Financing and Valuation of Intangible Assets

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1 Introduction

Firms produce with more than just traditional physical assets. They rely increasingly on intangible assets—i.e., assets without physical presence—such as technological know-how, software and databases, customer relations, or production and distribution processes. In this paper, we review financing arrangements for intangible assets and provide an assessment of the valuation methods in the financing context.

Among intangible assets, some are separable and can be transferred to alternative users on a standalone basis. Examples include licenses, patents, software, and some brands. These separable intangibles can be directly pledged as collateral for borrowing, in a way that resembles pledging traditional physical assets on a standalone basis. In this case, lenders focus on the value of the particular asset pledged. Some intangible assets are not separable from the company, such as organizational capital. These nonseparable intangibles require other contractual designs, either debt contracts against the business as a whole or equity financing. In this case, lenders predominantly focus on the enterprise value of the business, and valuation analyses follow standard methods (e.g., discounted cash flows, multiples).

Valuation for separable intangibles Compared to traditional physical assets, a smaller set of intangible assets are likely to be separable. When they are separable, secondary market trading and readily available prices can be sparse. As a result, when traditional physical assets are pledged directly for borrowing (e.g., equipment, structures, inventories), lenders typically underwrite the debt using estimated resale values based on historical secondary market transactions data (market-based methods). However, when intangibles are pledged directly for borrowing, lenders and appraisers can find it difficult to rely on historical secondary market transactions data alone. Instead, they may also need to consider estimated discounted future benefits the intangible assets can generate for potential buyers (income-based methods), to complement or substitute market-based (or cost-based) methods.

The income-based valuation methods can suffer from several pitfalls. In particular, since many assets

are used jointly for production, it is not easy to isolate the cash flows attributed to a given asset or to impute the cost of complementary inputs. We briefly summarize our assessment of the main types of income-based methods.

- The with-and-without method estimates the difference between the value of the business if it uses the intangible asset of interest and the value of the business if it does not use the intangible asset. From an economics perspective, when this method can be implemented, it is the preferable approach among the income-based valuation methods.

- The relief-from-royalty method estimates the discounted value of the royalty that a company would pay to license the intangible asset. This estimate depends on the royalty or licensing rate the licensor would charge. The upper bound is the total value the intangible asset can generate for the licensee. If the licensee has some bargaining power over the licensor, then the relief-from-royalty method would produce a valuation lower than the incremental value of the intangible asset (the with-and-without valuation). Take together, this method is likely to underestimate the value of the intangible asset.

- The excess earnings method calculates the net present value of the cash flows that the buyer would generate, minus charges for complementary assets. The greenfield method calculates the net present value of the cash flows that the buyer would generate from the business, minus the cost of building or renting all other assets except the intangible asset of interest (the greenfield method is often used to estimate the value of "enabling" intangible assets such as franchise agreements and broadcast spectrum). If the cost of assets other than the intangible are computed using the internal user cost, the rental rate, or the purchasing/reproduction cost, then they can be too low relative to the actual rate of return the buyer can generate with the physical asset alone. This happens when the business can generate economic rents (e.g., due to market power in the output market, market power in the input market, or decreasing returns to scale).[1] Accordingly, these methods attribute some or all of the rents that the buyer would be able to generate even without the intangible asset to the value of intangible asset. As a result, they are likely to overestimate the value of the intangible asset.

We also discuss the choice of discount rates in the valuation process. There is a common intuition that intangibles are "risky" and therefore valuations should use high discount rates. In principle, discount rates should depend on systematic risks (e.g., comovement with the market) not idiosyncratic risks. The systematic risks of intangibles depend on how much the cash flows they generate comove with market fluctuations, which may not necessarily be higher than the overall riskiness of the company. The idiosyncratic risks of intangibles may arise from obsolescence or property right infringement. Such idiosyncratic risks typically should not be priced in discount rates. However, if there is some chance of obsolescence

[1] For a more complete discussion of rents and their relationship to intangible capital, see Crouzet and Eberly (2019) and Crouzet and Eberly (2023).

or infringement and *the cash flows used in income-based methods focus on scenarios where such risks do not materialize*, then one can adjust the discount rate to allow for the probability of such risks. This would be equivalent to using the usual discount rate and adjusting the cash flows to take these risks into account.

Valuation for nonseparable intangibles For nonseparable intangibles that are pledged with the firm as a whole for either debt or equity financing, valuation is a relatively standard task (these financing arrangements assess the enterprise value using commonly used valuation methods). However, these financing arrangements that allow companies to pledge the business as a whole rely on more advanced financial markets and institutional infrastructure. For example, reliable accounting is important for investors' ability to assess the cash flows of the business, and restructuring-oriented bankruptcy is important for debt claims against the business as a whole (so that companies in financial distress can survive as an operating entity instead of being liquidated in which case only separable assets remain).

Finally, we summarize the bottlenecks for the financing and valuation of intangible assets. First, for separable intangibles, the development and enforcement of clear property rights would be useful. Clear property rights will make these assets easier to transfer and easier to pledge directly for borrowing; less uncertainty around ownership can also reduce the challenges for valuation. Second, the development of secondary markets will be helpful for the valuation of separable intangibles. With more data from transactions, it would be useful to compare the transaction prices with the estimated values from income-based methods, which suffer from a number of inherent pitfalls as described above. Third, for nonseparable intangibles, debt financing options can be limited if companies are likely to be liquidated in times of financial distress. Therefore, efficient restructuring in bankruptcy is important. Finally, improvements in accounting that make cash flow information more reliable and transparent will be useful for valuation and financing in general.

The rest of this article is organized as follows. Section 2 defines intangible assets and discusses their economic properties. Section 3 summarizes the financing arrangements of intangible assets. Section 4 evaluates the valuation methods of intangible assets. Section 5 summarizes key aspects of institutional infrastructure that are important for supporting intangible financing.

2 What are intangible assets?

This section begins with the definition of intangible assets. We then discuss the key types of intangibles, and summarize their economic properties.

2.1 Definition

Firms commonly create and maintain resources such as scientific knowledge, production or sales processes, software, databases, product designs, or customers relations. These resources are not entirely embodied in a particular piece of physical asset such as equipment or structure. Accordingly, they are commonly referred to as intangible assets or intangible capital. Both academics and practitioners have long recognized the importance of intangibles for firms.[2] However, the content and scope of the definition of intangibles can vary across different settings.

Economic definition Economists define intangible assets as all non-physical resources that are currently controlled by a firm, and from which the firm expects to draw private economic benefits in the future. Financial assets are typically excluded although they are also non-physical. Additionally, economists generally require the resources to result from past expenditures by the firm, in order to distinguish intangible assets from public capital goods, such as non-patented scientific knowledge or open-source software. This expenditure is often referred to as "intangible investment."

Accounting definition International Accounting Standard (IAS) 38 defines intangible assets as "identifiable, non-monetary assets without a physical substance." The emphasis is on identifiability, with the standard stating that an asset is identifiable in two cases: if it can be divided from its controlling entity and sold, transferred, or licensed; or if it arises from specifically defined contractual and legal rights. Given the emphasis on identifiability, intangible assets are typically recognized on firms' balance sheets if they are acquired from transactions (internally generated intangibles are less easily identifiable and are often not capitalized on firms' balance sheets).

Our focus in this paper is the broader economic definition, not the accounting definition. The economic definition is consistent with the definition of intangible assets set forth in international valuation standards (see International Valuation Standards Council 2022, IVS 210, paragraph 20).

