Skill and Capacity Management in Large-scale Service Marketplaces

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Customers who need quick resolutions for their temporary problems are increasingly seeking help in large-scale, Web-based service marketplaces. Due to the temporary nature of the relations between customers and service providers (agents) in these marketplaces, customers may not have an opportunity to assess the ability of an agent before their service completion. On the other hand, the moderating firm has a more sustained relationship with agents, and thus it can provide customers with more information about the abilities of agents through skill screening mechanisms. The main goal of this paper paper is to explore how the moderating firm may use its skill screening tools while making skill and capacity management decisions. Surprisingly, we show that the dynamics of the service marketplace may force the moderating firm to limit the potential service capacity even when supply is significantly scarce. We also show that the impact of the marketplace dynamics is alleviated when the firm can isolate its business with one particular customer type from the other customer types, especially when skills required to serve different customer needs are correlated. Thus, the firm prefers to utilize the potential service capacity if skills of an agent are correlated.

Key words: Service marketplaces; fluid models; skill management; flexible resources; non-cooperative game theory; large games.

1. Introduction

Large-scale, Web-based service marketplaces have, recently, emerged as a new resource for customers who need quick resolutions for their temporary problems. In these marketplaces, many small service providers (agents) compete among themselves to help customers with diverse needs. Typically, an independent firm, which we shall refer to as the *moderating firm*, establishes the infrastructure for the interaction between customers and agents in these marketplaces. In particular, the moderating firm provides the customers and the agents with the information required to make their decisions. Besides their essential role, moderating firms can introduce operational tools which specify how the customers and the agents are matched together. For instance, some of the moderating firms allow customers to post their needs and let service providers apply, postponing the service provider selection decision of the customers until they obtain enough information about agents' availability. Moreover, moderating firms can provide strategic tools which allow communication and collaboration among the agents. These different involvements result in different economic and operational systems, and thus vary in their level of efficiency and the outcomes for both customers and service providers.

A notable example among many existing online marketplaces is oDesk.com. The web-site hosts around 2,000,000 programmers competing to provide software solutions. Considering such large-scale marketplaces, it is not surprising to see that the ability of agents to serve customers with a particular need varies significantly. Naturally, customers prefer to be serviced by a more skilled agent because a more capable agent is likely to generate more value for customers. Unfortunately, customers may not have an opportunity to assess the ability of an agent before their service completion because most of the relations between customers and providers are temporary in these marketplaces. On the other hand, the moderating firm has a more sustained relationship with agents, and thus it can obtain more information about their abilities. Particularly, the firm can constitute a *skill screening mechanism*. In general, these mechanisms take the form of skill tests and/or certification programs that are run by moderating firms. For instance, oDesk.com offers various exams to test the ability of the *candidate* providers. In fact, being successful in these exams is the first requirement for providers to be eligible to serve customers in the marketplace. oDesk.com (or any other moderating firm) freely decides on how comprehensive the exams are. The more comprehensive the exams become, the more value customers expect from the service. If necessary, oDesk.com can use these exams to disqualify some of the agents, and thus control the portfolio of different agent types (e.g., flexible, dedicated) and the service capacity in the marketplace. We use the term *skill-mix structure* to denote the portfolio of different agent types. oDesk.com, like many other online service marketplaces, receives 10% of the revenue obtained by the providers at service completion. Therefore, it is in great interest of oDesk.com to intervene in the marketplace by using its skill test in order to make sure that the "right" prices and customer demand emerge as the outcome in the marketplace.

Motivated by these online service marketplaces, we aim to study the impact of the moderating firm's skill screening mechanism on the behavior of the customers and the providers in a service marketplace where the objective of each individual player is to maximize his/her own utility. We use a game theoretical framework to study the interaction between the customers and the agents. Specifically, each agent announces a price for his service, and customers request service from agents based on the price and the expected waiting time. We focus on the setting where the moderating firm provides both an operational and a strategic tool. The operational tool introduced by the moderating firm efficiently matches customers interested in purchasing the service at a particular price with the agents charging that or a lower price, while the strategic tool allows limited pre-play communication among agents within a noncooperative structure. As we mention above, the moderating firm can use its screening mechanism as a tool to shape the skill-mix structure of the marketplace. Therefore, we are also interested in how the firm can use this mechanism to maximize its own profit, which is a predetermined share of the total revenue generated in the marketplace.

In this paper, we consider a service marketplace with two groups of customers, each of which has different needs. We use the term *class* to identify the group of customers with similar needs. On the supply side, we assume there is a finite but large number of agents, who are homogeneous in their service capacity and heterogeneous in the value that their services generate for each customer class. Specifically, the service of an agent generates a random value (with a known distribution) for each customer class. We refer to the vector that represent these two random values as *the skills of an agent*. In this setting, we study the moderating firm's decisions when customers cannot observe agent skills but the firm can assess whether the value that an agent generates for a given class is above a certain threshold through an exam. We characterize the equilibrium outcome of the game between customers and agents for any given pair of exam thresholds and find the pair of exam thresholds that maximizes the profit of the moderating firm.

In analyzing the model described above, we observe that the dynamics of the arising system are too complex for exact analysis, and thus we resort to asymptotic analysis. In particular, we study the fluid approximation of our model while characterizing the equilibrium outcome of the marketplace for any given skill-mix structure. We also observe that the optimization problem of the firm becomes analytically intractable when the skills of an agent follow a general joint probability distribution. Thus, we obtain the optimal skill-mix structure and capacity decisions of the firm under three different scenarios based on the relationship of agent skills: (a) perfectly positive correlation, (b) perfectly negative correlation, and (c) independence among the skills of an agent.

We next state our key findings along with the contributions of the paper:

1. Fixing the skill-mix structure in the marketplace, we show that when the service capacity allocated to a customer class is less then its demand rate, the competition among providers serving this class is mitigated and agents can agree on a price which extract all of the customer surplus. On the contrary, the competition is intensified when the service capacity exceeds the demand, and this leads to arbitrarily low profits for both the agents and the firm. Hence, if the total service capacity of candidate agents exceeds the total demand rate, the firm always fails some of the candidate agents to make sure the service capacity is not abundant. However, it is not obvious whether the firm would limit the service capacity by failing candidate agents when the total service capacity of candidate agents are initially lower than the total demand rate.

2. The moderating firm always trades off between the service capacity and the value of the service because customers expect more value when there are fewer eligible agents. We show that the gains from increasing the service capacity allocated to a customer class outweighs the losses from reducing the expected value of the service. Therefore, the firm's profit from a customer class increases in the service capacity allocated to this class.

3. In the cases of correlated agent skills, the service capacity allocated to a customer class affects only the firm's profit from this class. In other words, the capacity allocation problem of the firm can be de-coupled. As the firm's profit from a customer class increases in the capacity, the firm does not benefit from reducing the service capacity allocated to one class while keeping the capacity for the other class unchanged. Hence, the firm always lets all candidate agents be eligible to serve customers if agent skills are correlated and the service capacity is initially scarce. This translates into not failing any agent in both exams at the same time.

4. Unlike when agent skills are correlated, changing the service capacity allocated to one class affects the profit from the other class if the skills of an agent is independent. In fact, when the demand rate from one class is sufficiently low, we show that reducing the capacity allocated to the high-demand class (while not changing the capacity for the low-demand class) may improve the total profit of the moderating firm even though such a change deteriorates the profit from the high-demand class. Based on this observation, we show that the firm may need to fail some of the candidate agents in both exams in order to maximize its profit. The key insight of this result is that the dynamics of the service marketplace force the moderating firm to limit the potential service capacity even when supply is scarce.

2. Literature Review

Our paper lies in the intersection of various streams of research. The first line of papers related with our paper studies the applications of queueing theory in service systems. Service systems with customers who are both price and time sensitive have attracted the attention of researchers for many years. The analysis of such systems dates back to Naor's seminal work (See Naor, 1969), which analyzes customer behavior in a single-server queueing system. Motivated by his work, many researchers study the service systems facing price- and delay-sensitive customers in various settings. We refer the reader to Hassin and Haviv (2003) for an extensive summary of the early attempts in this line of research. More recently, Cachon and Harker (2002) and Allon and Federgruen (2007) studies the competition between multiple firms offering substitute but differentiated services by modeling the customer behavior implicitly via an exogenously given demand function. An alternative approach is followed in Chen and Wan (2003), where authors examine the customers' choice problem explicitly by embedding it into the firms' pricing problem. Other notable examples focusing on the customers' demand decision in competition models are Ha et al. (2003), and Cachon and Zhang (2007).

