Threshold Incentives over Multiple Periods and the Sales Hockey Stick Phenomenon

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Abstract: In this article, we study threshold-based sales-force incentives and their impact on a dealer's optimal effort. A phenomenon, observed in practice, is that the dealer exerts a large effort toward the end of the incentive period to boost sales and reach the threshold to make additional profits. In the literature, the resulting last-period sales spike is sometimes called the hockey stick phenomenon (HSP). In this article, we show that the manufacturer's choice of the incentive parameters and the underlying demand uncertainty affect the dealer's optimal effort choice. This results in the sales HSP over multiple time periods even when there is a cost associated with waiting. We then show that, by linking the threshold to a correlated market signal, the HSP can be regulated. We also characterize the variance of the total sales across all the periods and demonstrate conditions under the sales variance can be reduced. © 2010 Wiley Periodicals, Inc. Naval Research Logistics 57: 503–518, 2010

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1. INTRODUCTION

Quota-based compensation plans are widely used in practice to motivate the sales-force to increase sales. One example of a threshold-based[1] sales-force incentive mechanism is the so, called stair-step incentive offered by auto-manufacturer Chrysler to its dealers [18]. Under this incentive plan[2], Chrysler gave dealers cash based on the percentage of the monthly vehicle sales target met. It is commonly believed that such threshold-based incentives are effective in increasing a manufacturer's expected sales while motivating the dealer to exert effort appropriately. However, it has been observed that when such threshold-based incentives are offered over multiple time periods, they typically result in higher sales (and sales effort) toward the end of the incentive period [3]. This rise in sales is sometimes referred to as the "hockey stick phenomenon" (HSP[3]) in the literature. From the manufacturer's perspective, this resulting sales surge poses two problems. First, the ordering pattern of the dealer is not smooth, and second, it adds to the sales variance. Although the expected sales may increase, the dealer's sales pattern, during the incentive period, causes substantial difficulty in production planning and inventory management, thus increasing a manufacturer's operational cost. As is well known in the operations management literature, higher variability in the demand process affects overstocking and understocking costs.

The desire of this work is to study the impact of threshold-based contracts on the sales variance observed by the manufacturer. To this end, we have not explored the issue of the optimal contract to maximize the total profit (considering the effect of sales variance) of the manufacturer. Rather, we draw insights about the impact of these contracts on the mean and variance of the observed sales (demand), which hold regardless of the production costs at the manufacturer.

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[1] We use the terms threshold and quota interchangeably throughout this article.

[2] A dealer got no additional cash for sales below 75% of the sales target, \$150 per vehicle for sales between 75.1 and 99.9% of the sales target, \$250 per vehicle for sales between 100 and 109.9%, and \$500 per vehicle for reaching 110% of the sales target.

[3] We use the terms HSP and sales surge interchangeably throughout this article.

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To do so, we consider two types of such sales-force compensation plans: (i) when the dealer (salesperson) is paid an additional amount per unit when sales exceed a threshold value, and (ii) when instead of an additional per unit (marginal) payment, a fixed bonus is offered if the total sales exceed the threshold. These two forms are commonly observed in several industries. Sinha and Zoltners [22] provided empirical evidence of such sales-force compensation schemes. For example, Sinha and Zoltners [22] studied several compensation plans for a newspaper advertising sales force. The options differ in threshold level (which could be 70, 80, 90, or 95% of the territorial sales goal), rate of payout, and the percentage of payout attributable to individual (versus team) performance. Other examples include firms from the pharmaceutical and telecommunication industries. The bonus-based incentive plan is quite popular in industries such as electronics and retail [10]. Bonus payments have helped firms direct sales-force effort toward specific organizational goals and increased productivity. The Chrysler plan described earlier is a combination of the marginal payment and bonus plan[4]. In an example from the video rental business, Baker and Shimer [1] described the sales compensation plan at RKO Warner Video, a Manhattan-based company. Under the proposed plan, each store was classified as an "A," "B," or "C" store based upon store size, volume, hours of operation, and an assessment of the "difficulty" of managing the store. Each class of store was then assigned a "base bonus" to be split by store management, on a quarterly basis, if revenue targets were achieved. Some additional examples can be found in Ref. 24 (chapter 4). For example, to promote sales of storage switches, Cisco returns 5% of the wholesale price back to its resellers if the sales in a quarter exceed the previous quarter's sales by more that 20%. Similarly, Xerox, under its Xtra partner program, provides a 4% rebate for every unit of sales if the sales exceed \$1 million. Although some of the aforementioned examples are related to "make-to-stock" supply chains, it is noteworthy that sales compensation plans are also used within "made-to-order" supply chains. For example, Cisco also sells/customizes components based on customer requests, and its sales-force is compensated for generating orders.

In this article, we first show why such threshold-based contracts lead to uneven sales (ordering) patterns and result in the HSP. We characterize the dealer's optimal response (effort levels) when there is a cost associated with waiting and the underlying demand is uncertain. Note that if there was no cost to waiting, i.e., the effort could be costlessly substituted across periods, then the dealer could exert all his effort in the last period after all the demand uncertainty is resolved. This specific setting is studied in Ref. 3, where the dealer holds back on exerting effort until the last period. Note that this is true even when there is no underlying demand variability. In contrast, in our model, the dealer cannot costlessly substitute efforts across periods. We model this behavior using the concavity of the effort function and a linear stationary cost function. Essentially, this implies that it is "costly" to exert all the effort in the last period even though waiting until the last time period allows for more precise information regarding what it will take to exceed the threshold and by how much. In other words, if the dealer has access to all the future demand information in our setting (or if there is no demand uncertainty), he would exert equal effort in all periods. However, when the dealer is not clairvoyant (or demand is uncertain), we show that the dealer's optimal expected effort levels stochastically increase over time even when there is a cost associated with waiting. This results in the HSP and higher sales variance. Interestingly, under certain conditions, it is possible that the expected sales dip over time, in spite of increasing expected sales effort levels. We characterize these conditions as well.

Next, through numerical experiments, we study the effect of the incentive parameters and demand uncertainty on the HSP. These experiments illustrate that the magnitude of the HSP depends on the underlying demand variability. We exploit this observation and propose a mechanism that adjusts the incentive threshold level suitably using a correlated market signal. Specifically, we analytically show that linking the threshold to a correlated market signal is equivalent to a setting where the adjusted market demand is transformed to the difference between the original market demand and the threshold. We prove that this adjusted market demand has lower variance and, consequently, that the HSP can be regulated. The manufacturer typically designs both the contracting parameters (the threshold and the marginal payment) to induce higher sales effort. The novelty of the proposed mechanism is that we use the threshold to additionally regulate the HSP. We also discuss a mechanism to implement such a plan based on market-based sales information.

The rest of this article is organized as follows. First, we provide a literature review in Section 2. We describe the model and related assumptions in Section 3. In Section 4, we characterize the dealer's response for both forms of the incentive contract. We present the value of better information in setting the threshold level in Section 6. Finally, we conclude our article in Section 7. The proofs are provided in the Appendix.

2. LITERATURE REVIEW

The theoretical literature in Economics and Marketing provides evidence that threshold-based incentives are effective (and optimal in some cases) in increasing a manufacturer's expected sales while motivating the dealer to exert effort

[4] We show the similarity in Section 7.

appropriately [2, 11, 17, 20]. However, many of these models are single time period models and hence do not include intertemporal effects of such incentive schemes. The model discussed in this article, however, is a multiperiod model and captures intertemporal effects of sales effort.

Basu et al. [2] studied the sales-force compensation plan for a single period and showed that the optimal compensation plan is an increasing nonlinear function of the sales (BLSS plan). Basu et al. [2] also suggested that, for ease of implementation, a piece-wise linear approximation scheme could be used in practice. Such a scheme closely resembles the threshold-based incentive studied in this article. The analytical approach in Ref. 2 follows the moral hazard and agency theoretic framework adopted in Refs. 6 and 7. Raju and Srinivasan [19] compared the BLSS plan to a simple quota-based plan (fixed pay plus a commission rate) and showed, through numerical experiments, that their simple approximation results in minimal loss of optimality. The significant advantage, as argued earlier, is the simplicity of implementation in practice. Lal and Srinivasan [12] found that a linear compensation scheme may be optimal; however, such a scheme seems impractical because most sales-force incentive schemes in practice are nonlinear. In a related article, Oyer [17] showed that a quota-based plan with a bonus payment is optimal when the sales distribution function is assumed to have an increasing hazard rate. Through field experiments, Joseph and Kalwani [10] demonstrated the growing importance of bonus payments in aligning sales incentives and increasing productivity in a wide variety of firms.

More recently, the literature in operations management has focused on the operational impact of such quota-based contracts [3] and suggested strategies to smoothen the demand process. Chen [3] discussed the impact of sales-force incentives (over multiple time periods) on a manufacturing firm's production and inventory decisions and proposed a moving-window plan to induce selling effort to smoothen the demand process. The HSP is due to the fact that the dealer does not experience any additional cost of postponing the effort decision and, hence, exerts all the effort in the last period. In this case, the dealer waits for the exogenous stochasticity in the demand to resolve itself before making any effort. This results in the sales spike that is observed only in the last period, whereas all the earlier period sales are unaffected. In a following article Chen [4] related sales-force compensation to the manufacturing firm's production and inventory costs and compared Gonik's scheme with a menu of linear contracts. Gonik [5] proposed a scheme to extract maximum effort from the sales-force and induced them to truthfully reveal the underlying demand by forecasting accurately. Xiao [24] studied all-unit target rebates (similar to the Chrysler example) and incremental-unit target rebates (similar to our additional marginal payment) in a single period setting where the manufacturer is a Stackelberg leader specifying the terms of the incentives. In their stylized setting, the dealer makes his effort decision after the market condition is realized. Xiao [24] found that, from the manufacturer's perspective, the expected profit from an all-unit rebate is dominated by a wholesale price contract, which in turn is dominated by an incremental-unit target rebate. As mentioned earlier, in this article, we focus on a multiperiod setting when there is a cost associated with waiting and the underlying market demand is uncertain.

