Policy Dynamics and Inefficiency in a Parliamentary Democracy with Proportional Representation

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Abstract

This paper presents a dynamic model of election, government formation, and legislation in a parliamentary democracy with proportional representation in which the policy chosen in one period becomes the status quo for the next period. The electorate votes strategically by taking into account the likely governments that parties would form and the policies they would choose as a function of the status quo. The status quo also affects the bargaining power of the parties during government formation and their respective policy choices. A formateur party thus has incentives to strategically position the current policy to gain an advantage in both the next election and the subsequent government formation. These incentives can give rise to centrifugal forces that result in policies that are outside the Pareto set of the parties.

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1 Introduction

How political constitutions influence policy choices is of central importance in the field of political economy. Considerable progress has been made in developing comparative models of political institutions to predict the induced policy choices (Persson and Tabellini 2000, 2003). While these models have been successful in comparing certain dimensions of political institutions, e.g., by comparing parliamentary and presidential systems, other constitutional differences have received less attention. For example, there is only a limited understanding of whether there is an institutional explanation for the significant variation in economic policies and performance across parliamentary democracies. One reason for this is the absence of general models of policy choice in parliamentary democracies, especially for the modal case of multi-party systems under proportional representation. Policy choice in a parliamentary democracy is affected by each of the three principal government institutions–elections, government formation, and parliamentary authority. This paper provides a dynamic theory of policy choice, representation, and government formation to explain how policy in one period is affected by the incentives in future periods arising from the three institutions.

The perspective taken is one of bargaining among political parties over government formation and policy choice with representation determined by proportional representation. The bargaining in the theory is strongly coalition efficient in the sense that the policy maximizes the aggregate multi-period utility of the governing coalition, yet the policy can be strongly inefficient. That is, the policy can be outside the single-period Pareto set of the parties and of the voters. In contrast to political systems with plurality elections in which institutions exert a centripetal force on policy, the institutions of multi-party parliamentary systems with proportional representation exert a centrifugal forces on policy choices. Centrifugal forces arise from coalition bargaining and from elections, but electoral incentives also limit the extent of those forces and of the resulting inefficiency. Pareto inefficiency is always associated with majoritarian governments and never with consensus governments, but even with a consensus government the centrifugal force can move policy away from the center of preferences.

The institutions of multi-party parliamentary democracies create incentives for both parties and voters to act strategically. Such incentives are well-understood in the case of plurality rule elections, but they are also present under proportional representation. First, incentives for strategic voting are created by minimum representation thresholds (Cox 1997, Austen-Smith and Banks 1988, Baron and Diermeier 2001). Supporters of parties with a small effective vote share may abandon their most preferred party and vote for a party with a larger expected vote share to avoid wasting their vote. These incentives are similar to those present in plurality systems that lead to phenomena such as Duverger's Law.[1] Second, with proportional representation it is rare for one party to capture a majority of seats,[2] which means that parties must form coalitions to govern. Because governments and their policies are the consequence of multi-party coalition bargaining, voters base their vote not on a party's announced platform or policy preferences but on the policies expected to be chosen by the governing coalitions that may form once a new parliament has been elected. A moderate supporter of a conservative party, for example, may prefer a coalition government of the conservative party with a centrist party over a single-party conservative government. So, in cases in which the conservative party is close to gaining an absolute majority of seats, the voter may be (weakly) better off voting for the (second preferred) centrist party. Finally, if there is an expectation that no party will obtain a majority of seats, supporters of a small party may choose to vote for a larger party instead if they expect that this will make it more likely that the small party will be included in the governing coalition. This may occur, for example, if the large party is expected to be selected as the formateur. A supporter of a small party may then rationally vote for the larger party.[3] This paper identifies the policy consequences of such sophisticated voting by comparing it to sincere voting for the closest party.

These incentives have been considered in full-equilibrium models of parliamentary democracies by Austen-Smith and Banks (1988), Baron and Diermeier (2001), and Schofield and Sened (2006). If these models agents are strategic, and the outcome corresponds to a subgame perfect Nash equilibrium in government formation, policy choice, and elections. This literature, however, has not considered a potentially important incentive for strategic behavior. Incumbent governments may strategically position the current government policy

[1] See Cox (1997) for a detailed overview.

[2] The empirical regularity that under proportional representation there seldom is a majority party needs to be explained. Baron and Diermeier provide an explanation in the context of a one-period model. This issue is discussed here in the final section.

[3] For empirical evidence on the use of sophisticated voting strategies in proportional representation systems see Cox (1997) and especially Bawn (1999).

to influence the outcome of the next election, as well as the subsequent government formation and policy choice.[4] This requires a dynamic analysis, where dynamics means that the policy chosen in one period become the status quo for the next period and any new policy must defeat that status quo. An incumbent government thus chooses a policy in the current period that both responds to its policy preferences and positions the status quo to its advantage in the next election and government formation cycle.

Few political economy models have addressed the dynamics of policy choice, and fewer yet have incorporated elections. Baron (1996) considers a dynamic model with a unidimensional policy space and without elections and shows that the policy choices converge to the ideal policy of the median legislator. Baron and Herron (2003) consider a two-dimensional policy space with no elections and both analytically and computationally study the government coalitions that form and their policies in a model with a finite horizon. Discontinuities in the value functions, however, precluded a general characterization of equilibria. In contrast to these models, the present paper incorporates transferable benefits in addition to policy choice, and this allows explicit characterization of equilibria.

Kalandrakis (2004) characterizes a Markov perfect equilibrium for a repeated divide-the-dollar game in which the status quo for one round is the allocation of the dollar in the previous round. He shows that the equilibrium transitions to allocations in which the proposer in each period takes all of the dollar. In his model preferences are linear, however, so there is no efficiency incentive to attain an allocation that responds to the policy preferences of coalition members. Bernheim, Rangel, and Rayo (2006) consider a more general, finite-horizon model in which the status quo in a round is the winner from the previous round. They show that the last proposer essentially obtains its ideal outcome, and hence the outcome is not efficient unless the model is pure distribution. Neither of these models includes an election. In the model presented in this paper the parties also choose a policy that must be voted against the status quo, which is the policy in place in the previous period, but policy preferences are not linear and an election provides additional incentives for strategic behavior.

[4] See, however, Fong (2006) who uses the Baron-Diermeier model to analyze the incentives of parties to strategically position the current policy to influence the bargaining over government formation and legislation in the next period. In contrast to Baron and Diermeier, however, Fong assumes sincere voting. This model is discussed in more detail below.

Battaglini and Coate (2005) characterize a Markov perfect equilibrium for a dynamic model in which a legislature chooses the stock of a public good financed by distortionary taxes. In each period the legislature can increase or decrease the stock of the public good, so the legislature votes between the change in the stock and no change. Battaglini and Coate (2006) consider a legislature that can spend on both pork and a public good and can finance the spending with distortionary taxes and debt. Because of shocks to the value of the public good, inefficiency can result with too little spending on the public good and taxes and debt that are too high. The political system they consider is a legislature in office, so elections are not considered.

Riboni (2005) characterizes a Markov perfect equilibrium for a model in which the current status quo is the state variable, and any new policy must defeat that status quo. In contrast to Riboni, who assumes that the agenda setter is fixed over time, in the model presented here the identity of the agenda setter is determined endogenously as a result of an election and the selection of a formateur. Riboni shows that the heterogeneity of preferences of voters on a committee can give rise to effective commitment for monetary policy.

Penn (2005) takes a different approach to studying dynamics by considering the long-run preferences of players in a collective choice setting. In her model a proposal in the current period is voted against the status quo, which is the outcome of the vote in the previous period. In contrast to the model considered here, proposals in her model are generated exogenously rather than by the players. Her focus is on generating the value functions for players as the game continues indefinitely. She presents a computed spatial example in which policy outcomes are all in the Pareto set and are close to the stable set. In the model presented here the formateur can have incentives to propose policies outside the Pareto set to advantage itself in the subsequent election and government formation process.

Besley and Coate (1998) consider a two-period citizen-candidate model in which the citizen elected in the first period can invest in a public good that has a cost in the first period and a return in the second period. In the equilibrium, the identity of the citizen who will be elected in the second period may not be known with certainty because of the possibility of a tie. In that case, the first-period elected official may not undertake a Pareto-improving investment in a public good for fear that in the second period a citizen with opposing preferences will be elected and not compensate those who bore the cost of the public good in the first period. They view this as a political failure. In the model considered here, costs and benefits are contemporaneous, and any inefficiency is due to the incentives provided by the institutions of the parliamentary system.

To analyze the dynamics of representation, government formation, and policy choice, a multi-period equilibrium model of parliamentary democracies is developed based on Baron and Diermeier. A period corresponds to an interelection period and consists of an election, a government formation stage, and the choice of a policy by parliament. A key feature of that model is that the equilibrium policies in a period are completely determined by that period's initial status quo policy. This induces a dynamic process of policy change where the current government's policy determines the status quo for the next period. Parties recognize that the current policy choice has consequences for the next period, so the formateur of the government in the current period has an incentive to choose the policy strategically to create an advantage in the next period election and government formation process. The formateur could choose a policy that yields it a majority in the election, but if the parties are sufficiently impatient, the optimal policy choice for the formateur is not to position the policy so that it receives a majority in the next period. Rather, it is optimal for the formateur to position the status quo to balance its current period policy preferences with the anticipated electoral outcome and its bargaining power in the next coalition government. The model thus can account for the fact that parliamentary democracies under proportional representation rarely yield a parliament with a majority party, even if the current government can strategically position the status quo for future elections and voters act strategically. If the parties are sufficiently patient over the interelection period, however, the formateur can prefer to position the current period policy so that it receives a majority in the next election. Thus, the theory is consistent with the absence of majority parties provided parties are politically impatient.[5]

Parties that take into account future political choices in addition to the present ones have centrifugal incentives that result in more extreme policies than would be chosen by myopic parties. These policies are chosen not only to favor the formateur in the current period but also to disadvantage rival parties in the next election and the subsequent government formation phase. How extreme the policies are depends on how important the future is relative to the present. The more patient the parties, the more extreme are the policies

[5] Alternative explanations of the absence of majority parties are considered in the final section. See Diermeier, Eraslan, and Merlo (2003) for empirical support for impatient parties.

chosen in the first period. Policies outside the Pareto set, however, are only identified with majoritarian and single-party governments. These extreme policies can lead to a consensus government and a centrist policy in the next period. For example if the formateur in the current period chooses a policy, such that if selected as the formateur, it would form a consensus government whereas the other parties if selected as formateur would form governments with less central policies, a majority of voters prefer to vote for that party.

This paper thus identifies a strong form of inefficiency in policy-making that stems from the interaction of two institutions–elections and government formation. Parties have electoral incentives to obtain greater representation in parliament, since the likelihood of being selected as the formateur is weakly increasing in representation. Parties also have government formation incentives to position themselves favorably for the bargaining over policy and office-holding benefits in the next period. The instrument available to parties to respond to these incentives is the policy chosen in the current period.

2 The Model

The model is specified to identify how incentives from political institutions can exert a centrifugal force on policies and lead to inefficiency. This requires that the model be formulated in a neutral manner so that no political party has a natural electoral advantage and no coalition is more likely to form than another because, for example, of a preference alignment between parties.[6] The extent, as opposed to the existence, of the centrifugal force depends on a number of model specifications, such as the bargaining protocol, so the paper focuses on identifying a maximal effect of institutions on policy. The bargaining protocol for government formation thus is specified as the selection of a formateur with the formateur then making a take-it-or-leave-it offer to the other parties. The bargaining and electoral effects are present in the absence of these specifications, and only their magnitudes are affected by the modeling choices. Informally, these magnitudes are bounded by the equilibrium identified.

The political system consists of large but finite (and even) number N of voters, and three political parties labeled a, b, and c. The political system selects a two-dimensional policy $x \in \Re^2$ in each of two periods, where a period corresponds to an interelection period.

[6] A unidimensional policy space thus cannot be used.

The choice in a period is made by a government formed among those parties that have representation in parliament as determined by a proportional representation election. A government consists of a coalition of parties with a majority of seats in parliament. Neither voters nor parties can commit to their future actions. A two-period model is sufficient to identify the incentives for strategic behavior on the part of the political parties, and the analysis begins after the first-period election and government formation, since the policy choice in period one depends only on the coalition and the formateur.

A party may be thought of as consisting of a leader supported by a group of party activists with similar preferences. In a period t party $i \in \{a, b, c\}$ derives utility from both policy x_t and the redistribution of office-holding benefits $y_t^i \in \Re$, where $y_t^a + y_t^b + y_t^c = 0$ and $y_t^i = 0$ if party i is not in government.[7] An important assumption in this model is that the reallocation of office-holding benefits can only be made among the parties in the government. That is, the parties in government can neither collect benefits from nor credibly promise to compensate the out party. The office-holding benefits are assumed to matter to parties but not to voters, for example, the benefits could be fixed in the aggregate. They could take a variety of forms - Baron and Diermeier (p. 935) give some illustrative examples: "These benefits include jobs for party stalwarts, board seats on public companies or the national television system, and transfers to interest groups and party foundations. Again, consider Germany; all the major parties (as well as interest groups like churches and labor unions) occupy seats on the supervisory boards of the national television system and major corporations (such as Volkswagen). Moreover, each major party receives substantial amounts of public money for its research and education foundations. Similar arrangements are common in many other parliamentary democracies, especially Austria and Italy."

Party preferences are assumed to be quasilinear, and the expected discounted sum of utility of party i is by

$$E\left[\sum_{t=1}^{2} \beta^{t-1}\left(y_t^i + u^i\left(x_t\right)\right)\right],$$

where $u^i(\cdot)$ represents single-period policy preferences and $\beta \in [0, 1]$ is a common discount factor and the expectation is over the selection of formateur in the second period and any possible mixing of strategies. The discount factor may be interpreted as political patience

[7] It is implicitly assumed that each party is originally endowed with sufficient office-holding benefits to satisfy any proposal made by the formateur. This assumption simplifies the analysis and yields efficient bargaining within coalitions.

by party leaders. Since an interelection period may extend for four years, for example, β can be low if party leaders are impatient. Political patience could differ across countries and also within a country depending on the tenure, age, and other factors associated with party leaders.

A party is assumed to have lexicographic preferences over its status as formateur and utilities derived from policies and office-holding benefits. A party desires a higher level of utility regardless of the chances that it will be selected as formateur, but whenever the utility is the same for two distinct combinations of policies and benefits, it prefers the proposal in which it is more likely to be selected as formateur in the following period. This assumption is technical and helps eliminate additional equilibria in the first period. Substantively, it amounts to a party's lexicographic preference to head a government.