2.2 Commonly recognized types of intangibles

Three sets of resources used by firms are commonly considered in discussions of intangible assets: first, technology-related assets (e.g., patents, databases, or software); second, artistic-related assets (e.g., copyrights of books, music, or film); third, marketing and customer-related assets (e.g., trademarks and brand names, internet domain names, customer lists and customer contracts). Another set of intangible assets relate to contractual rights. Examples include licences, import quotas, franchising rights, mortgage

[2] Dicksee and Tillyard (1906) is an early reference that mentions "intangible property" in the context of accounting statements. For recent work on the importance of intangibles in national accounting data, see Corrado et al. (2009), McGrattan and Prescott (2010), and Crouzet and Eberly (2021). For work using firm-level data, see Eisfeldt and Papanikolaou (2013), Peters and Taylor (2017), Alexander and Eberly (2018), and Crouzet and Eberly (2023).

servicing rights, broadcasting rights, and many other types of usage rights.[3]

The economics definition also encompasses a broader set of intangible assets, often referred to as "organization capital." Examples include firm-specific know-how, such as managerial or organizational principles, production processes, distribution systems, or customer acquisition methods.[4] These resources are costly to develop and they create economic benefits to the firm over extended periods of time, so they meet the economic definition. However, they may not necessarily meet the identifiability requirement in accounting standards, so they are generally not included in intangible assets for accounting purposes (e.g., in financial statements except for the case where they contribute towards goodwill).[5]

2.3 Economic properties of intangible assets

Intangible assets can differ from physical assets along three key dimensions that are relevant for financing and valuation: separability, specificity, and uncertainty.

Separability An asset is separable if it can exist independently from its controlling entity, in which case it can be sold, transferred, or licensed to a third party on a standalone basis. Most types of physical assets are separable. Many types of intangibles are not. Intangibles that are generally considered to have a higher degree of separability include IP assets such as patents and intellectual originals. IP assets are relatively easy to sell or license to alternative users. On the other hand, organization capital (e.g., core competencies developed by a firm) often cannot be separated from the firm.

Specificity Asset specificity refers to the extent to which an asset can generate economic benefits when redeployed to alternative users. For example, intangible assets like broadband spectrum or airline gate rights are relatively less specific (equivalently, more generic): they perform similar functions and generate similar values for a number of users. Intangible assets like technologies might be more specific, especially if they are customized for the original user. Since asset specificity affects the value of the asset to alternative users, it is important for the amount of financing that companies can obtain by pledging assets on a standalone basis.

[3] At the firm level, these licenses require purchases, so they resemble investment. In the aggregate, however, creating these rights does not necessarily require spending resources. Accordingly, they are often included in financial accounting but not in national accounting.

[4] An example of the importance of these assets in the context of the growth and structural transformation of the U.S. retail sector after 1990 is discussed in Foster et al. (2006) and Crouzet and Eberly (2018).

[5] Accounting standards acknowledge that nonseparable intangible assets, such as organization capital, can exist and contribute to firm value. They are generally recognized for accounting purposes as part of goodwill in mergers. The accounting value of goodwill is given by the difference between the value of the target paid by the acquirer and the accounting value of the target's recognizable assets (both physical and separable intangibles); this difference can arise from the value of the target's organizational capital. Organizational capital generated internally (e.g., expenditures leading to the creation of a new distribution system for a wholesaler) is not recognized as an accounting asset under generally accepted accounting principles because it is not easily identifiable and the cost of its creation is hard to measure (see IAS 38, paragraph 49).

Kermani and Ma (2023a) collect information about asset specificity, for physical assets and identifiable intangible assets using data on the net liquidation value (after transaction costs and transportation costs) relative to the book value (i.e., replacement cost). They find that the liquidation value (per dollar of book value) is 0.32 for identifiable intangibles, compared to 0.35 for property, plant, and equipment, but the value for identifiable intangibles has more dispersion. Some identifiable intangibles can be easier to transfer than physical assets because they do not incur transportation costs, but other intangibles may be highly customized. In addition, nonseparable intangibles tend to be highly specific (almost by definition).

Uncertainty Two types of uncertainty stand out for intangibles relative to physical assets: uncertainty regarding ownership and exclusivity, and uncertainty regarding useful life.

Certain types of intangible assets can be used by multiple parties at once, both inside of the controlling entity and outside of it. In particular, usage by outsiders might be difficult to detect or preclude.[6] For instance, a firm may develop software or databases internally, only for its employees to then share these assets with competitors, resulting in a loss of economic benefits to the firm. This form of uncertainty can be mitigated by institutions that protect intellectual property rights. Indeed, a long literature in economics has emphasized that property rights institutions are important for innovation and growth. Separability interacts with this issue: separable assets can be more easily identified and therefore registered, and registration reduces uncertainty around ownership.

Certain types of intangibles also face uncertainty regarding the duration of their useful lives. For example, although IP assets do not suffer from wear and tear (e.g., physical forms of deterioration), they can be superseded by others that perform the same function at lower cost or better quality. This uncertainty implies that the useful life of the asset may be substantially shorter than its legal life. Computer software, artistic originals, and patents provide examples of intangible assets for which this could be the case.[7]

Relation to financing and valuation Separability, specificity, and uncertainty all have an impact on financing and valuation methods, which we summarize below. Sections 3 and 4 provide more detail.

Separability is particularly important because it determines whether financing for the asset on a standalone basis is feasible. In particular, separability is the prerequisite for transferring the asset to alternative users on its own. In this case, investors can provide financing on the merit of the particular

[6] Crouzet et al. (2022b) discuss these two related properties in more detail, namely (1) their non-rival nature, and (2) the difficulty to exclude other firms from using them. Non-rivalry refers to the idea that the same intangible can have multiple users at once. This can affect returns to scale within firm. However, if it is difficult to exclude other firms from using the asset, non-rivalry can also benefit potential competitors. Both of these effects are relevant in the context of asset valuation and financing to the extent that they affect future cash flows as well as the potential useful life of the intangible. Crouzet et al. (2022a) explores the implications of these properties for long-term growth.

[7] An influential example was the case Tennesse Fibre Co v. C.I.R. The patent involved in the case protected a process to separate cotton fiber from its seeds. This process became very useful for producing gun cotton, which was sold to munitions manufacturers during the 1910s. However, after the end of the war in 1918, demand for gun cotton collapsed, making the patent worthless, and prompting its owner to file for a tax deduction for obsolescence; see Sumer (2020).

asset (without having to screen or closely monitor the quality of the company's business) and can resort to the standalone value of the asset even if the company no longer exists. Accordingly, separable intangibles can be directly pledged for borrowing. On the other hand, nonseparable assets need to rely on financing arrangements that pledge the business as a whole. This can be done via equity financing, or via debt financing in some countries with the necessary institutional infrastructure.

Relatedly, the specificity of an asset affects how much financing can be obtained if it is pledged directly (Kermani and Ma, 2023b). Generic assets (e.g., aircraft as a prototypical example) are most often pledged on a standalone basis. Specialized assets, meanwhile, are not very valuable on a standalone basis and cannot raise much financing if pledged directly. Therefore, they are often pledged as part of the business as a whole even if they are in principle separable.

Uncertainty around ownership and economic life may also impact the value that a prospective investor can attach to the future economic benefits that the asset might generate. As we discuss more in Section 4, the valuation of intangibles on a standalone basis often relies on estimating the future benefits these assets can generate. Uncertainty about these benefits can play an important role in the valuation analysis.

3 The financing of intangible assets

This section discusses several approaches for financing intangibles. We organize the discussion around separable versus nonseparables intangibles, given they typically rely on different financing arrangements.

3.1 Separable intangibles

Separable intangibles can be defined, valued, and pledged on a standalone basis. This makes it possible to pledge them directly for borrowing, in a way that resembles borrowing against physical assets. It is also possible to arrange sale-leaseback.

Directly pledging separable intangibles for borrowing Several types of separable intangibles have been used as collateral for asset-based lending, where companies pledge separable assets directly based on their appraised liquidation values. Examples include trademarks and brands, domain names, and data. Lenders obtain a lien against the particular assets pledged, and have a secured (i.e., senior) claim up to the value of these assets in the event of default.[8] In this case, lenders conduct appraisals of the assets that are being pledged. Valuation by lenders could be based on orderly liquidation value (OLV) from simulations of liquidation scenarios (taking into account the possible primary buyers and secondary buyers) using past transactions data, or estimates from discounted cash flows analyses. The valuation then determines the

[8] In default, if the value of the assets pledged is less than the debt amount, then the difference becomes a general unsecured claim against the firm.

borrowing capacity.

