The Non-Bank Credit Channel

Nicolas Crouzet[1] and Olivier Darmouni[2]

[1]Kellogg School of Management, Northwestern University
[2]Columbia Business School

This version: January 2023.*

[Preliminary and incomplete; please do not circulate or cite.]

Abstract

This paper studies the macroeconomic implications of the rise of non-bank financial intermediaries (NBIs) — institutional investors ranging from insurance companies to bond mutual funds — in U.S. corporate credit markets. Contrary to commercial banks, NBIs are not levered financial intermediaries, and do not face default risk. Some NBIs, because of their funding structure, nevertheless face redemption risk. We provide a framework to analyze the implications of these differences in funding structure for macroeconomic and financial stability. Relative to banks, NBIs do not necessarily promote macroeconomic and financial stability, especially when redemption risk is high. However, regulatory attempts at constraining redemptions have little macroeconomic impact unless NBIs hold sufficient capital buffers.

*We thank Ian Dew-Becker, Yueran Ma, and Fabrice Tourre for useful discussions, and Yudan Ying for outstanding research assistance.

1 Introduction

The credit channel holds that endogenous fluctuations in the cost of credit to firms can amplify cyclical movements in economic activity (Bernanke and Gertler, 1989, 1995). An extensive literature has argued that a mechanism central to this amplification is the so-called bank lending channel, whereby the source of variation in the cost of credit to firms are frictions between banks and depositors (Bernanke and Blinder, 1988).

A challenge for this literature is that, in recent years, the role of banks in U.S. corporate capital markets has shrunk. Figure 1 shows that bank loans now only account for approximately 1 of every 3 dollars of non-financial corporate debt outstanding, with bonds making up the lion's share of the rest. The share of non-financial corporate debt in the asset porfolio of U.S. commercial banks has also declined. As banks have withdrawn from corporate capital markets, they have progressively been replaced by non-bank intermediaries (NBIs). Though NBIs are often lumped under the umbrella term of "institutional investors", the group include a variety of intermediary types, ranging from insurance companies and pension funds, to loan and bond mutual funds. While the role of NBIs in bond markets has always been central, in recent years, they have also become central players in loan markets (Berg et al., 2020; Cordell et al., 2021).

In this paper, we ask whether the rise of NBIs in corporate capital markets should prompt academics and policymakers to revisit their understanding of how the credit channel works.

We focus on one important difference between banks and NBIs: their funding structure. The traditional bank lending channel relies on the idea that bank deposit-taking, and the leverage and default risk it creates, leads to frictions in intermediation. But NBIs are not funded using deposits. As we discuss in Section 2, the liabilities issued by NBIs are instead closer to equity contracts, with one crucial difference: they often feature embedded put options. These put options, which effectively allow investors to redeem the liabilities at pre-set prices, can force NBIs to liquidate assets to meet redemptions, potentially at inopportune times. Thus, while banks are levered intermediaries that face default risk, NBIs do not use leverage, and instead face redemption risk.

We ask three specific questions. First, do these differences in funding structure matter at all for macroeconomic activity? Second, and related, should we expect the rise of NBIs to be good or bad for macroeconomic and financial stability, relative to a world where banks are the main intermediaries? Third, should financial regulation approach BIs and NBIs in the same way?

The answer to these questions is not obvious for at least two reasons. First, a traditional view of NBIs is that they are a "veil", in the sense that they intermediate funds from savers to borrowers with little or no frictions. A bond mutual fund, for instance, issues shares to investors and simply invests the proceeds in corporate debt. It is not necessarily clear why redemption risk should create distortions in the investment decisions of NBIs, especially if it remains idiosyncratic. Second, even if redemption risk does create distortions, it is not clear how these distortions would compare with those created by levered intermediaries like banks. While the investor base of bond mutual funds may be more volatile that bank depositors (as the events of March 2020 illustrated), pension funds, for instance, may face less redemption risk, and be a source of financial stability compared to banks.

In Section 2, we discuss the broad anatomy of corporate credit markets in the US, highlighting the growing importance of non-bank intermediaries. While these intermediaries differ from banks in many ways, we pay particular attention to differences in their funding structure with respect to banks, since funding structure is at the heart of the bank lending channel. As hinted at above, we isolate redemption risk as the key funding problem for NBIs, and show that, in this respect, NBIs live on a continuum: while pension funds and insurance companies face limited redemption risk, the risk is much more substantial for other types of NBIs, like loan or bond mutual funds.

In order to address our three core questions, we then construct a macroeconomic model with a financial sector. The model is described in Section 3. The main contribution, relative to the literature, is to design the financial intermediation block so that it can nest different types of financial intermediaries as special cases. We focus on two specific cases, consistent with the discussion of Section 2. First, we consider bank-like intermediary (BI), which resembles intermediaries in existing models: it issues deposits and faces default risk. Second, we introduce a non-bank intermediary, which issues redeemable shares to fund its investments. The non-bank intermediary is subject to redemption risk, which we model by assuming that a fraction of investors can choose to redeem the shares at a pre-determined price in any period. We refer to the two versions of the model — the one with a bank-like, levered intermediary, and the one with an intermediary funded through redeemable shares — as the "BI" and "NBI" models, respectively. While the framework is fully dynamic, we design it so that it can be solved in closed-form, making it possible to exactly compare its predictions for macroeconomic activity in the BI and NBI case.

We then use the model to answer the three questions formulated above. The first question is whether, in general, differences in the funding structure of intermediaries matter for macroeconomic activity. The answer to this question, in our model, is stark: they do, but *only* to the extent that they lead to different equilibrium probabilities of intermediary liquidation. More precisely, the equilibrium of the model has a simple Markovian structure: at any date, the intermediary (whether BI or NBI) is either active (when its portfolio returns are sufficiently high), or inactive (when its portfolio returns are sufficiently low). Conditional on being in a state where the intermediary is active or inactive, all macroeconomic outcomes are therefore the same, and depend only on the transition probability between states, which is the probability of intermediary liquidation. The somewhat more surprising result is that the mapping is *independent* of the details of the intermediation process, and in particular, of the funding structure of the intermediary. In other words, the default probability of the intermediary is a sufficient statistic for understanding the macroeconomic effects of financial frictions in this model. While this result is particularly stark, and depends on assumptions on preferences and adjustment costs, it helps separate cleanly the macroeconomic and the financial blocks of the model. In turn, this separation makes the comparison of the macroeconomic effects of different types of financial intermediaries simple, since one only needs to compare equilibrium liquidation probabilities across models.

This helps us answer our second question: relative to banks, do NBIs amplify or mitigate the macroeconomic distortions created by financial intermediation? The answer to this question, in the model, depends mainly on the relative magnitude of two key structural parameters: the fraction λ of investors that can redeem their shares early in the NBI model; and the leverage constraint of banks in the BI model.

There are two broad possibilities. If bank leverage constraints are very slack, so that banks have very low capital buffers, the NBI model leads to lower intermediary liquidation risk for *any* value of λ. The intuition is that in the NBI model, liquidation can only be triggered by redemptions, and redemptions only happen in states of the world in which asset returns are low. This implies that the intermediary liquidation probability in the NBI model is always bounded from above. By contrast, in the BI model, the liquidation probability of the intermediary becomes arbitrarily large as the capital buffer of the intermediary goes to zero. This is because, as the capital buffer of the bank shrinks, depositors require an increasingly large default risk premium on their deposits, in turn leading the intermediary to default even when portfolio returns are high.

If, on the other hand, bank leverage constraints are tighter, then the model shows that there is a unique threshold for the share of early investors, λ, above which the NBI model leads to a higher liquidation probability — and therefore stronger macroeconomic distortions — than the BI model. In other words, the degree of funding fragility of the NBI, λ, relative to the amount of bank leverage, is sufficient to determine whether non-bank intermediation improves or worsens outcomes, relative to a world where banks are the main intermediaries.

This naturally leads to our third question: how should policymakers approach the regulation of the funding structure of NBIs? Here, the model has two main implications. The first and natural implication is that while higher capital requirements can help improve macroeconomic stability in the BI model, in the NBI model, there are two possible tools. The first is capital requirements (such as constraints on holdings of liquid assets), which make redemptions less likely to lead to liquidations. The second are direct restrictions on redemptions (such as redemption gates), which directly reduce the frequency with which investors redeem their shares. Both forms of financial regulation can improve outcomes in the NBI model.

However, the second implication of the model is that restrictions on redemptions have no significant impact *unless* the NBIs are already sufficiently well-capitalized. In the limit where the NBI is fully funded through redeemable shares, and has no capital buffers, we show that liquidity regulation has *no* impact on macroeconomic and financial stability. Without capital buffers, any redemption by even a small fraction of investors will force asset liquidations. As mentioned earlier, the redeemable shares issued by NBIs embed a put option on the underlying asset portfolio. Investors optimally exercise this put option when portfolio returns have fallen below the strike price of the put. Without a capital buffer, this is also precisely when the NBI cannot meet the redemption calls, even if it were to sell assets. More generally, when NBIs have narrow capital buffers, the model suggests that the best way to reduce macroeconomic and financial stability risk is to encourage them to build up these buffers. In other words, capital requirements are more impactful than redemption limits. The reverse intuition holds when NBIs have larger capital buffers; in that case, redemption limits are the more impactful tool.

Overall, the framework proposed in this paper leads to three results on the link between funding structure of intermediaries, and macroeconomic and financial stability. On the positive side, we show that details of the funding structure of intermediaries can have macroeconomic consequences, but also proposes a simple sufficient statistic to measure measure these distortions. Additionally, we show that intermediation through NBI will only lead to improved macroeconomic outcomes, compared to a world with banks, if the redemption risk that NBIs face is sufficiently low, or if bank leverage is sufficiently high, and we characterize exactly what "sufficiently" means in terms of structural parameters of the model. Finally, on the normative side, it shows that while liquidity regulation might be a useful prudential tool in a world of non-bank intermediation, it can only be impactful if intermediaries are already well-capitalized.

Related literature This paper contributes to three main strands of literature.

First, it relates to the macroeconomic literature on the credit channel. Recent empirical work has shown that frictions or shocks affecting banks can propagate to the real economy (Chodorow-Reich, 2014; Chodorow-Reich et al., 2022), particularly so in the context of economies where intermediation to firms primarily runs through the banking sector (Jiménez et al., 2012). This provides support for recent models of the bank lending channel, where leverage and risk-taking by banks is the key friction that leads to macroeconomic distortions (Gertler and Kiyotaki, 2010; Gertler et al., 2020). The contribution of this paper is to show that other funding structures, in which leverage plays a muted role, can still produce large macroeconomic distortions. Intermediation friction can remain an important source of amplification of macroeconomic risk even in a world where intermediaries are not levered — something we refer to as the "non-bank credit channel".

Second, the paper relates to a literature in corporate finance that highlights the rise of non-bank intermediaries as one of the key recnt trends in corporate borrowing. This literature is discussed in more detail in Section 2. Within this literature, recent empirical work has established that funding shocks to non-bank financial intermediaries can have substantial effects on their portfolio allocation decisions, and that these effects feed through to corporate investment (Siani, 2021; Coppola, 2021). However, estimates provided in this literature are cross-sectional; their macroeconomic implications are unclear. Our paper contributes to this literature by identifying the theoretical conditions under which NBI funding risk could indeed have macroeconomic effects.

Third, the paper relates to a literature in macroeconomic and finance on the role of intermediaries for asset prices (He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014; Muir, 2017; Haddad and Muir, 2021). Our contribution is to allow for general payoff structures for intermediary liabilities in this class of models, and study their macroeconomic implications in two particular cases — deposits, and redeemable shares. Relative to existing work, our framework makes some simplifications, but the payoff is that we can characterize and compare macroeconomic outcomes across models analytically, and derive a number of sufficient statistics for the size of macroeconomic distortions. Our approach thus complements existing work, which generally focuses on numerical solution methods.

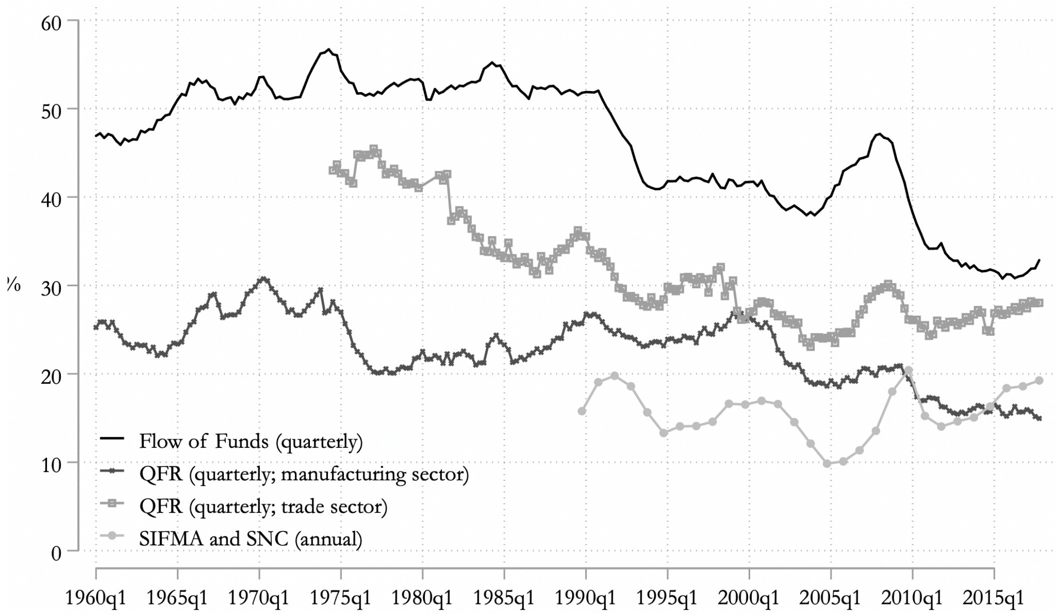


Figure 1: The share of loans in total debt for US non-financial firms. Each line plots the ratio of aggregate loans to aggregate stock of debt outstanding for non-financial firms, using a different data sources. The figure is reproduced from Crouzet (2021).

2 An Anatomy of Non-Bank Credit to Corporations

This section describes the role played by NBIs in U.S. corporate capital markets. NBIs are also often referred to as "institutional investors". They consist of large institutions, like insurance companies, pension funds or mutual funds, that provide credit intermediation, potentially in addition to other financial services. They contrast with "retail investors" that mainly consist of wealthy individuals that save in financial markets.

We start by describing their broad importance, relative to banks. We then discuss their role in two specific corporate credit markets: the bond market and the loan market. Finally, we highlight the key common economic features of the different types of NBIs.

2.1 The relative importance of NBIs

In aggregate, loans represent only about 30% of credit to non-financial firms in the US, implying that there are about two dollars of bonds outstanding for each dollar of loans outstanding. Figure 1, reproduced from Crouzet (2021), shows the loan share of aggregate non-financial corporate debt outstanding, using a variety of data sources. The survey of Berg et al. (2020) also shows that aggregate balance-sheet borrowing of firms is driven by bond financing.[1] There are nevertheless some differences in the cross-section of firms: larger and safer "investment-grade" (IG) firms use bonds for balance sheet financing, while non-IG ("leveraged") firms are much reliant on term loans,

[1] On the other hand, undrawn credit lines/revolver have become increasingly important since the global financial crisis of 2008–2009.

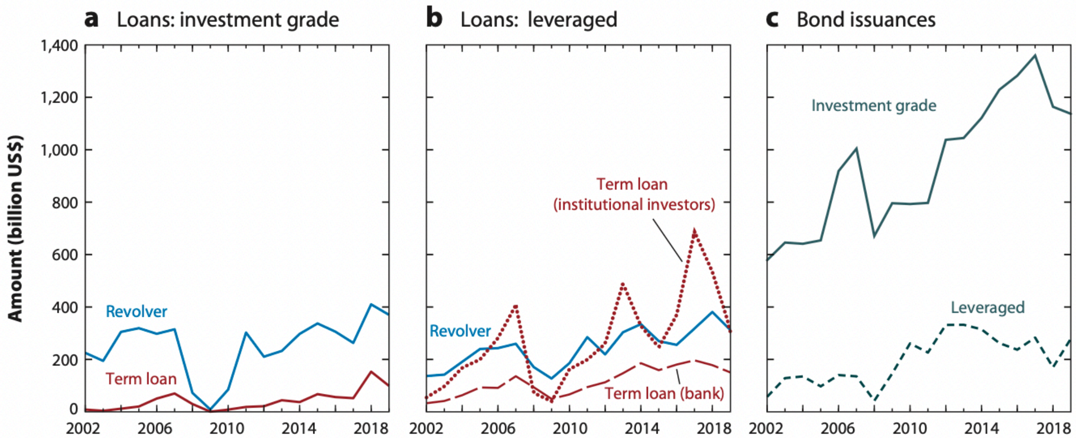


Figure 2: New borrowing in the investment grade loan market (left panel), the leverage loan market (middle panel), and the bond market (right panel). The figure is reproduced from Berg et al. (2020).

as shown in Figure 2.

While their preponderant role in bond markets is well understood, NBIs are also key players in the loan market. Indeed, term loans are largely funded by non-banks in the syndicated loan market, not banks, the survey of Berg et al. (2020) also establishes. We next discuss in more detail their role in each market, starting with the bond market.

