Information Transmission and the Bullwhip Effect:
Online Supplement

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Appendix

A. Proposition Proofs

Proof of Proposition 1: We first consider the case in which all signals are mutually independent. We use the following orders and inventory formulations:

$$o_t = \mu + \sum_{l=0}^{\infty} \tilde{o}_{t,l}, \qquad \tilde{o}_{t,l} = \sum_{i=-\infty}^{\infty} (w_{i,l} - w_{i+1,l})\epsilon_{t+L+i,l},$$
$$i_t = m + \sum_{l=0}^{\infty} \tilde{i}_{t,l}, \qquad \tilde{i}_{t,l} = \sum_{i=0}^{\infty} w_{i+1,l}\epsilon_{t+i+1,l} + (w_{-i,l} - 1)\epsilon_{t-i,l}.$$

Accordingly, the firm's objective is:

$$\min_{w_{i,l}} \; k_i \sqrt{\sum_{l=0}^{\infty} Var\big(\tilde{i}_{t,l}\big)} + k_p \sqrt{\sum_{l=0}^{\infty} Var\big(\tilde{o}_{t,l}\big)} \tag{1}$$

s.t.

$$Var\big(\tilde{i}_{t,l}\big) = e_l{}'\Sigma e_l \cdot \sum_{i=0}^{\infty} \left[w_{i+1,l}^2 + (w_{-i,l} - 1)^2\right]$$
$$Var\big(\tilde{o}_{t,l}\big) = e_l{}'\Sigma e_l \cdot \sum_{i=-\infty}^{\infty} (w_{i,l} - w_{i+1,l})^2$$
$$w_{i,l} = 0 \quad \text{for } i > l - L.$$

Inventory variability grows quadraticly in $w_{i,l}$, so a global minimum must exist in a compact region about the origin. At this global minimum, the following first-order conditions must hold:

$$(\theta + 2)w_{i,l} = w_{i-1,l} + w_{i+1,l} \quad i \geq 1, \tag{2}$$

$$(\theta+2)\tilde{w}_{i,l} = \tilde{w}_{i-1,l} + \tilde{w}_{i+1,l} \quad i \leq 0, \tag{3}$$

$$\theta = k_i/k_p \sqrt{\frac{Var(o_t|w_{i,l})}{Var(i_t|w_{i,l})}}, \tag{4}$$

where $\tilde{w}_{i,l} = w_{i,l} - 1$. Expressions (2) and (3) describe second order difference equations, with characteristic polynomial $x^2 - (\theta+2)x + 1 = 0$. This polynomial has roots λ and λ^{-1}, where $\lambda = \theta/2 + 1 - \sqrt{(\theta/2+1)^2 - 1}$. First, we consider the case in which $L \geq l$. Here, the constraints set the left-hand-side variables of (2) to zero. Applying the $\tilde{w}_{l-L+1,l} = 1$ boundary condition to (3) yields, for some constant κ:

$$\tilde{w}_{i,l} = \kappa\lambda^{l-L+1-i} + (\kappa-1)\lambda^{i-l+L-1}, \quad L \geq l \text{ and } l - L \geq i. \tag{5}$$

Notice, for all $w_{i,l}$ to be finite—an optimality requirement—κ must equal one. Hence, we find:

$$w_{i,l} = 1 - \lambda^{l-L+1-i}, \quad L \geq l \text{ and } l - L \geq i. \tag{6}$$

Second, we consider $l > L$. Now we apply the $w_{l-L+1,l} = 0$ boundary condition to (2), to get:

$$w_{i,l} = \kappa(\lambda^{i+-l+L-1} - \lambda^{l-L+1-i}), \quad l > L \text{ and } l - L \geq i > 0. \tag{7}$$

Expression (7), in turn, yields $\tilde{w}_{1,l} = \kappa(\lambda^{-l+L} - \lambda^{l-L}) - 1$ and $\tilde{w}_{0,l} = \kappa(\lambda^{-l+L-1} - \lambda^{1+l-L}) - 1$, which we apply as boundary conditions to (3) to get:

$$\tilde{w}_{i,l} = \left[-\kappa\lambda^{l-L} - 1/(1+\lambda)\right]\lambda^{-i+1} + \left[\kappa\lambda^{-l+L} - \lambda/(1+\lambda)\right]\lambda^{i-1}, l > L \text{ and } i \leq 0. \tag{8}$$

Now, for each $w_{i,l}$ to be finite, κ must equal $\frac{\lambda^{l+L+1}}{1+\lambda}$, because $\lambda^{-1} > 1$. Plugging this κ value into (7) and (8) yields:

$$w_{i,l} = \begin{cases} \frac{\lambda^i - \lambda^{2+2l-2L-i}}{1+\lambda} & l > L \text{ and } l - L \geq i > 0, \\ 1 - \frac{1+\lambda^{2l-2L+1}}{1+\lambda}\lambda^{-i+1} & l > L \text{ and } i \leq 0. \end{cases} \tag{9}$$

From $w_{i,l}$ we can then derive A.