Our paper is also related to the papers focusing on the economic trade-offs between investing on flexible resources, which provide the ability to satisfy a wide variety of customer needs, and dedicated resources, which can respond to only a specific demand type. This line of literature studies a two-stage decision problem with recourse, which is also known as the Newsvendor Network problem, and dates back to Fine and Freund (1990). Fine and Freund (1990) considers a firm that invests in a portfolio of multiple dedicated resources and one flexible resource in the first stage where the market demand for its products is uncertain. After making the capacity investments, the demand uncertainty is resolved, and the firm makes the production decisions to maximize its profits. Fine and Freund (1990) argues that the flexible resource is not preferred when demand distributions are perfectly and positively correlated. Gupta et al. (1992) studies a similar model where the firm initially has some existing capacity and presents results parallel to Fine and Freund (1990). Contrary to the examples provided in these two papers, Callen and Sarath (1995) and Van Mieghem (1998) show that it can be optimal for a firm to invest in a flexible resource even if demand distributions are perfectly and positively correlated. Recent papers extends the model in Fine and Freund (1990) by studying the optimal pricing decision of a monopolist (See Chod and Rudi (2005) and Bish and Wang (2004)), competition between two firms (See Goyal and Netessine (2007)), and more detailed configurations of flexibility (See Bassamboo et al. (2010)). In all of these papers, the firm chooses its price and allocates its flexible capacity in order to maximize its profits. However, in the service marketplaces we consider, the (moderating) firm does not have a direct control over the pricing and the service decisions of the service providers.

The pricing and the capacity planning problem of the service systems can easily become analytically intractable when trying to study more complex models, such as a multi-server queueing systems. Recognizing this difficulty, many researchers seek robust and accurate approximations to analyze multi-server queues. Halfin and Whitt (1981) is the first paper that proposes and analyzes a multi-server framework. This framework is aimed at developing approximations, which are asymptotically correct, for multi-server systems. It has been applied by many researchers to study the pricing and service design problem of a monopoly in more realistic and detailed settings. Armony and Maglaras (2004), and Maglaras and Zeevi (2005) are examples of recent work using the asymptotic analysis to tackle complexity of these problems. Furthermore, Garnett et al. (2002), Ward and Glynn (2003), and Zeltyn and Mandelbaum (2005) extends the asymptotic analysis of markovian queueing system by considering customer abandonments.

The idea of using approximation methods can also be applied to characterize the equilibrium behavior of the firms in a competitive environment. To our knowledge, Allon and Gurvich (2010) and Chen et al. (2008) are the first papers studying competition among complex queueing systems by using asymptotic analysis to approximate the queueing dynamics. There are two main differences between these two papers and our work. First, both of them study a service environment with a fixed number of decision makers (firms) while the number of decision makers in our marketplace (agents) is infinite. Second, they only consider a competitive environment where the firms behave individually. In contrast, we study a marketplace where the agents have a limited level of collaboration. Another recent paper that studies the equilibrium characterization of a competitive marketplace using asymptotic analysis is Allon et al. (2012). Like our paper, Allon et al. (2012) studies a service marketplace with large number of service provider. However, it focuses on a marketplace with a fixed skill-mix structure without incorporating the skill and capacity management decisions of the moderating firm.

In the field of operations management (OM), the majority of the papers employing game-theoretic foundations study non-cooperative settings. For an excellent survey, we refer to Cachon and Netessine (2004). There is also a growing literature that studies the OM problems in the context of cooperative game theory. Nagarajan and Sosic (2008) provide an extensive summary of the applications of cooperative game theory in supply chain management. Notable examples are the formation of coalitions among retailers to share their inventories, suppliers, and marketing powers (See Granot and Sosic (2005), Sosic (2006), and Nagarajan and Sosic (2007)). This body of research is related with our work as we look for the limited collaboration among agents.

Our work may also be viewed as related to the literature on labor markets that studies the wage dynamics (See Burdett and Mortensen (1998), Manning (2004), and Michaelides (2010)). In our model, service seekers trade-off the time they need to wait until their job starts and the price, the phenomenon generally disregarded in labor economics literature. Further, our focus is on a market for temporary help, which means that the engagement between sides ends upon the service completion. This stands in contrast to the labor economics literature in which the engagement is assumed to be permanent. Our paper also differs from the literature on market microstructure. This body of literature studies market makers who can set prices and hold inventories of assets in order to stabilize markets (See Garman (1976), Amihud and Mendelson (1980), Ho and Stoll (1983), and a comprehensive survey by Biais et al. (2005)). However, the moderating firm considered in our paper has no direct price-setting power and cannot respond to customers' service requests. Furthermore, papers studying market microstructure disregard the operational details such as waiting and idleness.

3. Model Formulation

Consider a service marketplace where agents and customers make their decisions in order to maximize their individual utilities. There are two groups of customers, each of which has different needs. We use the term "*class*" to identify the group of customers with similar needs. We refer to one customer class as class-A and the other one as class-B. Class-I customers' need for the service is generated according to a Poisson process with rate Λ_I for all $I \in \{A, B\}$. This forms the "potential demand" for the marketplace. A customer decides whether to join the marketplace or not. The customers who join the marketplace form the "effective demand" for the marketplace. If a customer decides not to join the system, her utility is zero. If she joins the system, she decides who would process her job. The exact nature of this decision depends on the specific structure of the marketplace decided upfront by the moderating firm. We shall elaborate on the choices of customers while we discuss the role of the moderating firm. We assume that service time required to satisfy the requests of a given customer is exponentially distributed with rate μ. Without loss of generality, we let $\mu = 1$. When the service of a class-I customer is successfully completed, she pays the price of the service, earns a reward, which depends on the skills of the agent serving her, and incurs a waiting cost of c_I per unit time until her service commences for all $I \in \{A, B\}$. Because the customers who visit the marketplace seek temporary help, a customer joining the system may become impatient while waiting for her service to start and abandon. The abandoning customer does not pay any price or earn any reward but she incurs a waiting cost for the time she spends in the system. We assume that customers' abandonment times are independent of all other stochastic components and are exponentially distributed with mean m_a. Customers decide whether to request service or not and by whom to be served according to their expected utility. The expected utility of a customer is based on the reward, the price and the anticipated waiting time.

The above summarizes the demand arriving to the marketplace. Next, we discuss the capacity provision in the marketplace. There are k *candidate* agents endowed with different processing skills. Particularly, the value that an agent's service generates for a class-I customer is S_I for all $I \in \{A, B\}$. S_A and S_B are random variables with a joint probability density function $f_{A,B}(\cdot,\cdot)$ on the support $[0, \bar{R}_A] \times [0, \bar{R}_B]$. We refer to (S_A, S_B) as *the skills of an agent.* The skills of an agent are not observable but the moderating firm can verify whether the value generated by an agent is above a certain threshold through a skill screening process. The decisions of an agent are to set a price for his service and choose the customer class to serve among the classes he is allowed to serve; each agent makes these decisions independently in order to maximize his expected revenue. The expected revenue of an agent depends on the price he charges and his demand volume. We normalized the operating cost of the agents to zero for notational convenience.

We refer to the ratio Λ_I/k as the demand-supply ratio of class-I and denote it by $\rho_I > 0$ for all $I \in \{A, B\}$. The demand-supply ratio is a first order measure for the mismatch between aggregate demand and the total processing capacity. We broadly categorize marketplaces into two: Buyer's market where $\rho_A + \rho_B < 1$, and seller's market where $\rho_A + \rho_B \geq 1$.

4. The Role of the Moderating Firm

The essential role of the moderating firm in a large scale marketplace is to construct the infrastructure for the interaction between players. This is crucial because all players have to be equipped with the necessary information, such as prices to make their decisions, yet individual players cannot gather this information on their own. There are also other ways for moderating firms to be involved in a marketplace. For instance, moderating firms can provide mechanisms, which improve the operational efficiency of the whole system by efficiently matching customers and agents. They may also complement their operational tools with strategic tools, which enable communication among agents. Furthermore, because agents' skills are not observable to the customers, moderating firms may provide customers with further information about the candidate agents by screening agents' abilities. In this section, our goal is to build a model where we capture these different roles of the moderating firms. To this end, we first introduce a screening mechanism which consists of skill tests determining whether a candidate agent is eligible to serve customers. Next, we provide a detailed description of the interaction between customers and agents in a marketplace when operational inefficiencies are minimized and agents are allowed to communicate.

