Strategic Argumentation

Wioletta Dziuda[1]
Kellogg School of Management,
Managerial Economics and Decision Sciences,
2001 Sheridan Road, Evanston, IL 60208, USA
wdziuda@princeton.edu

May 8, 2007

[1]I thank Dilip Abreu, Marco Battaglini, Mehmet Ekmekci, Jane Garrison Fortson, Kenneth Fortson, Paolo Ghirardato, Faruk Gul, Navin Kartik, Massimo Marinacci, Marc Martos-Vila, Eric Maskin, Giovanni Mastrobuoni, Stephen Morris, Wolfgang Pesendorfer, Hyun Shin, Jonathan Vogel, and especially my adviser, Markus Brunnermeier, for helpful comments. Seminar participants at Bocconi University, Collegio Carlo Alberto, Harvard University, IESE Business School, INSEAD, Kellogg School of Management, London School of Economics, Princeton University, University of Maryland and Washington University in St. Louis provided further useful comments.

Abstract

I analyze a communication game in which an uninformed decision maker chooses an action based on the advice of an informed but possibly biased expert. The quality of each alternative is described by a set of arguments, and each argument favors one of two alternatives. Each argument is verifiable, but the number of arguments is not. The expert selects a subset of arguments to reveal to the decision maker.

In all equilibria the biased expert exaggerates his reports in favor of his preference, yet he does not suppress all of the unfavorable information. If the expert reports many arguments, the decision maker can infer the expert's potential bias and bases her choice solely on the number of arguments that favor the expert; otherwise the expert's report is ignored. If the decision maker expects the expert to be honest, the biased expert inflates his reports more.

Keywords Strategic communication, persuasion, argumentation, expert, hard evidence

1 Introduction

Consider an investor who consults a financial adviser to help her choose between two investment options. The quality of each investment depends on many characteristics: average predicted returns, risk, correlation between the returns and the investor's income, liquidity, etc. The investor does not know the values of these characteristics, but more importantly, she does not even know what the characteristics are and how many are relevant. For example, she may not know whether a given investment is risky, but she also may not realize that some investments offer tax breaks or differ in liquidity. The financial adviser can credibly reveal any characteristics of the investments. However, since the investor does not know how many characteristics there are, the adviser can conceal some of them. Additionally, the interests of the investor may not coincide with the interests of the adviser. The adviser may be honest, or he may have an agenda. For example, the adviser may always recommend an investment fund that pays him a commission for each persuaded investor.

Many situations share similar features. A patient does not know what factors she should consider when choosing a treatment and she relies on the information provided by her doctor, but the doctor may benefit financially if the patient chooses a particular option. An author of an academic article will not fabricate results, but he can present them selectively. A journalist may omit unfavorable information about his favorite candidate in an election. A lobbyist may hide an unfavorable analysis.

In this paper I formalize these situations in a communication game, focusing on two primary questions. First, I am interested in how much information is transmitted and whether there is room for persuasion when lying is not allowed and information manipulation takes place through selective disclosure. Second, I am interested in the details of communication. Anecdotal evidence suggests that even a biased expert reveals some information that is unfavorable to him. Even if the financial adviser wants the investor to choose a particular option, he is likely to mention some positive characteristics of the alternative option as well. Many commercials use two-sided messages; for example, an advertisement for dBase IV software attempting to persuade consumers of the software's superiority disclosed that it was more costly and poorer at handling errors than the competing products.[1] I am interested in whether

[1] See Pechmann (1992). Other examples include Continental Airlines admitting a variety of past problems such as canceled flights and lost luggage when trying to persuade the clients about its new commitment to quality (Crowley and Hoyer, 1994).

these observations can be explained in a simple, game-theoretical framework.

The existing literature is not well suited to analyze these questions. Information is usually modeled as a single variable, which can either be credibly disclosed[2] or is non-verifiable.[3] My motivating examples fit neither of these settings. The information of a financial adviser is not perfectly verifiable–he cannot prove that he has no arguments besides the ones he has disclosed–but each argument is verifiable. Additionally, to explain the use of mixed arguments by a biased expert I need to model arguments explicitly. And finally, in contrast to a large body of the literature, I want to analyze situations in which the preference of the expert is unknown to the decision maker.[4]

The communication game in this paper has the following structure. There is one expert and one decision maker. The decision maker chooses one of two alternatives–*Right* or *Left*. The quality of the alternatives is described by a set of arguments, each of which favors one alternative. The number of arguments is a random variable and is known to the expert, but is unknown to the decision maker. The expert can credibly reveal any subset of the arguments; that is, he cannot misrepresent any argument, but he can hide some of them. The expert can be either an honest type, who reveals all of the arguments, or a persuader, who wants the decision maker to choose one particular alternative, independent of its quality. The type of the expert is private information.

There are many reasons why experts cannot lie about some types of information. First, it may be illegal to misrepresent the facts. Second, decision makers may be able to verify the information revealed by an expert. On the Internet almost any information is available, and the role of experts is to identify information that is relevant. For example, an investor may not think about looking at tax breaks, but once informed about their existence, she can easily verify whether a given investment offers a tax break. And third, sometimes information takes the form of an argument describing implications of well-known facts. In such a situation, if rational decision

[2] See Grossman and Hart (1979) and Grossman (1981).

[3] See Crawford and Sobel (1982), Krishna and Morgan (2001), Levy and Razin (2006), Battaglini (2002), and Chakraborty and Harbaugh (2005).

[4] A large body of the literature assumes that the preferences of the expert are common knowledge; see, for example, Crawford and Sobel (1982), Krishna and Morgan (2001), and Battaglini (2002). Sobel (1985), Benabou and Laroque (1992), Morris (2001), and Morgan and Stocken (2003) assume uncertainty about the expert's preferences, but allow the bias to be only one-sided. Dimitrakas and Sarafidis (2006) and Li and Madarász (2006) allow for two-sided bias in cheap-talk games. Only Wolinsky (2003) allows for two-sided bias in a game with partially verifiable signals.

makers can evaluate the logical consistency of arguments, experts are constrained to be truthful.

I characterize all equilibria in a version of the game in which the expert can be either honest or biased in favor of *Right* and show that there is a subset of equilibria for which the belief function is continuous. Moreover, across all equilibria in this set the belief function is the same, which implies that all continuous equilibria are outcome equivalent. Apart from the obvious attractiveness of continuity, these equilibria exhibit other appealing properties. First, they are the most informative equilibria of the game. Second, only continuous equilibria are robust to a small cost of concealing information. For most of the analysis I focus on the properties of the continuous equilibria.

The model delivers three main findings. First, in every equilibrium, the persuader biases his reports in favor of his preference, but he does not completely suppress all unfavorable information. In terms of my investor example, a financial adviser who receives a commission from one particular investment fund will reveal all positive aspects of this investment, but will mention some negative ones as well; for example, he may mention its low liquidity. Why is it the case that the expert reveals arguments that seemingly work against his interest? It must be the case that in equilibrium the expert cannot gain by completely suppressing the unfavorable arguments. If he could, he would obviously do that. But then the decision maker would heavily discount one-sided arguments and would take the mixed arguments at face value. In such a situation the biased expert would strictly prefer to disclose at least one opposing argument, which is a contradiction.

This finding is interesting because it confirms the casual observation about the behavior of the experts. But it is also interesting from a purely theoretical perspective. Shin (1994) analyzes a similar problem, but he assumes that the expert's preference is common knowledge and focuses on the equilibrium in which biased experts completely suppress the unfavorable information. In my model the persuader reveals unfavorable arguments even as the probability of facing the honest expert goes to 0, which implies that an equilibrium in which unfavorable information is completely suppressed is not robust to any uncertainty about the expert's preference.

Second, in a situation in which the expert can be either honest or a persuader toward *Right,* the decision maker bases her decision solely on the number of arguments that favor *Right,* unless she observes a balanced report. It is in the interest of both types of experts to reveal all arguments that favor *Right*; and therefore, the decision maker knows that she observes all arguments of this type. It is surprising, however, that she ignores the arguments that favor *Left.* Although these arguments carry some information about the alternatives, they do not carry sufficient information about how many arguments of this sort have been concealed, and therefore, they do not help the decision maker with her decision.

When the expert can be biased in favor of any alternative, the decision maker does not know whether the expert wants to conceal arguments that favor *Right* or the ones that favor *Left.* This may suggest that little information is revealed in equilibrium. However, if the expert uses many arguments, in equilibrium the decision maker is able to infer the potential bias of the persuader; only the honest expert and the persuader toward *Right* report many arguments that favor *Right.* On the other hand, when the expert reveals few arguments, the decision maker cannot infer the direction of the potential bias. She does not know whether she faces a persuader toward *Right,* in which case he is likely to have concealed many arguments that favor *Left,* or she faces a persuader toward *Left,* in which case he is likely to have concealed many arguments that favor *Right.* As a result, when she observes few arguments, she ignores them completely.

In Section 5 I show that the ability of the decision maker to infer information from the expert's report depends crucially on her uncertainty about the choice problem. If the decision maker is familiar with the problem, she has a more precise estimate of how many arguments she should consider. For example, an experienced investor knows more about the complexity of investing than an inexperienced one; therefore, the former will make better decisions after listening to the same expert.

This paper makes three main contributions. First, it complements the existing literature. The papers that are closest to mine are Milgrom (1981), Milgrom and Roberts (1986), and Shin (1994). Milgrom (1981) analyzes a model in which a salesman can credibly disclose attributes of a product, but the number of attributes is common knowledge; this generates full information disclosure. Shin (1994) assumes that experts may have verifiable but imperfect signals about the state of nature, but the decision maker does not know how precise the expert's information is. Unlike me, he assumes that the preference of the expert is common knowledge and allows for signals that convey any information about the state of nature, while the argument structure of my model restricts the signal space considerably. From a modelling perspective, the model in this paper is related to Glazer and Rubinstein (2001) and Glazer and Rubinstein (2004). They model information as a collection of arguments, each of which either can be credibly disclosed or can be verified by the decision maker. They focus on information transmission when there are exogenous constraints on how much information can be revealed.

Second, my paper provides insights into the structure of communication. It formalizes the casual observation that even a biased expert is likely to use arguments that oppose his interests. And third, the model presented here is a good starting point to analyze communication in more elaborate settings. The argument structure of information may be useful when analyzing two-sided communication in debates or bargaining situations.

The paper is organized as follows. In Section 2, I describe the game. In Section 3, I analyze a version of the model in which the expert is either honest or a persuader toward *Right.* In Section 4, I extend the analysis to the case in which the expert can be of three types. In Section 5, I analyze the impact of the uncertainty that the decision maker faces on the equilibrium outcome. Section 6 concludes.

2 The Model

The environment

There are two alternatives: *Right* and *Left.* Information about these alternatives is summarized in a state of nature (L, R). L is the number of arguments that favor *Left* and R is the number of arguments that favor *Right.* The interpretation of arguments follows the interpretation of alternatives. If alternative *Left* is "investing in option *Left*" and alternative *Right* is "investing in option *Right,*" then arguments are the relevant aspects of those investments. For example, an argument that "option *Left* had historically higher returns than option *Right*" favors option *Left.*

The state space is $S = \{(L, R) \in [0, 1] \times [0, 1] : R + L \leq 1\}$: the number of arguments is continuous and the maximum number of arguments is normalized to 1.[5] My motivating examples suggest that the number of arguments is an integer, but for tractability I model it as a continuous variable. If all arguments are equally important, only the number of arguments that favor each alternative matters, not their

[5]This normalization is without loss of generality. All results would still hold for $L + R \in [0, \infty]$ if the same regularity conditions as imposed below hold.

identity, and then assuming that L and R are continuous is without much loss of generality. [6]

The state of nature is distributed according to the probability density function $f(L,R)$ which is continuous with full support. Let $f(L|R)$ be the conditional density of L given R and $f(R|L)$ be the conditional density of R given L. I assume that the following regularity conditions are satisfied:

$$\frac{dF(L|R)}{dR} > 0,\ \frac{dF(R|L)}{dL} > 0. \tag{1}$$

Intuitively, condition 1 says that observing an additional argument that favors one alternative does not make the opposing arguments more likely.[7]

The expert

The expert observes the state of nature (L,R) and sends a report (λ,ρ), where λ is the number of arguments that favor *Left* that the expert reveals, and ρ is the number of arguments that favor *Right* that the expert reveals. Let $\Sigma = S$ denote the set of all reports. A report $(\lambda,\rho) \in \Sigma$ is *feasible* in state (L,R) if $\lambda \leq L$ and $\rho \leq R$. The structure of reporting implies the following: First, the expert must be truthful. Whenever he discloses an argument that favors *Right,* he cannot claim that it favors *Left.* Second, he cannot create arguments. Additionally, the expert cannot credibly convey to the decision maker that he has disclosed all of the arguments.

There are three types of experts. The expert may be biased toward *Right,* P_r, biased toward *Left*, P_l, or honest, H. An honest expert is *non-strategic* and reveals all of the arguments in each state of nature.[8] Biased experts are called *persuaders.* A persuader toward *Right* wants the decision maker to choose *Right,* independent of the state of nature; that is, he maximizes $\Pr\{$*Right* is chosen$\}$. The probability that the expert is of type $i \in \{P_l, P_r, H\}$ is π_i.

The *reporting strategy* of an expert of type i is denoted by $m_i((\lambda,\rho)\,|\,(L,R))$. If the strategy puts positive probability only on a countable set of reports then

[6] Glazer and Rubinstein (2001) show that if the decision maker commits to conditioning her actions on the identities of the arguments, then even if the arguments are ex ante identical, she can extract more information from the expert. In my model such a commitment is infeasible since the decision maker does not know ex ante the identities of the arguments. She cannot commit to choosing *Right* if she receives argument A and *Left* if she receives argument B, because she does not distinguish between arguments A and B, ex ante.

[7] These conditions are sufficient but not necessary.

[8] Later, I show that many of the results hold if the honest expert is strategic and maximizes the utility of the decision maker.

$m_i((\lambda,\rho)\,|\,(L,R))$ is the probability of sending (λ,ρ) when the state is (L,R). If the support of the strategy is an uncountable set, then m_i is a conditional probability density function over feasible reports. A report is *full* if $m_i((L,R)\,|\,(L,R)) = 1$. Let $\sigma_i(L,R)$ denote the set of all reports that lie in the support of the strategy of an expert of type i, that is, $\sigma_i(L,R) = \{(\lambda,\rho) : m_i((\lambda,\rho)\,|\,(L,R)) > 0\}$.

