Intangible capital, non-rivalry, and growth

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Introduction

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- Intellectual property assets [patents, trademarks]
- Organization capital [managerial know-how, production processes]

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2. Intangibles have distinct economic characteristics?

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This paper: Model emphasizing 2, with an application to long-run growth

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- Technology determines how easy reproduction is — e.g. writing vs. digital

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Different types of intangible assets $\leftrightarrow$ different (ρ, δ)

Classifying intangibles

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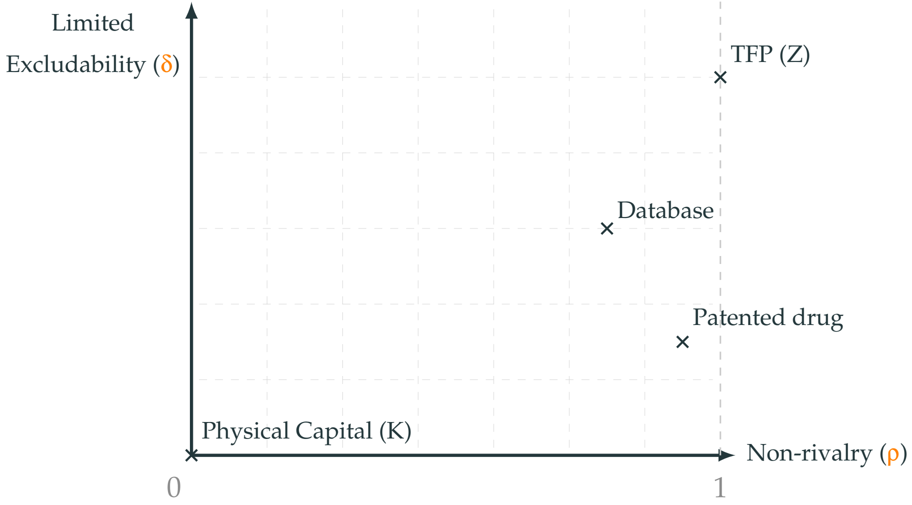
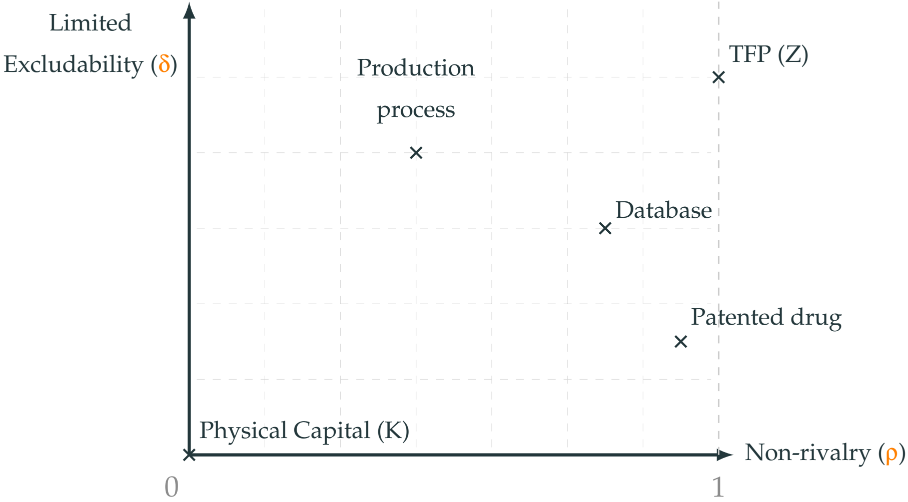


Classifying intangibles



Long-run growth implications

Q: How does growth change when $\uparrow \rho$?

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Benchmarks:

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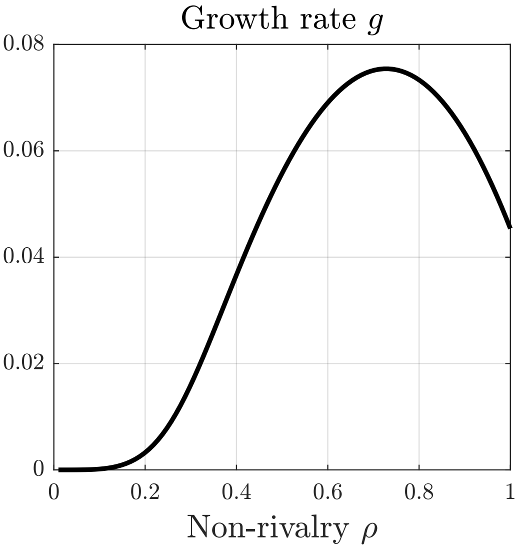
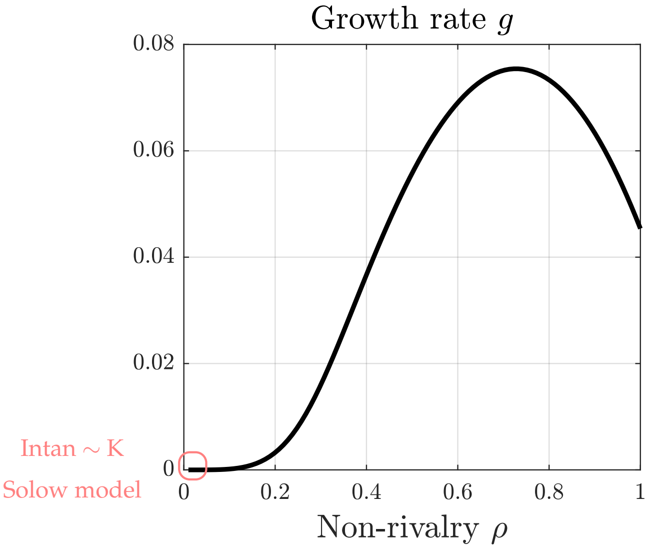
Real A: Non-monotonic relationship between ρ and growth


Growth rate g
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
Non-rivalry ρ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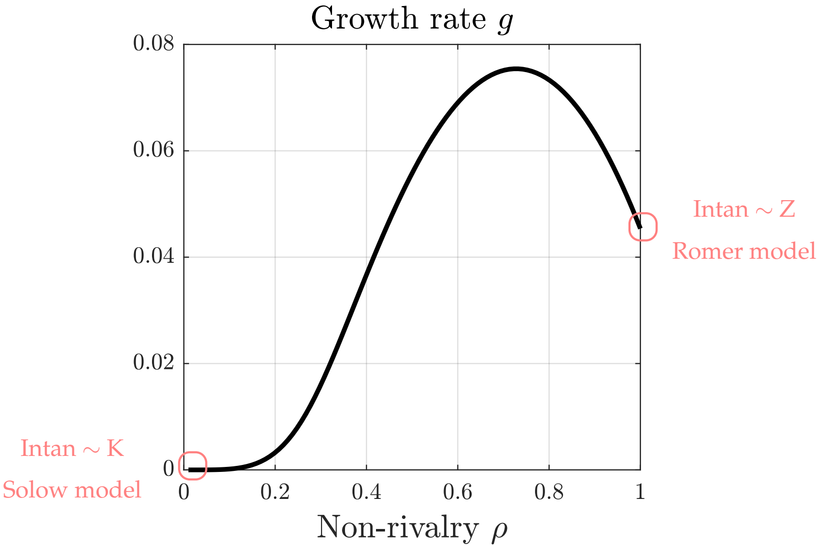
Non-rivalry ρ
Intan ~ K
Solow model



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Main Mechanism

$\uparrow \rho$

Main Mechanism

$$\uparrow \rho \implies$$

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$$\uparrow \rho \implies \begin{cases} \uparrow \text{scale economies} \\ \\ \end{cases}$$

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Entrants are larger, have more intangibles to build on (↑ incentive to enter)

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Implications: $\uparrow$ profits, valuations, concentration