For the sake of tractability the policy preferences are assumed to be quadratic

$$u^i(x) = -\left\|x - z^i\right\|^2,$$

where $z^i \in \Re^2$ denotes party i's ideal point. Thus, parties not only prefer policies closer to their ideal points, but they are more averse to policy changes the farther those changes are from their ideal points.

To avoid preference alignments among the parties, the ideal points of the three parties are assumed to be located at the vertices of an equilateral triangle. As indicated above, this specification allows the dynamics induced by the institutions to be isolated. Without loss of generality the policy space is normalized so that $\left\|z^i - z^j\right\| = 1$ for all $i, j = a, b, c$, and $i \neq j$. The policy $\overline{z} \equiv \frac{1}{3}\sum_{i=1}^{3} z^i$ is the center of party preferences (or the centroid).

Voters care only about policy outcomes. The preferences of voter v in period t are represented by a time-separable utility function $u^v(x_t)$ of the same form as those of the parties. That is, the parties are formed among the electorate. The expected discounted utility of a voter is given by

$$E\left[\sum_{t=1}^{2} \delta^{t-1} u^v(x_t)\right],$$

where $\delta \in [0,1]$ is a discount factor that may differ from that of the political parties and their leaders. A voter v is characterized by his ideal point $z^v \in \Re^2$. So that voter preferences do not favor a particular party or coalition, the ideal points of voters are assumed to be uniformly distributed on a disk $\mathcal{Z} \equiv \left\{z^v \in \Re^2 : \|z^v - \overline{z}\| < L\right\}$, where $L > \frac{1}{\sqrt{3}}$.[8]

[8] The assumption $L > \frac{1}{\sqrt{3}}$ guarantees that the ideal points of the parties are not more extreme than those

Timing

An interelection period consists of three stages. The first stage is a parliamentary election that determines the seat shares of the parties in the parliament. The second stage is government formation, and the third stage is legislative and involves the choice of a policy by the parliament. Let q_{t-1} denote the status quo at the beginning of period t, where q_0 is the initial status quo. The game has complete information, and no player can commit to an action in the next period.

Parliamentary Election Stage The electoral system is proportional representation with a minimum vote share m required for representation, where $m \in \left(0, \frac{1}{4}\right)$. The restriction $m < \frac{1}{4}$ allows all three parties to be represented in a parliament with a majority party. If the vote shares ρ_t^i of all parties are at least m, their seat shares s_t^i are $s_t^i = \rho_t^i$. If only party j's vote share is less than m, it is not represented in parliament and the other parties have seat shares $s_t^i = \frac{\rho_t^i}{1-\rho_t^j}$, $i \neq j$. If two parties have vote shares less than m, the other party has a seat share of 1.

Government Formation Stage After an election one party is selected as the formateur. Selection is governed by a proportionality rule with the probability of selection equal to the party's seat share in parliament, unless one party has a majority of seats in which case it is selected as the formateur.[9] The formateur in period t has the opportunity to form a government, which is a coalition C_t, i.e., a non-empty subset of the parties represented in parliament such that $\sum_{i \in C_t} s_t^i > \frac{1}{2}$. We assume that if a formateur is indifferent between two two-party governments, it flips a coin. Moreover, if it is indifferent between a two-party government and a consensus government, it chooses the latter. As will be evident in the analysis, the equilibrium policy choice conditional on a consensus government leads to a greater aggregate utility of all parties than the policy choice conditional on any two-party government. This assumption thus strengthens our inefficiency results.

Legislative Bargaining Stage As indicated above, consistent with the objective of identifying the maximal effect on policy, in forming a government the formateur is assumed to make a take-it-or-leave-it offer to the other members of the coalition. The offer specifies a policy proposal $x_t \in \Re^2$ the government will implement if formed, and an alloca-

of the most extreme voters.

[9] Diermeier and Merlo (2004) present empirical evidence supporting a proportionality rule with some support for an incumbency advantage.

tion of office-holding benefits; i.e., who is to transfer office-holding benefits to whom.[10] The formateur as head of government has an incentive to make an offer the coalition prefers to the status quo, so when the government proposes the new policy x_t, the coalition votes for it and the office-holding benefits are allocated as proposed. A new period $t+1$ then begins with the status quo $q_t = x_t$. If any party in the coalition rejects the offer, the status quo q_{t-1} is the policy in period t, and no redistribution of the office-holding benefits is made.[11] The status quo for period $t+1$ then is $q_t = q_{t-1}$.

Terminology A parliament in which no party has a majority is referred to as a "minority parliament," whereas in a "majority parliament" one party has a majority of the seats. A "consensus government" includes all three parties, a "majoritarian government" is composed of two parties, and a "single-party government" is composed of a single majority party.[12]

3 Results

The subgame perfect equilibrium is characterized by backward induction. A legislative equilibrium is the outcome of government formation and policy choice supported by optimal strategies of the parties, and an electoral equilibrium is the party seat shares supported by strong Nash equilibrium voting strategies of the voters.

3.1 Electoral and Legislative Equilibrium in the Second Period

The lemmata below summarize results in Baron and Diermeier that characterize the equilibria for the second period. To facilitate our statement, we define $D^i \equiv \left\{ x \in \Re^2 : u^i(\mathbf{x}) > -\frac{1}{2} \right\}$, for all $i = a, b, c$, as the set of alternatives that yield party i a period utility greater than $-\frac{1}{2}$. If the status quo is in D^i, party j as formateur prefers to form a majoritarian government with party k rather than a consensus government.

Lemma 1 ***Legislative equilibrium for the second period*** *Consider the legislative bargaining stage in the second period with a status quo q_1 and a parliament with all three*

[10] The government may be understood as being of cabinet form in which all government parties must agree on the policy choice.

[11] An offer to form a consensus government is thus conditional on both coalition partners accepting the offer. If either rejects the offer, the status quo remains.

[12] The model effectively precludes the formation of minority governments.

parties represented. I:(A) A consensus government chooses the center of preferences $\overline{z}$. (B) A majoritarian government chooses the mid-point $z^{ij} = \frac{1}{2}\left(z^i + z^j\right)$, of the contract curve of the government parties' ideal points for all $i, j = a, b, c$, $i \neq j$. (C) A single-party government chooses the ideal point z^i of its member party i, for all $i = a, b, c$.[13] *II: For all $i, j, k \in \{a, b, c\}$, $i \neq j \neq k$, party i as formateur (1) forms a consensus government if $q_1 \in \Re^2 \setminus \left(D^j \cup D^k\right)$, (2) forms a majoritarian government with party j if $q_1 \in D^j \cup D^k$ and $u^j(q_1) < u^k(q_1)$, and (3) forms a majoritarian government with party j or party k with probability $\frac{1}{2}$ if $q_1 \in D^j \cup D^k$ and $u^j(q_1) = u^k(q_1)$. III: The joint utility of all three parties is -1 when a consensus government is formed, $-\frac{5}{4}$ when a majoritarian government is formed, and -2 when a single-party government is formed. In a minority parliament a majority party i chooses (A) a consensus government with policy $\overline{z}$ if $q_1 \notin D^j \cup D^k, j, k \neq i$, (B) a majoritarian government with the party j that is the more disadvantaged by q_1 (for $q_1 \notin D^j$) and with policy at the midpoint z^{ij} of the contract curve if $q_1 \in D^j \cup D^k$ and $q \notin D^j \cap D^k, j, k \neq i$, and (C) a single-party government with policy z^i if $q_1 \in D^j \cap D^k, j, k \neq i$.*

Lemma 2 ***Electoral equilibrium for the second period*** *Consider the parliamentary election stage in the final (second) period with a status quo q_1. (A) If only party i as formateur would form a consensus government, every strong Nash electoral equilibrium results in a majority parliament with three parties represented, where i is the majority party. The consensus government chooses policy $\overline{z}$. (B) If party i as formateur would form a majoritarian government with some of the other parties without randomization, and the other two parties j and k would form majoritarian governments with each other, a minority parliament results with a strong electoral equilibrium with vote shares $\rho_2^i = \frac{1}{2}$, $\rho_2^j + \rho_2^k = \frac{1}{2}$, and $\rho_2^j, \rho_2^k \in \left[m, \frac{1}{2} - m\right]$. (C) If all parties would form a consensus government, election of any three-party parliament is a strong electoral equilibrium and the policy is $\overline{z}$. (D-1) If party i as formateur would randomize between z^{ij} and z^{ik} and the other two parties would form governments with party i, a minority parliament results, and the strong electoral equilibrium yields a vote share $\rho_2^i = m$, equal vote shares for the other two parties, majoritarian governments, and policy outcomes z^{ij} and z^{ik} with probability one-half. (D-2) If party i as formateur would form a majoritarian government with each of the other parties with*

[13] A majority party, however, may not choose to form a single-party government.

probability $\frac{1}{2}$*, and the other two parties* j *and* k *would form majoritarian governments with each other, a minority parliament results with a strong electoral equilibrium with vote shares* $\rho_2^i = \frac{1}{2}$*,* $\rho_2^j + \rho_2^k = \frac{1}{2}$*, and* $\rho_2^j, \rho_2^k \in \left[m, \frac{1}{2} - m\right]$*. (E) If* $q_1 = \overline{z}$*, the unique strong equilibrium is equal vote shares for all three parties and an even lottery over* z^{ab}*,* z^{ac}*, and* z^{bc}*.*[14]

Lemmata 1 and 2 identify the intuition underlying both the bargaining within the coalition and the incentives to position the status quo strategically for the final period. A formateur in the final period could form a government with a policy at its ideal policy. It prefers, however, to form a majoritarian government or a consensus government because bargaining with the other two parties generates the transfer of office-holding benefits from those parties in exchange for a policy closer to their ideal points. For a consensus government this bargaining continues until the policy is equidistant from the three ideal points; i.e., at $\overline{z}$. For a majoritarian government the formateur chooses a policy at the midpoint z^{ij} of the contract curve with its coalition partner. Whether the formateur prefers to form a consensus government or a majoritarian government depends on the status quo q_1 for the second period.

If, for example, party a were the formateur in the second period and the status quo q_1^o were in $D^a \setminus \left(D^b \cup D^c\right)$, the formateur prefers to form a consensus government at $\overline{z}$ by Lemma 1-II(1), since both parties b and c are disadvantaged in the bargaining by the status quo. This is illustrated in Figure 1. If the status quo $\hat{q}_1$ were in $D^a \cup D^b$, party c is more disadvantaged by the status quo than is party b, so by Lemma 1-II(2) the formateur a prefers to form a government with c at z^{ac}, as illustrated in Figure 1. From the perspective of the formateur in the first period, consider the choice between governments with policies $x_1^o = q_1^0$ or $\hat{x}_1 = \hat{q}_1$. To examine this choice suppose party a is the period-one formateur. If party a forms a government with $\hat{x}_1$, in period 2 either party a or b as formateur would form a majoritarian government and choose a policy z^{ab} or z^{ac}, respectively, whereas party c as formateur in period 2 would randomize between majoritarian governments ac or bc with policies z^{ac} and z^{bc}, respectively. Instead of forming a government with policy $\hat{x}_1$, party a

[14] A couple of remarks. First, (B) has been restated and (E) added. Second, the equilibrium vote shares in case (D-2) are different from Proposition 4 of Baron and Diermeier (2001) due to a different population structure. Baron and Diermeier assumed that voters' ideal points were uniformly distributed in the single-period Pareto set of the parties, whereas here voters' ideal points are assumed to be uniformly distributed in the disk $\mathcal{Z} \equiv \left\{x \in \mathsf{R}^2 : \|\mathbf{z}^v - \overline{\mathbf{z}}\| < L\right\}$.

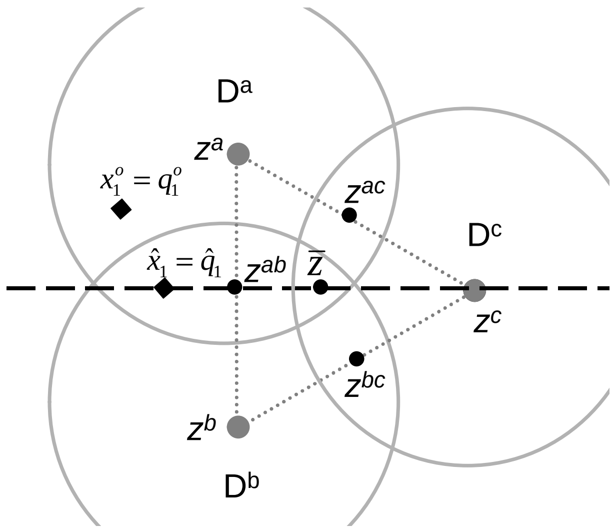

Figure 1: Second-period equilibrium: Bargaining effect.

could form a government with policy x_1^o. That policy would lead party a as a formateur in period 2 to choose a consensus government with policy $\bar{z}$. Since x_1^o is close to a's ideal policy, parties b and c would, if selected as the formateur in period 2, find each other to be more attractive government partners than a. They would then form a government with each other at z^{bc}. By Lemma 2-(A) party a would receive a majority of the vote in the election. A majority of voters vote for party a because they anticipate that as formateur it would choose the centrist policy $\bar{z}$ and the other parties would choose more extreme policies. Whether the government in the first period prefers to position the status quo so that one party receives a majority depends on the discount factor of the parties and is the subject of Sections 3.2.2 and 3.2.3.

To illustrate the election result, when a majoritarian government is anticipated with parties a and b having large vote shares, voters give party c just enough votes so that it is represented in the parliament. If it were not in parliament, parties a and b would each have half the seats. Each would, if selected as the formateur, form a government with the other at z^{ab}. Thus, if a voter with an ideal point near that of party c voted for a or b the policy z^{ab} would be implemented, whereas if she voted for c the policies would be z^{ac} and z^{bc} with probability one-half each, which is to her benefit.

