = \bar{R} = 0.92402$.

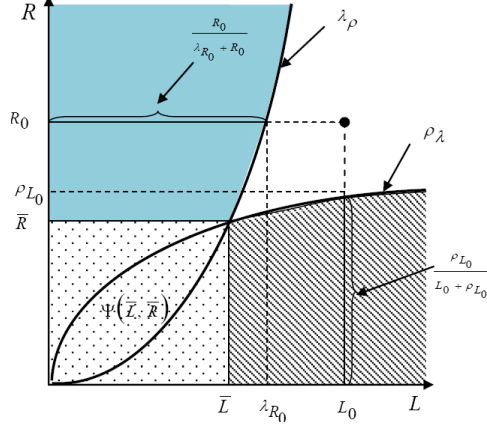


Figure 4: Two-sided bias.

mimic the persuader of the other type. To avoid this, in equilibrium the decision maker must hold a constant belief for all reports in $\Psi(\bar{L}, \bar{R})$. In Figure 4, $\Psi(\bar{L}, \bar{R})$ is represented by the dotted area. Due to the symmetry of the example, this area is a square; that is, $\bar{L} = \bar{R}$, and $\eta(\lambda, \rho) = \frac{1}{2}$ for all $(\lambda, \rho) \in \Psi(\bar{L}, \bar{R})$.

When the model is not symmetric, either because $f(L, R)$ is asymmetric or one type of persuaders is more likely, $\Psi(\bar{L}, \bar{R})$ is no longer a square as can be seen from expression (5). The continuity of the belief function requires that \bar{L} and \bar{R} are such that Eq. (6) holds: the belief induced by short reports must be equal to the lowest belief that P_r induces when he separates from P_l , and the highest belief that P_l induces when he separates from P_r . Eq. (7) says that this belief must be also equal to the expected q_R conditional on knowing that the report is $\Psi(\bar{L}, \bar{R})$ and taking into account the strategy of the expert (right-hand side). For a fixed \bar{L} , \bar{R} , and the belief induced by short reports, the shape of $\Psi(\bar{L}, \bar{R})$ is determined by the continuity of the belief function. The boundary of $\Psi(\bar{L}, \bar{R})$ with the area of reports sent in equilibrium only by the honest expert must be such that $\frac{\lambda}{\lambda + \rho} = \frac{\bar{R}}{\lambda \bar{R} + \bar{R}}$.

Proposition 5 does not specify whether persuaders reveal all favorable arguments when they send short reports. The reason is that there may be equilibria in which both persuaders reveal all favorable arguments, and there may be equilibria in which they do not. To see this, let us use the symmetric example depicted in Figure 4, and consider the following equilibrium of the former type. The persuader P_r always reveals all favorable arguments. For each $R \leq \bar{R}$, he reveals all unfavorable arguments if $L \leq R$, but if $L > R$, he uses the following mixed

strategy. With some probability $\gamma_R < 1$, he sends $\lambda = R$, and with the remaining probability $1 - \gamma_R$ he randomizes over $\lambda < R$, mixing in a way that justifies a belief constant in λ for $\lambda < \rho = R$. The strategy of P_l is constructed analogously. Given these strategies, for the reports with $\lambda = \rho$, the states in which P_r sends such reports are offset by the states in which P_l sends them; hence, the decision maker must hold a belief equal to the prior. For the remaining reports in $\Psi(\bar{L}, \bar{R})$, by construction the strategy of each persuader is such that the decision maker holds a constant belief, and for each R it is possible to find γ_R for which this belief is $\frac{1}{2}$.¹⁸

However, it is easy to perturb the equilibrium described above in such a way that the persuaders do not always reveal all favorable arguments. Consider some $x_2 < x_1 < \bar{R}$. Assume that the persuaders use the strategies described above, with the difference that when the persuader of any type is supposed to send (x_2, x_2) , he sends (x_1, x_1) instead. Given that $\eta(x_1, x_1) = \eta(x_2, x_2) = \frac{1}{2}$, the persuaders are indifferent between these two reports, and given that the strategies of the persuaders still offset each other, the decision maker's belief remains unchanged. Hence, these strategies are also part of some equilibrium.

The fact that there exist equilibria with a continuous belief function in which the persuaders conceal some favorable arguments distinguishes the two-sided case from the one-sided one. To understand the intuition for this difference, think of a persuader with a given R as a type, with higher types associated with higher R . Consider the one-sided case first. If in equilibrium no types of P_r pool, Lemma 1 implies that the decision maker's belief is strictly increasing in the revealed type of P_r . If in equilibrium some types of P_r pool, some higher types must induce the same belief as some lower types. Since the average belief must remain unchanged, all pooling types must induce a belief that is a weighted average of the beliefs that they would generate if they did not pool. These two observations imply that the highest pooling type generates a strictly lower belief than he would in an equilibrium with no pooling; and therefore, the belief function is discontinuous at this type. The same does not have to happen in the two-sided case because it is possible to have all pooling types induce the same belief that the highest pooling type would if there was no pooling. The reason for this is the presence of P_l . By

¹⁸From Section 4, we know that there exists a strategy of P_r in which he reveals all R , which induces a constant belief for a range of λ . The proof of Proposition 2 shows that this belief is at most $\frac{R}{R+\lambda_R}$, which, as Figure 4 shows, for $R \leq \bar{R}$ is smaller than $\frac{1}{2}$. If in states in which q_R is low, P_r sends $\lambda < R$ only with probability $1 - \gamma_R$, the belief formed after such reports goes up, and one can find γ_R high enough that brings this belief up to $\frac{1}{2}$.

pooling with low P_r types when he has a lot of arguments in favor of *Right*, P_l induces a lower belief than he would if he revealed all these arguments. This, however, raises the belief of the decision maker for all types that pool, and if it raises this belief to the level of the highest pooling type, there is no discontinuity at this type.

One may ask what would happen if the persuader could reveal his potential bias, for example, if a doctor could disclose who sponsored his research. Revealing the sponsor does not prove that the expert is biased, but only suggests the direction of the potential bias. Hence, within the model, disclosing the potential bias is equivalent to moving from the two-sided to the one-sided case. The discussion above implies that if the persuader has few arguments that favor his preferred alternative, he would not like to disclose his bias. In the current equilibrium, he pools with the persuader of the other type, and as a result, the decision maker believes that both type of arguments could have been concealed. If the persuader revealed his potential bias, for example, toward *Right*, the decision maker would suspect that he had concealed many arguments in favor of *Left*. She would hence revise her beliefs downwards. If the expert has many favorable arguments, however, such disclosure is completely harmless, as his reports reveal his potential bias anyway. This finding sheds some light on why a party presenting a case, for example, an author of an op-ed article, sometimes discloses his conflict of interest. Given that this does not hurt the author, we may see him disclosing the potential bias voluntarily if he has many arguments in favor of his position.

5.2 Comparative statics

To focus attention, assume that $f(L, R)$ is symmetric as in Figure 4, and consider varying the probabilities of the different types of experts. Since λ_ρ is determined by the same equation as in the one-sided case, the comparative statics from section 4.2 applies to it. That is, for each $\rho > \bar{R}$, as the probability of H increases at the expense of the other types, λ_ρ decreases, and $\eta(\lambda_\rho, \rho)$ increases. Using the same argument, it is easy to establish that for each $\lambda > \bar{L}$, as the probability of H increases, ρ_λ decreases, and $\eta(\lambda, \rho_\lambda)$ decreases. This implies that if a given persuader has enough of favorable arguments to separate himself from the persuader of the other type, he benefits from an increase in the probability of the honest type and suffers from an increase in the probability of his type.

The comparative statics for the belief that the persuaders induce when they do not have

enough arguments, i.e., when they send short reports, is more complex; therefore let me start with a special case in which $\pi_{P_r} = \pi_{P_l}$. By symmetry, in this case we have that $\bar{R} = \bar{L}$ and $\eta^* = \frac{1}{2}$. As argued above, when π_H decreases while keeping $\pi_{P_r} = \pi_{P_l}$, λ_ρ and ρ_λ increase, and the symmetry implies that they increase by the same amount. Therefore, they intersect at higher $\bar{R} = \bar{L}$. In other words, the belief induced by short reports remains unchanged, but more reports are considered short.

Proposition 6 analyzes how the belief induced by short reports changes as the relative probability of the types of persuaders changes.

Proposition 6 *Let η^* be the belief induced by all $(\lambda, \rho) \in \Psi(\bar{L}, \bar{R})$, and consider a change in π_{P_r} which is offset by a change in π_{P_l} ; that is, $d\pi_{P_r} = -d\pi_{P_l}$. In equilibrium, $\frac{d\eta^*}{d\pi_{P_r}} < 0$. Moreover, $\lim_{\pi_H \rightarrow 0} \frac{d\bar{R}}{d\pi_{P_r}} < 0$, and $\lim_{\pi_H \rightarrow 0} \frac{d\bar{L}}{d\pi_{P_r}} > 0$.*

Proposition 6 says that as P_r becomes more likely, the decision maker's belief after short reports goes down. It is P_r who sends short reports when q_R is low; hence, if the probability of P_r increases, short reports signal lower q_R . What is considered a short report changes as well, but this change is unambiguous only when $\pi_H \rightarrow 0$. In this case, Proposition 6 says that \bar{R} decreases and \bar{L} increases. The reason for this is as follows. In the limit as $\pi_H \rightarrow 0$, the functions λ_ρ and ρ_λ do not depend on π_{P_r} and π_{P_l} (see Eqs. (2) and (4) with $\pi_H = 0$), which implies that the belief that P_l induces when he sends $\lambda > \bar{L}$ and the belief that P_r induces when he sends $\rho > \bar{R}$ do not depend on them either. Consider now an increase in π_{P_r} , and assume that \bar{R} and \bar{L} remain unchanged. In this case, since η^* decreases, it becomes profitable for P_r to pretend that he is P_l and send a report with λ slightly above the initial \bar{L} instead of sending a short report. As a result, reports with λ slightly above the initial \bar{L} now signal a lower quality of q_R ; and therefore, the decision maker treats them as short. That is, \bar{L} must increase. Also, since now a lower number of arguments in favor of *Right* is enough to convince the decision maker that the expected $q_R > \eta^*$, \bar{R} must decrease. When $\pi_H > 0$, however, there is an additional effect. When π_{P_l} goes down, the probability of the expert being honest conditionally on him being either H or P_l increases, which implies that for each level of $\lambda > \bar{L}$ the belief of the decision maker goes down (ρ_λ decreases). This decreases the incentive of P_r to deviate to reports with $\lambda > \bar{L}$, and as a result \bar{L} may move in either direction.