4.1. Setting up the Skill-Mix

As we mentioned in the introduction, moderating firms can obtain more information about the ability of candidate agents through a screening process. We model this by assuming that the moderating firm runs two skill tests on each candidate agent, say Exam-A and Exam-B, in order to screen his abilities. In particular, in Exam-I, the firm picks a threshold level ω_I (a measure for comprehensiveness) and test whether the value that an agent's service generate for class-I is above ω_I for all $I \in \{A, B\}$. The firm publicly announces the results of the tests, and a candidate agent will be *eligible* to serve customers if he passes at least one exam. Candidate agents, who pass both exams, will be eligible to serve both classes of customers. We refer to this type of agents as *flexible* agents, and denote the fraction of flexible agents by α_F. Since a flexible agent can serve both classes, he makes a service decision by choosing which customer class he serves in addition to his pricing decision. On the other hand, an agent who passes only Exam-J, for some $J \in \{A, B\}$, will be eligible to serve only class-J, and thus his only decision will be to set a price for his service. We refer to this type of agents as *dedicated* agents, and denote the fraction of dedicated agents for class-I by α_I, for all $I \in \{A, B\}$. Once the firm chooses a pair of exam thresholds (ω_A, ω_B), the fraction of flexible and dedicated agents in marketplaces with large number of agents can be approximated as follows:

$$\alpha_F(\omega_A,\omega_B)\simeq\int_{\omega_A}^{\bar{R}_A} ds_A \int_{\omega_B}^{\bar{R}_B} f_{A,B}(s_A,s_B)ds_B \text{ and } \alpha_I(\omega_A,\omega_B)\simeq\int_{\omega_I}^{\bar{R}_I} ds_I \int_{0}^{\omega_J} f_{A,B}(s_I,s_J)ds_J, \tag{1}$$

for all $I,J\in\{A,B\}$ with $J\neq I$. Throughout the paper, we use the term *skill-mix structure* to denote the portfolio of different agent types in the marketplace. For instance, a marketplace may consist of all three types of agents: flexible agents, and dedicated agents for each class. We refer to such a skill-mix structure as "*M-Network*". On the other hand, there may be only flexible agents in a marketplace. This skill-mix structure will be referred to as "*V-Network*". In addition to these two, there may be other skill-mix structures such as "*N-Network*" and "*I-Network*". We illustrate these different structures in Figure 1. The moderating firm can set up various skill-mix structure by changing the threshold levels ω_A and ω_B.

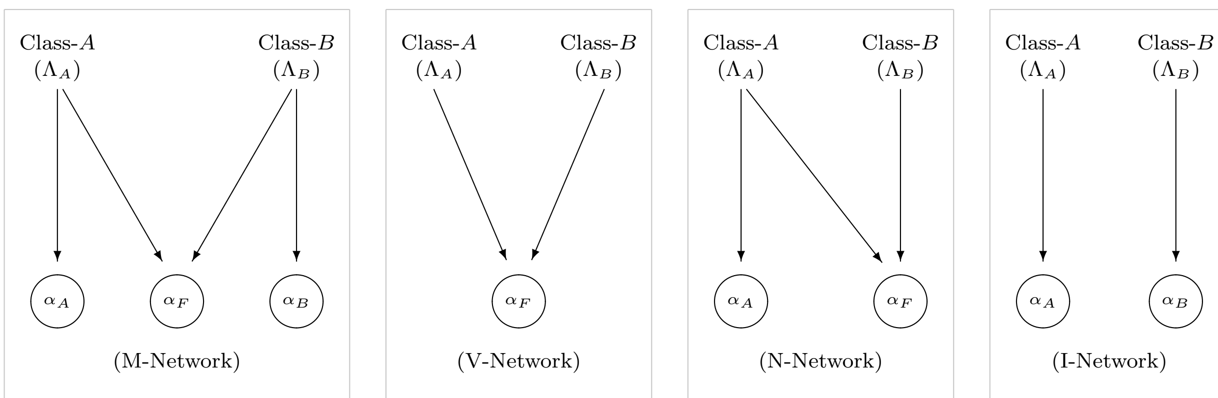


Figure 1 Different skill-mix structures that can be set up by the moderating firm.

In addition to changing the skill-mix structure, exam thresholds impact the expected reward that a customer earns upon her service completion. If there is no further information about the agents' skills, a class-I customer expects to earn the average value that agents generate for her class, which can be approximated by $\mathbf{E}[S_I]$, for all $I\in\{A,B\}$. However, after customers learn the exam results, they update the reward that they expect from agents who are eligible to serve them. More specifically, for any exam threshold pair (ω_A,ω_B), the expected reward that a class-I customer earns from a dedicated agent becomes $\mathbf{E}[S_I|S_I\geq\omega_I,S_J<\omega_J]$ can be written as follows:

$$R_I(\omega_A,\omega_B)=\Big(\int_{\omega_I}^{\bar{R}_I} ds_I \int_{0}^{\omega_J} s_I f_{A,B}(s_I,s_J)ds_j\Big)/\alpha_I(\omega_A,\omega_B), \tag{2}$$

for all $I,J\in\{A,B\}$ with $I\neq J$. Likewise, a class-I customer expects to earn $R_{IF}(\omega_A,\omega_B)$ from a flexible agent given the exam thresholds (ω_A,ω_B), where

$$R_{IF}(\omega_A,\omega_B) = \Big(\int_{\omega_I}^{\bar{R}_I} ds_I \int_{\omega_J}^{\bar{R}_J} s_I f_{A,B}(s_I,s_J)ds_J\Big)/\alpha_F(\omega_A,\omega_B), \tag{3}$$

4.2. Operational Efficiency

In addition to setting up the skill-mix, the moderating firm provides a mechanism that improves the operational efficiency of the whole system by efficiently matching customers and agents. This mechanism aims at reducing inefficiency due to the possibility of having a customer waiting in line for a busy agent while an agent who can serve her is idle. For instance, oDesk.com achieves this goal by allowing customers to post their needs and allowing service providers to apply to these postings. When a customer posts a job at oDesk.com, agents that are willing to serve this customer apply to the posting. Among the applicants charging less than what the customer wants to pay, the customer will favor agents based on their immediate availability. The main driver of the operational efficiency in this setting is the fact that customers no longer need to specify an agent upon their arrival because the job posting mechanism allows customers to postpone their service request decisions until they have enough information about the availability of the providers. Thus, we assume that the mechanism introduced by the moderating firm ensures that customers do not stay in line when there is an idle agent willing to serve them for a price they want to pay or less.

Note that the expected utility of a customer will depend on both the price and the type of the agent who serves her because each agent type may provide a different expected reward. To account for that, we define the "*net reward*" of a class-I customer from a dedicated and a flexible agent charging p as $R_I(\omega_A,\omega_B)-p$ and $R_{IF}(\omega_A,\omega_B)-p$, respectively, for all $I\in\{A,B\}$ and any given pair of thresholds (ω_A,ω_B). Then, we model the efficiency improvement in the system by considering the marketplace as a queuing network where the agents announcing the same net reward and serving the same customer class are virtually grouped together, regardless of their types. We assume that customers decide which agents to choose based on the net reward and they treat all agents as the same when they offer the same net reward because the nature of the tasks is simple, benefits are tangible, features are clear, and thus rewards are easily quantifiable.

Once each agent announces a price per customer to be served, we can construct a resulting net reward vector $(r_{I_n})_{n=1}^{N_I}$ for class-I where $N_I\le k$ is the number of different net rewards announced by the agents serving class-I for all $I\in\{A,B\}$. We refer to the agents announcing the net reward r_{I_n} as sub-pool-I_n and denote the number of agents in the sub-pool-I_n by y_{I_n}. Hence, the vectors $(\mathbf{r_A},\mathbf{y_A})\equiv(r_{A_n},y_{A_n})_{n=1}^{N_A}$ and $(\mathbf{r_B},\mathbf{y_B})\equiv(r_{B_n},y_{B_n})_{n=1}^{N_B}$ summarizes the strategy of all agents.

Under the mechanism provided by the moderating firm, we model the customer decision making and experience as follows: If there are different net rewards announced by the agents serving class-I, i.e. $N_I>1$, a class-I customer chooses a sub-pool from which she requests the service for all $I\in\{A,B\}$. We refer to the net reward offered by this sub-pool as the "*preferred net reward.*" Each customer who decides to join the system enters the service immediately if there is an available agent either in the sub-pool she chooses or any sub-pool announcing a net reward more than the sub-pool she chooses. Moreover, the customer is served by the sub-pool offering the highest net reward among all available sub-pools. Otherwise, she waits in queue in front of the sub-pool she chooses until an agent who offers a net reward higher than or equal to her preferred net reward becomes available. We denote the fraction of customers requesting service from sub-pool-I_n by D_{I_n} and summarize the decisions of the class-I customers by the vector $\mathbf{D_I} \equiv (D_{I_n})_{n=1}^{N_I}$ for all $I \in \{A, B\}$. In this model of customer experience, there are two crucial features: 1) The service of an arriving customer commences immediately when there are available agents offering higher than or equal to her preferred net reward, 2) If they have to wait, customers no longer wait for a specific agent rather for an available agent.