2.2 NBIs in the corporate bond market

Two contrasting types of NBIs play a key role in the corporate bond market: insurers and pension funds (IPs) and mutual funds (MFs). Together, they account for the lion's share of the market: two-thirds of aggregate U.S. corporate bonds outstanding, including 85% of domestic holdings. MFs (including exchange-traded-funds) have grown from less than 8% during the early 2000s to more than 20% today. IPs are on the decline, but still hold about 40% of total corporate bonds outstanding, as illustrated in Figure 3, reproduced from Li and Yu (2021).[2]

The key economic differences between IPs and MFs are their investment horizon and their liability structure. IPs have long-term liabilities, such as insurance or retirement policies. They thus face little outflow risk, and have low trading needs. MFs tend to have open-ended capital structures, meaning their shares can be redeemed by investors with little or no notice. In bad times, these redemptions can lead to outflows and fire-sales. While IPs help stabilize funding to non-financial firms (Coppola, 2021), MFs were at the center of the modern bank runs observed in Spring 2020 (Ma et al., 2020; Falato et al., 2021).

Several additional details are worth highlighting. First, MFs do their own liquidity management

[2] One reason that has been hypothesized for the rise of MFs is the decline in risk-free rates. By triggering reaching-for-yield, the decline in risk-free rates may have encouraged investors to move towards riskier/more illiquid assets. Li and Yu (2021) present a simple model in which, as yields fall, investors with high liquidity needs move away from risk-free assets towards higher-yielding but more illiquid corporate bonds. This can help explain the rise of MFs. In addition, reaching-for-yield can also account for the slow decline in the IP share, as IPs move away from traditional fixed income into alternative private investments.

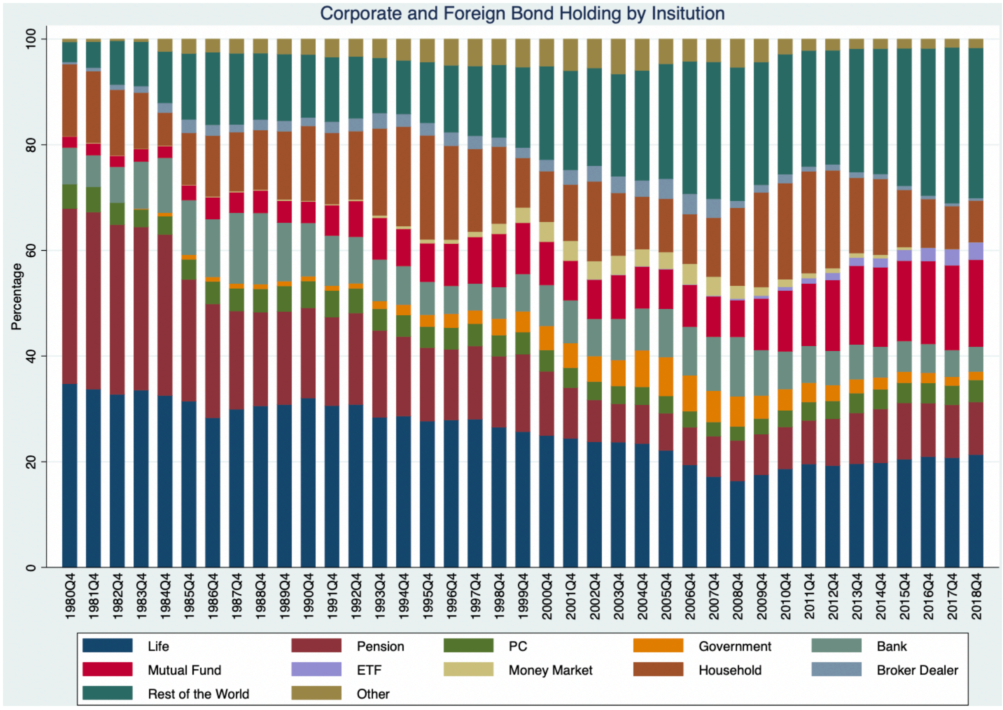


Figure 3: Share of total corporate and foreign bond holdings by investor type. The data are from the flow of funds. The figure is reproduced from Li and Yu (2021).

by holding cash or Treasuries as buffer. The reverse flight to liquidity in 2020 was largely due to the balance sheet structure of these new non-banks intermediaries (Ma et al., 2020; Falato et al., 2021; Vissing-Jorgensen, 2020). The effect was dampened during the GFC because MFs were significantly smaller. IPs are tightly regulated: they face capital charges that essentially force them to hold only investment-grade bonds. The IP sector is also generally more concentrated than the MF sector. Finally, insurers also have a preference for holding long-term bonds in order to reduce the duration mismatch with their (very) long-term liabilities.

2.3 NBIs in the corporate loan market

The leveraged loan market has grown dramatically since the financial crisis: from just under \$500 billion at year end 2010 to over \$1.7 trillion at the end of 2020. Most leveraged loans are syndicated and, except for revolving lines of credit, are not typically retained by the originating banks. According to Berg et al. (2020), NBIs hold more than 80% of leveraged loans originated in 2019. Moreover, a deep and liquid secondary market has developed in the United States over the last 20 years, enabling banks to sell their exposure after loans have been originated (Irani et al., 2021), supporting the view that the investors' base of leveraged loans has become more diverse and that leveraged loans have thus become more like high-yield bonds.

The two main types of NBIs in loan markets are Collateralized Loan Obligations (CLOs) and loan mutual funds (LMFs). CLOs are the largest investor by a significant margin: two-thirds, or \$2.1 trillion, of leveraged loan issuance since the 2008 financial crisis has been funded by CLOs

(Cordell et al., 2021). They have grown tremendously in recent years. By contrast, loan mutual funds held about 15 percent of leveraged loans outstanding at the end of 2018.

The asset side of CLOs' balance sheets consist primarily of floating-rate, senior secured term loans with maturities between five and seven years. On the liability side, their debt tranches are differentiated by priority in the CLO capital structure — senior (AAA and AA), mezzanine (A and BBB), and junior (BB and B) —, and consequently the interest rate spread they are promised. Equity investors receive unsecured, unrated claims.

A large share of CLO funding comes from banks. Banks invest primarily in AAA-rated senior tranches. Insurance companies and pension funds invest across the capital structure, while hedge funds and other alternative asset managers concentrate in mezzanine and junior debt. The equity tranche is usually funded in part by a private credit fund raised by the CLO manager's parent company, with outside investors contributing as well.

2.4 Funding fragility and relation to the credit channel

The previous discussion highlighted two key points. First, NBIs are central players in corporate debt markets. Second, the funding structure of NBIs varies across investor types — IPs, MFs, CLOs —, and also differs from the traditional deposit-taking model that commercial banks rely on for their funding.

A common feature of the funding structure of NBIs is that they rely on liabilities that are directly exposed to the underlying asset portfolio — as opposed to traditional bank deposits, which pay a fixed interest rate —, but that also offers redemption options for investors. Thus while these liabilities may appear to be more equity-like — in contrast with the debt-like liabilities of commercial banks —, they still expose create redemption risk for NBIs.

However, the degree of redemption risk is different across NBIs. In the bond market, we can expect IPs to be less cyclical than commercial banks (since they have more long-term liabilities). MFs are likely more cyclical: their liabilities are redeemable on demand but there is no equivalent to deposit insurance and they have no access to central bank funding.

Likewise, in the loan market, CLOs are more stable investors, while loan mutual funds are potentially more fragile. Indeed, CLOs are closed-end vehicles in which capital inflows and outflows are limited, and are financed by issuing long-term debt with maturities in excess of seven years and fixed credit spreads.[3] Loan funds are more concerning: they have open-ended liquid liabilities, and can have to sell loans quickly to meet redemptions.

Because redemption risk is heterogeneous among investors types, it is thus ambiguous whether the rise of NBIs, should increase or decrease financial and macroeconomic stability overall. The rise of bond mutual funds, for instance, does raise concerns that were in plain sight during COVID, triggering a drastic shift in the Fed Credit policy of purchasing corporate bonds for the first time

[3]Moreover, "coverage tests" used for risk management are based on bond par values and credit ratings instead of market prices. Consequently, market volatility does not cause the diversion of cash flows to pay down debt tranches unless the volatility coincides with rating downgrades and defaults.

ever. In the following section, we develop a simple model to tackle this question.

3 A framework for comparing financial intermediaries

This section describes a macroeconomic framework that nests two possible types of financial intermediaries: bank-like intermediaries (BIs), which are funded through deposit issuance, and are subject to default risk; and non-bank intermediaries (NBIs), which are instead funded through the issuance of redeemable shares, and are subject to redemption risk.

The goal of the model is to describe the potential implications of the rise of NBIs for macroeconomic and financial stability. Consistent with the discussion in the previous section, a particular focus will be to understand whether the degree of redemption risk faced by NBIs tends to amplify or mitigate macroeconomic and financial risk, relative to a world in which banks intermediate credit.

The model has three agents: a household; a non-financial corporation (NFC); and a financial intermediary. The rest of the section describes the problem of each agent in turn. At the end of the section, we discuss in more detail the motivation for the key assumptions of the model. Throughout the section, we use the notation:

$$\zeta_t = \mathbf{1}\left\{\text{intermediary active in period } t\right\}.$$

3.1 Household and non-financial corporation

Household In any period t where $\zeta_t = 1$, the household solves following problem:

$$\begin{aligned} W_t &= \max \quad \log(C_t) - \frac{\chi}{2(1+\rho)} S_{t+1}^2 + \mathbb{E}_t\left[\frac{W_{t+1}}{1+\rho}\right] \\ \text{s.t.} & \quad C_t + P_t L_{t+1} + S_{t+1} V_t + S_{t+1}^{(i)} V_t^{(i)} \leq S_t(\Pi_t + V_t) + S_t^{(i)}(\Pi_t^{(i)} + V_t^{(i)}) + F_t L_t, \\ & \quad L_{t+1} \geq 0, \quad S_{t+1} \geq 0, \quad S_{t+1}^{(i)} \geq 0. \end{aligned}$$

In periods when the intermediary is not liquidated, the household can invest in three possible assets. First, it can buy non-equity liability contracts issued by the intermediary. These cost P_t and offer a (possibly state-contingent) gross payoff of F_{t+1} per contract; the household purchases L_{t+1} of them. For instance, in the case of the bank-like intermediary, these liabilities will be deposit contracts, which will have a price of $P_t = 1$ and earn a pre-determined rate of return $R_{t+1}^{(d)}$ if the intermediary is not liquidated.

Second, the household can buy equity in the financial intermediary. Here, $V_t^{(i)}$ is the ex-dividend (or end-of-period) value of intermediary equity; $\Pi_t^{(i)}$ are net dividend payouts, which can be negative (corresponding to equity issuance); and $S_{t+1}^{(i)} \in [0,1]$ is the number of intermediary shares owned by the household (we normalize the total number of equity shares to 1).

Finally, the household can buy equity in the NFC. Here, $S_{t+1} \in [0,1]$ is the number of shares of the NFC directly held by the household (we have again normalized the total to 1); Π_t are net dividend payouts from the NFC, which can be negative; and V_t is the ex-dividend value of NFC equity. The household's holdings of NFC equity are the object of our first key assumption.

Assumption 1 (Frictions in direct financing)**.** *The household incurs a flow utility cost of holding NFC equity, whose size is determined by the parameter* $\chi \geq 0$*.*

Notice that this cost only affects the household's holdings of NFC equity; holdings of equity or non-equity liabilities of the intermediary are not subject to this cost. This creates scope for intermediation to improve macroeconomic outcomes, relative to a world without intermediary; we discuss this assumption in more detail in Section 3.4.

In any period t where $\zeta_t = 0$, the intermediary has been liquidated and remains inactive. Therefore, it does not issue non-equity liabilities or equity, so that the household solves following problem:

$$\begin{aligned} W_t &= \max \quad \log(C_t) - \frac{\chi}{2(1+\rho)} S_{t+1}^2 + \mathbb{E}_t \left[\frac{W_{t+1}}{1+\rho} \right] \\ \text{s.t.} & \qquad C_t + S_{t+1} V_t \leq S_t(\Pi_t + V_t) + F_t L_t, \\ & \qquad S_{t+1} \geq 0. \end{aligned}$$

Here, F_t is the payoff received from the intermediary's non-equity liabilities at dates when the intermediary is liquidated. Because no intermediary is active at these dates, the household will have to hold all of the NFC equity.

Let $\lambda_t = C_t^{-1}$ be the marginal utility of consumption, $\tilde{\nu}_t \geq 0$ be the Lagrange multiplier associated with the constraint $S_{t+1} \geq 0$, and $\nu_t \equiv (\lambda_t V_t)^{-1} \tilde{\nu}_t \geq 0$. The necessary first-order condition for holdings of NFC equity are:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}^{(e)} \right] \qquad \text{and} \qquad 0 = \nu_t S_{t+1},$$

where:

$$\Lambda_{t,t+1} \equiv \frac{1}{(1+\rho)} \frac{C_{t+1}^{-1}}{(1-\nu_t) C_t^{-1} + \frac{\chi}{1+\rho} S_{t+1} V_t^{-1}} \qquad \text{and} \qquad R_{t+1}^{(e)} \equiv \frac{\Pi_{t+1} + V_{t+1}}{V_t}.$$

First-order conditions relating to the intermediary only hold in periods where $\zeta_t = 1$. The first-order condition for intermediary equity is:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1}^{(u)} \zeta_{t+1} R_{t+1}^{(i)} \right]$$

and the first-order condition for non-equity liability contracts is:

$$P_t = \mathbb{E}_t \left[\Lambda_{t,t+1}^{(u)} F_{t+1} \right],$$

where:

$$\Lambda_{t,t+1}^{(u)} \equiv \frac{1}{(1+\rho)} \frac{C_{t+1}^{-1}}{C_t^{-1}} \qquad \text{and} \qquad R_{t+1}^{(i)} \equiv \frac{\Pi_{t+1}^{(i)} + V_{t+1}^{(i)}}{V_t^{(i)}}.$$

The notation (u) in the discount factor $\Lambda^{(u)}_{t,t+1}$ is for "undistorted", since, as mentioned above, holdings of intermediary liabilities or equity are not subject to the utility cost χ.[4]

Non-financial corporation In any period, the NFC faces the following problem:

$$\begin{aligned} V^c_t(K_t) &= \max \quad AK_t - \Phi\left(\frac{I_t}{K_t}\right)K_t + \mathbb{E}_t\left[\Lambda^{(f)}_{t,t+1}V^c_{t+1}\left(\xi_{t+1}\tilde{K}_{t+1}\right)\right] \\ \text{s.t.} & \quad \tilde{K}_{t+1} \le I_t + (1-\delta)K_t. \end{aligned}$$

Here, $V^c_t(K_t)$ denotes the cum-dividend (or beginning-of-period) value of NFC equity. Capital quality shocks, ξ_{t+1}, create a wedge between the planned capital stock, $\tilde{K}_{t+1}$, and the realized one,

$$K_{t+1} = \xi_{t+1}\tilde{K}_{t+1}.$$

We assume that the discount factor used by the NFC is:

$$\Lambda^{(f)}_{t,t+1} = \zeta_t\Lambda^{(u)}_{t,t+1} + (1-\zeta_t)\Lambda_{t,t+1}.$$

Since the $\Lambda^{(u)}_{t,t+t}$ is the effective discount rate of the intermediary, this assumption says that outside of liquidation dates, the intermediary is pricing NFC equity, while at liquidation dates, the household is. We discuss this implications of this assumption in more detail in Section 3.4. Finally, the (ex-dividend) value of NFC shares is given by:

$$V_t = \mathbb{E}_t\left[\Lambda^{(f)}_{t,t+1}V^c_{t+1}\right],$$

while dividends are given by:

$$\Pi_t = AK_t - \Phi\left(\frac{I_t}{K_t}\right)K_t.$$

The optimality condition for investment is the standard Q-theory condition:

$$\Phi'\left(\frac{I_t}{K_t}\right) = Q_t,$$

where:

$$Q_t = \mathbb{E}_t\left[\Lambda^{(f)}_{t,t+1}\xi_{t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}(\xi_{t+1}\tilde{K}_{t+1})\right].$$

Additionally, standard derivations show that the envelope condition can be expressed as:[5]

$$V_t = Q_t\tilde{K}_{t+1}.$$

[4] We omitted Lagrange multipliers associated with the non-negativity constraints on $S^{(i)}_{t+1}$ and L_{t+1} because we will focus on equilibria where $S^{(i)}_{t+1} = 1$ and $L_{t+1} > 0$.

[5] The envelope condition can also be written as $\mathbb{E}_t\left[\Lambda^{(f)}_{t,t+1}R^{(I)}_{t+1}\right] = 1$, where the return on investment $R^{(I)}_{t+1}$ is defined as $R^{(I)}_{t+1} \equiv Q_t^{-1}\left(\xi_{t+1}\left(A - \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) + \left(\frac{I_{t+1}}{K_{t+1}} + (1-\delta)\right)Q_{t+1}\right)\right)$.

3.2 Financial intermediary

We now describe the financial intermediary. We start with a general model, and then specialize it to the two cases of interest: a bank-like intermediary, and a non-bank intermediary.

3.2.1 A general financial intermediary

The intermediary's problem is:

$$V_t^{(i,c)}(L_t, E_t) = \max_{L_{t+1}, E_{t+1}} R_t^{(e)} E_t - E_{t+1} + P_t L_{t+1} - F_t L_t + \mathbb{E}_t \left[\Lambda_{t,t+1}^{(u)} \zeta_{t+1} V_{t+1}^{(i,c)}(L_{t+1}, E_{t+1}) \right],$$
$$\text{s.t.} \quad P_t L_{t+1} \leq \overline{x} E_{t+1} \quad [\tilde{\psi}_t],$$

where the indicator function ζ_t gives the liquidation decision:

$$\zeta_{t+1} = \mathbf{1} \left\{ V_{t+1}^{(i,c)}(L_{t+1}, E_{t+1}) \geq 0 \right\}.$$

As in the household's problem, L_{t+1} denotes the number of non-equity liabilities issued by the intermediary during period t. F_{t+1} is the gross payoff to each of these liabilities, and P_t is the price at which these liabilities are issued. For instance, in the case of the bank-like intermediary, we will have $F_{t+1} = R_{t+1}^{(d)}$, where $R_{t+1}^{(d)}$ is the rate of return on deposits, which is determined at time t. In general, F_{t+1} could be state-contingent. The intermediary takes both F_{t+1} and P_t as given. E_{t+1} is the value of the intermediary's asset portfolio at the end of period t. The intermediary's portfolio is entirely invested in NFC equity, and earns a rate of return $R_{t+1}^{(e)}$.