3. MODEL ASSUMPTIONS

In this section, we outline a multiperiod model to investigate the HSP. We consider a setting where a single manufacturer sells her product through a single dealer. The selling price of the product is fixed, i.e., the dealer is not empowered to give price discounts to customers. Thus, the only way for the dealer to grow sales is to exert effort. For example, the dealer could expend selling effort through advertising. As the retail price is held fixed, the dealer makes a standard margin (excluding cost of effort) p for every unit sold. However, to a limited extent, the dealer's ability to increase sales by lowering the retail price (and decreasing his standard margin) is approximated by the additional cost of exerting sales effort to increase sales.

The threshold incentive is offered over T time periods, i.e., the incentive horizon comprises T time periods. In any time period, t ($t = 1, \ldots, T$), the dealer chooses a selling effort e_t resulting in a sales growth of $g(e_t)$, where the impact-of-effort function, $g(\cdot)$, is positive, differentiable, increasing, and concave in the effort e_t. Therefore, the marginal impact of sales effort is diminishing. The sale, S_t, in period t is assumed to be

$$S_t = g(e_t) + X_t, \tag{1}$$

where X_t is the random market demand observed in period t. We assume that the market demand, X_t, follows a continuous and twice differentiable, cumulative distribution function, F, with a probability density function f such that $f(x_t) = 0$ for all $x_t < 0$. We also assume that the random market demand is independent and identically distributed (iid) in every time period t. However, we show that some results hold even if this assumption is relaxed. The distribution of the market demand is common knowledge (publicly known.) Such a sales-response function, as shown in Eq. (1), has also been used by Lal and Staelin [13], Oyer [17], and Lal and Srinivasan [12]. Here, we assumed an additive model, where the period sales is sum of a random demand and an increasing function of the effort deployed by the dealer. In reality, it is plausible that the impact of the effort is more complex (such as a combination of an additive and a multiplicative form); however, the additive response model is commonly used in the previous literature to capture the key essence and also to have analytical tractability. Further, one can envision a more generalized model where the dealer effort results in random increase in demand. If this increase is additive, then this general model can be reduced to the model described above.

The total sales across all the T periods are defined by

$$S = \sum_{t=1}^{T} S_t, \tag{2}$$

and the sales up to any period $t = 1, \ldots, T$ are defined by

$$\mathcal{D}_t = \sum_{j=1}^{t-1} S_j. \tag{3}$$

Let the function $h(\cdot)$ be defined as follows:

$$h(\cdot) \equiv \frac{1}{g'}(\cdot). \tag{4}$$

The function $h(\cdot)$ essentially represents the effort required to raise the demand by one unit. We make the following assumption for the remainder of this article:

ASSUMPTION 1: The function $h(\cdot)$ is increasing and concave.

Assumption 1 is not restrictive. For example, it is satisfied by any concave polynomial function $g(e_t) \propto e_t^{\alpha}$, where $0 < \alpha < 1$. The dealer's cost of exerting effort is assumed to be linear in his effort, i.e.,

$$v(e_t) = \beta e_t. \tag{5}$$

The threshold incentive is organized as follows: for every additional unit sold above the threshold, K, the manufacturer pays the dealer an amount Δ in addition to his standard margin p. In a multiperiod model, K applies to the entire horizon. Thus, the dealer's margin (excluding the cost of effort) increases to $p + \Delta$ for every unit sold above the threshold K. We refer to this contract as the "Δ-contract." In the case of a bonus contract (instead of the additional marginal payment, Δ), the manufacturer offers a fixed bonus, $D > 0$, if the total sales, S, reach or exceed the threshold K. We refer to this bonus contract as the "D-contract" and consider it separately. The incentives offered are represented by their defining parameters, i.e., (p, Δ, K) represents the Δ-contract and (p, D, K) represents the D-contract.

The dealer's objective is to maximize his expected profit, which is a function of the effort exerted, the incentive parameters, and the profit margin. The sequence of events unfolds as follows. Prior to the first time period, i.e., at $t = 0$, the manufacturer offers the threshold incentive. In every time period $t = 1, \ldots, T$, the dealer decides on his optimal effort level $e_t^\star$ and observes the random demand X_t. The dealer orders S_t units from the manufacturer, which also allows the manufacturer to observe the sales in that period. The manufacturer keeps finished-goods inventory and is responsible for replenishing the dealer's inventory. At the end of the last period, i.e., when $t = T+1$, the payoffs are determined. We do not model the manufacturer's objective function explicitly. As mentioned earlier, we draw insights about the impact of such contracts on mean and variance of the observed sales, which hold regardless of the production costs at the manufacturer. For the remainder of this article, we use the following definitions to characterize the sales HSP when the threshold-based incentive horizon spans multiple time periods:

DEFINITION 1: The sales HSP is defined by a sales pattern such that the expected sales in a period, $\mathbb{E}[S_t]$, increase as t increase, i.e., $\mathbb{E}[S_t] \leq \mathbb{E}[S_{t+1}]$, for all t.

The operator, $\mathbb{E}$, represents the expectation over the random variable X_t[5]. However, in some instances, the expected sales may decrease over time in spite of higher dealer effort. We define this situation as the reverse HSP.

DEFINITION 2: The sales HSP is reversed when the sales pattern is such that the expected sales in a period, $\mathbb{E}[S_t]$, decrease as t increases, i.e., $\mathbb{E}[S_t] \geq \mathbb{E}[S_{t+1}]$, for all t.

4. THE GENERALIZED HSP

In this section, we study the sales HSP under both the contract forms, i.e., the Δ-contract and the D-contract. First, we characterize the dealer's optimal response when the incentive horizon consists of $T > 1$ time periods. We set up the dealer's optimization problem as a stochastic dynamic programing problem and show that the dealer's optimal effort levels form a submartingale. We also characterize the conditions under which the sales HSP is nontrivially observed.

4.1. The Dealer's Optimal Response with a Δ-Contract

Consider the case when the manufacturer offers the dealer a (p, Δ, K) contract prior to the first time period. At the beginning of every time period, the dealer must decide on the optimal effort level, e_t, such that his future expected profit in time periods $t, \ldots, T$ is maximized. Recall the definition of

[5] We use X_t as a subscript to indicate over which random variable we are computing the expectation. However, when this is clear from the context we omit it for brevity.

$\mathcal{D}_t$, the sales up to period t, from Eq. (3). We now define the adjusted threshold, K_t, as follows:

$$\begin{aligned} K_t &= \max\{0, K - \mathcal{D}_t\} \\ &= \max\{0, K_{t-1} - X_{t-1} - g(e_{t-1})\}. \end{aligned} \tag{6}$$

Thus, K_t measures the number of units the dealer needs to sell between now and the end the horizon to reach the threshold. By definition $K_1 = K$. The recursive relationship in Eq. (6) follows from the definition of $\mathcal{D}_t$.

First, observe that the dealer will at least exert a minimum effort, $e_{\rm l}^\star$, such that the marginal gain from exerting such an effort is equal to the marginal cost of exerting such an effort.

$$p = \beta\, h\big(e_{\rm l}^\star\big). \tag{7}$$

Furthermore, it is important to also note that if in any period t, $\mathcal{D}_t > K$ (i.e., $K_t = 0$), then, for all time periods $t, \ldots, T$, the dealer exerts a constant effort, $e_{\rm h}^\star$, such that the marginal benefit equals the marginal cost, i.e.,

$$(p + \Delta) = \beta\, h\big(e_{\rm h}^\star\big). \tag{8}$$

Thus, once the total sales surpass the threshold K, the dealer's profit, from all future time periods, is a constant, $\mathcal{C}_t$, given by

$$\mathcal{C}_t = (p+\Delta)(T-t)\big(\mathbb{E}[X_t] + g\big(e_{\rm h}^\star\big)\big) - \beta(T-t)e_{\rm h}^\star. \tag{9}$$

Let $V_t(K_t)$ be the dealer's optimal expected profit from time periods $t, \ldots, T$ beginning in time period t. If $K_t > 0$, then $V_t(K_t)$ is given by the following equation:

$$\begin{aligned} V_t(K_t) = \max_{0 \le e_t} \int_0^{\min\{0, K_t - g(e_t)\}} & \big\{p[X_t + g(e_t)] \\ & + V_{t+1}(K_t - X_t - g(e_t))\big\} f(X_t)dX_t \\ + \int_{K_t - g(e_t)}^{\infty} & \big\{(p+\Delta)[X_t + g(e_t)] - \Delta K_t + \mathcal{C}_t\big\} \\ & \times f(X_t)dX_t - \beta e_t \end{aligned} \tag{10}$$

else, if $K_t = 0$, then $V_t(K_t) = C_t$. The other terminal condition is $V_{T+1}(K_{T+1}) = 0\ \forall\ K_{T+1}$. The first term in Eq. (10) corresponds to the profit when the sale in the current period, S_t, is below the adjusted threshold K_t. As a result of the effort exerted in the current time period, e_t, and the random demand observed thereafter, the adjusted threshold for the next time period, K_{t+1}, reduces to $K_t - X_t - g(e_t)$. Hence, the optimal expected profit from time period $t+1, \ldots, T$ is given by $V_{t+1}(K_{t+1})$. The second term in Eq. (10) corresponds to the case when the sales, S_t, exceed the adjusted threshold K_t. In this case, the total future period profit is a constant $\mathcal{C}_t$, because the total sales have already exceeded the incentive threshold, K, and hence the adjusted threshold $K_j \le 0$ for all time periods, $j = t+1, \ldots, T$. In this case, the dealer exerts a constant effort $e_{\rm h}^\star$. It is easy to verify the concavity of the profit function in the last period, i.e., $t = T$, if the sales up to the last period, $\mathcal{D}_t < K$. However, in general, the expected profit function depends on the concavity of the impact-of-effort function, $g(\cdot)$, and the demand distribution, $F(\cdot)$. To derive conditions on $g(\cdot)$ under which $V_t(K_t)$ is concave, we assume that the probability density function, $f(\cdot)$, is bounded above by a scalar $M < \infty$. Let us denote $\sup_{e_t \in [e_{\rm l}^\star, e_{\rm h}^\star]} \left\{\frac{g''(e_t)}{g'(e_t)^2}\right\}$ by M_2. It is easy to verify that $M_2 < \infty$ for polynomial functions such e_t^α, where $0 < \alpha < 1$. Consequently, if $p\, M_2 \le -\Delta M$ then $V_t(K_t)$ is a concave function. We formalize this assumption:

ASSUMPTION 2: There exist M_2 and M such that $p\, M_2 \le -\Delta M$, where $M_2 = \sup_{e_t \in \left[e_{\rm l}^\star, e_{\rm h}^\star\right]} \left\{\frac{g''(e_t)}{g'(e_t)^2}\right\}$ and M is bounded on the probability distribution function for the demand.