As we model the marketplace as a queuing network, the operations of each sub-pool depend on the operations of the other sub-pools. For instance, each sub-pool may handle customers from the other sub-pools (giving priority to its own customers) while some of the other sub-pools are serving its customers. Therefore, given the strategies of agents, $(\mathbf{r_I}, \mathbf{y_I})$, and the service request decisions of class-I customers, $\mathbf{D_I}$, the expected utility of a class-I customer choosing the sub-pool-I_ℓ, for all $\ell \in \{1, \dots, N_I\}$ and $I \in \{A, B\}$, depends on all of these given decisions, and can be written as:

$$P_{I_{\ell\ell}}\big[(r_{I_\ell} + cm_a)(1 - \beta_{I_\ell}) - cm_a\big] + \sum_{m \neq \ell} P_{I_{\ell m}} r_{I_n}, \tag{4}$$

where $\beta_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}, \mathbf{D_I})$ denotes the probability of abandonment for customers choosing the sub-pool-I_ℓ given that they are not served by another sub-pool, and $P_{I_{\ell m}}(\mathbf{r_I}, \mathbf{y_I}, \mathbf{D_I})$ denotes the probability that a customer choosing the sub-pool-I_ℓ is served by the sub-pool-I_m for all $m \in \{1, \dots, N_I\}$. We want to note that for any sub-pool-I_ℓ, $P_{\ell m} = 0$ for any m such that $r_{I_m} < r_{I_\ell}$ since customer choosing sub-pool-I_ℓ cannot be serve by a sub-pool offering less than r_{I_ℓ}, which is their preferred net reward. Furthermore, given the exam thresholds (ω_A, ω_B), the revenue of a dedicated agent in the sub-pool-I_ℓ is

$$\big[R_I(\omega_A, \omega_B) - r_{I_\ell}\big]\sigma_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}, \mathbf{D_I}), \tag{5}$$

where $\sigma_{I\ell}(\mathbf{r_I}, \mathbf{y_I}, \mathbf{D_I})$ is utilization of agents in sub-pool-ℓ. Similarly, the revenue of a flexible agent in the sub-pool-I_ℓ is

$$\big[R_{IF}(\omega_A, \omega_B) - r_{I_\ell}\big]\sigma_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}, \mathbf{D_I}). \tag{6}$$

After describing the operational tool provided by the moderating firm, we now discuss the moderating firm's strategic tool which changes the nature of the competition among agents. In a marketplace such as oDesk.com, service providers are offered discussion boards where they are allowed to exchange information. Moreover, the firm supports the creation of affiliation groups, which are self-enforcing entities. Motivated by these examples, we assume that the moderating firm allows agents to make non-binding communication prior to making their decisions, so that they can try to self-coordinate their actions in a mutually beneficial way despite the fact that each agent selfishly maximizes his own utility. As it is discussed in Allon et al. (2012), this can be modeled by using an equilibrium concept, which allows several agents to deviate together. Since the marketplaces we consider tend to be large, the size of the group deviating together is restricted. We denote the largest fraction of agents that is allowed to deviate together by δ such that $1/k < \delta \leq 1$.

Given the setup of Section 3, along with the above mentioned roles of the moderating firm, we model the strategic interaction between the agents and the customers as a sequential move game. The agents first make their service and pricing decisions. Then, in the second stage, each arriving customer observes these decisions and decides whether to request service or not. Moreover, customers specify the sub-pool from which they request service. Customers make their service request in order to maximize their utility. Therefore, in the equilibrium of the second stage, a class-I customer, for all $I \in \{A, B\}$, chooses a sub-pool only if the utility she obtains from this sub-pool (weakly) dominates her utility from any other sub-pool. On the other hand, the equilibrium of the first stage requires that service and pricing decisions of the agents should be immune to any deviations formed by a group with less than or equal to δk agents. In other words, any group of agents with less than or equal to δk agents should not have any incentive to change their service and/or pricing decisions.

In analyzing the equilibrium outcome of marketplaces with finite number of agents, we would like to highlight the following two observations: 1) The arising system dynamic is too complex for the exact analysis. 2) Asymptotic analysis is applicable since these marketplaces tends to be large. Thus, in the next section, we shall approximate the original system in a parametric regime, where the demand and the number of agents are large.

5. Fluid Model

In this section, we consider a sequence of marketplaces indexed by the number of agents, i.e. there are k agents in the k^{th} marketplace. The arrival rate of class-I in the k^{th} marketplace is assumed to be $\Lambda_I^k = \rho_I k$. This ensures that the demand-supply ratios are constant along the sequence of marketplaces. Furthermore, instead of characterizing the behavior of the equilibrium along the sequence of marketplaces as in Allon et al. (2012), we describe the limiting game formally, and characterize the behavior of the equilibrium in the limiting game. We will also show that the equilibrium characterization in the limiting game constitutes a precise approximation for the behavior of the equilibrium along the sequence of marketplaces.

To describe our limiting game, we assume that the number of agents k goes to infinity, and we consider our original model defined in the previous sections as a fluid model. Note that in the limiting game, we can summarize the strategy of all agents by $(p_n, y_n)_{n=1}^N$ where y_n is the fraction of the agents in sub-pool-n instead of the number of agents. The benefit of using a fluid model is that it provides an accurate yet simple approximation for the abandonment and utilization functions which helps us to derive the utility of the customers and agents in simple form. In particular, denoting the probability of abandonment as $\beta^M(\lambda, k)$ and agent utilization as $\sigma^M(\lambda, k)$ in an $M/M/k+M$ system with arrival rate λ, service rate 1, and abandonment rate $1/m_a$, Whitt (2006) shows that these two functions can be approximated as follows:

$$\beta^M(\lambda,k) \simeq \begin{cases} 0 & \textit{if } \lambda \le k \\ (\lambda - k)/\lambda & \textit{if } \lambda > k. \end{cases}, \qquad \sigma^M(\lambda,k) \simeq \begin{cases} (\lambda)/k & \textit{if } \lambda \le k \\ 1 & \textit{if } \lambda > k. \end{cases}$$

In the limiting game, we replace each sub-pool of agents offering the same net reward by a fluid model where the arrival and service rates depend on the strategies of customers and agents. However, we cannot use the results in Whitt (2006) directly because the sub-pools in our model are interdependent. Particularly, a sub-pool can serve customers from other sub-pools as long as there are idle agents in the sub-pool, and customers requesting service from a sub-pool can be served by other sub-pools offering a higher net reward. Figure 5 illustrates an example sub-pool where the fraction of agents is y, the rate of customers who request service from this sub-pool is λ, the probability that its customers are served by other sub-pools is p, and the rate of customers coming from other sub-pools is λ'.

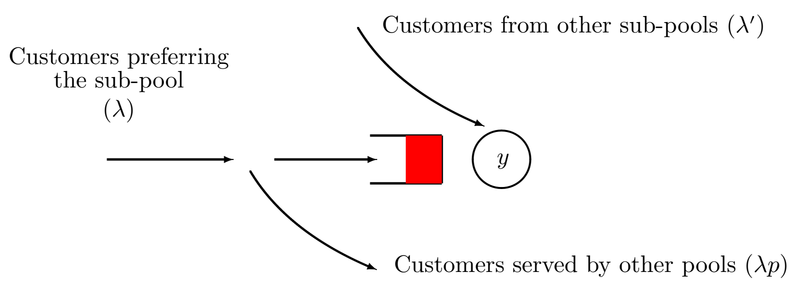


Figure 2 An example sub-pool.

Note that the queue accumulated in front of the above sub-pool depends only on the rate of customers requesting and getting the service from the sub-pool because customers coming from other sub-pools never queue in front of it. Therefore, the fluid approximation of this sub-pool operates as a system where the arrival rate is $\lambda(1-p)$ and the service capacity is y. Using Whitt (2006), the rate at which customers abandon such a system is $\max\{\lambda(1-p)-y, 0\}$. Based on this observation, the probability of abandonment for customers choosing the above sub-pool given that they are not served by another sub-pool is:

$$\overline{\beta}(\lambda,p,y) = \begin{cases} 0 & \textit{if } \lambda(1-p) < y \\ [\lambda(1-p) - y]/[\lambda(1-p)] & \textit{if } \lambda(1-p) \ge y, \end{cases} \tag{7}$$

Similarly, the utilization of the agents can be written as:

$$\overline{\sigma}(\lambda,\lambda',p,y)=\begin{cases}[\lambda(1-p)+\lambda']/y & \text{if } \lambda(1-p)+\lambda'<y\\ 1 & \text{if } \lambda(1-p)+\lambda'\geq y.\end{cases} \tag{8}$$