$\downarrow$ entry and investment

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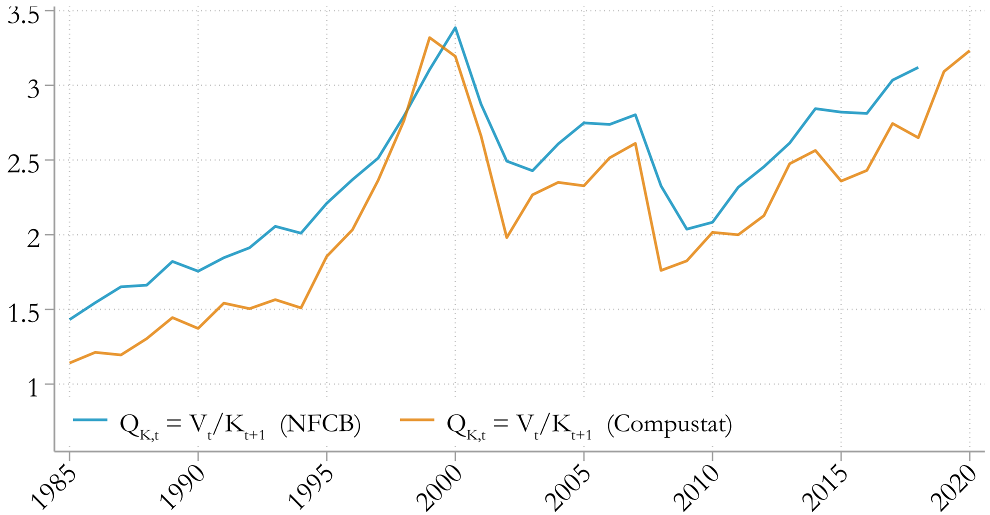
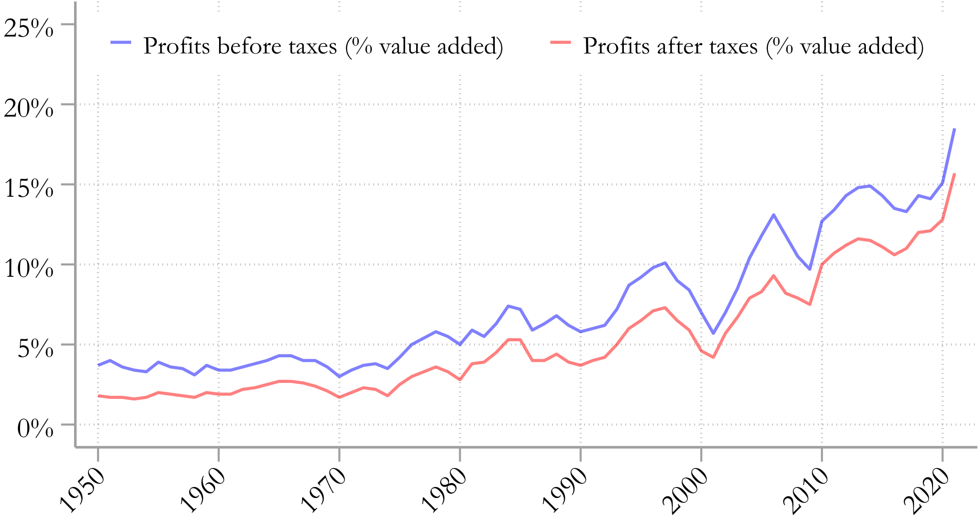
Why is this interesting?

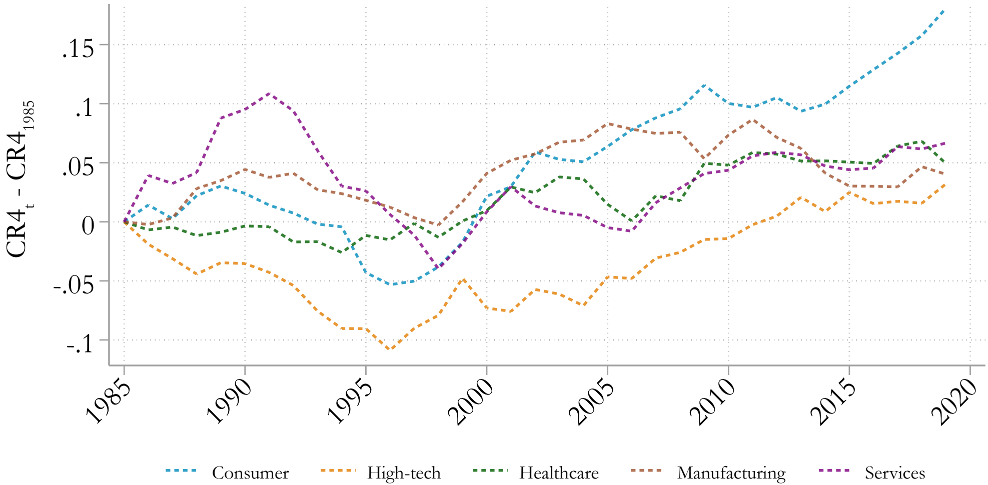
Market valuations have increased



Corporate profits as a share of GDP have increased



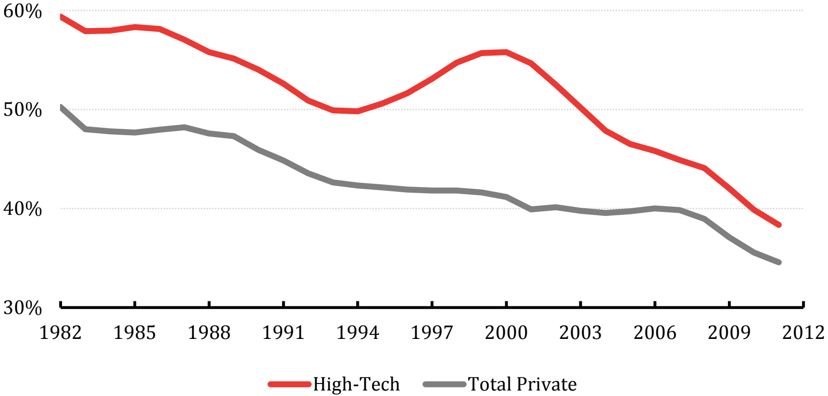
Concentration has been increased



Compustat; NAICS-3D sectors weighted by sales.

New entry has declined

Fig. 4: Young Firms (aged five years or younger) as a Share of Total Firms by Sector (1982–2011)



[Haltiwanger, Hathaway, Miranda, 2014]

Related literature

Macro and financial implications of rising intangibles

Hall (2001), Atkeson and Kehoe (2005), McGrattan and Prescott (2010), Eisfeldt and Papanikolaou (2013), Bhandari and McGrattan (2021), Crouzet and Eberly (2021)

Contribution: formalize non-rivalry and limited excludability

Endogenous technological change

Lucas and Moll (2014), Stokey (2015); Jones and Tonetti (2020), Farboodi and Veldkamp (2022)

Contribution: non-rivalry facilitates imitation; not limited to data

Competition and returns to innovation

Aghion, Bloom, Blundell, Griffith, Howitt (2005), Aghion, Bergeaud, Boppart, Klenow, Li (2022)

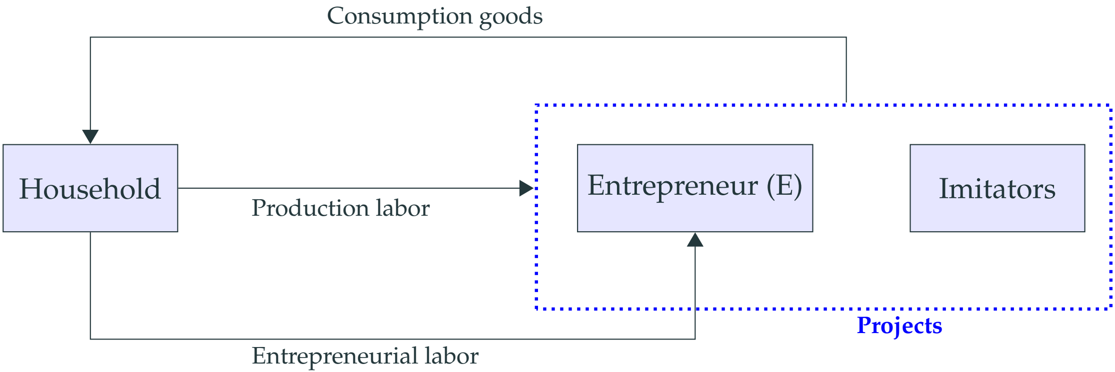
Contribution: non-rivalry creates *both* returns to scale and competitive risk