6 Robustness

6.1 Selection of continuous equilibria

For most of the paper, I have focused on equilibria with the continuous belief function. Since all equilibria of this model share most of the qualitative features, this is without too much loss of generality. Proposition 7 establishes, however, that continuous equilibria are attractive to the decision maker.

Proposition 7 *In the one-sided case, the ex-ante utility of the decision maker is highest in continuous equilibria.*

In any continuous equilibrium the belief of the decision maker is strictly increasing in ρ , while in any discontinuous equilibrium there are ranges of ρ for which it is not. Hence, there exist states that lead to two different decisions in the continuous equilibrium but lead to the same decision in the discontinuous equilibrium. This means that less information is revealed in the latter one.

6.2 Continuity of arguments

The intuition for Proposition 1 makes it clear that the assumption of the continuity of arguments is not completely innocuous. If arguments are discrete, revealing one unfavorable argument proves that the quality of a given alternative is discretely lower than 1, and this may be enough to discourage a persuader from doing so. However, if the distribution of arguments is not very skewed toward states with only one type of arguments— $(0, R)$ or $(L, 0)$ —and the probability that the expert is a persuader is high enough, then in a discrete model the persuaders would still use two-sided messages, and the decision maker would ignore unfavorable arguments revealed by the persuaders.

6.3 The benevolent expert

In this model, the honest expert is assumed to reveal all of the arguments. Alternatively, the honest expert may want to maximize the utility of the decision maker; i.e., he may be benevolent. It is easy to show that any continuous equilibrium of the original game is still an equilibrium of a game with the honest type replaced by the benevolent type. However, the

set of equilibria in a model with the honest type replaced by the benevolent type is neither a subset nor a superset of the equilibria in the current model. First, there are some discontinuous equilibria in the current model in which the honest expert could do better by concealing some arguments; hence, these equilibria would not exist if the honest expert were benevolent.¹⁹ On the other hand, when the expert is benevolent, it is no longer true that all reports must be used in equilibrium, and by constructing off-equilibrium beliefs carefully, one can support equilibria in which the benevolent expert suppresses some arguments.

7 Conclusion

In many economically relevant situations almost any piece of information is verifiable. For example, documents exist that can prove what transactions occurred or what degree a person obtained; one can construct a mathematical proof or make a measurement. Even if verifiable evidence does not exist, informed parties may reveal pieces of information truthfully, be it for legal or moral reasons. However, it is more difficult to verify how many documents exist or how many features a certain product has. This creates room for manipulation, especially in environments with uncertainty about conflict of interests. The current paper investigates interactions in such environments.

First, the paper shows that not all information is revealed. Second, it sheds light on the type of information the expert reveals. The biased expert may reveal all arguments that are favorable to him, but he also often provides some arguments that seemingly do not work to his advantage. Third, the paper shows that when the expert is biased, the decision maker is influenced only by the arguments that are favorable to this expert. The paper also provides insight on when persuasion is successful. If the biased expert has sufficiently many favorable arguments, he is able to convince the decision maker to choose the expert's preferred alternative even if it is inferior. However, if he has few favorable arguments, he may not be able to convince the decision maker to choose his preferred alternative even if that would be better for the decision maker.

One of the features of the model is that although all pieces of information are ex ante symmetric, they are treated asymmetrically in equilibrium. If the decision maker suspects that

¹⁹Examples available upon request.

the expert is biased toward one alternative, she treats arguments that favor this alternative as hard evidence: the more of them the expert reveals, the more likely the decision maker is to choose this alternative. However, unless the expert reveals himself to be honest by providing a lot of arguments for both sides, the decision maker chooses the same alternative independent of how many unfavorable arguments the expert reveals. Hence, it is as if the first few unfavorable arguments were cheap talk.

To prove the relevance of my results, in accordance with my model I should also mention some of its limitations. First, the model assumes that the decision maker is uninformed, but it would be interesting to analyze the case in which the decision maker observes some subset of arguments herself. Second, the model explains the use of two-sided messages, but it predicts that two-sided messages are not more persuasive. Although I do not think that two-sided messages are always more persuasive in economic interactions, some classic experiments on mass communication indicate that they might be more effective.²⁰ Slight modifications of the model, such as the introduction of a small fraction of decision makers who take the experts' reports at face value, could generate this effect; this, however, is beyond the scope of this paper.

²⁰See Hovland, Lumsdaine, and Sheffield [14] and Lumsdaine and Janis [20].

A Appendix. Example of a discontinuous equilibrium

Let L and R be independent random variables distributed according to a Pareto distribution, i.e., $f(L, R) = \frac{1}{(1+L)^2} \frac{1}{(1+R)^2}$.²¹ Fix some $r > 0$ and $\lambda_r > 0$. Assume that all properties outlined in Proposition 1 are satisfied, and additionally the following holds.

Beliefs:

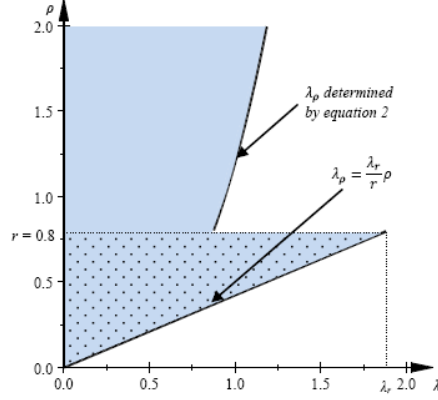
1. For $\lambda \leq \lambda_\rho$, $\eta(\lambda, \rho) = \frac{\rho}{\lambda_\rho + \rho}$, where
 - (a) λ_ρ solves Eq. (2) for $\rho > r$;
 - (b) $\lambda_\rho = \frac{\lambda_r}{r} \rho$ for $\rho \leq r$.
2. For any (λ, ρ) with $\lambda > \lambda_\rho$, $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$.

The strategy of the persuader:

1. For $R > r$, the persuader uses the same strategy as in Proposition 2.
2. For $R \leq r$,
 - (a) when $L \leq \lambda_{\rho=R}$, then with probability $\gamma_R \leq 1$ the persuader sends $(\lambda, \rho) = (L, R)$, and with probability $1 - \gamma_R$, the persuader sends $\lambda = 0$ and randomizes over $\rho \leq R$ with some p.d.f. $s_R(\rho)$;
 - (b) when $L > \lambda_{\rho=R}$, then the persuader sends $\rho = R$, with probability β_R he randomizes over $\lambda \leq \lambda_\rho$ with some probability density function $m_{P_r}^\rho(\lambda)$, and with probability $1 - \beta_R$, the persuader sends $\lambda = 0$.

The beliefs and the strategy profiles outlined above form an equilibrium with a discontinuous belief function. The figure below shows this equilibrium for $r = 0.8$, $\lambda_r = 1.89$, and $\pi_{P_r} = 0.9$. The boundary of the shaded area represents λ_ρ as defined in part (1a), and the persuader sends only reports from the shaded area. All reports in the dotted area generate $\eta(\lambda, \rho) = \frac{\rho}{\lambda_\rho + \rho} = \frac{r}{\lambda_r + r}$. Graphically, λ_ρ is discontinuous at $\rho = r$, which by part (1a) implies that the decision maker's belief is discontinuous at $\rho = r$.

²¹Pareto distribution is chosen because of its relative tractability for this problem.



It follows directly from Proposition 2 and the discussion thereafter that the belief function for $\rho > r$ and the strategy of the persuader for $R > r$ form an equilibrium. When $R < r$, the persuader receives the same payoff from any feasible report (λ, ρ) with $\lambda \leq \lambda_\rho$, and since for all $\lambda > \lambda_\rho$, $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho} < \frac{\rho}{\lambda_\rho + \rho}$, the persuader strictly prefers to send only $\lambda \leq \lambda_\rho$. It remains to show that the strategy of the persuader justifies a constant belief equal to $\frac{r}{\lambda_r + r}$ for all reports in the dotted area and must result in discontinuity of the belief function at $\rho = r$.

Since all reports in the dotted area induce the same belief $\frac{r}{\lambda_r + r}$, this belief must equal the expected q_R conditional on knowing that the report lies in the dotted area, without knowing the exact report. Clearly, this cannot be satisfied for arbitrary r and λ_r , but in the appendix I show that this is satisfied for $r = 0.8$ and $\lambda_r = 1.89$. From Proposition 2 and the discussion thereafter, we know that if $\rho = R$, then one can find a probability density function $m_{P_r}^\rho(\lambda)$ that supports a constant belief for all (λ, ρ) with $\lambda \leq \lambda_\rho$. However, Lemma 1 says that this belief is strictly increasing in ρ . Hence, to generate a belief that is constant in ρ , the persuader must use a strategy that will decrease the belief of the decision maker when she sees high ρ and increase it when she sees low ρ . This can only happen if the persuader conceals some favorable arguments when R and q_R are relatively high. In such a case the decision maker will not form a very high belief when observing high ρ , as this is likely to happen when P_r conceals many unfavorable arguments, and will not be very pessimistic about q_R when observing low ρ , as this is likely to happen when P_r conceals some favorable arguments. This is exactly what the strategy outlined in (2b) does; when $R \leq r$ and q_R is high, the persuader reports fully only with probability γ_R . In the appendix, I show that for each $R \leq r$, one can find $\gamma_R \leq 1$ that

supports the equilibrium, and such γ_R is strictly smaller than 1 for R close to r . Since for R slightly higher than r , the persuader reveals all favorable arguments, while for R slightly lower than r the persuader reveals all favorable arguments mainly when the quality of *Right* is low, the belief of the decision maker must be discontinuous at $\rho = r$.