The intermediary faces a balance sheet constraint. This constraint limits the overall amount of financing that the intermediary can raise through non-equity liabilities to some fraction $\overline{x} \in [0,1]$ of its overall portfolio. $\tilde{\psi}_t$ is the associated Lagrange multiplier. We take this constraint as given; it could reflect either regulatory requirements, or agency frictions. We discuss the interpretation of this assumption below, in Section 3.4.

The intermediary is owned by the household and it uses the undistorted discount factor $\Lambda_{t,t+1}^{(u)}$ to price cash flows, adjusted for the possibility that the intermediary will be liquidated. Define intermediary net worth as:

$$N_t \equiv R_t^{(e)} E_t - F_t L_t.$$

We can rewrite the intermediary problem as:

$$\begin{aligned} V_t^{(i,c)}(N_t) &= \max_{x_{t+1}, E_{t+1}} N_t - (1 - x_{t+1}) E_{t+1} + \mathbb{E}_t \left[\Lambda_{t,t+1}^{(u)} \zeta_{t+1} V_{t+1}^{(i,c)}(N_{t+1}) \right] \\ \text{s.t.} \quad & N_{t+1} = \left(R_{t+1}^{(e)} - \frac{x_{t+1}}{P_t} F_{t+1} \right) E_{t+1} \\ & x_{t+1} \leq \overline{x} \quad [\tilde{\psi}_t] \end{aligned}$$

where x_{t+1} denotes the share of total intermediary funding coming from non-equity liabilities:

$$x_{t+1} = \frac{P_t L_{t+1}}{E_{t+1}}.$$

Appendix A.1 shows that the cum-dividend value of the intermediary and its net worth coincide:

$$V_t^{(i,c)}(N_t) = N_t.$$

Therefore, we have:

$$\zeta_{t+1} = \mathbf{1}\left\{N_{t+1}^{(i,c)} \geq 0\right\} = \mathbf{1}\left\{R_{t+1}^{(e)} \geq \frac{F_{t+1}}{P_t} x_{t+1}\right\}.$$

The necessary first-order conditions for E_{t+1} and x_{t+1} are then:

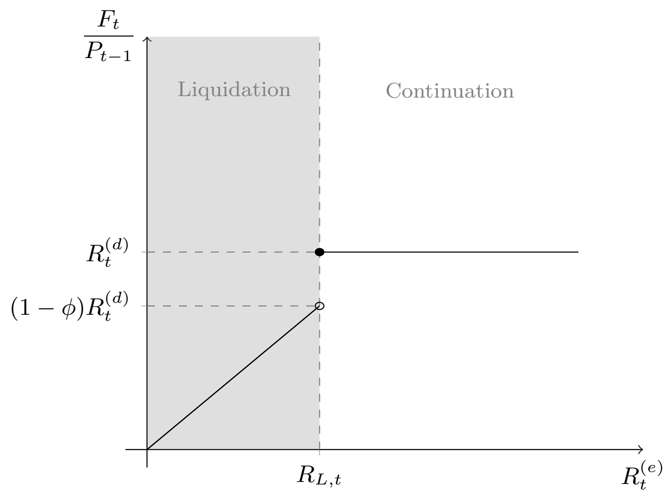
$$\begin{aligned}
1 - x_{t+1} &= \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)} \zeta_{t+1}\left(R_{t+1}^{(e)} - \frac{F_{t+1}}{P_t} x_{t+1}\right)\right], \\
1 &= \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)} \zeta_{t+1} \frac{F_{t+1}}{P_t}\right] + \psi_t, \\
0 &= \psi_t\left(\overline{x} - x_{t+1}\right),
\end{aligned}$$

where we defined $\psi_t = \tilde{\psi}_t / E_{t+1}$.

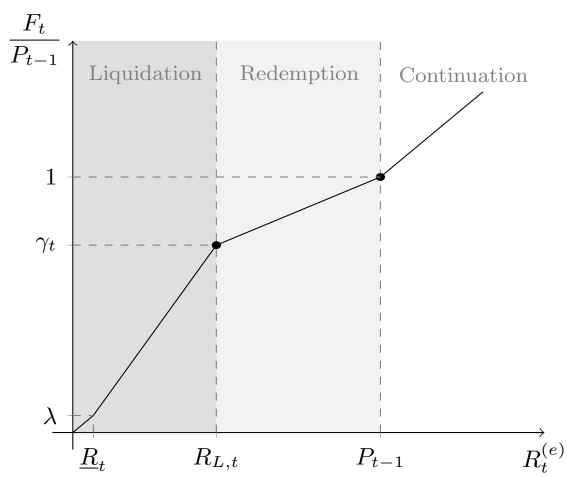
If the intermediary is liquidated, its asset portfolio is transferred to the household. The transfer possibly involves deadweight losses; we specify this in more detail in each of the two specific cases considered below. Note that intermediary dividends can be negative, corresponding to an issuance of equity. This will occur when $N_t - (1 - x_{t+1})E_{t+1} \leq 0$, which could be negative if E_{t+1} is large enough, or if N_t is small enough. In principle, following liquidation, the intermediary could therefore immediately be re-capitalized (that is, issue equity and non-equity liabilities to the household). We rule this out in the following assumption.

Assumption 2 (Intermediary recapitalization)**.** *When the intermediary is liquidated, it does not issue any liabilities, and remains inactive until the following period.*

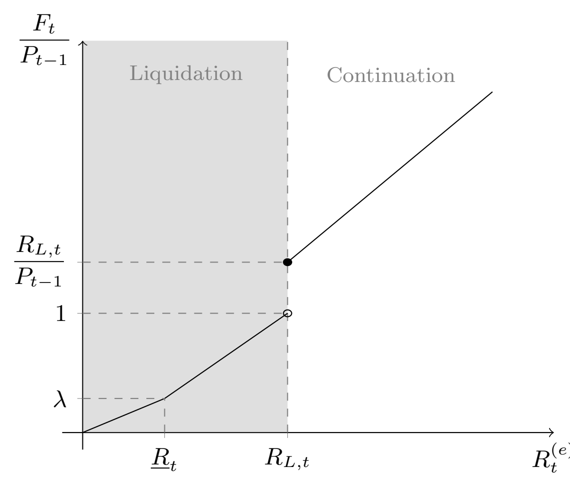
Thus, in periods where the intermediary is liquidated, the household must hold all of the shares issued by the NFC. However, in the subsequent period, the intermediary re-enters and can again issue equity and non-equity liabilities to fund its purchases of NFC shares. The assumption that the recapitalization is immediate in the period following a liquidation helps make the analysis more tractable, as we discuss in more detail below, in Section 3.4.[6]



(a) BI model



(b) NBI model, case $x_t < \overline{x}(P_{t-1})$



(c) NBI model, case $x_t > \overline{x}(P_{t-1})$

Figure 4: Payoff functions for the intermediary liabilities in the bank (BI) model (panel a) and in the non-bank intermediary (NBI) model (panels b and c). The horizontal axis, $R_t^{(e)}$ is the return on the intermediary's assets, and the vertical axis, F_t/P_{t-1}, is the return on each liability issued by the intermediary. In the BI model, the intermediary issues deposits, and we normalize $P_{t-1} = 1$, so $F_t = R_t^{(d)}$, where $R_{d,t}$ is the promised return on deposits conditional on no liquidation. $R_{L,t} = x_t R_t^{(d)}$ is the liquidation threshold. In the NBI model, the intermediary issues redeemable shares. Panels (b) and (c) plot the average gross return per share, $F_t/P_{t-1} = \lambda F_t^{(1)}/P_{t-1} + (1-\lambda)F_t^{(2)}/P_{t-1}$, where λ is the share of early investors, $F_t^{(1)}$ is the payoff to each early investor, and $F_t^{(2)}$ is the payoff to each late investor. $R_{L,t}$ is the liquidation threshold. The left panel describes the case where $x_t \leq \overline{x}(P_{t-1})$, and redemptions without intermediary liquidation can occur, while the right panel describes the case where $x_t \leq \overline{x}(P_{t-1})$ and redemption always lead to intermediary liquidations. On the left panel, the coefficient γ_t is given by $\gamma_t = \lambda + (1-\lambda)\frac{R_{L,t}}{P_{t-1}} < 1$. The expression for the threshold $R_{L,t}$ is reported in Result 1. The expression for the threshold $\underline{R}_t$ is reported in Appendix A.1.

3.2.2 A bank-like intermediary

The intermediary model just described can first be specialized to represent a bank, with:

$$F_t = \begin{cases} R_t^{(d)} P_{t-1} & \text{if} \quad \zeta_t = 1 \\ (1-\phi)\dfrac{R_t^{(e)} P_{t-1}}{x_t} & \text{if} \quad \zeta_t = 1 \end{cases}$$

In this version of the model, the non-equity liabilities of the intermediary can be interpreted as deposit contracts. Each dollar of deposit pays a rate of return $R_t^{(d)}$ conditional on the intermediary not being liquidated, where $R_t^{(d)}$ is determined at date $t-1$. Upon liquidation, the intermediary's assets are shared equally among depositors, with a deadweight loss in liquidation represented by the parameter $\phi \geq 0$. Liquidation follows a threshold rule,

$$\zeta_t = \mathbf{1}\left\{R_t^{(e)} \geq R_{L,t}\right\}, \qquad \text{where} \qquad R_{L,t} = R_t^{(d)} x_t.$$

Panel a of Figure 4 shows the return on deposits, F_t/P_{t-1}, as a function of $R_t^{(e)}$. Note that when $\phi > 0$, there is a discontinuity at the liquidation boundary, $R_{L,t}$. Note that, in this case, the price of the non-equity liabilities, P_t, is irrelevant; only the return per dollar of deposit, $R_{t+1}^{(d)}$, is pinned down. We therefore normalize the price of deposits to $P_t = 1$.

3.2.3 A non-bank intermediary

Next, we specialize the previous model to represent a non-bank financial intermediary. Instead of deposits, the non-equity liabilities issued by this intermediary are shares that are redeemable on demand by the household. We first describe the key assumptions regarding the intermediary problem. We then derive the possible redemption and liquidation outcomes. Finally, we map the outcomes of the non-bank intermediary model to the general model of Section 3.2.1.

Timeline and key assumptions The timeline of events is described in Figure 5. The intermediary starts the period with L_t redeemable shares outstanding.

Each redeemable share entitles its owner to a payoff of $R_t^{(e)}$ dollars.[7] Additionally, at the start of each period, a random fraction λ of investors receive the option to redeem each share for P_{t-1}\$, the price at which the redeemable shares were issued at time $t-1$. We call the group of investors that receive this option the "early investors", and denote variables referring to them using a superscript (1). We call the remaining investors the "late investors", and denote variables referring to them with a superscript (2).

[6]Finally, note that the ex-dividend value of intermediary equity is $V_t^{(i)} = \mathbb{E}_t\left[\zeta_{t+1}\Lambda_{t,t+1}^{(u)} V_{t+1}^{(i,c)}\right]$, while intermediary dividends are $\Pi_{t+1}^{(i)} = N_t - (1-x_{t+1})E_{t+1}$.

[7]Recall that the price of a claim on NFC equity has effectively been normalized to 1, so that the price of redeemable shares is effectively expressed relative to the price of equity shares.

Investors all belong to the household. However, in their decisions of whether to redeem their shares early, we assume that they behave independently from one another, and myopically, in the sense that they maximize only time-t payoffs.[8]

Redemptions can create deadweight losses. Specifically, a fraction $\phi \in [0,1]$ of the proceeds from any sale made by the intermediary to meet redemptions from early investors is destroyed, so that in order to meet a redemption call from early investors in the amount of X\$, the intermediary must sell $X/(1-\phi)$\$ worth of assets.

If proceeds from these sales are sufficient to meet the redemption demands of early investors, *and* if the remaining value of the intermediary's portfolio is sufficient to meet the promise made to remaining households of a rate of return $R_t^{(e)}$ per security, the intermediary is not liquidated. Otherwise, the intermediary is liquidated.

As in the bank intermediary case, if the intermediary is liquidated, their assets are transferred to investors. No intermediary operates for the rest of period t, so there is no possible intermediary liquidation in period $t+1$. A new intermediary is created only at the start of period $t+1$. Finally, note that in liquidation, the deadweight losses ϕ apply to all the assets of the intermediary, not only those sold to meet redemption by early investors.

If the intermediary is not liquidated, and once payments to existing investors have been made, the intermediary can issue new redeemable shares, in the amount L_{t+1}, and at price P_t. At the same time that the intermediary issues redeemable shares, they can also issue standard equity shares, which do not have a redemption option. Using the funds from the issuance of redeemable and standard shares, the intermediary then purchases NFC shares.

Finally, note that we assume that deadweight losses are *only* incurred on sales made to meet redemptions by early investors, *even if* the intermediary is liquidated. Deadweight losses, in this version of the model, should therefore be interpreted as representing fire sales costs. An alternative approach is to assume that, once the intermediary is liquidated, the same deadweight losses that affect early sales apply to the entire intermediary portfolio. We discuss this alternative model in Appendix A.1.2.

Redemption and liquidation Given a realization of returns on NFC shares, $R_t^{(e)}$, we now determine whether early investors will choose to redeem their shares at the beginning of the period. We derive the conditions under which, if all other early investors choose redemption, it is privately optimal for an individual early investor to also redeem their shares. In those situations, we assume that redemption is the outcome that will prevail. This assumption is discussed in Section 3.4.

The following lemma describes the two types of redemption and liquidations outcomes that can prevail. The proof for this lemma is reported in Appendix A.1.

[8] I think this could be micro-founded somewhat better, but the model would need to be modified earlier on, by introducing explicitly a "family" of investors belonging to the household.

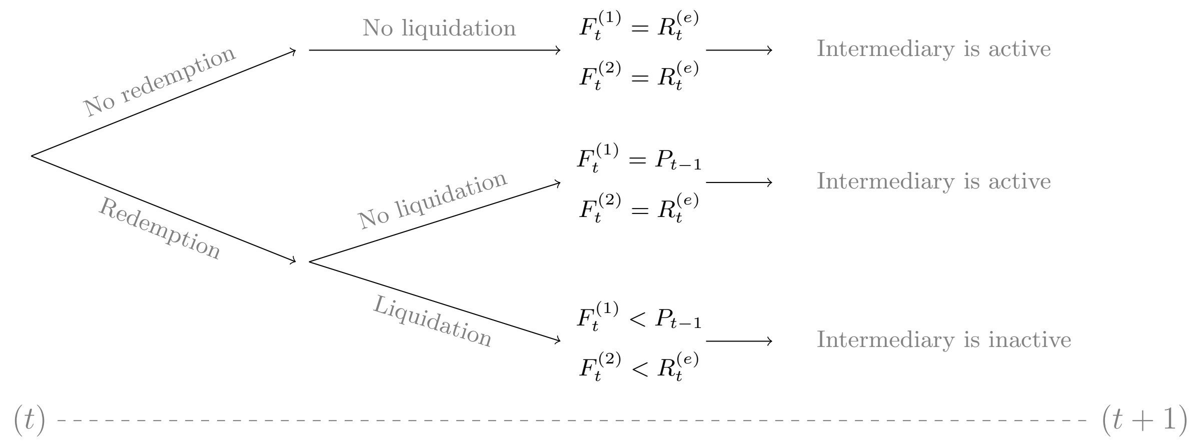


Figure 5: Timeline of redemption, liquidation, and share issuance in the non-bank intermediary's problem. $F_t^{(1)}$ denotes the payoff per share for early investors, while $F_t^{(2)}$ denotes the payoff per share for late investors.

Lemma 1 (Redemption and liquidation)**.** *Define:*

$$\overline{x}(P_t) \equiv \frac{1-\phi}{1-\phi+\lambda\phi}P_{t-1}$$

$$R_{L,t} \equiv \begin{cases} \dfrac{\lambda x_t}{(1-\phi)(P_{t-1}-(1-\lambda)x_t)}P_{t-1} & \text{if} \quad x_t \leq \overline{x}(P_{t-1}), \\ \left(1+\dfrac{\lambda\phi}{1-\phi}\right)x_t & \text{if} \quad x_t \geq \overline{x}(P_{t-1}). \end{cases}$$

The intermediary is liquidated, if and only if $R_t^{(e)} \leq R_{L,t}$. Moreover, if $x_t \leq \overline{x}(P_{t-1})$, then early investors do not redeem their shares when $R_t^{(e)} \geq P_{t-1}$, and when $R_t^{(e)} \in \left[\underline{R}_{L,t}, P_{t-1}\right)$, they redeem their shares but the intermediary is not liquidated. By constrast, if $x_t \geq \overline{x}(P_{t-1})$, the intermediary is liquidated whenever early investors attempt to redeem their shares, and early investors recover strictly less than P_{t-1} per share in liquidation.

Panels b and c of Figure 4 illustrates the rate of return on the redeemable shares, F_t/P_{t-1}, in the two possible cases: $x_t \leq \overline{x}(P_{t-1})$ and $x_t \geq \overline{x}(P_{t-1})$, which correspond, respectively, to a low and a high share of non-equity financing for the intermediary. Here, by "rate of return on redeemable shares", we mean the average rate of return across early and late investors:

$$\frac{F_t}{P_{t-1}} = \lambda\frac{F_t^{(1)}}{P_{t-1}} + (1-\lambda)\frac{F_t^{(2)}}{P_{t-1}}. \tag{1}$$

Exact expressions for F_t/P_{t-1} are reported in Appendix A.1.