The solution easily extends to the case in which $\epsilon_{t+l,l}$ and $\epsilon_{t+j,j}$ are correlated, when l and j are less than or equal to L. All unmet demands impose the same per-period inventory costs, so the firm finds it optimal to adopt the same order schedule for $\epsilon_{t,l}$ and $\epsilon_{t-l+L,L}$, for $l > L$, as these signals are both realized within the product lead time. The only difference between these signals is the former's additional $L - l$ periods of delinquency. Hence, as long as we properly account for these delinquent periods, we can treat $\epsilon_{t,l}$ as if it where $\epsilon_{t-l+L,L}$. Consequently, we can use the solution outlined above, with three small changes: first, we let l-lead-time signals, for $l > L$, have zero variance; second, we let L-lead time signals have variance $\left(\sum_{l=0}^{L} e_l'\right)\Sigma\left(\sum_{l=0}^{L} e_l\right)$; third, we add $\sum_{i=0}^{L-1} e_0^i{}'\Sigma e_0^i$ to $Var(i_t)$, where $e_0^i = \sum_{j=0}^{i} e_j$. Doing so yields $w_{i,l}$, for $l \geq L$; we get the rest with the following: $w_{i,l} = w_{i+L-l,L}$, for $l < L$. The result has the same form as (6).

□

Proof of Propositions 2 and 3: One can use a simple substitution argument to demonstrate that the optimal inventory and production standard deviations weakly decrease and increase in k_i/k_p, respectively, which means θ and λ strictly increase and decrease in k_i/k_p, respectively. Then one can show: 1) $\lambda \in (0,1)$; 2) β and $\lim_{l\to\infty} \beta_l$ strictly decrease in λ; 3) if $\left(\sum_{l=0}^{L} e_l'\right)\Sigma\left(\sum_{l=0}^{L} e_l\right) > \sum_{l=0}^{L} e_l{}'\Sigma e_l$ then β is negative for λ in a neighborhood of one, and positive for λ in a neighborhood of zero; 4) if $\left(\sum_{l=0}^{L} e_l'\right)\Sigma\left(\sum_{l=0}^{L} e_l\right) \le \sum_{l=0}^{L} e_l{}'\Sigma e_l$ then β converges to a negative or zero value as λ approaches zero; 5) $\lim_{l\to\infty} \beta_l$ converges to zero with λ. These conditions are sufficient. $\square$

B. Additional Product Aggregation Robustness Checks

We present two additional product-aggregation-bias analyses: the first further aggregates products with merger and acquisitions data, and the second proxies for the degree of aggregation with business segment counts. Neither analysis suggests the presence of a meaningful product aggregation bias.

B.1. Mergers and Acquisitions

Mergers and acquisitions, aggregate two companies' products into a single firm. Since a company's product assortment increases after merging with another, the bullwhip change, following a merger, should point in the direction of the aggregation bias. Our null hypothesis is that product aggregation does not meaningfully impact bullwhip estimates (i.e. that pre- and post-merger bullwhips are the same). Naturally we hope to fail to reject the null (i.e. find no significant difference between pre- and post-merger bullwhips). This investigation has several limitations: company mergers are not exogenous, many other unobserved changes transpire when two companies become one, and the ability to forecast sales and demand likely changes post merger.

Combining the COMPUSTAT and Thompson SDC M&A datasets, we create a panel of 882 acquisitions, made by 400 acquiring companies. We treat each acquisition as a separate event, so some acquiring companies have replicated data (our standard errors account for these replications). In constructing our sample, we allocate the same number of pre- and post-merger observations to all acquiring companies (at least 25), so forecasts are equally accurate, before and after mergers. We regress the squared forecast errors on a post-merger indicator variable and control variables (quarter dummies, inventory levels, and total assets).[1] Table A's estimates are not significantly positive, which demonstrates that M&A aggregation does not significantly accentuate bullwhip estimates. Thus we fail to reject the null hypothesis that the pre- and post-merger bullwhips are the same: viz., we find no evidence of a meaningful product aggregation bias.

[1] This regression, unlike the others, block bootstraps for standard errors, as two-way cluster robust standard errors are not valid with quarter dummy regressors.