Roadmap

1. Economic environment
2. The effects of non-rivalry on growth
3. Other macro implications

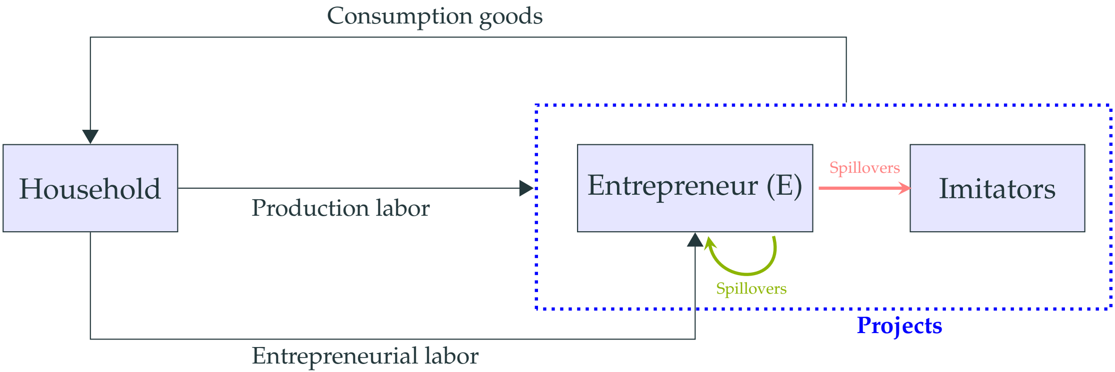
1. Economic environment

Overview



Consumption goods
Household
Production labor
Entrepreneur (E)
Imitators
Projects
Entrepreneurial labor

Overview



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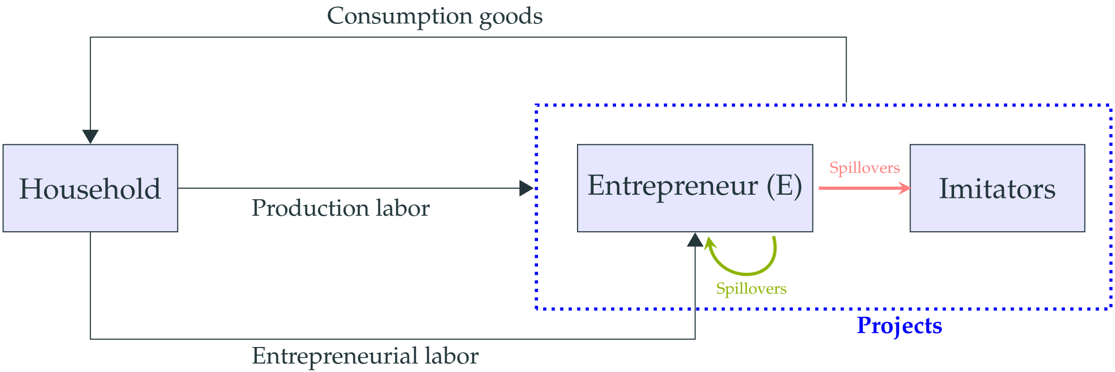
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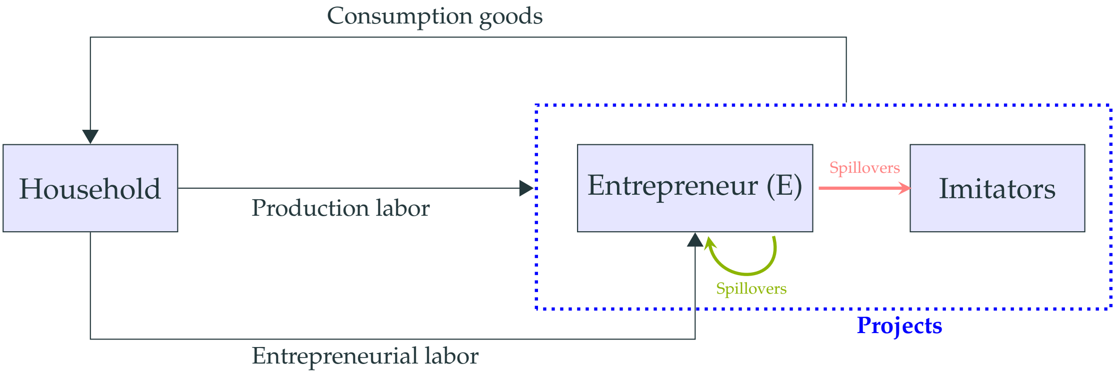

project = { product streams $s \in [0, x_t]$ }

x_t: project "span"

Allocating intangible capital within a project

$$\Pi(x_t, N_t) = \max_{\{N(s),L(s)\},L_t} \int_0^{x_t} N(s)^{1-\zeta} L(s)^{\zeta} ds - W_t L_t$$

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$$\Pi(x_t, N_t) \propto x_t^{\rho} N_t$$

What does ρ capture?

$$\left(\int_0^{x_t} N(s)^{\frac{1}{1-\rho}}\, ds\right)^{1-\rho} \leq N_t$$

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N is rival within the project

e.g. machines, structures

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N is <u>non-rival</u> within the project

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e.g. a patent for a touchscreen

using it for one product not reduce its availability for other products

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$\rho \in (0, 1)$

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N is imperfectly rival within the project

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e.g. an inventory management process for an online retailer

implementation in a new warehouse may be imperfect
it may require allocating

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non-rivalry of intangibles (ρ) $\leftrightarrow$ returns to scale

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$\Pi_t \propto x_t^{\rho} N_t$

if $\rho > 0$, N_t raises marginal returns to x_t

Imperfect excludability and spillovers

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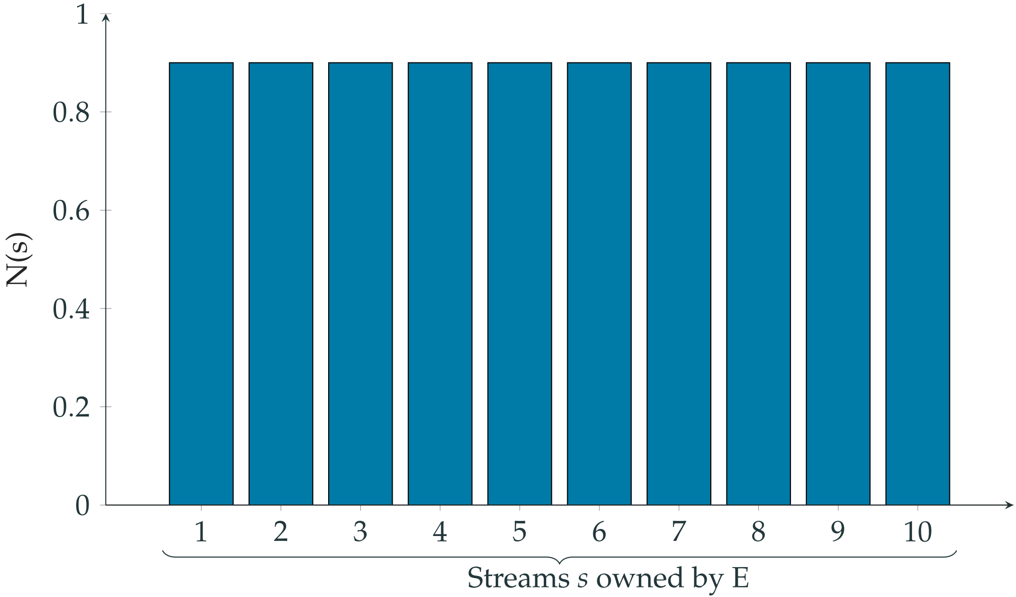
Imitators progressively appropriate streams initially created by E