In the first case ($x_t \leq \overline{x}(P_{t-1})$), the liquidation threshold $R_{L,t}$ satisfies;

$$R_{L,t} \leq P_{t-1}.$$

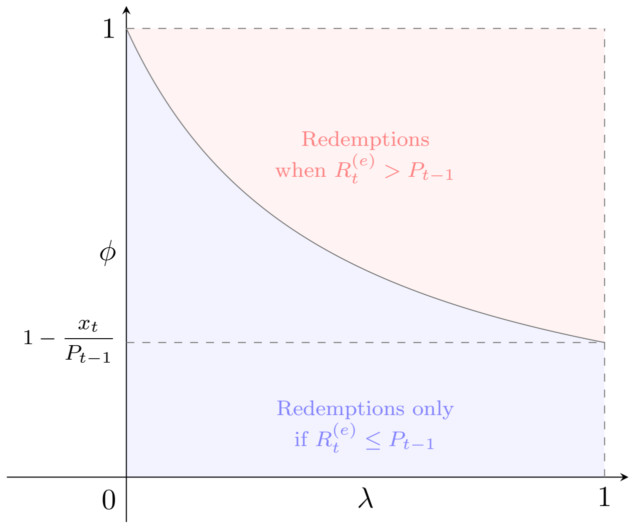


Figure 6: Regions of low and high leverage, and the corresponding redemption decisions, as a function of the share of early investors, λ, and deadweight losses, ϕ. The graph is drawn for a particular value for leverage $x_t \in (0, 1)$. The frontier between the light blue region (where all redemptions are driven by fundamentals) and the light red regions (where redemptions can be driven by coordination problems) is given by $\phi_f(\lambda, x_t/P_{t-1}) = (1 - x_t/P_{t-1})/(1 - (1 - \lambda)x_t/P_{t-1})$.

In this case, early investors' decision to redeem their shares is entirely driven by fundamentals: redemption occurs only when $R_t^{(e)} \leq P_{t-1}$. Redemptions without intermediary liquidation occur in an intermediate range of realizations of portfolio returns, while, when portfolio returns are sufficiently low, the intermediary is liquidated.

By constrast, when the external financing share is high ($x_t \geq \overline{x}(P_{t-1})$), we have:

$$R_{L,t} \geq P_{t-1}.$$

In this case, there is no region where early investors redeem their shares without triggering the liquidation of the intermediary. The reason is that when intermediary is mostly funded through redeemable shares (x_t is high), their capital buffer is insufficient to absorb the deadweight losses that redemptions create. Thus redemptions make intermediary net worth negative, and the intermediary is liquidated as a result.

Thus when $x_t \geq \overline{x}_{t-1}$, redemption decisions are driven by strategic motives. Early investors, understanding that the intermediary's net worth buffer is insufficient to meet all redemptions, find it optimal to redeem if all other early investors are doing so, even if returns are higher than the redemption value of the share. This decision leads to the liquidation of the intermediary.

A difference between the two cases $x_t \geq \overline{x}(P_{t-1})$ and the case $x_t \leq \overline{x}(P_{t-1})$ is that total returns to investors are discontinuous in the former case, while they are continuous in the latter case. When $x_t \geq \overline{x}(P_{t-1})$, returns are discontinuous because early investors start redeeming even though portfolio returns are strictly above P_{t-1}, so that their returns jumps discretely downward upon redemption. By contrast, in the case when $x_t \leq \overline{x}(P_{t-1})$, early investors start redeeming exactly at $R_t^{(e)} = P_{t-1}$, making their returns a continuous function of intermediary portfolio returns.[9]

Thus, with a non-bank financial intermediary, there are two possible types of outcomes each period, which depend on the ratio x_t/P_{t-1}. Figure 6 illustrates these outcomes. In the blue shaded area, redemptions are fundamentally-driven: they occur only when $R_t^{(e)} < P_{t-1}$. On the other hand, in the red shaded area, redemptions can be driven by coordination problems, and occur even though $R_t^{(e)} > P_{t-1}$. This is more likely to be the case when there are more early investors (for a given level of deadweight losses), and where there are more deadweight losses associated with redemptions (for a given share of early investors).

3.3 Equilibrium

Equilibrium requires that the market for goods clears:

$$C_t + \Phi\left(\frac{I_t}{K_t}\right) K_t = AK_t.$$

By assumption, when $\zeta_t = 0$ (the intermediary is liquidated), no intermediary shares trade, and the household holds all NFC equity, so that:

$$S_{t+1} = 1.$$

When $\zeta_t = 1$ (the intermediary is not liquidated), equity markets clear:

$$S_{t+1} + \frac{E_{t+1}}{V_t} = 1, \quad S_{t+1}^{(i)} = 1.$$

In what follows, we will refer to the model with a bank-like model as the BI model, and the model with non-bank intermediary as the NBI model. Appendix A.1 reports the equilibrium conditions for both models.

3.4 Discussion of key assumptions

Frictions in direct financing The first key assumption of the model is that there are frictions to the direct financing of the NFC by the household, and moreover that these frictions do not apply to the household's holding of intermediary equity or non-equity liabilities. The role of this friction is to allow for financial intermediaries to improve macroeconomic outcomes, relative to a world in which all financing is direct. Consider, for instance, the version of the model with no adjustment

[9] Thus, the discontinuity in payoffs in the case reported on the right-hand side is a manifestation of deadweight losses associated with sales for redemptions. In the case where $x_t \leq \overline{x}(P_{t-1})$, the deadweight losses also arise, but they are absorbed first by intermediary net worth, then by the payoff to late investors. In the case where $x_t \geq \overline{x}(P_{t-1})$, the net worth of the intermediary is immediately wiped out by deadweight losses, and the late investors absorb any residual. The kink in the two payoff functions in the liquidation region corresponds to the point where payoffs to the late investors reach zero, so that the entirety of the asset portfolio of the intermediary is subjected to deadweight losses, as it is entirely used to pay off early investors.

costs to investment, which Section 4.2 focuses on. In that model, the equilibrium growth rate of the economy would be:

$$1 + g_D(\chi) = \frac{A + 1 - \delta}{1 + \frac{\rho}{1-\chi}} \leq \frac{A + 1 - \delta}{1 + \rho} \equiv 1 + g^*,$$

where g^* is equilibrium growth in the frictionless model, that is, the model with $\chi = 0$. The expression above is decreasing with χ; χ is an index of the distortions induced by the friction χ in the model. This friction captures, in a tractable way, any comparative advantage in providing funding to the NFC that financial intermediaries may have, including a superior ability to screen potential borrowers, to monitor their actions after funding, or to mitigate deadweight losses in situations of financial distress of borrowers. Assuming this sort of comparative advantage is common in the macroeconomic literature on financial intermediaries; see, for instance, Gertler and Kiyotaki (2010), Brunnermeier and Sannikov (2014), or He and Krishnamurthy (2018).

Intermediary recapitalization The second key assumption is that intermediaries cannot be immediately re-enter in periods when they are liquidated, but, can otherwise issue equity frictionlessly. The first part of this assumption is also common to macroeconomic models with financial intermediaries; see, for instance, Gertler and Kiyotaki (2010) or Gertler et al. (2020). Without this assumption, intermediary failure would be irrelevant to macroeconomic outcomes. One interpretation of it is that in periods of financial panics, financial intermediaries may face higher cost of capital than in normal times.

The second part of the assumption is non-standard relative to the literature: we allow for flexible recapitalization of the intermediary immediately after default. This assumption is primarily to keep the analysis tractable. It eliminates the need to of keep track intermediary net worth as a macroeconomic state variable (other than whether it is positive or negative), and allows for simple closed-form solutions of the model in specific cases. Allowing for costly recapitalization of intermediaries would require the analysis to be mostly numerical, making it more challenging to clearly contrast the BI and NBI models.

Other assumptions While we have assumed that the NFC discount factor is equal to that of the intermediary in non-liquidation dates, we will see below that this is without loss of generality, since at those dates, the discount factor of the intermediary and the household coincide.

We have also imposed a balance sheet constraint of the form $x_{t+1} \leq \overline{x}$ in both intermediaries' problem. This constraint is meant to capture actual regulatory constraints faced by certain types of intermediaries, such as for instance capital requirements for banks. As we will see below, in the NBI case, we can let $\overline{x} = 1$, so that all intermediary funding can come from the issuance of non-equity liabilities. Constraints of this type can also be micro-founded from a principal-agent problem between the intermediary and its investors, as highlighted in He and Krishnamurthy (2018).

In the NBI model, we have focused on situations in which early redemptions are a Nash equilibrium among early investors. Two remarks on this are important. First, the approach relies on early investors being atomistic, or not internalizing the effects of their choices on the intermediary and on other investors. Second, the approach assumes that redemption is the outcome even when waiting might also be a Nash equilibrium among early investors. Standard sunspot techniques could be used to eliminate the potential for equilibrium multiplicity. Nevertheless, our results in the NBI model should be thought of as an upper bound on the potential for redemption risk to create macroeconomic distortions.

4 The credit channel with non-bank intermediaries

This section compares macroeconomic and financial outcomes in the two versions of the model described above: the bank (BI) model, and the non-bank intermediary (NBI) model. We start by stating results that hold generally, and then specialize the model to a particular case, linear adjustment costs to capital, for which results can be established analytically.

4.1 General results

We establish two general preliminary results. First, if their are no deadweight losses in liquidation ($\phi = 0$), then when the intermediary is active, the household discount factor is undistorted. Second, when the intermediary is active, their balance sheet constraint must bind.

Result 1 (Equilibrium SDF)**.** *Assume that there are no deadweight losses in liquidation: $\phi = 0$. Then, when the intermediary is active ($\zeta_t = 1$), it must be that $\nu_t = 0$ and $S_{t+1} = 0$. Therefore, when $\zeta_t = 1$, the household discount factor is undistorted: $\Lambda_{t,t+1} = \Lambda^{(u)}_{t,t+1}$.*

The proof of this result is reported in Appendix A.2. In general, the first-order conditions for the intermediary and the household's problem can be combined to yield:

$$1 = \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}R^{(e)}_{t+1}\right] - \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}(1-\zeta_{t+1})D(R^{(e)}_{t+1}, x_{t+1})\right]$$

where D is a function which measures the extent of deadweight losses when the intermediary is liquidated. Appendix A.2 shows that this function is given by:

$$D(R^{(e)}_{t+1}, x_{t+1}) = \phi \times \begin{cases} R^{(e)}_{t+1} & \text{in the BI model} \\ \min\left(\dfrac{\lambda}{1-\phi}x_{t+1}, R^{(e)}_{t+1}\right) & \text{in the NBI model} \end{cases}$$

When $\phi = 0$, this function is identically zero, so that the undistorted first-order condition $1 = \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}R^{(e)}_{t+1}\right]$ must hold. This requires that the household hold zero NFC equity, $S_{t+1} = 0$, *and* that the Lagrange multiplier on the constraint $S_{t+1} \geq 0$ be zero, leading to the result.

An implication of this result is that when the intermediary is active ($\zeta_t = 1$), the two first-order conditions for pricing intermediary shares — from the intermediary's problem, and from the household's problem — coincide, and boil down to:

$$1 = \mathbb{E}_t \left[\frac{1}{1+\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-1} R^{(e)}_{t+1} \right].$$

This equation formally identical to the one that would hold in a model where the household own all shares, but is not subject to the direct intermediation cost $\chi = 0$.

Crucially, while this first-order condition holds, it doesn't follow that all equilibrium outcomes will be undistorted; in other words, this is not a model where intermediaries are a "veil", even if $\phi = 0$. Indeed, if there are periods when the intermediary defaults ($\zeta_t = 0$), then the household *will* have to own the capital stock, which will create distortions in the cost of capital, and therefore in growth. However, this result suggests that the *only* way in which these distortions will affect investment in non-liquidation dates is through the likelihood of future liquidation. We formalize this intuition below, in the analytical version of the model.

Finally, the result implies that the discount factor used by the NFC simplifies to:

$$\Lambda^{(f)}_{t,t+1} = \Lambda_{t,t+1}.$$

Thus in equilibrium, the NFC is effectively using the same discount factor as the household, which includes a penalty for direct intermediation when $S_{t+1} > 0$, and no penalty otherwise.

Result 2 (No interior solution for x_{t+1})**.** *When the intermediary is active* ($\zeta_t = 1$)*, its share of non-equity funding must be either* $x_{t+1} = 0$ *or* $x_{t+1} = \bar{x}$*.*

The proof of this result is also reported in Appendix A.2. When the intermediary is active, there are two possibilities. First, the intermediary can be entirely equity-financed. From Result 1, the intermediary must still hold all NFC equity; so it effectively acts as an equity mutual fund.

Second, the intermediary can exhaust its non-equity financing capacity. To understand why, consider the BI model. Absent any constraint on x_{t+1}, there would be an interior solution to the bank's problem only if:

$$\overline{R}^{(d)}_{t+1} = \left(\mathbb{E}_t \left[\Lambda^{(u)}_{t,t+1} \zeta_{t+1} \right] \right)^{-1}.$$

By contrast, the required return on deposits for the household is:

$$R^{(d)}_{t+1} = \left(\mathbb{E}_t \left[\Lambda^{(u)}_{t,t+1} \zeta_{t+1} \right] \right)^{-1} \left(1 - \mathbb{E}_t \left[\Lambda^{(u)}_{t,t+1} (1-\zeta_{t+1}) \frac{(1-\phi) R^{(e)}_{t+1}}{x_{t+1}} \right] \right).$$

This is lower than $\overline{R}^{(d)}_{t+1}$, since the household recovers a positive value in liquidation states. Therefore, the low required rate of return on deposits gives an incentive for the bank to issue as many deposits as possible. In turn, this creates liquidation risk, which itself leads to distortions when $\chi > 0$, since the household is forced to hold the liquidated asset portfolio in liquidation states.

In what follows, we will focus on equilibria where the balance sheet constraint is binding at all dates when the intermediary is active:

$$x_{t+1} = \overline{x}.$$

Though they are efficient, we do not focus on the equilibria with zero intermediary leverage since, in this case, both the BI and NBI intermediary effectively act as stock mutual funds. Ruling out these equilibria would require that the household face a cost of holding equity in the financial intermediary that is lower than or equal to the cost χ of directly financing the NFC. Introducing this additional friction would complicate the analysis without yielding substantially different insights.

4.2 An analytical model

4.2.1 Assumptions and solution

For the rest of the paper, we impose the following restrictions on the primitives of the model.

Assumption 3. *Investment adjustment costs are linear:*

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t}.$$

Moreover, there are no deadweight losses when the intermediary is liquidated: $\phi = 0$. Finally, capital quality shocks are independently and identically distributed, with mean $\mathbb{E}[\xi_t] = 1$.

Intuitively, the first assumption implies that the demand for capital from the NFC is infinitely elastic given the household's discount rate. It also implies that the gross returns on intermediary equity are simply given by:

$$R^{(e)}_{t+1} = z\xi_{t+1}, \quad z \equiv A + 1 - \delta,$$

where z is a measure of gross returns to capital. Thus the shocks to capital quality can equivalently be thought of as shocks to the rate of return on the intermediary's asset portfolio. In what follows, we will use the notation:

$$\xi_{L,t} \equiv \frac{R_{L,t}}{z}$$

to express the liquidation threshold, that is, the realization of capital quality shocks below which the intermediary is liquidated.

Following Result 1, the assumption that $\phi = 0$ implies that in normal times ($\zeta_t = 1$), the undistorted asset pricing equation for NFC equity will hold. Additionally, in the NBI model, redemptions and liquidations will be driven by fundamentals, and not coordination problems. The case $\phi = 0$ is useful to analyze because it provides a lower bound on the distortions caused by intermediation frictions in this model. We come back to the potential implications of the model in the case $\phi > 0$ in the conclusion. Finally, the *i.i.d.* assumption gives the equilibrium a simple recursive structure, but it is not crucial to the main results derived in what follows.

Result 3 (Equilibrium with intermediary default)**.** *There exists an equilibrium where K_t follows:*

$$K_{t+1} = \xi_{t+1}(1 + g_t)K_t$$

$$g_t = \begin{cases} g_L & \textit{if} \quad \zeta_t = 0 \\ g_N & \textit{if} \quad \zeta_t = 1 \end{cases}$$

where $g_L < g_N$ are two constants. Additionally, the liquidation threshold is constant: $\xi_{L,t} = \xi_L$.

Let $F(.)$ be the cumulative distribution function (CDF) of the capital quality shocks ξ_{t+1}. In what follows, we will denote:

$$p_L = F(\xi_L)$$

the probability of liquidation of the intermediary. In the equilibrium described above, there exist two discount rates ρ_N and ρ_L such that the price-to-earning ratio of NFCs is given by:

$$\frac{V_t}{\Pi_t} = \begin{cases} \frac{1}{\rho_L} & \text{if } \zeta_t = 1 \\ \frac{1}{\rho_N} & \text{if } \zeta_t = 0 \end{cases}$$

These discount rates satisfy $\rho_L \geq \rho_N$, with strict inequality if $\xi_L > 0$ (that is, if the intermediary may be liquidated in equilibrium). Equilibrium growth rates satisfy:

$$1 + g_L = \frac{1}{1+\rho_L} z \leq \frac{1}{1+\rho_N} z = 1 + g_N \leq g^*,$$

where g^* is the first-best growth rate of output, that is, the growth rate of output in a model with $\chi = 0$. Thus in this model, growth will be weakly lower than first-best, and moreover, at dates when the intermediary is liquidated, growth will be lower than at dates when it is active. Intuitively, when the intermediary is liquidated, the household must hold the stock of NFC equity. Because of the direct financing friction, $\chi > 0$, the household requires a higher return than usual on NFC equity. This depresses investment and growth. Because intermediaries are immediately recapitalized in the period following liquidation, this effect only persists for a period, giving the equilibrium the simple structure described above.