Implied by Lemma 2 is a relationship between the bargaining advantage from the status quo and the advantage in a parliamentary election. In particular, the party that in the

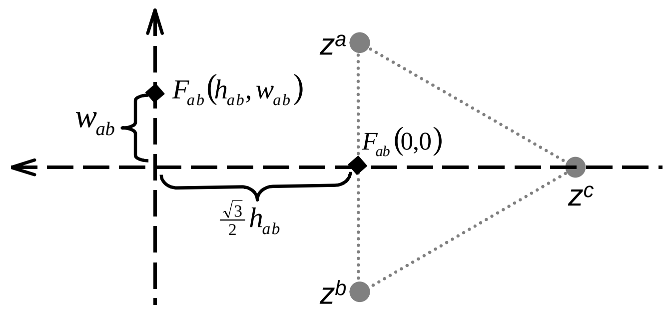

Figure 2: Transformation of coordinates: the F-representation.

bargaining is relatively disadvantaged by the status quo obtains a (weakly) lower expected seat share than the others. Similarly, the party that in the bargaining is relatively favored by the status quo obtains a (weakly) higher expected seat share than the other parties and therefore a greater chance of being recognized as formateur. This identifies an incentive for the period-one formateur to position strategically the policy for electoral advantage; i.e., an incentive to position the policy to disadvantage the out party and advantage itself.

3.2 Legislative Equilibrium in the First Period

3.2.1 Notation

To characterize the legislative equilibrium in the first period, let $H_i(C)$ denote the optimal policy choice by party i as formateur when it forms a government coalition C in the first period. Since parties care about their status as formateur (or heads of government) in addition to the policy, a formateur may propose a policy that yields a greater chance of getting more votes in the subsequent parliamentary election.

To facilitate the presentation of results, a notation system that locates the positions of different policies is used. For any $x \in \Re^2$ and for any distinct $i, j = a, b, c$, there exist $h_{ij}, w_{ij} \in \Re$ such that

$$x = F_{ij}(h_{ij}, w_{ij}) \equiv \frac{1}{2}\left(z^i + z^j\right) + \left(\frac{1}{2}\left(z^i + z^j\right) - z^k\right) h_{ij} + \left(z^i - z^j\right) w_{ij}.$$

(See Figure 2 for an illustration.) With the coordinate system of $F_{ij}(\cdot)$, any policy is described according to its position relative to the ideal points of all three parties. Note that if $h_{ij}>0$, the policy x is outside the Pareto set. For example, $F_{ab}(\frac{1}{\sqrt{3}}, 0) = (-\frac{1}{2}, 0)$.

3.2.2 Sincere Voting in Parliamentary Elections

Fong (2006) analyzed a version of this game in which in the parliamentary election each party receives one-third of the vote. Given the location of voter ideal points, his results correspond to those in a model with parliamentary elections in which each voter loyally casts her votes for the party whose ideal point is closest to hers.[15] In this case, voters are said to vote sincerely. Considering the case of sincere voting allows the incentives provided by sophisticated voting in the election to be identified separately from the effect of bargaining.

With sincere voting voters vote independently of the status quo, so parties cannot position the status quo to gain electoral advantage. Despite a fixed election outcome, parties have an incentive to position the status quo strategically for the next period so as to obtain a bargaining advantage if they are selected as the formateur in the next period. The equilibrium policy choices by different governments are illustrated in Figure 3.

Proposition 1 *Suppose that in the second period all voters vote sincerely; i.e., every party receives the vote from its natural constituency and therefore has a vote share of one-third. In the first period: (A) A consensus government chooses the center of preferences* $\overline{z}$*, and a majoritarian government then is formed in period two and chooses the mid-point of the government parties' contract curve as the policy. (B) Any majoritarian government* $C = \{ij\}$ *chooses a policy that is far from the ideal point of the out party* k*. Moreover, this policy is outside the single-period Pareto set of the three parties. In particular, there exists* $\overline{\beta} \in (0,1)$ *such that*

$$H_i(ij) = \begin{cases} F_{ij}\left(\frac{2\beta}{6-\beta}, 0\right) & \text{if } \beta \in \left[0, \overline{\beta}\right) \\ F_{ij}\left(\frac{1}{\sqrt{3}}, 0\right) & \text{if } \beta \in \left[\overline{\beta}, 1\right]. \end{cases}$$

For all $\beta \in \left[0, \overline{\beta}\right)$ *the policy choice is such that in the second period party* i *or* j *as formateur will form a majoritarian government with party* k*, whereas party* k *as formateur will randomize between majoritarian governments* ik *and* jk*. For all* $\beta \in \left[\overline{\beta}, 1\right]$*, the policy choice is such that in the second period a consensus government with policy* $\overline{z}$ *is formed with probability one.*

Proposition 1 identifies the formateur's incentive to position the status quo in the first period to gain a bargaining advantage. Suppose party a with $z^a = (0, \frac{1}{2})$ is the formateur in

[15] Baron (1993) makes this assumption in a model of endogenous party formation.

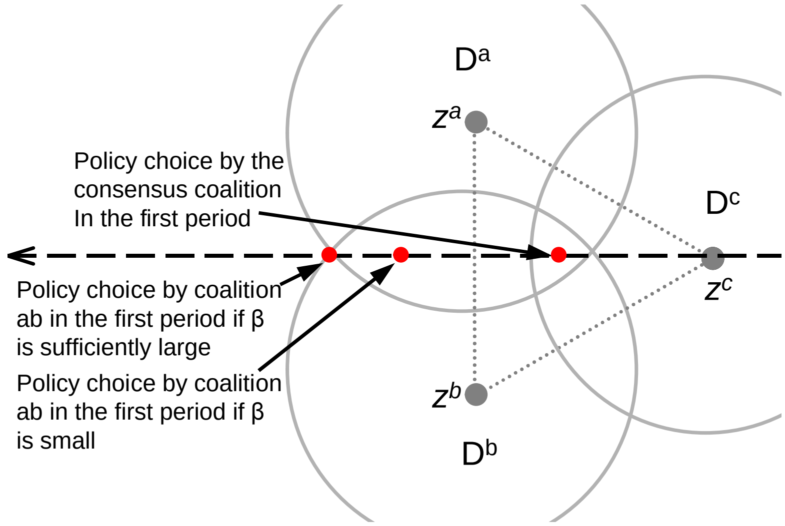

Figure 3: Equilibrium policy choices in the first period with sincere voting in parliamentary elections.

the first period, and suppose that a prefers to form a majoritarian government with party b with $z^b = (0, -\frac{1}{2})$. In the absence of a second period ($\beta = 0$), a myopic party a chooses $x = z^{ab} = (0, 0)$, the midpoint of the contract curve of the two parties. As the future becomes more important (β increases), party a chooses a policy equidistant from z^a and z^b but farther from the ideal policy of the out party c and thus outside the Pareto set of party preferences. Doing so reduces the joint period-one utility of parties a and b. However, this allows party a, if it is the formateur in period two, to form a government with party c and obtain greater office-holding, since the status quo disadvantages party c. Similarly, party b, if it is selected as the formateur in period two, chooses the policy z^{bc}. Party b is advantaged in period two, but in period one party a extracts the gain from b in the form of additional office-holding benefits, since the (expected) discounted gain to b in the second period is fully anticipated.[16] The formateur thus has an intertemporal tradeoff. As the parties care more about the future, party a as the period-one formateur chooses a more extreme policy. If the discount factor is sufficiently high (i.e., $\beta \geq \overline{\beta}$), party a chooses a sufficiently extreme policy that as the status quo it would trigger the formation of a consensus government. This allows party a, once recognized as the period-two formateur, to extract office-holding

[16] If party c is selected as formateur, it forms a majoritarian government with either party a or b and is able to extract more office-holding benefits from its government partner than if the status quo were $(0, 0)$. However, due to strict concavity of the utility functions, this is a second-order effect and is always dominated.

benefits from both of the other parties.

3.2.3 Strategic Voting in Parliamentary Elections

In this section the first-period legislative equilibrium is characterized for the case in which voters are strategic in parliamentary elections. That is, voters anticipate not only the election outcome, but also the possible outcomes of the government formation and policy choice stages. Voters then can reward parties for the policies the government coalitions they might form would choose, and this gives the government in the first period incentives to position the status quo to gain an advantage in the election. These incentives add an electoral effect to the bargaining effect identified with sincere voting and result in less efficient policies. Except in the case of a majority government, each voter is pivotal for representation in parliament and hence for the probability that a party is selected as formateur in the second period.

To identify the incentives and resulting behavior of the parties and to trace the effects on policy, it is sufficient to begin the analysis after the election and coalition choice in the first period. Equilibria are characterized for each type of government the formateur might form. Propositions 2, 3, and 4 characterize, respectively, the policy choice made by a consensus government, majoritarian government, and single-party government in the first period. In the second period for a large number of status quo alternatives, there are multiple electoral equilibria, which may lead to different distributions of period-two utilities of the parties. A complete description of a legislative equilibrium in the first period thus requires a specification of expectations formed by the parties about the period-two election outcome. Instead of enumerating all possible combinations of expectations and period-one legislative strategies, in what follows two assumptions are made about the selection of period-two electoral equilibria. First, if a period-two status quo is such that party i as formateur would form a consensus government with the central policy $\overline{z}$ whereas any of the other parties as formateur would be indifferent between forming a majoritarian government and a consensus government, the period-two electoral equilibrium is such that party i is elected as the majority party. This assumption applies only to a period-two status quo on the boundary of D^i for some i and outside the set of D^j for all $j \neq i$; for example, policy $\widetilde{x}_1$ in Figure 5 below. Second, for any other period-two status quo, if there are multiple electoral equilibria, all equilibria identified in Lemma 2 occur with equal probability. These selection rules assure that each period-two status quo is associated with unique expected period-two utilities of the parties and thus simplify the analysis.

Which type of government forms in the first period depends on the initial status quo q_0 and the identity of the formateur. The mapping from the initial status quo to the first-period election outcome, selection of formateur, and choice of government is both complex and discontinuous. A complete characterization of this mapping is not informative about the dynamics of either governments or policy, so the choice of first-period government is not considered here. The following propositions are thus conditional on the type of government and the formateur selected. The proofs are provided in the Appendix.

Proposition 2 *(A) If the first-period formateur forms a consensus government, the policy is* $\overline{z}$ *if the parties are sufficiently impatient; i.e.,* $H_i(abc) = \overline{z}$ *for* $i = a, b, c$ *and* $\beta \in \left[0, \widehat{\beta}\right)$ *where* $\hat{\beta} \equiv 4 - 2\sqrt{3} \approx 0.536$*. In the second period there is a minority parliament, and each majoritarian government is formed with probability one-third. The policy for any chosen government* jk *is the midpoint* z^{jk} *of the contract curve. (B) If* $\beta \in \left[\hat{\beta}, 1\right]$ *and the first-period formateur* i *forms a consensus government, the policy is* $H_i(abc) = F_{jk}(-\frac{1}{\sqrt{3}}, 0)$, $j, k \neq i$*. In the second period party* i *receives a majority vote share and as formateur forms a consensus government with policy* $\overline{z}$*.*[17]

The policies identified in Proposition 2 are illustrated in Figure 4. If parties are impatient ($\beta < \widehat{\beta}$), a consensus government in the first period chooses the centroid $\bar{z}$, and from Lemma 1 a majoritarian government results in the second period. With impatient parties the period-one formateur of a consensus government cannot gain enough in the second period by strategically positioning the status quo and is better off by choosing the efficient policy. As a consequence in the second period each party receives one-third of the vote, and a majoritarian government is formed with policy at the midpoint of the contract curve of the government parties.

If parties are patient ($\beta \geq \widehat{\beta}$), the future is sufficiently important that a consensus government sacrifices efficiency in the first period to position the status quo to its electoral advantage in the second period. It chooses a first-period policy close to its ideal point;

[17] The policy $F_{jk}\left(-\frac{1}{\sqrt{3}}, 0\right)$ is equi-distant from the ideal points of parties j and k at the intersection of the boundaries of D^j and D^k and relatively closer to that of formateur i.

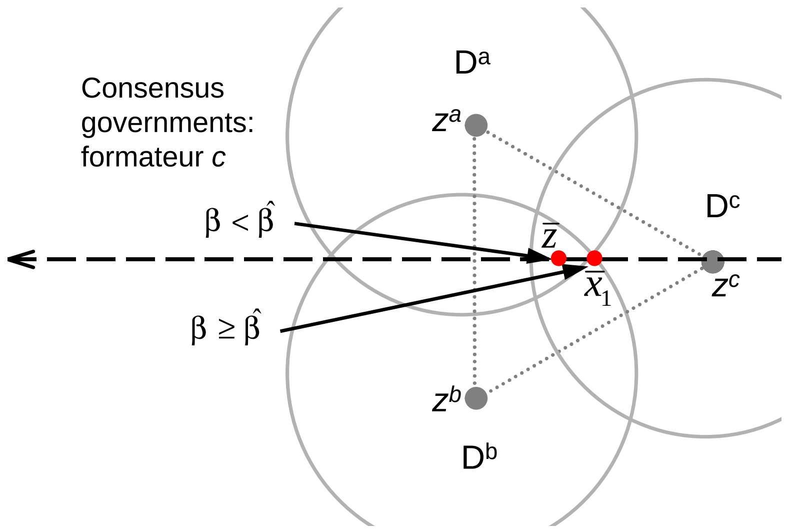

Figure 4: Policy choice by party c that forms a consensus government in the first period.

e.g., if party c with ideal point $z^c = (\frac{\sqrt{3}}{2}, 0)$ is the formateur, the policy is $\overline{x}_1 = (\frac{1}{2}, 0)$. This policy as the status quo would lead party c to choose $\overline{z}$ in the second period, whereas parties a and b would form majoritarian governments with a more extreme policy. Voters then give c a majority. The electoral effect is thus fully exploited for all $\beta \geq \widehat{\beta}$. The policy $\overline{x}_1$ disadvantages parties a and b in the next election, and it also disadvantages parties a and b in the bargaining over government formation and policy choice in the second period. Party c must provide the other parties with sufficient benefits to obtain $\overline{x}_1$ rather than $\overline{z}$. Consensus governments always choose a first-period policy that is interior to the single-period Pareto set, but that policy maximizes aggregate welfare in the first period only if the parties are impatient.

The next result characterizes the equilibrium when the first-period formateur forms a majoritarian government.