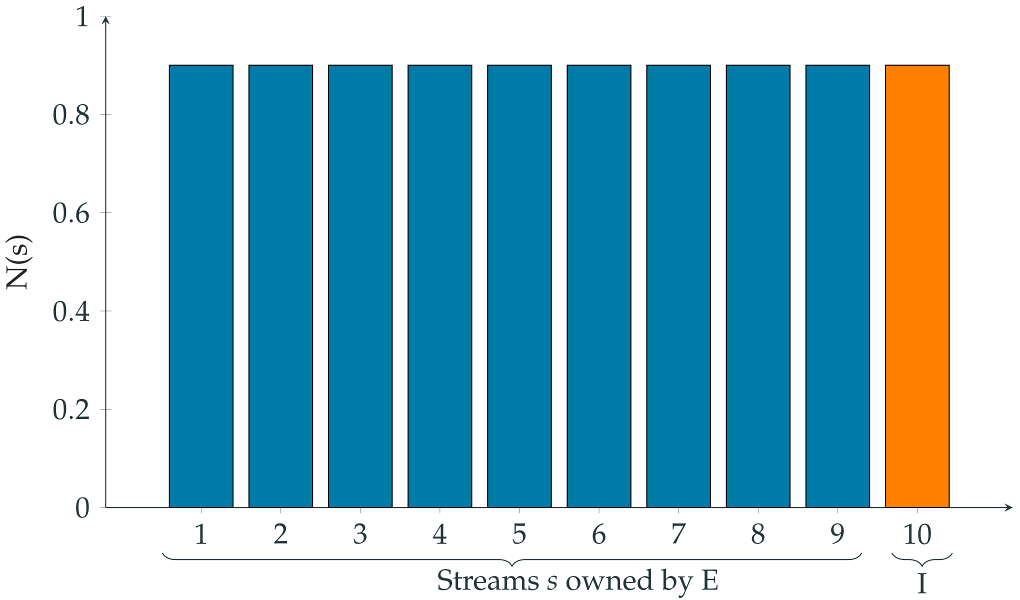
Imperfect excludability and spillovers

Imitators progressively appropriate streams initially created by E

Imperfect excludability: E loses each stream $\tilde{\delta} dt$



N(s)
0
0.2
0.4
0.6
0.8
1
1
2
3
4
5
6
7
8
9
10
Streams *s* owned by E



N(s)
1
0.8
0.6
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0.2
0
1
2
3
4
5
6
7
8
9
10
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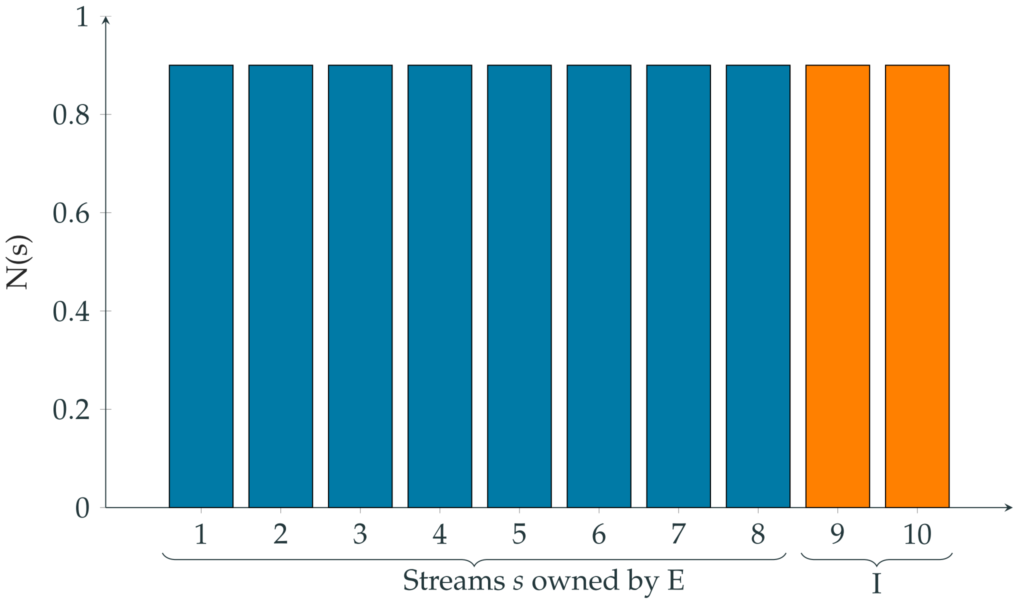
I



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9
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Streams s owned by E
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Imperfect excludability and spillovers

Imitators progressively appropriate streams initially created by E

Imperfect excludability: E loses each stream $\tilde{\delta}\,dt$

$$\begin{aligned} dx_t &= -\tilde{\delta}\,x_t\,dt \\ \Longrightarrow \; dN_t &= -\tilde{\delta}(1-\rho)\,N_t\,dt \end{aligned}$$

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$$
\begin{aligned}
dx_t &= -\tilde{\delta} x_t \, dt \\
\Longrightarrow dN_t &= -\tilde{\delta}(1-\rho) N_t \, dt
\end{aligned}
$$

Spillovers: Spillovers S_t = intangibles in expropriated streams

$$
\text{Initial intangible stock} = N_\tau = \left(N_t^{\frac{1}{1-\rho}} + S_t^{\frac{1}{1-\rho}} \right)^{1-\rho}
$$

ρ determines how fast spillovers accumulate

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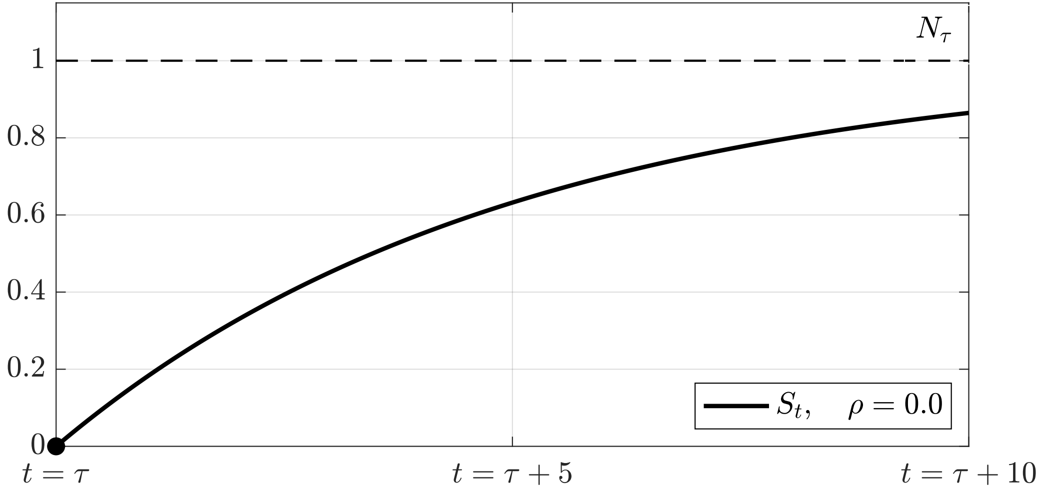
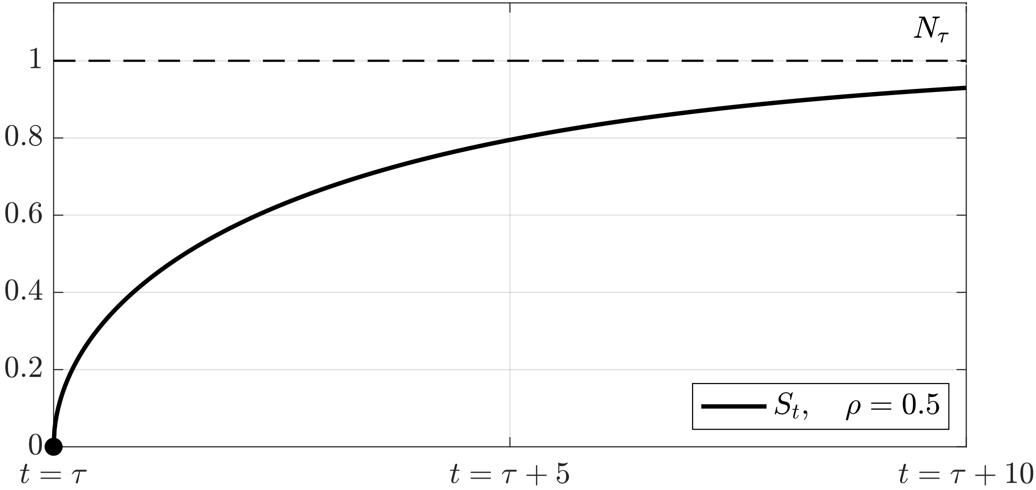