4.2.2 Implications for the credit channel

We now compare in more detail the implications of the model when the intermediary is either a bank (the BI model) or a non-bank (the NBI model).

Does the funding structure of intermediaries affect macroeconomic outcomes? We first ask a basic question: do details regarding the way in which intermediary are funded affect macroeconomic outcomes? In particular, does it matter to distinguish between BIs, who use leverage and face default risk, and NBIs, who use redeemable shares and face redemption risk? The following result gives a qualified answer within the context of the model.

Result 4 (A sufficient statistic for macroeconomic outcomes)**.** *The two equilibrium discount factors* ρ_N *and* ρ_L *are givne by:*

$$\rho_N = \rho_N(p_L; \chi, \rho), \quad \rho_L = \rho_L(p_L; \chi, \rho). \tag{2}$$

where the expressions for the functions $\rho_N(.; \chi, \rho)$ *and* $\rho_L(.; \chi, \rho)$ *are independent of the details of the intermediary problem. Thus, intermediary funding structures leading to the same equilibrium liquidation probability* p_L *produce the same macroeconomic outcomes.*

The result is proved in Appendix A.2, which also reports the expressions for the functions $\rho_N(.; \chi, \rho)$ and $\rho_L(.; \chi, \rho)$. An immediate consequence is that the state-contingent growth rates g_L and g_N themselves only depend on p_L, and that the mapping is independent of the details of how the intermediary is funded. For a given value of the liquidation probability p_L, macroeconomic outcomes do not depend on the details of how intermediation works.

Intuitively, this result comes from the fact that when $\phi = 0$, the first-order condition:

$$E_t\left[\Lambda_{t,t+1} R_{t+1}^{(e)}\right] = 1$$

holds when $\zeta_t = 1$. Returns $R_{t+1}^{(e)}$ are independent from the structure of financial intermediaries, and the discount factor $\Lambda_{t,t+1}$ only depends on consumption growth, and therefore on the probability of intermediary liquidation in the following period.

The two functions ρ_N and ρ_L satisfy $\rho_L(p_L; 0, \rho) = \rho_N(p_N; 0, \rho) = \rho$, implying that when there are no frictions to direct financing, $\chi = 0$, growth is at its first-best level, as described above. Finally, the function ρ_N satisfies $\rho_N(0; \chi, \rho) = \rho$, implying that even if there are frictions to direct financing ($\chi > 0$), growth can be at its first-best level, so long as the funding structure of the intermediary is such that it is never liquidated in equilibrium.

This result does not say that intermediary funding structure is irrelevant. Rather, it highlights the fact that the probability of intermediary liquidation p_L is a sufficient statistic for the macroeconomic effects of intermediary funding structure on macroeconomic outcomes, such as average output growth or average investment rates.

The equilibrium value of p_L is itself determined through the first-order condition that prices non-equity liabilities in the household's problem. That first-order condition will differ substantially between the BI and the NBI models, leading to potentially different macroeconomic and financial stability implications. We next turn to discussing the determination of ξ_L in both models.

Does the rise of NBIs pose a risk to macroeconomic and financial stability? Next, we turn to discussing whether, in the context of the model, NBIs are likely to improve or worsen macroeconomic and financial stability.

In order to do this, Result 4 indicates that we need to compare the implications of each model for the liquidation probability of intermediaries, and therefore the liquidation threshold ξ_L in each version of the model. Appendix A.2 shows that, in the BI model, the threshold for intermediary liquidation is the unique solution to:

$$1 = \frac{H(\xi_L^{(b)})}{\overline{x}},$$

$$H(\xi_L^{(b)}) \equiv F(\xi_L^{(b)}) + \int_{\xi_L^{(b)}}^{+\infty} \frac{\xi_L^{(b)}}{\xi} dF(\xi).$$

The right-hand side of the first equation captures the return on deposits for the household, in both liquidation and non-liquidation states, after taking into account the bank's leverage. $H(.)$ is strictly increasing with respect to $\xi_L^{(b)}$, with $H(0) = 0$ and $H(+\infty) = 1$.[10] Thus, the intermediary liquidation probability only depends on bank leverage, and it increases strictly with it. In particular, when $\overline{x} = 0$, the liquidation probability of the bank is zero: $p_L = 0$, and there are no macroeconomic distortions. Otherwise, the liquidation probability is strictly positive, and equilibrium growth rates are lower than first-best.

In the NBI model, the threshold for intermediary liquidation, $\xi_L^{(nb)}$, and the threshold for redemption by early investors:

$$\xi_R^{(nb)} \equiv \frac{P}{z},$$

are the joint solution to:

$$\xi_L^{(nb)} = \frac{\lambda \overline{x}}{z\xi_R^{(nb)} - (1-\lambda)\overline{x}} \xi_R^{(nb)},$$

$$z\xi_R^{(nb)} = 1 + \left(\frac{z\xi_R^{(nb)}}{\overline{x}} - 1 \right) F(\xi_L^{(nb)}) + \lambda G(\xi_L^{(nb)}, \xi_R^{(nb)}),$$

$$G(\xi_R^{(nb)}, \xi_L^{(nb)}) \equiv \int_{\xi_L^{(nb)}}^{\xi_R^{(nb)}} \frac{\xi_R^{(nb)} - \xi}{\xi} dF(\xi).$$

In the second equation, the right-hand side represents the expected return on redeemable shares. The last term, which depends on the function $G(.,.)$, captures the payoff in states of the world where early investors redeem their shares, but late investors do not.

As in the bank model, if the NBI is entirely funded through equity ($\bar{x} = 0$), there is no liquidation risk: $\xi_L^{(nb)}$, and therefore no macroeconomic distortions. However, different from the non-bank intermediary, we see that even when $\bar{x} > 0$, if the share of early investors is zero, $\lambda = 0$, the intermediary does not default: $\xi_L = 0$. Thus in the NBI model, macroeconomic frictions require both that $\overline{x} > 0$ (the intermediary is financed at least in part through redeemable shares) and $\lambda > 0$ (at least some of the investors can exercise their redemption option). Therefore, in the discussion that follows, we focus on the case $\lambda > 0$.

[10]The equations that determine the liquidation threshold ξ_L in the two versions of the model are expressed here in the case where the discount factor, ρ, is small. Appendix A.2 contains more detailed expressions in the general case; the qualitative implications of the model discussed here largely follow through.

A limit case of the NBI model is when $\overline{x} = 1$, that is, when the NBI is entirely funded through redeemable shares. In this case, the expressions above imply that:

$$\forall \lambda \in (0,1], \quad \xi_R^{(nb)} = \xi_L^{(b)} = z^{-1}. \tag{3}$$

This implies, in turn, that the price of redeemable shares is $P = 1$. This remark has two implications. First, when $\overline{x} = 1$, redemption always leads to liquidation. The intuition is that without some capital buffer, the NBI cannot honor the redemptions from early investors, since they only occur when the return on assets $R_{t+1}^{(e)}$ falls below the price of redeemable shares. Second, this is independent of the share of early investors.

Note, however, that the redemption and liquidation probabilities remain finite: $p_L^{(nb)} = F(z^{-1}) < +\infty$. The reason is that even if the intermediary is fully funded by redeemable shares, redemptions will only occur if $R_{t+1}^{(e)}$ falls below 1, the redemption value of the share when the intermediary has no equity buffer.

By contrast, in the BI model, as $\overline{x}$ approaches 1, the liquidation probability approaches $p_L^{(b)} = 1$. The intuition for this is that when $\overline{x} = 1$, it becomes costless for the bank to expand its balance sheet. An equilibrium is only possible if there are also no benefits of expanding the balance sheet, which only occurs when the probability of liquidation is 1. Thus in the case where $\overline{x}^{(nb)} = \overline{x} = 1$, intermediary liquidation probabilities will strictly larger in the BI model than in the NBI model.

Figure 7 reports the comparative statics of the two models with respect to $\overline{x}$, the share of intermediary funding that is obtained through either deposits (in the BI model) or non-redeemable shares (in the NBI model). For the NBI model, we report these comparative statics for two values of the share of early investors, $\lambda = 0.5$ and $\lambda = 0.9$. In both model, distortions are amplified when the share of non-equity funding is higher. However, when $\overline{x}$ approaches 1, distortions remain bounded in the NBI model, while in the BI model, the probability of intermediary liquidation converges to 1. As a result, average output growth converges to their lower bound in the BI model, but not in the NBI model.

The following result shows that the intuition that NBIs produce less macroeconomic distortions does not hold more broadly, that is, for arbitrary values of $(\overline{x}^{(b)}, \overline{x}^{(nb)}, \lambda)$.

Result 5 (Macroeconomic and financial stability with BIs vs. NBIs)**.** *Let bank leverage be $\overline{x}^{(b)}$, let the share of external funding of NBIs be $\overline{x}^{(nb)}$, and let λ be the share of early investors. If:*

$$\overline{x}^{(b)} \geq H\left(\frac{\overline{x}^{(nb)}}{z}\right),$$

then the liquidation probability in the NBI model is always weakly smaller than in the BI model:

$$\xi_L^{(nb)} \leq \xi_L^{(b)} \quad \forall \lambda \in (0,1].$$

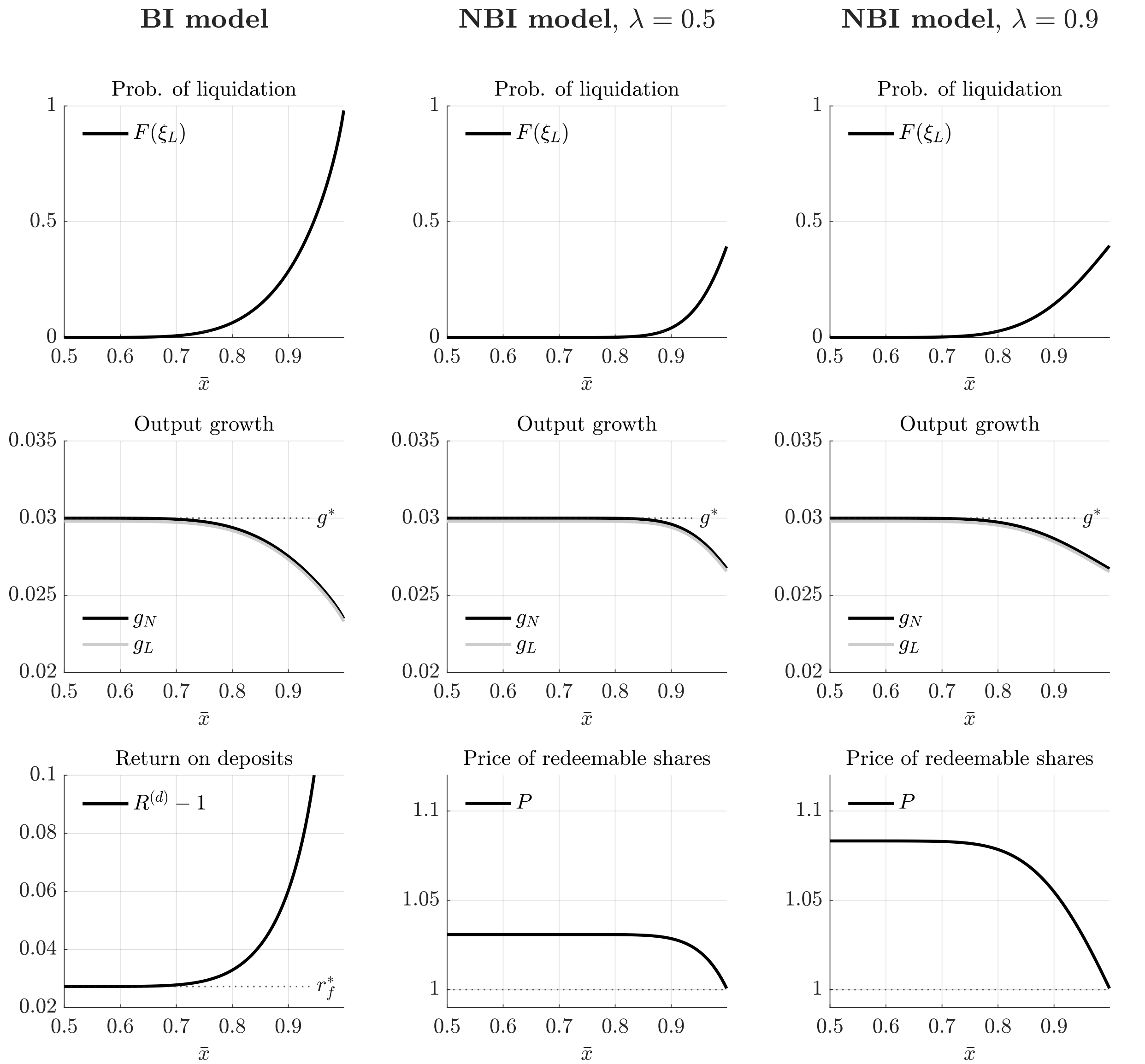


Figure 7: Comparative statics of the two models with respect to the share of external financing of the intermediary, $\bar{x}$. The left column shows the bank intermediary (BI) model. The middle and right columns show the non-bank intermediary (NBI) model, with two different values of the share of early investors: $\lambda = 0.5$ (middle column) and $\lambda = 0.9$ (right column). In all models, the calibration is $\sigma = 0.15$, $\rho = 0.2$, $\chi = 0.50$, $\delta = 0.15$, and $A = 0.2006$, leading to a frictionless growth rate of $g^* = 0.03$, a frictionless risk-free rate of $r_f^* = 0.0272$, and a frictionless risk premium of $rp^* = 0.0234$.

Otherwise, there exists a mapping $\gamma(\overline{x}^{(b)}, \overline{x}^{(nb)})$ *such that:*

$$\xi_L^{(nb)} \leq \xi_L^{(b)} \quad \iff \quad \lambda \in \left(0, \gamma(\overline{x}^{(b)}, \overline{x}^{(nb)})\right].$$

This result has two parts. The first part says that if bank leverage, $x^{(b)}$, is sufficiently high, relative to the fraction of external funding of NBIs, then the liquidation probability in the BI model will be generically higher than in the NBI model, regardless of the fraction of early investors. Therefore, macroeconomic distortions will be larger in the BI model. In this parameter region, NBI thus promote macroeconomic stability, compared to levered financial intermediaries. Note

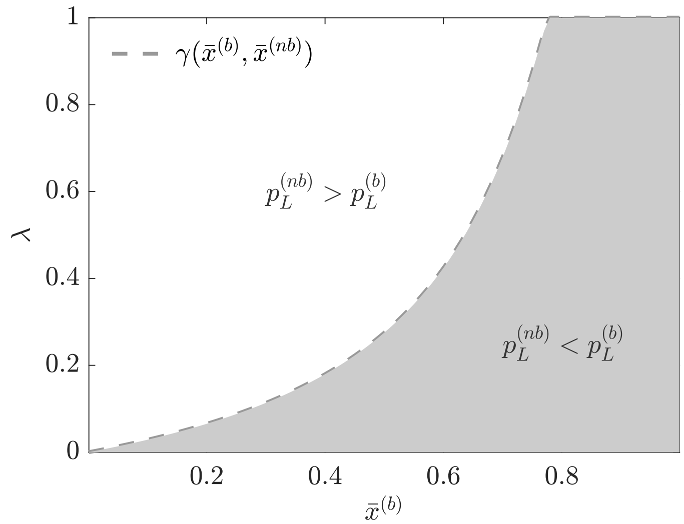


Figure 8: Threshold $\gamma(\overline{x}^{(b)}, \overline{x}^{(nb)})$ for the share of early investors such that, when $\lambda > \gamma(\overline{x}^{(b)}, \overline{x}^{(nb)})$, the likelihood of intermediary liquidation is lower in the NBI model than in the BI model. The value of $\overline{x}^{(nb)}$ is $\overline{x}^{(nb)} = 0.8$. The calibration is $\sigma = 0.15$, $\rho = 0.2$, $\chi = 0.50$, $\delta = 0.15$, and $A = 0.2006$, leading to a frictionless growth rate of $g^* = 0.03$, a frictionless risk-free rate of $r_f^* = 0.0272$, and a frictionless risk premium of $rp^* = 0.0234$.

that this does not require that $x^{(nb)} < 1$. Even if $x^{(nb)} = 1$, so that the NBI is entirely funded through redeemable shares, macroeconomic distortions will be larger in the BI model so long as $x^{(b)} \geq H(z^{-1})$.

The second part of the result says that, when the capital buffer of banks is sufficiently large, *and* the share of early investors in NBIs is sufficiently high, macroeconomic distortions can be larger in the NBI model. This will occur for values of the early investor share above the threshold $\gamma(\overline{x}^{(b)}, \overline{x}^{(nb)})$.

Figure 8 illustrates the result for a particular value of the share of external funding of the NBI, $\overline{x}^{(nb)} = 0.8$. The parameter space $(\overline{x}^{(b)}, \lambda)$ is partitioned in two regions by the threshold $\gamma(\overline{x}^{(b)}, \overline{x}^{(nb)})$. For values of the share of early investors, λ, above the threshold, the liquidation probability in the NBI model is higher than in the BI model, whereas for values below the threshold, intermediary liquidation is more likely in the BI model than in the NBI model.