Proposition 3 *In the first period, the formateur of a majoritarian government ij chooses a policy that is distant from the ideal point of the out party k. Moreover, the policy choice is outside the single-period Pareto set of the three parties for all positive β. In particular,*

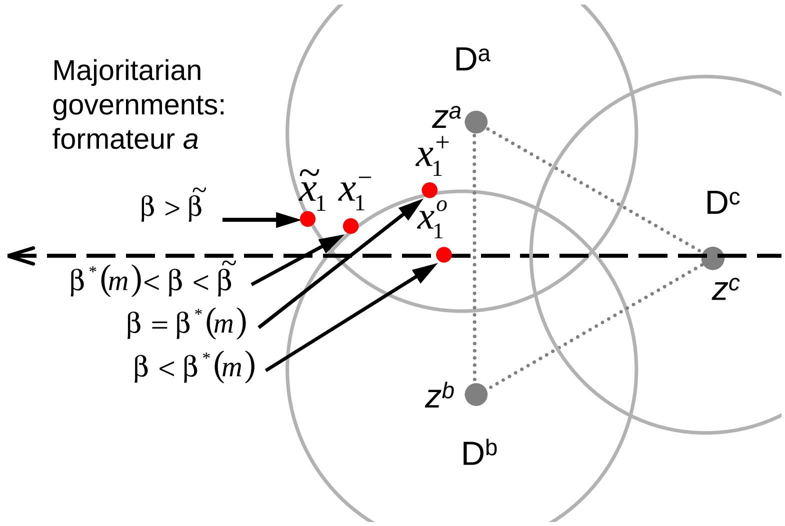

Figure 5: Policy choice by party a that forms majoritarian government ab in the first period.

for any $m \in \left(0, \frac{1}{4}\right)$, *there exists a decreasing function* $\beta^*(m)$ *such that*[18]

$$H_i(ij) = \begin{cases} F_{ij}\left(\frac{(1-m)\beta}{2-(1-2m)\beta}, 0\right) & \text{if } \beta \in [0, \beta^*(m)) \\ F_{ij}\left(\kappa\left(\frac{\beta}{2-\beta}\right), |\kappa - 1|\frac{1}{2}\right) & \text{if } \beta \in [\beta^*(m), 1], \end{cases}$$

$$\text{where } \kappa \equiv \left(\tfrac{1}{2}\right)^{\frac{1}{2}}\left[\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\left(\tfrac{\beta}{2-\beta}\right)\right)^2\right]^{-\frac{1}{2}}.$$

For all $\beta \in [0, \beta^*(m))$, *the policy choice is such that in the second period both parties* i *and* j *obtain a vote share* $\frac{1-m}{2}$ *and party* k *obtains a vote share* m. *In period two the formateur forms a majoritarian government and the policy outcomes are* $H_i(ik)$ *and* $H_j(jk)$ *with probability one-half each. For all* $\beta \in [\beta^*(m), 1]$, *the policy choice favors the period-one formateur. As a consequence, in the second period there is a majority parliament with all parties represented, where the period-one formateur receives a majority vote share and forms a consensus government with policy* $\bar{z}$.

The formateur (for example, party a) of a majoritarian government ab chooses the first-period policy strategically to position the status quo for the second period to its advantage. When the parties are very impatient, the policy is equidistant from the ideal points of the government parties as illustrated by x_1^o in Figure 5 and both parties receive $\frac{1-m}{2}$ of the

[18] The value of $\beta^*(m)$ ranges from 0.17 to 0.26. The function $\beta^*(m)$ is characterized in the Appendix.

votes.[19] At $\beta = \beta^*(m)$ the policy chosen by the formateur a jumps to the boundary of the coalition partner's set D^b, which yields a majority of votes for a in the election. This is illustrated by x_1^+ in Figure 5. When the future is more important ($\beta > \beta^*(m)$), the period-one formateur chooses a policy farther from the out party but on the boundary of D^j, as illustrated by x_1^- in Figure 5. At $\beta = \widetilde{\beta} = \sqrt{3} - 1$, the policy is at the intersection of the boundaries D^i and D^j.[20] For $\beta > \widetilde{\beta}$ the policy is on the boundary of D^i, as illustrated by $\tilde{x}_1$ in Figure 5, and disadvantages both of the other parties in the period-two election and coalition bargaining.

Proposition 3 identifies the limiting effect of elections on policy choice. When parties are sufficiently patient ($\beta > \widetilde{\beta}$), in the absence of electoral incentives the formateur of a majoritarian government in period one would prefer a policy outside $D^a \cup D^b \cup D^c$ to put both of the other parties in a weaker position for coalition bargaining in period two. Doing so, however, would lead each party if selected as formateur to form a consensus government with policy $\overline{z}$. The formateur in period one then would loose its electoral advantage. The formateur prefers to retain that electoral advantage and hence restrains its period-one policy choice. Electoral considerations thus bound the extent of the inefficiency.

The next proposition characterizes the equilibrium for a single-party government formed by a majority party.

Proposition 4 *In the first period any single-party government i chooses a policy that favors itself but is far and equally distant from the ideal points of the other parties. In particular, for all $\beta \in [0,1]$ and all distinct $j, k \neq i$,*

$$H_i(i) = \begin{cases} F_{jk}\left(-\frac{1}{1-2\beta}, 0\right) & \text{if } \beta \leq \beta^o \equiv \frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}\right) \\ F_{jk}\left(-1-\sqrt{\frac{2}{3}}, 0\right) & \text{if } \beta > \beta^o. \end{cases}$$

As a consequence, in the second period there is a majority parliament with all three parties represented, and the majority party i forms a consensus government with policy $\overline{z}$.

A majority party i in the first period chooses a policy that yields it a majority in the period-two election. For $\beta = 0$ the party chooses its ideal policy, and as β increases it

[19] Voters give the out party a vote share of m to keep it in parliament so that the other two parties will form governments with it rather than with each other.

[20] At $\beta = \widetilde{\beta}$, $\kappa = 1$.

chooses a policy equidistant but farther from the ideal points of the other two parties, but the policy remains in D^i, so i remains a majority party in the second period. It receives a majority because as formateur it would choose the centrist policy $\bar{z}$, whereas the other two parties as formateur would form majoritarian governments with less central policies in period two. For $\beta > 0$ it is outside the single-period Pareto set because a policy away from the ideal points of the other two parties allows the majority party to obtain more office-holding benefits in the second period. For $\beta > \beta^o$ the policy $H_i(i) = F_{jk}\left(-1 - \sqrt{\frac{2}{3}}, 0\right)$ is on the boundary of D^i; i.e., as far as possible from the ideal points of the other parties and still have party i receive a majority of the vote in period two. Elections again limit the extent of the inefficiency.

Proposition 5 indicates that a majority party in period one can remain the majority party in period two by forming a single-party government in period one. Given any initial status quo q_0, however, any elected majority party in the first period can be shown to prefer to form either a majoritarian or a consensus government rather than a single-party government. Hence, single-party governments will not be considered further.

Coalition and Legislative Bargaining

To illustrate the bargaining, suppose that party a is the formateur in the first period. It knows that if it forms a government with policy x_1 in the first period, that policy will be the status quo q_1 in the second period. Suppose that $q_1 \in D^a \setminus (D^b \cup D^c)$, so by Lemma 1 party a prefers to form a consensus government in period two with $x_2 = \overline{z}$. To obtain $\overline{z}$ rather than z^a party $j = b, c$ will provide benefits $(-y_2^j)$ satisfying the coalition participation condition

$$u^j(\overline{z}) + y_2^j \ \geq \ u^j(q_1),$$

so

$$y_2^j = u^j(q_1) - u^j(\overline{z}).$$

Note that j's office-holding benefits are strictly lower as the status quo q_1 is farther from its ideal point. That is, the formateur's bargaining power is greater the farther the status quo is from the ideal points of its possible government partners.

The utility or continuation value $v^a(q_1)$ of party a is

$$\begin{aligned} v^a(q_1) &= u^a(\overline{z}) - y_2^b - y_2^c \\ &= u^a(\overline{z}) + u^b(\overline{z}) + u^c(\overline{z}) - u^b(q_1) - u^c(q_1), \end{aligned}$$

and the continuation values $v^j(q_1)$ for the other parties are

$$v^j(q_1) = u^j(q_1),\ j = b, c.$$

The period-two policy maximizes the aggregate utility of the government parties and thus is efficient from the perspective of the parties; i.e., it is coalition efficient. When that policy is the centroid, it also maximizes the aggregate utility of the voters, since their ideal points are symmetrically located in a disk centered around the ideal points of the parties.

In the first period suppose that the formateur a were to form a majoritarian government with party b with policy $x_1 \in D^a \setminus (D^b \cup D^c)$. Party b prefers to be in government if

$$u^b(x_1) + y_1^b + \beta v^b(x_1) \geq u^b(q_0) + \beta v^b(q_0),$$

since the status quo for period 2 will be $q_1 = x_1$. The government is formed with policy x_1 and a transfer $y_1^b = u^b(q_0) + \beta v^b(q_0) - (1+\beta)u^b(x_1)$ of office-holding benefits from party b to party a.

The utility $W^a(x_1)$ of party a for the two periods then is

$$\begin{aligned} W^a(x_1) &= u^a(x_1) - y_1^b + \beta v^a(x_1) \\ &= u^a(x_1) + u^b(x_1) - u^b(q_0) - \beta v^b(q_0) + \beta(u^a(\bar{z}) + u^b(\bar{z}) + u^c(\bar{z}) - u^c(x_1)). \end{aligned}$$

If it prefers to have a majority in period two, party a chooses the policy x_1^* given by

$$x_1^* \in \underset{x_1 \in D^a \setminus (D^b \cup D^c)}{\arg\max} W^a(x_1),$$

which is the point (for example, either x_1^+, x_1^-, or $\widetilde{x}_1$) illustrated in Figure 5 for $\beta \geq \beta^*(m)$. This policy is chosen to disadvantage party c both electorally and in government formation in period 2, since in that period c is willing to provide sufficient office-holding benefits to obtain $\bar{z}$ rather than x_1. As in a single-period model, the period-one policy x_1^* maximizes the aggregate utility of the period-one government parties despite the formateur preferring to position the status quo for period two to advantage itself in the election and government formation in period two.

In equilibrium the first-period formateur a chooses x_1^*, party b joins the government, and party c does not. Given $x_1^* \in D^a \setminus (D^b \cup D^c)$, voters understand that if recognized as the formateur in period two, parties b and c would form majoritarian governments with policy z^{bc}. Voters also recognize that party a as the formateur in period 2 would choose a consensus government with the central policy $\overline{z}$. A majority of voters prefers $\overline{z}$ to z^{bc}, and hence a majority vote for a with the others voting for b and c. As the majority party, a is selected as the formateur in period 2, and bargaining with the other two parties results in the policy $\overline{z}$, as voters anticipated.

The period-one policy favors the formateur in both the election and the bargaining, and may also favor its government partner as well. To illustrate this, consider a's policy choice as a function of β. For $\beta = 0$ the policy is $x_1^* = (0, 0)$, which is the midpoint of the contact curve of parties a and b. As β increases, the formateur chooses a policy equidistant from z^a and z^b but farther from z^c, and hence outside the single-period Pareto set. This yields both government parties a $\frac{1-m}{2}$ vote share as well as improving their period-two bargaining position relative to party c. For $\beta \geq \beta^*(m)$ the future is sufficiently important that the formateur forms a government with b at a policy sufficiently far from z^b and z^c that a receives a majority in the next election. As β increases the policy moves along the boundary of D^b until at $\widetilde{\beta} = \sqrt{3} - 1$ it reaches $x_1^* = (-\frac{1}{2}, 0)$, which is the intersection of the boundaries of D^a and D^b. As β increases further, the policy moves along the boundary of D^a, but farther from both parties b and c. This disadvantages both parties b and c in the period-two bargaining while ensuring a majority for a in the election. At $\beta = 1$ the policy is $x_1^* = (-\frac{\sqrt{3}}{2\sqrt{2}}, \frac{1}{2}(1 - \frac{1}{\sqrt{2}}))$.

This analysis identifies the importance of strategic voting. With sincere voting the vote shares (and seat shares) of the parties are each one-third. Strategic voters choose the party to support based on the policies they anticipate will be chosen by the governments that would form in the second period. A party that will form a consensus government will receive a majority of the vote if the other two parties would form majoritarian governments with less central policies. In essence, a majority of voters rewards the party that will choose a central policy. Similarly, for $\beta < \beta^*(m)$ a party that will form a majoritarian government chooses a policy that yields it and its government partner a $\frac{1-m}{2}$ vote share.

3.3 Dynamics of Government Coalition and Policy Choice

Propositions 2, 3, and 4 identify a rich set of dynamics of government coalition and policy choice. Given any representation hurdle m, three regions of political patience can be identified, each of which has a different pattern of dynamics.

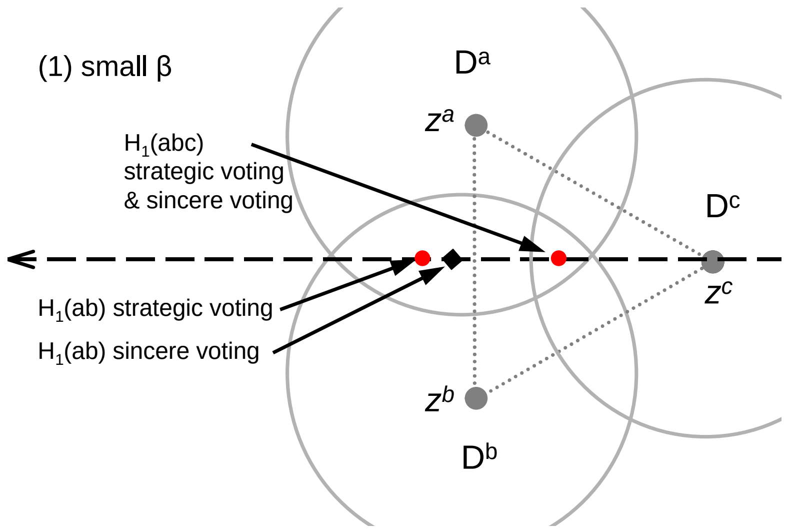

Figure 6: Equilibrium policy choices with $\beta \in [0, \beta^*(m))$.

1. $\beta \in [0, \beta^*(m))$: This case is illustrated in Figure 6. Properties: (i) A formateur of a consensus government in the first period chooses the central policy $\overline{z}$, and neither the government nor the policy is sustainable in the second period. (ii) A majoritarian government in the first period chooses a policy that is outside the single-period Pareto set and equidistant from the ideal points of the government parties. This policy disadvantages the out party in the next election and in government formation and treats the government parties identically. (iii) Conditional on either a consensus government or a majoritarian government in the first period, in the second period there is a minority parliament, and a majoritarian government forms and chooses $z^{ij}, \forall i, j$; i.e., the mid-point of the contract curve of the government parties.

2. $\beta \in \left[\beta^*(m), \widehat{\beta}\right)$: This case is illustrated in Figure 7. Properties: (i) A formateur of a consensus government in the first period chooses the central policy $\overline{z}$, and neither the government nor the policy is sustainable in the second period. A minority parliament results in the second period, and majoritarian governments are formed with policies z^{ij}. (ii) A formateur of a majoritarian government in the first period chooses a policy that yields it a majority in the election. In the second period the period-one formateur forms a consensus government with policy $\overline{z}$.