The decision maker

The decision maker does not know the realization of (L,R), but she holds a correct probabilistic belief. Define quality of a given alternative to be the fraction of arguments that favor it, $q_R = \frac{R}{R+L}$ and $q_L = \frac{L}{R+L}$. The decision maker chooses one of the alternatives, and her utility function is:

$$U(Right|L,R) = q_R - \theta_i,\ U(Left|L,R) = q_L + \theta_i - 1 \tag{2}$$

where $\theta_i \in [0,1]$ is a preference parameter. This utility function implies that the decision maker is risk neutral, and chooses alternative *Right* if and only if $E[q_R|\lambda,\rho] \geq \theta_i$. She cares only about the quality of each alternative; the total number of arguments does not enter into her utility function directly because it does not carry any relevant information. Keeping the quality constant, choosing alternative *Right* when the total number of arguments is high does not result in a different utility level than when the total number of arguments is low.

The parameter θ_i describes an ex ante preference of the decision maker. For example, the decision maker may have some intrinsic preference for Honda over Toyota when buying a car; an investor may prefer stocks of environmentally friendly companies; a voter may prefer a Republican candidate because of family tradition, other things being equal. In other words, θ_i is the smallest quality of alternative *Right* that will make the decision maker choose *Right*. $\theta_i = \frac{1}{2}$ means that the decision maker has no prior preference bias.

The decision maker knows her θ_i, but the expert does not. This assumption makes the model more interesting and realistic because it implies that the expert does not know exactly when the decision maker prefers *Right*. Let $h(\theta_i)$ be the probability density function of θ_i with the corresponding distribution function $H(\theta_i)$. $h(\theta_i)$ is continuous and has full support over $[0,1]$.

The game

The triangle in Figure 1 represents the state space S and the report space Σ.

Define $Z(\lambda, \rho) = \{(L, R) \in S : L \geq \lambda \text{ and } R \geq \rho\}$ and let $V(L, R)$ be the set of all feasible reports in state (L, R). If the state of nature is (L_0, R_0), then the shaded region $V(L_0, \rho_0)$ is the set of all feasible reports. The shaded region $Z(L_0, R_0)$ is the set of all states of nature that allow the expert to send a report (L_0, R_0).

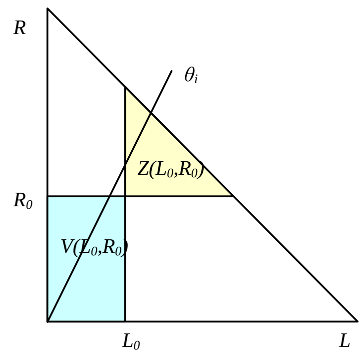


Figure 1: The state space. Each point in the triangle represents a state of nature. The ray from the origin represents the states in which $q_R = \theta_i$.

The line θ_i represents the states of nature that generate the same quality of alternative *Right,* $q_R = \theta_i$. The decision maker of type θ_i prefers to choose alternative *Right* if the state of nature lies above the θ_i line and alternative *Left* otherwise.

The game proceeds as follows. First, nature determines the type of the expert $i \in \{P_l, P_r, H\}$ and the set of arguments (L, R). The expert observes his type and the state of nature (L, R) and sends a report (λ, ρ) to the decision maker. After observing the report, the decision maker chooses one of the alternatives.

I look for a perfect Bayesian equilibrium of this game. The decision maker in this game has a very limited role: she chooses *Right* if her beliefs are above θ_i. Given that, from the perspective of the expert the probability that the decision maker chooses *Right* is a strictly increasing function of the belief induced by the strategy of the expert. Therefore, one can transform this game into a game with one player only in which the expert of type i maximizes the belief of the decision maker about q_i, where the belief is formed using Bayes' rule whenever possible. Let $\eta(\lambda, \rho)$ be the equilibrium belief of the decision maker about q_R if she observes a report (λ, ρ). A perfect Bayesian equilibrium is characterized by m_i for $i \in \{P_l, P_r\}$ and $\eta(\lambda, \rho)$ such that

1. m_i satisfies

$$\int_0^R \int_0^L m_i \left((\lambda, \rho) \,|\, (L, R) \right) d\lambda d\rho = 1 \text{ for all } (L, R) \in S,$$

2. if (λ^*, ρ^*) is in the support of $m_{P_r} \left(\cdot | \, (L, R) \right)$, then (λ^*, ρ^*) solves

$$\max_{(\lambda,\rho) \in V(L,R)} \eta \left(\lambda, \rho \right),$$

and if (λ^*, ρ^*) is in the support of $m_{P_l} \left(\cdot | \, (L, R) \right)$, then (λ^*, ρ^*) solves

$$\min_{(\lambda,\rho) \in V(L,R)} \eta \left(\lambda, \rho \right),$$

3. $\eta \left(\lambda, \rho \right)$ is derived using Bayes' rule whenever possible.

3 One-sided Bias

In this section, I consider a situation in which the decision maker knows the direction of the potential bias of the expert. Let π be the probability that the expert is biased toward *Right,* i.e., is of type P_r, and $1-\pi$ be the probability that the expert is honest, i.e., is of type H.

Often, the decision maker knows which alternative the expert favors. A sales representative may honestly advise the customer about the quality of his product, but certainly he is not interested in increasing the sales of competing products. A lawmaker may propose a particular policy because it is beneficial to his constituency, but in any case he prefers this policy to be adopted rather than rejected. The financial adviser may be honest, but if he is known to receive a higher commission from a specific investment fund for each persuaded investor, the investor will be suspicious whenever the adviser recommends this fund. I analyze the case with three types of the expert in Section 4.

3.1 The properties of the equilibria

Proposition 1 states that there are many equilibria in this game, which is a typical feature of communication games. Based on the continuity of the equilibrium belief function, $\eta(\lambda, \rho)$, all equilibria can be divided into two groups. Proposition 1 describes common properties of all equilibria and provides the details of the set of continuous equilibria.

Proposition 1

a) *There are infinitely many equilibria in this game. There is no fully revealing equilibrium and no babbling equilibrium. The set of equilibria does not depend on the distribution of the decision maker's preference* θ_i.

b) *In each equilibrium, for all* ρ, *there exists* $\lambda_\rho > 0$ *such that* $\eta(\rho, \lambda)$ *is constant for all* $\lambda \in [0, \lambda_\rho]$. *This implies that the persuader is indifferent between suppressing the unfavorable information completely and revealing any small number of unfavorable arguments. The belief function is weakly increasing in* ρ.

c) *There is a unique equilibrium belief function* $\eta(\lambda, \rho)$ *that is continuous in reports* (λ, ρ). *All equilibria characterized by this function are outcome equivalent. In any such equilibrium* $\eta(\lambda, \rho)$ *is strictly increasing in* ρ, *and for each* R', *there exists* $\lambda_{R'} > 0$, *defined by*

$$\eta^*_{R'} \equiv \frac{R'}{R' + \lambda_{R'}} = \Pr(H|\lambda \leq \lambda_{R'}, \rho = R')\, E[q_R|L \leq \lambda_{R'}, R = R'] + \Pr(P_r|\lambda \leq \lambda_{R'}, \rho = R')\, E[q_R|R = R'] \quad (3)$$

and

i. P_r *reveals all arguments that favor* Right, $\rho = R'$,

ii. P_r *reveals a subset of arguments that favor* Left, $\lambda \leq \min\{L, \lambda_{R'}\}$, *using a strategy that results in* $\eta(\lambda, R') = \eta^*_{R'}$ *for all* $\lambda \leq \lambda_{R'}$.

Proof The proof of this proposition and a more detailed description of the discontinuous equilibria are in the Appendix. ■

Part (a) of Proposition 1 is straightforward. A fully revealing equilibrium cannot exist because a message sent in a state associated with high q_R and few arguments is feasible in a state associated with low q_R and many arguments, and this would be exploited by the persuader. A babbling equilibrium cannot exist because by revealing all arguments in states in which $L+R=1$ the persuader can induce the correct belief, and he would use this opportunity in a babbling equilibrium if R is high enough. The equilibria do not depend on the distribution of the decision maker's preference as long as this distribution has full support because each decision maker forms the same belief independent of her type, which implies that the persuader simply maximizes the induced belief.[9]

Proposition 1 says that all equilibria have the following two properties. First, the strategy of the persuader includes revealing arguments unfavorable to him, a property predicted by casual observation. In terms of the motivating example, a financial adviser who is biased toward investment *Right,* while saying that this investment has high returns and is relatively less risky, will mention its low liquidity. Second, when the expert reveals only a small number of unfavorable arguments, the decision maker forms her beliefs using only the number of arguments that favor *Right* that have been revealed to her. This is an interesting finding given that the arguments that favor *Left* carry some information about each alternative. However, because the persuader does not necessarily reveal all arguments that favor *Left,* the decision maker does not know how many arguments of this type are concealed from her, and therefore, λ does not help her in the decision making.

Proposition 1 says that there is a unique equilibrium belief function that is continuous in reports. Given that the persuader always tries to induce the highest possible belief, all continuous equilibria are outcome equivalent. Figure 2 represents all of these equilibria. The triangle represents all states of nature, S, and all reports, Σ, at the same time. The white area, which I call the *revealing area*, represents reports that are used in equilibrium only by H. The shaded region, which I call the *ambiguity area,* includes all reports used by P_r. Hence, the ambiguity area includes all reports that do not allow the decision maker to identify the type of the expert. The boundary of the ambiguity area is determined by λ_R defined by equation (3).

In any continuous equilibrium, the persuader reveals all of the arguments that favor *Right* and *some* arguments that favor *Left.* The highest number of arguments that favor *Left* that the persuader reveals for any R is λ_R. After observing a report from the revealing area, the decision maker knows that the expert has revealed all arguments, and she believes $\eta(\lambda,\rho)=\frac{\rho}{\rho+\lambda}$. After observing a report from the ambiguity

[9] If $h(\theta)$ is degenerate, the set of equilibria becomes larger as there are states in which the strategy of the expert and the beliefs of the decision maker do not matter.

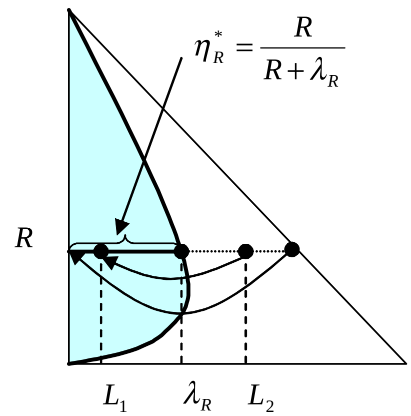


Figure 2: The details of the continuous equilibria. The shaded region represents the ambiguity area.

area, she forms her belief based only on $\rho : \eta(\lambda, \rho) = \frac{\rho}{\rho + \lambda_\rho}$. Given this belief function, the persuader is indifferent between sending any report (λ, ρ) such that $\rho = R$ and $\lambda \leq \lambda_R$, however, in equilibrium he must use a strategy that supports the decision maker's beliefs.

There are many strategies of the persuader that support the continuous belief function, but they must satisfy the following properties: A smaller number of arguments that favor *Left* implies a higher quality of alternative *Right* if the expert happens to report fully. For the decision maker to hold the same belief independent of λ, this effect must be offset by the strategy of the persuader. Hence, it must be that a smaller number of arguments that favor *Left* implies that either more arguments of this type are concealed from the decision maker if the expert is not reporting fully, or it is more likely that the expert is a persuader. One such equilibrium strategy is represented in Figure 2, where the arrows connect a state with a message. It is important to stress that there is no equilibrium with a monotonic strategy of the expert.

The complete proof of Proposition 1 is in the Appendix, but I provide below the intuition for the shape of this equilibrium. First, in any equilibrium, for each ρ, there must exist some λ_ρ such that the decision maker holds the same belief for any report of a form $(\lambda \leq \lambda_\rho, \rho)$. The reason for this is the following. Assume that for some ρ

the belief is strictly decreasing in λ, as shown in the first triangle in Figure 3.[10] In such a case P_r never sends any reports of a form $(\lambda \in (0, \varepsilon), \rho)$ (any reports that lie on the arrow); he prefers to send $(0, \rho)$ instead. Therefore, the belief that the decision maker forms when she sees $(0, \rho)$ must be strictly smaller than 1. This also implies that when the decision maker sees a report (ε, ρ), she knows that it was sent by the honest expert and she believes $\eta(\varepsilon, \rho) = \frac{\rho}{\rho+\varepsilon}$. But as $\varepsilon \to 0$, we have $\eta(\varepsilon, \rho) \to 1$, and that contradicts the finding that $\eta(\varepsilon, \rho) < \eta(0, \rho) < 1$. Alternatively, the belief may be strictly increasing in λ, as shown in the triangle on the right. In such a case, the persuader, when sending reports of a form $(\lambda \in [0, \varepsilon), \rho)$ (from the arrow), sends the highest λ possible. In particular, he may send (ε, ρ) only in the states of nature that lie on the thick dashed line: when (L, R) are such that $\frac{R}{R+L} \geq \frac{\rho}{\rho+\varepsilon}$. That implies that $\eta(\varepsilon, \rho) \geq \frac{\rho}{\rho+\varepsilon}$. But then, for ε small enough, the belief must be arbitrarily close to 1, and that contradicts the assumption that the belief is strictly increasing in λ because the belief can never be greater than 1.

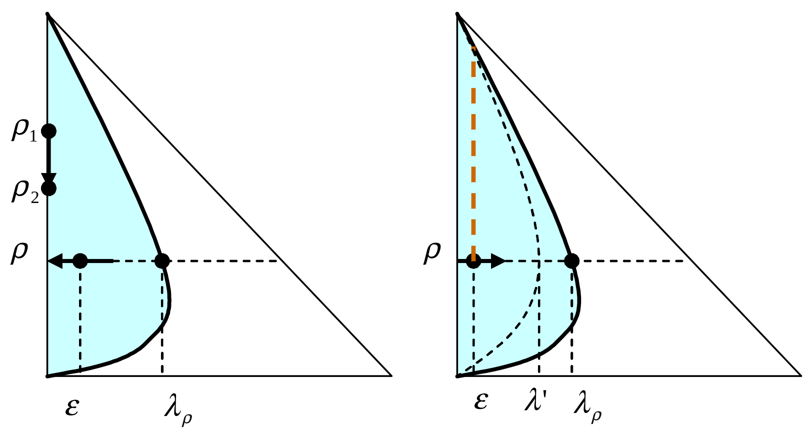


Figure 3: The shape of the equilibrium.