More generally, the lesson from Result 5 is that, compared to banks, non-bank intermediaries do not necessarily improve macroeconomic stability. To the extent that λ proxies for the fragility of the funding base of the NBI, this result can be interpreted as saying that NBIs pose a risk to macroeconomic stability *only when* their funding base is sufficiently fragile.

An additional but important difference between the BI and NBI models is that even when macroeconomic risk is lower in the NBI model, *financial stability* risk might be higher, in the sense that redemptions by early investors might occur frequently, without necessarily triggering a liquidation. This is already visible in the comparative statics of Figure 7, which shows that even in the case $\lambda = 0.1$, redemptions remain frequent event. Figure 9 reports the comparative statics of the NBI model with respect to λ, the share of early investors. These comparative statics show that

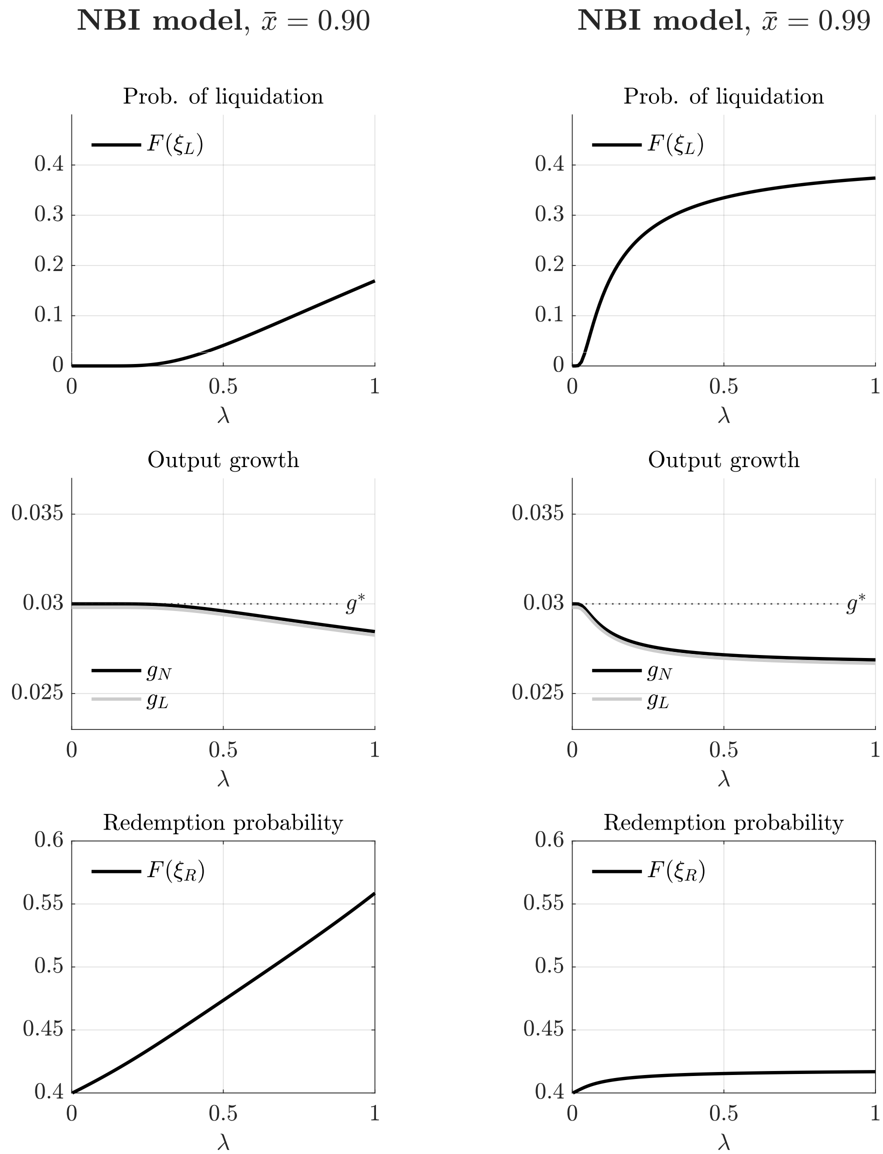


Figure 9: Comparative statics of the NBI model with respect to the share of early investors, λ. The left columns shows a calibration with a share of external financing of $\bar{x} = 0.90$, and the right column shows a calibration with a share of external financing of $\bar{x} = 0.99$. In both columns, the calibration is $\sigma = 0.15$, $\rho = 0.2$, $\chi = 0.50$, $\delta = 0.15$, and $A = 0.2006$, leading to a frictionless growth rate of $g^* = 0.03$, a frictionless risk-free rate of $r_f^* = 0.0272$, and a frictionless risk premium of $rp^* = 0.0234$.

even if the probability of liquidation of the intermediary in the NBI model remains bounded, the probability of redemptions can be elevated even when λ is small, especially when the intermediary has a large capital buffer ($x^{(nb)}$ is lower than 1).[11] Thus, in the model, while the fragility of the funding base of NBIs might not necessarily amplify macroeconomic risk, it does amplify financial risk, in the sense that it increases the likelihood of investor redemptions.

Which tools should policymakers use to regulate NBIs? Finally, we discuss whether, in the BI and NBI models, the regulation of financial intermediaries should operate differently.

[11]It can be shown that the probability of redemptions is bounded from below in the NBI model, by $p_R = F(\frac{\overline{x}^{(nb)}}{z})$.

In the BI model, the only possible target for regulation is leverage. In the NBI model, there are two possible targets for regulators, the share of external funding of intermediaries, and the share of early investors λ.[12] The following result summarizes the effects of regulation in the two models.

Result 6 (Regulation with BIs vs. NBIs)**.** *In the BI model, tighter capital requirements reduce liquidation risk and increase average output growth. Moreover, the marginal effect of tigher capital requirements on the liquidation threshold are given by:*

$$\frac{\overline{x}^{(b)}}{\xi_L^{(b)}} \frac{\partial \xi_L^{(b)}}{\partial \overline{x}^{(b)}} = \frac{p_L^{(b)}}{\overline{x}^{(b)} - p_L^{(b)}} > 0.$$

In the NBI model, if the intermediary is entirely funded through non-redeemable shares ($\overline{x}^{(nb)} = 1$), tighter limits on redemptions have no effect on macroeconomic outcomes. Moreover, when $\overline{x}^{(nb)} = 1$, the effect of introducing a capital requirement on NBIs is given by:

$$\frac{1}{\xi_L^{(nb)}} \frac{\partial \xi_L^{(nb)}}{\partial \overline{x}^{(nb)}} = \left(\frac{1}{\lambda} - 1\right) \frac{1}{1 + p_L^{(nb)}} > 0.$$

On the other hand, when $\overline{x}^{(nb)} < 1$, both tighter capital constraints and tighter limits on redemptions increase average output growth.

The proof of this result is reported in Appendix A.2. The first part of the result, on the effect of tightening capital constraints in the bank model, is already visible in the comparative statics of Figure 7. The expression in the result additionally shows that $p_L^{(b)}$ is a sufficient statistic not only for macroeconomic stability overall (as already discussed in Result 4), but also for the effects of financial regulation: it summarizes the effects in capital requirements on macroeconomic stability. The expression in Result 6 indicates that these effects are weaker when the banking sector is already well capitalized (that is, when $\overline{x}^{(b)}$ is high).

In the NBI model, when the intermediary is fully funded by redeemable shares ($\overline{x}^{(nb)} = 1$), its probability of liquidation is independent of the share of early investors, as discussed above. As a result, increasing redemption limits (that is, lowering λ) has no macroeconomic effects. However, when $\overline{x}^{(nb)} < 1$, tighter capital constraints and tighter limits on redemption both improve macroeconomic outcomes in the model.

The result also indicates that when $\overline{x}^{(nb)} = 1$, introducing capital requirements would lower the liquidation threshold, though the effects depend on the share of early investors. The real-world counterpart of such an intervention could, for instance, consist of forcing NBIs to hold a fraction of their portfolio in risk-free assets, which can provide a buffer against early redemptions.

In the limit where $\lambda = 1$, the effects of such an intervention vanish, because introducing a small capital requirement would be insufficient to offset the large amounts of redemption that occur when

[12] The discussion above shows that imposing $\overline{x} = 0$ — that is, forcing intermediaries to act as stock mutual funds — would eliminate default risk and lead to the first-best level of growth. This conclusion strongly depends on our assumption that the recapitalization of intermediaries is costless. We focus on the effects of a marginal reduction in $\overline{x}$ or λ because it offers a more plausible account of policies than might be considered by policymakers.

$R^{(e)}_{t+1}$ falls below P_t. On the other hand, when the funding base of intermediaries is relatively stable (λ is close to 0), introducing a small capital requirement helps reduce the likelihood of liquidation.

Thus, a broad message of Result 6 is that with NBIs, not only tighter capital requirements, but also tighter limits on redemptions, can both help improve macroeconomic stability. However, the impact of limits on redemption only have an impact when the intermediary has substantial capital buffers. Conversely, capital requirements only have a significant impact on NBI liquidation risk — and therefore on macroeconomic stability — when their investor base is relatively stable.

5 Conclusion

Non-bank financial intermediaries (NBIs) are gradually replacing banks as the main source of credit in many segments of U.S. corporate capital markets. Does this development matter for macroeconomic and financial stability? Should academic and policymakers update their understanding of how the credit channel works?

In this paper, we focus on a particular dimension along which banks and NBIs differ: their funding structure. While banks are levered and subject to default risk, NBIs tend to fund themselves using equity-like instruments, making them less exposed to default. However, the liabilities of NBIs often embed a redemption option, which can force them to liquidate their asset portfolios. Whether this risk mitigates or amplifies macroeconomic instability, relative to banks, is an open question.

Our contribution is to provide a simple framework to compare the macroeconomic effects of intermediaries with different funding structures. This framework produces three new insights. First, while intermediary funding structures generally affect macroeconomic outcomes, their impact can be summarized by a single sufficient statistic, the probability of intermediary liquidation. Second, if bank leverage is sufficiently high, NBIs will act as macroeconomic stabilizers regardless of how much redemption risk they face. By contrast, when banks are well-capitalized, NBIs become macroeconomic de-stabilizers when redemption risk is sufficiently high. Third, with NBIs, limits to redemptions only reduce macroeconomic distortions when NBIs are sufficiently well capitalized; otherwise, capital requirements are a more efficient way of promoting macroeconomic stability.

Our analysis leaves a number of questions open. First, we focused on the case of no deadweight losses in liquidation, $\phi = 0$. This case is useful because it shows that macroeconomic distortions can arise even without these deadweight losses, but it also limits the NBI model to equilibria in which all redemptions are driven by fundamentals. Second, by assuming that the recapitalization of intermediaries is frictionless, the model eliminates some of the dynamics that can be generated in models where recapitalization is costly and intermediary net worth is a state variable. Finally, the model does not feature any heterogeneity in the funding costs or the returns to capital of intermediaries, making it difficult to confront it directly to the microeconomic evidence on the real effects of shocks to the supply of NBI capital on firms. We leave these questions to future research.

References

Berg, T., A. Saunders, and S. Steffen (2020). Trends in corporate borrowing. Annual Review of Financial Economics.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. The American Economic Review 79(1), 14.

Bernanke, B. S. and A. S. Blinder (1988). Credit, money, and aggregate demand. The American Economic Review 78(2), 435–439.

Bernanke, B. S. and M. Gertler (1995). Inside the black box: the credit channel of monetary policy transmission. Journal of Economic perspectives 9(4), 27–48.

Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. American Economic Review 104(2), 379–421.

Chodorow-Reich, G. (2014). The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis. The Quarterly Journal of Economics 129(1), 1–59.

Chodorow-Reich, G., O. Darmouni, S. Luck, and M. Plosser (2022). Bank liquidity provision across the firm size distribution. Journal of Financial Economics 144(3), 908–932.

Coppola, A. (2021). In safe hands: The financial and real impact of investor composition over the credit cycle. Technical report.

Cordell, L., M. R. Roberts, and M. Schwert (2021). CLO performance. Technical report, National Bureau of Economic Research.

Crouzet, N. (2021). Credit disintermediation and monetary policy. IMF Economic Review.

Falato, A., I. Goldstein, and A. Hortaçsu (2021). Financial fragility in the covid-19 crisis: The case of investment funds in corporate bond markets. Journal of Monetary Economics 123, 35–52.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. In Handbook of monetary economics, Volume 3, pp. 547–599. Elsevier.

Gertler, M., N. Kiyotaki, and A. Prestipino (2020). A macroeconomic model with financial panics. The Review of Economic Studies 87(1), 240–288.

Haddad, V. and T. Muir (2021). Do intermediaries matter for aggregate asset prices? The Journal of Finance 76(6), 2719–2761.

He, Z. and A. Krishnamurthy (2012). A model of capital and crises. The Review of Economic Studies 79(2), 735–777.

He, Z. and A. Krishnamurthy (2013). Intermediary asset pricing. American Economic Review 103(2), 732–70.

He, Z. and A. Krishnamurthy (2018). Intermediary asset pricing and the financial crisis. Annual Review of Financial Economics 10(1), 173–197.

Irani, R. M., R. Iyer, R. R. Meisenzahl, and J.-L. Peydro (2021). The rise of shadow banking: Evidence from capital regulation. The Review of Financial Studies 34(5), 2181–2235.

Jiménez, G., S. Ongena, J.-L. Peydró, and J. Saurina (2012). Credit supply and monetary policy: Identifying the bank balance-sheet channel with loan applications. American Economic Review 102(5), 2301–26.

Li, J. and H. Yu (2021). The importance of investor heterogeneity: An examination of the corporate bond market.

Ma, Y., K. Xiao, and Y. Zeng (2020). Mutual fund liquidity transformation and reverse flight to liquidity. Available at SSRN 3640861.

Muir, T. (2017). Financial crises and risk premia. The Quarterly Journal of Economics 132(2), 765–809.

Siani, K. (2021). Raising bond capital in segmented markets. Technical report, Working Paper.

Vissing-Jorgensen, A. (2020). Bond markets in spring 2020 and the response of the federal reserve. Technical report.

Appendix

A.1 Appendix to Section 3

A.1.1 Proofs

Proof that $V_t^{(i,c)}(N_t) = N_t$*.* Let $\alpha > 0$. By making the change of variable:

$$\tilde{E}_{t+1} = \frac{E_{t+1}}{\alpha}, \quad \tilde{N}_{t+1} = \frac{N_{t+1}}{\alpha}, \tag{4}$$

in the Bellman equation defining $V_t^{(i,c)}(N_t)$, we can write $V_t^{(i,c)}(\alpha N_t)$ as:

$$\begin{aligned} V_t^{(i,c)}(\alpha N_t) &= \max_{x_{t+1}, E_{t+1}} \quad \alpha(N_t - (1 - x_{t+1})\tilde{E}_{t+1}) + \quad \mathbb{E}_t \left[\Lambda_{t,t+1}^{(u)} \zeta_{t+1} V_{t+1}^{(i,c)}(\alpha \tilde{N}_{t+1}) \right] \\ \text{s.t.} \qquad & \tilde{N}_{t+1} = \left(R_{t+1}^{(e)} - \frac{x_{t+1}}{P_t} F_{t+1} \right) \tilde{E}_{t+1} \\ & x_{t+1} \leq \overline{x} \quad [\tilde{\psi}_t] \end{aligned}$$

This implies that:

$$\forall \alpha > 0, \quad \frac{V_t^{(i,c)}(\alpha N_t)}{\alpha} = V_t^{(i,c)}(N_t),$$

which, in turn, implies that:

$$V_t^{(i,c)}(0) = 0.$$

Moreover, the envelope condition for N_t can be written as:

$$\frac{\partial V_t^{(i,c)}}{\partial N_t} = 1,$$

implying that:

$$V_t^{(i,c)}(N_t) = V_{0,t}^{(i,c)} + N_t$$

for some constant $V_{0,t}^{(i,c)}$. Since $V_t^{(i,c)}(0) = 0$, it must be that $V_{0,t}^{(i,c)} = 0$, establishing the result. ■

Proof of Result 1. Assume that all early investors redeem their shares. Then the payoffs to early investors, late investors, and the net worth of the intermediary are given by:

$$\begin{aligned} F_t^{(1)} &= \min\left(P_{t-1}, \frac{(1-\phi)R_t^{(e)}}{\lambda x_t} P_{t-1} \right) \\ F_t^{(2)} &= \min\left(R_t^{(e)}, \max\left(\left\{ \frac{R_t^{(e)}}{(1-\lambda)x_t} - \frac{\lambda}{(1-\phi)(1-\lambda)} \right\} P_{t-1}, 0 \right) \right) \\ N_t &= \max\left(\left\{ \left(1 - (1-\lambda)\frac{x_t}{P_{t-1}} \right) R_t^{(e)} - \frac{\lambda x_t}{1-\phi} \right\} E_t, 0 \right) \end{aligned}$$

To make notation easier, let:

$$\begin{aligned}
\underline{R}_t &= \frac{\lambda x_t}{1-\phi} \\
\hat{R}_t &= \left(1+\frac{\lambda\phi}{1-\phi}\right) x_t \\
\tilde{R}_t &= \frac{\lambda x_t}{(1-\phi)\left(1-(1-\lambda)\frac{x_t}{P_{t-1}}\right)}
\end{aligned}$$

The first threshold is the threshold below which late investors receive no payoff. The third threshold is the intermediary liquidation threshold if all early investors redeem their shares. Note that the threshold $\hat{R}_t$ can be written as:

$$\hat{R}_t = w_t \tilde{R}_t + (1-w_t)\underline{R}_t,$$

where:

$$w_t \equiv \frac{(1-\phi)(P_{t-1}-(1-\lambda)x_t)}{\lambda x_t}.$$

We have $w_t \geq 0$, and:

$$w_t \leq 1 \quad \Longleftrightarrow \quad x_t \geq \frac{1-\phi}{1-\phi+\lambda\phi} P_{t-1} \equiv \overline{x}(P_{t-1}).$$

Thus if $x_t \leq \overline{x}(P_{t-1})$, we have:

$$\hat{R}_t \geq \tilde{R}_t > \underline{R}_t,$$

while if $x_t \geq \overline{x}(P_{t-1})$, we have:

$$\tilde{R}_t \geq \hat{R}_t > \underline{R}_t.$$

Additionally, straightforward algebra shows that when $x_t \leq \overline{x}(P_{t-1})$, it cannot be the case that $P_{t-1} \leq \tilde{R}_t$. Finally, using the payoffs reported above, note that, if all early investors choose redemption:

- if $R_t^{(e)} > \tilde{R}_t$, the intermediary has sufficient funds to meet the promises to early and late investors, and is not liquidated;
- if $R_t \in \left[\underline{R}_t, \tilde{R}_t\right)$, the intermediary is liquidated; early investors can redeem their shares; and late investors earn a return that is below the promised rate of return, $R_t^{(e)}$;
- if $R_t \in < \underline{R}_t$, the intermediary is liquidated; early investors cannot redeem their shares, and instead receive all the intermediary's assets; and late investors receive no payoff.