Loosely speaking, $|M_2|$ measures the curvature of the function g and requires that it is suitably large to ensure that the value function is concave. In Lemma 5 in the appendix section, we show that if Assumption 2 holds, then $V_t(K_t)$ is concave.

Using Eq. (10) we characterize the dealer's optimal effort in Lemma 1. Let $\mathbf{1}_{\{a>b\}}$ denote the indicator function that is true when the condition $a > b$ is satisfied. Further, let $\mathbb{E}_{X_t,\ldots,X_T}$ denote the expectation operator over all the random variables X_t through X_T.

LEMMA 1: Given a (p, Δ, K) contract the dealer's optimal effort, $e_t^\star$, in any time period $t < T$ is characterized by the following optimality condition

$$h\big(e_t^\star\big) = \frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g(e_j^\star)+X_j\right) > K_t\right\}}\right]\right). \tag{11}$$

Further, if Assumption 2 holds, then there is a unique solution to (11).

Observe that, in Lemma 1, the left-hand term is an increasing function (by Assumption 1) of effort e_t. Similarly, the right-hand side is also an increasing function of e_t. It is possible to find at least one solution to Eq. (11). Assumption 2 provides sufficient conditions under which there is a unique optimal effort.

Lemma 1 allows us to characterize the relationship between the dealer's optimal effort in any two consecutive periods. We state the results, which follow directly from Lemma 1, in Lemma 2.

LEMMA 2: Given a (p, Δ, K) contract the dealer's optimal efforts in any two consecutive time periods are related as follows:

$$h\big(e_t^\star\big) = \mathbb{E}\left[h\big(e_{t+1}^\star\big)\right] \quad \forall \quad 1 \le t < T. \tag{12}$$

Furthermore, the dealer's optimal effort is bounded as follows:

$$e_l^\star \le e_t^\star \le e_h^\star \quad \forall \quad 1 \le t \le T. \tag{13}$$

Lemma 2 essentially states that, given a Δ-contract, the dealer exerts optimal effort such that the marginal cost of increasing demand by one unit is equalized across all the periods. A priori, given that the impact-of-effort function is concave, the HSP is not an obvious outcome. However, if Assumption 1 holds, then the dealer's expected sales effort in any time period is stochastically dominated by the expected sales effort in future time periods. We state the dominance relationship, between the dealer's expected optimal effort in any two consecutive time periods, in Theorem 1.

THEOREM 1: The dealer's optimal effort in time period t is stochastically dominated by the optimal effort in period $t+1$ for all $t = 1, \ldots, T-1$, i.e.,

$$\mathbb{E}[e_{t+1}^\star] \ge \mathbb{E}\big[e_t^\star\big] \quad \forall \ t = 1, \ldots, T-1. \tag{14}$$

A consequence of Theorem 1 is that the dealer's optimal efforts form a submartingale. If the demand signals, X_t $(t = 1, \ldots, T)$, are assumed to be iid, then the expected sales, $\mathbb{E}[S_t] = \mathbb{E}[g(e_t^\star) + X_t]$, in each time period also follow a similar stochastic dominance relationship resulting in the sales HSP. However, even if we do not assume that the demand signals are iid, we show that the sales HSP exists under certain conditions in Theorem 2.

THEOREM 2: If the composite function $g \circ h^{-1}$ is convex, the sales HSP exists. However, if $g \circ h^{-1}$ is concave, then we observe a reversal in the sales HSP (see Definition 2) even though the dealer's optimal efforts follow the stochastic dominance relationship shown in Eq. (14).

To gain some intuition into Theorem 2, observe that the composite function $g \circ h^{-1}$ measures the relative concavity of the impact-of-effort function $g(\cdot)$. Essentially, the concavity of $g(\cdot)$ directly results in the sales HSP or its reversal. For example, consider any polynomial impact-of-effort function e_t^α, where $0 < \alpha < 1$. It is easy to verify that, for $0 < \alpha < \frac{1}{2}$, the composite function $g \circ h^{-1}$ is concave, but for all other values of α it is convex. Thus, the sales HSP is observed in the former case, whereas for the latter case, the reverse HSP is observed.

Theorems 1 and 2 are generalizable to the setting where the demand across time periods is not iid. To this end, consider a filtration $\mathcal{F} = \{\mathcal{F}_n : n = 1, 2, \ldots, T\}$ and assume that the demand process is adapted with respect to this filtration. Thus, at the end of period m, the demand distribution for period n is the conditional distribution of D_n w.r.t. to $\mathcal{F}_m$. This model captures the fact that the demand from period to period is dependent and also that the dealer can update his belief about the demand in future time periods as he gets closer to that time period. If $e_t^\star$ represents the optimal effort of the dealer in time period t, then Theorem 3 summarizes the effort dominance result.

THEOREM 3: The dealer's optimal effort process $\{e_t : t = 1, \ldots, T\}$ forms a submartingale with respect to the filtration $\mathcal{F}$, i.e.,

$$\mathbb{E}\big[e_{t+1}^\star | \mathcal{F}_t\big] \ge e_t^\star \quad \forall \ t = 1, \ldots, T-1. \tag{15}$$

It is noteworthy that, as a consequence of Theorem 2, the dealer's order levels (or sales) form either a submartingale (exhibiting the HSP) or a supermartingale (exhibiting a reverse HSP) depending on the composite function $g \circ h^{-1}$.

4.1.1. Sales Variance with a Δ-Contract

A further consequence of Theorem 2 is that the manufacturer experiences a higher variance in the orders placed by the dealer when threshold-based contracts are offered across multiple periods. We now characterize the variance of total sales, $\mathbb{V}[S]$, observed by the manufacturer.

THEOREM 4: Suppose the market demand in every period is iid with variance σ^2. Then, $\mathbb{V}[S] \ge T\sigma^2$.

Theorem 4 suggests that the observed variance of total sales is larger than that of the underlying market demand. This follows from the fact that the random demand in period t, X_t, is positively correlated with the random optimal effort, $e_{t+1}^\star$, in period $t+1$, i.e., $\text{cov}(X_t, e_{t+1}) \ge 0$. Such variance can pose problems for the manufacturer. For example, inventory and production scheduling becomes challenging see Ref. 3 or transportation logistics costs could be higher.

If the resulting operational costs are high enough, the manufacturer may be better off offering a constant-margin contract. In such a situation, the manufacturer may lose on higher expected profit but gain on lower variance. The manufacturer may also choose to devise other strategies that enable her to reduce the impact of the sales HSP by adjusting the Δ-contract parameters appropriately. In Section 6, we discuss how additional information and the use of correlated signals can help in such situations.

4.2. Offering a Bonus Contract

In this section, we study the other form of prevalent threshold-based contracts in which the manufacturer offers the dealer a bonus payment, D, on reaching the threshold K, instead of an additional marginal payment of Δ. We denote the bonus contract by its parameters (p, D, K). We first characterize the dealer's optimal response when the incentive horizon consists of $T > 1$ time periods. Similar to the Δ-contract, we set up the dealer's optimization problem as a dynamic programing problem and show that the dealer's optimal effort levels form a submartingale. We also characterize the conditions when sales HSP is observed.

4.2.1. The Dealer's Optimal Response with a D-Contract

Consider the case when the manufacturer offers the dealer a (p, D, K) contract prior to the first time period. Similar to the Δ-contract, we can express the dealer's optimal expected profit function as a DP formulation for the D-contract too. When $K_t \geq 0$, under a D-contract, the optimal expected profit function, $V_t(K_t)$ is as follows:

$$\begin{aligned} V_t(K_t) = \max_{0 \leq e_t} \quad & p\,\mathbb{E}[X_t + g(e_t)] \\ & + \int_0^{\min\{0, K_t - g(e_t)\}} V_{t+1}(K_t - X_t - g(e_t)) f(X_t) dX_t \\ & \quad + \int_{\min\{0, K_t - g(e_t)\}}^{\infty} [D + \mathcal{C}_t] f(X_t) dX_t - \beta e_t, \quad (16) \end{aligned}$$

where $\mathcal{C}_t$ is now defined as

$$\mathcal{C}_t = p(T-t)\big(\mathbb{E}[X_t] + g\big(e_l^\star\big)\big) - \beta(T-t)e_l^\star. \quad (17)$$

In Eq. (17) the effort level $e_l^\star$ is defined by Eq. (7). Furthermore, $V_t(K_t)$ takes a value of 0 when $K_t = 0$. We characterize the dealer's optimal effort in Lemma 3. We omit the proof as it is very similar to the proof of Lemma 1.

LEMMA 3: Given a (p, D, K) contract the dealer's optimal effort, $e_t^\star$, in any time period $t < T$ is characterized by the following optimality condition:

$$h\big(e_t^\star\big) = \frac{1}{\beta}\left(p + D\mathbb{E}_{X_t,\ldots,X_T} \times \left[f_{X_T}\left(K_t - \sum_{j=t}^{T-1} X_j - \sum_{j=t}^{T} g\big(e_j^\star\big)\right)\right]\right). \quad (18)$$

Using Assumption 2 and Lemma 3, it is easy to verify that the dealer's optimal efforts form a submartingale similar to the one discussed earlier in Theorem 1. Therefore, Lemma 2 holds in this case too, and the sales HSP is also observed with a D-contract. Furthermore, Theorem 2 continues to hold for the bonus contract.