Proof that the above is an equilibrium. Clearly, by Proposition 2 the behavior of P_r and the belief function constitute an equilibrium for $R > r$. Eq. (2) can be rewritten as $\frac{1}{(\rho+\lambda\rho)} = \frac{1}{(\pi_{P_r}+\lambda\rho)(1-\rho)} \left(\ln \frac{1}{\rho} - (1-\pi_{P_r}) \ln \frac{(\lambda\rho+1)}{(\lambda\rho+\rho)} \right)$. The curve for $\rho > r$ in the figure above is the plot of $(\lambda\rho, \rho)$ that satisfy this equation for $\pi_{P_r} = 0.9$. Denote $\Psi(r, \lambda_r) \equiv \left\{ (\lambda, \rho) : \rho \leq r, \lambda \leq \frac{\lambda_r R}{R} \right\}$, which is depicted by the dotted triangle in the figure above. By (1a), it must be that $E_b[q_R | (\lambda, \rho) \in \Psi(r, \lambda_r)] = \frac{r}{r+\lambda_r}$; which using Bayes' rule can be written as:

$$\frac{r}{r+\lambda_r} = \frac{\pi_H \int_0^r \int_0^{\frac{\lambda_r R}{r}} \frac{R}{R+L} \frac{1}{(1+L)^2} \frac{1}{(1+R)^2} dL dR + \pi_{P_r} \int_0^r \int_0^\infty \frac{R}{R+L} \frac{1}{(1+L)^2} \frac{1}{(1+R)^2} dL dR}{\pi_H \int_0^r \int_0^{\frac{\lambda_r R}{r}} \frac{1}{(1+L)^2} \frac{1}{(1+R)^2} dL dR + \pi_{P_r} \int_0^r \int_0^\infty \frac{1}{(1+L)^2} \frac{1}{(1+R)^2} dL dR}.$$

For $\pi_{P_r} = 0.9$ and $r = 0.8$, the unique λ_r that satisfies the above equation is 1.8921. It remains to show that the belief of the decision maker is constant for all $(\lambda, \rho) \in \Psi(r, \lambda_r)$. The belief will satisfy Bayes' rule for all non-zero measure sets if for each $\rho \geq r$, $m_{P_r}^\rho(\lambda)$, β_ρ ($\equiv \beta_R$ for $\rho = R$) and γ_ρ ($\equiv \gamma_R$ for $\rho = R$) must satisfy:

$$\frac{r}{r+\lambda_r} = \frac{\pi_H \frac{\rho}{\lambda+\rho} f^L(\lambda|\rho) + \pi_{P_r} \left(\frac{\rho}{\lambda+\rho} f^L(\lambda|\rho) \gamma_\rho + m_{P_r}^\rho(\lambda) \beta_\rho \int_{\frac{\lambda_r}{r}\rho}^\infty \frac{\rho}{\lambda+\rho} f^L(L|\rho) dL \right)}{\pi_H f^L(\lambda|\rho) + \pi_{P_r} \left(f^L(\lambda|\rho) \gamma_\rho + (1 - F^L(\frac{\lambda_r}{r}\rho|\rho)) m_{P_r}^\rho(\lambda) \beta_\rho \right)},$$

which will happen if

$$m_{P_r}^\rho(\lambda) = \frac{\left(\frac{r}{r+\lambda_r} - \frac{\rho}{\lambda+\rho} \right) (\pi_H + \pi_{P_r} \gamma_\rho) f^L(\lambda|\rho)}{\pi_{P_r} \beta_\rho \int_{\frac{\lambda_r}{r}\rho}^\infty \left(\frac{\rho}{\lambda+\rho} - \frac{r}{r+\lambda_r} \right) f^L(L|\rho) dL}.$$

It remains to show that we can always find $\gamma_R \in [0, 1]$, $\beta_R \in [0, 1]$ and $m_{P_r}^\rho(\lambda)$ such that for each $\rho \leq r$, $\int_0^{\frac{\lambda_r}{r}\rho} m_{P_r}^\rho(\lambda) d\lambda = 1$. But we have

$$\int_0^{\frac{\lambda_r}{r}\rho} m_{P_r}^\rho(\lambda) d\lambda = 1 \Leftrightarrow$$

$$\frac{r}{r + \lambda_r} = \frac{(\pi_H + \pi_{P_r} \gamma_\rho) \int_0^{\frac{\lambda_r}{r} \rho} \frac{\rho}{\lambda + \rho} f^L(\lambda|\rho) d\lambda + \pi_{P_r} \beta_\rho \int_{\frac{\lambda_r}{r} \rho}^\infty \frac{\rho}{\lambda + \rho} f^L(L|\rho) dL}{(\pi_H + \pi_{P_r} \gamma_\rho) \int_0^{\frac{\lambda_r}{r} \rho} f^L(\lambda|\rho) d\lambda + \pi_{P_r} \beta_\rho \int_{\frac{\lambda_r}{r} \rho}^\infty f^L(L|\rho) dL}.$$

For $\beta_\rho = 0$ and $\gamma_\rho = 1$, the *RHS* is $\frac{\int_0^{\frac{\lambda_r}{r} \rho} \frac{\rho}{\lambda + \rho} f^L(\lambda|\rho) d\lambda}{\int_0^{\frac{\lambda_r}{r} \rho} f^L(\lambda|\rho) d\lambda} > \frac{r}{r + \lambda_r}$. For $(\beta_\rho = 1, \gamma_\rho = 0)$ the *RHS* is $\frac{\pi_H \int_0^{\frac{\lambda_r}{r} \rho} \frac{\rho}{\lambda + \rho} f^L(\lambda|\rho) d\lambda + \pi_{P_r} \int_{\frac{\lambda_r}{r} \rho}^\infty \frac{\rho}{\lambda + \rho} f^L(L|\rho) dL}{\pi_H \int_0^{\frac{\lambda_r}{r} \rho} f^L(\lambda|\rho) d\lambda + \pi_{P_r} \int_{\frac{\lambda_r}{r} \rho}^\infty f^L(L|\rho) dL}$, and from the proof of Proposition 2 we know that this is at most equal to $\frac{\rho}{\frac{\lambda_r}{r} \rho + \rho} = \frac{r}{r + \lambda_r}$. Since the *RHS* is continuous, one can find the appropriate parameters. ■

B Appendix. Proofs

Lemma 2 *There is no equilibrium in which the decision maker's belief is independent of the expert's report (babbling equilibrium).*

Proof. Let $\bar{\eta} \equiv E \left[\frac{R}{R+L} \right]$. Fix λ_0 , and consider a set $X(\rho_0) = [0, \lambda_0] \times [\rho_0, \infty)$ with ρ_0 such that $\frac{\rho_0}{\rho_0 + \lambda_0} > \bar{\eta}$. I will show that for any feasible strategy of the persuaders, the corresponding belief and for ρ_0 high enough, the belief of the decision maker conditional on observing a report in $X(\rho_0)$ is higher than the prior, that is, $E_b \left[\frac{R}{R+L} | (\lambda, \rho) \in X(\rho_0) \right] > \bar{\eta}$; which cannot happen in a babbling equilibrium. Let $Y(\rho_0)$ be the largest set of states such that if $(L, R) \in Y(\rho_0)$, then at least one persuader sends some $(\lambda, \rho) \in X(\rho_0)$. Define $Y^H(\rho_0) \equiv X(\rho_0) \cup \left\{ Y(\rho_0) \cap \left\{ (L, R) : L < \frac{1-\bar{\eta}}{\bar{\eta}} R \right\} \right\}$ and $Y^L(\rho_0) \equiv Y(\rho_0) \cap \left\{ (L, R) : L \geq \frac{1-\bar{\eta}}{\bar{\eta}} R \right\}$. The decision maker's belief must satisfy

$$\begin{aligned} E_b [q_R | (\lambda, \rho) \in X(\rho_0)] &= \\ &= \Pr_b \left((L, R) \in Y^H(\rho_0) | (\lambda, \rho) \in X(\rho_0) \right) E_b [q_R | (L, R) \in Y^H(\rho_0), (\lambda, \rho) \in X(\rho_0)] + \\ &+ \Pr_b \left((L, R) \in Y^L(\rho_0) | (\lambda, \rho) \in X(\rho_0) \right) E_b [q_R | (L, R) \in Y^L(\rho_0), (\lambda, \rho) \in X(\rho_0)]. \end{aligned}$$

By construction of $Y^H(\rho_0)$, we have $E_b [q_R | (L, R) \in Y^H(\rho_0), (\lambda, \rho) \in X(\rho_0)] > \bar{\eta}$. I will show that $\lim_{\rho_0 \rightarrow \infty} \Pr_b \left((L, R) \in Y^L(\rho_0) | (\lambda, \rho) \in X(\rho_0) \right) = 0$, which together with the fact that the expectation of q_R must be weakly smaller than 1, proves that for ρ_0 large enough,

$E_b [q_R | (\lambda, \rho) \in X(\rho_0)] > \bar{\eta}$. We have

$$\begin{aligned} \Pr_b((L, R) \in Y^L(\rho_0) | (\lambda, \rho) \in X(\rho_0)) &= \frac{\Pr_b((L, R) \in Y^L(\rho_0), (\lambda, \rho) \in X(\rho_0))}{\Pr_b((\lambda, \rho) \in X(\rho_0))} \leq \\ &\leq \frac{(1-\pi_H) \Pr_b((L, R) \in Y^L(\rho_0))}{\pi_H \Pr_b((L, R) \in X(\rho_0))} \leq \frac{1-\pi_H}{\pi_H} \frac{\int_{\rho_0}^{\infty} \int_{\frac{1-\bar{\eta}}{\eta} R}^{\infty} f^L(L|R) f^R(R) dL dR}{\int_{\rho_0}^{\infty} \int_0^{\lambda_0} f^L(L|R) f^R(R) dL dR}, \end{aligned}$$

where the first inequality follows from the fact that reports in $X(\rho_0)$ are sent by H at least whenever $(L, R) \in X(\rho_0)$, and at most by the persuaders when $(L, R) \in Y^L(\rho_0)$, and the second inequality follows from the definition of Y^L . But the limit of the last expression is

$$\begin{aligned} \lim_{\rho_0 \rightarrow \infty} \frac{\int_{\rho_0}^{\infty} \int_{\frac{1-\bar{\eta}}{\eta} R}^{\infty} f^L(L|R) f^R(R) dL dR}{\int_{\rho_0}^{\infty} \int_0^{\lambda_0} f^L(L|R) f^R(R) dL dR} &\stackrel{\text{by L'Hospital's Rule}}{=} \\ &= \lim_{\rho_0 \rightarrow \infty} \frac{\int_{\frac{1-\bar{\eta}}{\eta} \rho_0}^{\infty} f(L|\rho_0) dL}{\int_0^{\lambda_0} f^L(L|\rho_0) dL} = \lim_{\rho_0 \rightarrow \infty} \frac{1-F^L\left(\frac{1-\bar{\eta}}{\eta} \rho_0 | \rho_0\right)}{F^L(\lambda_0 | \rho_0)} = 0. \end{aligned}$$

The last equality comes from the regularity conditions (1); since $\frac{dF^L(L|R)}{dR} \geq 0$, we have $\lim_{\rho_0 \rightarrow \infty} F^L(\lambda_0 | \rho_0) > 0$, while $\lim_{\rho_0 \rightarrow \infty} F^L\left(\frac{1-\bar{\eta}}{\eta} \rho_0 | \rho_0\right) = 1$. ■

Proof of Proposition 1. Note that in equilibrium, in any state, P_r sends the report that generates the highest belief from the set of feasible reports. Throughout this proof and the proof of Proposition 2, I refer to this as property (*). Also, if for some set of non-zero measure X , in equilibrium only H sends reports from X , then Bayes' rule requires that for almost all $(\lambda, \rho) \in X$, $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$. I refer to this as property (**). Both properties will be used also in the subsequent proof.