Next, we consider the two cases $x_t \leq \overline{x}(P_{t-1})$ and $x_t \geq \overline{x}(P_{t-1})$ separately.

The case $x_t \leq \overline{x}(P_{t-1})$ We check whether redemption is a Nash equilibrium for early investors in each of the three regions for realized returns defined above. By Nash equilibrium, we mean that, if all other early investors decide to redeem their shares, it is optimal for an individual early investor to also redeem their share, instead of waiting to get the same payoff as late investors.

First, if $R_t^{(e)} < \underline{R}_t$, then redemption by early investors is clearly a Nash equilibrium, since late investors receive zero payoff. Second, if $R_t^{(e)} \in \left[\underline{R}_t, \tilde{R}_t\right)$, redemption by early investors is a Nash equilibrium, if and only if:

$$P_{t-1} \geq \left(\frac{R_t^{(e)}}{(1-\lambda)x_t} - \frac{\lambda}{(1-\phi)(1-\lambda)} \right) P_{t-1} \iff R_t^{(e)} \leq \hat{R}_t.$$

But, in this case, $\hat{R}_t > \tilde{R}_t$. Thus, if $R_t^{(e)} \in \left[\underline{R}_t, \tilde{R}_t\right)$, redemption by early investors is a Nash equilibrium. Finally, when $R_t^{(e)} > \tilde{R}_t$, redemption by early investors is an equilibrium, if and only if:

$$R_t^{(e)} < P_{t-1}.$$

As mentioned above, when $x_t \leq \overline{x}(P_{t-1})$, it must be that $P_{t-1} \geq \tilde{R}_t$. So, early redemption will occur for $R_t^{(e)} \in \left[\tilde{R}_t, P_{t-1}\right)$, and otherwise, early investors will not redeem their shares.

The case $x_t \geq \overline{x}(P_{t-1})$ Again, we check whether redemption is a Nash equilibrium for early investors in each of the three regions for realized returns defined above. First, if $R_t \in < \underline{R}_t$, then redemption by early investors is clearly a Nash equilibrium. Second, if $R_t \in \left[\underline{R}_t, \tilde{R}_t\right)$, redemption by early investors is a Nash equilibrium, if and only if:

$$P_{t-1} \geq \left(\frac{R_t^{(e)}}{(1-\lambda)x_t} - \frac{\lambda}{(1-\phi)(1-\lambda)} \right) P_{t-1} \iff R_t^{(e)} \leq \hat{R}_t.$$

In this case, we have that $\hat{R}_t \in \left(\underline{R}_t, \tilde{R}_t\right]$. So, redemption is a Nash equilibrium only when $R_t^{(e)} \in \left[\underline{R}_t, \hat{R}_t\right)$. Finally, if $R_t \geq \tilde{R}_t$, redemption is a Nash equilibrium only if:

$$R_t^{(e)} < P_{t-1}.$$

For this to be possible, it would need to be the case that $\tilde{R}_t < P_{t-1}$, which implies $x_t < \overline{x}(P_{t-1})$, a contradiction. Thus, when $R_t \geq \tilde{R}_t$, redemption cannot be a Nash equilibrium. ■

Equilibrium returns on redeemable shares When $x_t \leq \overline{x}(P_{t-1})$, the rate of return on redeemable shares is given by:

$$\frac{F_t}{P_{t-1}} \equiv \lambda \frac{F_t^{(1)}}{P_{t-1}} + (1-\lambda) \frac{F_t^{(2)}}{P_{t-1}}$$

$$
= \begin{cases} \dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } R_t^{(e)} \geq P_{t-1} \\ \lambda + (1-\lambda)\dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } R_t^{(e)} \in [R_{L,t}, P_{t-1}) \\ \dfrac{R_t^{(e)}}{x_t} - \dfrac{\lambda\phi}{1-\phi} & \text{if } R_t^{(e)} \text{ if } R_t^{(e)} \in \left[\dfrac{\lambda x_t}{(1-\phi)}, R_{L,t}\right) \\ \dfrac{(1-\phi)R_t^{(e)}}{x_t} & \text{if } R_t^{(e)} < \dfrac{\lambda x_t}{(1-\phi)} \end{cases}
$$

When $x_t \leq \overline{x}(P_{t-1})$, the rate of return on redeemable shares is given by:

$$
\begin{aligned}
\frac{F_t}{P_{t-1}} &\equiv \lambda\frac{F_t^{(1)}}{P_{t-1}} + (1-\lambda)\frac{F_t^{(2)}}{P_{t-1}} \\
&= \begin{cases} \dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } R_t^{(e)} \geq R_{L,t} \\ \dfrac{R_t^{(e)}}{x_t} - \dfrac{\lambda\phi}{1-\phi} & \text{if } R_t^{(e)} \in \left[\dfrac{\lambda x_t}{(1-\phi)}, R_{L,t}\right) \\ \dfrac{(1-\phi)R_t^{(e)}}{x_t} & \text{if } R_t^{(e)} < \dfrac{\lambda x_t}{(1-\phi)} \end{cases}
\end{aligned}
$$

A.1.2 An alternative model for the NBI

[To be added.]

A.1.3 Equilibrium conditions for the BI model

The equilibrium conditions are:

$$\zeta_{t+1} = \mathbf{1}\left\{R_{t+1}^{(e)} \geq R_{L,t}\right\} \tag{1}$$

$$1 - x_{t+1} = \mathbb{E}_t\left[\zeta_{t+1}\left(R_{t+1}^{(e)} - x_{t+1}F_{t+1}\right)\right] \tag{2}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)}\zeta_{t+1}F_{t+1}\right] + \psi_t \tag{3}$$

$$0 = \psi_t\left(\overline{x} - x_{t+1}\right) \tag{4}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)}F_{t+1}\right] \tag{5}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}R_{t+1}^{(e)}\right] \tag{6}$$

$$Q_t = \Phi'\left(\frac{I_t}{K_t}\right) \tag{7}$$

$$V_t = Q_t \tilde{K}_{t+1} \tag{8}$$

$$\Pi_t = AK_t - \Phi\left(\frac{I_t}{K_t}\right) K_t \tag{9}$$

$$K_t = \xi_t \tilde{K}_t \tag{10}$$

$$\tilde{K}_{t+1} = I_t + (1-\delta) K_t \tag{11}$$

$$0 = \nu_t S_{t+1} \tag{12}$$

$$1 = (1-\zeta_t) + \zeta_t \left(S_{t+1} + \frac{E_{t+1}}{V_{t+1}}\right) \tag{13}$$

$$\Lambda_{t,t+1} = \frac{1}{1+\rho} \frac{C_{t+1}^{-1}}{(1-\nu_t) C_t^{-1} + \frac{\chi}{1+\rho} V_t^{-1} S_{t+1}} \tag{14}$$

$$\Lambda_{t,t+1}^{(u)} = \frac{1}{1+\rho} \frac{C_{t+1}^{-1}}{C_t^{-1}} \tag{15}$$

$$R_{t+1}^{(e)} = \frac{\Pi_{t+1} + V_{t+1}}{V_t} \tag{16}$$

$$F_t = \begin{cases} R_t^{(d)} & \text{if} \quad \zeta_t = 1 \\ (1-\phi)\dfrac{R_t^{(e)}}{x_t} & \text{if} \quad \zeta_t = 0 \end{cases} \tag{17}$$

$$R_{L,t} = x_t R_t^{(d)} \tag{18}$$

$$P_t = 1 \tag{19}$$

The corresponding 19 endogenous variables are:

$$\Lambda_{t,t+1}, \Lambda_{t,t+1}^{(u)}, R_{t+1}^{(e)}, I_t, K_{t+1}, \tilde{K}_{t+1}, Q_t, V_t, \Pi_t, S_{t+1}, \nu_t, \zeta_t, \psi_t, x_{t+1}, E_{t+1}, F_t, R_{L,t}, R_{t+1}^{(d)}, P_t.$$

A.1.4 Equilibrium conditions for the NBI model

The equilibrium conditions are:

$$\zeta_{t+1} = \mathbf{1}\left\{R_{t+1}^{(e)} \geq R_{L,t}\right\} \tag{1}$$

$$1 - x_{t+1} = \mathbb{E}_t \left[\zeta_{t+1}\left(R_{t+1}^{(e)} - x_{t+1}\frac{F_{t+1}}{P_t}\right)\right] \tag{2}$$

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1}^{(u)} \zeta_{t+1} \frac{F_{t+1}}{P_t}\right] + \psi_t \tag{3}$$

$$0 = \psi_t \left(\overline{x} - x_{t+1}\right) \tag{4}$$

$$P_t = \mathbb{E}_t \left[\Lambda^{(u)}_{t,t+1} F_{t+1}\right] \tag{5}$$

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R^{(e)}_{t+1}\right] \tag{6}$$

$$Q_t = \Phi' \left(\frac{I_t}{K_t}\right) \tag{7}$$

$$V_t = Q_t \tilde{K}_{t+1} \tag{8}$$

$$\Pi_t = A K_t - \Phi \left(\frac{I_t}{K_t}\right) K_t \tag{9}$$

$$K_t = \xi_t \tilde{K}_t \tag{10}$$

$$\tilde{K}_{t+1} = I_t + (1-\delta) K_t \tag{11}$$

$$0 = \nu_t S_{t+1} \tag{12}$$

$$1 = (1-\zeta_t) + \zeta_t \left(S_{t+1} + \frac{E_{t+1}}{V_{t+1}}\right) \tag{13}$$

$$\Lambda_{t,t+1} = \frac{1}{1+\rho} \frac{C_{t+1}^{-1}}{(1-\nu_t) C_t^{-1} + \frac{\chi}{1+\rho} V_t^{-1} S_{t+1}} \tag{14}$$

$$\Lambda^{(u)}_{t,t+1} = \frac{1}{1+\rho} \frac{C_{t+1}^{-1}}{C_t^{-1}} \tag{15}$$

$$R^{(e)}_{t+1} = \frac{\Pi_{t+1} + V_{t+1}}{V_t} \tag{16}$$

$$F_t = P_{t-1} \times \begin{cases} \dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } x_t \leq \overline{x}_t \text{ and } R_t^{(e)} \geq P_{t-1} \\ \lambda + (1-\lambda)\dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } x_t \leq \overline{x}_t \text{ and } R_t^{(e)} \in [R_{L,t}, P_{t-1}) \\ \dfrac{R_t^{(e)}}{x_t} - \dfrac{\lambda\phi}{1-\phi} & \text{if } x_t \leq \overline{x}_t \text{ and } R_t^{(e)} \in \left[\dfrac{\lambda x_t}{(1-\phi)}, R_{L,t}\right) \\ \dfrac{(1-\phi)R_t^{(e)}}{x_t} & \text{if } x_t \leq \overline{x}_t \text{ and } R_t^{(e)} < \dfrac{\lambda x_t}{(1-\phi)} \\ \dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } x_t \geq \overline{x}_t \text{ and } R_t^{(e)} \geq R_{L,t} \\ \dfrac{R_t^{(e)}}{x_t} - \dfrac{\lambda\phi}{1-\phi} & \text{if } x_t \geq \overline{x}_t \text{ and } R_t^{(e)} \in \left[\dfrac{\lambda x_t}{(1-\phi)}, R_{L,t}\right) \\ \dfrac{(1-\phi)R_t^{(e)}}{x_t} & \text{if } x_t \geq \overline{x}_t \text{ and } R_t^{(e)} < \dfrac{\lambda x_t}{(1-\phi)} \end{cases} \tag{17}$$

$$R_{L,t} = \begin{cases} \dfrac{\lambda x_t}{(1-\phi)(P_{t-1} - (1-\lambda)x_t)} P_{t-1} & \text{if} \quad x_t \leq \overline{x}_t \\ \left(1 + \dfrac{\lambda\phi}{1-\phi}\right) x_t & \text{if} \quad x_t \geq \overline{x}_t \end{cases} \tag{18}$$

$$\overline{x}_t = \frac{1-\phi}{1-\phi+\lambda\phi} P_{t-1} \tag{19}$$

The corresponding 19 endogenous variables are:

$$\Lambda_{t,t+1}, \Lambda_{t,t+1}^{(u)}, R_{t+1}^{(e)}, I_t, K_{t+1}, \tilde{K}_{t+1}, Q_t, V_t, \Pi_t, S_{t+1}, \nu_t, \zeta_t, \psi_t, x_{t+1}, E_{t+1}, F_t, P_t, R_{L,t}, \overline{x}_t.$$

A.1.5 Other model variables

The rest of the variables in the model can be recovered from the following 8 conditions:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1}\zeta_{t+1}R_{t+1}^{(i)}\right] \tag{1}$$

$$R_{t+1}^{(i)} = \frac{\Pi_{t+1}^{(i)} + V_{t+1}^{(i)}}{V_t^{(i)}} \tag{2}$$

$$\Pi_t^{(i)} = N_t - (1 - x_{t+1})E_{t+1} \tag{3}$$

$$N_t = (R_t^{(e)} - x_t R_t)E_t \tag{4}$$

$$C_t = \Pi_t \tag{5}$$

$$R_{f,t+1} = (1+\rho)\mathbb{E}_t\left[\left(\frac{\Pi_{t+1}}{\Pi_t}\right)^{-1}\right]^{-1} \tag{6}$$

$$rp_t^{(e)} = \mathbb{E}_t\left[R_{t+1}^{(e)}\right] - R_{f,t+1} \tag{7}$$

$$rp_t^{(d)} = R_{t+1} - R_{f,t+1} \tag{8}$$

These conditions pin down the 8 remaining variables:

$$R_{t+1}^{(i)}, V_t^{(i)}, \Pi_t^{(i)}, N_t, C_t, R_{f,t+1}, rp_t^{(e)}, rp_t^{(d)}.$$

A.2 Appendix to Section 4

A.2.1 Proofs of general results

Recall that the first-order conditions from the intermediary and household problem are:

$$1 - x_{t+1} = \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)}\zeta_{t+1}\left(R_{t+1}^{(e)} - \frac{F_{t+1}}{P_t}x_{t+1}\right)\right] \tag{9}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)}\zeta_{t+1}\frac{F_{t+1}}{P_t}\right] + \psi_t \tag{10}$$

$$0 = \psi_t\left(\overline{x} - x_{t+1}\right) \tag{11}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)}\frac{F_{t+1}}{P_t}\right] \tag{12}$$

while intermediary liquidation is governed by:

$$\zeta_{t+1} = \mathbf{1}\left\{R_{t+1}^{(e)} \geq \frac{F_{t+1}}{P_t}x_{t+1}\right\}. \tag{13}$$

Recall that all these equations only hold in periods when the intermediary is active, $\zeta_t = 1$.