Now we can show that $\eta(0, \rho)$ is strictly increasing in ρ. Assume that $\eta(0, \rho)$ is strictly decreasing in ρ. That is, there exist ρ_1 and $\rho_2 < \rho_1$ such that $\eta(0, \rho_1) < \eta(0, \rho_2)$. But then only the honest type sends $(0, \rho_1)$ which implies that $\eta(0, \rho_1) = 1$, which is a contradiction. The strict monotonicity of $\eta(0, \rho)$ is a feature of the continuous equilibria only and comes from the regularity condition 1.

[10] Although the argument presented here assumes a certain degree of continuity of beliefs, the formal proof presented in the Appendix does not.

Given that $\eta(0,\rho)$ is strictly increasing, the persuader will always send $\rho = R$ and will be indifferent between sending any $\lambda \leq \lambda_R$. For the equilibrium to be continuous, λ_R must be such that for each R, $\eta(0,R) = \frac{R}{R+\lambda_R}$. In the Appendix I show that such λ_R is unique for each R.

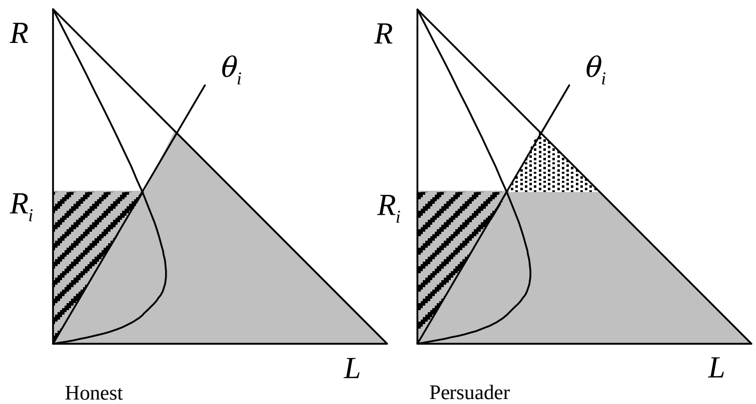


Figure 4: The behavior of the decision maker. The triangles represent the choice of the decision maker given the state of nature and given that the expert happens to be an honest type or a persuader, respectively.

Figure 4 shows the behavior of the decision maker of type θ_i in any continuous equilibrium. The first triangle represents her decision as a function of state if she happens to face the honest expert, and the second triangle represents her decisions if she happens to face the persuader. The decision maker prefers alternative *Right* in the states that lie above the θ_i line and prefers alternative *Left* otherwise. R_i is the number of arguments that favor *Right* for which $\eta^*_{R_i} = \theta_i$. This means that if the decision maker observes R_i, she is indifferent between the alternatives. Seeing a report from the ambiguity area, the decision maker chooses *Right* if $R \geq R_i$ and *Left* if $R < R_i$. If she observes a report from the revealing area, she chooses *Right* if the report lies above the θ_i line.

In Figure 4 the shaded areas represent the states in which the decision maker chooses *Left*. When the number of arguments that favor *Right* is low enough, the decision maker chooses *Left* even if she receives an extreme report. The states in which *Right* is optimal but *Left* is chosen are represented by the striped area, which I call a *skeptic mistake.* The decision maker is skeptical about the quality of *Right* if the expert claims that it is high, but provides few arguments to support his claim. It is interesting to note that in the states of nature from the striped area every player of the game prefers the decision maker to choose alternative *Right,* but she nevertheless chooses *Left* in equilibrium. The dotted area represents *successful persuasion;* that is, the states in which *Left* is actually better but the decision maker nonetheless is persuaded to choose *Right*.

3.2 Robustness

Among all equilibria, the set of equilibria with the continuous belief function stands out. First, all continuous equilibria are outcome equivalent, that is, they are characterized by the same belief function and by the same choice of the decision maker in each state of nature; they differ only in the details of the strategy of the expert. The same is not true about the set of discontinuous equilibria. Second, the proof of Proposition 1 reveals that all discontinuous equilibria require the persuader to play a rather elaborate strategy even in the states of nature in which he is indifferent between doing that and revealing all of the arguments. This feature of the discontinuous equilibria is not very attractive; it seems natural to expect that an indifferent persuader would reveal all of the arguments. We can expect that revealing all of the arguments is easier or cheaper, for example, this strategy may require a smaller mental effort than constructing an elaborate one, or there may be a fixed legal or reputation cost of concealing information, or the expert may experience guilt if he conceals information (see, for example, Gneezy 2005).

Proposition 2 says that if we perturb the game by adding a small fixed cost, $\xi > 0$, of sending a report that is not full, measured in terms of utility, then there is a unique equilibrium belief function in the perturbed game, and it converges to a continuous belief function in the original game.[11,12] Let Γ be the original game and $\Gamma(\xi)$ the perturbed game.[13]

[11] One may argue that the cost should depend on the number of arguments that are being concealed. The perturbed game with variable cost is more complicated to analyze because any equilibrium in the pertubed game depends critically on the shape of the cost function. I conjecture, however, that if the variable cost is concave, there is a unique equilibrium in the perturbed game, and it converges to the same equilibrium. The same is not true, however, for a convex cost.

[12] Kartik (2005) introduces an increasing cost of lying in a Crawford and Sobel (1982) type game and shows that only the most informative equilibrium can be a limit of monotone equilibria as the cost goes to zero. Kartik, Ottaviani and Squintani (2006) show that an increasing cost of lying when the state space is unbounded leads to a fully revealing equilibrium.

[13] Many economists have proposed refinements that restrict the set of equilibria in communication games (Farrell 1993, Sobel 1985, Rabin 1990, Matthews, Okuno-Fujiwara and Postlewaite 1991,

Proposition 2 *The perturbed game* $\Gamma(\xi)$ *has a unique equilibrium belief function, and as* $\xi \to 0$*, it converges to the continuous equilibrium belief function with the properties described in Proposition 1, and with* $m_{P_r}((L,R) \mid (L,R)) = 1$ *for all* $L \leq \lambda_R$.

Proof In the Appendix. ■

In the rest of the paper I focus on the continuous equilibria. First, the only equilibrium that survives the introduction of a small cost of concealing information is continuous. Second, since in every discontinuous equilibrium we can find two reports arbitrarily close to each other that generate very different beliefs, it seems that any discontinuous equilibrium requires a lot of rationality on the part of the decision maker who has to observe and interpret the reports perfectly. Based on previous research one can expect that any noise in sending or interpreting the arguments should exclude the discontinuous equilibria.[14] Finally, focusing only on the continuous equilibria does not result in significant loss of generality since all discontinuous equilibria display many qualitative features of the continuous equilibria, as seen in Proposition 1. In all equilibria the persuader biases his reports, but does not suppress all of the unfavorable arguments, and the belief in the ambiguity area is a function of the arguments that favor *Right* only.

Proposition 3 establishes another attractive feature of the continuous equilibria.

Proposition 3 *All continuous equilibria are equally informative, and they are the most informative equilibria of the unperturbed game (measured by the decision maker's ex ante utility).*

Proof In the Appendix. ■

All continuous equilibria are equally informative because they are outcome equivalent. One can understand the intuition for the latter result by investigating the main difference between every discontinuous and every continuous equilibrium. Figure 5 shows an example of a discontinuous equilibrium. All other equilibria differ mainly in the number, size, and location of the shaded trapezoids, but the trapezoids must be such that the boundary of the ambiguity area for the continuous equilibrium must intersect the side of each trapezoid. All reports lying in each shaded trapezoid induce

Blume 1994).

[14] See, for example, Carlsson and van Damme (1993) and Battaglini (2002).

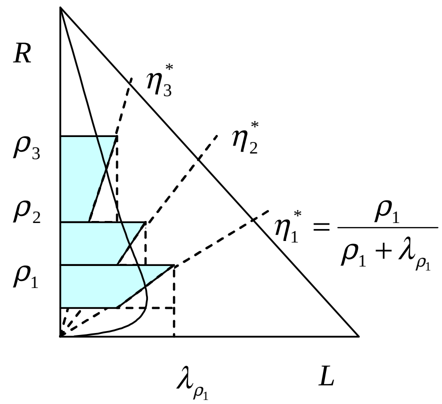


Figure 5: An example of a discontinuous equilibrium. Each shaded trapezoid consists of reports that induce the same belief. The curve represents a continuous equilibrium.

the same belief.[15] In all equilibria of this game, the belief given a report from the ambiguity area depends only on the number of revealed arguments that favor *Right*. Figure 5 shows that η_ρ^* is not strictly increasing in ρ in any discontinuous equilibrium. In the continuous equilibria each ρ carries different information, while in any discontinuous equilibrium some ρs carry the same information; therefore, more information is revealed in the continuous equilibria.

3.3 Benevolent expert

So far, I have assumed that the honest expert reveals all of the arguments. Alternatively, the honest expert may want to maximize the utility of the decision maker, i.e., he may be benevolent. One can easily see, however, that any continuous equilibrium of the original game is still an equilibrium of a game with a benevolent type of the expert, and the benevolent expert behaves like an honest expert. To see this, note that if the state of nature lies in the revealing area, the benevolent expert cannot do better than to report fully, because in this way he induces the correct belief. If the state of nature lies in the ambiguity area, the benevolent expert would like to induce a higher belief than the one induced in equilibrium, but there is no feasible report that can achieve that; therefore, again, full reporting is optimal.

[15] Focus on one trapezoid from Figure 5. In an equilibrium it may be that also some reports from the rectangle that completes this trapezoid can generate the same belief as the one generated by the reports from the trapezoid or a trapezoid above.

The proposition below provides an even stronger result; it says that the continuous belief function is the unique limit of the equilibrium belief function in the perturbed game with the strategic, benevolent expert.

Proposition 4 *Let the expert be either a persuader or a benevolent expert who maximizes the utility of the decision maker. Consider a perturbed game* $\Gamma'(\xi, \epsilon, B)$*, where* ξ *is a fixed cost of concealing information and* ϵ *is a fraction of honest experts. There exists a unique equilibrium belief function in the perturbed game, and it converges to the continuous belief function of the original game.*

Proof In the Appendix. ■

4 Two-sided Bias

In this section, I consider a situation in which the expert can be biased toward either alternative. The expert can be P_r, P_l, or H with probabilities π_r, π_l, and π_H, respectively.

Sometimes the decision maker is not only uncertain whether the expert is honest, but she also does not know the potential bias of the persuader. A financial adviser may give honest advice, or he may have interest in promoting a particular investment fund, but the decision maker may not know which investment fund offers the adviser the highest commission. A scientist publishing a comparison of the performance of two drugs may be honest or biased, and the reader may not know which pharmaceutical company funded the research.

We have seen in the previous section that when the expert is either honest or biased toward *Right*, the decision maker knows that all arguments that favor *Right* are revealed and uses those arguments to form her beliefs. Unless the expert reveals himself to be honest, she disregards the arguments that favor *Left* completely. When the expert can be biased in either direction, the decision maker cannot use the same logic; therefore, we can expect that much less information is revealed. This is only partially true, however. Proposition 5 describes the unique continuous equilibrium outcome. In equilibrium, the persuader toward *Right* and the persuader toward *Left* separate themselves if they happen to receive many arguments that favor their preferred alternatives. In these states, the decision maker can use the same skeptical approach as in the one-sided case to infer information.

Proposition 5 *There is a unique equilibrium belief function that is continuous.*[16] *Any continuous equilibrium is characterized by the same parameters* $\bar{R}, \bar{L}$ *and by the functions* λ_R, ρ_L*, such that*

i. for all (L, R) *such that* $R \geq \bar{R}$*,* P_r *reveals all arguments that favor* Right*, and reveals a subset of arguments that favor* Left*:* $\lambda \leq \min\{L, \lambda_R\}$*,*

ii. for all (L, R) *such that* $L \geq \bar{L}$*,* P_l *reveals all arguments that favor* Left*, and reveals a subset of arguments that favor* Right*:* $\rho \leq \min\{R, \rho_L\}$*,*

iii. there exists a double ambiguity area such that when $R < \bar{R}$*,* P_r *sends reports from this area only, and when* $L < \bar{L}$*,* P_l *sends reports from this area only. The belief is constant for all reports in the double ambiguity area,*

iv. λ_R *and* ρ_L *solve the following equations:*

$$\eta^*_{R'} \equiv \frac{R'}{R' + \lambda_{R'}} = \Pr(H|\rho = R', \lambda \leq \lambda_{R'}) E[q_R|R = R', L \leq \lambda_{R'}] + \Pr(P_r|\rho = R', \lambda \leq \lambda_{R'}) E[q_R|R = R'], \quad (4)$$

$$\eta^*_{L'} \equiv \frac{\rho_{L'}}{\rho_{L'} + L'} = \Pr(H|\lambda = L', \rho \leq \rho_{L'}) E[q_R|L = L', R \leq \rho_{L'}] + \Pr(P_l|\lambda = L', \rho \leq \rho_{L'}) E[q_R|L = L']. \quad (5)$$

Proof in the Appendix. ■

Figure 6 represents a continuous equilibrium for symmetric $f(L, R)$ and for $\pi_l = \pi_r$. In this equilibrium $\bar{R} = \bar{L}$ and $\eta(\bar{L}, \bar{R}) = \frac{1}{2}$. The striped area represents the ambiguity area for P_r, and the dotted area represents the ambiguity area for P_l. The ambiguity area for P_r contains all reports used by P_r and H only, while the ambiguity area for P_l contains all reports used by P_l and H only. The shaded square represents the set of reports that in equilibrium are used by all three types of the expert, the *double ambiguity area.*

As before, each type of persuader biases his reports; therefore, reports that consist of many relatively balanced arguments are sent only by the honest expert. Because each persuader biases the report toward his preferred alternative, only H and P_r

[16] In fact, this is the only equilibrium belief function that is robust to a small cost of concealing information, as in Proposition 1. The proof of this is available upon request.

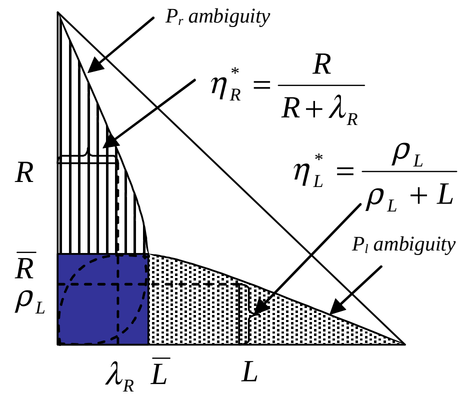


Figure 6: Two-sided bias.

reveal many arguments that favor *Right,* and only H and P_l reveal many arguments that favor *Left.* Hence, reports that consist of many arguments reveal the potential bias of the expert. After observing many arguments that favor *Right* (reports from P_r's ambiguity area), the decision maker knows that she does not face P_l, and hence, she knows that all arguments that favor *Right* have been revealed to her. Similarly, when she observes many arguments that favor *Left* (reports from P_l ambiguity area), she knows that she does not face P_r, and therefore, she knows that all arguments that favor *Left* have been revealed to her.