Using these probability of abandonment and utilization functions, we can simplify the expected utility and the revenue function described in (4)-(6). Given the strategies of agents, $(\mathbf{r_I},\mathbf{y_I})$, and the service decisions of class-I customers, $\mathbf{D_I}$, the expected utility of a class-I customer choosing the sub-pool-I_ℓ, for $\ell\in\{1,\dots,N_I\}$ and $I\in\{A,B\}$ can be written as:

$$U_{I_\ell}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})=\overline{P}_{I_{\ell\ell}}\big[(r_{I_\ell}+cm_a)(1-\overline{\beta}(\rho_I D_{I_\ell},\overline{P}_{I_{\ell\ell}},y_{I_\ell}))-cm_a\big]+\sum_{m\neq\ell}\overline{P}_{I_{\ell m}}r_{I_n},$$

where $\overline{P}_{I_{\ell m}}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})$ denotes the probability that a customer choosing the sub-pool-I_ℓ is served by the sub-pool-I_m for all $m\in\{1,\dots,N_I\}$. Moreover, the utilization of agents in sub-pool-ℓ can be written as

$$\overline{\sigma}_{I_\ell}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})=\begin{cases}\rho_I\big(\sum_{n=1}^{N_I}\overline{P}_{I_{n\ell}}D_{I_n}\big)/y_{I_\ell} & \text{if } \rho_I\sum_{n=1}^{N_I}\overline{P}_{I_{n\ell}}D_{I_n}<y_{I_\ell}\\ 1 & \text{if } \rho_I\sum_{n=1}^{N_I}\overline{P}_{I_{n\ell}}D_{I_n}\geq y_{I_\ell}.\end{cases}$$

Finally, given the exam thresholds ω_A and ω_B, the revenue of a dedicated agent in the sub-pool-I_ℓ is $[R_I(\omega_A,\omega_B)-r_{I_\ell}]\overline{\sigma}_{I_\ell}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})$, while the revenue of a flexible agent in the sub-pool-I_ℓ is $[R_{IF}(\omega_A,\omega_B)-r_{I_\ell}]\overline{\sigma}_{I_\ell}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})$.

Now, using these utility and revenue functions, we introduce the definitions of customer and market equilibrium. We first start with the equilibrium among customers given the strategy of agents.

Definition 1 (Fluid Customers Equilibrium). Given any $(\mathbf{r_I},\mathbf{y_I})$ for $I\in\{A,B\}$, we say that $\mathbf{D_I}\equiv(D_{I_n})_{n=1}^{N_I}$ is a *Fluid Customers Equilibrium (FCE)* if the following conditions are satisfied:

1. For any ℓ with $D_{I_\ell}>0$ and for all $n\leq N_I$, we have that $\overline{U}_{I_\ell}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})\geq\overline{U}_{I_n}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})\geq 0$.
2. If $\overline{U}_{I_n}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})>0$ for some $n\leq N_I$, then $\sum_{n=1}^{N_I}D_{I_n}=1$.
3. If $\overline{U}_{I_n}(\mathbf{r_I},\mathbf{y_I},\mathbf{D_I})=0$ and $\rho D_{I_n}<y_{I_n}$ for some $n\leq N_I$, then $\sum_{n=1}^{N_I}D_{I_n}=1$.

The first condition of the *FCE* requires that customers request service from a sub-pool in equilibrium only if that sub-pool maximizes their expected utility. Moreover, the second condition ensures that all customers join the system if it is possible to earn strictly positive utility by requesting service from a sub-pool. Finally, the last condition states that all customers join the system when there is an under-utilized sub-pool even if the expected utility of customers is not strictly positive. Note that an underutilized sub-pool should be offering zero net-reward if the expected utility of customers is zero. Therefore, customers are indifferent between requesting service from such a sub-pool and leaving the system immediately. This condition essentially breaks ties for a customer who is indifferent between joining the system and leaving immediately in favor of joining.

Using Rath (1992) and the fact that utility functions are continuous imply that a *FCE* always exits. However, the uniqueness of a *FCE* is not guaranteed for a given strategy of agents. Despite the fact that there may be multiple *FCE* for a given strategy of agents, Proposition 1 proves that the equilibrium utilization of an agent can be characterized irrespective of the multiplicity in *FCE*.

PROPOSITION 1. *Given any* $(\mathbf{r_I}, \mathbf{y_I})$ *such that* $r_{I_n} > r_{I_{n+1}}$ *for all* $n \in \{1, \ldots, N_I\}$ *and* $I \in \{A, B\}$, *let* $\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I})$ *be the utilization of an agent in sub-pool-*I_ℓ *in a Fluid Customer Equilibrium for all* $\ell \in \{1, \ldots, N_I\}$. *Then, we have that*

$$\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}) = \begin{cases} 1 & \text{if } \rho_I > \rho^0_\ell + y_{I_\ell}, \\ (\rho_I - \rho^0_n)/y_{I_n} & \text{if } \rho \in [\rho^0_\ell, \rho^0_\ell + y_{I_\ell}], \\ 0 & \text{if } \rho_I < \rho^0_\ell. \end{cases}$$

Here, we let $\rho^0_\ell = \frac{\sum_{n=1}^{\ell-1}(r_{I_n} + c_I m_a) y_{I_n}}{r_{I_\ell} + c_I m_a}$, *and* $\rho^0_1 = 0$.

The above proposition shows that agents in a sub-pool can earn a strictly positive revenue only if the demand from the customer class they serve is greater than a critical demand level. The critical demand level for each sub-pool depends only on the decisions of the sub-pools offering a net reward that is higher than what the given sub-pool offers. Furthermore, Proposition 1 shows that a sub-pool can be fully utilized when demand rate exceeds the critical demand level for this sub-pool by at least its capacity. In Proposition 1, we focus only on the utilization of the agents in the customer equilibrium. We can also characterize the fraction of customers choosing each sub-pool explicitly (See Proposition 3 in Appendix A). The main driver of the result in Proposition 1 is that customers are either always or never served by sub-pools other than their preferred one in equilibrium. This establishes that the interdependency between the sub-pools announcing different net rewards is minimal in a *FCE*. In fact, the marketplace operates "almost like" the combination of independent sub-pools.

Once we characterize the equilibrium among customers and derive the revenues of agents in this equilibrium, we now focus on the game among the agents. As we mentioned before, agents simultaneously make their service and pricing decisions in the first stage of the game. In the game among agents, we need to account for two types of agent deviations: Agents can either choose to deviate by joining an existing sub-pool or announce a new price. Furthermore, a limited size of agents are allowed to deviate together since the moderating firm enables communication among agents. Therefore, an equilibrium in the first stage should be immune to any of these two types of deviations formed by at most δ fraction of agents. In the large-scale marketplace we study, it is possible that a small group of agents can find profitable deviation from every price in some cases. However, the gains from some deviations may be arbitrarily small. As in Allon et al. (2012), we ignore deviations which result in small gains by employing a somewhat weaker notion of equilibrium, which allows us to characterize the market outcome in the fluid game even when a Nash equilibrium does not exist. To be more specific, we focus on an equilibrium concept, which requires immunity against only deviations that improve the revenue of an agent by at least $\epsilon > 0$ as formally stated in Definition 2 (see below). We refer to ϵ as the level of equilibrium approximation and suppose it is arbitrarily close to zero. To ease notation, we denote $R_I(\omega_A, \omega_B)$ and $R_{IF}(\omega_A, \omega_B)$ by R_I and R_{IF}, respectively, for any given exam threshold pair (ω_A, ω_B) and any $I \in \{A, B\}$. We also let $\mathbf{e_I}^x \equiv (e^x_{I_n})_{n=1}^{N_I}$ denote the N_I-dimensional vector with a 1 in the x^{th} coordinate and 0 elsewhere.

Definition 2 (Fluid Market Equilibrium). Let $(\mathbf{r_I}, \mathbf{y_I}) \equiv (r_{I_n}, y_{I_n})_{n=1}^{N_I}$ summarize the strategy of all agents in the marketplace for any $I \in \{A, B\}$. Then, $(\mathbf{r_I}, \mathbf{y_I})$ is a (ϵ, δ)-*Fluid Market Equilibrium ((ϵ, δ)-FME)* if the following conditions are satisfied.

1. For any $\ell \le N_I$, $m \le N_I$, $0 < d \le \min\{y_{I_\ell}, \delta\}$, and $I \in \{A, B\}$, we have that

$$[R_I - r_\ell]\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}) \ge [R_I - r_m]\sigma^{FCE}_{I_m}(\mathbf{r_I}, \mathbf{y'_I}) - \epsilon \tag{9}$$

$$[R_{IF} - r_\ell]\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}) \ge [R_{IF} - r_m]\sigma^{FCE}_{I_m}(\mathbf{r_I}, \mathbf{y'_I}) - \epsilon, \tag{10}$$

where $\mathbf{y'_I} = \mathbf{y_I} - d\mathbf{e_I}^\ell + d\mathbf{e_I}^m$.

2. For any $\ell \le N_I$, $0 < d \le \min\{y_{I_\ell}, \delta\}$, $I \in \{A, B\}$, and $r' \ne r_{I_n}$ for all $n \le N_I$, we have that

$$[R_I - r_\ell]\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}) \ge [R_I - r']\sigma^{FCE}_{I_{N_I+1}}(\mathbf{r'_I}, \mathbf{y'_I}) - \epsilon, \tag{11}$$

$$[R_{IF} - r_\ell]\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}) \ge [R_{IF} - r']\sigma^{FCE}_{I_{N_I+1}}(\mathbf{r'_I}, \mathbf{y'_I}) - \epsilon, \tag{12}$$

where $\mathbf{r'_I} = (\mathbf{r_I}, r')$, and $\mathbf{y'_I} = (\mathbf{y_I}, d)$.