ρ determines how fast spillovers accumulate



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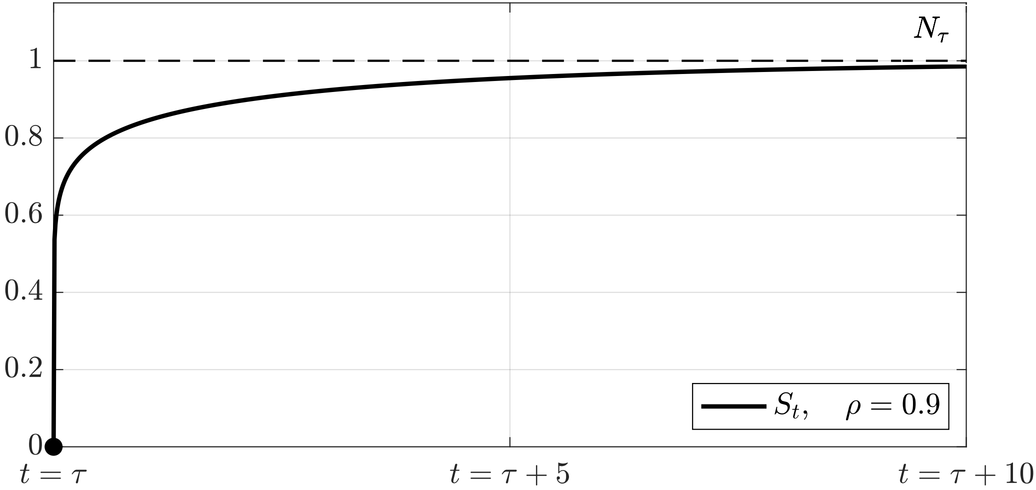

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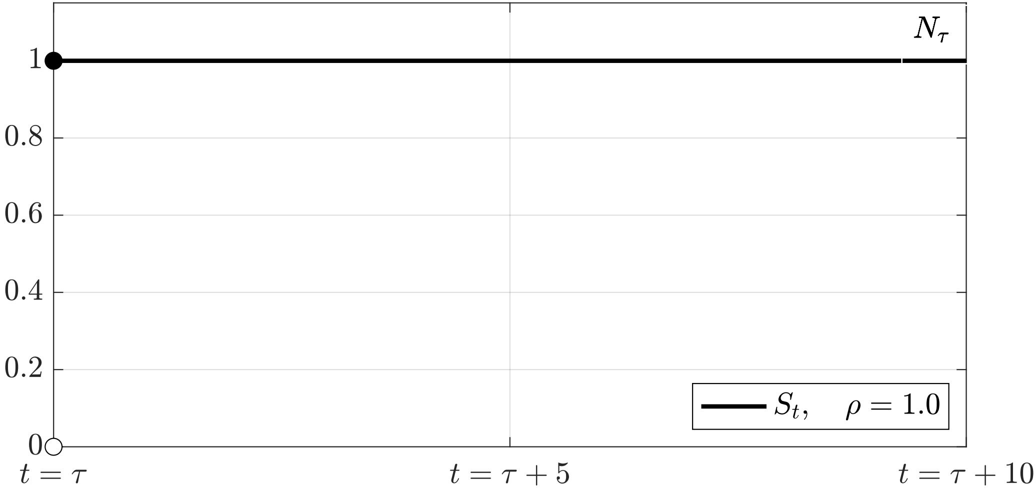

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Labor markets and equilibrium

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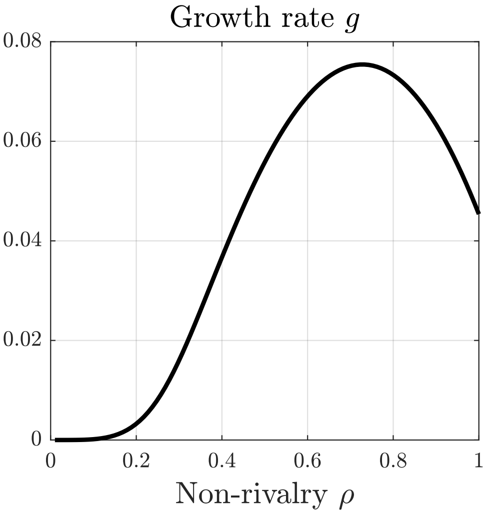
Result 1 (Balanced growth path)

For any $\rho \in [0,1]$, if ν is sufficiently high, there exists a unique equilibrium where $(x_t, L_{e,t})$ are constant and $(\overline{S}_t, N_t)$ grow at the same constant rate g.

2. The Effects of Non-Rivalry

The effects of non-rivalry



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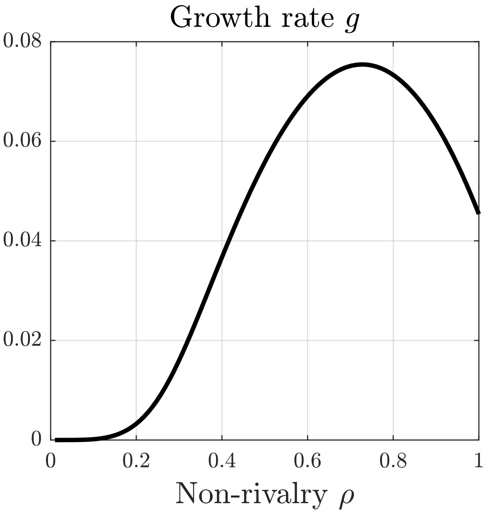

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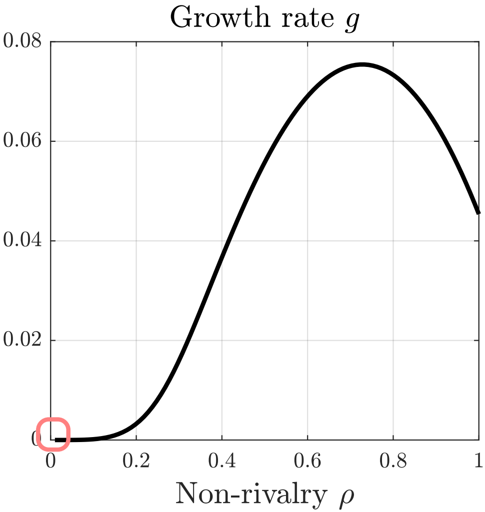

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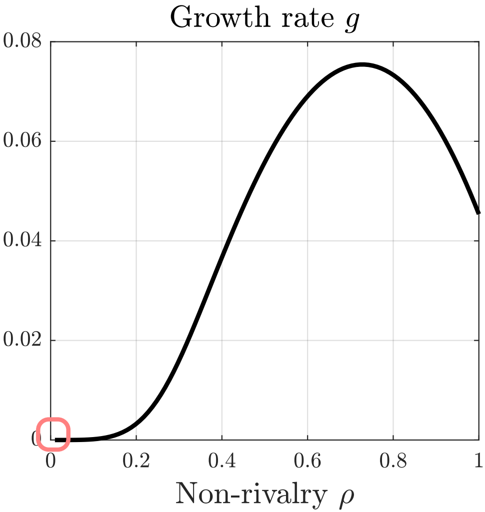

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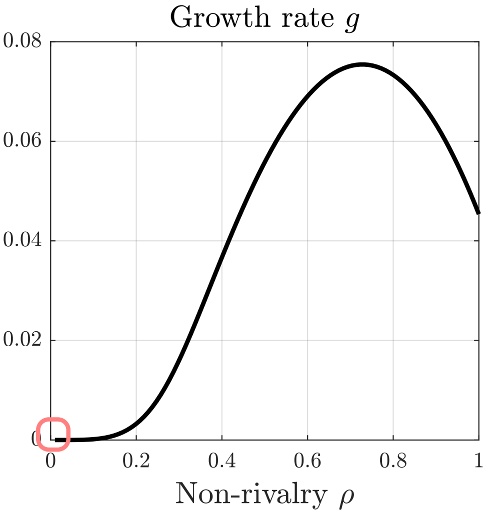

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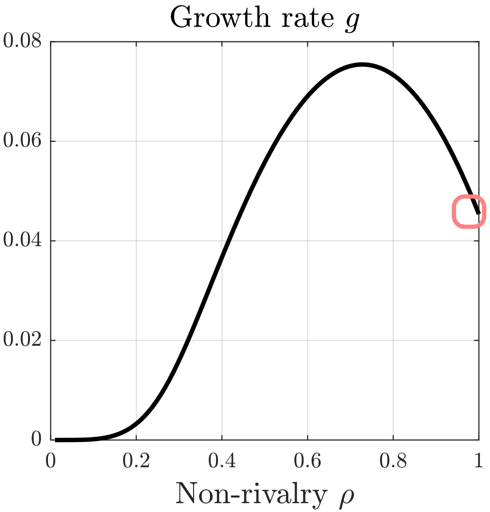