When the expert supports his recommendation with few arguments only, the decision maker ignores what the expert says. The intuition behind this finding is the following. If many arguments are revealed, the decision maker expects that the expert has hidden only few arguments. So even if the expert reveals himself to be biased toward *Right*, if he reveals many supporting arguments, the decision maker is still likely to choose *Right.* On the other hand, if only few arguments are revealed, it is more likely that the expert has concealed a lot of arguments. Now, if the expert reveals himself to be biased toward *Right*, the decision maker would rather choose *Left.* This is why the experts have no incentive to separate themselves in equilibrium when they have few favorable arguments. Therefore, for small R and L the equilibrium resembles a pure babbling equilibrium.

5 Comparative statics

In this section, I analyze how the parameters of the model, such as the probabilities of different types of the expert and the prior distribution of arguments, affect the equilibrium.

5.1 Varying the probability of facing the persuader

This section analyzes how changes in the probability of facing the persuader affect the agents' utilities and, more generally, the whole equilibrium. First, I look at what happens when the fraction of honest experts becomes negligible and what happens when the expert is honest with probability of almost 1. Second, I analyze how the probability of facing a particular type of the persuader impacts the bias of his reports and the probability of persuading the decision maker.

Since there are three types of the expert, it is necessary to specify how the remaining probabilities change when the probability of facing the expert of type i changes. In the proposition below, I vary the probability of P_r and keep the conditional probability of facing the honest type given that the expert is not P_r constant. In such a case the shape of the ambiguity area for P_l remains the same.

Proposition 6 *In every continuous equilibrium,* $\lim_{\pi_H \to 1} \lambda_R = 0$, $\lim_{\pi_H \to 1} \rho_L = 0$, *and* $\lim_{\pi_H \to 0} \lambda_R = \bar{\lambda}_R$, $\lim_{\pi_H \to 0} \rho_L = \bar{\rho}_L$, *where* $\bar{\lambda}_R$ *and* $\bar{\rho}_L$ *are such that* $\frac{R}{R+\bar{\lambda}_R} = E\left[q_R|R\right]$ *and* $\frac{\bar{\rho}_L}{\bar{\rho}_L+L} = E\left[q_R|L\right]$. *Keeping* $\frac{\pi_H}{1-\pi_r}$ *constant, as the probability of facing* P_r *decreases*

i. the reports of P_r *become more extreme,*

ii. the utility of P_r *increases,*

iii. the utility of P_l *decreases,*

iv. the expected utility of the decision maker increases.

Proof In the Appendix. ■

Proposition 6 states that as the probability of facing the honest expert increases, the ambiguity areas for both persuaders disappear, and the equilibrium converges to a fully informative equilibrium. On the other hand, as the probability of facing the honest expert goes down, λ_R converges to $\bar{\lambda}_R < 1 - R$ and ρ_L converges to $\bar{\rho}_L < 1 - L$. That means that the ambiguity areas are always strict subsets of Σ and that the equilibrium never becomes a pure babbling equilibrium. The assumption that arguments are verifiable prevents equilibria from becoming completely uninformative. It is worth noting that even when $\pi_H = 0$, revealing unfavorable information is a part of some equilibria.

Figure 7 shows how the equilibrium changes as π_r decreases. π_1 and $\pi_0 > \pi_1$ are two different probabilities of facing P_r. The thick curves represent the initial equilibrium in which $\pi_r = \pi_0 = \pi_l$. Since the conditional probability of facing H is kept constant, the shape of the ambiguity area for P_l remains unchanged as π_r changes. As $\pi_r \to 0$, the ambiguity area for P_r becomes smaller, as represented by the thinner curve. The double ambiguity area for $\pi_r = \pi_1$ is the shaded region.

When π_r decreases, the reports of P_r become more extreme. The reason for this is the following. Sending a report that is not extreme works in two opposite directions. If the report comes from H, a less extreme report implies a lower q_R, hence it is less likely that the decision maker will choose *Right*. On the other hand, if the report comes from P_r, a less extreme report implies that P_r has concealed fewer arguments that favor *Left*, and therefore, the more likely it is that the decision maker will choose *Right.* As π_r decreases, the first effect dominates, and P_r has a higher incentive to bias his reports. In terms of my motivating example this result says that the financial advisor biased toward an investment option that is not very popular among other advisers will not use very many arguments that work against this option.

From Figure 7 one can see that when the decision maker faces P_r, she chooses *Right* more often when π_r is low (a decision maker with θ_i chooses *Right* whenever $R \geq R_i^{\pi_0}$ for π_0 and whenever $R \geq R_i^{\pi_1}$ for π_1). Therefore, the utility of the persuader toward *Right* increases. If the expert happens to be P_l, the decision maker chooses *Right* more often, which decreases the utility of the persuader toward *Left.* However, the utility of the decision maker increases because she is more likely to face the honest expert.

Proposition 6 implies that a financial adviser biased toward a stock that is unpopular among other advisers is better at persuading the investor, while a financial adviser biased toward a popular stock is unlikely to be successful.

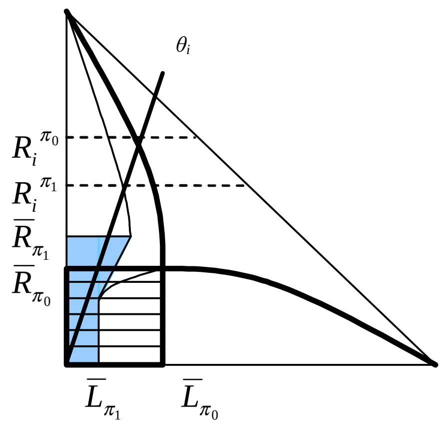


Figure 7: The effect of decreasing π_r, keeping $\frac{\pi_H}{1-\pi_r}$ constant.

5.2 Varying the familiarity of the problem

In this section, I analyze how the prior distribution of arguments affects the utility of the decision maker in cases in which the potential bias of the persuader is known, i.e., when the expert can be either P_r or H.

From the probability density function of the arguments, $f(L, R)$, one can derive the probability density function of the total number of signals $g(N)$, where $N \equiv L + R$. The prior distribution of N reflects the decision maker's knowledge about the choice problem. It describes how the total number of arguments varies from situation to situation for the same decision problem. For example, in each election campaign a different number of issues is important, which in the model can be represented by a relatively dispersed prior belief over N. Other choice problems are likely to be characterized by roughly the same number of arguments every time the decision maker faces them, for example, choosing an investment option or buying a car, and this is captured by a distribution of N concentrated around the mean. Alternatively, the prior distribution of N may describe the decision maker's knowledge about the problem. An investor with a dispersed distribution of N knows little about the nature of the problem, while an experienced or educated investor is likely to have a concentrated distribution of N.

To isolate the effect of changing the distribution of N while keeping the distribution of the quality of the alternatives unchanged I reformulate the problem in terms of $(q_R, N) \equiv \left(\frac{R}{R+L}, L+R\right)$, and assume that q_R is uniformly distributed and independent of N. This implies that the joint density of q_R and N is equal to the density of $N: g(q_R, N; z) = g(N; z)$. Let $g(N; z)$ be symmetric around $\frac{1}{2}$ and z be a parameter that measures stochastic dominance: if $z_1 > z_2$ then $g(N; z_1)$ second-order stochastically dominates $g(N; z_2)$, and as $z \to \infty$, $g(\cdot)$ becomes degenerate at $N = \frac{1}{2}$.

Proposition 7 *For every preference type of the decision maker θ_i and every $\pi > 0$, if $z_1 > z_2$, then the decision maker's utility is higher for $g(N; z_1)$ than for $g(N; z_2)$. As $z \to \infty$, there is full revelation of information.*

Proof in the Appendix. ■

Proposition 7 says that the lower the uncertainty about N is, the better-off the decision maker is. When the decision maker knows more about how many arguments are available to the expert, she can more easily extract his information: when she receives a report, she can estimate rather precisely how many arguments have been concealed from her. The decision maker is better-off in situations that are standard or familiar to her. When faced with an unfamiliar choice, or a choice that is familiar but very different every time it presents itself, the decision maker chooses the suboptimal alternative more often, even when faced with an honest expert.

6 Conclusion

One of the main findings of this paper is that even a biased expert uses two-sided reports that include arguments which oppose his interests. There is an extensive research on using two-sided arguments in marketing literature which usually finds that two-sided messages are more effective and increase the perceived truthfulness of the expert.[17] In my model, the persuader uses two-sided messages to gain credibility, but in equilibrium one-sided and two-sided messages induce the same belief. This is, however, only because the theoretical model assumes away some realistic interactions. For example, if we allow for a small fraction of naive decision makers who take the reports at their face value, in equilibrium the belief function would be such that a rational decision maker would be persuaded more often by two-sided reports and

[17] See, for example, Smith and Hunt (1978) and Anderson and Golden (1984).

those reports would be deemed more credible. Also, if we assumed that biased experts collect information in a biased way, revealing opposing arguments might be strictly profitable as it could increase the expert's credibility.

At this point, I should mention some limitations of my model. First, the model assumes that the decision maker is uninformed, but clearly, it would be interesting to analyze the case where the decision maker has some prior information. Experiments on mass communication indicate that two-sided arguments are more effective when the audience is initially opposed to the expert's position, while one-sided arguments are more successful with listeners who are already disposed toward the expert's position.[18] Moreover, if the audience is later provided with arguments favoring the other position, those who were previously exposed to two-sided argumentation are less likely swayed away from this position than those initially exposed to one-sided argumentation.[19] These issues could be analyzed within my model if we allow the experts to observe only a subset of the existing arguments. Based on the analysis presented in this paper, I conjecture that such an extension would produce results that are consistent with these studies. Decision makers with information favoring one alternative should weight the arguments that reinforce their knowledge more than before, because they should expect the expert to have few opposing arguments. Those who have some opposing arguments should expect the honest expert to reveal such arguments, and if that is not the case, they should believe that the expert is likely to be a persuader. A counterpropaganda should be less effective if the expert first establishes himself to be honest by using two-sided messages.

Another limitation of my model is that it does not explain the trend toward polarization that seems to have been occurring recently. Many media outlets and politicians rely more on one-sided messages. This may be a result of competition or information overload–issues that are not addressed in this paper.

The model presented in this paper is a good starting point to analyze communication in more elaborate settings. Because of the argument structure, it is easy to define the differences in information that different players have, which makes this model especially well suited to analyze two-sided communication. Moreover, since disclosing arguments requires time, the model has some natural timing structure built in, which means that it can be easily applied to debates and communication in bargaining.

[18] See Hovland, Lumsdaine and Sheffield (1949).

[19] See Lumsdaine and Janis (1953).

A Appendix

Proof of Proposition 1

Part a) It is straightforward and omitted.

Part b) Every equilibrium takes the following form: 1. For each ρ, there exists $\lambda_\rho : 0 < \lambda_\rho < 1 - \rho$, such that for all $(\lambda, \rho) : \lambda \leq \lambda_\rho$, we have $\eta(\lambda, \rho) = \eta(0, \rho) \equiv \eta^*_\rho$, 2. η^*_ρ is weakly increasing in ρ, 3. $\eta^*_\rho \geq \frac{\rho}{\rho + \lambda_\rho}$.

The proof of these statements proceeds with the following steps.

Step 1 If for some (λ, ρ) there exists no (L, R) such that $(\lambda, \rho) \in \sigma_{P_r}(L, R)$, then $\eta(\lambda, \rho) = \frac{\lambda}{\lambda + \rho}$.

This follows directly from the existence of H type. This step will be used extensively in the proof.

Step 2 $\eta(0, \rho) < 1$ for all $\rho < 1$.

The existence of H type assures that $\eta(\lambda, \rho) < 1$ for all $\lambda > 0$. Assume that there exists $\rho < 1$, such that $\eta(0, \rho) = 1$. Then for all $(L, R) \in Z(0, \rho)$, P_r can induce belief $\eta = 1$ by sending $(0, \rho)$ (or sending some other feasible report $\eta(0, \rho')$ such that $\eta(0, \rho') = 1$ if it exists). But since a set of points of the form $(0, \rho)$ such that $\eta(0, \rho) = 1$ is at most one dimensional, and the set $Z(0, \rho)$ is two-dimensional, a rational decision maker must form a belief $\eta(0, \rho) < 1$ (or a belief $\eta(0, \rho') < 1$), which is a contradiction.

Step 3 For all ρ, there exists $\lambda_\rho > 0$ such that $\eta(\lambda, \rho) = \eta(0, \rho)$ for all $\lambda \leq \lambda_\rho$.

Assume not. This means that for each ρ and for all $\varepsilon > 0$, we can find $\lambda_0 < \varepsilon$ such that $\eta(\lambda_0, \rho) \neq \eta(0, \rho)$. Assume first that $\eta(\lambda_0, \rho) < \eta(0, \rho)$. By Step 2 we have $\eta(\lambda_0, \rho) < \eta(0, \rho) < 1$. But then P_r always prefers to send $(0, \rho)$ instead of (λ_0, ρ), which implies that (λ_0, ρ) is sent only by H. Then Step 1 implies that $\eta(\rho, \lambda_0) = \frac{\rho}{\lambda_0 + \rho}$. But $\eta(\rho, \lambda_0) = \frac{\rho}{\lambda_0 + \rho}$ becomes arbitrarily close to 1 as we take $\varepsilon \to 0$. This contradicts the assumption that $\eta(\lambda_0, \rho) < \eta(0, \rho) < 1$.

Assume then that for all $\varepsilon > 0$ we can find $\lambda_0 < \varepsilon$ such that $\eta(\lambda_0, \rho) > \eta(0, \rho)$. Then P_r may send $(0, \rho)$ only if $L = 0$ because for any $L > 0$ he would find $\lambda_0 \leq L$ and send (λ_0, ρ) instead. But then $\eta(0, \rho) = 1$, which contradicts Step 2.

Define $\eta^*_\rho \equiv \eta(0, \rho)$.

Step 4 η^*_ρ is weakly increasing in ρ.