3. $\beta \in \left[\widehat{\beta}, 1\right]$: This case is illustrated in Figure 8 for $\beta \in \left[\widehat{\beta}, \widetilde{\beta}\right)$ and Figure 9 for

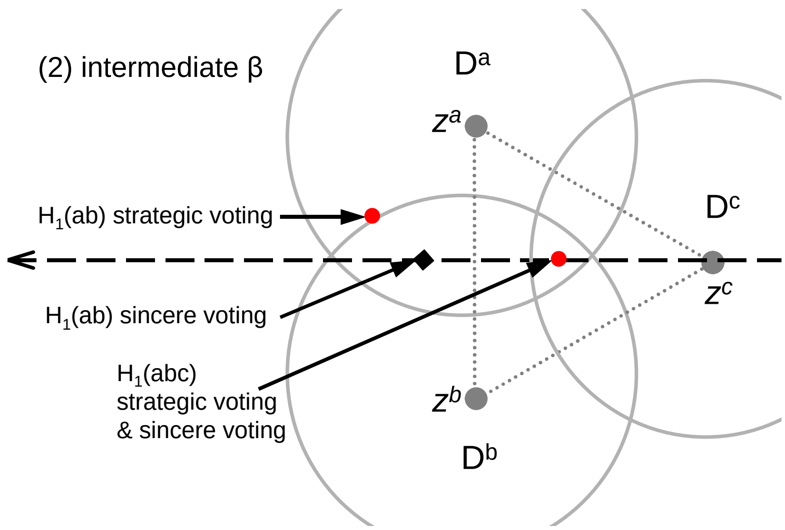

Figure 7: Equilibrium policy choices with intermediate $\beta \in \left[\beta^*(m), \widehat{\beta}\right)$.

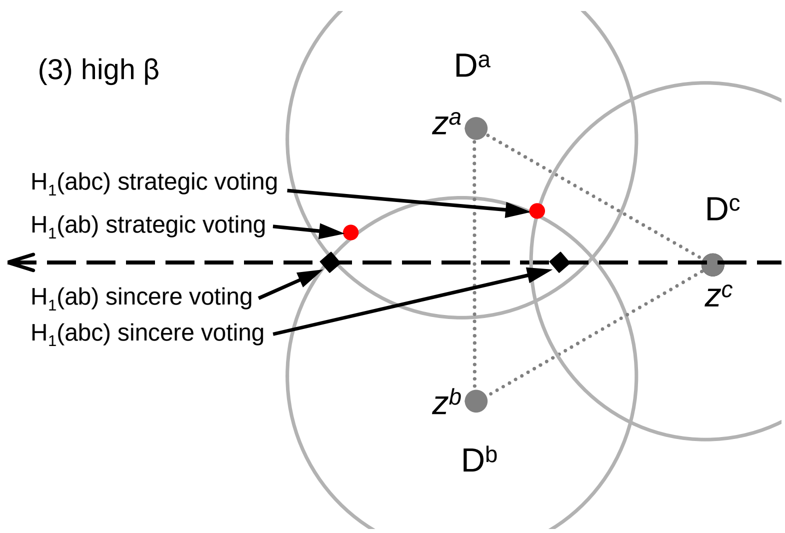

Figure 8: Equilibrium policy choices with high $\beta \in \left[\widehat{\beta}, \widetilde{\beta}\right)$.

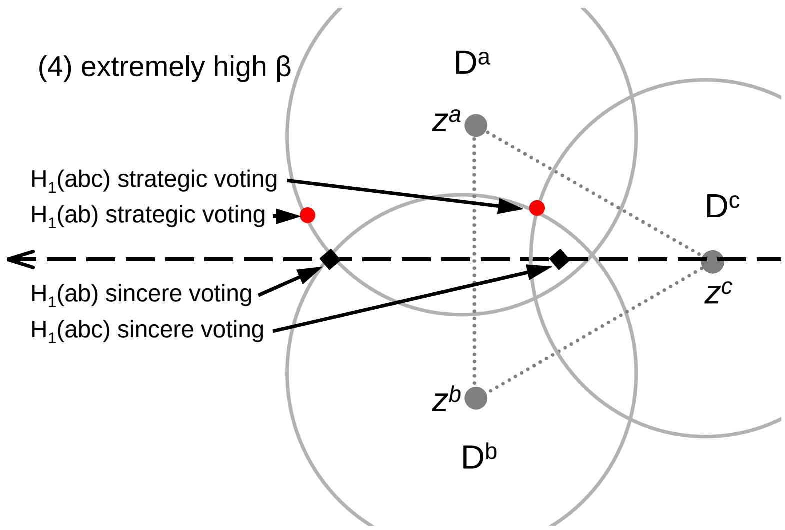

Figure 9: Equilibrium policy choices with high $\beta \in \left[\widetilde{\beta}, 1\right]$.

$\beta \in \left[\widetilde{\beta}, 1\right]$. Properties: (i) A formateur of a consensus government in the first period chooses a non-central policy in the Pareto set that yields it a majority in the election. As a consequence, the consensus government is sustainable in the sense that it is formed again in the second period. The policy in period 2 is the centroid $\overline{z}$. (ii) A formateur that forms a majoritarian government in the first period chooses a policy outside the Pareto set that yields it a majority in the election. In the second period the period-one formateur forms a consensus government with policy $\overline{z}$. (iii) In the second period there is a majority parliament, regardless of the government formed and the policy in the first period.

3.4 Comparative Statics–Centrifugal Forces

The incentives created by the institutions of a parliamentary democracy exert centrifugal forces on the policy in the first period,[21] and those forces are generally stronger, and the period-one policies more extreme, the higher is the discount factor. These forces reflect a tradeoff between contemporaneous and future utilities. For example, with strategic voters and β sufficiently high, a first-period formateur of a majoritarian government does not choose a policy that is equidistant from the ideal points of its member parties. Instead, it chooses a policy that yields a majority in the second period. This reduces the aggregate

[21] Schofield and Sened, (2006) p. 63, also identify a centrifugal force.

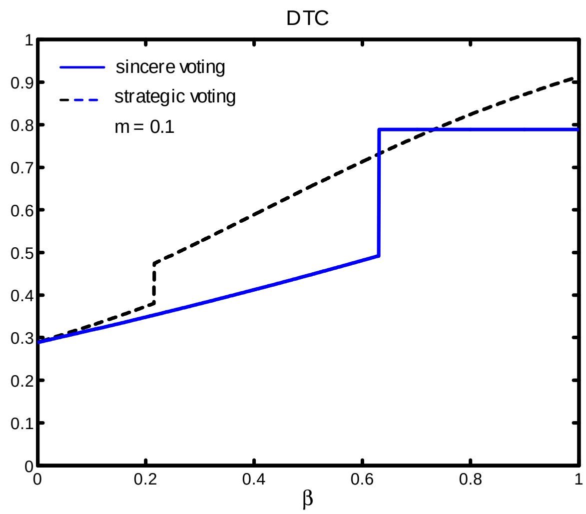

Figure 10: Distance to centroid.

period-one utility of the two parties but guarantees that the first-period formateur obtains a majority in the period-two election and thus is the formateur in the second period. A consensus government then is formed in the second period. An asymmetric and Pareto inferior policy thus is associated with a majoritarian government if the parties care sufficiently about the future. To measure the extremeness of the policy choice in the first period, define a metric, the distance to the centroid (DTC), where for any policy $x \in \Re^2$, $DTC(x) \equiv \|x - \overline{z}\|$.

One centrifugal force is due to the incentives created by the institutions of government formation and legislation. This bargaining centrifugal force can be identified from the properties of the equilibrium with sincere voting. As illustrated in Figure 3, the formateur of a period-one majoritarian government has an incentive to choose an extreme policy far from the out party's ideal point. This lowers the reservation value of the out party and hence increases the bargaining power of the formateur, allowing the two parties in the incumbent government to have a cheaper coalition partner in the subsequent period. This bargaining force (with sincere voting) is illustrated by the solid DTC line in Figure 10 and is weakly increasing in β.

The second centrifugal force is due to the institution of elections. With sophisticated voters the period-one formateur chooses a policy that advantages itself not only in the bargaining over governments and policy in period two but also in the election. That is, the party disadvantaged by the period-one bargaining effect is also disadvantaged in the election because both parties in the incumbent government would form a majoritarian government with the out party from the first period. This attracts some of the voters located close to the out party and induces the incumbent to choose an even more extreme policy in the first period. The DTC for a majoritarian government with sophisticated voting is illustrated by the dashed line in Figure 10 and for $\beta > 0$ is everywhere above the line for sincere voting except for the jump in the bargaining effect identified in Proposition 1.

Parliamentary elections with proportional representation thus provide a centrifugal force on policy in the first period, and that force is stronger the higher is β. Government formation also provides a centrifugal force on policy because a formateur in one period can strengthen its bargaining power in the next period by choosing a more extreme policy. These two forces interact, so their effects are not additive.

4 Empirical Implications

This section summarizes a set of refutable predictions derived from the model that are potentially testable with data on governments, the policies they implement, and other observable parameters such as the average duration of coalitions or party leaders. Duration could serve as a measure of political patience and can vary across countries or among coalition types across time. Some of the predictions pertain to a cross-section of countries, whereas others pertain to variations over time.

Representation Threshold Proposition 3 implies that, ceteris paribus, a higher representation threshold leads to a less extreme policy choice by a majoritarian government when the discount factor of the parties is sufficiently small. Consider the natural constituents of a party i that is relatively disadvantaged in the second period by the status quo. To have their party represented in parliament, at least an m proportion of voters have to vote for party i. Therefore, a higher threshold restricts the extent to which the natural constituents of the disadvantaged party vote strategically. This then lowers the probability that any of the parties in the period-one majoritarian government will be recognized as formateur. This, in turn mitigates the incentive of a period-one majoritarian government to choose a more extreme policy. The representation threshold, however, does not affect the policy outcome if the discount factor of the parties is sufficiently large. In that case, the period-one formateur chooses a policy that ensures that it is elected as the majority party in the second period, and outcomes are independent of the representation threshold. The testable comparative static then is that the policies of majoritarian governments are more distant from the center of preferences in countries with a lower representation threshold, controlling for other factors.

Incumbency Advantage and Strategic Voting Whether voters vote sincerely or strategically in parliamentary elections is an empirical question.[22] The model sheds light on how such studies could be conducted. Propositions 2 and 3 imply that with strategic voting on the equilibrium path any party included in an incumbent government receives on average a (weakly) greater vote share than the out party in the subsequent election and therefore a higher probability of being recognized as the next formateur.[23] Moreover, an incumbent formateur receives a (weakly) larger vote share than any other party in the subsequent election and therefore it is (weakly) the most likely party to head the government in the next period. This results because the parties disadvantaged by the status quo have less bargaining power in the parliament and thus are more likely to be included in the new government after the election. Foreseeing this, some natural constituents of the disadvantaged parties strategically vote for the party favored by the status quo. At the same time an incumbent government has an incentive to propose a policy to advantage itself and disadvantage the out parties to gain more votes (from strategic voters) and more bargaining power. As a consequence, the incumbent has an electoral advantage. This advantage is not present if voters sincerely vote for parties whose ideal points are closest to theirs.[24] Evidence of incumbency advantage in proportional representation systems is therefore consistent with

[22]The few existing empirical studies (Cox 1997, Bawn 1999) have focused on the effects of variations in voting rules on voting behavior. Examples include representation thresholds or mutiple-ballot systems. To our knowledge there is no empirical work that has directly studied strategic voting in a full-equilibrium context with government formation.

[23]There, however, exist equilibria in which the out party receives a greater vote share than the non-formateur party in a majoritarian government. For example, consider any $m \in \left(0, \frac{1}{4}\right)$, any $\beta \in (\beta^*(m), 1]$, and suppose that in the first period party a as formateur forms a majoritarian government with party b. A majority parliament then results and for some $\varepsilon > 0$ sufficiently small, $\left(\rho_2^a, \rho_2^b, \rho_3^c\right) = \left(\frac{1}{2} + \varepsilon, m, \frac{1}{2} - m - \varepsilon\right)$ constitute a period-two electoral equilibrium in which party c receives more votes than b.

[24]Degan and Merlo (2006) assess whether voter behavior is consistent with sincere voting in U.S. national elections in the postwar period.

the implications of strategic voting.

An Alternative Interpretation of β In the model, a government cannot fail before the next regularly scheduled election, so the life span of a government corresponds to the length of an interelection period. Yet, a defining feature of most parliamentary systems is that an incumbent government can be removed by parliament at any time during the inter-election period, e.g., by a successful no confidence motion. In many counties this can also lead to the dissolution of parliament and early elections. In western European multiparty parliamentary systems in the postwar period, the average duration of governments has varied from 13 months in Italy to 45 months in Luxembourg (Laver and Schofield 1990). If politicians all (subjectively) discount their future utility at similar *annual* rates, politicians in countries with shorter expected life spans of governments should have a higher *per period* discount factor, i.e., a larger β, than politicians in countries with longer life spans.[25] In the model this corresponds to Italians having higher β's than Luxembourgers. Such differences could also result from some unmodeled constitutional feature, such as the requirements for confidence and censure procedures, that may affect the stability and therefore duration of governments. For example, it may be more difficult to replace a government in a country with a constructive vote of confidence than in a country in which a successful no confidence motion can end a government.[26] If these factors affect β and are reflected in the average duration of government, a higher (lower) discount factor can be interpreted not only as more (less) patience of the parties, but also as a political system that leads to more (less) frequent government turnover. While the former may be difficult to measure, the latter is easily measurable.[27]

Average Duration of Governments and Dynamics of Policy and Coalition Government The model also provides a set of refutable predictions about dynamics of policy and coalition government. First, more extreme policies are chosen by any type of coalition government in countries with shorter average durations of governments or more patient parties, regardless of voter behavior. If indices regarding extremeness of economic

[25] Suppose that a political party has an *annual* discount factor $\beta^o \in [0, 1]$. If it expects a government to last for T years, its *per period* discount factor is $\beta = (\beta^o)^T$, which is decreasing in T.

[26] There is a large empirical literature on the factors that influence cabinet duration including constitutional features. See Diermeier, Eraslan, and Merlo (2003) for a recent example.