3. For any $\ell \le N_I$, $m \le N_J$, $0 < d \le \min\{y_{I_\ell}, \delta\}$, $I, J \in \{A, B\}$, and $I \ne J$, we have that

$$[R_{IF} - r_\ell]\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}) \ge [R_{JF} - r_m]\sigma^{FCE}_{J_m}(\mathbf{r_J}, \mathbf{y'_J}) - \epsilon, \tag{13}$$

where $\mathbf{y'_J} = \mathbf{y_J} + d\mathbf{e_J}^m$.

4. For any $\ell \le N_I$, $0 < d \le \min\{y_{I_\ell}, \delta\}$, $I, J \in \{A, B\}$, $I \ne J$, $r' \ne r_{J_n}$ for all $n \le, N_J$, we have that

$$[R_{IF} - r_\ell]\sigma^{FCE}_{I_\ell}(\mathbf{r_I}, \mathbf{y_I}) \ge [R_{IJ} - r']\sigma^{FCE}_{J_{N_J+1}}(\mathbf{r'_J}, \mathbf{y'_J}) - \epsilon, \tag{14}$$

where $\mathbf{r'_J} = (\mathbf{r_J}, r')$, and $\mathbf{y'_J} = (\mathbf{y_J}, d)$.

Moreover, $(\mathbf{r_I}, \mathbf{y_I})$ is a *Fluid Market Equilibrium (FME)* if there exists a sequence $(\mathbf{r_I}^k, \mathbf{y_I}^k)$ such that $(\mathbf{r_I}^k, \mathbf{y_I}^k)$ is a (ϵ^k, δ^k)-*FME* for any $k = 1, 2, \ldots$ where $\epsilon^k \to 0$, $\delta^k \to 0$, and $(\mathbf{r_I}^k, \mathbf{y_I}^k) \to (\mathbf{r_I}, \mathbf{y_I})$ for all $n \le N_I$ as $k \to \infty$.

(9) and (11) in the (ϵ, δ)-*FME* definition state that dedicated agents have no incentive to deviate. Note that dedicated agents cannot change the customer class they serve. Therefore, (ϵ, δ)-*FME* accounts for two possible deviations for dedicated agents: joining an existing sub-pool or creating a new one. On the other hand, flexible agents have the option of changing the customer class they serve. Thus, (ϵ, δ)-*FME* ensures that flexible agents cannot improve their revenues whether they change the customer class they serve or not. In particular, (10) and (12) focus on flexible agent deviations when they keep the customer class they serve the same, whereas (13) and (14) consider deviations where flexible agents change the class they serve. Finally, we conclude that a strategy

profile is a *Fluid Market Equilibrium (FME)* if it is the limit of (ϵ,δ)-*FME* as both ϵ and δ becomes arbitrarily small.

The above definition of the market equilibrium has no restriction on the structure of the emerging equilibrium outcome. Particularly, we do not rule out an asymmetric equilibrium outcome, where the same type of agents (dedicated or flexible) charge different prices to serve the same customer class. Despite the quite general equilibrium definition, we show that the possibility of asymmetric equilibrium is very limited. Proposition 2 proves that the same type of agents serving the same customer class in any asymmetric *FME* must earn zero revenue. In other words, any asymmetric *FME* is outcome-equivalent (from the agents' point of view) to a symmetric *FME* where the same type of agents serving the same customer class charge their operating costs, which is normalized to zero. It also shows that the equilibrium revenues of flexible agents must be the same regardless of the customer class they serve. Hence, Proposition 2 establishes that the same type of agents earn the same revenue in any *FME*. This nice property of the equilibrium eases our analysis significantly.

Proposition 2. *Given any FME* $(\mathbf{r_I},\mathbf{y_I})$*, let* $\overline{V}_{ID_n}^{FME}$ *and* $\overline{V}_{IF_n}^{FME}$ *be the equilibrium revenue of a dedicated and a flexible agents in sub-pool-*I_n *for all* $I\in\{A,B\}$*, respectively.*

1. *If the number of different prices announced by the dedicated (flexible) agents serving class-I is two or more for some* $I\in\{A,B\}$*, then we have that* $\overline{V}_{ID_n}^{FME}=0$ $(\overline{V}_{IF_n}^{FME}=0)$ *for all* $n\in\{1,\dots,N_I\}$*.*
2. $V_{AF_n}^{FME}=V_{BF_m}^{FME}$ *for any* $n\in\{1,\dots,N_A\}$*,* $m\in\{1,\dots,N_B\}$*.*

We next start to characterize the equilibrium outcome in a marketplace with a special market structure, namely one customer class and two types of agents. This structure constitutes the building block of a marketplace with two classes. Carrying out our analysis in this building block model is a fundamental step towards finding the equilibrium outcome of the whole marketplace. It also allows us to discuss the intuition behind our results in a more clear way.

5.1. A Special Marketplace Structure

As we mentioned before, the moderating firm may set up the skill-mix structure in the marketplace (i.e., the capacity of dedicated and flexible agents) by changing the thresholds levels in each exam. We illustrate the possible skill-mix structures that can arise after the firm's choice of exam threshold pair in Figure 1. We observe that "*Inverted V-Network*," where there is only one customer class and two types of agents, is the key to characterize the equilibrium outcome in any skill-mix structure. For instance, once the agents make their service decisions in an M-Network, we need to analyze two separate Inverted V-Networks, in each of which one customer class can be served by both dedicated and flexible agents. Similarly, we can obtain the equilibrium outcome in other skill-mix structures by analyzing two separate Inverted V-Networks. Therefore, in this section, we study the equilibrium outcome of this special market structure, the Inverted V-Network.

We consider a marketplace, where there is only one class of customers, but there are two types of agents, say dedicated and flexible, who can serve these customers. We assume that the arrival rate of customers is ρ. As we have two types of agents, we let α_D be the capacity of dedicated agents, and α_F be the capacity of flexible agents. Furthermore, we assume that customers earn a reward of R_D when their service is completed by a dedicated agent, and they earn R_F when a flexible agent serves them. Finally, we denote the waiting cost by c. We keep all other assumptions we made in Section 3, and we will use the equilibrium concepts introduced in Section 5. We refer to the marketplace as buyer's market if $\rho < \alpha_D + \alpha_F$ and seller's market otherwise.

As we show in Proposition 2, agents in the same pool (dedicated or flexible) earn zero revenue in any asymmetric Fluid Market Equilibrium (*FME*). In other words, any emerging asymmetric *FME* should be outcome-equivalent to a symmetric *FME*, where agents in the same pool charge the same price and earn zero revenue in equilibrium. Based on this relationship, our approach to characterize the equilibrium outcome is as follows: We first derive the equilibrium revenues of agents in any symmetric *FME*, where all dedicated agents charge the same price p_D, and all flexible agents charge p_F. If there is a symmetric equilibrium where the equilibrium revenue of agents in a pool (dedicated or flexible) is zero, we can ignore any asymmetric equilibrium where agents in this pool charge different prices because such an asymmetric equilibrium will not provide any further information about the revenues of the agents. Otherwise, we will check if there is any asymmetric equilibrium, where agents in this pool charge different prices.

As a first step towards characterizing the symmetric equilibrium, we derive the revenue of agents when all the agents in the same pool charge the same price as follows:

Corollary 1. *Let $\overline{V}_D^{FCE}(p_D, p_F)$ and $\overline{V}_F^{FCE}(p_D, p_F)$ be the revenue of a dedicated and a flexible agent, respectively, when all the dedicated agents charge p_D and all flexible agents charge p_F. If $R_D - p_D \neq R_F - p_F$, then we have that*

$$\overline{V}_h^{FCE}(p_D, p_F) = \begin{cases} p_h(\rho/\alpha_h) & \text{if } \rho \le \alpha_h \\ p_h & \text{if } \rho > \alpha_h \end{cases}, \text{ and } \overline{V}_l^{FCE}(p_D, p_F) = \begin{cases} 0 & \rho < \rho_l^0 \\ p_l(\rho - \rho_l^0)/\alpha_l & \rho \in [\rho_l^0, \rho_l^0 + \alpha_l] \\ p_l & \rho > \rho_l^0 + \alpha_l, \end{cases}$$

where $\rho_l^0 = \frac{R_h - p_h + c m_a}{R_l - p_l + c m_a}\alpha_h$ $h = \arg\max_{i\in\{D,F\}}(R_i - p_i)$, and $l = \arg\min_{i\in\{D,F\}}(R_i - p_i)$.