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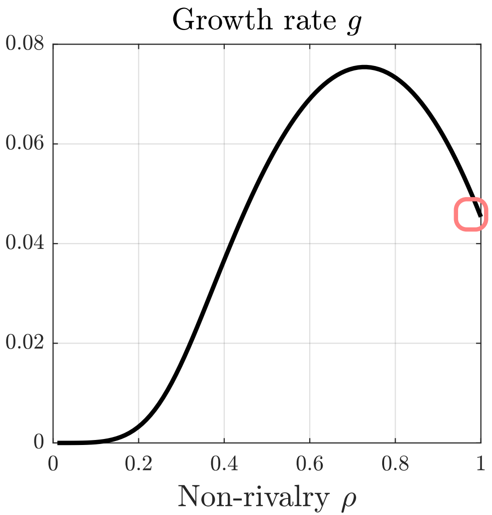

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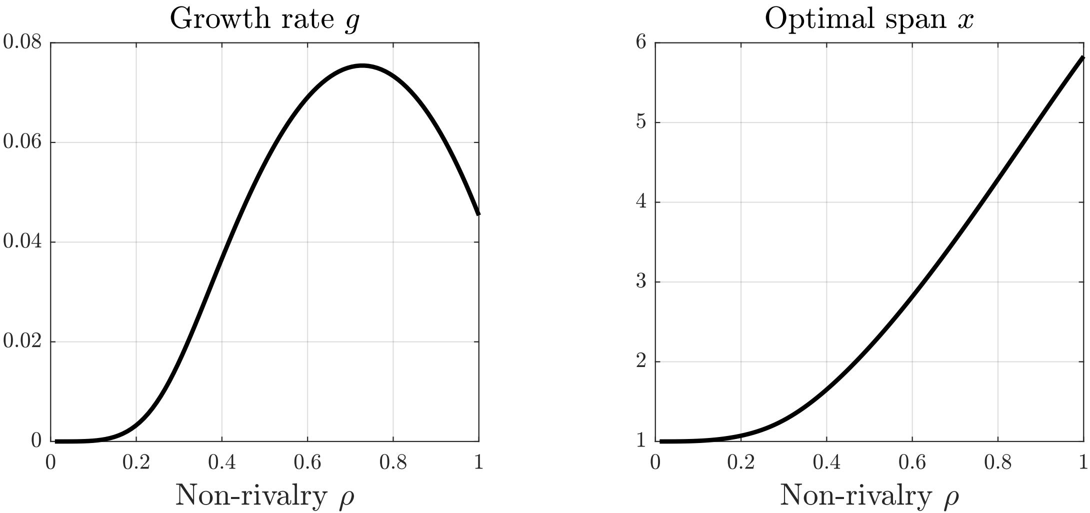

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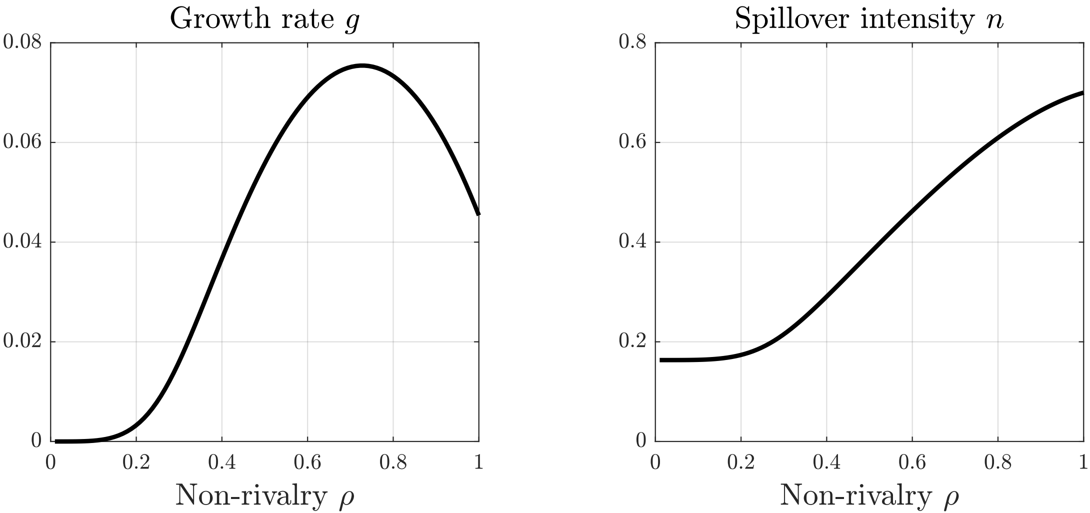

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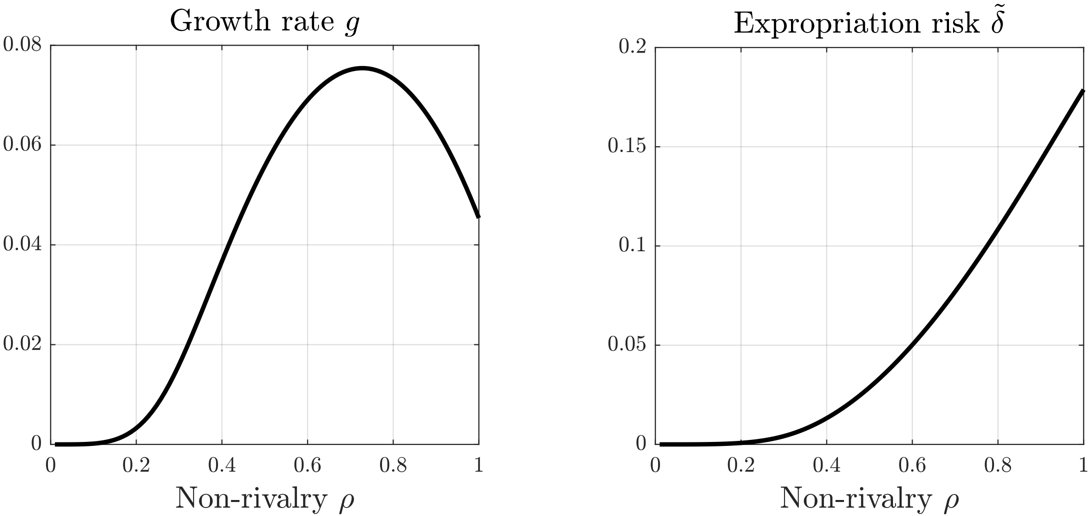

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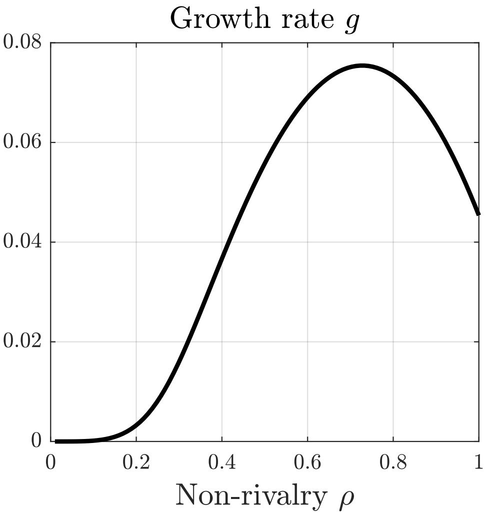