[27] A third possibility may be to measure the expected tenure of party leaders across parties and countries.

and social policies are constructed, this hypothesis can be directly tested. Second, consensus governments choose more central policies than do majoritarian governments. Third, consensus governments have policies interior to the convex hull of the ideal points of all parties in government, whereas majoritarian governments may have policies outside the convex hull of the ideal points of the government parties. Fourth, conditional on strategic voting, in countries with sufficiently frequent government turnover, a consensus government chooses a policy away from the center of preferences, and it perpetuates itself. In contrast, in countries with sufficient long life spans of governments or sufficiently impatient parties, a consensus government implements a central policy and is replaced by a majoritarian government. Fifth, both majoritarian and consensus governments can precede a consensus government formed by a majority party, but only in political systems with patient parties or short durations of governments. Sixth, a majoritarian government is followed by a different majoritarian government only in political systems with impatient parties or long government life spans. Seventh, any majority party implements a central policy. This is because any party would not have received a majority of votes if the voters expected that party as formateur to carry out an extreme policy.

5 Welfare Implications

5.1 Pareto Inefficiency and Political Failure

Proposition 3 identifies a political failure when the parties care about the future, i.e., $\beta > 0$. The term "political failure" is used here to refer to incentives inherent in the political system that lead to a policy that is outside the single-period Pareto set of party preferences.[28] In the theory presented here, political failures are associated with the institution of elections and with government formation and legislation in parliament. These failures are unavoidable, since voting is an inalienable right and voters and parties are unable to commit to future actions. The commitment problem in principle could be resolved by repetition, but the coordination problems among voters and among parties with divergent preferences seem insurmountable, particularly when political leaders are impatient. Moreover, a period in the model can be as long as four years. A theory of commitment supported by reputation and long-term relationships implicitly assumes that political parties base their current strategies

[28] Besley and Coate (1998) define a political failure as the failure to make a Pareto-improving investment.

on history, which may potentially date back ten years ago. This is theoretically plausible but not likely in real-life politics.

The political failure associated with elections is manifested by the incentive to position the status quo for the next period to advantage the period-one formateur in the election. This political failure is associated with majoritarian and single-party governments. The incentive is (weakly) stronger the higher is the discount factor. For high β the period-one formateur positions the status quo so that voters anticipate that it will choose a central policy in the next period if chosen as the formateur, whereas the other parties would choose less central policies. A majority of voters then prefers to vote for the current formateur, and anticipating this, the formateur will position the policy, and hence the next status quo, outside the single-period Pareto set. The incentives provided by future elections are also present when parties are patient and a consensus government is formed in the first period. The resulting policy is interior to the Pareto set but not at the centroid.

One source of the political failure associated with elections is voters, who are willing to reward centrist policies in the final period even though it induces inefficiency in the previous period. This political failure results because voters cannot commit to how they will vote in future elections. If all voters were loyal to a party, and hence voted sincerely, this source of political failure would be eliminated. The presence of voters who condition their vote on what the parties would do if in government, however, precludes universal sincere voting.

A second source of the political failure associated with elections lies with parties, which may have difficulty committing to enact, or not to enact, particular policies. A party's platform or a pre-announced electoral coalition could be credible, but only if voters were to punish a party for deviating. Parties may be able to develop reputations for fulfilling promises, but political temptations to exploit a reputation for short-term gains can be strong, particularly when voters are sophisticated and respond to the anticipated future actions by the parties. Moreover, the centrifugal force is stronger when the future is more important to the parties, which could make reputations more difficult to sustain.

The political failure associated with government formation and legislative bargaining is evident from Proposition 1. To improve its bargaining position, the period-one formateur has an incentive to position the status quo for the second period outside the single-period Pareto set. This increases its bargaining power in the second period. This failure is (weakly) more severe the more patient are the parties, so building a reputation may not be a remedy.

This bargaining failure is exacerbated by the political failure associated with elections, which can lead to even more extreme policies as indicated in Proposition 3 for majoritarian governments and Proposition 4 for single-party governments. Again, this failure becomes more severe as the patience of parties increases.

5.2 The Utilitarian Measure of Welfare

This section examines how the institutions of parliamentary systems affect the welfare of voters and how social welfare responds to the impatience of political parties. Social welfare is defined as the average two-period utility of all voters. As shown in Appendix B, the average per-period utility is approximately a constant plus that of a hypothetical voter with ideal point $\overline{z}$. Given a policy x, the aggregate per-period utility of all voters is then measured by $-\left\|x-\overline{z}\right\|^2$. Similarly, the aggregate two-period utility of all voters is measured by $-\left\|x_1-\overline{z}\right\|^2-\delta\left\|x_2-\overline{z}\right\|^2$, where $\delta \in (0,1]$ is the discount factor of voters.

As shown above, for most of the domain $[0,1]$ of β and conditional on any form of the government, the period-one policy outcome is more extreme if the voters vote strategically in the second period than if they vote sincerely. This implies that, conditional on any type of period-one government, on average voters are worse off if they vote strategically than sincerely. This results because sincere voting implicitly amounts to a commitment to vote for the closest party regardless of the government that would be formed or the policy it would choose in the second period. Strategic voting reflects the inability of voters to commit to how they will vote, so their votes depend on the policy in the first period. The parties anticipate this and choose a policy that is more extreme.

By Proposition 3 conditional on a majoritarian government being formed in the first period, the voters on average are worse off for higher $\beta \in [0, \beta^*(m))$ or $\beta \in (\beta^*(m), 1]$. This implies that social welfare is lower as the parties care more about the future. This results because the period-one formateur has an incentive to choose a more extreme policy as the future becomes more important. A more extreme policy achieves two purposes for the formateur. First, it reduces the vote shares the other parties are likely to receive in the subsequent election. This increases the probability that the period-one formateur will again be recognized as the formateur in the second period. Second, if the period-one formateur is recognized as the period-two formateur, it obtains greater office-holding benefits from its future coalition partners, since they are more disadvantaged in the bargaining by the status quo. Note that when β moves from slightly below to slightly above $\beta^*(m)$, on average the voters are better off. This is because for $\beta \in (\beta^*(m), 1]$ the period-one policy is so extreme that in the second period a consensus government with a central policy $\overline{z}$ will result with probability one. This leads to a discrete jump of period-two utility for the average voter. The same result obtains if in the first period a consensus government is formed; i.e., replace $\beta^*(m)$ in the above statement by $\widehat{\beta}$.

As discussed in Section 4, the representation threshold m affects the policy outcome only if the discount factor is sufficiently small. In this case, social welfare is strictly increasing in m. A higher representation threshold benefits an average voter because it serves as a commitment device and reduces strategic voting. In the model, each party is assumed to have natural constituents of equal size. Although unmodeled, a potential cost of a higher representation hurdle is that small parties – parties that represent minority groups – may be unable to obtain sufficient votes for representation in parliament.

6 Conclusions

The principal institutions of parliamentary democracies are elections, government formation, and legislatures. Since the government serves with the confidence of the parliament, government formation and legislation are necessarily intertwined and a bargaining perspective is a natural approach to studying policy choice. Both government formation and legislation depend on representation in parliament, and the modal electoral institution is proportional representation. Political incentives arise from all three institutions, and both political parties and voters respond to those incentives. The present and the future are linked by both long-lived players and the feature of political systems that the status quo policy remains in effect until it is replaced by a new policy. The policy chosen in the present period is the status quo for the next period, so the future shapes the incentives in the present period.

This paper identifies how the incentives present in a multi-party parliamentary system affect the dynamics of representation, governments, and policy. The bargaining over government formation and policy choice creates intertemporal incentives, since the current policy choice affects the bargaining power of parties in the next period.[29] When parties are polit-

[29] This insight is distinct from the literature on cabinet stability (e.g. Diermeier, Eraslan, and Merlo 2003,

ically patient, bargaining incentives can lead a majoritarian government to choose a policy outside the single-period Pareto set of the parties, and the inefficiency is increasing in political patience. Elections determine the representation of parties and also the likelihood that a party will be selected as formateur. This provides incentives for parties to position the current policy to gain an advantage in the next election. The incentives arise because voters anticipate both the governments that could form in the next period and the policies they would choose as a function of representation and the status quo. These electoral incentives can lead to policies farther from the center of voter preferences. Political failures thus result from both government formation and elections, and those failures provide centrifugal forces on policy choice. These forces are generally stronger the more patient are political parties.

The incentives present in a parliamentary system also affect the continuity of governments and policy. These incentives are sufficiently strong that governments generally do not persist from one interelection period to the next and neither do their policies. Government transition and policy change thus should be expected in parliamentary systems. For example, inefficiency of the present policy can be followed by efficiency in the next period. The causation runs in the opposite direction, however. The incentives for a party to choose a central policy in the next period can lead a majority of voters to vote for that party, and in the present period that party has an incentive to position the status quo so that if it is the formateur in the next period it will choose a central policy. This advantages the party in both the election and the subsequent bargaining over government formation.

The incentives leading to policy inefficiency and transitions in government and policy are due in part to commitment problems. If voters could commit to loyalty to a party, the centrifugal force arising from elections would not be present. If parties could commit credibly to the governments they would form and the policies they would choose, the centrifugal force arising from bargaining over government formation and policy would be mitigated but not eliminated.

The theory predicts that when parties are politically patient and voters vote strategically, majority parties can arise. This prediction is contrary to empirical evidence: majority parties are rare in proportional representation systems. In the context of the model the absence of majority parties is implied by three factors: politically impatient parties, a centrally located status quo, or voters who vote sincerely or are loyal to a party. Several

Warwick 1994) where the issue is the duration of a governing coalition during an interelection period.

extensions of the model could make it less likely that a majority party would emerge in equilibrium when none of these factors is present. For example, the model considers a parliamentary system with only three parties and without entry. With more parties or endogenous entry, a majority would be more difficult to obtain.

The model considered here is sufficient to identify the incentives arising from the institutions of parliamentary systems and provide predictions of the consequences of those incentives for government and policy continuity and for policy efficiency, but it is not fully dynamic. The analysis of a stationary equilibrium in an infinite horizon framework is the subject of future research. As in any dynamic theory the political patience of voters and parties is important. The incentives for extreme policies identified here are stronger for more patient parties, which works against norms or implicit agreements based on repeated interaction that could overcome the policy inefficiencies.

Appendix

A Proofs of Propositions 2-4

To simplify the notation in the proofs, let $U^i(q_0) \equiv u^i(q_0) + \beta v^i(q_0)$ be the reservation value of party i in the first period given an initial status quo q_0, so $U^i(q_0)$ is the sum of party i's period-one utility with the status quo policy q_0 and no transfers, plus its discounted continuation value for period two with a status quo $q_1 = q_0$. The expected discounted sum of utilities of party i in the first period is therefore $y_1^i + U^i(x_1)$ if a policy x_1 is chosen and it receives a redistribution of office-holding benefits of y_1^i.

Proof of Proposition 2. Consider a consensus government formed by party a in period one. Note that a formateur will propose a policy that maximizes the joint two-period utility of all members in the government, since it can use redistributions of office-holding benefits as instruments to reallocate the utilities of the parties. Therefore, the proof involves the $H_a(abc)$ that maximizes the joint two-period utility of all three parties and yields party a the highest probability of being recognized as period-two formateur among all policy alteratives that maximize this joint utility.

Partition the policy space into two regions: $R_1^C \equiv \left(D^a \cap D^b\right) \cup \left(D^a \cap D^c\right) \cup \left(D^b \cap D^c\right)$ and $R_2^C \equiv \Re^2 \setminus R_1^C$. Note that neither R_1^C nor R_2^C is convex. In the second period, a status quo in R_1^C or R_2^C leads to a different joint utility for all three parties. The approach is to characterize local maxima in these regions separately and then compare them to identify the globally optimal policy choice for party a. An optimal policy in a region R is denoted by $H_a(abc|R)$.

Region R_1^C Suppose that a consensus government is restricted to choose a policy from region R_1^C. Then by Lemmata 1 and 2, in the second period a majoritarian government will be formed, and the joint period-two utility of all three parties will be $-\frac{5}{4}$. Therefore,

$$\sum_{i=a,b,c} U^i\left(H_a\left(abc|R_1^C\right)\right) = \max_{x' \in R_1^C}\left\{\sum_{i=a,b,c} u^i\left(x'\right) + \beta\left(-\frac{5}{4}\right)\right\} = -1 - \frac{5}{4}\beta,$$

and $H_a\left(abc|R_1^C\right) = \overline{z}$.

Region R_2^C Suppose that a consensus government is restricted to choose a policy in region R_2^C. Then by Lemmata 1 and 2 in the second period all three parties will be represented in parliament and a consensus government will be formed with policy $\overline{z}$. This implies that the joint period-two utility of all three parties will be -1. Therefore,

$$\sum_{i=a,b,c} U^i\left(H_1\left(abc|R_2^C\right)\right) = \max_{x'\in R_2^C}\left\{\sum_{i=a,b,c} u^i\left(x'\right) + \beta\left(-1\right)\right\} = \frac{\sqrt{3}}{2} - 2 - \beta,$$

and the maximum is attained at $F_{ab}\left(-\frac{1}{\sqrt{3}}, 0\right)$, $F_{ac}\left(-\frac{1}{\sqrt{3}}, 0\right)$ and $F_{bc}\left(-\frac{1}{\sqrt{3}}, 0\right)$. Note that only if the last policy alternative is chosen, in the second period party a will receive a majority vote share and be recognized as formateur for certain. Therefore, given our assumption of lexicographic preferences, $H_a\left(abc|R_2^C\right) = F_{bc}\left(-\frac{1}{\sqrt{3}}, 0\right)$.

Comparison Finally, it can be shown that

$$\sum_{i=a,b,c} U^i\left(H_1\left(abc|R_1^C\right)\right) > \sum_{i=a,b,c} U^i\left(H_1\left(abc|R_2^C\right)\right)$$

if and only if $\beta \in \left[0, \widehat{\beta}\right)$ where $\widehat{\beta} \equiv 4 - 2\sqrt{3}$. ■

Proof of Proposition 3. Consider majoritarian government ab formed by party a in period one. The goal is to identify the $H_a\left(ab\right)$ that maximizes the joint two-period utility of parties a and b and yields party a the highest probability of being recognized as period-two formateur among all policy alteratives that maximize this joint utility.