Assume that there exist $\rho_2 > \rho_1$ such that $\eta^*_{\rho_2} < \eta^*_{\rho_1}$. Then P_r never sends $(0, \rho_2)$, which by Step 1 implies that $\eta^*_{\rho_2} = 1$, which contradicts Step 2.

Step 5 $\lambda_\rho < 1 - \rho$.

Assume $\lambda_\rho = 1-\rho$. Then a report (λ_ρ, ρ) reveals the state of nature, and $\eta(\lambda_\rho, \rho) = \rho = \eta^*_\rho$. But for all $(\lambda', \rho') \in Z(0, \rho)$ other than $(1-\rho, \rho)$ we have $\frac{\rho'}{\lambda' + \rho'} > \rho$, hence it is impossible to have $\eta^*_\rho = \rho$.

Step 6 $\eta^*_\rho \geq \frac{\rho}{\rho + \lambda_\rho}$.

Assume $\eta^*_\rho < \frac{\rho}{\rho+\lambda_\rho}$. By definition of λ_ρ, we can find an arbitrarily small $\varepsilon > 0$ such that $\eta(\lambda_\rho + \varepsilon, \rho) > \eta^*_\rho$ or $\eta(\lambda_\rho + \varepsilon, \rho) < \eta^*_\rho$. The former cannot happen as it would be impossible to generate $\eta(\lambda, \rho) = \eta^*_\rho$ for all $\lambda \in [0, \lambda_\rho]$. The latter would mean that only H sends a report $(\lambda_\rho + \varepsilon, \rho)$ in equilibrium, which in turn would imply that $\eta(\lambda_\rho + \varepsilon, \rho) = \frac{\rho}{\rho + \lambda_\rho + \varepsilon} \to \frac{\rho}{\rho+\lambda_\rho} > \eta^*_\rho$, which is a contradiction.

By Step 1 and Step 3 we know that P_r does not always suppress unfavorable information; otherwise $\eta(\lambda, \rho)$ would be a strictly decreasing function of λ.

Part c)

Now, I will characterize all equilibria in which the belief function $\eta(\lambda, \rho)$ is continuous in λ and ρ. Note first that continuity requires that $\eta^*_\rho = \frac{\rho}{\lambda_\rho + \rho}$; therefore, there exists a unique λ_ρ for a given η^*_ρ. This, together with continuity, implies that there is no $\lambda > \lambda_\rho$ such that $\eta(\lambda, \rho) \geq \eta^*_\rho$, which implies that only H sends reports of a form $(\lambda > \lambda_\rho, \rho)$.

Continuity implies also that η^*_ρ is strictly increasing in ρ. To see this assume that η^*_ρ is constant over $\rho \in [\rho_1, \rho_2]$ and strictly increasing for $\rho \in (\rho_1 - \epsilon, \rho_1)$ and $\rho \in (\rho_2, \rho_2 + \epsilon)$ for some ϵ. Then for all $\varepsilon < \epsilon$ and all $R = \rho_1 - \varepsilon$, the persuader sends $\rho = \rho_1 - \varepsilon$, and for all $R = \rho_2 + \varepsilon$, the persuader sends $\rho = \rho_2 + \varepsilon$. Therefore

$$
\begin{aligned}
\eta^*_{\rho_2+\varepsilon} \quad &= \quad \Pr\left(H | \rho = \rho_2 + \varepsilon, \lambda \leq \lambda_{\rho_2+\varepsilon}\right) E\left[q_R | R = \rho_2 + \varepsilon, L \leq \lambda_{\rho_2+\varepsilon}\right] \\
&\quad + \Pr\left(P_r | \rho = \rho_2 + \varepsilon, \lambda \leq \lambda_{\rho_2+\varepsilon}\right) E\left[q_R | R = \rho_2 + \varepsilon\right],
\end{aligned}
$$

$$
\begin{aligned}
\eta^*_{\rho_1-\varepsilon} \quad &= \quad \Pr\left(H | \rho = \rho_1 - \varepsilon, \lambda \leq \lambda_{\rho_1-\varepsilon}\right) E\left[q_R | R = \rho_1 - \varepsilon, L \leq \lambda_{\rho_1-\varepsilon}\right] \\
&\quad + \Pr\left(P_r | \rho = \rho_1 - \varepsilon, \lambda \leq \lambda_{\rho_1-\varepsilon}\right) E\left[q_R | R = \rho_1 - \varepsilon\right],
\end{aligned}
$$

where $\lambda_\rho = \frac{1-\eta_\rho}{\eta_\rho}\rho$. If $\eta(\rho, \lambda)$ is continuous then $\lim_{\varepsilon \to 0} \eta^*_{\rho_2+\varepsilon} = \lim_{\varepsilon \to 0} \eta^*_{\rho_1-\varepsilon}$. If we take the limit of both sides of both equations as $\varepsilon \to 0$, we get the expressions for $\eta^*_{\rho_1}$ and $\eta^*_{\rho_2}$. I will show that $\frac{d\eta^*_\rho}{d\rho} > 0$, which implies that $\eta^*_{\rho_1} < \eta^*_{\rho_2}$, which is a contradiction.

The limit of the first expression can be rearranged as follows:

$$Y = \eta^*_{\rho_2}\left(F\left(\frac{1-\eta^*_{\rho_2}}{\eta^*_{\rho_2}}\rho_2|R=\rho_2\right)(1-\pi)+\pi\right)+$$
$$-(1-\pi)\int_0^{\frac{1-\eta^*_{\rho_2}}{\eta^*_{\rho_2}}\rho_2}\frac{\rho_2 f(L|R=\rho_2)}{\rho_2+L}dL \quad -\pi\int_0^{1-\rho_2}\tfrac{\rho_2}{\rho_2+L}f(L|R=\rho_2)\,dL = \quad 0.$$

Then $\frac{d\eta^*_{\rho_2}}{d\rho_2} = -\frac{\frac{\partial Y}{\partial\rho_2}}{\frac{\partial Y}{\partial\eta^*_{\rho_2}}}$, and

$$\frac{\partial Y}{\partial\eta^*_{\rho_2}} = F\left(\frac{1-\eta^*_{\rho_2}}{\eta^*_{\rho_2}}\rho_2|R=\rho_2\right)(1-\pi)+\pi>0,$$

$$\begin{aligned}\frac{\partial Y}{\partial\rho_2} \quad = \quad & -(1-\pi)\int_0^{\frac{1-\eta^*_{\rho_2}}{\eta^*_{\rho_2}}\rho_2}\frac{Lf(L|R=\rho_2)}{(\rho_2+L)^2}dL-\pi\int_0^{1-\rho_2}\frac{Lf(L|R=\rho_2)}{(\rho_2+L)^2}dL \\ & -(1-\pi)\int_0^{\frac{1-\eta^*_{\rho_2}}{\eta^*_{\rho_2}}\rho_2}\frac{\rho_2F_R(L|R=\rho_2)}{(\rho_2+L)^2}dL-\pi\int_0^{1-\rho_2}\frac{\rho_2F_R(L|R=\rho_2)}{(\rho_2+L)^2}dL,\end{aligned}$$

where the expression above was obtained by first applying integration by parts to the formula for Y, taking the derivative and applying integration by parts again. Using the regularity condition (1) we get $\frac{dY}{d\rho_2}<0$, which in turn implies that $\frac{d\eta^*_{\rho_2}}{d\rho_2}>0$.

When η^*_ρ is strictly increasing, the strategy of P_r is to send $\rho=R$. He is indifferent between sending any number of arguments that support *Left*, as long as $\lambda\leq\lambda_\rho$. In equilibrium, however, his strategy must support the constant belief. Given this strategy define $\eta^*_R\equiv\eta^*_\rho$.

It remains to show that for each R, η^*_R described by equation (3) exists and is unique. Equation (3) can be rewritten as follows:

$$\eta^*_R = \frac{\pi\int_0^{1-R}\frac{R}{R+L}f(L|R)\,dL+(1-\pi)\int_0^{\frac{1-\eta^*_R}{\eta^*_R}R}\frac{R}{R+L}f(L|R)\,dL}{\pi+(1-\pi)F\left(\frac{1-\eta^*_R}{\eta^*_R}R|R\right)} \tag{6}$$

The left hand side (LHS) goes from R to 1. The right hand side (RHS) is continuous, and we have $RHS(\eta^*_R=R)=\bar{\eta}_R>R$ and $RHS(\eta^*_R=1)=\bar{\eta}_R<1$, where $\bar{\eta}_R=\int_0^{1-R}\frac{R}{R+L}f(L|R)\,dL$; therefore, the solution to equation (6) exists.

The LHS is strictly increasing. If we differentiate the RHS with respect to η_R^*, we get

$$\begin{aligned}\frac{dRHS}{d\eta_R^*} &= \frac{(1-\pi) f\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)}{\left(\pi+(1-\pi)F\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)\right)^2}\frac{-1}{(\eta_R^*)^2}R\cdot \\ &\cdot\left(\pi\left(\eta_R^*-\bar{\eta}_R\right)+(1-\pi)\left(\eta_R^*F\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)-\int_0^{\frac{1-\eta_R^*}{\eta_R^*}R}\frac{Rf(L|R)}{R+L}dL\right)\right),\end{aligned}$$

and evaluating it at η_R^* that satisfies equation (6) we get

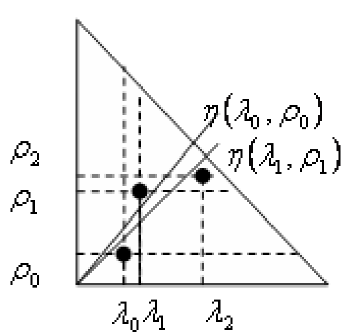
$$\frac{dRHS}{d\eta_R^*}=\frac{(1-\pi) f\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)}{\left(\pi+(1-\pi)\left(\frac{1-\eta_R^*}{\eta_R^*}R|R\right)\right)}\left(\eta^*-\eta^*\right)=0.$$

This implies that the solution to equation (6) is unique. ■

Proof of Proposition 2

Step 1 In any equilibrium $\eta(\lambda,\rho)\leq\frac{\rho}{\rho+\lambda}$.

Assume that there exists $(L,R)=(\lambda_0,\rho_0)$ such that $\eta(\lambda_0,\rho_0)>\frac{\rho_0}{\rho_0+\lambda_0}$ (see the picture below). The presence of H implies that in order to generate such a belief there must exist $(L,R)=(\lambda_1,\rho_1)\in Z(\lambda_0,\rho_0)$ such that $\frac{\rho_1}{\rho_1+\lambda_1}\geq\eta(\lambda_0,\rho_0)$, and $(\lambda_0,\rho_0)\in\sigma_{P_r}(\lambda_1,\rho_1)$. But this implies that $\frac{\rho_1}{\rho_1+\lambda_1}\geq\eta(\lambda_0,\rho_0)>\eta(\lambda_1,\rho_1)$, otherwise P_r would prefer to send (λ_1,ρ_1) over (λ_0,ρ_0). This in turn means that there exists $(L,R)=(\lambda_2,\rho_2)\in Z(\lambda_1,\rho_1)$ such that $\frac{\rho_2}{\rho_2+\lambda_2}\leq\eta(\lambda_1,\rho_1)$ and $(\lambda_1,\rho_1)\in\sigma_{P_r}(\lambda_2,\rho_2)$. But this cannot happen: if P_r does not report fully (λ_2,ρ_2), he prefers to send (λ_0,ρ_0) instead and induce higher belief.



Step 2 If $(\lambda_0, \rho_0) \in \sigma_{P_r}(L, R)$ only if $\frac{R}{R+L} \geq \frac{\rho_0}{\rho_0+\lambda_0}$, then $\eta(\lambda_0, \rho_0) = \frac{\rho_0}{\rho_0+\lambda_0}$.

This follows directly from Step 1.

Step 3 If $\eta(\rho, \lambda_0) = \frac{\rho}{\rho+\lambda_0}$, then $\eta(\rho, \lambda) = \frac{\rho}{\rho+\lambda}$ for all $\lambda \geq \lambda_0$.

Assume that there exists $\lambda_1 > \lambda_0$, and $\eta(\rho, \lambda_1) < \frac{\rho}{\rho+\lambda_1}$. This means that there exists $(\rho_2, \lambda_2) \in Z(\rho, \lambda_1)$ such that $\frac{\rho_2}{\rho_2+\lambda_2} \leq \eta(\rho, \lambda_1) < \frac{\rho}{\rho+\lambda_1}$, and $(\rho, \lambda_1) \in \sigma_{P_r}(\rho_2, \lambda_2)$. But P_r would rather send (ρ, λ_0) because sending (ρ, λ_1) also costs him ξ. This is a contradiction.

Step 4 $\eta(0, \rho) < 1$ for all $\rho < 1 - \xi$.

For $\rho < 1-\xi$, there exist many $(L, R) \in Z(0, \rho)$ such that $\eta(L, R)$ must be smaller than $1 - \xi$. By the same logic as in Step 2 of Proposition 1 we get that $\eta(0, \rho) < 1$.

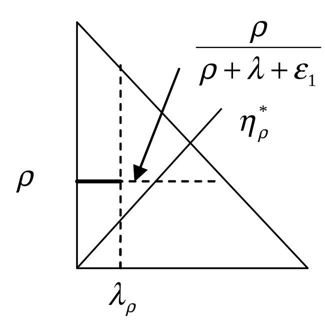
Step 5 For all $\rho < 1 - \xi$, there exists $\lambda_\rho > 0$ such that $\eta(\lambda, \rho) = \eta(0, \rho)$ for all $\lambda \leq \lambda_\rho$.

The proof is identical to Step 3 of Proposition 1.

Let $\eta^*_\rho \equiv \eta(0, \rho)$, and define λ_ρ to be such that $\eta(\rho, \lambda) = \eta^*_\rho$ for all $\lambda \leq \lambda_\rho$. By Step 1 we have $\lambda_\rho \leq \frac{1-\eta^*_\rho}{\eta^*_\rho}\rho$.

Step 6 $\lambda_\rho = \frac{1-\eta^*_\rho}{\eta^*_\rho}\rho$, and $\eta(\lambda, \rho) = \frac{\rho}{\rho+\lambda}$ for all $\lambda > \lambda_\rho$.