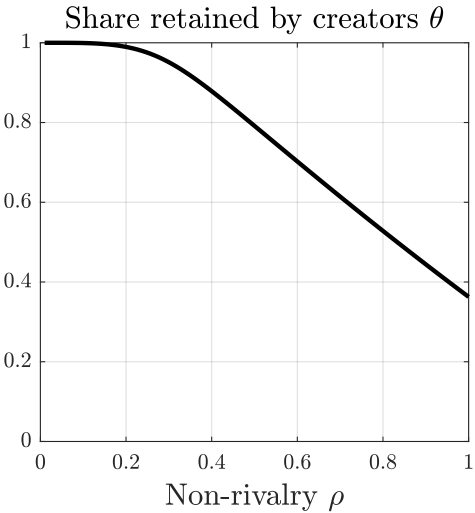

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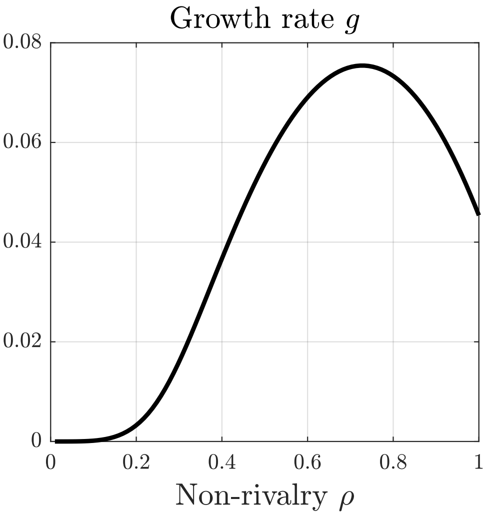

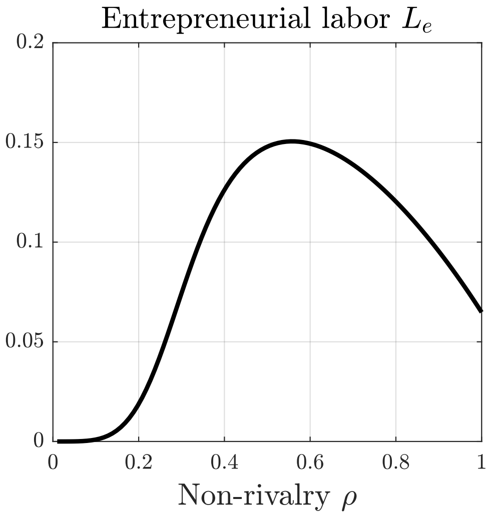

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There exists $\underline{\lambda}$ such that $\forall\, \lambda \geq \underline{\lambda}$, growth is maximized at $\hat{\rho} \in (0,1)$.

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When λ is large enough, spillovers to imitators $\gg$ spillovers to new firms at $\rho = 1$

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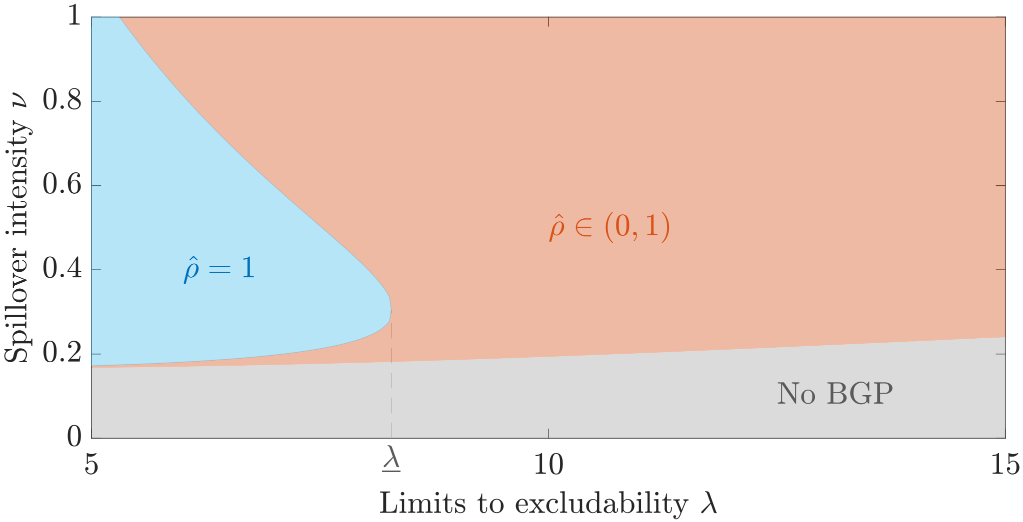

3. Model Implications

Valuations and profits

Valuations

$$V_t = \underbrace{V_t^e}_{\text{creators}} + \underbrace{(1-\theta)V_t}_{\text{imitators}}$$

Valuations and profits

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$$
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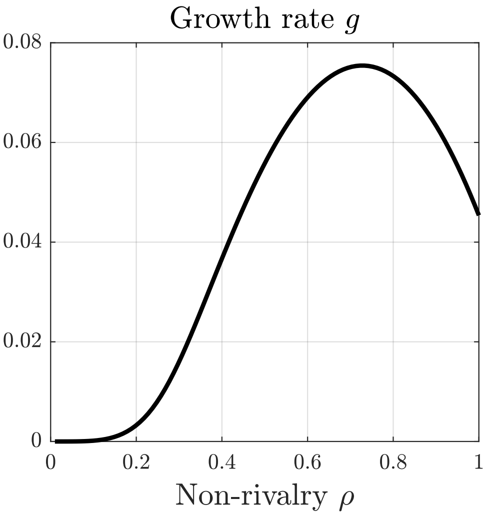

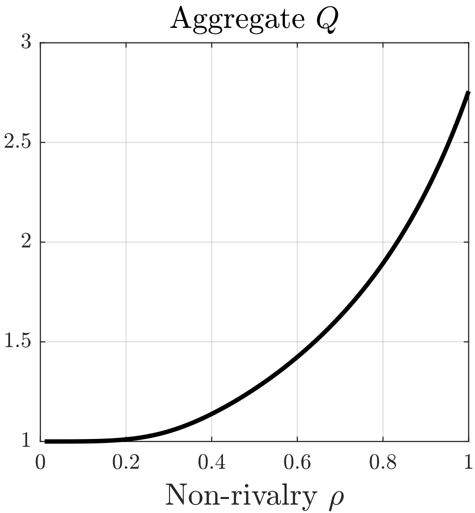

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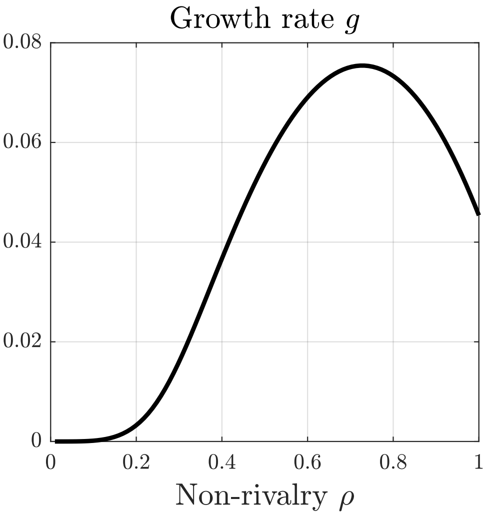

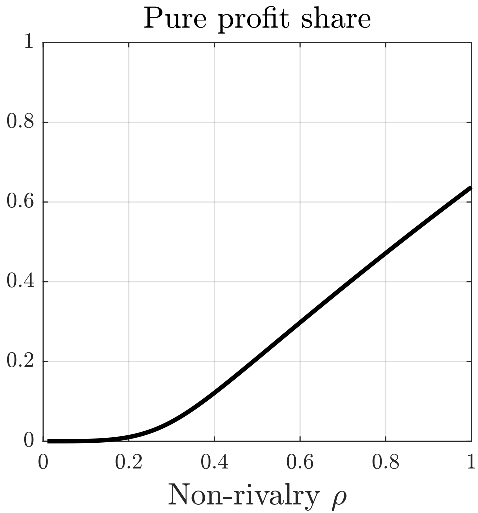

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Sales share for project i

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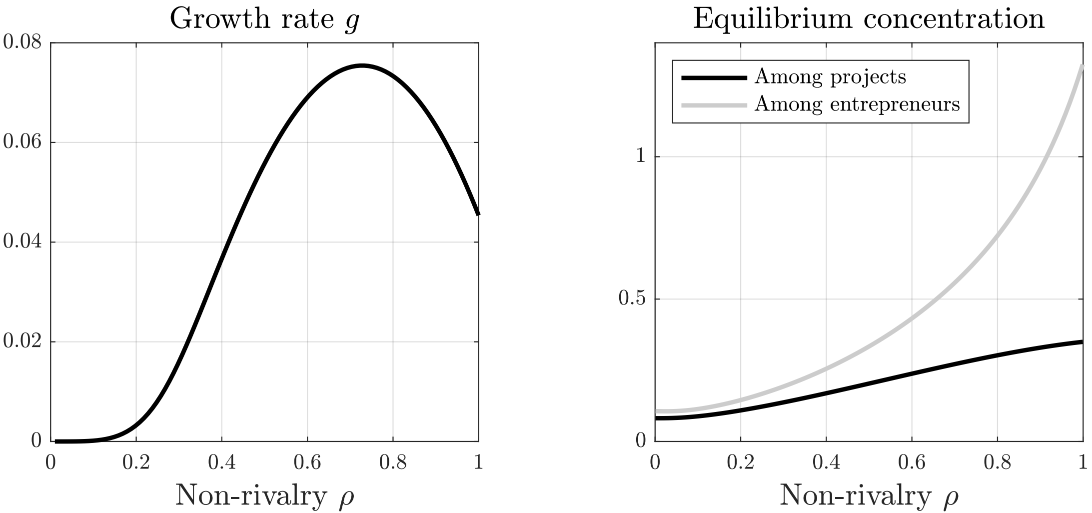