Let $Q_{ij} \equiv \left\{x \in \Re^2 : u_i\left(x\right) > u_j\left(x\right) > u_k\left(x\right)\right\}$ for all $i, j, k = a, b, c$, $i \neq j \neq k$, and partition the policy space into 10 regions. In the second period, a status quo in a different region will lead to a different joint expected utility of parties a and b. These regions are:[30]

$R_1^T \equiv \left(\overline{D}^a \backslash \left(D^b \cup D^c\right)\right) \cup \left(\overline{D}^b \backslash \left(D^a \cup D^c\right)\right)$,

$R_2^T \equiv D^a \cap D^b \cap \left\{\mathbf{x} : u_a\left(\mathbf{x}\right) = u_b\left(\mathbf{x}\right) > u_c\left(\mathbf{x}\right)\right\}$,

$R_3^T \equiv \Re^2 \backslash \left(\overline{D}^a \cup \overline{D}^b \cup \overline{D}^c\right)$,

$R_4^T \equiv D^a \cap D^b \cap \left(Q_{ab} \cup Q_{ba}\right)$,

$R_5^T \equiv D^a \cap D^b \cap \left\{\mathbf{x} : \max_{i=a,b}\left\{u_i\left(\mathbf{x}\right)\right\} > \min_{i=a,b}\left\{u_i\left(\mathbf{x}\right)\right\} = u_c\left(\mathbf{x}\right)\right\}$,

$R_6^T \equiv \left[\left(D^a \cap Q_{ac}\right) \cup \left(D^b \cap Q_{bc}\right)\right] \cap D^c$,

$R_7^T \equiv \left(D^a \cup D^b\right) \cap D^c \cap \left\{\mathbf{x} : \max_{i=a,b}\left\{u_i\left(\mathbf{x}\right)\right\} = u_c\left(\mathbf{x}\right) > \min_{i=a,b}\left\{u_i\left(\mathbf{x}\right)\right\}\right\}$,

$R_8^T \equiv \left(D^a \cup D^b\right) \cap D^c \cap \left\{x : u^a\left(x\right) = u^b\left(x\right) < u^c\left(x\right)\right\}$,

$R_9^T \equiv \left\{\overline{z}\right\}$, and

$R_{10}^T \equiv \overline{D}^c \backslash \left(D^a \cup D^b\right)$.

[30] For any $i = a, b, c$, we denote the closure of D^i by $\overline{D}^i$.

The approach is again to characterize local maxima or suprema in these regions separately and then compare them to identify the globally optimal policy choice for party a. An optimal policy in a region R is denoted by $H_a(ab|R)$.

Region R_1^T Suppose that government ab is restricted to choose a policy from region R_1^T. In particular, suppose that a policy $x' \in \overline{D}^a \backslash \left(D^b \cup D^c\right)$ is chosen in the first period. Then by Lemma 2 and the equilibrium selection rules assumed, in the second period all three parties will be represented, and party a will receive a majority vote share.[31] By Lemma 1 party a as formateur will form a consensus government with policy $\overline{z}$, and the joint period-two utility of parties a and b will be $(-1) - u^c(x)$. Therefore, the joint two-period utility of parties a and b is

$$\sum_{i=a,b} U^i(x') = \sum_{i=a,b} u_i(x') + \beta\left(-1 - u_c(x')\right), \tag{1}$$

and

$$\max_{x' \in \overline{D}^a \backslash \left(D^b \cup D^c\right)} \sum_{i=a,b} U^i(x') = \begin{array}{ll} \max\limits_{x' \in \mathbb{R}^2} & -(2-\beta)\left\|x' - F_{ab}\left(\frac{\beta}{2-\beta}, 0\right)\right\|^2 - \frac{(1+\beta)(1-\beta)}{2-\beta} \\ s.t. & \|x' - z^a\|^2 \leq \frac{1}{2}, \left\|x' - z^b\right\|^2 \geq \frac{1}{2}, \|x' - z^c\|^2 \geq \frac{1}{2}, \end{array}$$

where the objective function on the right-hand side is a simplification of equation (1), and the three constraint inequalities correspond to the constraints that $x' \in \overline{D}^a$, $x' \notin D^b$, and $x' \notin D^c$ respectively. The maximum is attained at

$$H_a\left(ab|\overline{D}^a \backslash \left(D^b \cup D^c\right)\right) = F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right), |\kappa - 1|\left(\frac{1}{2}\right)\right),$$

where

$$\kappa \equiv \left(\frac{1}{2}\right)^{\frac{1}{2}}\left[\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\left(\frac{\beta}{2-\beta}\right)\right)^2\right]^{-\frac{1}{2}},$$

and

$$\sum_{i=a,b} U^i\left(H_1\left(ab|\overline{D}^a \backslash \left(D^b \cup D^c\right)\right)\right) = -\left(\frac{2-\beta}{2}\right)\left(\frac{1}{\kappa} - 1\right)^2 - \frac{(1+\beta)(1-\beta)}{2-\beta}.$$

This is a corner solution, since either the constraint $\left\|x' - z^b\right\|^2 \geq \frac{1}{2}$ or that $\|x' - z^a\|^2 \leq \frac{1}{2}$ is binding. Due to symmetry,

$$H_a\left(ab|\overline{D}^b \backslash \left(D^a \cup D^c\right)\right) = F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right), -|\kappa - 1|\left(\frac{1}{2}\right)\right)$$

attains the same maximum in R_1^T. However, if this policy is chosen in the first period, by Lemma 2 and the equilibrium selection rules assumed, party a will never be recognized as

[31] The equilibrium selection rules apply only if $q_1 = x'$ is on the border of $\overline{D}^a$ and outside D^b and D^c.

period-two formateur since party b will be elected as the majority party. Therefore, by the assumption of lexicographic preferences, $H_a\left(ab|R_1^T\right) = F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right), |\kappa-1|\left(\frac{1}{2}\right)\right)$.

Region R_2^T Suppose that a policy $x' \in R_2^T$ is chosen in the first period. Then by Lemma 2, in the second period party c will receive a vote share of m, and both parties a and b will receive $\frac{1-m}{2}$. As a consequence, with probability m party c will be recognized as formateur and randomize between majoritarian governments ac and bc. If, for example, a majoritarian government ac is formed, the joint period-two utility of parties a and b will be $u^a(x') + \left(-\frac{3}{4}\right)$; party a gets its reservation value since it is included in the new government, and party b gets $\left(-\frac{3}{4}\right)$ since it is excluded from the new government coalition. With probability $1-m$, either party a or b will be recognized, and the formateur will form a majoritarian coalition with party c. The joint period-two utility of parties a and b then will be $\left(-\frac{1}{2} - u^c(x')\right) + \left(-\frac{3}{4}\right)$. Therefore, the joint two-period utility of parties a and b is

$$\sum_{i=a,b} U^i(x') = \sum_{i=a,b} u^i(x') + \beta\left((1-m)\left(\left(-\tfrac{1}{2} - u^c(x')\right) + \left(-\tfrac{3}{4}\right)\right) + m\left(u^a(x') + \left(-\tfrac{3}{4}\right)\right)\right),$$

and

$$\begin{aligned}
&\sup_{x' \in R_2^T} \sum_{i=a,b} U_1^i(x') \\
=& \sup_{h \in \left(-\frac{1}{3}, \frac{1}{\sqrt{3}}\right)} \Phi(h) \equiv -\tfrac{3(2-(1-2m)\beta)}{4}\left(h - \tfrac{(1-m)\beta}{2-(1-2m)\beta}\right)^2 - \tfrac{4+2(1+4m)\beta-\left(5-8m-m^2\right)\beta^2}{4(2-(1-2m)\beta)} \\
=& \begin{cases} -\frac{4+2(1+4m)\beta-\left(5-8m-m^2\right)\beta^2}{4(2-(1-2m)\beta)}, & \text{if } \beta \in \left[0, \widehat{\beta}(m)\right), \\ -1 + \left(\frac{\left(2\sqrt{3}-1\right)-2\left(2+\sqrt{3}\right)m}{4}\right)\beta, & \text{otherwise,} \end{cases}
\end{aligned}$$

where

$$\widehat{\beta}(m) = \begin{cases} 1, & \text{if } m \in \left[\widehat{m}, \frac{1}{4}\right), \text{ where } \widehat{m} \equiv 3\sqrt{3} - 5, \\ \frac{2}{1+\sqrt{3}-\left(2+\sqrt{3}\right)m}, & \text{otherwise.} \end{cases}$$

For all $\beta \in \left[0, \widehat{\beta}(m)\right)$, the local supremum is attained by an interior policy $F_{ab}(h^*, 0) \in R_2^T$ where $h^* \equiv \frac{(1-m)\beta}{2-(1-2m)\beta}$, and therefore a supremum is a maximum. On the other hand, for all $\beta \in \left[\widehat{\beta}(m), 1\right]$, a maximum does not exist in region R_2^T. To see this, pick any policy $F_{ab}(h, 0) \in R_2^T$ and define $h(\varepsilon) \equiv \varepsilon h^* + (1-\varepsilon) h$. Since R_2^T is an open region, $F_{ab}(h(\varepsilon), 0) \in R_2^T$ for $\varepsilon > 0$ sufficiently small. Note that $\Phi(h)$ is strictly concave and $h^* = \arg\max_{h' \in \mathsf{R}} \Phi(h')$. Therefore, $\sum_{i=a,b} U^i(F_{ab}(h(\varepsilon), 0)) > \sum_{i=a,b} U^i(F_{ab}(h, 0))$. Finally, $\sup_{x' \in R_2^T} \sum_{i=a,b} U^i(x') \leq \max_{x' \in R_1^T} \sum_{i=a,b} U^i(x')$ for all $\beta \in [\beta^*(m), 1]$ and all $m \in \left(0, \frac{1}{4}\right)$, where $\beta^*(m)$ is a decreasing function in m and $\beta^*(m) \in \left(0, \widehat{\beta}(m)\right)$ for all m. This can be verified by comparing the functional forms of local maxima and/or suprema in regions R_1^T and R_2^T. The function $\beta^*(m)$ will be characterized in the last part of the proof.

Region R_3^T Suppose that a policy $x' \in R_3^T$ is chosen in the first period. Then by Lemmata 1 and 2, in the second period all three parties will be represented, each party will be recognized as formateur with probability one-third (which is the probability they perceive before the period-two election), and a consensus government will be formed. Therefore, with probability one-third, party c will be recognized and the joint period-two utility of parties a and b will be $u^a(x')+u^b(x')$ because both of them will be included in the consensus coalition and receive their period-two reservation values. With probability two-thirds, either party a or b will be recognized, and their joint period-two utility will be $(-1) - u^c(x')$, which is the joint utility of all three parties (that is, -1) net of party c's reservation value. Thus,

$$\sum_{i=a,b} U^i\left(x'\right) = \sum_{i=a,b} u^i\left(x'\right) + \tfrac{2}{3}\beta\left((-1) - u^c\left(x'\right)\right) + \tfrac{1}{3}\beta \sum_{i=a,b} u^i\left(x'\right),$$

and

$$\begin{aligned}\sup_{x'\in R_3^T}\sum_{i=a,b} U^i\left(x'\right) = \quad & \sup_{x'\in \mathsf{R}^2} \quad -2\left\|x' - F_{ab}\left(\tfrac{\beta}{3},0\right)\right\|^2 - \tfrac{1}{6}(1+\beta)(3-\beta) \\ & s.t. \quad \left\|x'-z^a\right\|^2 > \tfrac{1}{2}, \left\|x'-z^b\right\|^2 > \tfrac{1}{2}, \left\|x'-z^c\right\|^2 > \tfrac{1}{2}.\end{aligned}$$

The supremum is $-1+\frac{1}{3}\left(\sqrt{3}-1\right)\beta$, which is strictly less than $\sum_{i=a,b} U^i\left(H_1\left(ab|R_1^T\right)\right)$ for all m and all β. Therefore, $H_a(ab) \notin R_3^T$.

Regions R_4^T to R_{10}^T The procedures to characterize local maxima (or suprema) in regions R_4^T to R_{10}^T are similar to those for regions R_1^T, R_2^T and R_3^T. To save space, we only summarize the final utility calculations:

$$\sup_{x'\in R_4^T}\sum_{i=a,b} U^i\left(x'\right) = -\tfrac{64+64\beta-47\beta^2}{32(4-\beta)},$$

$$\max_{x'\in R_5^T}\sum_{i=a,b} U^i\left(x'\right) = -\tfrac{40+48\beta-7\beta^2}{8(8-\beta)},$$

$$\sup_{x'\in R_6^T}\sum_{i=a,b} U^i\left(x'\right) = -\tfrac{5+6\beta}{8},$$

$$\sup_{x'\in R_7^T}\sum_{i=a,b} U^i\left(x'\right) = -\tfrac{16+(19-2m)\beta}{24},$$

$$\sup_{x'\in R_8^T}\sum_{i=a,b} U^i\left(x'\right) = -\tfrac{16+19\beta}{24},$$

$$\sum_{i=a,b} U^i\left(\overline{z}\right) = -\tfrac{4+5\beta}{6},\ \left(R_9^T = \{\overline{z}\}\right), \text{ and}$$

$$\max_{x'\in R_{10}^T}\sum_{i=a,b} U^i\left(x'\right) = -\left(1+\beta\right).$$

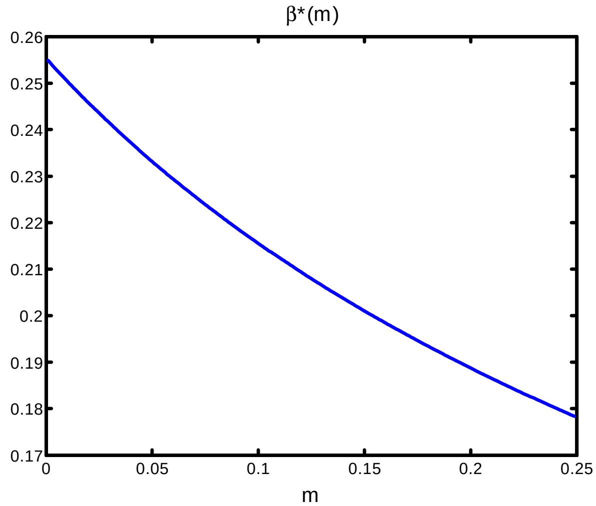

Figure 11: $\beta^*(m)$

All these local maxima or suprema can be verified to be strictly smaller than $\sup_{x'\in R_2^T}\sum_{i=a,b}U_i^i(x')$ for all m and all $\beta\neq 0$. Therefore, $H_a(ab)\notin\bigcup_{r=4}^{10}R_r^T$.