Proof of Result 1. Assume that $\zeta_t = 1$, so that conditions (9)-(12) hold. First, suppose that $x_{t+1} = 0$. Then, using Equation (13), $\zeta_{t+1} = 1$ almost surely. Using Equation (9), we then have that:

$$1 = E_t\left[\Lambda_{t,t+1}^{(u)}R_{t+1}^{(e)}\right].$$

Next, suppose that $x_{t+1} > 0$. Multiplying Equation (12) by x_{t+1} and re-arranging, we have:

$$x_{t+1} = \mathbb{E}_t\left[\Lambda_{t,t+1}^{(u)}\left((1-\zeta_{t+1})\frac{F_{t+1}}{P_t}x_{t+1} + \zeta_{t+1}\frac{F_{t+1}}{P_t}x_{t+1}\right)\right].$$

Taking the sum with Equation (9):

$$1 = \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}R^{(e)}_{t+1}\right] - \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}(1-\zeta_{t+1})\left(R^{(e)}_{t+1} - \frac{F_{t+1}}{P_t}x_{t+1}\right)\right].$$

In both models, we have:

$$R^{(e)}_{t+1} - \frac{F_{t+1}}{P_t}x_{t+1} = D(R^{(e)}_{t+1}, x_{t+1}),$$

In the BI model, the function $D(R_{t+1})^{(e)}$ is:

$$D(R^{(e)}_{t+1}, x_{t+1}) = \phi R^{(e)}_{t+1}$$

while in the NBI model, the function is:

$$D(R^{(e)}_{t+1}, x_{t+1}) = \phi \min\left(\frac{\lambda}{1-\phi}x_{t+1}, R^{(e)}_{t+1}\right).$$

In both cases, when $\phi = 0$, the function is identically zero, so that:

$$1 = \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}R^{(e)}_{t+1}\right].$$

Comparing this with the first-order condition for NFC equity from the household's problem,

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}R^{(e)}_{t+1}\right],$$

and using the expressions for the discount factors $\Lambda_{t,t+1}$ and $\Lambda^{(u)}_{t,t+1}$, we must have that:

$$C_t^{-1} = (1-\nu_t)C_t^{-1} + \frac{\chi}{1+\rho}S_{t+1}V_t^{-1}.$$

Therefore,

$$\nu_t = \frac{\chi}{1+\rho}\frac{V_t}{C_t}S_{t+1}.$$

Since $\nu_t S_{t+1} = 0$, this equation then implies that $\nu_t = S_{t+1} = 0$. ■

Proof of Result 2. Assume that $x_{t+1} > 0$. Equation (12) can be rewritten as:

$$1 = \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}\left((1-\zeta_{t+1})\frac{F_{t+1}}{P_t} + \zeta_{t+1}\frac{F_{t+1}}{P_t}\right)\right].$$

Comparing this to condition (10), we see that:

$$\psi_t = \mathbb{E}_t\left[\Lambda^{(u)}_{t,t+1}(1-\zeta_{t+1})\frac{F_{t+1}}{P_t}\right].$$

When $x_{t+1} > 0$, because $R^{(e)}_{t+1}$ has full support, the probability of intermediary liquidation in the following period is non-zero. Therefore, the right-hand side of the equation above is strictly positive, $\psi_t > 0$, and $x_{t+1} = \bar{x}$. Note that when $x_{t+1} = 0$, the intermediary is not issuing any liabilities, so that the first-order condition (12) need not hold. ■

A.2.2 Proofs of for the analytical model

A.2.2.1 Preliminaries

Using Results 1 and 2, we first re-state the set of equilibrium conditions when $\phi = 0$ as follows. First, in both models, we have:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R^{(e)}_{t+1} \right] \tag{1}$$

$$Q_t = \Phi' \left(\frac{I_t}{K_t} \right) \tag{2}$$

$$V_t = Q_t \tilde{K}_{t+1} \tag{3}$$

$$\Pi_t = A K_t - \Phi \left(\frac{I_t}{K_t} \right) K_t \tag{4}$$

$$K_t = \xi_t \tilde{K}_t \tag{5}$$

$$\tilde{K}_{t+1} = I_t + (1-\delta) K_t \tag{6}$$

$$\Lambda_{t,t+1} = \begin{cases} \frac{1}{1+\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-1} & \text{if} \quad \zeta_t = 1 \\ \frac{1}{1+\rho} \frac{C^{-1}_{t+1}}{C^{-1}_t + \frac{\chi}{1+\rho} V^{-1}_t} & \text{if} \quad \zeta_t = 0 \end{cases} \tag{7}$$

$$R^{(e)}_{t+1} = \frac{\Pi_{t+1} + V_{t+1}}{V_t} \tag{8}$$

$$P_t = \mathbb{E}_t \left[\Lambda_{t,t+1} F_{t+1} \right] \tag{9}$$

$$\zeta_{t+1} = \mathbf{1} \left\{ R^{(e)}_{t+1} \geq R_{L,t} \right\} \tag{10}$$

$$1 = \mathbb{E}_t \left[\Lambda^{(u)}_{t,t+1} \zeta_{t+1} \frac{F_{t+1}}{P_t} \right] + \psi_t \tag{11}$$

In the BI model, the endogenous variables are:

$$\Lambda_{t,t+1}, R^{(e)}_{t+1}, I_t, K_{t+1}, \tilde{K}_{t+1}, Q_t, V_t, \Pi_t, \zeta_t, \psi_t, F_t, R_{L,t}, R^{(d)}_{t+1}, P_t.$$

The three additional equations are:

$$F_t = \begin{cases} R_t^{(d)} & \text{if} \quad \zeta_t = 1 \\ \dfrac{R_t^{(e)}}{\overline{x}} & \text{if} \quad \zeta_t = 0 \end{cases} \tag{12}$$

$$R_{L,t} = \overline{x} R_t^{(d)} \tag{13}$$

$$P_t = 1 \tag{14}$$

In the NBI model, the endogenous variables are:

$$\Lambda_{t,t+1}, R_{t+1}^{(e)}, I_t, K_{t+1}, \tilde{K}_{t+1}, Q_t, V_t, \Pi_t, \zeta_t, \psi_t, F_t, R_{L,t}, \overline{x}_t, P_t.$$

The three additional equations are:

$$\frac{F_t}{P_{t-1}} = \begin{cases} \dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } \overline{x} \leq \overline{x}_t \text{ and } R_t^{(e)} \geq P_{t-1} \\ \lambda + (1-\lambda)\dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } \overline{x} \leq \overline{x}_t \text{ and } R_t^{(e)} \in [R_{L,t}, P_{t-1}) \\ \dfrac{R_t^{(e)}}{P_{t-1}} & \text{if } \overline{x} \geq \overline{x}_t \text{ and } R_t^{(e)} \geq R_{L,t} \\ \dfrac{R_t^{(e)}}{\overline{x}} & \text{if } R_t^{(e)} \leq R_{L,t} \end{cases} \tag{15}$$

$$R_{L,t} = \begin{cases} \dfrac{\lambda \overline{x}}{(P_{t-1} - (1-\lambda)\overline{x})} P_{t-1} & \text{if} \quad \overline{x} \leq \overline{x}_t \\ \overline{x} & \text{if} \quad \overline{x} \geq \overline{x}_t \end{cases} \tag{16}$$

$$\overline{x}_t = P_{t-1} \tag{17}$$

Next, we show that in the NBI model with $\phi = 0$, if $\overline{x} \leq 1$, then we must have $P_{t-1} \geq \overline{x}$. Assume otherwise: $P_{t-1} < 1$. Suppose that $1 \geq \overline{x} > P_{t-1}$. Then, the first-order condition of the household with respect to non-equity liabilities can be written as:

$$\begin{aligned} 1 &= \mathbb{E}_t \left[\Lambda_{t,t+1} \left((1-\zeta_{t+1}) \frac{R_t^{(e)}}{\overline{x}} + \zeta_{t+1} \frac{R_t^{(e)}}{P_{t-1}} \right) \right] \\ &> \mathbb{E}_t \left[\Lambda_{t,t+1} \left((1-\zeta_{t+1}) R_{t+1}^{(e)} + \zeta_{t+1} R_{t+1}^{(e)} \right) \right] && (18) \\ &= \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}^{(e)} \right] = 1, && (19) \end{aligned}$$

where the last line uses Results 1. This is a contradiction, so it must be that $P_{t-1} \geq \overline{x}$. Therefore, it must also be that $\overline{x}_t \geq \overline{x}$. So the last three conditions in the NBI problem further simplify to:

$$\frac{F_t}{P_{t-1}} = \begin{cases} \frac{R_t^{(e)}}{P_{t-1}} & R_t^{(e)} \geq P_{t-1} \\ \lambda + (1-\lambda)\frac{R_t^{(e)}}{P_{t-1}} & R_t^{(e)} \in [R_{L,t}, P_{t-1}) \\ \frac{R_t^{(e)}}{\overline{x}} & \text{if } R_t^{(e)} \leq R_{L,t} \end{cases} \tag{20}$$

$$R_{L,t} = \frac{\lambda \overline{x}}{(P_{t-1} - (1-\lambda)\overline{x})} P_{t-1}, \tag{21}$$

and we can omit $\overline{x}_t = P_{t-1}$ from the description of equilibrium.

A.2.2.2 Returns to NFC equity

Next, assume that:

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} = \frac{\tilde{K}_{t+1}}{K_t} - (1-\delta)$$

In this case, the resource constraint can be written as:

$$\frac{\Pi_t + \tilde{K}_{t+1}}{K_t} = A + 1 - \delta \equiv z.$$

Moreover:

$$Q_t = 1.$$

Therefore,

$$V_t = \tilde{K}_{t+1}$$

and so we have:

$$\frac{\Pi_t + V_t}{K_t} = z,$$

and so:

$$\frac{\Pi_t + V_t}{\tilde{K}_t} = \frac{\Pi_t + V_t}{V_{t-1}} = z\xi_t,$$

so that:

$$R_{t+1}^{(e)} = z\xi_t. \tag{22}$$

A.2.2.3 Equilibrium

Redemption and liquidation thresholds In what follows, for both model, we define:

$$\xi_{L,t} \equiv \frac{R_{L,t}}{z}. \tag{23}$$

Additionally, for the NBI model, we define:

$$\xi_{R,t} \equiv \frac{P_{t-1}}{z}. \tag{24}$$

These define, respectively, liquidation and redemption thresholds in terms of the realizations of the fundamental shock, ξ_t.

Equilibrium structure We next show that in each model, there is an equilibrium such that the growth rate of capital K_{t+1} is given by:

$$K_{t+1} = (1+g_t)K_t \tag{25}$$

$$g_t = \begin{cases} g_L & \text{if} \quad \zeta_t = 0 \\ g_N & \text{if} \quad \zeta_t = 1 \end{cases} \tag{26}$$

Moreover, all endogenous variables are proportional to K_t, and that the proportionality factor only depends on whether the economy is in a liquidation state or not. For any variable Y_t, we denote the ratio Y_t/K_t by y_X, where $X \in \{L, N\}$ indexes whether the ratio corresponds to date when the intermediary is liquidiated ($X = L$), or a date when the intermediary is not liquidated ($X = N$). Finally, the redemption and liquidation thresholds are constant: $\xi_{E,t} = \xi_E$ and $\xi_{L,t} = \xi_L$.

Equilibrium existence With this guess, the fact that $\Pi_t + V_t = (\zeta\xi_t)V_{t-1}$ can be rewritten as:

$$\begin{aligned} \pi_N + v_N &= z, \\ \pi_L + v_L &= z. \end{aligned} \tag{27}$$

Moreover, the condition $V_t = \tilde{K}_t$ can be rewritten as:

$$\begin{aligned} 1 + g_N &= v_N, \\ 1 + g_L &= v_L. \end{aligned} \tag{28}$$

We next define:

$$\begin{aligned} \rho_N &\equiv \frac{\pi_N}{\rho_N}, \\ \rho_L &\equiv \frac{\pi_L}{\rho_L}, \end{aligned} \tag{29}$$

so that the conditions pinning down equilibrium returns can be written as:

$$
\begin{aligned}
v_N &= \frac{1}{1+\rho_N} z, \quad \pi_N = \frac{\rho_N}{1+\rho_N} z, \\
v_L &= \frac{1}{1+\rho_L} z, \quad \pi_L = \frac{\rho_L}{1+\rho_L} z.
\end{aligned} \tag{30}
$$

Next, we express the equilibrium condition:

$$
1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R^{(e)}_{t+1} \right]
$$

under this equilibrium guess, using the notation above.

When $\zeta_t = 0$, or equivalently, $\xi_t < \xi_L$ (the intermediary is liquidated), no other intermediary re-enters in period t, so that no deposits are issued in period t. Therefore, it must be the case that, at time $t+1$, $\zeta_{t+1} = 1$. Using the definition of the household discount fact, the asset pricing condition for NFC equity can then be written as:

$$
1 + \frac{1}{\rho_N} = \chi + (1+\rho) \frac{1}{\rho_N}
$$

When there is no liquidation, tedious computation shows that the asset pricing condition for NFC equity be written as:

$$
1 + \rho = (1 - F(\xi_L))(1+\rho_N) + F(\xi_L) \frac{\rho_N}{\rho_L} (1+\rho_L).
$$

We then make the change of variable:

$$
\rho_N = \rho(1-y_N)^{-1}, \quad \rho_L = \rho(1-y_L)^{-1},
$$

to rewrite the two asset pricing conditions for NFC equity as:

$$
\begin{aligned}
\rho y_N &= (y_L - y_N) F(\xi_L) \\
y_L - y_N &= \rho(\chi - y_L).
\end{aligned}
$$

These equations can be used to express y_N and y_L as a function of the equilibrium liquidation threshold only:

$$
\begin{aligned}
y_N &= \frac{F(\xi_L)}{1+\rho+F(\xi_L)} \chi, \\
y_L &= \frac{\rho + F(\xi_L)}{1+\rho+F(\xi_L)} \chi,
\end{aligned}
$$

and these two expressions can in turn be used to solve for ρ_N and ρ_L:

$$\begin{aligned}\rho_N = \rho_N(\xi_L) &= \rho\left(1 - \frac{F(\xi_L)}{1+\rho+F(\xi_L)}\chi\right)^{-1}, \\ \rho_L = \rho_L(\xi_L) &= \rho\left(1 - \frac{\rho+F(\xi_L)}{1+\rho+F(\xi_L)}\chi\right)^{-1}.\end{aligned} \tag{31}$$

To complete the characterization of equilibrium, we need to solve for the default threshold ξ_L. The default threshold is pinned down by the first-order condition of the household with respect to intermediary's non-equity liabilities, which is different in the two versions of the model.

Liquidation threshold in the BI model In this version of the model, the pricing condition for deposits, $1 = \mathbb{E}_t\left[\lambda_{t,t+1}F_{t+1}\right]$ can be written as:

$$\frac{1+\rho}{1+\rho_N(\xi_L)}(1-\overline{x}) = H(\xi_L),$$

where:

$$H(\xi) \equiv \int_{\xi_L}^{+\infty} \frac{\xi - \xi_L}{\xi} dF(\xi).$$

Using the expression for $\rho_N(\xi_L)$ derived above, the liquidation threshold must therefore be a solution to:

$$H(\xi_L) = (1-\overline{x})\frac{1 - \chi\dfrac{F(\xi_L)}{1+F(\xi_L)+\rho}}{1 - \dfrac{\chi}{1+\rho}\dfrac{F(\xi_L)}{1+F(\xi_L)+\rho}}$$

The function $H(y)$ is strictly decreasing with $H(0) = 1$ and $\lim_{y\to+\infty} H(y) = 0$. Let $R(y)$ be the function on the right-hand size of Equation (32). It is decreasing with $R(0) = 1-\overline{x} < 1$ and:

$$\lim_{y\to+\infty} R(y) = (1-\overline{x})\frac{2+\rho-\chi}{(1+\rho)(2+\rho)-\chi} > 0$$

Therefore, there is at least one strictly positive solution. The smallest weakly positive solution to the equation above is an equilibrium default threshold; therefore, an equilibrium of the form described exists.

Liquidation and redemption thresholds in the NBI model In the NBI model, computation shows that the first-order conditions for pricing the non-equity liabilities of the intermediary can be written as:

$$\xi_L = \frac{\lambda\bar{x}}{z\xi_E - (1-\lambda)\overline{x}}\xi_E$$

$$\xi_E = \frac{1}{z}\left(1+\left(\frac{z\xi_E}{\overline{x}}-1\right)\frac{\rho_N(\xi_L)}{\rho_L(\xi_L)}\frac{1+\rho_L(\xi_L)}{1+\rho}F(\xi_L)+\lambda\frac{1+\rho_N(\xi_L)}{1+\rho}G(\xi_L,\xi_E)\right) \tag{32}$$

where the function $G(\xi_E, \xi_L)$ is given by:

$$\forall \xi_L \leq \xi_E, \quad G(\xi_L, \xi_E) = \int_{\xi_L}^{\xi_E} \frac{\xi_E - \xi}{\xi} dF(\xi). \tag{33}$$

Using the expressions for $\rho_N(\xi_L)$ and $\rho_L(\xi_L)$ given above, we can rewrite this as:

$$\begin{aligned}
\xi_L &= \frac{\lambda\bar{x}}{z\xi_E-(1-\lambda)\overline{x}}\xi_E \\
\xi_E &= \frac{1}{z}\Bigg(1 \\
&\quad +\left(\frac{z\xi_E}{\overline{x}}-1\right)\frac{1-\dfrac{\chi(\rho+F(\xi_L))}{(1+\rho)(1+F(\xi_L)+\rho)}}{1-\dfrac{(1+\rho)\chi F(\xi_L)}{(1+\rho)(1+F(\xi_L)+\rho)}}F(\xi_L) \\
&\quad +\lambda\frac{1-\dfrac{\chi F(\xi_L)}{(1+\rho)(1+F(\xi_L)+\rho)}}{1-\dfrac{(1+\rho)\chi F(\xi_L)}{(1+\rho)(1+F(\xi_L)+\rho)}}G(\xi_L,\xi_E)
\end{aligned} \tag{34}$$

Expressions for the log-normal case We next consider the case when the shocks ξ_t are independenty and identically log-normally distributed over time, with $\mu = \frac{-\sigma^2}{2}$, where σ^2 is μ is the mean and σ the variance of $\log(xi_t)$, so that $\mathbb{E}[\xi_t] = 1$. In this case we have:

$$\begin{aligned}
F(\xi_L) &= \Phi\left(\frac{ln(\xi_L)+\frac{1}{2}\sigma^2}{\sigma}\right) \\
H(\xi_L) &= 1-\Phi\left(\frac{ln(\xi_L)+\frac{1}{2}\sigma^2}{\sigma}\right)-\xi_L\exp(\sigma^2)\left(1-\Phi\left(\frac{ln(\xi_L)+\frac{3}{2}\sigma^2}{\sigma}\right)\right) \\
G(\xi_L,\xi_E) &= \xi_E\exp(\sigma^2)\left(\Phi\left(\frac{ln(\xi_E)+\frac{3}{2}\sigma^2}{\sigma}\right)-\Phi\left(\frac{ln(\xi_L)+\frac{3}{2}\sigma^2}{\sigma}\right)\right) \\
&- \left(\Phi\left(\frac{ln(\xi_E)+\frac{1}{2}\sigma^2}{\sigma}\right)-\Phi\left(\frac{ln(\xi_L)+\frac{1}{2}\sigma^2}{\sigma}\right)\right)
\end{aligned} \tag{35}$$