How much companies can borrow against separable intangibles depends on how generic the assets are (the same holds for borrowing against separable physical assets). Kermani and Ma (2023a) find that the orderly liquidation value for separable intangibles is 32% of book value on average, compared to 35% for property, plant, and equipment (PPE). PPE can have low liquidation values because they are specialized or costly to transport. Separable intangibles can avoid transportation costs, given their lack of physical presence by definition. This can reduce both the direct transportation costs and possibly allow them to access a wider range of buyers. The pledgeability of separable intangibles on a standalone basis also depends on the development of secondary markets for trading and transferring these assets. Such markets may continue to grow and improve in the future.

A number of companies have pledged IP assets for asset-based lending. One example is Levi Strauss, which entered into a $850 million asset-based facility in 2011 where the borrowing availability is based on the value of the U.S. Levi's trademarks as well as accounts receivable and inventory. Similarly, Chiquita Brands International also reports using a substantial amount of its intellectual property as collateral for asset-based lending facilities.

Securitization Intangibles can also be used for securitization, especially for those assets that generate well-defined income streams such as royalties. In the case of securitization backed by royalties, the asset is essentially the right to receive royalties from licensees, and the right is often transferred to a bankruptcy-remote special purpose vehicle (Madden and Rungpry, 2007). As an example, DreamWorks set up securitization backed by copyright license receivables on the company's films. Securitization against patent royalties is also used in the pharmaceutical industry. In addition to copyright and patent royalties, securitization of trademark royalties is also used, but it can face more complications in terms of treatment in bankruptcy.

Sale-leaseback Sale-leaseback for separable intangibles is conceptually similar to sale-leaseback for tangible assets. The seller pays a lease, license, or fee to use the asset. The sale-leaseback may have a purchase option allowing the seller to buy back the ownership during or at the end of the contract period. The repurchase option may have a fixed price, or it may require a fair market value appraisal at the time of repurchase. Some examples of sale-leaseback transactions include companies selling their domain names and leasing back the right to use the domain name.

Component in blanket liens Separable intangibles can also be part of a blanket lien that encompasses a wide collection of assets. Patents and other types of intellectual properties are often included in blanket liens. In the case of blanket liens, the assets pledged are not valued separately; the collateral value is given by the value of the company as a whole, including the value of synergies and nonseparable intangibles, which we discuss in more detail below. One notable issue for blanket lien lenders is that they need to make

sure the loan document does not have loopholes for borrowers to transfer collateral away (such as in the J. Crew case where the borrower transferred intellectual property assets away from the term loan lenders).

3.2 Nonseparable intangibles

Although nonseparable intangibles cannot be easily defined, valued, or pledged on a standalone basis, companies can raise financing against the value of the firm as a whole, which includes the contribution of nonseparable intangibles.

Cash flow-based debt For debt financing, in countries like the U.S., companies can borrow based on the value of their cash flows from business operations. Lenders often refer to such borrowing as "cash flow-based debt." In this case, debt capacity is typically a function of firms' EBITDA (earnings before interest, taxes, depreciation, and amortization). Nonseparable intangibles such as organizational capital can contribute to firms' cash flows, and in turn their debt capacity. In the U.S. , lenders of cash flow-based debt have claims against the value of the company as a whole in the event of bankruptcy, minus the collateral value of assets separately pledged. Cash flow-based lenders can have either secured (i.e., senior) claims against the firm value through blanket liens, or unsecured claims.[9] Lian and Ma (2021) find that cash flow-based debt accounts for around 80% of U.S. nonfinancial corporate debt outstanding.

Equity Equity holders also derive value from cash flows generated by firms' operations, which can benefit from nonseparable intangibles. In particular, cash flow-based debt is more common for companies that already generate high and stable cash flows, which are verifiable to lenders. Equity, on the other hand, is appealing to companies that do not yet generate high or stable cash flows in the near term, but may have high growth potential in the future.

3.3 Special cases

There are some cases where the assets that companies use for financing straddle between separable and nonseparable assets. For example, in 2020, airlines like United and Delta took out loans against their frequent flyer programs. In principle, frequent flyer programs do not have value if separated from the operations of an airline—they cannot be seized or transferred on a standalone basis. Meanwhile, there are ways to assign value to them according to the cash flows they generate (e.g., often times in connection with credit card programs). Therefore, frequent flyer programs are in between nonsperable and separable assets:

[9] In Chapter 11 restructuring, the total payment to claimants are given by the estimated going-concern value of the company, which is often calculated with DCF using cash flow projections or with EBITDA multiples. In Chapter 7 liquidation (which is not common for large firms but more common for small firms), the total payment to claimants are given by the total liquidation value of all assets. Cash flow-based lenders would end up getting paid by the total liquidation value minus the liquidation value of assets separately pledged (again the secured cash flow-based debt would have priority ahead of unsecured debt).

operationally they are not separable from the company, but valuation-wise there is some separability.

In the U.S., creditors do not necessarily count on asset seizure; indeed, the automatic stay provision prohibits asset seizure to prevent disruptions to orderly restructuring. By taking security in the frequent flyer programs, lenders are not looking to seize these programs and liquidate them if the airlines default. Instead, taking security gives them high priority claims up to the value of the frequent flyer programs. As a result, separability in terms of valuation is sufficient for this purpose, whereas separability in terms of operations is not necessary in this case.

In these cases that feature separability in terms of valuation but not necessarily separability in operations, valuation is typically based on cash flows generated in *current use*. In comparison, for separable intangibles that are pledged on a standalone basis for asset-based lending, valuation is typically from the perspective of *alternative buyers* (e.g., liquidation value if the intangible is transferred to other users) and based on estimated cash flows that can be generated by the buyers as we discuss in the next section.

4 The valuation of intangible assets

This section discusses the different methods used to value intangible assets, with a focus on valuation for the purpose of financing activities.

4.1 Overview

When separable intangibles are pledged on a standalone basis, the objective is to assess the value of the asset to alternative buyers if it is sold by itself. There are three types of approaches for valuing separable assets: market-based methods, cost-based methods, and income-based methods.

Market-based methods rely on data about past transactions involving comparable assets. Cost-based methods start with the book value, which reflects expenditures on obtaining the asset (net of depreciation) and can indicate the quantity of the asset; adjustments are then made (e.g., use a given fraction of the book value) to reflect the value to alternative users. Finally, income-based methods estimate the expected value the asset can create for buyers, typically by discounting the expected future income they would generate with the asset.

Among these three types of approaches for the valuation of separable intangibles, market-based methods would be ideal if sufficient information on comparable intangible assets is available. However, transactions data for comparable assets can be sparse or proprietary. In addition, as International Valuation Standards Council (2022) points out, the heterogeneous nature of intangible assets and the fact that intangible assets seldom transact separately from other assets means that it is often difficult to find market evidence of transactions involving identical assets.