4.2.2. Sales Variance with a D-Contract

Unlike the Δ-contract case, we are not guaranteed a positive covariance between the optimal effort in a period and the random demand in the previous period. In some cases, the covariance could be negative, and hence the variance of total sales, $\mathbb{V}[S]$, could be lower than that of the underlying demand. We show an example when $T = 2$.

An Example. Let the demand be uniform between 0 and M. The second period effort then solves the following optimality equation:

$$h\big(e_2^\star\big) = \frac{1}{\beta}\big[p + Df_U\left(K - X_1 - g\big(e_1^\star\big) - g\big(e_2^\star\big)\right)\big],$$

where $f_U(\cdot)$ is the probability density function for the uniform distribution with support $[0, M]$. Thus, one can verify that there exists a threshold C such that if $X_1 > C$, then the optimal $e_2^\star$ is given by $h(e_2^\star) = \frac{1}{\beta}p$, else it is given by $h(e_2^\star) = \frac{1}{\beta}\left[p + \frac{1}{M}\right]$. Hence, $e_2^\star$ is negatively correlated with X_1, and because $g(e_2^\star)$ is an increasing function of $e_2^\star$, the overall variance of the sales in this setting is reduced.

5. WHAT DRIVES THE HSP?

In Section 4, we showed that the manufacturer observes a larger sales variance (when compared with the variance of the underlying demand) with a threshold-based contract due to dealer's optimal effort decisions. In this section, our goal is to study what drives the dealer's effort decisions and its impact on the magnitude of the HSP. Developing an analytical solution is extremely challenging, and hence, we resort to numerical experiments to analyze the magnitude of the resulting HSP.

For all these experiments, we consider both forms of the threshold-based contracts with $p = 1$, $g(e_t) = 2(e_t)^{0.8}$, $v(e_t) = e_t$, and $T = 6$. The market demand in each time period is assumed to be independent and uniformly distributed between 1 and 100. The first set of experiments consider the impact of the incentive parameters. Figure 1 shows the effect of varying the threshold level K under a Δ-contract. For these experiments, we keep Δ fixed at 0.4. In Fig. 1, the numbers besides the plots represent different threshold levels. It is evident that the HSP is more pronounced (magnitude is larger) at intermediate levels of K, i.e., between 450 and 500. In this range, the ratio $\mathbb{E}\left[\frac{g(e_6^\star)}{g(e_1^\star)}\right]$ is 6% at $K = 450$, 11% at $K = 475$, and 7% at $K = 500$. At lower values of K,

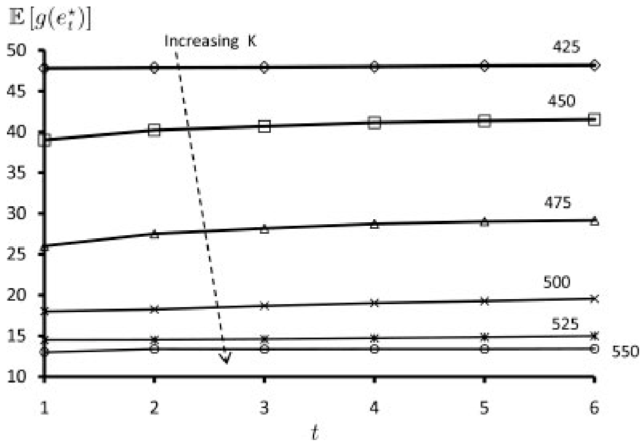

Figure 1. The HSP for a Δ-contract for different levels of K ($\Delta = 0.4$).

the HSP is diminished with the dealer exerting higher effort consistently across all the six periods. Similarly, at higher values of K the dealer exerts lower effort levels consistently, and the HSP is less pronounced. Consequently, the manufacturer faces a critical tradeoff between choosing an appropriate threshold level to induce higher effort while paying a low premium and controlling the sales variance.

Figure 2 shows the results of the experiment with a D-contract. The numbers besides the plots represent the threshold level K. As is readily observed in this case, the HSP is present for all threshold levels. Furthermore, it is noteworthy that, for any time period t, as K increases from 425 to 475, the effort level also increases. However, as K increases further, from 500 to 550, the effort level starts decreasing. Essentially, as K increases, the effort level, in any time period t, first increases and then decreases (nonmonotone).

Even with the D-contract, similar to the case with the Δ-contract, the same tradeoff exists for the manufacturer while choosing K. However, it is important to note that, in the case of Δ-contract, the effort the dealer exerts when K is very low and very high are very different, whereas for the bonus contract these levels would be the same. This is due to the fact that, once the quantity sold exceeds the threshold, the marginal gain is different in the Δ-contract, whereas in the D-contract the marginal gain is identical. Because of this we observe that the effort is monotone in the threshold for the Δ-contract; it is not the case for the bonus-based D-contract. The choice of the appropriate incentive threshold is different, yet remains crucial in controlling the HSP, for both forms of the contract. Sinha and Zoltners [22] and Zoltners et al. [25] corroborated this fact through several other qualitative arguments.

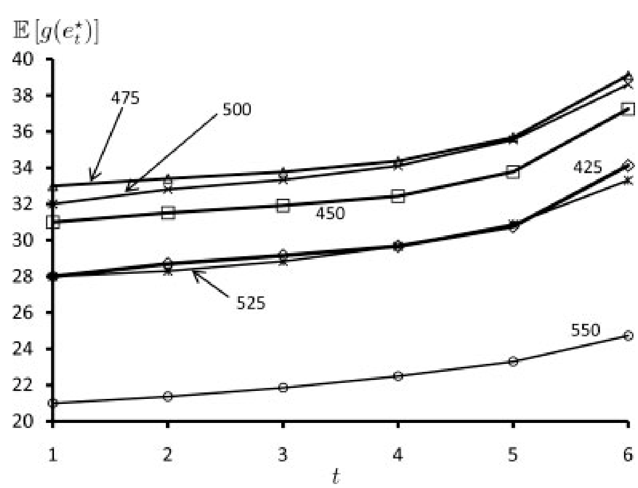

Figure 2. The HSP for a D-contract for different levels of K ($D = 50$).

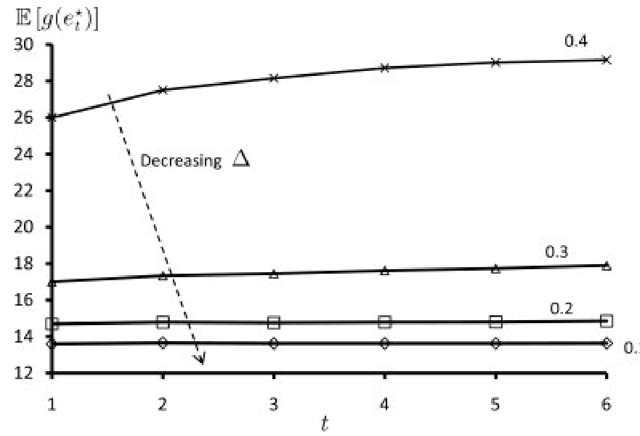

Figure 3. The HSP for a Δ-contract for different levels of Δ ($K = 475$).

Figures 3 and 4 measure the impact of varying Δ and D, respectively. In these sets of experiments, the threshold level K is fixed at 475. The numbers besides the plots in these figures indicate the values of Δ and D, respectively.

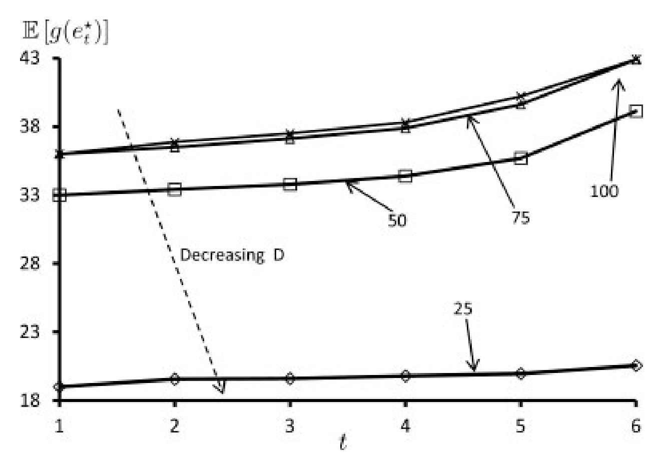

Figure 4. The HSP for a D-contract for different levels of D ($K = 475$).

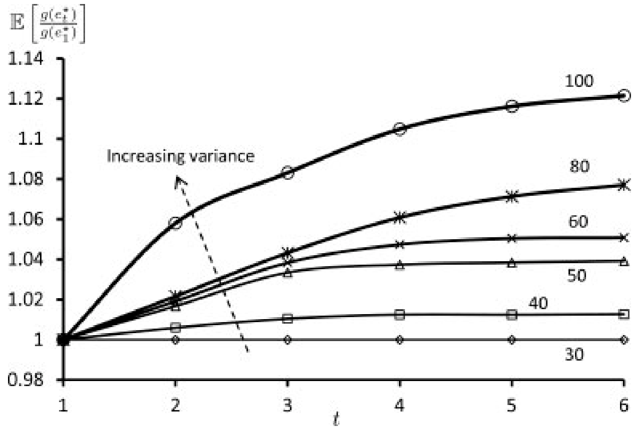

Figure 5. Increase in the HSP as variance increases for a Δ-contract ($K = 475$).

It is readily observable that as Δ and D increase, the HSP is pronounced (larger in magnitude), whereas it is diminished for lower levels. From a manufacturer's perspective, a higher marginal or bonus payment induces higher dealer effort.

An important observation in all the earlier experiments is the effect of the underlying demand uncertainty on the dealer's optimal effort. We study this effect of demand uncertainty on the resulting sales variance in the following numerical experiments. For these experiments, we hold K fixed at 475 and calculate the demand generated by the sales effort for a family of uniform distributions: $[\ell, 100 - \ell]$, where $\ell \in \{0, 10, 20, 25, 30, 35\}$. Figure 5 shows the results for the Δ-contract, and Fig. 6 shows the results for the D-contract.