First, it must be that $\sup_{\lambda} \eta(\lambda, \rho) < 1$ for all (λ, ρ) . Assume, by contradiction, that $\sup_{\lambda} \eta(\lambda, \rho) = 1$. Then there exists (λ, ρ) such that $\eta(\lambda, \rho) = 1$. This is because either $\max_{\lambda} \eta(\lambda, \rho) = \sup_{\lambda} \eta(\lambda, \rho)$, or the maximum does not exist, in which case the problem of P_r has a solution only if he can induce 1 by sending some other report (and in equilibrium the problem of P_r must have a solution). Therefore, assume that there exists (λ, ρ) such that $\eta(\lambda, \rho) = 1$, and note for all $(L, R) \in Z(\lambda, \rho)$, P_r sends some reports that generate belief equal to 1, but this would make the decision maker's belief wrong for at least some of such reports.

Part i: I will show that part i holds for almost all ρ . Let $X = X_1 \cup X_2$, where $X_1 \equiv \left(0, \frac{\varepsilon}{2}\right) \times (\rho', \rho'')$ and $X_2 \equiv \left(\frac{\varepsilon}{2}, \varepsilon\right) \times (\rho', \rho'')$ for some ε and some ρ' and ρ'' , and assume that part i does not hold on X . That is, for any ε and almost any ρ , we have that $\eta(\lambda_1, \rho) \neq \eta(\lambda_2, \rho)$

for almost all $(\lambda_1, \rho) \in X_1$ and $(\lambda_2, \rho) \in X_2$. Assume first that for some sets of reports of non-zero measure, $\eta(\lambda_1, \rho) > \eta(\lambda_2, \rho)$. By property (*), P_r strictly prefers to send the former over the latter. Since the former are feasible when the latter are, by property (**), almost all of the latter generate $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$. But as $\varepsilon \rightarrow 0$, $\frac{\rho}{\lambda + \rho}$ becomes arbitrarily close to 1 for all those reports, hence it must exceed $\sup_\lambda \eta(\lambda, \rho) < 1$. But this is a contradiction. Assume then that $\eta(\lambda_1, \rho) < \eta(\lambda_2, \rho)$. By property (*), P_r always prefers to send (λ_2, ρ) instead of (λ_1, ρ) . That means that the latter reports are sent only by H or by P_r when the former are not feasible. But this implies that as $\lambda \rightarrow 0$, the beliefs generated by the former reports are arbitrarily close to 1, again exceeding $\sup_\lambda \eta(\lambda, \rho) < 1$. This is a contradiction.

Define η_ρ^* to be the belief induced by most (λ, ρ) with $\lambda \leq \lambda_\rho$.

Part ii: η_ρ^* is weakly increasing in ρ up to measure 0 perturbations.

Define $X_1 \equiv (0, \lambda) \times (\rho', \rho' + \varepsilon)$ and $X_2 \equiv (0, \lambda) \times (\rho'', \rho'' + \varepsilon)$ with $\rho'' > \rho' + \varepsilon$, and assume that the belief of the decision maker is higher for some non-zero measure set of reports in X_1 than for some non-zero measure set of reports in X_2 . Then by property (*), P_r never sends reports from the latter, which implies that $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$ for almost all of those reports. But as $\lambda \rightarrow 0$, $\frac{\rho}{\lambda + \rho} \rightarrow 1$, which in turn contradicts that reports from X_1 generate higher beliefs.

Part *i* and part *ii* combined with property (**) imply part *iii*. For each ρ , define λ_ρ to be the supremum of λ_ρ that satisfies part *i* of Proposition 1. ■

Proof of Proposition 2. Define λ_ρ and η_ρ^* as in the previous proof.

Lemma 3 *In any equilibrium $\eta_\rho^* \geq \frac{\rho}{\rho + \lambda_\rho}$ apart from a set of zero measure on R .*

Proof *If $\lambda_\rho = \infty$, this is immediate. If $\lambda_\rho < \infty$, then assume that $\eta_\rho^* < \frac{\rho}{\rho + \lambda_\rho}$ for some set of non-zero measure on R . By definition of λ_ρ , in a set $\left\{ (\lambda, \rho) : \lambda_\rho < \lambda < \frac{1 - \eta_\rho^*}{\eta_\rho^*} \rho \right\}$ one can find a set X of non-zero measure for which $\eta(\lambda, \rho) \neq \eta_\rho^*$. Assume first that $\eta(\lambda, \rho) < \eta_\rho^*$ for those reports. Then by property (*) defined in proof of Proposition 1, only H sends those reports, and by property (**), also defined in proof of Proposition 1, $\eta(\lambda, \rho) = \frac{\rho}{\rho + \lambda} > \eta_\rho^*$ for most of those reports, but this is a contradiction. Assume now that $\eta(\lambda, \rho) > \eta_\rho^*$ for those reports. Then by property (*), P_r sends reports with $\lambda \leq \lambda_\rho$ only when X is not feasible, therefore those reports must generate a belief higher than the face value of reports in X , which by definition of X is higher than η_ρ^* . But this is a contradiction.*

Lemma 3 implies that in any equilibrium in which $\rho = R$ for all R , $\eta_\rho^* = \frac{\rho}{\rho + \lambda_\rho}$ for almost all ρ . To see this, assume the other possibility allowed by Lemma 3, namely, $\eta_\rho^* > \frac{\rho}{\rho + \lambda_\rho}$, and take a set of reports of non-zero measure $X = \left\{ (\lambda, \rho) : \frac{1 - \eta_\rho^*}{\eta_\rho^*} \rho < \lambda < \lambda_\rho \right\}$. To generate a belief $\eta(\lambda, \rho) = \eta_\rho^*$ for most of $(\lambda, \rho) \in X$, those reports must be used by P_r (otherwise, by property (**), $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$ for most of those reports). But since $\rho = R$, and feasibility requires that $\lambda \leq L$, P_r sends those reports only when $\frac{R}{L + R} \leq \frac{\rho}{\lambda + \rho}$, which by construction of X is smaller than η_ρ^* . This implies that the belief those reports generate must be lower than η_ρ^* , which is a contradiction. The feasibility requirement $\lambda \leq L$ together with $\rho = R$, imply that for all (λ, ρ) with $\lambda > \lambda_\rho$, we have $\eta(\lambda, \rho) \leq \frac{\rho}{\lambda + \rho} < \frac{\rho}{\lambda_\rho + \rho} = \eta_\rho^*$ for almost all $\lambda > \lambda_\rho$. This implies that $(\lambda, \rho) \in \Sigma_{P_r}(L, R)$ for some (L, R) only if $\lambda \leq \lambda_\rho$, and hence in turn $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$ for almost all (λ, ρ) with $\lambda > \lambda_\rho$. This proves parts *ii* and *iii*.

Given that $\eta(\lambda, \rho) = \eta_\rho^*$ for almost all (λ, ρ) with $\lambda \leq \lambda_\rho$, it must be that the expected q_R conditional on knowing that the report contains fewer unfavorable arguments than λ_ρ must be equal to η_ρ^* . Using Bayes' rule, this is equal to the probability that the expert is honest conditional on $R = \rho$ and $\lambda \leq \lambda_\rho$, times the expected quality of *Right* conditional on $R = \rho$ and $L \leq \lambda_\rho$, plus the probability that the expert is a persuader conditional on $R = \rho$ and $\lambda \leq \lambda_\rho$, times the expected quality of *Right* conditional on $R = \rho$. This is exactly the right-hand side of Eq. (2). Together with the fact that $\eta_\rho^* = \frac{\rho}{\rho + \lambda_\rho}$, this implies that λ_ρ solves Eq. (2) (this proves *i*). One can rewrite Eq. (2) in the following way:

$$\eta_\rho^* = \frac{\pi_{P_r} \int_0^\infty \frac{\rho}{\rho + L} f^L(L|R = \rho) dL + \pi_H \int_0^{\frac{1 - \eta_\rho^*}{\eta_\rho^*} \rho} \frac{\rho}{\rho + L} f^L(L|R = \rho) dL}{\pi_{P_r} + \pi_H F^L\left(\frac{1 - \eta_\rho^*}{\eta_\rho^*} \rho | R = \rho\right)}. \quad (8)$$

The left-hand side (*LHS*) is strictly increasing in η_ρ^* and ranges from 0 to 1. The right-hand side (*RHS*) is continuous, $\lim_{\eta_\rho^* \rightarrow 0} RHS(\eta_\rho^*) = \int_0^\infty \frac{\rho}{\rho + L} f^L(L|R = \rho) dL > 0$, and $RHS(1) = \int_0^\infty \frac{\rho}{\rho + L} f^L(L|R = \rho) dL < 1$; therefore, the solution to Eq. (8) exists. Moreover, if we differentiate the *RHS* with respect to η_ρ^* and evaluate it at the equilibrium η_ρ^* satisfying Eq. (8), we obtain $\frac{dRHS}{d\eta_\rho^*} = 0$, which together with the previous observation implies that *RHS* crosses η_ρ^* from above. This means that the solution to Eq. (8) is unique. The above proves also the following corollary, which I will use in the proof of Proposition (7).

Corollary 2 $\eta_\rho^* < RHS(\eta_\rho^*)$ when η_ρ^* is smaller than the unique η_ρ^* solving Eq. (8), and

$\eta_\rho^* > RHS(\eta_\rho^*)$ when η_ρ^* is bigger than the unique η_ρ^* solving Eq. (8).