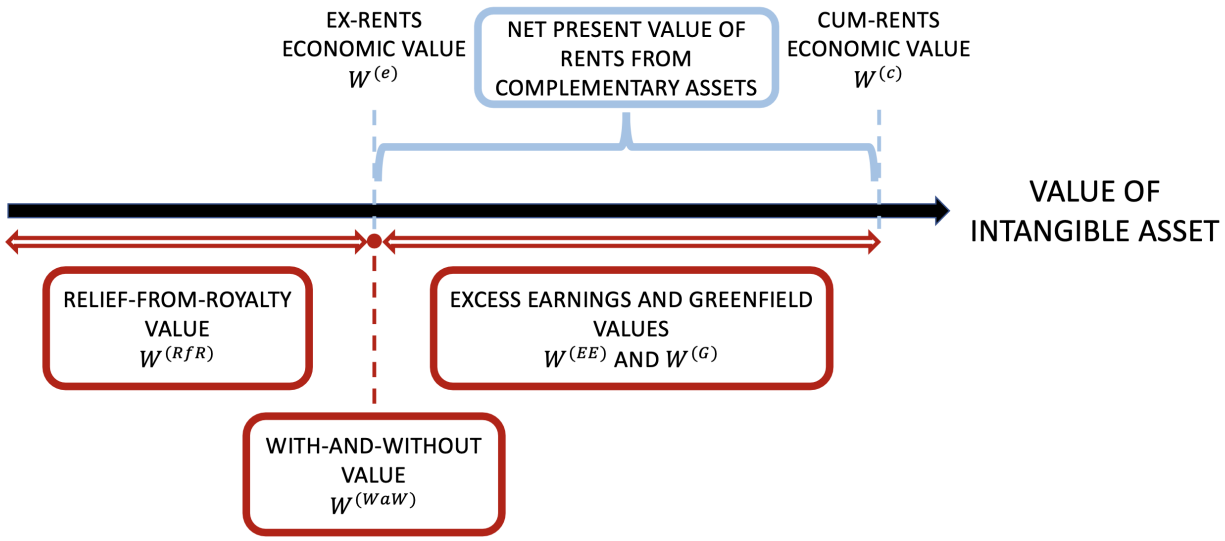

Figure 1: Summary of income-based valuation methods

Notes: This figure shows the relationship between the economic value of intangibles and the valuations produced by the four main income-based methods: relief-from-royalty, with-and-without, excess earnings, and greenfield. The rents mentioned in the figure are those that the buyer could generate even if they were to operate using only the complementary assets.

Cost-based methods can also be challenging because, for intangible assets, the relationship between investment expenditures and the economic value to alternative users can be highly uncertain.[10] Even when there is a predictable relationship between investment expenditures and economic value, it may be difficult to isolate these investment expenditures from accounting data, since not all intangible investment is reported in isolation by firms. As a result, cost-based methods may also face practical implementation issues around data availability.

Income-based methods can be used so long as information on prospective cash flows and useful life of the intangible asset are available. In situations where data availability on comparable transactions or past expenditures is limited, income-based methods can become the primary source for estimates of intangible value. However, there are several issues to keep in mind when applying income-based methods, as we explain in what follows.

When intangibles are pledged together with the business as a whole rather than separately, the focus is then to estimate the enterprise value of the business. The enterprise value calculations follow standard methods, such as discounted cash flows, multiples, or comparables analyses. Accordingly, we do not cover them in detail.

[10] For instance, the cost of research and development for new drugs may not be directly proportional to the economic value of the patent they create. The same expenditure may in some instances lead to a blockbuster drug, and in others to a compound that fails FDA approval or a product that quickly becomes obsolete.

4.2 Income-based valuation methods

We discuss the four main income-based valuation methods: the with-and-without (WaW) method, the relief-from-royalty (RfR), the excess earnings (EE) method and the greenfield (G) method.[11] We use a simple economic model to organize our discussion. The model aims to set the benchmark for defining the economic value of an intangible asset, and to illustrate the relationship among the different methods. We present the details of the model in Appendix A.[12]

The main findings are summarized in Figure 1. Within the context of the environment we study, the four methods will produce valuations that are ordered as:

$$W^{(RfR)} \leq W^{(WaW)} \leq W^{(G)}, W^{(EE)}. \tag{1}$$

The model also points out the feature of the economic environment that has a critical impact on valuations, namely the rents (i.e., earnings in excess of the competitive costs of complementary assets) the buyers can generate in the *absence* of the intangible asset. If such rents do not exist, then all four methods coincide, and moreover, they coincide with the natural (and unique definition) of the economic value of the intangible asset. However, if the buyer can earn rents even without the intangible asset, then the four methods do not coincide. In particular, the greenfield and excess earnings methods will tend to produce higher valuations, unless capital charges or rental rates appropriately take the rents into account.

Crouzet and Eberly (2023) study replacement costs of physical and intangible capital, the rents which each type of capital are expected to generate, and the contribution of each to total enterprise value, using both national accounts and firm financial statements. They find that after the early 2000s, rents generated by either physical or intangible assets account for more than 60% of the enterprise value of nonfinancial firms. Moreover, rents generated by physical assets still account for approximately two thirds of the difference between the enterprise value over the replacement cost.[13] This evidence suggests that the contribution of rents to valuations, including rents generated by traditional assets, remains substantial. Thus in practice, biases in valuations arising from rents from complementary assets are likely to be important.

4.2.1 Description of the economic environment

The four income-based valuation methods attempt to answer the same question: how much would the potential buyer of an intangible asset be willing to pay for that asset?

We consider this question in the context of a situation where the potential buyer is a firm which could

[11] See International Valuation Standards Council (2022) for more details on these four valuation methods.

[12] The Appendix also provides various generalizations of the results below, in particular to cases where cash flows are not constant and where there are other sources of uncertainty than the useful life of the intangible asset.

[13] However, in certain sectors, such as healthcare and tech, intangibles have become the dominant source of rents.

Notation	Meaning
$W^{(c)}$	Enterprise value including rents attributable to complementary assets
$W^{(e)}$	Enterprise value excluding rents attributable to complementary assets
$W^{(RfR)}$	Valuation using relief-from-royalty method
$W^{(WaW)}$	Valuation using with-and-without method
$W^{(G)}$	Valuation using greenfield method
$W^{(EE)}$	Valuation using excess earnings method
$\Pi^{(N)}$	EBITDA per period (e.g., year) from combining the intangible asset with complementary assets
$\Pi^{(0)}$	EBITDA per period from operating complementary assets alone
k	Replacement cost of complementary assets
δ	Rate of depreciation per period (e.g., year) of complementary assets
r	Buyer's discount rate per period (e.g., annual)
λ	Per period probability of termination of useful life of intangible asset ($\frac{1}{\lambda}$: expected useful life, in # of periods)
γ	Royalty rate in relief-from-royalty method
$R^{(EE)}$	Charge rate for complementary assets in excess earnings method
$R^{(G)}$	Charge rate for complementary assets in greenfield method

Table 1: List of mathematical symbols used in the model of Section 4.2 and their definition.

generate earnings before interest, deduction and amortization (EBITDA) of $\Pi^{(N)}$ per period (say, per year) if it were to combine the intangible asset with some fixed amount of complementary assets, which we denote by K, and $\Pi^{(0)}$ if it were to use *only* the complementary assets. In many situations, the complementary assets could be physical capital, but they could also include other intangible assets that are not the subject of the transaction.[14] We assume that $\Pi^{(N)} > \Pi^{(0)}$. Otherwise the buyer would not be interested in the transaction.

The complementary assets depreciate at a constant rate δ per period (e.g., due to wear and tear). To maintain a constant stock of complementary assets (as we have assumed), the buyer must incur δK in capital expenditures per period. We assume that the intangible asset does not suffer from wear-and-tear. However, its useful economic life is uncertain. We summarize this uncertainty with the parameter $\lambda > 0$, which is the per-period probability that the intangible stops producing economic benefits for the buyer (e.g., due to obsolescence or infringement). Once the intangible stops producing economic benefits to the buyer, cash flows becomes $\Pi^{(0)}$.

Finally, the buyer is risk neutral and uses a discount rate of r per period to value future cash flows. The rate r can be interpreted as the buyer's opportunity cost, minimum internal rate of return, or hurdle rate.

Note that the parameters of this model, $(\Pi^{(N)}, \Pi^{(0)}, K, \delta, \lambda, r)$, are buyer-specific. They reflect the buyer's use of the intangible and of the complementary assets, the uncertainty faced by the buyer regarding the useful life of the intangible asset, and so on. Different buyer characteristics would be expressed as different values for these parameters, and would lead to different valuations.[15] Table 1 contains a detailed

[14] To streamline the discussion we consider the case of a single complementary asset, but the statements we make below could be interpreted as referring to all complementary inputs (physical capital and otherwise) that the buyer combines with intangibles to generate EBITDA.