Assume that $\lambda_\rho < \frac{1-\eta^*_\rho}{\eta^*_\rho}\rho$. Then for any small ϵ we can always find $\epsilon_1 < \epsilon$ such that $\eta(\lambda_0 + \epsilon_1, \rho) > \eta^*_\rho$ or $\eta(\lambda_0 + \epsilon_1, \rho) < \eta^*_\rho$ (see the picture below). Consider the first case $\eta(\lambda_0 + \epsilon_1, \rho) > \eta^*_\rho$. Then P_r may send (λ_ρ, ρ) only if $\frac{R}{R+L} \geq \frac{\rho}{\rho+\lambda_\rho}$. But then by Step 2 $\eta(\lambda_\rho, \rho) = \frac{\rho}{\rho+\lambda_\rho} > \eta^*_\rho$, which is a contradiction. Assume then that $\eta(\lambda_0 + \epsilon_1, \rho) < \eta^*_\rho$ for some arbitrarily small $\epsilon_1 > 0$. This implies that P_r sends $(\lambda_0 + \epsilon_1, \rho)$ only if $(L, R) = (\lambda_0 + \epsilon_1, \rho)$, which implies that $\eta(\lambda_0 + \epsilon_1, \rho) = \frac{\rho}{\rho+\lambda_0+\epsilon_1} \to \frac{\rho}{\rho+\lambda_0} > \eta^*_\rho$, a contradiction. Therefore, $\lambda_\rho = \frac{1-\eta^*_\rho}{\eta^*_\rho}\rho$, and by Step 3 we have $\eta(\rho, \lambda) = \frac{\rho}{\rho+\lambda}$ for all $\lambda > \lambda_\rho$.



Step 7 $m_{P_r}((L, R) \mid (L, R)) = 1$ for $R \geq 1 - \xi$.

This is an immediate implication of the cost of concealing information and previous steps.

The steps above imply that each equilibrium is fully characterized by η^*_ρ. Below I show that η^*_ρ is strictly increasing.

Step 8 η^*_ρ is weakly increasing in ρ for $\rho < 1 - \xi$.

Assume there exist $\rho_2 > \rho_1$ such that $\eta^*_{\rho_2} < \eta^*_{\rho_1}$. Then no P_r ever sends any messages of a form $(0, \rho_2)$ whenever $(L, R) \neq (0, \rho_2)$. But then $\eta(0, \rho_1) = 1$, which contradicts Step 4.

Step 9 η^*_ρ is strictly increasing in ρ for $\rho < 1 - \xi$.

By Step 8 we know that if $\rho_2 > \rho_1$ and $\eta^*_{\rho_1} = \eta^*_{\rho_2}$, then $\eta^*_\rho = \eta^*_{\rho_1}$ for all $\rho \in [\rho_1, \rho_2]$. Assume that ρ_1 and ρ_2 are the lowest and the highest ρ such that $\eta^*_\rho = \eta^*_{\rho_1}$. Then for $R = \rho_1$ we have that P_r reports (λ, ρ) such that $\rho = \rho_1$ and $\lambda \leq \lambda_{\rho_1}$. If P_r reported ρ_1 only if $R = \rho_1$ then $\eta^*_{\rho_1}$ would be equal to the RHS of equation (7) below. However, P_r may report ρ_1 if $R \in [\rho_1, \rho_2]$, but only if $\lambda > \lambda_\rho$ (because of the cost of concealing information), which can only decrease η^*_ρ. Therefore we get

$$\begin{aligned} \eta^*_{\rho_1} \leq\ & \Pr\left(H|\rho = \rho_1, \lambda \leq \lambda_{\rho_1}\right) E\left[q_R|R = \rho, L \leq \lambda_{\rho_1}\right] + \\ & \Pr\left(P_r|\rho = \rho_1, \lambda \leq \lambda_{\rho_1}\right) E\left[q_R|R = \rho_1\right]. \end{aligned} \tag{7}$$

By a similar argument we get

$$\begin{aligned} \eta^*_{\rho_2} \geq\ & \Pr\left(H|\rho = \rho_2, \lambda \leq \lambda_{\rho_2}\right) E\left[q_R|R = \rho_2, L \leq \lambda_{\rho_2}\right] \\ & + \Pr\left(P_r|\rho = \rho_2, \lambda \leq \lambda_{\rho_2}\right) E\left[q_R|R = \rho_2\right]. \end{aligned}$$

But in Proof of Proposition 1 I have shown that under the regularity conditions (1) and when $\xi = 0$ we have that the RHS of the second equation is strictly higher than the RHS of the first one. The same argument would hold for small $\xi > 0$, so I omit it here. This leads to $\eta^*_{\rho_1} < \eta^*_{\rho_2}$, which is a contradiction.

The persuader P_r reports fully if $(L, R) : L \leq \lambda_{\rho=R}$ and conceals some arguments that favor *Left* if $L \geq \lambda_{\rho=R}$. ■

Proof of Proposition 3

This proof compares any discontinuous equilibrium with any continuous one. Let $\eta^{*c}_R \equiv \eta(0, R)$ in any continuous equilibrium and let $\eta^*_R \equiv \eta(0, R)$ in some discontinuous equilibrium. For each R, we have three cases: 1. $\eta^*_R < \eta^{*c}_R$, 2. $\eta^*_R > \eta^{*c}_R$ and 3.

$\eta_R^* = \eta_R^{*c}$. I will show that for each R, any type of the decision maker is better off in a continuous equilibrium. Recall that λ_R is the highest λ such that $\eta(\lambda, R) = \eta_R^*$ for all $\lambda \leq \lambda_R$, and let λ_R^c be the highest λ such that $\eta(\lambda, R) = \eta_R^{*c}$. From the proof of Proposition 1 we know that $\frac{R}{R+\lambda_R^c} = \eta_R^{*c}$ and $\frac{R}{R+\lambda_R} \leq \eta_R^*$. Clearly, for a given R and keeping η_R^* fixed, the decision maker is weakly better off when $\frac{R}{R+\lambda_R} = \eta_R^*$ than when $\frac{R}{R+\lambda_R} < \eta_R^*$ as in both cases she makes the same decision when facing P_r, but in the latter case she may make worse decisions when facing H. Therefore, in calculations below I assume that $\frac{R}{R+\lambda_R} = \eta_R^*$.

First, note that by the definition of η_R^{*c} we have (see the discussion at the end of the proof of Proposition 1)

$$\frac{\pi E[q_R|R] + (1-\pi) P(q_R > \theta_i) E[q_R | q_R > \theta_i, R]}{\pi + (1-\pi) P(q_R > \theta_i | R)} \begin{cases} > \theta_i \text{ if } \theta_i < \eta_R^{*c} \\ < \theta_i \text{ if } \theta_i > \eta_R^{*c} \end{cases}. \tag{8}$$

The argument below holds for all discontinuous equilibria in which $\eta_R^* \geq \eta(\lambda, R)$ for all λ. However, it can be shown that if $\eta_R^* < \eta(\lambda', R)$ for some λ', the utility of the decision maker is even lower in a discontinuous equilibrium, which makes it even less attractive.

Let U_d^R and U_c^R be the utilities of the decision maker in a discrete and in a continuous equilibrium respectively if the number of arguments that favor *Right* is R.

Case 1: $\eta_R^* < \eta_R^{*c}$. A decision maker with $\theta_i \geq \eta_R^{*c}$ or with $\theta_i \leq \eta_R^*$ makes the same decision in both equilibria. A decision maker with $\theta_i \in (\eta_R^*, \eta_R^{*c})$ chooses *Left* in the discrete equilibrium, and in the continuous equilibrium she chooses *Right* if she faces P_r and chooses optimally if she faces H. Therefore,

$$\begin{aligned} U_d^R - U_c^R &= EU(Left|R) - \pi EU(Right|R) - (1-\pi) E[\max\{U(Left|R), U(Right|R)\}] = \\ &= E[\theta_i - q_R|R] - \pi E[q_R - \theta_i|R] \\ &\quad - (1-\pi) P(q_R < \theta_i|R) E[\theta_i - q_R | q_R < \theta_i, R] \\ &\quad - (1-\pi) P(q_R > \theta_i|R) E[q_R - \theta_i | q_R > \theta_i, R] = \\ &= \underbrace{2\pi(\theta_i - E[q_R|R]) + 2(1-\pi) P(q_R > \theta_i)(\theta_i - E[q_R|q_R > \theta_i, R])}_{<0 \text{ using } (8)} < 0. \end{aligned}$$

Case 2: $\eta_R^* > \eta_R^{*c}$. A decision maker with $\theta_i \leq \eta_R^{c*}$ or with $\theta_i \geq \eta_R^*$ makes the same decision in both equilibria. A decision maker with $\theta_i \in (\eta_\rho^{*c}, \eta_\rho^*)$ chooses *Left* in the continuous equilibrium, and in the discrete equilibrium she chooses *Right* if she faces the persuader and chooses optimally if she faces H. Therefore,

$$\begin{aligned} U_d^R - U_c^R &= (1-\pi) E\left[\max\left\{U(Left|R), U(Right|R)\right\}\right] + \pi U(Right|R) - U(Left|R) = \\ &= (1-\pi) P(q_R < \theta_i|R)(\theta_i - E[q_R|q_R < \theta_i, R]) + \\ &\quad + (1-\pi) P(q_R > \theta_i|R)(E[q_R|q_R > \theta_i, R] - \theta_i) + \\ &\quad + \pi E[q_R - \theta_i|R] - E[\theta_i - q_R|R] \\ &= \underbrace{2\pi(E[q_R|R] - \theta_i) + 2(1-\pi) P(q_R > \theta_i|R)(E[q_R|q_R > \theta_i, R] - \theta_i)}_{<0 \text{ using } (8)} < 0. \end{aligned}$$

Case 3: $\eta_R^* = \eta_R^{*c}$. It is immediate that $U_d^R - U_c^R = 0$. ■

Proof of Proposition 4

This proof requires only slight modifications to the proof of Proposition 2.

First, let me prove the following Lemma.

Lemma 1 *There cannot exist an infinite sequence of reports* (λ_n, ρ_n) *such that* $(\lambda_n, \rho_n) \in Z(\lambda_{n-1}, \rho_{n-1})$ *and* $\eta(\lambda_n, \rho_n) \neq \frac{\rho_n}{\rho_n + \lambda_n}$.

Proof We have $Z(\lambda_n, \rho_n) \subset Z(\lambda_{n-1}, \rho_{n-1})$; therefore, for some n, $Z(\lambda_n, \rho_n)$ is so small that the difference between $\max_{(\lambda,\rho)\in Z(\lambda_n,\rho_n)} \frac{\rho}{\rho+\lambda}$ and $\min_{(\lambda,\rho)\in Z(\lambda_n,\rho_n)} \frac{\rho}{\rho+\lambda}$ is smaller than the cost of concealing arguments ξ; therefore, all experts report fully, which implies that $\eta(\lambda_n, \rho_n) = \frac{\rho_n}{\rho_n + \lambda_n}$. ■

Below, I repeat steps 1 to 6 of the proof of Proposition 2 for H being a strategic, benevolent expert, highlighting the necessary changes. All of the remaining steps of the proof of Proposition 2 then follow.

Step 1 $\eta(\lambda, \rho) \leq \frac{\rho}{\rho+\lambda}$.

Assume that there exists $(L, R) = (\lambda_0, \rho_0)$ such that $\eta(\lambda_0, \rho_0) > \frac{\rho_0}{\rho_0 + \lambda_0}$. Then there exists $(\lambda_1, \rho_1) \in Z(\lambda_0, \rho_0)$ such that $\frac{\rho_1}{\rho_1+\lambda_1} \geq \eta(\lambda_0, \rho_0) > \frac{\rho_0}{\rho_0+\lambda_0}$ and $(\lambda_0, \rho_0) \in \sigma_i(\lambda_1, \rho_1)$. There are three cases: 1. $\eta(\lambda_1, \rho_1) = \frac{\rho_1}{\rho_1+\lambda_1}$, 2. $\eta(\lambda_1, \rho_1) > \frac{\rho_1}{\rho_1+\lambda_1}$, 3. $\eta(\lambda_1, \rho_1) < \frac{\rho_1}{\rho_1+\lambda_1}$. If Case 1, $\eta(\lambda_1, \rho_1) = \frac{\rho_1}{\rho_1+\lambda_1}$, then only P_r could use a strategy $(\lambda_0, \rho_0) \in \sigma_{P_r}(\lambda_1, \rho_1)$, but only if $\eta(\lambda_0, \rho_0) > \eta(\lambda_1, \rho_1) = \frac{\rho_1}{\rho_1+\lambda_1}$, which is not the case. If Case 2, $\eta(\lambda_1, \rho_1) > \frac{\rho_1}{\rho_1+\lambda_1}$, then then call $(\lambda_0, \rho_0) = (\lambda_1, \rho_1)$ and start the analysis from the beginning. Eventually, however, one would have to exit case 2 because by Lemma 1 one cannot have an infinite sequence of reports such that $\eta(\lambda, \rho) > \frac{\rho}{\rho+\lambda}$.

Therefore, it must be that Case 3, $\eta(\lambda_1, \rho_1) < \frac{\rho_1}{\rho_1+\lambda_1}$, and moreover $\eta(\lambda_1, \rho_1) < \eta(\lambda_0, \rho_0)$, otherwise no expert would send (λ_1, ρ_1). But then there exists $(\lambda_2, \rho_2) \in Z(\lambda_1, \rho_1)$ such that $\frac{\rho_2}{\rho_2+\lambda_2} \le \eta(\lambda_1, \rho_1)$ and $\sigma_i(\lambda_2, \rho_2) = (\lambda_1, \rho_1)$. But since P_r would prefer to send (λ_0, ρ_0) it must be the case that $i = H$. Therefore, again, we have three cases. Case 1, $\frac{\rho_2}{\rho_2+\lambda_2} = \eta(\lambda_2, \rho_2)$, is impossible. If Case 2, $\eta(\lambda_2, \rho_2) > \frac{\rho_2}{\rho_2+\lambda_2}$, then it must be that $\eta(\lambda_2, \rho_2) > \eta(\lambda_1, \rho_1)$, otherwise H would send $\sigma_H(\lambda_2, \rho_2) = (\lambda_1, \rho_1)$. But then set $(\lambda_0, \rho_0) = (\lambda_2, \rho_2)$ and start the analysis from the beginning, which would again violate Lemma 1. Therefore, Case 3, $\eta(\lambda_2, \rho_2) < \frac{\rho_2}{\rho_2+\lambda_2}$, must be the case. If we continue this reasoning we will end up with an infinite sequence of reports such that $(\lambda_n, \rho_n) \in Z(\lambda_{n-1}, \rho_{n-1})$ and $\eta(\lambda_n, \rho_n) < \frac{\rho_n}{\rho_n+\lambda_n}$, but this violates Lemma 1.

Step 2 Identical to the one from the proof of Proposition 2.