The effects of excludability

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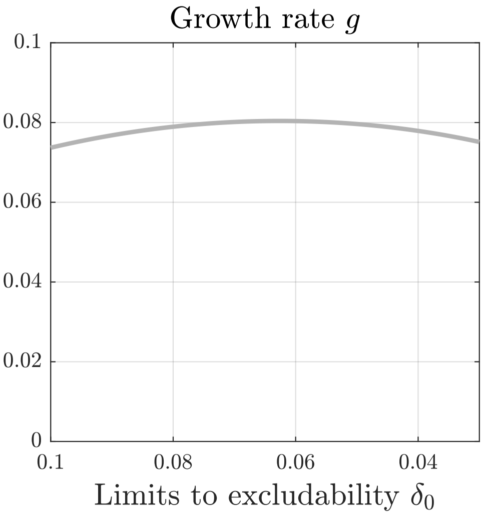

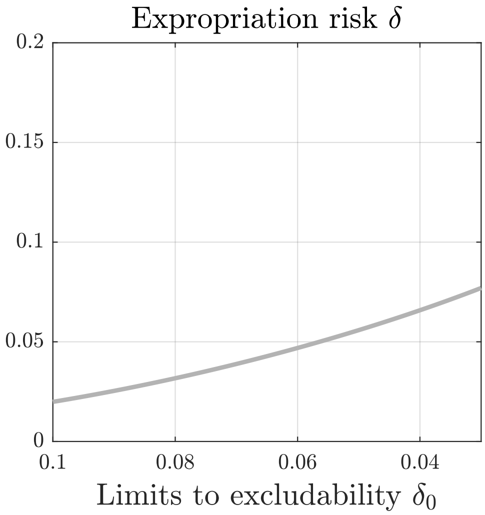

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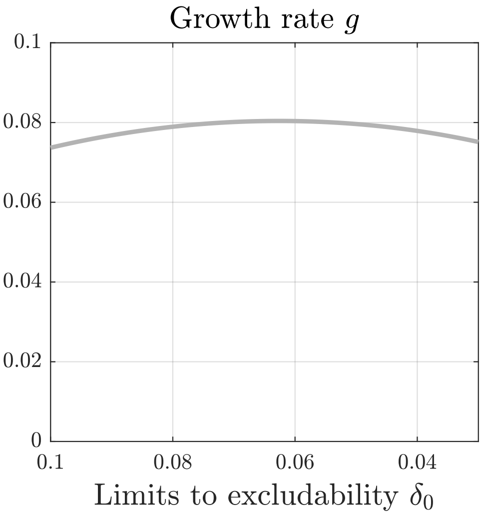

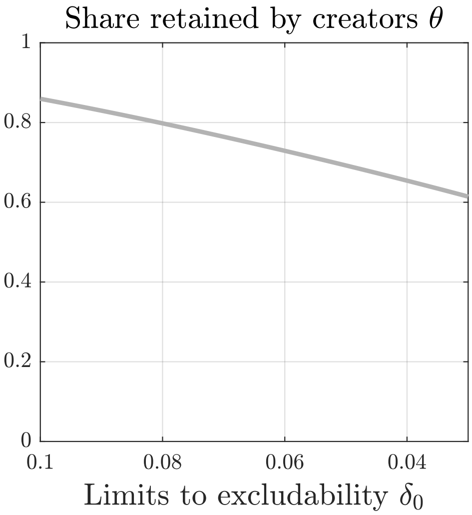

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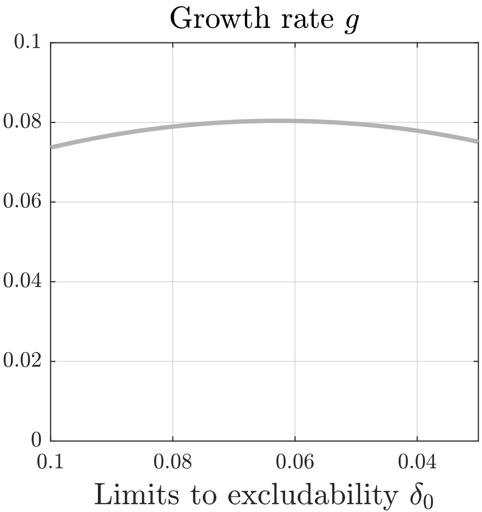

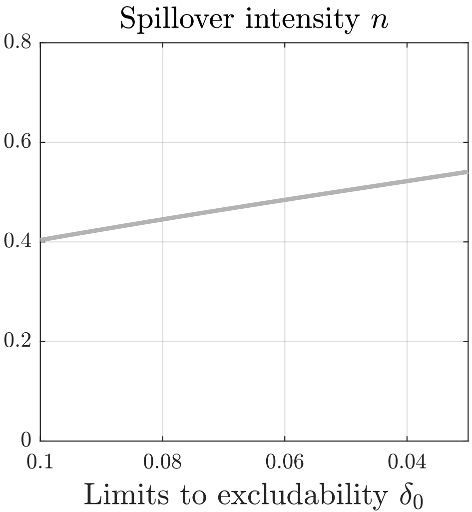

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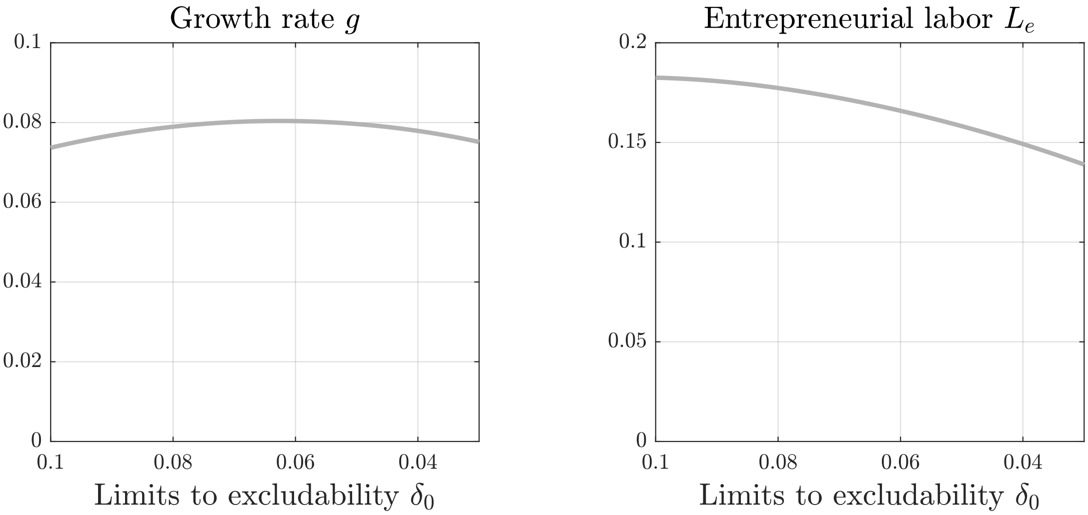

Conclusion

Q: Intangibles can be non-rival within firm. Does that matter for growth?

Scale + spillovers to new firms vs. spillovers to imitators

Non-monotonic relationship btw. ρ and growth

Next:

Transitional dynamics

Estimation of (ρ, δ)