[15] We do not explicitly include tax considerations in our discussion, but these could generally be captured by appropriate

list of the parameters and economic variables in this environment, along with the mathematical symbols we use to represent them.

4.2.2 Standalone economic value of the intangible asset

In the context of this model, two definitions of the standalone economic value of the intangible asset (which we refer to as the intangible value, for short, in what follows) can be proposed.

These definitions provide benchmarks for assessing the four income-based valuation methods. Thus, we summarize these benchmarks here to set the stage for analyzing the income-based valuation methods afterwards. As we show later, some valuation methods may generate results that differ from both of these benchmarks, but can be bounded by them.

First, the intangible value can be defined as the difference between (a) the enterprise value the buyer would generate by combining the intangible and the complementary assets; and (b) the enterprise value the buyer would generate by operating only the complementary assets. This difference is given by:

$$W^{(e)} = \frac{1}{r+\lambda}\left(\Pi^{(N)} - \Pi^{(0)}\right). \tag{2}$$

Second, it can be defined as the difference between (a) the enterprise value the buyer would generate by combining the intangible and the complementary assets; and (b) the *replacement cost* of the stock of complementary assets. In our simple example we assume that this replacement cost is simply equal to K.[16] Appendix A shows that this difference is given by:

$$W^{(c)} = W^{(e)} + \frac{1}{r}\left(\Pi^{(0)} - (r+\delta)K\right). \tag{3}$$

Definition (2) focuses on the incremental enterprise value associated with the intangible, for a buyer that already operates K units of the complementary asset. For instance, a business services firm might purchase customer relationship management software to increase EBITDA from existing relationships, and to create new relationships. In this case, the complementary assets (the firm's existing complementary assets and customer relations) are already in place.

Definition (3) can instead be thought of as describing situations where the buyer must purchase complementary assets along with the intangible. For instance, the purchase of a patent may require the buyer to add new specialized capital equipment or build new facilities. The purchase of franchise rights requires the buyer to obtain others assets to implement production.

modifications to $(\Pi^{(N)}, \Pi^{(0)}, \delta, \lambda, r)$.

[16] This simplification is without loss of generality, though the general case in which the complementary asset is not the numeraire and there may be costs of adjusting the stock requires more complex notation.

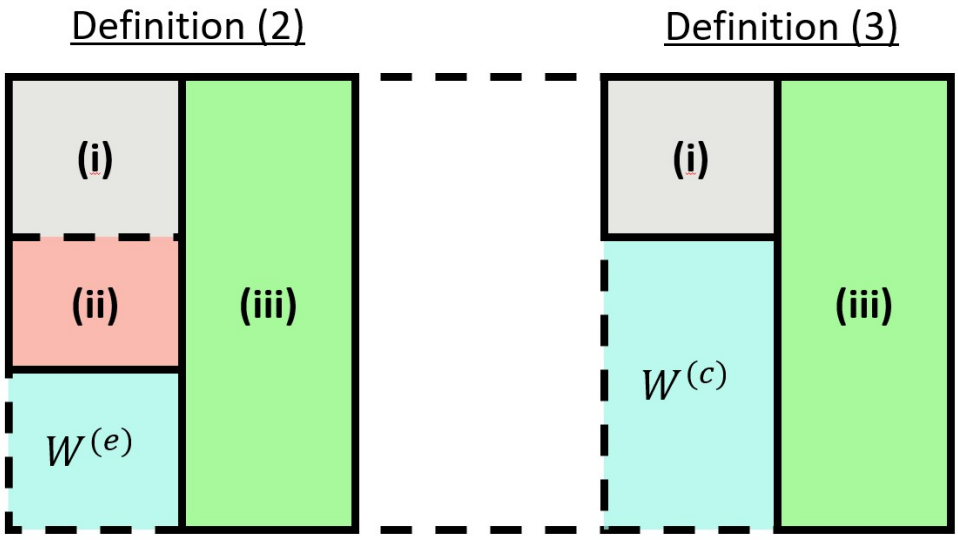

Figure 2: Ex-rents value $W^{(e)}$ (definition (2)) and cum-rents value $W^{(c)}$ (definition (3)) of intangible asset

Notes: This figure illustrates the definitions of the cum- and ex-rents value of the intangible asset, $W^{(e)}$ and $W^{(c)}$. In both panels, the green area marked (iii) is the buyer's enterprise value from combining the intangible asset with the complementary assets. The grey area marked (i) is the replacement cost of the complementary assets, given by K in the model. The red area marked (ii) is the present value of rents that the buyer would generate by operating the complementary assets on a standalone basis, given by $(1/r)(\Pi^{(0)} - (r+\delta)K)$ in the model.

Comparing the two definitions, we see that the second term to the right of Equation (3) is the present value of the difference between the flow profits generated by the buyer *without* the intangible, $\Pi^{(0)}$, and a capital cost of $(r+\delta)$ per unit of the complementary asset, which the economics literature refers to as the "user cost" of capital. This user cost includes the opportunity cost r and the wear-and-tear cost δ. It represents the lowest rate of return on *complementary* assets the buyer would accept; otherwise, the buyer would be better off not using the complementary assets at all.[17] Thus, it is natural to restrict attention to cases where:

$$\Pi^{(0)} \geq (r+\delta)K. \tag{4}$$

In those cases, we have:

$$W^{(c)} \geq W^{(e)}, \quad \text{with equality, if and only if} \quad \Pi^{(0)} = (r+\delta)K. \tag{5}$$

In other words, definition (2) leads to estimates of the value of intangibles that are *lower than* definition (3). Moreover, the difference between the two definitions reflects cash flows that the buyer would earn in excess of their internal user cost of capital when they *only* use the complementary assets.

In many economic models, the internal user cost is also equal to the competitive cost of capital. As

[17] Note that this is a gross rate of return, as it includes a depreciation charge.

a result, situations where $\Pi^{(0)} > (r+\delta)K$ correspond to the buyer earning rents, namely cash flows in excess of the competitive input costs. Rents could reflect different underlying economic mechanisms, such as market power in the output market, market power in the input market, or decreasing returns to scale. The valuation of the intangible in Equation (3) *includes* these rents — even if they accrue to the buyer without the intangible asset —whereas the valuation in Equation (2) does not. Therefore, in what follows, we refer to the valuation in Equation (2) as the "ex-rents" economic value (hence the subscript e), and the valuation in Equation (3) as the "cum-rents" economic value (hence the subscript c).

Figure 2 illustrates the two definitions of the intangible asset value, and how they relate to one another. As can be seen in that figure, the difference between the two values is equal to the area highlighted in red, which is the present value of future rents that the buyer would generate from operating complementary assets on a standalone basis.

4.2.3 Value of the intangible asset according to four income-based methods

Next, we apply the four income-based methods to the valuation problem above, and compare the result to the two potential definitions of the intangible value, $W^{(e)}$ and $W^{(c)}$.

The with-and-without method The with-and-without method calculates the present value of the difference between the flow profits the buyer generates when combining the intangible asset with the complementary asset, and the flow profits when using only the complementary assets.[18] By definition, this method produces a valuation that is equal to the ex-rents intangible value in Equation (2):

$$W^{(WaW)} = \frac{1}{r+\lambda}\left(\Pi^{(N)} - \Pi^{(0)}\right) = W^{(e)} \leq W^{(c)}. \tag{6}$$

One benefit of this method is it requires fewer inputs. For example, in the simple environment above, the incremental cash flows, the failure rate of the intangible when operated by the buyer ($\Pi^{(N)} - \Pi^{(0)}$ and λ), and the buyer's discount rate (r) are the only elements needed to produce the valuation; no further assumptions regarding royalty rates, rental rates, or capital charges are needed. In practice, the with-and-without method may not be always easy to implement. For instance, cash flows $\Pi^{(0)}$ generated by complementary assets on a standalone basis could be hard to estimate if they are generally used jointly with the intangible asset (or other similar assets).