The above corollary, which follows as an implication of Proposition 1, establishes that the agent pool offering the higher net reward (dedicated or flexible) will always be "over-utilized" as long the total demand exceeds their capacity. On the other hand, the pool offering the lower net reward will be "under-utilized" unless ρ is sufficiently high.

Using the above result, the agent pool offering the lower net reward will always be "under-utilized" in a buyer's market since we have that $\rho_l^0 + \alpha_l > \alpha_D + \alpha_F$. In other words, the demand rate, ρ, cannot be high enough to let the agent pool offering the lower net reward be "over-utilized." We show that this will create an opportunity for the members of that pool to improve their revenue by slightly decreasing their price. Hence, in a buyer's market, we can only have a symmetric equilibrium where only one of the agent pools (dedicated or flexible) earn positive revenue. In fact, Theorem 1 below establishes that the pool generating higher customer reward can earn positive revenue in a buyer's market once the demand exceeds their capacity.

The above corollary also states that both pools can be over-utilized in a seller's market. In this case, cutting the price does not help the agents to improve their revenues. However, we show that in this setting, a small fraction of agents from the group offering the higher net reward will have an opportunity to increase their prices and improve their revenues. Thus, in a seller's market, all agents, regardless of their group, offer the same net reward in a symmetric equilibrium. We formally present these observations in the following result:

THEOREM 1. *Let $\overline{V}_D^{FME}$ and $\overline{V}_F^{FME}$ be the revenue of a dedicated and a flexible agent, respectively, in a FME of a marketplace with one customer class and two agent pool. Then, letting $h = \arg\max_{i\in\{D,F\}} R_i$, and $l = \arg\min_{i\in\{D,F\}} R_i$, we have that:*

1. *If $\rho < \alpha_h$, then we have that $\overline{V}_h^{FME} = \overline{V}_l^{FME} = 0$.*
2. *If $\rho = \alpha_h$, then we have that $\overline{V}_h^{FME} \le R_h - R_l$, and $\overline{V}_l^{FME} = 0$.*
3. *If $\alpha_h < \rho < \alpha_h + \alpha_l$, then we have that $\overline{V}_h^{FME} = R_h - R_l$, and $\overline{V}_l^{FME} = 0$.*
4. *If $\rho = \alpha_h + \alpha_l$, then we have that $\overline{V}_h^{FME} = V_l^{FME} + (R_h - R_l)$, and $\overline{V}_l^{FME} \le R_l$.*
5. *If $\rho > \alpha_h + \alpha_l$, then we have that $\overline{V}_h^{FME} = R_h$, and $\overline{V}_l^{FME} = R_l$.*

When we focus on the case where customers earn more reward from the dedicated agents (which reduces to the setting $h = D$ and $l = F$ in Theorem 1), the above theorem shows that the dedicated agents can only charge their operating costs in a symmetric equilibrium when demand is low, and in such a setting, only the dedicated agents can serve customers. Thus, all agents earn zero in equilibrium when demand is low in a buyer's market. Once the demand rate exceeds the capacity of dedicated agents, α_D, the revenues of the dedicated agents in a buyer's market can be positive, yet below $R_D - R_F$, the difference between the customers rewards generated by different types of agents. When demand is equal to α_D, there are multiple equilibria and dedicated agents may charge any price less than or equal to $R_D - R_F$ in equilibrium. However, they can agree on charging $R_D - R_F$ in equilibrium when demand rate is strictly greater than their capacity. Dedicated agents are fully utilized in such an equilibrium and thus earn $R_D - R_F$. The intuition behind this result is the following: The rate of customers requesting service from the dedicated agents will exceed their processing capacity when dedicated agents charge a price lower than $R_D - R_F$. Therefore, customers experience significant waiting times and incur a strictly positive waiting cost. Using the fact that customers pay an extra cost, a small group of agents can increase their prices while ensuring that they are still over-utilized after the price increase. Since this small group of agents increases their prices without hurting their utilization, this deviation clearly improves their revenues.

Theorem 1 also shows that flexible agents can earn a strictly positive revenue only in a seller's market. When the demand rate is equal to the total processing capacity, there are multiple equilibria and the equilibrium revenue of flexible agents can be any value below R_F. However, once the demand rate is strictly greater than the total service capacity, revenues of both dedicated and flexible agents must be equal to the customer reward they generate in a symmetric equilibrium. In such an equilibrium, agents charge their highest prices, and the rate of customers requesting service is equal to the total capacity, $\alpha_D + \alpha_F$. In other words, the equilibrium behavior of the agents avoids any congestion in the system while extracting all of the customer surplus. The main driver of this result is similar to the intuition given above: There is congestion in the system when all agents earn less than the reward they generate, and this creates an incentive for agents to increase their prices. Once we obtain the revenues of agents in any symmetric equilibrium, we now discuss asymmetric equilibria. As we mentioned before, we only need to pay attention to asymmetric equilibria when agents in a pool earn strictly positive revenues in any symmetric equilibrium. In the proof of Theorem 1, we show that an asymmetric *FME* cannot emerge in such a circumstance. Particularly, we rule out the possibility of asymmetric equilibrium, where the dedicated agents charge different prices, for any demand rate $\rho > \alpha_D$. Similarly, there cannot be any asymmetric equilibrium, where the flexible agents charge different prices, when $\rho > \alpha_D + \alpha_F$.

After characterizing the equilibrium outcome in our building-block model, we next turn our attention to the moderating firm's skill-mix and capacity decisions.

Remark 1. The marketplace we consider in this section is similar to the one with nonidentical agents studied in Allon et al. (2012). Allon et al. (2012) develops an asymptotic theory to understand the behavior of the equilibrium along the sequence of marketplaces growing in size. The methodology developed in Allon et al. (2012) can easily become analytically intractable while analyzing more complex problems such as the moderating firm's skill and capacity management problem. Thus in this paper, we focus directly on the limiting game, whose results are easy to incorporate into the moderating firm's problem while capturing the main managerial insights obtained from an asymptotic analysis. Furthermore, given that we use a fluid model in the limiting game, we also investigate the outcomes of asymmetric equilibria, which are ignored in Allon et al. (2012).

6. The Skill-Mix and Capacity Management of the Moderating Firm

As we mentioned in the Introduction, the profit of the moderating firm is a share of the total revenue generated in the marketplace. Therefore, in order to maximize its profit, the firm has to maximize the total revenue in the marketplace by choosing the appropriate exam threshold pair (ω_A, ω_B).

Once the firm chooses the exam thresholds, a *Fluid Market Equilibrium* will emerge, and using the outcome of this equilibrium one can calculate the revenue generated in the marketplace. We denote the total revenue in the marketplace by $\Pi(\omega_A,\omega_B)$ given any exam threshold pair (ω_A,ω_B).

Notice that while Theorem 1 shows that there might be multiple equilibria, this multiplicity happens only if a given exam threshold pair (ω_A,ω_B) leads to a *knife-edge* case, i.e., the demand rate of a class is equal to the capacity of the dedicated and/or the flexible agents serving this class. For mathematical convenience, we will be focusing on the equilibrium generating the highest possible revenue among all possible equilibria[1]. We let $\overline{V}_F(\omega_A,\omega_B)$ be the highest equilibrium revenue of a flexible agent, and $\overline{V}_{ID}(\omega_A,\omega_B)$ be the highest equilibrium revenue of a dedicated agent serving class-I for all $I\in\{A,B\}$[2]. Then, we write the total revenue in the marketplace as $\Pi(\omega_A,\omega_B)=\sum_{I\in\{A,B\}}\left(\overline{V}_{ID}(\omega_A,\omega_B)\alpha_I(\omega_A,\omega_B)\right)+\overline{V}_F(\omega_A,\omega_B)\alpha_F(\omega_A,\omega_B)$.

We need to characterize the profit function $\Pi(\omega_A,\omega_B)$ for any given exam threshold pair (ω_A,ω_B) to find the optimal skill-mix and capacity decision of the moderating firm. Based on our results in Theorem 1, we know that the equilibrium revenue of an agent can change significantly even after the firm slightly modifies the exam thresholds in a marketplace. For instance, the revenue of an agent may go down from his highest price to zero when the moderating firm increases the service capacity available to the customer class he is serving by a small amount. Therefore, the firm's profit function has different functional forms in different regions of exam threshold space $[0,\bar{R}_A]\times[0,\bar{R}_B]$. Finding the optimal exam threshold pair requires comparing all these different functional forms, which is analytically intractable when the skills of an agent, S_A and S_B, follow a general joint probability distribution. To have a tractable problem, in this section, we study the skill-mix and capacity decision of the firm by imposing three different conditions on S_A and S_B:

1. S_A and S_B are perfectly positively correlated: $P(\{S_A=S_B\})=1$,
2. S_A and S_B are perfectly negatively correlated: $P(\{S_A+S_B=1\})=1$,
3. S_A and S_B are independent and identically distributed.