Step 3 In Step 3 it may be that there exists (λ_2, ρ_2) such that $(\lambda_1, \rho_1) \in \sigma_H(\lambda_2, \rho_2)$. But, using Step 1, we have $\eta(\lambda_2, \rho_2) < \frac{\rho_2}{\rho_2+\lambda_2}$. Continuing this argument we get an infinite sequence of $(\lambda_n, \rho_n) \neq \frac{\rho_n}{\rho_n+\lambda_n}$, which violates Lemma 1.

Step 4 Identical to Step 4 from the proof of Proposition 2.

Step 5 Assume not, which means that for each ρ and for all $\varepsilon > 0$, we can find $\lambda_0 < \varepsilon$ such that $\eta(\lambda_0, \rho) \neq \eta(0, \rho)$. Assume first that $\eta(\lambda_0, \rho) < \eta(0, \rho)$. Then by Step 2 we have $\eta(\lambda_0, \rho) < \eta(0, \rho) < 1$. Given that, P_r always prefers to send $(0, \rho)$ instead of (λ_0, ρ) when concealing information, by Step 2 we have $\eta(\rho, \lambda_0) = \frac{\rho}{\lambda_0+\rho}$. But $\eta(\rho, \lambda_0) = \frac{\rho}{\lambda_0+\rho}$ becomes arbitrarily close to 1 as we let $\varepsilon \to 0$. This contradicts the assumption that $\eta(\lambda_0, \rho) < \eta(0, \rho) < 1$.

Assume then that for all $\varepsilon > 0$, we can find $\lambda_0 < \varepsilon$ such that $\eta(\lambda_0, \rho) > \eta(0, \rho)$. Then P_r may send $(0, \rho)$ only if $L = 0$ since for any $L > 0$ he would find $\lambda_0 \le L$ and send (λ_0, ρ) instead. But then by Step 2 $\eta(0, \rho) = 1$, which contradicts Step 4.

Step 6 Holds with slight modifications. ■

Proof of Proposition 5

Step 1 If for some (λ, ρ) there exists no (L, R) such that $(\lambda, \rho) \in \sigma_{P_r}(L, R)$ or $(\lambda, \rho) \in \sigma_{P_l}(L, R)$ then $\eta(\lambda, \rho) = \frac{\lambda}{\lambda+\rho}$.

This follows directly from the existence of H type. This finding will be used extensively in the proof.

Step 2 $\eta(0, \rho) < 1$ for all $\rho < 1$ and $\eta(\lambda, 0) > 0$ for all $\lambda < 1$.

The proof of the first part is identical to Step 2 of the proof of Proposition 1. By a similar argument we get $\eta(\lambda, 0) > 0$.

Step 3 i) For all ρ, there exists $\lambda_\rho > 0$ such that $\eta(\lambda, \rho) = \eta(0, \rho)$ for all $\lambda \leq \lambda_\rho$.

ii) For all λ, there exists $\rho_\lambda > 0$ such that $\eta(\lambda, \rho) = \eta(\lambda, 0)$ for all $\rho \leq \rho_\lambda$

Assume that part (i) does not hold, which means that for some λ close to 0 the belief function $\eta(\lambda, \rho)$ is strictly increasing or decreasing in λ. If $\eta(\lambda, \rho)$ is decreasing in λ then P_r always prefers to send $(0, \rho)$ instead of any (λ, ρ) for any λ close to 0. Take (ε, ρ). P_l may send (ε, ρ) only if $L = \varepsilon$ and $R \geq \rho$, therefore it must be that $\eta(\varepsilon, \rho) \geq \frac{\rho}{\rho+\varepsilon}$. But $\eta(\varepsilon, \rho) \geq \frac{\rho}{\rho+\varepsilon} \rightarrow 1 > \eta(0, \rho)$, which contradicts the continuity assumption.

Assume now that (λ, ρ) is strictly increasing in λ for some λ close to 0. Then P_l never sends (λ, ρ) for small λ : he would prefer to send $(0, \rho)$ instead. P_r may send (λ, ρ) with small λ only if $L = \lambda_\rho$ and $R \geq \rho$. But then $\eta(\lambda, \rho) \geq \frac{\rho}{\rho+\lambda} \rightarrow_{\lambda \rightarrow 0} 1$, which again contradicts the assumption of continuity of $\eta(\lambda, \rho)$. A similar argument holds for part ii).

Define $\eta^*_\rho \equiv \eta(0, \rho)$ and $\eta^*_\lambda \equiv \eta(\lambda, 0)$.

Step 4 i) P_r sends only reports of the form $(\lambda \leq \lambda_\rho, \rho)$,

ii) P_l sends only reports of the form $(\lambda, \rho \leq \rho_\lambda)$.

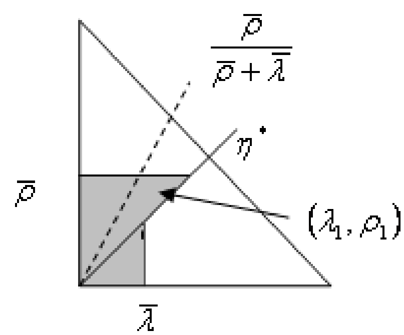
Assume that P_r sends (λ_0, ρ_0), where $\lambda_0 > \lambda_{\rho_0}$. This means that $\eta(\lambda_0, \rho_0) \geq \eta^*_{\rho_0}$. This also means that for some $\lambda \in (\lambda_{\rho_0}, \lambda_0)$, the belief function $\eta(\lambda, \rho_0)$ is strictly increasing in λ but $\eta(\lambda, \rho_0) < \eta^*_{\rho_0}$. But these reports would be sent by H only, which means that $\eta(\lambda, \rho_0) = \frac{\rho_0}{\lambda+\rho_0}$. This contradicts that $\eta(\lambda, \rho_0)$ was increasing in λ. An analogous proof holds for part (ii).

In proving Step 4 I have also proved that $\eta^*_\rho > \eta(\lambda, \rho)$ for all $\lambda > \lambda_\rho$ and $\eta^*_\lambda < \eta(\lambda, \rho)$ for all $\rho > \rho_\lambda$.

Step 5 Previous steps imply that there exist $\bar{\rho} > 0$ and $\bar{\lambda} > 0$ such that $\eta^*_\rho = \eta^*_\lambda \equiv \eta^*$ for all $\rho \leq \bar{\rho}$ and for all $\lambda \leq \bar{\lambda}$. If we take the highest such $\bar{\rho}$ and $\bar{\lambda}$, then P_l never sends reports $(0, \rho > \bar{\rho})$ and P_r never sends reports $(\lambda > \bar{\lambda}, 0)$. By the same argument as in the proof of Proposition 1 we have that η^*_ρ is strictly increasing in ρ for $\rho > \bar{\rho}$, and η^*_λ is strictly decreasing in λ for $\lambda > \bar{\lambda}$. Moreover, by continuity of $\eta(\lambda, \rho)$ for $\rho > \bar{\rho}$ we have $\eta^*_\rho = \frac{\rho}{\rho+\lambda_\rho}$ and for $\lambda > \bar{\lambda}$ we have $\eta^*_\lambda = \frac{\rho_\lambda}{\rho_\lambda+\lambda}$. The definition of η^*_ρ is the same as in equation (3) and η^*_λ is defined analogously.

Step 6 There are three possible situations, $\eta^* > \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$, $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$ or $\eta^* = \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$. Below I describe the shape of the equilibrium if $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$. For the remaining cases the discussion is analogous.

If $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\hat{\lambda}}$, then for all (λ, ρ) that lie in the shaded area in the figure below, and only for these reports (or when $\frac{R}{L+R} = \eta^*$), we have $\eta(\lambda, \rho) = \eta^*$.



To see this notice that by Step 3 $\eta(\lambda, \rho) = \eta^*$ for all $(\lambda, \rho) : \lambda \leq \bar{\lambda}$ and $\rho \leq \bar{\rho}$. Take report $(\lambda_1, \rho_1) : \frac{\rho_1}{\rho_1+\lambda_1} > \eta^*$ like in the figure above. By Step 4, $\eta(\lambda_1, \rho_1) \leq \eta^*$. Assume that $\eta(\lambda_1, \rho_1) < \eta^*$; then only P_l may send (λ_1, ρ_1), which by Step 4 implies that $\eta(\lambda_1, \rho_1) = \eta^*_{\lambda_1}$, and by Step 5 η^*_λ is strictly decreasing in λ, which in turn implies that $(\lambda_1, \rho_1) \in \sigma_{P_l}(L, R)$ only if $L = \lambda_1$ and $R \geq \rho_1$. But this implies that $\eta(\lambda_1, \rho_1) \geq \frac{\rho_1}{\rho_1+\lambda_1} > \eta^*$, which is a contradiction.

Step 6 allows us to summarize the shape of any equilibrium. If $\eta^* < \frac{\bar{\rho}}{\bar{\rho}+\bar{\lambda}}$, then the equilibrium looks like in the figure below.

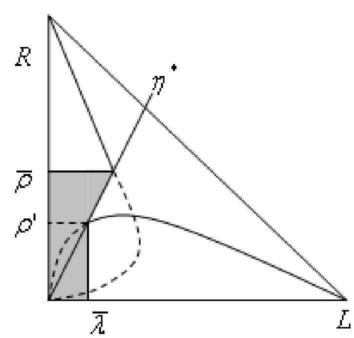


Figure A

The grey area represents all reports that generate the same belief η^*. The solid curves represent the areas in which only P_r and H (along the vertical axis) or only P_l and H (along the horizontal axis) send reports.

In what follows below I use $\bar{R}$ instead of $\bar{\rho}$ and $\bar{L}$ instead of $\bar{\lambda}$. In the proof of Proposition 1 I have shown that η^*_ρ (and therefore also η^*_λ) exists and is unique and strictly increasing for $\rho > \bar{R}$ (η^*_λ is strictly decreasing for $\lambda > \bar{L}$). I will show now that $\bar{R}$ and $\bar{L}$ and η^* are unique. Let all reports (λ, ρ) that generate η^* and either $\rho \leq \bar{R}$ or $\lambda \leq \bar{L}$ be called the double ambiguity area (DAA).

First, by continuity of $\eta(\lambda, \rho)$, $\bar{R}$ and $\bar{L}$ must be such that $\eta^*_{\bar{R}+\varepsilon} \to \eta^*$ and $\eta^*_{\bar{L}+\varepsilon} \to \eta^*$ as $\varepsilon \to 0$. This means that $\bar{R}$ and $\bar{L}$ must satisfy equations (4) and (5), which can be rewritten as

$$\eta^* = \frac{\frac{\pi_r}{1-\pi_l}\int_0^{1-\bar{R}} \frac{\bar{R}}{\bar{R}+L} f\left(L|\bar{R}\right) dL + \frac{\pi_H}{1-\pi_l}\int_0^{\lambda_{\bar{R}}} \frac{\bar{R}}{\bar{R}+L} f\left(L|\bar{R}\right) dL}{\frac{\pi_r}{1-\pi_l} + \frac{\pi_H}{1-\pi_l} F\left(\lambda_{\bar{R}}|\bar{R}\right)}, \tag{9}$$

$$\eta^* = \frac{\frac{\pi_l}{1-\pi_r}\int_0^{1-\bar{L}} \frac{R}{R+\bar{L}} f\left(R|\bar{L}\right) dR + \frac{\pi_H}{1-\pi_r}\int_0^{\rho_{\bar{L}}} \frac{R}{R+\bar{L}} f\left(R|\bar{L}\right) dR}{\frac{\pi_l}{1-\pi_r} + \frac{\pi_H}{1-\pi_r} F\left(\rho_{\bar{L}}|\bar{L}\right)}. \tag{10}$$

These equations uniquely determine $\bar{R}$ and $\bar{L}$ as a function of η^*. For all reports in DAA to generate the same belief η^*, this belief must satisfy

$$\begin{aligned} \eta^* &= P\left(P_r|DAA\right) E\left[\frac{R}{R+L}|DAA, P_r\right] + \\ &\quad + P\left(P_l|DAA\right) E\left[\frac{R}{R+L}|DAA, P_l\right] + P\left(H|DAA\right) E\left[\frac{R}{R+L}|DAA, H\right]. \end{aligned}$$

Recall Figure A; the equation above can be rewritten as follows (where f is used instead of $f(L,K)$ to shorten the formula):

$$\begin{aligned} \eta^* &= \frac{\pi_r \int_0^{\bar{R}}\int_0^{1-R} q_R f dL dR + \pi_l \int_0^{\bar{L}}\int_0^{1-L} q_R f dL dR}{\pi_r \int_0^{\bar{R}}\int_0^{1-R} f dL dR + \pi_l \int_0^{\bar{L}}\int_0^{1-L} f dL dR + \pi_H\left(\int_{\frac{\eta^*\bar{L}}{1-\eta^*}}^{\bar{R}}\int_0^{\frac{(1-\eta^*)R}{\eta^*}} f dL dR + \int_0^{\frac{\eta^*\bar{L}}{1-\eta^*}}\int_0^{\bar{L}} f dL dR\right)} \\ &+ \frac{\pi_H\left(\int_{\frac{\eta^*\bar{L}}{1-\eta^*}}^{\bar{R}}\int_0^{\frac{(1-\eta^*)R}{\eta^*}} q_R f dL dR + \int_0^{\frac{\eta^*\bar{L}}{1-\eta^*}}\int_0^{\bar{L}} q_R f dL dR\right)}{\pi_r \int_0^{\bar{R}}\int_0^{1-R} f dL dR + \pi_l \int_0^{\bar{L}}\int_0^{1-L} f dL dR + \pi_H\left(\int_{\frac{\eta^*\bar{L}}{1-\eta^*}}^{\bar{R}}\int_0^{\frac{(1-\eta^*)R}{\eta^*}} f dL dR + \int_0^{\frac{\eta^*\bar{L}}{1-\eta^*}}\int_0^{\bar{L}} f dL dR\right)} \end{aligned} \tag{11}$$

The LHS is continuous, strictly increasing and $LHS \in [0,1]$. The RHS is continuous, and as $\eta^* \to 0$, equations (9) and (10) imply that $\bar{R} \to 0$ and $\bar{L} \to 1$; therefore the $RHS \to \frac{\int_0^1\int_0^{1-L} \frac{R}{R+L} f dL dR}{\int_0^1\int_0^{1-L} f dL dR} > 0$. Similarly, as $\eta^* \to 1$, by equations (9) and (10)