The relief-from-royalty method The relief-from-royalty method (RfR) estimates the net present value of the savings that the buyer would generate by owning the asset, compared to a hypothetical scenario where they would license the intangible from the seller.[19] Appendix A establishes the following result regarding this method.

[18] See International Valuation Standards Council (2022) 60.22 to 60.28.
[19] See International Valuation Standards Council (2022) 60.18 to 60.21.

Result 1. *The ex- and cum-rents intangible values are upper bounds on the valuation obtained using the relief-from-royalty method:*

$$W^{(RfR)} \leq W^{(e)} \leq W^{(c)}. \tag{7}$$

The valuation $W^{(RfR)}$ obviously depends on the royalty or licensing rate the seller would charge to the buyer. Let γ denote this royalty rate, expressed as a fraction of the EBITDA earned by the buyer conditional on the intangible not having exhausted its useful life (so that licensing fees are $\gamma\Pi^{(N)}$ per period).[20] Then, as shown in the Appendix, the relief-from-royalty valuation coincides with the ex-rent valuation if and only if:

$$\gamma = 1 - \frac{\Pi^{(0)}}{\Pi^{(N)}}. \tag{8}$$

For any royalty rate lower than this, the RfR valuation will satisfy $W^{(RfR)} < W^{(e)}$: a lower royalty rate implies lower cost savings from owning the asset, and therefore a lower valuation.

The royalty rate in Equation (8) has a simple interpretation: it implies that the buyer will rebate all incremental cash flows associated with the intangible to the seller in the form of licensing fees. This is precisely the case when the relief-from-royalty valuation corresponds to the ex-rents valuation in Equation (2), and the with-and-without valuation in Equation (6). The cost saving is equal to the present value of licensing fees, which are in turn equal to the present value of the incremental cash flows the buyer generates from the intangible if licensing fees are given by Equation (8).

We also see that $W^{(RfR)}$ can never exceed $W^{(e)}$. If the licensing fees are higher than the value in Equation (8), then the license will be unattractive since it would be better to operate without the intangible asset. The value $W^{(RfR)}$ can be lower than the value of $W^{(e)}$. If the licensing fees are below the value, which might happen if the buyer has some bargaining power over the seller, then the relief-from-royalty method would produce a valuation lower than the incremental value of the intangible asset (the with-and-without valuation).

The excess earnings and the greenfield methods The excess earnings (EE) method calculates the net present value of the cash flows that the buyer would generate, minus appropriately computed charges for complementary assets.[21] The greenfield (G) method calculates the net present value of the benefits that the buyer would generate if they *only* owned the intangible asset and were to rent complementary inputs on competitive markets.[22]

In the context of the simple model described above, there is a single complementary asset, so the charge rate (in the excess earnings method) or the rental rate (in the greenfield method) would reflect only that input. The two methods are identical if the charge rate and the rental rate are the same, as shown in

[20] We assume that once the intangible reaches the end of its useful life, no royalty payments are due anymore.
[21] See International Valuation Standards Council (2022) 60.6 to 60.10.
[22] See International Valuation Standards Council (2022) 60.29 to 60.32.

Appendix A. The Appendix also establishes the following result.

Result 2. *The valuations under the excess earnings method and greenfield methods satisfy:*

$$\begin{aligned} W^{(e)} &\leq W^{(EE)} \leq W^{(c)}, \\ W^{(e)} &\leq W^{(G)} \leq W^{(c)}. \end{aligned} \tag{9}$$

Let $R^{(EE)}$ be the charge rate for the complementary asset in the excess earnings method, and $R^{(G)}$ be the rental rate of the complementary asset in the greenfield method. Then $W^{(EE)} = W^{(G)} = W^{(c)}$ if and only if:

$$R^{(EE)} = R^{(G)} = r + \delta, \tag{10}$$

while $W^{(EE)} = W^{(G)} = W^{(e)}$ if and only if:

$$R^{(EE)} = R^{(G)} = \frac{\Pi^{(0)}}{K}. \tag{11}$$

This result says that the two valuation methods will generally lead to estimates of the intangible value that are between the ex- and cum-rents value. This follows from the fact that capital charges will generally need to satisfy:

$$R^{(EE)}, R^{(G)} \in \left[r + \delta, \frac{\Pi^{(0)}}{K} \right]. \tag{12}$$

The lower bound on these rates follows from the fact that the (gross) required return the complementary asset for the buyer should be weakly larger than the user cost. The upper bound follows from the fact that the enterprise value associated with operating the complementary asset alone would be negative under higher rates, leading the buyer to simply shut down the firm.

The two valuation methods give rise to estimates of the intangible value that are higher than the ex-rent value for the same reason: they attribute some or all of the rents that the buyer would be able to generate even without the intangible asset to the value of intangible asset. This happens because the capital charge rate or the rental rate may be too low relative to the actual rate of return the buyer can generate with the complementary assets alone. In particular, the internal user cost of the complementary assets is a *lower bound* for the actual rate of return of the complementary assets for the buyer. Using it as the capital charge or rental rate in the valuation exercise will therefore implicitly transfer some of the rents the buyer might generate to the intangible asset's value.

The valuations coincide with the ex-rents economic value of the intangible asset if the capital charge or the rental rate used in the valuation exercise is sufficiently large. Specifically, the recommendation that follows from this model is that one should *not* apply the user cost of capital in the excess earnings or greenfield estimates. Instead, one should use estimates of the average return on complementary assets in

the absence of the intangible. That is, the capital charges or rental rates should reflect the internal rate of return on complementary assets that the buyer can generate, and not the user cost of these assets. The user cost and the internal rate of return will not coincide if the buyer earns rents even without the intangible.

4.3 The role of uncertainty and the choice of discount rates

A crucial input in valuation formulas such as Equations (2) or (3) is the discount rate r, which appears in the denominator of both formulas. Here we briefly discuss guidelines for the choice of discount rates in these formulas, and adjustments that might be made to it.

It is often argued that, because intangible assets are "more risky," using higher discount rates is generally appropriate. This is only true to the extent that cash flow risk associated with the intangible is systematic, as opposed to idiosyncratic. Here, by "systematic" risk, we refer to the possibility that cash flows of the intangible asset are correlated with cash flows of other assets owned by the buyer (or, more generally, with market returns). By contrast, "idiosyncratic risk" refers to the possibility that intangible cash flows are uncorrelated with cash flows of other assets owned by the buyer, or uncorrelated with the market.

In general, cash flows associated with the intangible asset can have a systematic risk component. In those cases, adding a risk premium to the discount rate of the buyer, r, in formulas such as Equations (2) or (3) is appropriate. A potential approach is to gather information on required returns for similar intangible assets, perhaps from buyers with similar characteristics to the one under consideration. Given the specificity of many intangible assets, this may be difficult to do accurately in practice, creating an additional hurdle to the implementation of income-based methods.

Nonetheless, some adjustments to the formulas such as Equations (2) and (3) may be appropriate even when all of the intangible asset's risk is idiosyncratic. Indeed, the example model that we described contains such an adjustment: the parameter λ appears in Equation (2). In the model we outlined, cash flow uncertainty is entirely idiosyncratic and the buyer is risk-neutral. The λ adjustment in Equation (2) should *not* be interpreted as an "intangible risk premium" reflecting higher required returns due to systematic risk. Instead, the λ adjustment simply reflects that higher obsolescence or infringement risk (or in general, a shorter expected useful life of the intangible) lowers the present value of expected cash flows it will generate.[23] Thus while an adjustment is still appropriate, it is important to remember that it does not represent a higher required return for the buyer. Therefore, it should not be estimated using data on expected returns for similar assets, but rather using information on effective useful life (or cash flow duration more generally) that is specific to the asset.

[23] In a version of the model with constant cash flow growth, the adjustment would take the form of a reduction in the growth rate used in the Gordon growth formula for valuations, highlighting perhaps more clearly that these adjustments should not, strictly speaking, be thought of as changes in the discount factor of the buyer.