Consider a strategy in which P_r reports fully if $L \leq \lambda_{\rho=R}$, and randomizes over $\lambda \in [0, \lambda_\rho]$ with some p.d.f. $m_{P_r}^\rho(\lambda)$ otherwise. The following Lemma completes the proof by showing that such a strategy supports the equilibrium belief, and it will be referred to in the subsequent proofs.

Lemma 4 Let $m_{P_r}^\rho(\lambda) = \frac{\left(\frac{1}{\rho+\lambda_\rho} - \frac{1}{\rho+\lambda}\right) f^L(\lambda|\rho)}{\pi_{P_r} \left(\int_{\lambda_\rho}^\infty \frac{1}{\rho+L} f^L(L|\rho) dL - \frac{1}{\rho+\lambda_\rho} (1 - F^L(\lambda_\rho|\rho)) \right)}$, and assume that the persuader plays the strategy described above. Then Bayes' rule implies that for almost all (ρ, λ) with $\lambda \leq \lambda_\rho$, $\eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda_\rho}$.

Proof Using Eq. (8) and $\eta_\rho^* = \frac{\rho}{\lambda_\rho+\rho}$, one gets $\int_0^{\lambda_\rho} m_{P_r}^\rho(\lambda) d\lambda = 1$, hence $m_{P_r}^\rho(\lambda)$ is a p.d.f. If the belief of the decision maker is derived from the prior beliefs $f(L, R)$ and π_H, π_{P_r} and the strategy of the persuader using conditional expectations as defined in DeGroot [6], then those belief will satisfy Bayes' rule on all sets of non-zero measure.²² For (λ, ρ) with $\lambda \leq \lambda_\rho$, the conditional expectation is

$$E \left[\frac{R}{R+L} | \lambda, \rho \right] = \frac{\frac{\rho}{\rho+\lambda} f(\lambda, \rho) + \pi_{P_r} m_{P_r}^\rho(\lambda) \int_{\lambda_\rho}^\infty \frac{\rho}{\rho+L} f(L, \rho) dL}{f(\lambda, \rho) + \pi_{P_r} m_{P_r}^\rho(\lambda) \int_{\lambda_\rho}^\infty f(L, \rho) dL},$$

which is equal to $\frac{\rho}{\rho+\lambda_\rho}$ for $m_{P_r}^\rho(\lambda)$ defined above.

■

Proof of Lemma 1. The "only if" part is immediate, since by the proof of Proposition 2, for any ρ , $\eta(\lambda, \rho)$ is continuous in λ up to perturbations of measure zero, and the fact that η_ρ^* is a unique solution to (8) together with the *RHS* and the *LHS* of it being continuous imply that η_ρ^* is continuous in ρ . To prove the "if" part, assume that $\eta(\lambda, \rho)$ is continuous. First, let me prove that $\eta_\rho^* = \frac{\rho}{\lambda_\rho+\rho}$. Assume first that $\lambda_\rho = \infty$ for all ρ . This, together with the fact that by Proposition 1 η_ρ^* is weakly increasing and by Lemma 2 there is no babbling equilibrium, implies that η_ρ^* must be strictly increasing for some ρ . But for those ρ s, property (*) implies that P_r reveals all favorable arguments, and hence by the same argument as in the proof of Proposition 2 $\lambda_\rho < \infty$ (and is actually determined by Eq. (2)). But then continuity requires that $\lambda_\rho < \infty$ for all ρ . Fix ρ , and note that by definition of λ_ρ and the continuity of the belief function,

²²Chapter 3, section 3.8.

$\eta(\lambda, \rho)$ is either strictly decreasing or increasing in $\lambda \in (\lambda_\rho, \lambda_\rho + \varepsilon)$ for some $\varepsilon > 0$. If it is strictly increasing, by property (*) P_r sends as high $\lambda \in (\lambda_\rho, \lambda_\rho + \varepsilon)$ as possible, therefore any $\lambda \in (\lambda_\rho, \lambda_\rho + \varepsilon)$ means that $L = \lambda$, and this cannot support a belief increasing in λ . Assume then that $\eta(\lambda, \rho)$ is strictly decreasing in $\lambda \in (\lambda_\rho, \lambda_\rho + \varepsilon)$. In such a case $\lambda \in (\lambda_\rho, \lambda_\rho + \varepsilon)$ is sent only by H , and property (**) implies that $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho} \rightarrow_{\lambda \rightarrow \lambda_\rho} \frac{\rho}{\lambda_\rho + \rho} = \eta_\rho^*$, which is what we wanted to establish.

By Proposition 1, η_ρ^* is weakly increasing. Assume then that η_ρ^* is constant for some ρ , and let $[\rho_1, \rho_2]$ be the largest interval for which this is true (hence, $\eta_{\rho_1}^* = \eta_{\rho_2}^*$). To focus the attention, assume that $\rho_1 > 0$, but an analogous argument holds if $\rho_1 = 0$. Given that $\eta(\lambda, \rho)$ is continuous, we can find ε such that η_ρ^* is strictly increasing in ρ for all $\rho \in (\rho_1 - \varepsilon, \rho_1)$ and $\rho \in (\rho_2, \rho_2 + \varepsilon)$. Clearly, for any $R \in (\rho_1 - \varepsilon, \rho_1)$ and $R \in (\rho_2, \rho_2 + \varepsilon)$, P_r sends ρ if and only if $R = \rho$. By the same argument as in the proof of Proposition 2, for any $\rho \in (\rho_1 - \varepsilon, \rho_1)$ and $\rho \in (\rho_2, \rho_2 + \varepsilon)$ the belief must satisfy Eq. (8). Since there is a unique solution to this equation, we can differentiate its *RHS*, obtaining:

$$\frac{\partial RHS}{\partial \rho} = \frac{\pi_{P_r} \left(\int_0^\infty \frac{L}{(L+\rho)^2} f^L(L|R=\rho) dL + \int_0^\infty \frac{\rho}{(\rho+L)^2} \frac{dF^L(L|R=\rho)}{dR} dL \right)}{\pi_{P_r} + \pi_H F^L \left(\frac{1-\eta_\rho^*}{\eta_\rho^*} \rho | R = \rho \right)} \\ \frac{\pi_H \left(\int_0^{\frac{1-\eta_\rho^*}{\eta_\rho^*} \rho} \frac{L}{(L+\rho)^2} f^L(L|R=\rho) dL + \int_0^{\frac{1-\eta_\rho^*}{\eta_\rho^*} \rho} \frac{\rho}{(\rho+L)^2} \frac{dF^L(L|R=\rho)}{dR} dL \right)}{\pi_{P_r} + \pi_H F^L \left(\frac{1-\eta_\rho^*}{\eta_\rho^*} \rho | R = \rho \right)}.$$

The last expression was obtained by applying first integration by parts to Eq. (8), taking the derivative, and applying integration by parts again. The regularity conditions (1) imply that $\frac{\partial RHS}{\partial \rho} > 0$, and in the proof of Proposition 2 we have established that at the equilibrium η_ρ^* , $\frac{\partial RHS}{\partial \eta_\rho^*} = 0$. Using these findings and the implicit function theorem, we obtain $\frac{d\eta_\rho^*}{d\rho} = -\frac{\frac{\partial RHS}{\partial \rho}}{1 - \frac{\partial RHS}{\partial \eta_\rho^*}} = \frac{\partial RHS}{\partial \rho}$, which implies that $\frac{d\eta_\rho^*}{d\rho} > 0$. Since $\frac{d\eta_\rho^*}{d\rho} > 0$, $\eta_{\rho_2+\varepsilon}^* > \eta_{\rho_1-\varepsilon}^*$ for all $\varepsilon > 0$, therefore by continuity $\lim_{\varepsilon \rightarrow 0} \eta_{\rho_2+\varepsilon}^* = \eta_{\rho_2}^* > \eta_{\rho_1}^* = \lim_{\varepsilon \rightarrow 0} \eta_{\rho_1-\varepsilon}^*$, which is a contradiction. Hence, $\eta(\lambda, \rho)$ is strictly increasing in ρ . This implies that P_r has a strict incentive to reveal all favorable arguments. ■

Proof of Proposition 3. Differentiating the *RHS* of Eq. (8) with respect to π_H , and

evaluating at the equilibrium η_ρ^* , we get
$$\frac{\eta_\rho^* \left(1 - F^L \left(\frac{1 - \eta_\rho^*}{\eta_\rho^*} \rho | R = \rho \right) \right) - \int_0^\infty \frac{\rho}{\eta_\rho^*} \frac{\rho}{\rho + L} f^L(L | R = \rho) dL}{\pi + (1 - \pi) F^L \left(\frac{1 - \eta_\rho^*}{\eta_\rho^*} \rho | R = \rho \right)} > 0.$$

Recall that at the equilibrium η_ρ^* , $\frac{\partial RHS}{\partial \eta_\rho^*} = 0$. By the implicit function theorem $\frac{d\eta_\rho^*}{d\pi_H} = -\frac{\frac{\partial RHS}{\partial \pi}}{-1 + \frac{\partial RHS}{\partial \eta_\rho^*}} > 0$. Since $\eta_\rho^* = \frac{\rho}{\rho + \lambda_\rho}$, we have $\frac{d\lambda_\rho}{d\pi_H} < 0$. This proves part (i). Recall that $\eta_\rho^* = \eta(\lambda_\rho, \rho)$; taking the limit of both sides of Eq. (8), one gets $\lim_{\pi_H \rightarrow 0} \eta(\lambda_\rho, \rho) = \int_0^\infty \frac{\rho}{\rho + L} f^L(L | R = \rho) dL = E \left[\frac{\rho}{\rho + L} | R = \rho \right]$, and $\lim_{\pi_H \rightarrow 1} \eta_\rho^* = \int_0^{\frac{\lim_{\pi_H \rightarrow 1} \eta_\rho^* \rho}{\lim_{\pi_H \rightarrow 1} \eta_\rho^* \rho}} \frac{\rho}{\rho + L} f^L(L | R = \rho) dL$. The latter can be satisfied only if $\lim_{\pi \rightarrow 1} \eta_\rho^* \rightarrow 1$. The fact that $\eta_\rho^* = \frac{\rho}{\rho + \lambda_\rho}$, implies that $\lim_{\pi_H \rightarrow 1} \lambda_\rho = \lim_{\pi_H \rightarrow 1} \frac{1 - \eta_\rho^*}{\eta_\rho^*} \rho = 0$. ■