In these figures, we plot $\mathbb{E}\left[\frac{g(e_t^*)}{g(e_1^*)}\right]$ against the time periods. The numbers besides the plots indicate the variance of the underlying demand. These numerical experiments clearly demonstrate that, as the variance of the demand distribution decreases, the magnitude of the dealer effort is also diminished. Consequently, the magnitude of the HSP is also diminished.

On the basis of the numerical study, we develop the following observations about the relationship between the threshold-based contract and the magnitude of the HSP.

1. We observe that the HSP is pronounced for moderate levels of K. This is due to the fact that if the threshold K is too small, or too large, then the dealer has an incentive to exert a constant effort over time. If K is too small, then the dealer realizes that he will be able to get the bonus under most demand realizations, whereas if K is too large, then the dealer realizes that, even if he puts in extra effort, the chances that he would be able to earn the addition bonus D or the additional margin Δ are very slim. Thus, he exerts almost constant effort for such thresholds.
2. As expected, we observe that if the additional bonus D or the additional margin Δ increases the HSP is pronounced. This is in line with Lemmas 1 and 3. Thus, even though the threshold-based contracts induce higher efforts, they also cause the HSP in our model.
3. We observe that, for demand distributions with higher variance, the magnitude of the HSP is larger. The numerical experiments with a uniform distribution demonstrate this effect, and this holds for other general demand distributions as well. It is noteworthy that, if there was no variability in demand, the dealer would exert a constant effort, and there would be no additional variability in sales.
4. Lastly, it is interesting to note the HSP is also driven by the shape of the effort function $g()$. Consider the case when $g(e_t) = e_t^{\alpha}, 0 < \alpha \leq 1$ and $\alpha < \frac{1}{2}$. In this case, we observe that, even though the dealer's effort is increasing, the sales HSP is reversed. In contrast, if $\alpha > \frac{1}{2}$, we observe the HSP for both the effort as well as the sales. This observation is in line with Theorem 2. The HSP is more severe as α increases, and in the special case when $\alpha = 1$, the dealer finds it optimal to exert all his effort in the last period when all the uncertainty in demand is resolved. The special case is similar to the setting studied in Ref. 3, where the dealer holds back on exerting effort until the last period.

These insights have important implications for the manufacturer. First, if the demand variability is reduced, the variability of observed sales can be regulated. Thus, the manufacturer should consider investing in instruments that help in controlling the underlying demand variability. This idea is similar to examples in the operations management literature that describe benefits of the reduction in demand variability

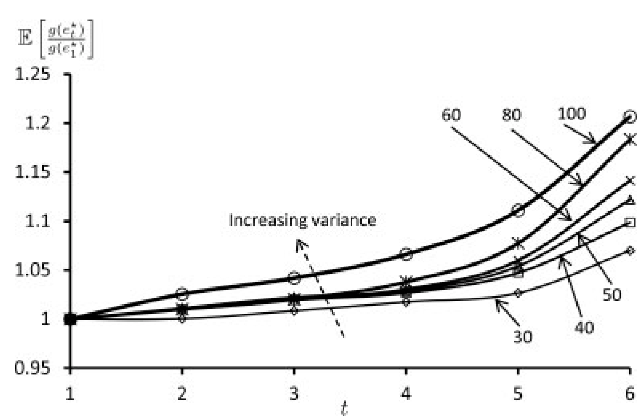

Figure 6. Increase in the HSP as variance increases for a D-contract ($K = 475$).

by modifying process or product attributes [14, 15]. Second, the manufacturer must design the incentive parameters appropriately to control the HSP while continuing to induce higher dealer effort (and expected sales). Consequently, an important question for the manufacturer is how to achieve both these goals simultaneously, i.e., control the underlying demand variability and hence the HSP (sales variance) and adjust the incentive parameters appropriately to induce higher dealer effort. We study this more extensively in the next section.

6. VALUE OF BETTER INFORMATION

It is well known in the supply chain literature that one way of aligning incentives [16] is to track additional business variables and use this additional information in designing the incentive. We build on this idea. Specifically, we show that when the manufacturer observes additional business variables, correlated with the underlying market demand, she is able to dampen the sales HSP. Recollect the numerical experiments in Section 5. We observe that by adjusting K and Δ (or D) appropriately, the manufacturer is able to induce higher dealer effort. In our proposed mechanism, we choose one of these levers to additionally regulate the HSP. The threshold K is a natural choice to do so because it is representative of the desired sales volume. Our goal in this section is to show that the manufacturer can achieve a reduction in observed sales variance by linking the threshold K to the underlying demand appropriately.

To accomplish this we consider a setting where there is a signal ζ_t commonly observable over the time horizon. Further, we assume that this signal is verifiable and contractible, i.e., the manufacturer can base the dealer's payoff on this signal. For the purpose of the threshold-based contract, the manufacturer can use the refined information of the underlying demand instead of simply its mean. It is noteworthy that examples of such market signals include macroeconomic indices observable to all entities in the economy.

To illustrate this point, for the purposes of our incentive, let us assume that the manufacturer sets the incentive threshold $K = \sum_{t=1}^{T} \bar{X}_t + \bar{K}$, where $\bar{X}_t$ is the mean demand in each time period. The idea behind using such a threshold is the fact that the manufacturer wants to reward the dealer only once he gets $\bar{K}$ units above the mean. Loosely speaking, such a threshold rewards the dealer for his own efforts and not just for the variability. Given a market signal ζ_t, let us define $Y_t = \mathbb{E}[X_t|\zeta_t], \forall t$. The manufacturer then sets the threshold level in the contract to be

$$K \equiv \sum_{t=1}^{T} Y_t + \bar{K}. \qquad (19)$$

In Eq. (19), Y_t measures the manufacturer's best estimate of the sales. It is interesting to note that, if there is no information, (19) reduces to the setting where the threshold is deterministic. Equation (19) is based on the idea that the manufacturer does not know the true demand realization yet would like to reward the dealer for the extra effort he exerts and not simply for a favorable draw in terms of demand. Thus, she defines a contract where the exact value of the threshold is computed at the end of the horizon using (19). However, she declares this method of computing the threshold to the dealer a priori. Essentially, in this setting, both the manufacturer and the dealer use a collaborative sales forecast (since both can observe Y_t) along with the computation rule (19), which the manufacturer announces at the beginning of the incentive period. Thus, at every intermediate time period, the dealer can form estimates about what the threshold will be at the end of the incentive horizon and choose the optimal effort to exert in each period. Note that as the signal varies from period to period, the dealer updates his estimate about the threshold K. This scheme is fair to the dealer too, as he is being rewarded/penalized for his effort and not for the stochastic fluctuations in the demand.

The sequence of events is the same as before except that, instead of providing the dealer with a fixed threshold, the manufacturer shares the threshold computation scheme [Eq. (19)] with the dealer. The remaining sequence is just as before. Notice that the dealer continues to observe his private market demand X_t as well as the correlated signal ζ_t (whereas the manufacturer only sees the latter) and decides on his optimal effort based on the difference (error), $Z_t = X_t - Y_t$. We now show that the optimal effort exerted by the dealer under this market signal-based contract is identical to the effort exerted under a constant threshold but with a different underlying demand.

PROPOSITION 1: The optimal effort $\{e_t^\star : t = 1, \ldots, T\}$ exerted by the dealer under the contract with $K = \sum_{t=1}^{T} Y_t + \bar{K}$ facing a demand $\{X_t : t = 1, \ldots, T\}$ is identical to the optimal effort exerted by the dealer under the contract with $K = \sum_{t=1}^{T} \bar{X}_t + \bar{K}$ facing a demand $\{X_t - Y_t + \bar{X}_t : t = 1, \ldots, T\}$[6].

Thus, Proposition 1 suggests that using the market signal ζ_t can be viewed as replacing the demand distribution by $\{X_t - Y_t + \bar{X}_t : t = 1, \ldots, T\}$. Consequently, by controlling the variance of the modified demand, the HSP can be effectively regulated. To illustrate this point further, we next consider the case when ζ_t has perfect information about

[6] Here, we allow $X_t - Y_t$ to be negative. For the setting where the distribution X_t is bounded by a constant M, one can translate the problem as follows: the threshold offered is $K + MT$, and the demand in each period is $\{X_t - Y_t + \bar{X}_t + M : t = 1, \ldots, T\}$.

the underlying demand signal, i.e., Y_t and X_t are perfectly correlated. We refer to this case as the ideal contract.

6.1. The Ideal Contract

When ζ_t has perfect information about the demand, the dealer's optimization problem remains the same as before [see Eq. (10)]. First, we state the result that characterizes the dealer's optimal action under perfect correlation. Specifically, under this ideal contract the HSP is eliminated completely. Recall our earlier definitions of the low effort level, $e_{\mathrm{l}}^{\star}$, and the high effort level, $e_{\mathrm{h}}^{\star}$, from Eqs. (7) and (8), respectively. Further, let us denote the threshold value where exerting $e_{\mathrm{l}}^{\star}$ in each period generates the same expected profit as exerting $e_{\mathrm{h}}^{\star}$ by $\tilde{K}$. We can then express $\tilde{K}$ as follows:

$$\tilde{K} = T\left[g\left(e_h^{\star}\right) - \frac{p\left(g\left(e_l^{\star}\right) - g\left(e_h^{\star}\right)\right) - \beta\left(e_l^{\star} - e_h^{\star}\right)}{\Delta}\right].$$

The next proposition summarizes the optimal action taken by the dealer facing a threshold in the ideal contract.