5 Institutional foundations and bottlenecks

This section discusses the institutional foundations necessary to support the financing arrangements for intangibles. We highlight four types of institutional requirements:

1. **Ownership and rights:** For separable intangibles, a particularly important dimension is the recognition and enforcement of liens on movable assets. For nonseparable intangibles, the recognition of blanket liens and well-defined cash flows and control rights can be important.

2. **Trading:** The development of secondary market trading is helpful for the plegeability of separable intangibles. These secondary markets often rely on large platforms that many buyers and sellers can access. Reducing information asymmetry and improving standard setting for transaction contracts can be useful. At the same time, markets for business combinations (e.g. M&A, private equity) may also be helpful for the financing of nonseparable intangibles.

3. **Measurement:** Financing capacity depends on valuations, which in turn require proper measurement of cash flows. For separable intangibles, it is useful to measure the cash flows that they generate. For nonseparable intangibles, it is important to measure the cash flows from the companies' operations. Commonly accepted accounting standards, along with public reporting requirements, are important input for cash flow measurement.

4. **Bankruptcy:** Legal institutions for bankruptcy are especially important for pledging firms' cash flows for borrowing, which is very useful for the financing of nonseparable intangible assets. In particular, if firms in financial distress are generally liquidated when they file for bankruptcy, then lenders will count on pledging separable assets for borrowing. If financial restructuring functions well, then viable firms in financial distress can continue to operate, and can borrow on the merit of their business. For U.S. Chapter 11, the total payments for creditors are given by the value of the cash flows of the restructured company. Accordingly, it is natural for creditors to care about borrowers' cash flows and to be willing to lend on the basis of cash flows.

6 Conclusion

Intangible assets have become increasingly important for firms' production activities. As we summarize in this article, pledging intangibles on a standalone basis faces a number of challenges. One of these challenges is assigning the value of a particular intangible asset, especially when secondary market transactions data are sparse and valuation depends to a large extent on income-based methods. Meanwhile, pledging intangibles as part of the firm as a whole also relies on a list of prerequisites. Pledging the firm as

a whole for borrowing is so far uncommon in countries where financial accounting is less reliable, firms in financial distress are likely to liquidate, or lenders do not have sufficient expertise to assess and monitor business performance. Even in settings where these conditions are met, such as the U.S., such borrowing is rare for firms that do not yet generate positive cash flows. Pledging the firm as a whole for equity financing is more conventional, but its capacity also depends much on financial development.

Going forward, we hope that further advancement in institutional infrastructure can improve firms' ability to obtain financing against intangible assets. It is also possible that countries with better institutional infrastructure will have an edge in promoting the growth of intangible-intensive industries as well as the overall growth of the economy in an intangible-intensive world.

A Model

In the following appendix, we describe a simple model to analyze the four main income-based valuation methods. The goal is to understand (a) what each method measures, (b) under which conditions the four methods measure the same object, and (c) when that object might be reasonably called the economic value of the intangible asset on a standalone basis.

In this appendix, we refer to the complementary asset as "physical capital," though the arguments we make could apply to other types of complementary assets.

A.1 Description

Time is discrete and denoted by $t = 0, 1, ...$ A firm wants to sell an intangible asset (the "seller"). There is a potential buyer for this asset (the "buyer"). The valuation question is: how much *should* the buyer be willing to pay for it?

The buyer will combine this asset with some physical capital in order to generate economic benefits. Let K_t be the buyer's stock of physical capital, which can be used in production during period $t + 1$.

Physical capital depreciates because of wear and tear, at a rate δ per period. We assume that the buyer can buy and sell units of physical capital on a competitive market, at a price that is constant and equal to 1, and that there are no additional costs associated with adjusting physical capital.

The intangible asset does not suffer from no wear and tear. However, the duration of its economic life is uncertain. We model this as follows: in each period, the intangible asset ceases to create any economic benefits with probability λ. Thus the expected useful economic life of the intangible to the buyer is $1/\lambda$. For now, this is the only form of uncertainty.

Combining physical capital with the intangible asset generates earnings before interest, taxes, deduction and amortization (EBITDA) given by:[24]

$$\Pi^{(N)}\left(K_{t+j}\right), \tag{13}$$

where the superscript N is a placeholder for the "producing with the intangible asset".[25] After the end of the intangible's useful life, EBITDA per period for the buyer is given by:

$$\Pi^{(0)}(K_{t+j}). \tag{14}$$

[24]Consistent with international standards, the valuation is estimated under the assumption that the buyer will make no future additions or substractions to the intangible asset. See, for instance International Valuation Standards Council (2022) paragraph 60.11. Given the absence of wear and tear or adjustments, the "quantity" of intangible asset is irrelevant to the economic benefits it generates, so long as the asset has not yet reached the end of its useful life.

[25]To simplify the discussion, we assume that the profit function $\Pi(.)$ is constant over time. In the generalization below, we show that the same results hold if it is time-varying. This could capture, for instance, time-varying synergies between intangibles and physical assets or time-varying demand for the firm's products.

Finally, we assume that the buyer is risk-neutral, and uses the constant discount rate r to value future EBITDA streams. In this environment, the different economic primitives, the depreciation rate of physical capital δ, the expected useful life of the intangible $1/\lambda$, the EBITDA functions $\{\Pi^{(N)}(.), \Pi^{(0)}(.)\}_{j\geq 1}$, and the discount rate r, are all specific to the buyer of the asset.

Summary of assumptions The key economic assumptions of the model are:

A1 Total investment costs are additive between physical and intangible capital;

A2 There is a perfectly competitive market for physical capital;

A3 Physical capital has a constant price in this market;

A4 Purchases or sales of capital involve no transaction costs; installing or uninstalling capital does not involve any adjustment costs;

A5 The buyer of the asset is risk-neutral;

A6 Only the useful life of the intangible asset is uncertain.

Additionally, in what follows, we assume that the buyer chooses an investment plan for physical capital such that:

$$\forall t \geq 0, \quad K_{t+1} = K. \tag{15}$$

This assumption helps streamline the discussion to highlight key points. We leave the cases of optimally chosen capital expenditures, which will depend more heavily on the details of the model, for future research.

A.2 Economic value of intangibles

If the input of capital is kept constant by the buyer after the purchase, then investment in physical capital per period is constant and equal to δK. Additionally, flow profits in each of the two possible situations (before and after the end of the useful life of the intangible asset) are constant. To make notation lighter we denote them as:

$$\Pi^{(N)} \equiv \Pi^{(N)}(K), \quad \Pi^{(0)} \equiv \Pi^{(0)}(K). \tag{16}$$

A.2.1 Enterprise value

Let $V^{(N)}$ denote the total enterprise value associated with combining K units of physical capital with the intangible asset, which is constant under our assumption. Additionally, let $V^{(0)}$ denote the enterprise value that the buyer can generate using *only* physical capital, which is also constant under our assumptions.

This value satisfies the following equation:

$$V^{(0)} = \frac{1}{1+r}\left(\Pi^{(0)} - \delta K + V^{(0)}\right), \tag{17}$$

which implies:

$$\begin{aligned} V^{(0)} &= \frac{1}{r}\left(\Pi^{(0)} - \delta K\right) && (18)\\ &= K + \frac{1}{r}\left(\Pi^{(0)} - (r+\delta)K\right). && (19) \end{aligned}$$

Similarly, total enterprise value to the buyer at purchase satisfies:

$$V = \frac{1}{1+r}\left(\Pi^{(N)} - \delta K + (1-\lambda)V^{(N)} + \lambda V^{(0)}\right), \tag{20}$$

which implies:

$$V = V^{(0)} + \frac{1}{r+\lambda}\left(\Pi^{(N)} - \Pi^{(0)}\right). \tag{21}$$

Equation (19) indicates that the enterprise value without the intangible has two components. Given assumptions **A2**-**A4**, K represents the replacement cost of the stock of physical capital. The other term represents the present value of surpluses generated by K, relative to the *buyer*'s internal user cost, which is the sum of an opportunity cost per unit of capital, r, and a cost of wear-and-tear per unit of capital, δ.