$\bar{R} \to 1$ and $\bar{L} \to 0$; therefore, $RHS \to \frac{\int_0^1 \int_0^{1-R} \frac{R}{R+L} f dL dR}{\int_0^1 \int_0^{1-R} f dL dR} < 1$. Therefore, there exists η^* that solves equation (11). To show the uniqueness we can take the derivative of the RHS of equation (11) with respect to η^* and evaluate it at the point at which $\eta^* = RHS(\eta^*)$. We have $\frac{dRHS}{d\eta^*} = \frac{\partial RHS}{\partial \eta^*} + \frac{\partial RHS}{\partial \bar{R}} \frac{d\bar{R}}{d\eta^*} + \frac{\partial RHS}{\partial \bar{L}} \frac{d\bar{L}}{d\eta^*}$, and using equation (9) and equation (10) we can show that for $\eta^* = RHS(\eta^*)$ we have $\frac{\partial RHS}{\partial \eta^*} = 0$, $\frac{\partial RHS}{\partial \bar{R}} = 0$, and $\frac{\partial RHS}{\partial \bar{L}} = 0$. Every time $\eta^* = RHS(\eta^*)$, the derivative $\frac{dRHS}{d\eta^*} = 0$; therefore, there is at most one solution. ■

Proof of Proposition 6

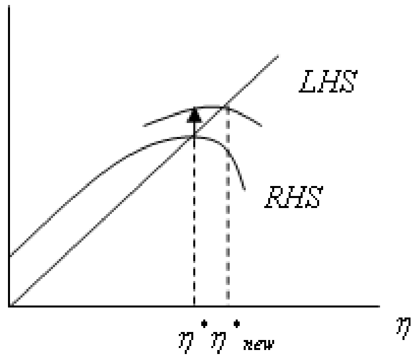
Equation (6) characterizes η_R^*. It does not depend on π_l; therefore, we can take the limit of equation (6) keeping π_r constant. We get $\lim_{\pi_H \to 1} \eta_R^* = E\left[\frac{R}{R+L}|R\right]$, and $\lim_{\pi_H \to 0} \eta_R^* \to 1$. The definition of λ_R, $\eta_R^* \equiv \frac{R}{R+\lambda_R}$, implies that $\lim_{\pi_H \to 0} \lambda_R = 0$; that is, the reports of the persuader become more extreme, and that $\lim_{\pi_H \to 1} \lambda_R = \bar{\lambda}_R = \frac{1-E[q_R|R]}{E[q_R|R]} R < 1 - R$. The analogous holds for P_l.

If we keep $\frac{\pi_H}{1-\pi_r}$ constant, the shape of the ambiguity area for P_l remains unchanged, which can be seen if we investigate the analog of equation (6) for P_l. Keeping $\frac{\pi_H}{1-\pi_r}$ constant implies that π_H increases as π_r decreases; therefore, equation (6) implies that the ambiguity area of P_r shrinks. That means that $\lim_{\pi_r \to 0} \lambda_R \to 0$, which means that the reports of P_r become more extreme. Now, I will show that η^* increases as π_r goes down.

η^* is determined by equation (11), where $\bar{R}$ and $\bar{L}$ are determined by equations (9) and (10). If we take the derivative of the RHS of (11) with respect to π_r and evaluate at η^*, we get

$$\begin{aligned}
\frac{dRHS}{d\pi_r}|_{\eta^*} &=_{sign} \frac{1}{(1-\pi_r)} \int_0^{\bar{R}} \int_0^{1-R} \left(\frac{R}{R+L} - \eta^* \right) f dL dR + \\
&+ \left(\pi_r \int_0^{1-\bar{R}} \left(\frac{\bar{R}}{\bar{R}+L} - \eta^* \right) f dL + \pi_H \int_0^{\frac{1-\eta^*}{\eta^*}\bar{R}} \left(\frac{\bar{R}}{\bar{R}+L} - \eta^* \right) f dL \right) \frac{d\bar{R}}{d\pi_r} \\
&= \frac{1}{(1-\pi_r)} \int_0^{\bar{R}} \int_0^{1-R} \left(\frac{R}{R+L} - \eta^* \right) f dL dR < 0,
\end{aligned}$$

where the last equality comes from using equation (9). Recall that η^* is the point of intersection of the $LHS(\eta)$ and the $RHS(\eta)$ of equation (11) for the initial π_r, like in the picture below.



The fact that $\frac{dRHS}{d\pi_r} < 0$ implies that as π_r decreases the function $RHS\left(\eta\right)$ shifts up, which implies that the new $\eta^*_{new} > \eta^*$.

This implies that when π_r goes down, P_r is better-off, and P_l is worse-off. As π_r decreases, η^* and η^*_R for each R increase, which implies that for each R the persuader toward *Right* can induce a higher belief. Since the shape of the ambiguity area for P_l is not affected for big L, P_l can induce the same belief. However, the belief in the double ambiguity area is higher, and it is achieved for lower $\bar{L}_{new} < \bar{L}$ which means that for $L < \bar{L}$, P_l induces higher beliefs than before.

Showing that the utility of the decision maker increases requires some tedious algebra, which I omit here, but the result is intuitive, since it is more likely that the decision maker faces the honest expert. ■

Proof of Proposition 7

Given the assumptions on $g\left(q_R, N; z\right)$, we can derive the distribution of $\left(N, R\right)$: $g\left(N, R; z\right) = \frac{g(N;z)}{N}$, and the conditional distribution of N given R is $g\left(N|R; z\right) = \frac{\frac{g(N;z)}{N}}{\int_R^1 \frac{1}{N} g(N;z) dN}$. The threshold R for which the decision maker with parameter θ_i is indifferent between alternatives, R_i, is defined by $\theta_i = \eta^*_{R_i}$, and by the definition of $\eta^*_{R_i}$ one gets

$$\theta_i > E\left[\frac{R_i}{N}\right] = \frac{\int_{R_i}^1 \frac{R_i}{N} \frac{1}{N} g\left(N; z\right) dN}{\int_{R_i}^1 \frac{1}{N} g\left(N; z\right) dN}.$$

This can be rewritten as

$$\theta_i > \frac{\int_{R_i}^1 \frac{R_i}{N} \frac{1}{N} g\left(N; z\right) dN}{\int_{R_i}^1 \frac{1}{N} g\left(N; z\right) dN} = \frac{R_i E\left[\frac{1}{N^2} | N > R_i; z\right]}{E\left[\frac{1}{N} | N > R_i; z\right]} > \frac{R_i}{\bar{N}} \Rightarrow 2R_i < \theta_i.$$

The utility of the decision maker with the preference parameter θ_i is

$$
\begin{aligned}
U &= \int_0^{R_i}\int_0^1 (1+\theta_i - q_R)\, g(N;z)\, dN dq_R \\
&\quad +\pi \int_{R_i}^1 \int_0^{\frac{R_i}{q_R}} (1+\theta_i - q_R)\, g(N;z)\, dN dq_R + \pi \int_{R_i}^1 \int_{\frac{R_i}{q_R}}^1 (1-\theta_i + q_R)\, g(N;z)\, dN dq_R \\
&\quad +(1-\pi)\left(\int_{R_i}^{\theta_i}\int_0^1 (1+\theta_i - q_R)\, g(N;z)\, dN dq_R + \int_{\theta_i}^1 \int_0^{\frac{R_i}{q_R}} (1+\theta_i - q_R)\, g(N;z)\, dN dq_R\right) \\
&\quad +(1-\pi)\int_{\theta_i}^1 \int_{\frac{R_i}{q_R}}^1 (1-\theta_i + q_R)\, g(N;z)\, dN dq_R.
\end{aligned}
$$

R_i is chosen to maximize U; therefore, $\frac{dU}{dz} = \frac{\partial U}{\partial z} + \frac{\partial U}{\partial R_i}\frac{dR_i}{dz} = \frac{\partial U}{\partial z}$. We get

$$
\frac{dU}{dz} = \pi \int_{R_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R + (1-\pi)\int_{\theta_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R.
$$

The second part is positive because $2(\theta_i - q_R) < 0$ and because $G_z\left(\frac{R_i}{q_R};z\right) < 0$ if $2R_i < \theta_i$. The first part can be rewritten as

$$
\begin{aligned}
\int_{R_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R &= \int_{R_i}^{2R_i} 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R + \\
+\int_{2R_i}^{\theta_i} 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R + &\quad \int_{\theta_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R
\end{aligned}
$$

We have $G_z\left(\frac{R_i}{q_R};z\right) < 0$ if and only if $q_R > 2R_i$. Hence, the first and the last parts are positive. Note that $R_i < \frac{R_i}{1-\frac{R_i}{\theta_i}} < 2R_i$; therefore, we can rewrite

$$
\begin{aligned}
&\int_{R_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R \\
&= \int_{R_i}^{\frac{R_i}{1-\frac{R_i}{\theta_i}}} 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R + \int_{\frac{R_i}{1-\frac{R_i}{\theta_i}}}^{2R_i} 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R \\
&\quad + \int_{2R_i}^{\theta_i} 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R + \int_{\theta_i}^1 2(\theta_i - q_R)\, G_z\left(\frac{R_i}{q_R};z\right) dq_R.
\end{aligned}
$$

By symmetry of G we have

$$\int_{\frac{R_i}{1-\frac{R_i}{\theta_i}}}^{2R_i} G_z\left(\frac{R_i}{q_R};z\right) dq_R + \int_{2R_i}^{\theta_i} G_z\left(\frac{R_i}{q_R};z\right) dq_R = 0,$$

which completes the argument that $\frac{dU}{dz} > 0$. Full revelation of information in the case of $\lim z = \infty$ is a straightforward result. ■

References

[1] Anderson, Thomas W. and Linda L. Golden (1984), "Bank Promotion Strategy," *Journal of Advertising Research,* 24, 53-65.

[2] Battaglini, Marco (2002), "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica,* 70, 1379-1401.

[3] Benabou, Roland and Jean Tirole (2002), "Self-confidence and personal motivation," *Quarterly Journal of Economics,* 117, 871–916.

[4] Blume, Andreas (1994), "Equilibrium Refinements in Sender-Receiver Games," *Journal of Economic Theory,* 64, 66-77.

[5] Carlsson, Hans and Eric van Damme (1993), "Global Games and Equilibrium Selection," *Econometrica,* 61, 989-1018.

[6] Chakraborty, Archishman and Rick Harbaugh (forthcoming), "Comparative Cheap Talk," *Journal of Economic Theory.*

[7] Crawford, Vincent P. and Joel Sobel (1982), "Strategic Information Transmission," *Econometrica,* 50, 1431-1451.

[8] Crowley, Ayn E. and Wayne D. Hoyer (1994), "An Integrative Framework for Understanding Two-sided Persuasion," *Journal of Consumer Research,* 20, 561-574.

[9] Dimitrakas, Vassilios and Yianis Sarafidis (2006), "Cheap talk from a Sender with Unknown Motives," mimeo, INSEAD.

[10] Farrell, Joseph (1993), "Meaning and Credibility in Cheap Talk Games," *Games and Economic Behavior,* 5, 514-531.

[11] Glazer, Jacob and Ariel Rubinstein (2001), "Debates and Decisions: On a Rationale of Argumentation Rules," *Games and Economic Behavior,* 36, 158-173.

[12] Glazer, Jacob and Ariel Rubinstein (2004), "On Optimal Rules of Persuasion," *Econometrica,* 72, 1715-1736.

[13] Gneezy, Uri (2005), "Deception: The Role of Consequences," *American Economic Review,* 95, 384-394.

[14] Grossman, Sanford J. (1981) "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law and Economics,* 24, 561-483.

[15] Grossman, Sanford, J. and Olivier D. Hart (1979), "Disclosure Laws and Takeover Bids," *The Journal of Finance,* 35, 323-334.

[16] Hovand, Carl I., Arthur A. Lumsdaine and Frederick D. Sheffield, *Experiments on Mass Communication*, Princeton University Press, 1949, 201-227.

[17] Kartik, Navin (2005), "Information Transmission with Almost-Cheap Talk," mimeo, University of California, San Diego.

[18] Kartik, Navin, Marco Ottaviani and Francesco Squintani (2006), "Credulity, Lies, and Costly Talk," *Journal of Economic Theory,* forthcoming.

[19] Krishna, Vijay and John Morgan (2001), "A Model of Expertise," *The Quarterly Journal of Economics,* 116, 747-775.

[20] Levy, Gilat and Ronny Razin (2006), "On the Limits of Communication in Multidimensional Cheap Talk: A Comment," *Econometrica,* 75, 885–893.

[21] Li, Ming and Kristóf Madarász (2006), "Does Disclosure Help? Cheap Talk and Conflict of Interest," mimeo.

[22] Lumsdaine, Arthur A. and Irving L. Janis (1953), "Resistance to 'Counterpropaganda' Produced by One-sided and Two-sided 'Propaganda' Presentations," *The Public Opinion Quarterly,* 17, 311-318.

[23] Matthews, Steven A., Masahiro Okuno-Fujiwara and Andrew Postlewaite (1991), "Refining Cheap-Talk Equilibria," *Journal of Economic Theory,* 55, 247-273.

[24] Milgrom, Paul R. and John Roberts (1986), "Relying on the Information of Interested Parties," *The RAND Journal of Economics,* 17, 18-32.

[25] Milgrom, Paul R. (1981), "Good News and Bad News: Representation Theorems and Applications," *The Bell Journal of Economics,* 12, 380-391.

[26] Morgan, John and Phillip C. Stocken (2003), "An Analysis of Stock Recommendations," *RAND Journal of Economics,* 34, 183–203.

[27] Morris, Stephen (2001), "Political Correctness," *Journal of Political Economy,* 109, 231–265.

[28] Pechmann, Cornelia (1992), "Predicting When Two-Sided Ads Will Be More Effective Than One-Sided Ads: The Role of Correlational and Correspondent Inferences," *Journal of Marketing Research,* 29, 441-453.

[29] Rabin, Matthew (1990), "Communication between Rational Agents," *Journal of Economic Theory,* 51, 144-170.

[30] Shin, Hyun Song (1994), "The Burden of Proof in a Game of Persuasion," *Journal of Economic Theory,* 64, 253-264.

[31] Smith, Robert E. and Shelby D. Hunt (1978), "Attributional Processes and Effects in Promotional Situations," *Journal of Consumer Research,* 5, 149-158.

[32] Sobel, Joel (1995), "A Theory of Credibility," *Review of Economic Studies*, 52, 557-573.

[33] Wolinsky, Asher (2003), "Information Transmission when the Sender's Preferences are Uncertain," *Games and Economic Behavior,* 42, 319-326.