PROPOSITION 2: Suppose Y_t is perfectly correlated with X_t, $\forall\ t = 1, \ldots, T$. With a Δ-contract the dealer's optimal effort, in time period t, is as follows:

$$e_t^{\star} = \begin{cases} e_l^{\star} \ :\ \bar{K} > \tilde{K} \\ e_h^{\star} \ :\ \bar{K} \leq \tilde{K} \end{cases}.$$

Proposition 2 states that the dealer either exerts a low effort, $e_{\mathrm{l}}^{\star}$, or a high effort, $e_{\mathrm{h}}^{\star}$, in all time periods. Thus, irrespective of the realizations of the underlying market demand, X_t, the HSP is completely eliminated. Moreover, there exists a cutoff value $\tilde{K}$ such that if the manufacturer chooses a sufficiently high $\bar{K}$ (i.e., $\bar{K} > \tilde{K}$), then the dealer chooses to exert lesser effort. However, for lower values of $\bar{K}$ the dealer's optimal effort is high. The intuition behind Proposition 2 is as follows: as the impact-of-effort function, $g(\cdot)$, is concave and the cost-of-effort function is linear, it is optimal for the dealer to smoothen the total effort equally between all the periods.

As the impact-of-effort function, $g(\cdot)$, is an increasing function, the following inequality relating the optimal efforts across all the periods must hold:

$$T\ g\left(e_l^{\star}\right) \leq \sum_{t=1}^{T} g\left(e_t^{\star}\right) \leq T\ g\left(e_h^{\star}\right).$$

It is easy to see that the choice of $\bar{K}$ (which depends on the dealer's impact-of-effort function) controls the total sales in the perfectly correlated case. In Proposition 3, we show how a manufacturer gains when the signal is perfectly correlated. But first, we define path-wise dominance:

DEFINITION 3: Given two random variables Z_1 and Z_2, we say Z_1 dominates Z_2 path-wise if $Z_1 \geq Z_2$ for all realizations of Z_1 and Z_2.

PROPOSITION 3: If the manufacturer uses a perfectly correlated signal, then

1. If $\bar{K} < \tilde{K}$, then the total sales across all the periods are path-wise higher compared with any threshold and any signal used by the manufacturer.
2. However, if $\bar{K} > \tilde{K}$, then the total sales are path-wise smaller compared with any threshold and any signal used by the manufacturer.

From Propositions 2 and 3, we observe the following: setting a threshold under the ideal contract (i.e., based on a perfectly correlated signal) allows the manufacturer to gain by (i) eliminating the HSP and (ii) increasing total expected sales. However, the manufacturer needs to pay a premium starting at a relatively low threshold to obtain this high sales and low variance. In contrast, if the threshold is set to a high level that is beyond $\tilde{K}$, then the sales volume is low but there would be no sales HSP. Thus, using a perfectly correlated signal eliminates the HSP. In addition, the threshold chosen by the manufacturer dictates whether the sales observed would be enhanced or not.

6.2. Towards the Ideal Contract (Partially Observable Case)

In reality, however, choosing a market signal that is perfectly correlated for such an ideal contract may be difficult. Next, we consider the setting when ζ_t has some information about the demand but it is not perfect information. We next show that even under partial correlation, variance of $Z_t = X_t - Y_t$ is lower.

LEMMA 4: The variance of $Z_t = X_t - Y_t$ is less than variance of X_t for every $t = 1, \ldots, T$.

As a consequence of Lemma 4 and Proposition 1, we note the fact that if the manufacturer uses such a contractible signal to set $\bar{K}$, then the dealer's optimal efforts are identical to the case where the demand variability is reduced. Thus, the HSP can be effectively regulated as shown by our numerical experiments (Figs. 5 and 6) in Section 5.

However, it is challenging to derive closed-form analytical expressions for the reduction in variance and dampening of the HSP in a general setting. We demonstrate the effect of partial correlation through numerical experiments. For the numerical experiments, we simulate a Δ-contract offered over two time periods, i.e., the case when $T = 2$. We assume that $Z_t = (X_t - Y_t)$, where $t \in \{1, 2\}$ is normally distributed

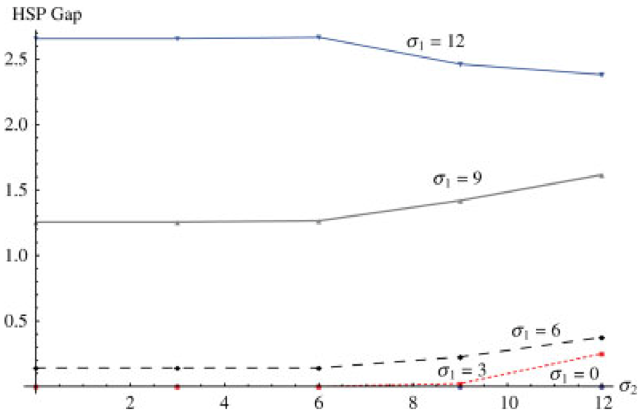

Figure 7. The HSP gap when $T = 2$ for different σ_1 and σ_2 ($p = 1$, $\Delta = 0.8$, $\bar{K} = 10$, $g(e_t) = 8\sqrt{e_t}$). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

with mean 50. We vary the standard deviations σ_t in both periods, from 12 to 0 in steps of 3, i.e., $\sigma_t \in \{0, 3, 6, 9, 12\}$. The impact-of-effort function is defined as $g(e_t) \equiv 8\sqrt{e_t}$. The linear cost of exerting effort is defined as $v(e_t) = e_t$. The dealer's constant margin $p = 1$ and the additional marginal profit Δ is set to 0.8. The value of $\bar{K}$, in Eq. (19), is set to 10. Figure 7 summarizes the results of our numerical experiments. In Fig. 7, we plot the HSP gap between the efforts across the two periods versus σ_2. The HSP gap is measured by the difference between the expected optimal efforts in the two periods. Values are plotted for different levels of σ_1. As observed, for any level of σ_2, as σ_1 decreases the effort gap across the period diminishes, implying that the HSP also diminishes. For the perfectly correlated case, i.e., when $\sigma_1 = 0$, the HSP vanishes. In general, as the variance of the error term, Z_t, increases, the HSP is larger. These numerical experiments demonstrate the benefits of setting the threshold based on a correlated market signal.

7. DISCUSSION

In this article, we show that the sales HSP is nontrivially observed when a manufacturer offers a quota-based contract (either a Δ-contract or a D-contract) over multiple periods. We draw insights about the impact of these contracts on the mean and variance of the observed sales, which hold regardless of the production costs at the manufacturer. We identified, through analytical results and numerical experimentation, two important reasons for the observed sales pattern: (i) the choice of the threshold level and (ii) the underlying demand uncertainty. As a consequence, the variance of the resulting sales pattern is higher for the manufacturer. This is especially true with a Δ-contract. Under such circumstances, the manufacturer faces a key tradeoff between higher expected sales versus larger variance in the orders placed by the dealer throughout the selling season.

In Section 1, we described Chrysler's quota-based incentive scheme[7]. In response to this incentive scheme, several Chrysler dealers complained [23] that the sales targets were not based on market demand but "on the amount of cars Chrysler had jammed down the regional business center's throat." In this case, the Chrysler dealers were complaining that threshold levels were too high, and the incentive was not designed to elicit higher sales effort given the depressed market conditions. As anecdotal evidence suggests that a manufacturer should choose a threshold level that aligns the dealer's sales effort appropriately given the market conditions. As we shown in this article, it is possible to do so and simultaneously regulate the HSP by linking the threshold to a correlated market signal. The transformed incentive is equivalent to a setting where the underlying market demand variance is reduced.

Sinha and Zoltners [22] and Zoltners et al. [25] provided an elaborate discussion on why setting incorrect sales goals can be very costly to a firm. They argue that using statistical techniques might help in designing better goals. For example, estimating future sales using positively correlated competitor information or correlated surrogates for market potential (see Chapter 9, Ref. 25) might help in setting better targets. We build on these ideas. In our setting, one possible way to implement such a mechanism is to link a dealer's incentive threshold to the sales of other dealers (a market-based incentive scheme). Such a scheme allows the manufacturer to link a dealer's incentive threshold to a correlated market signal without the dealer's effort and information influencing its computation. This idea is similar to the setting where a manufacturer decides on the wholesale price for retailers based on information received from other retailers [9,21]. For example, consider the case when a manufacturer works with M dealers and each dealer is in an independent geographical area where there are no (minimal) competition effects. If the sales from the dealers are publicly disclosed, then it is plausible that the incentive threshold of each dealer can be linked to the average (or sum of) sales of other dealers. $\bar{K}$ could be chosen appropriately to spur effort. Note that here we use the fact that the threshold value for a particular dealer is independent of his own effort, and the analysis performed in Section 4 holds. Essentially, the average of sales of all the other dealers is correlated with the underlying demand. The manufacturer must disclose this threshold computation

[7] The Chrysler incentive scheme described in Section 1 is essentially a combination of a Δ- and D-contract, i.e. a (p, Δ, D, K) contract. In this particular instance, $D = \Delta\ K$, and the dealer receives the additional marginal payment of Δ only on reaching and surpassing the sales threshold K.

scheme to the dealer a priori, and both parties could enforce adherence to this scheme through a contractual agreement.

In conclusion, in this article, we show that threshold incentives are poorly designed if the threshold level is unrelated to the underlying market demand. The findings of this article can be extended in multiple directions. In the current setting, we consider a single manufacturer selling through a single dealer. We assume that the manufacturer is not capacitated. It is plausible that in a capacitated system, because of lost sales, the HSP would be dampened. However, the analysis in this article provides an upper bound on the magnitude of the HSP. Another important issue not considered in our setting is the presence of multiple dealers. This will change the nature of the analysis. Similarly, dealer competition may also affect the effort exerted by a dealer in earlier time periods. Whether the effort exerted by a competing dealer aids or intensifies competition will also impact the resulting sales surge observed by the manufacturer.

Additionally, in our setting, we have ignored issues related to inventory and lead time. Considering these issues will change the analysis and provide additional avenues for future research. An important characteristic worth exploring is the setting where the dealer can carry inventory and incurs holding cost. In such a case, apart from deciding how much effort should he exert, the dealer must decide on how much inventory to buy from the manufacturer. In this setting, we expect that if the dealer's ordering frequency is low, then the HSP would be dampened due to the fact that the firm experiences holding cost on the products and would be less hesitant to exert effort to generate demand. However, if the ordering frequency is high, then the results are not clear and are worth studying.