Proof of footnote 16. Recall that γ is defined implicitly by

$$\int_1^\infty N^\gamma e^{\frac{z}{\gamma}(1 - N^\gamma)} dN + \frac{z - 2}{(z - 1)z} + \frac{1}{(z - 1)^2} \ln(z). \quad (9)$$

The p.d.f. is

$$g(N; z) = \begin{cases} \frac{1}{2} \frac{z}{((z - 1)(1 - N) + 1)^2} & \text{if } N < 1 \\ z N^{\gamma - 1} \frac{1}{2} e^{\frac{z}{\gamma}(1 - N^\gamma)} & \text{if } N > 1 \end{cases}.$$

Clearly, $g(N; z)$ is continuous in N . For condition 1 we have

$$\begin{aligned} \int_0^\infty N dG(N; z) &= \int_0^1 N \frac{1}{2} \frac{z}{((z - 1)(1 - N) + 1)^2} dN + z \frac{1}{2} \int_1^\infty N^\gamma e^{\frac{z}{\gamma}(1 - N^\gamma)} dN \\ &= \frac{1}{2} z \frac{z - \ln(z) - 1}{(z - 1)^2} + \frac{z - 2}{(z - 1)} \frac{1}{2} + \frac{1}{2} \frac{z}{(z - 1)^2} \ln(z) = 1, \end{aligned}$$

where the second equality follows from equation (9). For $N < 1$, condition 2 is immediate as $G_z(N; z) = \frac{1}{2} N \frac{N - 1}{(-N - z + Nz)^2} < 0$. For $N > 1$, condition 2 is equivalent to the assumption that for any $z \neq z'$, $G(N; z)$ and $G(N; z')$ do not cross for $N > 1$. To prove this, let us define an auxiliary family $G(N; z, \gamma) \equiv 1 - \frac{1}{2} e^{\frac{z}{\gamma}(1 - N^\gamma)}$. I will show that for any $(z, \gamma) \neq (z', \gamma')$ which satisfy equation (9), $G(N; z, \gamma)$ never crosses $G(N; z', \gamma')$ for $N > 1$. Assume without loss of generality that $z > z'$. Then

$$G(N; z, \gamma) - G(N; z', \gamma') = 0 \Leftrightarrow \frac{1}{\gamma} (1 - N^\gamma) - \frac{z'}{z} \frac{1}{\gamma'} (1 - N^{\gamma'}) = 0.$$

The above is satisfied for $N = 1$. Taking the derivative with respect to N , we obtain

$$\frac{d(G(N; z, \gamma) - G(N; z', \gamma'))}{dN} = -N^{\gamma-1} + \frac{z'}{z} N^{\gamma'-1}, \quad (10)$$

which is negative for all $N > 1$ if $\gamma > \gamma'$. Hence, if $\gamma > \gamma'$, then $G(N; z, \gamma) < G(N; z', \gamma')$. This implies that if the solution to equation (9) is increasing in z , then condition 2 is satisfied. I will show now that $\gamma > \gamma'$. Suppose to the contrary that $\gamma < \gamma'$. Then the derivative (10) is equal to 0 for at most one N . This means that $G(N; z, \gamma) - G(N; z', \gamma')$ has at most one interior extremum. Since $G(N; z, \gamma) - G(N; z', \gamma') = 0$ at $N = 1$, $\frac{d(G(N; z, \gamma) - G(N; z', \gamma'))}{dN} \Big|_{N=1} = -1 + \frac{z'}{z} < 0$, and $\lim_{N \rightarrow \infty} \left(\frac{1}{\gamma} (1 - N^\gamma) - \frac{z'}{z} \frac{1}{\gamma'} (1 - N^{\gamma'}) \right) \rightarrow \infty$, then there exists $\hat{N} > 1$ such that $G(N; z, \gamma) < G(N; z', \gamma')$ if and only if $N \in (1, \hat{N})$. But the last finding implies that compared to $G(N; z', \gamma)$, $G(N; z, \gamma')$ shifts probability to the right, and since for $N < 1$, $G(N, z)$ shifts the probability to the right with respect to $G(N, z')$, \bar{N} must be higher for z than for z' . But this would violate condition 1, which is shown to hold when equation (9) is satisfied; a contradiction.

The left-hand side of Eq. (9) is continuously differentiable with respect to z and γ ; hence γ , is a differentiable function of z almost everywhere. That implies that for any N , $G(N; z)$ is differentiable with respect to z almost everywhere. Condition 3 is straightforward.

■

Proof of Proposition 4. Let R_θ be such that $\frac{R_\theta}{R_\theta + \lambda_{R_\theta}} = \theta$. For a given q_R , the expected utility of the decision maker with parameter θ is

$$\begin{aligned} \text{for } q_R < \theta : \quad U(q_R) &= \pi_H \int_0^\infty (\theta - q_R) g(N; z) dN + \pi_{P_r} \int_0^{\frac{R_\theta}{q_R}} (\theta - q_R) g(N; z) dN + \\ &\quad + \pi_{P_r} \int_{\frac{R_\theta}{q_R}}^\infty (q_R - \theta) g(N; z) dN = (1 - 2\pi_{P_r}) (\theta - q_R) + 2\pi_{P_r} (\theta - q_R) G\left(\frac{R_\theta}{q_R}; z\right); \\ \text{for } q_R > \theta : \quad U(q_R) &= \int_0^{\frac{R_\theta}{q_R}} (\theta - q_R) g(N; z) dN + \int_{\frac{R_\theta}{q_R}}^\infty (q_R - \theta) g(N; z) dN = \\ &= 2(\theta - q_R) G\left(\frac{R_\theta}{q_R}; z\right) + (q_R - \theta). \end{aligned}$$

Let $E[U] \equiv \int_0^\theta U(q_R) dq_R$. The threshold R_θ is such that the decision maker is indifferent between *Right* and *Left*; therefore, $\frac{dE[U]}{dz} = \frac{\partial E[U]}{\partial z} + \frac{\partial E[U]}{\partial R_\theta} \frac{dR_\theta}{dz} = \frac{\partial E[U]}{\partial z}$. Differentiating $E[U]$:

$$\frac{\partial E[U]}{\partial z} = \int_0^\theta 2\pi_{P_r} (\theta - q_R) G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R + \int_\theta^1 2(\theta - q_R) G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R. \quad (11)$$

Notice that assumption 1 on $G(N; z)$ implies that $\int_0^1 G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R = 0$. **Case 1:** If $\frac{R_\theta}{N} < \theta$, the second expression of 11 is positive. Also, $\int_0^{\frac{R_\theta}{N}} G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R + \int_{\frac{R_\theta}{N}}^\theta G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R \geq 0$, $(\theta - q_R)$ is positive for $q_R \leq \theta$ and strictly decreasing in q_R ; therefore, the first expression of 11 is also positive. **Case 2:** If $\frac{R_\theta}{N} > \theta$, the first expression of 11 is positive. Also, $\int_\theta^{\frac{R_\theta}{N}} \pi G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R + \int_{\frac{R_\theta}{N}}^1 G_z\left(\frac{R_\theta}{q_R}; z\right) dq_R < 0$, $(\theta - q_R)$ is negative for $q_R > \theta$ and strictly decreasing in q_R ; therefore, the second expression of 11 is also positive. Full revelation of information in the case of $z = \infty$ is a straightforward result. ■

Proof of Proposition 5. In equilibrium, in any state, P_r sends some report that generates the highest belief from the set of feasible reports, and P_l sends some report that generates the lowest belief from the set of feasible reports. Throughout the proof, I refer to this as property (*). Also, if for some set X of non-zero measure, in equilibrium only H sends reports from X , then for almost all $(\lambda, \rho) \in X$, $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$. I refer to this as property (**).

Step 1 By a similar argument as in the proof of Proposition 1, $0 < \eta(\lambda, \rho) < 1$ for all (λ, ρ) .

Step 2 a) For all ρ , there exists $\lambda_\rho > 0$ such that $\eta(\lambda, \rho) = \eta(0, \rho)$ for all $\lambda \leq \lambda_\rho$. b) For all λ , there exists $\rho_\lambda > 0$ such that $\eta(\lambda, \rho) = \eta(\lambda, 0)$ for all $\rho \leq \rho_\lambda$. I will show that (a) holds for any interval of ρ , which together with continuity of η implies that it holds for any ρ . Assume that part (a) does not hold for an interval of $\rho \in (\rho_1, \rho_2)$, which means that for each such ρ , there exists $\lambda_0 > 0$ such that $\eta(\lambda, \rho)$ is strictly increasing or decreasing in λ for all $\lambda \leq \lambda_0$. But by property (*), in any of these situations, one persuader strictly prefers to send $(0, \rho)$ instead of any (λ, ρ) with $\lambda \leq \lambda_0$, and the other persuader, when sending ρ and $\lambda \in (0, \lambda_0)$ strictly prefers to send as high λ as possible. That is, this persuader will not send $(\lambda \in (0, \lambda_0), \rho)$ unless $L = \lambda$ and $R \geq \rho$; therefore, it must be that for most of the reports with $\lambda \in (0, \lambda_0)$ and $\rho \in (\rho_1, \rho_2)$, $\eta(\lambda, \rho) \geq \frac{\rho}{\rho + \lambda}$. But $\eta(\lambda, \rho) \geq \frac{\rho}{\rho + \lambda} \rightarrow_{\lambda \rightarrow 0} 1 > \eta(0, \rho)$, which contradicts the continuity assumption. A similar argument holds for part (b). For each ρ define λ_ρ to be the highest λ_ρ that satisfies (a) and for each λ define ρ_λ to be the highest ρ_λ satisfying (b). Continuity requires that λ_ρ is a continuous function of ρ and ρ_λ is a continuous function of λ . Define $\eta_\rho^* \equiv \eta(0, \rho)$ and $\eta_\lambda^* = \eta(\lambda, 0)$. Step 2 implies that there exist $\bar{R} > 0$ and $\bar{L} > 0$ such that $\eta_\rho^* = \eta_\lambda^* = \eta(0, 0)$ for all $\rho \leq \bar{R}$ and $\lambda \leq \bar{L}$. Take the highest \bar{R} and \bar{L} that satisfy this property. They exist by an argument virtually identical to the proof of Lemma 2

with $\eta(0, 0)$ used in place of $\bar{\eta}$.