Comparison This analysis has shown that $H_a(ab)\notin R_r^T$ for $r=3,4,...,10$. The analysis of local maxima in regions R_1^T and R_2^T also implies that for all $m\in\left(0,\frac{1}{4}\right)$,

$$H_a(ab)=\begin{cases}F_{ab}\left(\frac{(1-m)\beta}{2-(1-2m)\beta},0\right), & \text{if } \beta\in[0,\beta^*(m)),\\ F_{ab}\left(\kappa\left(\frac{\beta}{2-\beta}\right),|\kappa-1|\left(\frac{1}{2}\right)\right), & \text{if } \beta\in[\beta^*(m),1].\end{cases}$$

Characterization of $\beta^*(m)$ Consider the claim that $\sup_{x'\in R_2^T}\sum_{i=a,b}U^i(x')\leq\max_{x'\in R_1^T}\sum_{i=a,b}U^i(x')$ for all $\beta\in[\beta^*(m),1]$ and all $m\in\left(0,\frac{1}{4}\right)$, where $\beta^*(m)$ is a decreasing function in m and $\beta^*(m)\in\left(0,\widehat{\beta}(m)\right)$ for all m. To show this, first of all, observe that for all m and all $\beta\in\left[\widehat{\beta}(m),1\right]$,

$$\begin{aligned}&\sum_{i=a,b}U^i\left(H_a\left(ab|R_1^T\right)\right)-\lim_{x'\to H_a\left(ab|R_2^T\right)}\sum_{i=a,b}U^i(x')\\ =&\left[\left(1+\frac{\sqrt{3}}{2}\right)m-\frac{\sqrt{3}}{2}+\frac{3}{4}\right]\beta+\sqrt{2\left(\beta-\frac{1}{2}\right)^2+\frac{3}{2}}-1>0.\end{aligned}$$

Therefore, $\beta^*(m)<\widehat{\beta}(m)$. Second, for all $m\in\left(0,\frac{1}{4}\right)$ and all $\beta\in\left[0,\widehat{\beta}(m)\right)$,

$$\begin{aligned}&\sum_{i=a,b}U^i\left(H_a\left(ab|R_1^T\right)\right)\geq\lim_{x'\to H_a\left(ab|R_2^T\right)}\sum_{i=a,b}U^i(x')\\ \Leftrightarrow\quad&4\left(2-(1-2m)\beta\right)\sqrt{2\left(\beta-\frac{1}{2}\right)^2+\frac{3}{2}}\geq\left(7-12m-m^2\right)\beta^2-2\left(7-4m\right)\beta+12,\end{aligned}$$

which is equivalent to

$$\begin{aligned}\Omega(\beta,m) &\equiv -\left(17-40m+2m^2+24m^3+m^4\right)\beta^4+4\left(9-16m+9m^2+4m^3\right)\beta^3\\ &\quad -4\left(19-32m-22m^2\right)\beta^2+16(5+4m)\beta-16\\ &\geq 0.\end{aligned}$$

Note that for all $m \in \left(0, \frac{1}{4}\right)$, (1) $\Omega(0,m) < 0$, (2) $\lim_{\beta\to\infty}\Omega(\beta,m) < 0$, (3) $\Omega(1,m) = 7+168m+122m^2-8m^3-m^4 > 0$, and (4) $\Omega(\beta,m)=0$ is a biquadratic equation in β with four roots. There are standard procedures of solving a biquadratic equation, and it can be verified that (5) two of its roots are real and the other two are imaginary. Call the two real roots $\beta_1^*(m)$ and $\beta_2^*(m)$ such that $\beta_1^*(m) \leq \beta_2^*(m)$. By (1), (2), (3) and (5), it follows that $0 < \beta_1^*(m) < 1 < \beta_2^*(m)$, and for all $\beta \in [\beta_1^*(m), 1]$, $\Omega(\beta,m) \geq 0$ and therefore $\sum_{i=a,b} U^i\left(H_a\left(ab|R_1^T\right)\right) \geq \lim_{x'\to H_a\left(ab|R_2^T\right)} \sum_{i=a,b} U^i(x')$. Then, define $\beta^*(m) = \beta_1^*(m)$ and $\beta^*(m) > 0$. Finally, it can be verified that the relationship between $\beta^*(m)$ and m is as shown in Figure 11. ■

Proof of Proposition 4. Consider a single-party government formed by party c in the first period. Party c chooses a policy to maximize its expected discounted sum of utilities. To analyze this maximization problem, partition the policy space into two regions: $R_1^S \equiv \overline{D}^a \backslash \left(D^b \cup D^c\right)$ and $R_2^S \equiv \Re^2 \backslash R_1^S$.

Suppose that party c is restricted to choose a policy x' from the set of R_1^S. Then in the second period the parliamentary election leads to a majority parliament, and the majority party c forms a consensus government with policy $\overline{z}$. This implies that party c's expected discounted sum of utility is

$$\begin{aligned}U^c(x') &= u^c(x')+\beta\left[(-1)-u^b(x')-u^c(x')\right]\\ &= \tfrac{3}{4}(2\beta-1)h^2-\tfrac{3}{2}h+(2\beta-1)w^2-\tfrac{3}{4}-\tfrac{1}{2}\beta,\end{aligned}$$

where $h, w \in \Re$ are such that $F(h,w) = x'$. The first-order condition for h is

$$\frac{\partial U^c}{\partial h} = \tfrac{3}{2}\left[(2\beta-1)h^2-1\right] \leq 0.$$

For $\beta \geq \frac{1}{2}$, the policy is as extreme as possible while still leading to a consensus government in the second period formed by c. That is, $h^* = \widehat{h} \equiv -1-\sqrt{\frac{2}{3}}$. For $\beta < \frac{1}{2}$, the maximum is attained at an interior solution $h^* = -\frac{1}{1-2\beta}$ if $\beta \leq \beta^o \equiv \frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$, and at a corner solution $h^* = \widehat{h}$ otherwise.

Suppose that party c is restricted to choose a policy x' from the set of R_2^S. Then compared to the case with a policy choice in R_1^S, the probability that party a is recognized as period-two formateur substantially dropped from one to below one-half. Therefore, party a loses some of its expected utility in the second period. At the same time, by choosing a policy outside R_1^S, party a makes the policy farther away from its ideal point and thus lowers its period-one utility. Thus, in equilibrium party a does not choose a policy in region R_2^S. ■

B The Measure of Social Welfare

The following proposition facilitates calculating (or approximating) the average two-period utility of all voters, given any sequence of policies (x_1, x_2).

Proposition 5 *Suppose that the ideal points of all voters are located symmetrically with respect to those of the three parties; i.e., for any voter v_1, whose ideal point is $F_{ab}(h, w)$, there exists voters v_2 and v_3 such that their ideal points are $z^{v_2} = F_{bc}(h, w)$ and $z^{v_3} = F_{ca}(h, w)$. Then, for any policy $x \in \Re^2$,*

$$\begin{aligned} \tfrac{1}{N}\sum_v u^v(x) &= \tfrac{1}{N}\sum_v \|z^v - \overline{z}\|^2 - \|x - \overline{z}\|^2 \\ &= \textit{constant} + u^{v^*}(x), \end{aligned}$$

where v^* is a hypothetical voter whose ideal point is at the center of preferences.

Proof. Take any voter v_1 and let $(h, w) \in \Re^2$ be such that $F_{ab}(h, w) = z^{v_1}$. By the assumption of symmetry, there exists v_2 and v_3 such that $z^{v_2} = F_{bc}(h, w)$ and $z^{v_3} = F_{ca}(h, w)$. Define $r \equiv \|x - \overline{z}\|$, $d \equiv \|z^{v_1} - \overline{z}\| = \|z^{v_2} - \overline{z}\| = \|z^{v_3} - \overline{z}\|$, $\theta_1 = \angle x\overline{z}z^{v1}$, $\theta_2 \equiv \angle x\overline{z}z^{v2}$, and $\theta_3 \equiv \angle x\overline{z}z^{v3}$. Observe that $\theta_2 - \theta_1 = \frac{2}{3}\pi$, $\theta_1 + \theta_3 = \frac{2}{3}\pi$, and as a consequence,

$$\begin{aligned}\sum_{i=1}^{3}\cos\theta_i &= 2\cos\tfrac{1}{2}(\theta_1-\theta_2)\cos\tfrac{1}{2}(\theta_1+\theta_2)+\cos\theta_3\\ &= 2\cos\tfrac{1}{3}\pi\cos\tfrac{1}{2}(\theta_1+\theta_2)+\cos\theta_3\\ &= \cos\tfrac{1}{2}(\theta_1+\theta_2)+\cos\theta_3\\ &= 2\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1+\tfrac{1}{2}\theta_2+\theta_3\right)\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1+\tfrac{1}{2}\theta_2-\theta_3\right)\\ &= 2\cos\tfrac{1}{2}\left(\tfrac{1}{2}(\theta_1-\theta_2)+(\theta_1+\theta_3)\right)\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1+\tfrac{1}{2}\theta_2-\theta_3\right)\\ &= 2\cos\tfrac{1}{2}\left(\tfrac{1}{2}\left(\tfrac{2}{3}\pi\right)+\tfrac{2}{3}\pi\right)\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1+\tfrac{1}{2}\theta_2-\theta_3\right)\\ &= 2\cos\tfrac{1}{2}\pi\cos\tfrac{1}{2}\left(\tfrac{1}{2}\theta_1+\tfrac{1}{2}\theta_2-\theta_3\right)\\ &= 0\end{aligned}$$

Given policy x, voter v_i's per-period utility is

$$\begin{aligned}u^{v_i}(x) &= -\left\|x-z^{v_i}\right\|^2\\ &= -\left[(d-r\cos\theta_i)^2+(r\sin\theta_i)^2\right]\\ &= -\left[d^2+r^2\left(\cos^2\theta_i+\sin^2\theta_i\right)-2dr\cos\theta_i\right]\\ &= -\left(d^2+r^2\right)+2dr\cos\theta_i.\end{aligned}$$

Therefore, the aggregate per-period utility of voters v_1, v_2, and v_3 is

$$\begin{aligned}\sum_{i=1}^{3}u^{v_i}(x) &= -3\left(d^2+r^2\right)+2dr\sum_{i=1}^{3}\cos\theta_i\\ &= -3\left(d^2+r^2\right)\\ &= -\sum_{i=1}^{3}\left\|z^{v_i}-\overline{z}\right\|^2-3\left\|x-\overline{z}\right\|^2.\end{aligned}$$

■

Recall that there are sufficiently many voters, and their ideal points are uniformly distributed in a disk around the center of preferences. Therefore, voters can be grouped in triplets, such that for any voter v_1, whose ideal point is $F_{ab}(h,w)$, there exists voters v_2 and v_3 whose ideal points are *sufficiently close to* $F_{bc}(h,w)$ and $F_{ca}(h,w)$. Since the number of voters may not be a multiple of 3, there might be remainders of 1 or 2 voters. However, since there are a large number of voters, the effect of these remainder voters' utilities on the average is negligible. Therefore, $\frac{1}{N}\sum_v\left\|z^v-\overline{z}\right\|^2-\left\|x-\overline{z}\right\|^2$ is a good approximation of the average per-period utility of all voters. Note that the first term is just a constant, given any set of voters and the second term is the per-period utility of a hypothetical voter, whose ideal point is at the center of preferences. This provides a convenient measure of the social welfare.

References

[1] Austen-Smith, D. and Banks, J. 1988. "Elections, Coalitions, and Legislative Outcomes." *American Political Science Review*, 82 (June), 405-422.

[2] Baron, David P. 1993. "Government Formation and Endogenous Parties." *American Political Science Review*, 87 (March): 34-47.

[3] Baron, David P. 1996. "A Dynamic Theory of Collective Goods Programs." *American Political Science Review*. 90 (June): 316-330.

[4] Baron, David P. and Michael Herron. 2003. "A Dynamic Model of Multidimensional Collective Choice." In *Computational Models of Political Economy*, Ken Kollman, John H. Miller, and Scott E. Page, ed., MIT Press, pp. 13-47.

[5] Baron, David P. and Daniel Diermeier. 2001. "Elections, Governments, and Parliaments in Proportional Representation Systems." *Quarterly Journal of Economics*. (August): 933-967.

[6] Battaglini, Marco and Stephen Coate. 2005. "Inefficiency in Legislative Policy-Making: A Dynamic Analysis." Working paper, Princeton University.

[7] Battaglini, Marco and Stephen Coate. 2006. "A Dynamic Theory of Public Spending, Taxation and Debt." Working paper, Princeton University.

[8] Bawn, Kathleen. 1999. "Voter Responses to Electoral Complexity: Ticket Splitting, Rational Voters and Representation in the Federal Republic of Germany." *British Journal of Political Science* 29 (3): 487-505.

[9] Bernheim, Douglas, Antonio Rangel, and Luis Rayo. 2006. "The Power of the Last Word in Legislative Policy Making." *Econometrica*. 74: 1161-1190.

[10] Besley, Timothy and Stephen Coate. 1998. "Sources of Inefficiency in a Representative Democracy." *American Economic Review*. 88: 139-156.

[11] Cox, Gary W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge, UK: Cambridge University Press.

[12] Degan, Arianna and Antonio Merlo. 2006. “Do Voters Vote Sincerely?” Working paper, University of Pennsylvania.

[13] Diermeier, Daniel, Eraslan, Hülya, and Antonio Merlo. 2003. “A Structural Model of Government Formation.” *Econometrica*. 71: 27-70.

[14] Diermeier, Daniel and Antonio Merlo. 2004. “An empirical investigation of coalitional bargaining procedures.” *Journal of Public Economics*. 88: 783-797.

[15] Fong, Pohan. 2006. “Dynamics of Government and Policy Choice," Mimeo.

[16] Kalandrakis, Anastassios. 2004. “A Three-Player Dynamic Majoritarian Bargaining Game.” *Journal of Economic Theory*. 16: 294-322.

[17] Laver, M. and Schofield, N. 1990. *Multiparty Governments*. Oxford: Oxford University Press.

[18] Penn, Elizabeth Maggie. 2005. “A Model of Farsighted Voting.” Working paper, Harvard University.

[19] Persson, Torsten and Guido Tabellini. 2000. *Political Economics: Explaining Economic Policies*. Cambridge, MA: MIT Press.

[20] Persson, Torsten and Guido Tabellini. 2003. *The Economic Effects of Constitutions. The Munich Lectures in Economics*. Cambridge, MA: MIT Press.

[21] Riboni, Allesandro. 2005. “Committees as Substitutes for Commitment.” Working paper, Universite de Montreal.

[22] Schofield, Norman and Itai Sened. 2006. *Multiparty Democracy: Elections and Legislative Politics*. Cambridge University Press: Cambridge.

[23] Warwick, Paul. 1994. *Government Survival in Parliamentary Democracies*. Cambridge: Cambridge University Press.