APPENDIX: PROOFS

PROOF FOR LEMMA 1: The optimal profit function is given by

$$V_t(K_t) = \max_{0\le e_t} \int_0^{K_t-g(e_t)} \{p[X_t+g(e_t)]+V_{t+1}(K_t-X_t-g(e_t))\} f(X_t)dX_t + \int_{K_t-g(e_t)}^{\infty} \{(p+\Delta)[X_t+g(e_t)] - \Delta K_t + \mathcal{C}_t\} f(X_t)dX_t - \beta e_t.$$

To compute the optimal effort, we will need to first compute the partial derivative of $V_t(K_t)$ with respect to K_t, i.e., $\frac{\partial V_t(K_t)}{\partial K_t}$. To do so, we first look at the maximum achievable profit in the T^{th} period, which is given by:

$$V_T(K_T) = \max_{0\le e_T} \int_0^{K_T-g(e_T)} \{p[X_T+g(e_T)]\} f(X_T)dX_T + \int_{K_T-g(e_T)}^{\infty} \{(p+\Delta)[X_T+g(e_T)] - \Delta K_T\} f(X_T)dX_T - \beta e_T.$$

Observe that the optimal effort $e_T^\star$ is a function of K_T and $V_{T+1}(K_{T+1}) \equiv 0$. First, we compute the last-period optimal effort, $e_T^\star$, as follows. Using the first-order optimality conditions it is easy to verify that

$$e_T^\star = \begin{cases} e_T^\star : \beta h(e_T^\star) = \left[p + \Delta\left(1 - F\left(K_T - g(e_T^\star)\right)\right)\right] & \text{when } \mathcal{D}_t < K \\ e_h^\star : (p+\Delta) = \beta h(e_h^\star) & \text{when } \mathcal{D}_t \ge K. \end{cases} \tag{20}$$

Notice that when $K_T \le 0$, the partial derivative of $V_T(K_T)$ with respect to K_T, $\frac{\partial V_T(K_T)}{\partial K_T}$, is zero. However, when $K_T > 0$, then we must use the Envelope Theorem to compute the derivative. Using the Envelope Theorem, it is easy to verify that

$$\frac{\partial V_T(K_T)}{\partial K_T} = \begin{cases} -\Delta \mathbb{E}_{X_T}\left[\mathbf{1}_{\left\{g\left(e_T^\star\right)+X_T>K_T\right\}}\right] & : K_t > 0 \\ 0 & : K_T \le 0 \end{cases}. \tag{21}$$

We now make an assumption about the form of the partial derivative, $\frac{\partial V_t(K_t)}{\partial K_t}$, in any period t. Then, using the first-order optimality condition and an inductive argument, we will verify that our assumed form is correct. Let us suppose that

$$\frac{\partial V_t(K_t)}{\partial K_t} = \begin{cases} -\Delta \mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\right] & : K_t > 0 \\ 0 & : K_t \le 0 \end{cases}. \tag{22}$$

Equation (22) will also lead us to the desired optimality condition for the dealer's effort level in time period t. Let us assume that the partial derivative form is valid for time period $t+1$, i.e.,

$$\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}} = \begin{cases} -\Delta \mathbb{E}_{X_{t+1},\dots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t+1}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_{t+1}\right\}}\right] & : K_{t+1} > 0 \\ 0\ K_{t+1} \le 0 \end{cases}. \tag{23}$$

Using an inductive argument, we will show that the partial derivative holds for time period t too. Earlier, we have already shown that the partial derivative in the last period has a similar form. First, we compute the partial derivative, with respect to effort e_t, which is given by $\frac{\partial V_t(K_t)}{\partial e_t}$

$$\frac{\partial V_t(K_t)}{\partial e_t} = pg'(e_t) + \Delta g'(e_t)\mathbb{E}_{X_t}[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}] - g'(e_t)\int_0^{K_t-g(e_t)} \left.\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}}\right|_{(K_{t+1}=K_t-g(e_t)-X_t)} f(X_t)dX_t - \beta.$$

Substituting for the value of $\left.\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}}\right|_{(K_{t+1}=K_t-g(e_t)-X_t)}$ from Eq. (23) we get

$$\begin{aligned}\frac{\partial V_t(K_t)}{\partial e_t} &= pg'(e_t) + \Delta g'(e_t)\mathbb{E}_{X_t}\left[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}\right] + \Delta g'(e_t)\mathbb{E}_{X_t}\left[\left.\frac{\partial V_{t+1}(K_{t+1})}{\partial K_{t+1}}\right|_{(K_{t+1}=K_t-g(e_t)-X_t)} \times \mathbf{1}_{\{(g(e_t)+X_t)\le K_t\}}\right] - \beta \\ &= pg'(e_t) + \Delta g'(e_t)\mathbb{E}_{X_t}[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}] + \Delta g'(e_t)\mathbb{E}_{X_t,\dots,X_T} \\ &\quad \times \left[\mathbf{1}_{\left\{g(e_t)+X_t+\sum_{j=t+1}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}} \times \mathbf{1}_{\{(g(e_t)+X_t)\le K_t\}}\right] - \beta \\ &= pg'(e_t) + \Delta g'(e_t)\mathbb{E}_{X_t}[\mathbf{1}_{\{g(e_t)+X_t>K_t\}}] \\ &\quad + \Delta g'(e_t)\mathbb{E}_{X_t,\dots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\right] - \beta.\end{aligned}$$

Thus, the first-order optimality condition implies that the first-order condition must satisfy:

$$p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\right] = \beta h\left(e_t^\star\right). \tag{24}$$

However, we still need to verify that if Eq. (24) holds, then $\frac{\partial V_t(K_t)}{\partial K_t}$ follows the same form mentioned earlier. Again, we use the Envelope Theorem to compute $\frac{\partial V_t(K_t)}{\partial K_t}$. It is easy to verify that if $K_t > 0$, then

$$\begin{aligned}\frac{\partial V_t(K_t)}{\partial K_t} &= -\Delta \mathbb{E}_{X_t,\ldots,X_T}\\ &\quad\times\left[\mathbf{1}_{\left\{g\left(e_t^\star\right)+X_t+\sum_{j=t+1}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\times \mathbf{1}_{\left\{(g(e_t)+X_t)\le K_t\right\}}\right]\\ &= -\Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\right].\end{aligned}$$

However, when $K_t < 0$, then $\frac{\partial V_t(K_t)}{\partial K_t} = 0$. Thus, we have the proof by induction. □

Conditions for Concavity of the Dealer's Profit Under the Δ-Contract

LEMMA 5: If Assumption 2 holds, then the value function $V_t(\cdot)$ is concave.

PROOF: Suppose $f_J(\cdot)$ is the joint distribution of random variables $X_t,\ldots,X_T$. Furthermore, we assume that $X_1,\ldots,X_T$ are iid random variables. Now, the first partial derivative of the profit function with respect to e_t (in any time period t), $\frac{\partial V_t(K_t)}{\partial e_t}$, is

$$\frac{\partial V_t(K_t)}{\partial e_t} = g'(e_t)\left[p + \Delta E_{X_t\ldots X_T} 1_{\sum_{j=t}^{T}(g(e_j)+X_j)>K_t}\right] - \beta.$$

The second partial is given by

$$\frac{\partial^2 V_t(K_t)}{\partial e_t^2} = g''(e_t)\mathrm{Q} + g'(e_t)\frac{\partial \mathrm{Q}}{\partial e_t}, \tag{25}$$

where $\mathrm{Q} \equiv p + \Delta E_{X_t\ldots X_T}\left[1_{\left\{\sum_{j=t}^{T}(g(e_j)+X_j)>K_t\right\}}\right]$. We know that

$$p + \Delta \ge \mathrm{Q} \ge p. \tag{26}$$

Let us define $Z \equiv \sum_{j=t}^{T} X_j$ and the convolution function of random variables $X_t,\ldots,X_T$ as $\phi(Z)$. As $f_J(\cdot)$ is the joint density function of $X_t,\ldots,X_T$, we must have

$$\begin{aligned}&\int_{\sum_{j=t}^{T}(g(e_j)+X_j)>K_t} f_J(X_t,\ldots,X_T)\,dX_t\ldots dX_T\\ &\qquad = 1 - \int_0^{K_t-\sum_{j=t}^{T} g(e_j)} \phi(Z)\,dZ.\end{aligned} \tag{27}$$

Using Eq. (27) we compute

$$\frac{\partial \mathrm{Q}}{\partial e_t} = \Delta\, g'(e_t)\phi\left(K_t - \sum_{j=t}^{T} g(e_j)\right). \tag{28}$$

Observe that, as the pdf, $f(\cdot)$, is bounded above by M, and the variables are iid, we must have

$$\begin{aligned}\phi(Z) &= \int_{X_t=0}^{\infty}\cdots\int_{X_{T-1}=0}^{\infty} f\left(Z - \sum_{j=t}^{T-1} X_j\right)\\ &\quad\times f(X_{T-1})\ldots f(X_t)\,dX_{T-1}\ldots dX_t\\ &\le M\int_{X_t=0}^{\infty}\cdots\int_{X_{T-1}=0}^{\infty}\\ &\quad\times f(X_{T-1})\ldots f(X_t)\,dX_{T-1}\ldots dX_t \le M.\end{aligned} \tag{29}$$

Thus, Eqs. (29) and (28) imply

$$\frac{\partial \mathrm{Q}}{\partial e_t} \le \Delta M\, g'(e_t). \tag{30}$$

To ensure concavity, we must have $\frac{\partial^2 V_t(K_t)}{\partial e_t^2} \le 0$. Using Eq. (30) this condition implies $\frac{g''(e_t)}{g'(e_t)^2} \le -\frac{\Delta M}{\mathrm{Q}}$. Thus, from Eq. (26), $\frac{\partial^2 V_t(K_t)}{\partial e_t^2} \le 0 \iff \frac{g''(e_t)}{g'(e_t)^2} \le -\frac{\Delta M}{p}$. Hence, the profit function in any time period is concave if $\frac{g''(e_t)}{g'(e_t)^2} \le -\frac{\Delta M}{p}$. □