Step 3 a) $\eta(0, \rho)$ is strictly increasing in ρ for all $\rho \geq \bar{R}$. b) $\eta(\lambda, 0)$ is strictly decreasing in λ for all $\lambda \geq \bar{L}$. Assume, that for some $\rho \in [\rho_1, \rho_2]$ with $\rho_1 \geq \bar{R}$, $\eta(0, \rho)$ is strictly decreasing in ρ . This implies that reports with $\rho \in (\rho_1, \rho_2]$ and $\lambda \leq \lambda_\rho$ do not belong to the support of the strategy of P_r , as he strictly prefers to send ρ_1 and $\lambda \leq \lambda_{\rho_1}$ instead (property (*)), and this is feasible when the former are. But for the belief of the decision maker to be constant in λ and decreasing in ρ , it must be that reports with $\rho \in (\rho_1, \rho_2]$ and $\lambda \leq \lambda_\rho$ belong to the support of the strategy of P_l (property (**)). Consider $\rho \in (\rho_1, \rho_2]$ and $(\lambda_\rho + \varepsilon, \rho)$. By definition of λ_ρ , it must be that for any small $\varepsilon > 0$, $\eta(\lambda_\rho + \varepsilon, \rho) \neq \eta(\lambda_\rho, \rho)$. By continuity, for $\rho \in (\rho_1, \rho_2]$, $\eta(\lambda_\rho + \varepsilon, \rho) < \eta(0, \rho_1)$; hence reports $(\lambda_\rho + \varepsilon, \rho)$ do not belong to the support of P_r 's strategy. If $\eta(\lambda_\rho + \varepsilon, \rho) < \eta(\lambda_\rho, \rho)$, then P_l sends (λ, ρ) with $\lambda \leq \lambda_\rho$ only if $L \leq \lambda_\rho$, which in turn implies that it is impossible to have $\eta(\lambda, \rho) = \eta(\lambda_\rho, \rho)$ for all $\lambda \leq \lambda_\rho$. If, however, $\eta(\lambda_\rho + \varepsilon, \rho) > \eta(\lambda_\rho, \rho)$, $(\lambda_\rho + \varepsilon, \rho)$ with $\rho \in (\rho_1, \rho_2]$ and small $\varepsilon > 0$ belong only to the support of H 's strategy, and therefore $\eta(\lambda_\rho + \varepsilon, \rho) = \frac{\rho}{\lambda_\rho + \varepsilon + \rho}$. But that means that $\eta(\lambda_\rho + \varepsilon, \rho)$ is decreasing in ε , and since by continuity $\lim_{\varepsilon \rightarrow 0} \eta(\lambda_\rho + \varepsilon, \rho) = \eta(\lambda_\rho, \rho)$, we have $\eta(\lambda_\rho + \varepsilon, \rho) < \eta(\lambda_\rho, \rho)$, which is a contradiction. Hence, for all $\rho \geq \bar{R}$, $\eta(0, \rho)$ is weakly increasing in ρ , and by definition of \bar{R} , $\eta(0, \rho)$ is strictly increasing for some ρ slightly higher than \bar{R} . This implies that for all $\rho > \bar{R}$, $\eta(0, \rho) > \eta(0, \bar{R})$, which in turn implies that only H and P_r may send reports (λ, ρ) with $\rho > \bar{R}$ and $\lambda \leq \lambda_\rho$. Therefore, by the same argument as in the proof of Proposition 2, for each $\rho \geq \bar{R}$, λ_ρ must satisfy Eq. (2) with $\frac{\pi_H}{\pi_H + \pi_{P_r}}$ in place of π_H , and this λ_ρ is unique. Part (b) of Step 3 is proven using a similar argument, which implies that for each $\lambda \geq \bar{L}$, ρ_λ satisfies Eq. (4), and such ρ_λ is unique. Proposition 2 establishes that $\eta(0, \rho) = \frac{\rho}{\rho + \lambda_\rho}$, and by a similar argument $\eta(\lambda, 0) = \frac{\rho_\lambda}{\rho_\lambda + \lambda}$. This implies that $\eta(0, \bar{R}) = \frac{\bar{R}}{\bar{R} + \lambda_{\bar{R}}}$ and $\eta(\bar{L}, 0) = \frac{\rho_{\bar{L}}}{\bar{L} + \rho_{\bar{L}}}$, and since $\eta(0, \bar{R}) = \eta(\bar{L}, 0)$, we have $\frac{\bar{R}}{\bar{R} + \lambda_{\bar{R}}} = \frac{\rho_{\bar{L}}}{\bar{L} + \rho_{\bar{L}}}$, which is Eq. (6). Denote $\eta^* \equiv \frac{\rho_{\bar{L}}}{\bar{L} + \rho_{\bar{L}}}$. By continuity $\eta(\lambda, 0) = \eta(0, \rho) = \eta^*$ for all $\lambda \leq \bar{L}$ and $\rho \leq \bar{R}$. This completes the proof of part (i) and the first parts of part (iv) and (v).

Assume without loss of generality that $\eta^* < \frac{\bar{R}}{\bar{R} + \bar{L}}$, as in Figure A below. By the fact that $\eta^* = \frac{\rho_{\bar{L}}}{\bar{L} + \rho_{\bar{L}}}$, $\rho_{\bar{L}} < \bar{R}$. By the definition of \bar{R} and \bar{L} , $\eta(\lambda, \rho) = \eta^*$ for all (λ, ρ) with $\lambda \leq \bar{L}$ and $\rho \leq \bar{R}$. Hence, it remains to find the appropriate λ_ρ for all $\rho \in [\rho_{\bar{L}}, \bar{R}]$. I will show that this λ_ρ is such that $\eta(\lambda, \rho) = \eta^*$ for all reports in the shaded region of Figure A, which for this case is exactly $\Psi(\bar{L}, \bar{R})$ described in Eq. (5).

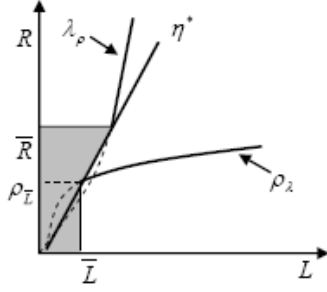


Figure A

Step 4 For all $(\lambda, \rho) \notin \Psi(\bar{L}, \bar{R})$ and $\lambda > \lambda_\rho$ and $\rho > \rho_\lambda$, $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$ (part *iii*). By contradiction, assume, that for some such reports we have $\eta(\lambda, \rho) \neq \frac{\rho}{\lambda + \rho}$. By continuity there is a non-zero measure set $X = (\lambda_1, \lambda_2) \times (\rho_1, \rho_2)$ of such reports, and by property (**), these reports must be in the support of the strategy of at least one persuader. Assume, w.l.o.g., that X is in the strategy of P_r , which requires that $\eta(\lambda, \rho) \geq \eta^*$ if $\rho > \bar{R}$ or $\eta(\lambda, \rho) \geq \eta^*$ if $\rho < \bar{R}$. But this requires that for all $\rho \in (\rho_1, \rho_2)$, $\eta(\lambda, \rho)$ is increasing in λ for some $(\lambda > \lambda_\rho, \rho)$ in a set on non-zero measure. By the same argument as in Step 2 (which uses property (*)), the reports in that set are sent by H only, which implies $\eta(\lambda, \rho) = \frac{\rho}{\lambda + \rho}$, which violates the assumption that $\eta(\lambda, \rho)$ is increasing in λ . The fact $\lambda_\rho = \frac{1 - \eta^*}{\eta^*} \rho$, follows immediately from Step 4, property (**), and the continuity of the belief function.

Step 5 Hence, if $\bar{R} > \bar{L}$, then $\lambda_\rho = \frac{1 - \eta^*}{\eta^*} \rho = \frac{\lambda_{\bar{R}}}{\bar{R}} \rho$, which implies that $\eta(\lambda, \rho) = \eta^*$ for all (λ, ρ) such that $(\lambda, \rho) \in [0, \bar{L}] \times [0, \bar{R}]$ or $\rho \in [\rho_{\bar{L}}, \bar{R}]$ and $\lambda \in [\bar{L}, \frac{\lambda_{\bar{R}}}{\bar{R}} \rho]$. Analogously, if $\bar{R} < \bar{L}$, then $\eta(\lambda, \rho) = \eta^*$ for all (λ, ρ) such that $(\lambda, \rho) \in [0, \bar{L}] \times [0, \bar{R}]$ or $\rho \in [\bar{R}, \rho_{\bar{L}}]$ and $\lambda \in [\frac{\lambda_{\bar{R}}}{\bar{R}} \rho, \bar{L}]$. Hence $\eta(\lambda, \rho) = \eta^*$ for all $(\lambda, \rho) \in \Psi(\bar{L}, \bar{R})$, where $\Psi(\bar{L}, \bar{R})$ is defined in Eq. (5) in section 5. This completes the proof part (*ii*) of this proposition.