B.2. Product Segments:

Since 1976, the SFAS No. 14 of the US GAAP has mandated public companies to produce segment reports for product divisions that constitute at least 10% of firm operating revenues. Using these segment reports, we create product aggregation proxies, with which we explore the effect of aggregation. The first proxy is company segment sales concentrations, measured by the Herfindahl index: dispersed sales suggests high product assortment. The second is company segment counts: more product divisions suggests more products. Our null hypothesis is that the estimated bullwhip effect is not decreasing in segment sales concentrations, nor increasing in the segment count. Rejecting the null would provide evidence that the bullwhip is increasing in product aggregation. (As before, we hope to fail to reject.)

Merging our forecast error panel with the COMPUSTAT product-segment dataset yields 31,777 observations from 3,979 companies. The segment counts and sales concentrations have means of 2.00 and .68, respectively, and standard deviations of 1.56 and .32. We regress the squared forecast errors on segment sales concentrations and segment count indicator variables, and present the results in Table B. The concentration coefficients are all insignificant, so we fail to reject the hypothesis that the degree of concentration has no meaningful impact on our bullwhip estimates. Similarly, there is no consistent trend amongst the segment-count coefficients, and all but three of these coefficients are insignificant, and those that are significant are negative, which indicates bullwhips that are smaller than their counterparts at single-segment companies. Thus we fail to reject the null hypothesis that product aggregation does not exaggerate our bullwhip estimates. (Apologies for the double negative.)

Table A Mergers and Acquisitions

Combining the COMPUSTAT and Thompson SDC M&A datasets, we create a panel of 882 acquisitions, made by 400 acquiring companies. We treat each acquisition a separate event, so some acquiring companies have replicated data (our standard errors account for these replications). In constructing our sample, we allocate the same number of pre- and post-merger observations to all acquiring companies (at least 23), so forecasts are equally accurate, before and after the mergers. We regress square forecast errors on a post-merger indicator variable and control variables (quarter dummies, inventory levels, and total assets). The following are the post-merger indicator variable coefficients, the mean bullwhip changes attributable to mergers.

	$\widehat{\beta}$	$\widehat{\beta}_0$	$\widehat{\beta}_1$	$\widehat{\beta}_2$	$\widehat{\sum_{i=3}^{\infty}\beta_i}$	$\widehat{\beta}_\infty$
Sample	−3.79	−3.18	−0.57	−1.11	−1.14	3.14
	(6.09)	(2.88)	(1.36)	(1.33)	(3.07)	(3.72)
Retail	−25.08***	−10.77***	−2.55	−1.87	−6.42*	−2.30
	(6.84)	(2.75)	(2.06)	(1.60)	(3.54)	(4.97)
Wholesale	−3.68	−5.43	−5.05**	−1.11	−3.60	11.78***
	(7.66)	(3.53)	(2.26)	(1.99)	(3.68)	(4.32)
Manufacturing	−1.49	−2.26	−0.09	−1.03	−0.46	3.32
	(6.79)	(3.23)	(1.35)	(1.38)	(3.28)	(3.68)
Extraction	−7.51	−3.81	−2.59	−1.10	−1.24	0.58
	(7.97)	(3.71)	(3.75)	(2.58)	(3.98)	(5.07)

*, **, and *** indicate significance levels $p \leq .1$, $p \leq .05$, and $p \leq .01$, respectively.

Table B Product Segments

We regress squared forecast errors on sales concentrations within firm business segments, measured by the Herfindahl Index, and on business segment count indicator variables, whose coefficients measure bullwhip differences from firms with one business segment. We control for inventory and total assets, and remove firm fixed effects.

Herfindal	−1.53	−1.34	−0.91	−1.51	2.62	−0.94
	(5.83)	(2.64)	(1.60)	(1.45)	(2.70)	(2.59)
2 segments	5.29	3.45	−1.56	−1.76	0.91	0.82
	(4.76)	(2.42)	(1.78)	(1.20)	(2.60)	(2.75)
3 segments	−1.61	−1.69	−4.09**	1.17	0.22	1.49
	(4.87)	(2.14)	(1.74)	(1.12)	(2.65)	(2.80)
4 segments	−3.87	−1.88	−0.81	0.74	−5.11	0.89
	(5.47)	(2.76)	(1.93)	(1.46)	(3.39)	(2.94)
5 segments	−2.38	−2.15	−3.44	−3.75*	1.83	2.08
	(6.65)	(3.52)	(2.36)	(2.02)	(3.58)	(3.53)
6 segments	−0.98	−0.79	−3.58	−3.59	2.72	2.45
	(8.46)	(4.34)	(2.95)	(2.73)	(4.60)	(4.30)
7+ segments	−15.41**	−5.61	−4.59	−0.99	3.94	−7.56
	(7.78)	(4.19)	(3.29)	(3.19)	(4.44)	(4.68)

*, **, and *** indicate significance levels $p \leq .1$, $p \leq .05$, and $p \leq .01$, respectively.

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