Equation (21) then says that, relative to the no-intangible case, the intangible purchase creates an additional enterprise value that is equal to the capitalized difference in the flow profits generated by combining the intangible with physical capital, and the flow profits generate by only using physical capital. The discount rate is adjusted for the risk associated with the useful life of the asset.

A.2.2 The standalone economic value of the intangible asset

In the context of this model, there are two potential definitions of the economic value of the intangible asset on a standalone basis. First, the standalone economic value of the intangible can be defined as the incremental enterprise value:

$$\begin{aligned} W^{(e)} &= V^{(N)} - V^{(0)} \\ &= \frac{1}{r+\lambda}\left(\Pi^{(N)} - \Pi^{(0)}\right). \end{aligned} \tag{22}$$

Second, the standalone economic value of the intangible could be defined as the difference between enterprise value and the replacement cost of physical capital:

$$\begin{aligned} W^{(c)} &= V^{(N)} - K \\ &= \frac{1}{r}\left(\Pi^{(0)} - (r+\delta)K\right) + \frac{1}{r+\lambda}\left(\Pi^{(N)} - \Pi^{(0)}\right). \end{aligned} \tag{23}$$

The difference between the two definitions arises from the present value of rents that can be generated by the buyer from physical assets alone:

$$W^{(c)} - W^{(e)} = \frac{1}{r}\left(\Pi^{(0)} - (r+\delta)K\right). \tag{24}$$

The second definition, in Equation (23), attributes any EBITDA generated by the buyer using physical capital in excess of the *buyer*'s own internal user cost of physical capital to the purchased intangible.

If the buyer enjoys some rents even without using the intangible asset, we have:

$$\Pi^{(0)} > (r+\delta)K \quad \Longrightarrow \quad W^{(c)} > W^{(e)}, \tag{25}$$

On the other hand, if the buyer enjoys no rents from using only the physical asset, then the two valuations coincide:

$$\Pi^{(0)} = (r+\delta)K \quad \Longrightarrow \quad W^{(c)} = W^{(e)}. \tag{26}$$

In this case, the value of operating the physical asset alone is simply equal to its replacement cost:

$$V^{(0)} = K, \tag{27}$$

a fact that follows from our assumption that there are no adjustment costs to physical capital.

We do not consider the third case, $\Pi^{(0)} < (\delta + r)K$, because in that case the standalone value of the physical capital to the buyer, $V^{(0)}$, would be strictly negative. If the physical capital could be scrapped at no cost, the buyer would choose to do so upon the end of the intangible's useful life.

Finally, note that in both definitions, any rent generated *only when* the intangible asset is in used is attributed to the value of the intangible asset.

A.2.3 Comparing the four income approaches

Next, we derive the value of the intangible asset under the four income-based valuation methods: the with-and-without (WaW) method; the relief-from-royalty (RfR) method; the excess earnings (EE) method; and the greenfield (G) method.

The with-and-without method The with and without method defines the value of the intangible asset as the incremental enterprise value realized through its use, so it is the same as the ex-rent standalone value of the intangible asset defined in Equation (22):

$$W^{(WaW)} = \frac{1}{r+\lambda}\left(\Pi^{(N)} - \Pi^{(0)}\right) = W^{(e)}. \tag{28}$$

Even when $\Pi^{(0)} > (r+\delta)K$, the valuation in this method does not reflect any of the rents generated by the buyer without the intangible (which is a desirable feature).

The relief-from-royalty method Let $\gamma^{(RfR)}$ be the per-period licensing rate, expressed as a fraction of total EBITDA. Let S be the present value of savings that the buyer would realize from owning the intangible, rather than licensing it from the seller, so long as the intangible has not yet reached the end of its useful life. Let $S^{(0)}$ be the present value of savings from owning the intangible rather than licensing it once it has reached the end of its useful life. Clearly, $S^{(0)} = 0$ since once the intangible has reached the end of its useful life, no more licensing fees are due. Thus S must satisfy:

$$S = \frac{1}{1+r}\left(\gamma\Pi + (1-\lambda)S\right), \tag{29}$$

leading to:

$$S = \frac{\gamma}{r+\lambda}\Pi. \tag{30}$$

Thus the RfR valuation is:

$$W^{(RfR)}(\gamma) = \frac{\gamma}{r+\lambda}\Pi. \tag{31}$$

We now have:

$$W^{(RfR)}(\gamma)\begin{cases} = W^{(e)} & \text{if} \quad \gamma = \underline{\gamma} \\ \in \left(W^{(e)}, W^{(c)}\right) & \text{if} \quad \gamma \in \left(\underline{\gamma}, \overline{\gamma}\right) \\ = W^{(c)} & \text{if} \quad \gamma = \overline{\gamma} \end{cases}, \tag{32}$$

where the royalty rates $\underline{\gamma}$ and $\overline{\gamma}$ are given by:

$$\underline{\gamma} = 1 - \frac{\Pi^{(0)}}{\Pi}, \quad \overline{\gamma} = \underline{\gamma} + \frac{r+\lambda}{r}\left(\frac{\Pi^{(0)} - (r+\delta)K}{\Pi}\right). \tag{33}$$

Under the licensing fee $\underline{\gamma}$, the licensor is exactly absorbing the incremental cash flows generated by the intangible asset, so that the present value of the licensing fees is equal to the ex-rent value of the intangible and to $W^{(2)}$. For higher values of the licensing fee, the licensor would be absorbing more than the incremental cash flows attributable to the intangible; thus the resulting valuation would be higher than the ex-rent value of the intangible.

The licensing fee should not be higher than $\underline{\gamma}$. Indeed, for any licensing fee higher than $\underline{\gamma}$, the enterprise value associated with licensing the asset is strictly *less* than operating with physical capital alone. If so, the buyer would be better off by operating without the intangible asset.

The economic environment we have described does not allow us to establish conclusively whether licensing fees *lower than* $\underline{\gamma}$ would be acceptable. This would require specifying more completely the outside options of each party in the licensing agreement. In particular, if the party receiving the license has some degree of bargaining or monopsony power in the licensing relationship, it may be able to obtain a share of the surplus created by licensing the asset, which is equal to $W^{(RfR)}(\underline{\gamma})$. This surplus might then be shared through licensing fees lower than $\underline{\gamma}$.

Thus in general, the licensing fee should be no higher than $\underline{\gamma}$, implying that the relief-from-royalty method provides an *upper bound* on the value of the intangible asset.

The excess earnings and the greenfield methods As discussed in the main text, the two methods are formally equivalent in the context of this model. We discuss only the excess earnings method but everything would be identical for the greenfield method, by simply replacing the capital charge with a rental rate.

Let R denote the capital charge that is associated with the use of the physical asset. Then, following similar steps to the computation of $W^{(c)}$, the valuation is given by:

$$W^{(EE)}(R) = W^{(e)} + \frac{1}{r}\left(\Pi^{(0)} - RK\right). \tag{34}$$

Note that:

$$W^{(EE)}(R) \begin{cases} = W^{(e)} & \text{if} \quad R = \frac{\Pi^{(0)}}{K} \\ \in \left(W^{(2)}, W^{(1)}\right) & \text{if} \quad R \in \left(r+\delta, \frac{\Pi^{(0)}}{K}\right) \\ = W^{(c)} & \text{if} \quad R = r + \delta \end{cases} \tag{35}$$

Thus, the capital charge that will produce a valuation consistent with both the with-and-without method, and with $W^{(e)}$ (the ex-rents value of the intangible), is $\Pi^{(0)}/K$, the buyer's average return to the physical asset without intangible capital.

By contrast, if the capital charge used in the valuation is equal to the buyer's cost (that is, their discount rate plus an appropriate deduction for depreciation), then the valuation will also incorporate any rents generated by the buyer even in the absence of the intangible.[26]

[26] A capital charge lower than the user cost would be inappropriate: it would artificially inflate profits by failing to reflect at least the opportunity and wear-and-tear cost of the physical asset.

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