PROOF FOR THEOREM 1: Using Jensen's inequality it is easy to show that $h(e_t^\star) \le h(\mathbb{E}[e_{t+1}^\star])$. As $h(\cdot)$ is an increasing function, this implies that $e_t^\star \le \mathbb{E}[e_{t+1}^\star]$. This further implies that $\mathbb{E}[e_t^\star] \le \mathbb{E}[e_{t+1}^\star]$. □

PROOF FOR THEOREM 2: Let us first consider the case when $g\circ h^{-1}(\cdot)$ is a convex function. From Eq. (11) we know that the optimal effort, $e_t^\star$, satisfies the following condition:

$$e_t^\star = h^{-1}\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\right]\right)\right).$$

The expected sales in any period t, $\mathbb{E}[S_t]$, are given by $\mathbb{E}[X_t + g(e_t^\star)]$. Therefore,

$$\begin{aligned}\mathbb{E}[S_t] &= \mathbb{E}[X_t] + \mathbb{E}[g(e_t^\star)]\\ &= \mathbb{E}[X_t] + \mathbb{E}\left[g\circ h^{-1}\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\right.\right.\right.\\ &\qquad\left.\left.\left.\times\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\right]\right)\right)\right].\\ &\text{(by Jensen's inequality we have)}\\ &\le \mathbb{E}[X_t] + g\circ h^{-1}\left(\mathbb{E}\left[\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_t,\ldots,X_T}\right.\right.\right.\right.\\ &\qquad\left.\left.\left.\left.\times\left[\mathbf{1}_{\left\{\sum_{j=t}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_t\right\}}\right]\right)\right)\right]\right)\\ &= \mathbb{E}[X_t] + g\circ h^{-1}\left(h\left(e_{t+1}^\star\right)\right)\\ &= \mathbb{E}[X_t] + g\left(e_{t+1}^\star\right)\\ &= \mathbb{E}[S_{t+1}].\end{aligned}$$

Thus, the sales HSP exists under this condition. However, following similar arguments it can be shown that the inequality reverses when $g\circ h^{-1}(\cdot)$ is concave. □

PROOF FOR THEOREM 4: From Eq. (11) we know that the optimal effort, $e_{t+1}^\star$, satisfies the following condition:

$$e_{t+1}^\star = h^{-1}\left(\frac{1}{\beta}\left(p + \Delta \mathbb{E}_{X_{t+1},\ldots,X_T}\left[\mathbf{1}_{\left\{\sum_{j=t+1}^{T}\left(g\left(e_j^\star\right)+X_j\right)>K_{t+1}\right\}}\right]\right)\right). \tag{31}$$

Observe that, in Eq. (31), as X_t increases, K_{t+1} decreases because $K_{t+1} = K - D_{t+1}$ [see definition of adjusted threshold in Eq. (6)]. Furthermore, $h^{-1}(\cdot)$ is convex and increasing (by Assumption 1). So, $e^\star_{t+1}$ also increases as X_t increases. We now use the following result proved in Ref. 8.

LEMMA 6 (Horn): Let X be a real random variable with nondegenerate distribution P_X. Let f and g be real functions both either monotonically not increasing or monotonically not decreasing with $\mathbb{E}[f(X)] < \infty$ and $\mathbb{E}[g(X)] < \infty$. Then

$$\mathbb{E}[f(X)g(X)] \geq \mathbb{E}[f(X)]\mathbb{E}[g(X)].$$

Using Lemma 6 it is easy to verify that $\mathrm{cov}\big[g\big(e^\star_{t+1}\big), X_t\big] \geq 0$. This completes the proof. □

PROOF FOR PROPOSITION 1: Let the optimal effort exerted by the dealer under the contract with $K = \sum_{t=1}^T Y_t + \bar{K}$ facing a demand $\{X_t : t = 1, \dots, T\}$ be denoted by $\{e^\star_t : t = 1, \dots, T\}$. Using the arguments in Section 4 (Lemma 1), the optimality equation for $e^\star_t$:

$$h\big(e^\star_t\big) = \frac{1}{\beta}\left(p + \Delta\mathbb{E}_{X_t,\dots,X_T,\zeta_t,\dots,\zeta_T}\left[\mathbf{1}_{\left\{\sum_{j=1}^T \left(g\left(e^\star_j\right)+X_j\right) > \sum_{j=1}^T Y_j + \bar{K}\right\}}\right]\right). \tag{32}$$

Rearranging the terms, we get

$$\begin{aligned} h\big(e^\star_t\big) = \frac{1}{\beta}\Bigg(& p + \Delta\mathbb{E}_{X_t,\dots,X_T,\zeta_t,\dots,\zeta_T} \\ &\times \left[\mathbf{1}_{\left\{\sum_{j=1}^T \left(g\left(e^\star_j\right)+(X_j - Y_j)+\bar{X}_j\right) > \sum_{j=1}^T \bar{X}_j + \bar{K}\right\}}\right]\Bigg). \end{aligned} \tag{33}$$

Comparing the right-hand sides with the one described in Section 4, we obtain that the $\{e^\star_t : t = 1, \dots, T\}$ is the optimal effort exerted by the dealer under the contract with $K = \sum_{t=0}^T \bar{X}_t + \bar{K}$ facing a demand $\{X_t - Y_t + \bar{X}_t : t = 1, \dots, T\}$. This completes the proof. □

PROOF FOR PROPOSITION 2: With perfect correlation and perfect information, and the choice of the threshold $K = \sum_t^T x_t + \bar{K}$ [Eq. (19)], the dealer's profit function reduces to

$$\Pi_\Delta = \begin{cases} p(K - \bar{K}) + p\sum_{t=1}^T g(e_t) - \beta\sum_{t=1}^T e_t & : \sum_{t=1}^T g(e_t) < \bar{K} \\ (p + \Delta)(K - \bar{K}) + (p + \Delta) & \\ \quad \times \sum_{t=1}^T g(e_t) - \Delta K - \beta\sum_{t=1}^T e_t & : \sum_{t=1}^T g(e_t) \geq \bar{K} \end{cases}. \tag{34}$$

It is easy to verify that the dealer's profit function is a deterministic concave function of the vector of efforts, i.e., $(e_1, \dots, e_t, \dots, e_T)$. To maximize his profit the dealer must solve the following two optimization problems:

$$\begin{aligned} \max_{e_1,\dots,e_T} \; & p\sum_{t=1}^T g(e_t) - \beta\sum_{t=1}^T e_t \\ s.t. \quad & e_1, \dots, e_T \geq 0 \end{aligned} \tag{35}$$

$$\begin{aligned} \max_{e_1,\dots,e_T} \; & (p + \Delta)\sum_{t=1}^T g(e_t) - \beta\sum_{t=1}^T e_t \\ s.t. \quad & e_1, \dots, e_T \geq 0 \end{aligned} \tag{36}$$

and choose the effort levels that result in the maximum profit. As, both Eq. (35) and equation (36) are concave functions, the optimal effort in each time period is either $e^\star_\mathrm{l}$ [optimal solution to Eq. (35)] or $e^\star_\mathrm{h}$ [optimal solution to Eq. (36)], where $e^\star_\mathrm{l}$ is defined by Eq. (7) and $e^\star_\mathrm{h}$ is defined by Eq. (8). Hence, the dealer exerts the same effort in all time periods. Thus, the sales HSP is eliminated under perfect information. Further, to decide between the high effort $e^\star_\mathrm{h}$ and low effort $e^\star_\mathrm{l}$ the dealer would contrast his payoff under both policies. It can be easily verified that if the threshold exceeds $\tilde{K}$, then the dealer's payoff is higher by choosing a lower effort, and if the threshold is less than $\tilde{K}$, then the dealer's payoff is higher by choosing the higher effort. This completes the proof. □

PROOF FOR PROPOSITION 3: Earlier in Proposition 2, we proved that the dealer either exerts a constant low effort, $e^\star_\mathrm{l}$, in all time periods or a high effort, $e^\star_\mathrm{h}$, in all time periods. This depends on the value of $\bar{K}$ chosen. We saw that if $\bar{K} > \tilde{K}$, then the dealer always chooses the lower effort for every realization of X_t, $t = 1, \dots, T$. Hence, had the manufacturer chosen any arbitrary threshold, K, (not based on a perfectly correlated signal) the dealer's effort in any time period would be at least as large as $e^\star_l$ (see Lemma 2). However, if the manufacturer chose $\bar{K} < \tilde{K}$, the dealer exerts the highest effort in all time periods, and hence, the sales are the highest of any realization of the demand signals. This is not the case when K is chosen arbitrarily (not based on a correlated signal), because (see Lemma 2) the dealer's effort is strictly bounded as follows: $e^\star_\mathrm{l} \leq e^\star_t \leq e^\star_\mathrm{h}$. Thus, path-wise dominance is obtained based on the choice of $\bar{K}$ under perfect correlation and information. □

PROOF FOR PROPOSITION 4: Noting that

$$\mathbb{V}[X_t] = \mathbb{V}[Y_t] + \mathbb{V}[X_t - Y_t] + \mathbb{E}[Y_t(X_t - Y_t)] - \mathbb{E}[Y_t]\mathbb{E}[(X_t - Y_t)]. \tag{37}$$

Recollect $Y_t = \mathbb{E}[X_t|\zeta_t]$. Consequently, we have $\mathbb{E}[X_t] = \mathbb{E}[Y_t]$, and

$$\begin{aligned} \mathbb{E}[Y_t(X_t - Y_t)] &= \mathbb{E}[\mathbb{E}[Y_t(X_t - Y_t)|\zeta_t]] \\ &= \mathbb{E}[Y_t\mathbb{E}[X_t - Y_t|\zeta_t]] = 0. \end{aligned}$$

Thus,

$$\mathbb{V}[X_t] = \mathbb{V}[Y_t] + \mathbb{V}[X_t - Y_t].$$

Noting that the variance is always positive, completes the proof. □

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