Given the strategy of the persuaders, it must be that

$$\begin{aligned} \eta^* &= \frac{\bar{R}}{\bar{R} + \lambda_{\bar{R}}} = \Pr_b(P_r | (\lambda, \rho) \in \Psi(\bar{L}, \bar{R})) E_b \left[\frac{R}{R + L} | (\lambda, \rho) \in \Psi(\bar{L}, \bar{R}), P_r \right] + \\ &\quad + \Pr_b(P_l | (\lambda, \rho) \in \Psi(\bar{L}, \bar{R})) E_b \left[\frac{R}{R + L} | (\lambda, \rho) \in \Psi(\bar{L}, \bar{R}), P_l \right] \\ &\quad + \Pr_b(H | (\lambda, \rho) \in \Psi(\bar{L}, \bar{R})) E_b \left[\frac{R}{R + L} | (\lambda, \rho) \in \Psi(\bar{L}, \bar{R}), H \right], \end{aligned}$$

which by Bayes' rule gives Eq. (7). This equation can be also rewritten as

$$\eta^* = \frac{\pi_{P_r} \int_0^{\bar{R}} \int_0^\infty q_R f dLdR + \pi_{P_l} \int_0^{\bar{L}} \int_0^\infty q_R f dRdL}{\pi_{P_r} \int_0^{\bar{R}} \int_0^\infty f dLdR + \pi_{P_l} \int_0^{\bar{L}} \int_0^\infty f dLdR + \pi_H \left(\int_0^{\frac{\bar{R}}{1-\eta^*}} \int_0^{\frac{(1-\eta^*)R}{\eta^*}} f dLdR + \int_0^{\frac{\eta^* \bar{L}}{1-\eta^*}} \int_0^{\bar{L}} f dLdR \right)} + \frac{\pi_H \left(\int_0^{\frac{\bar{R}}{1-\eta^*}} \int_0^{\frac{(1-\eta^*)R}{\eta^*}} q_R f dLdR + \int_0^{\frac{\eta^* \bar{L}}{1-\eta^*}} \int_0^{\bar{L}} q_R f dLdR \right)}{\pi_{P_r} \int_0^{\bar{R}} \int_0^\infty f dLdR + \pi_{P_l} \int_0^{\bar{L}} \int_0^\infty f dRdL + \pi_H \left(\int_0^{\frac{\bar{R}}{1-\eta^*}} \int_0^{\frac{(1-\eta^*)R}{\eta^*}} f dLdR + \int_0^{\frac{\eta^* \bar{L}}{1-\eta^*}} \int_0^{\bar{L}} f dLdR \right)}$$

The *LHS* of the above equation is continuous, strictly increasing, and between 0 and 1. The *RHS* is continuous, and Eqs. (2) and (4) imply that $\eta^* \rightarrow 0$ only if $\bar{R} \rightarrow 0$ and $\bar{L} \rightarrow 1$; therefore $\lim_{\eta^* \rightarrow 0} RHS = \frac{\int_0^\infty \int_0^{\frac{R}{R+L}} f dLdR}{\int_0^\infty \int_0^\infty f dLdR} > 0$. Similarly, by Eqs. (2) and (4), $\eta^* \rightarrow 1$ only if $\bar{R} \rightarrow 1$ and $\bar{L} \rightarrow 0$; therefore, $\lim_{\eta^* \rightarrow 1} RHS = \frac{\int_0^\infty \int_0^{\frac{R}{R+L}} f dLdR}{\int_0^\infty \int_0^\infty f dLdR} < 1$. This means that there exists η^* that solves the above equation. To show the uniqueness, take the derivative of the *RHS* of the above equation with respect to η^* and evaluate it at the point at which $\eta^* = RHS(\eta^*)$. We have $\frac{dRHS}{d\eta^*} = \frac{\partial RHS}{\partial \eta^*} + \frac{\partial RHS}{\partial \bar{R}} \frac{d\bar{R}}{d\eta^*} + \frac{\partial RHS}{\partial \bar{L}} \frac{d\bar{L}}{d\eta^*}$, and using Eq. (2) and Eq. (4) we can show that for $\eta^* = RHS(\eta^*)$, we have $\frac{\partial RHS}{\partial \eta^*} = 0$, $\frac{\partial RHS}{\partial \bar{R}} = 0$, and $\frac{\partial RHS}{\partial \bar{L}} = 0$. Hence, at any η^* solving the above equation, $\frac{dRHS}{d\eta^*} = 0$, which implies that there is at most one solution. ■

Proof of Proposition 6. Using $\eta^* = \frac{\bar{R}}{\bar{R} + \lambda_{\bar{R}}}$, and combining Eq. (6) and (7), we obtain

$$\pi_{P_r} \int_0^{\bar{R}} \int_0^\infty (q_R - \eta^*) f dLdR + (1 - \pi_{P_r} - \pi_H) \int_0^\infty \int_0^{\bar{L}} (q_R - \eta^*) f dLdR \quad (12) \\ + \pi_H \left(\int_0^{\bar{R}} \int_0^{\bar{L}} (q_R - \eta^*) f dLdR + \int_{\min\{\frac{\eta^* \bar{L}}{1-\eta^*}, \bar{R}\}}^{\max\{\frac{\eta^* \bar{L}}{1-\eta^*}, \bar{R}\}} \int_{\min\{\bar{L}, \frac{(1-\eta^*)R}{\eta^*}\}}^{\max\{\bar{L}, \frac{(1-\eta^*)R}{\eta^*}\}} (q_R - \eta^*) f dLdR \right) = 0.$$

It is easy to show that the above expression is differentiable even at a point at which $\frac{\eta^* \bar{L}}{1-\eta^*} = \bar{R}$. Totally differentiating Eq. (12) with respect to $\bar{L}, \bar{R}, \pi_{P_r}, \pi_{P_l}$ and η^* , using $d\pi_{P_r} = -d\pi_{P_l}$, and using the fact that for $\rho = \bar{R}$ and $\lambda = \bar{L}$ Eqs. (2) and (4) hold, one gets:

$$\frac{d\eta^*}{d\pi_{P_r}} = \frac{\int_0^{\bar{R}} \int_0^\infty (q_R - \eta^*) f dLdR - \int_0^\infty \int_0^{\bar{L}} (q_R - \eta^*) f dLdR}{\pi_{P_r} \int_0^{\bar{R}} \int_0^\infty f dLdR + \pi_{P_l} \int_0^{\bar{L}} \int_0^\infty f dLdR + \pi_H \left(\int_0^{\frac{\bar{R}}{1-\eta^*}} \int_0^{\frac{(1-\eta^*)R}{\eta^*}} f dLdR + \int_0^{\frac{\eta^* \bar{L}}{1-\eta^*}} \int_0^{\bar{L}} f dLdR \right)} < 0.$$

Totally differentiating Eq. (2) with respect to η^*, \bar{L}, \bar{R} and π_{P_r} (recall that *RHS* denotes the

right-hand side of this equation), one gets

$$d\eta^* = \frac{\partial RHS}{\partial \eta^*} d\eta^* + \frac{\partial RHS}{\partial \bar{R}} d\bar{R} + \frac{\partial RHS}{\partial \pi_{P_r}} d\pi_{P_r}.$$

Recall from the proof of Proposition 2 that at the equilibrium η^* , $\frac{\partial RHS}{\partial \eta^*} = 0$, and $\frac{\partial RHS}{\partial \bar{R}} > 0$.

Therefore

$$\frac{\partial RHS}{\partial \bar{R}} \frac{d\bar{R}}{d\pi_{P_r}} = \frac{d\eta^*}{d\pi_{P_r}} - \frac{\int_0^\infty \frac{\bar{R}}{\bar{R}+L} f^L(L|R=\bar{R}) dL - \eta^*}{\pi_{P_r} + \pi_H F^L\left(\frac{1-\eta^*}{\eta^*} \bar{R}|R=\bar{R}\right)}.$$

When $\pi_H = 0$, Eq. (2) implies that the last expression is 0, therefore, we get $\lim_{\pi_H \rightarrow 0} \frac{d\bar{R}}{d\pi_{P_r}} < 0$.

Using similar steps one can show that $\lim_{\pi_H \rightarrow 0} \frac{d\bar{L}}{d\pi_{P_r}} > 0$. ■

Proof of Proposition 7. Let $\eta_\rho^* \equiv \eta(\lambda_\rho, \rho)$ for a continuous equilibrium, and $\eta_\rho^{*d} \equiv \eta(\lambda_\rho^d, \rho)$ for a discontinuous equilibrium, where λ_ρ^d denotes λ_ρ for the discontinuous equilibrium. I will show that for any fixed R , all types of the decision maker are at least as well off in expectation in the continuous equilibrium as in the discontinuous one. Fix some R . In the continuous equilibrium, P_r sends $\rho = R$ and induces η_ρ^* ; in the discontinuous equilibrium he may send $\rho \leq R$, but still induces η_ρ^{*d} . From Lemma 3 in the proof of Proposition 2, $\frac{\rho}{\rho+\lambda_\rho} = \eta_\rho^*$ and $\frac{\rho}{\rho+\lambda_\rho^d} \leq \eta_\rho^{*d}$. For a given $\rho = R$ and η_ρ^{*d} fixed, the decision maker is weakly better off when $\frac{\rho}{\rho+\lambda_\rho} = \eta_\rho^{*d}$ than when $\frac{\rho}{\rho+\lambda_\rho} < \eta_\rho^{*d}$, as in both cases she makes the same decision when facing P_r , but in the latter case she may make worse decisions when facing H , as the number of states in which H pools with P_r is larger in the latter case. Hence, I will assume that $\frac{\rho}{\rho+\lambda_\rho} = \eta_\rho^{*d}$. For each R , there are two cases: 1. $\eta_\rho^{*d} < \eta_\rho^*$, 2. $\eta_\rho^{*d} > \eta_\rho^*$. **Case 1:** $\eta_\rho^{*d} < \eta_\rho^*$. Only a decision maker with $\theta \in (\eta_\rho^{*d}, \eta_\rho^*)$ may choose different alternatives across those equilibria: in the discontinuous equilibrium she chooses *Left* for all L , but in the continuous equilibrium she chooses *Right* when she faces P_r , or she faces H and $\frac{R}{L+R} \geq \theta$. The expected q_R conditional on the events in which she chooses *Right* in the continuous equilibrium is

$$\frac{\pi_{P_r} E[q_R|R=\rho] + \pi_H P(q_R > \theta|R=\rho) E[q_R|q_R > \theta, R=\rho]}{\pi_{P_r} + \pi_H P(q_R > \theta|R=\rho)}. \quad (13)$$

Noting that this is the *RHS* of Eq. (8) but with θ in place of η_ρ^* , and using Corollary (2), we have that since $\theta < \eta_\rho^*$, expression (13) is greater than θ . This means that *Right* is the better choice for this decision maker, which means that she chooses the better alternative in

the continuous equilibrium. **Case 2:** $\eta_\rho^{*d} > \eta_\rho^*$. Only a decision maker with $\theta \in (\eta_\rho^*, \eta_\rho^{*d})$ may choose different alternatives across those equilibria: in the continuous equilibrium she chooses *Left* for all L , but in the discontinuous equilibrium she chooses *Right* when she faces P_r , or she faces H and $\frac{R}{L+R} \geq \theta$. The expected q_R conditional on the events in which she chooses *Right* in the discontinuous equilibrium is the same as in expression (13), but by Corollary (2) this is smaller than θ since $\theta > \eta_\rho^*$. This means that *Left* is the better choice for this decision maker, which means that she again chooses the better alternative in the continuous equilibrium for this R . ■

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