We also restrict our attention to a seller's market, where the total demand rate exceeds the total service capacity of candidate agents. We provide a brief discussion on the firm's optimal decisions when this assumption is relaxed.

[1] When there are multiple equilibria in a marketplace, the moderating firm can sustain the equilibrium with the highest revenue by perturbing the thresholds levels (ω_A,ω_B) slightly. For instance, if the marketplace were in a knife-edge case described in Theorem 1.4, the firm could increase the exam threshold by an arbitrarily small amount and move the marketplace to the case described in Theorem 1.5. Such a change would make sure that there is a unique equilibrium and the total revenue in the marketplace is arbitrarily close to the highest level among the multiple equilibria before the change.

[2] Proposition 2 shows that the same type of agents (dedicate or flexible) must earn the same revenue for any given equilibrium even if they charge different prices. Therefore, we can denote the revenues of the same type of agents in any given equilibrium by a unique value.

6.1. Positively Correlated Skills

We start with studying the skill-mix and capacity decision of the firm in a marketplace where $S_A = S_B$ and S_A is random with continuous distribution $F(\cdot)$ on the support $[0,1]$. We denote the mean of S_A by μ. Given this structure, we can derive the capacity of dedicated and flexible agents for any pair of thresholds (ω_A, ω_B) using (1). Similarly, the reward that a customer expects from a certain pool of agents can be calculated using (2)-(3). As we have $S_A = S_B$, the expected reward from flexible agents is the same for both classes of customers, and thus will be denoted by $R_F(\omega_A, \omega_B)$ in this subsection.

When the skills of an agent are positively correlated, the moderating firm can create three different skill-mix structures: a) one with only flexible agents, the so-called V-Network, (by setting $\omega_A = \omega_B$), b) one with flexible agents and dedicated agents only for class-A, the so-called N-Network, (by setting $\omega_A > \omega_B$), c) one with flexible agents and dedicated agents only for class-B (by setting $\omega_A < \omega_B$). In a marketplace with only flexible agents, agents can extract all of the customer surplus as discussed in Theorem 1 since the total demand rate is higher than potential service capacity, i.e., $\rho_A + \rho_B \geq 1$. Therefore, the total revenue generated in a V-Network is $R_F(\omega_A, \omega_B)\alpha_F(\omega_A, \omega_B)$. On the other hand, agents may not extract all of the customer surplus in an N-Network because they may not charge the highest price that they can charge in equilibrium. In fact, we show that even if they extract all of the customer surplus, the total revenue generated in a marketplace with dedicated agents cannot exceed the revenue in a marketplace with only flexible agents keeping the capacity of eligible agents constant. Thus, keeping the total service capacity constant, the moderating firm is (weakly) better off having only flexible agents.

Once we establish that the firm prefers V-Network over N-Network, it is sufficient to focus on the firm's profit on skill-mix structures with only flexible agents. When there are only flexible agents in the marketplace, i.e., $\omega_A = \omega_B = \omega$ for some $\omega \in [0,1]$, the total service capacity, $\alpha_F(\omega, \omega)$, decreases in ω whereas higher threshold levels lead to higher expected reward, $R_F(\omega, \omega)$. Thus, the firm has to trade-off between the total service capacity and the equilibrium revenue of agents. It turns out, the firm strictly prefers higher capacity since $R_F(\omega, \omega)\alpha_F(\omega, \omega)$, which is equal to $\int_\omega^1 sf(s)$, is decreasing in ω. This implies that the moderating firm always chooses to use all of its available service capacity. Specifically, it sets the exam threshold pair to $(0,0)$, i.e., all agents pass both exams. The following theorem formally presents these results.

THEOREM 2. *When $S_A = S_B$ and S_A is random with continuous distribution $F(n)$ on support $[0,1]$, we have that*

1. $\alpha_A(\omega_A^*, \omega_B^*) + \alpha_F(\omega_A^*, \omega_B^*) + \alpha_B(\omega_A^*, \omega_B^*) = 1$ *for any threshold levels (ω_A^*, ω_B^*) that maximize the total revenue in the marketplace.*
2. *The exam threshold pair $(0,0)$ maximizes the total revenue in the marketplace.*

6.2. Negatively Correlated Skills

After studying the positively correlated skills, we now characterize the optimal skill-mix and capacity decision of the moderating firm in a setting where $S_A + S_B = 1$ and S_A is random with continuous distribution $F(\cdot)$ on support $[0,1]$. We denote the mean of S_A by μ. For any given pair of threshold levels (ω_A, ω_B), the capacity of each agent pool and the reward that a customer expects from a certain pool can be calculated using (1)-(3).

In a marketplace where an agent's skills are negatively correlated, there are two different skill-mix structures that can arise: a) one with only dedicated agents for each class, the so-called I-Network, (by setting $\omega_A + \omega_B \geq 1$), and b) one with flexible agents and dedicated agents for each class, the so-called M-Network, (by setting $\omega_A + \omega_B < 1$). In a marketplace with only dedicated agents, there is a separate marketplace for each customer class. Then, by Theorem 1, the equilibrium revenue of an agent serving class-I is the expected reward his pool generates, $R_I(\omega_A, \omega_B)$, if the marketplace for class-I is a seller's market, i.e., $\alpha_I(\omega_A, \omega_B) \leq \rho_I$, and zero otherwise for all $I \in \{A, B\}$. Therefore, the firm never creates a marketplace where the service capacity of a dedicated pool exceeds its demand. Furthermore, the revenue generated in each separate seller's market increases by its service capacity, as we discuss in Section 6.1. Thus, the moderating firm always prefers to increase the service capacity in each separate marketplace while making sure that the dedicated capacity for each customer class is less than their demand. In fact, among all exam threshold pairs resulting in a skill-mix structure with only dedicated agents, the firm prefers the ones which use all candidate agents and create two separate seller's markets, i.e. any (ω_A, ω_B) with $\omega_A + \omega_B = 1$ and $F^{-1}(1-\rho_A) \leq \omega_A \leq F^{-1}(\rho_B)$. Note that the firm uses all of its available service capacity for any exam threshold pair resulting in a marketplace with both flexible and dedicated agents. Therefore, the moderating firm always uses all of its available service capacity to maximize its profits as in the case of positively correlated skills. This finding is summarized in Theorem 3.1. We also show that the revenue generated in a marketplace with both flexible and dedicated agents cannot exceed the revenue in a marketplace where the firm utilizes all candidate agents and creates two separate seller's markets. Hence, in a marketplace with negatively correlated agent skills, the moderating firm can maximize its profits by creating two separate seller's markets and using all of its available service capacity.

After showing that creating two separate seller's market maximizes the profits of the firm, the question that still remains is how the firm divides the service capacity between these two marketplaces. As the firm prefers to use all of its available service capacity, an increase in the capacity allocated to one class decreases the capacity for the other one. Since more capacity means higher revenue in a seller's market, the moderating firm trades-off between the gains from increasing the capacity for one class and the losses from decreasing the capacity for the other one. Also, note that some of the customers from a class do not request service when the capacity allocated to this class is strictly less than its demand rate as discussed before in Theorem 1. Therefore, the service capacity allocated to a class should not be less than its demand rate if the firm wants to serve all customers from this class. We say the firm *favors* a class when it allocates enough capacity to serve all customers from this class.

It turns out that the gains from increasing the service capacity for class-A dominate the firm's losses in the marketplace for class-B when the demand from class-A is lower than a critical demand level, which is $1-F(1/2)$. Thus, if $\rho_A < 1-F(1/2)$, the moderating firm increases the service capacity for class-A until the capacity equals the demand from this class and allocates the rest of the agents to class-B. As a result of this skill-mix structure, all of the class-A customers obtain service whereas some of the class-B customers do not request service since the demand from class-B is more than the service capacity allocated to this class. In other words, the moderating firm favors class-A when $\rho_A < 1-F(1/2)$. Similarly, the firm maximizes its profits by favoring class-B when the demand from class-B is lower than the critical demand level of $F(1/2)$. As a customer class is favored when its demand rate is lower than a critical level, the firm favors a class if its demand rate is too low. The driver of this result is the fact that only the agents generating very high customer rewards can serve the class with too low demand even if all of the customers from this class are served. Then, one may expect that the firm always favors the customer class with lower demand rate because the expected reward of the customers from this class is always higher but this intuition is not true. For instance, when $F(1/2) < 1/2$, the firm may favor class-A even if $\rho_A > \rho_B$. This, actually, proves the significance of the skill distribution function in the firm's skill-mix and capacity allocation problem.

It is also possible that the demand from both classes exceed the corresponding demand thresholds discussed above. In such a case, the moderating firm maximizes its profits at a pair of threshold levels where the marginal gains from increasing the service capacity for one class is equal to the marginal loss from decreasing the capacity for the other class. This happens when the firm allocates $F(1/2)$ fractions of agents to class-B and the rest to class-A. Under such a screening mechanism, the firm rejects customers from both classes.

Besides the optimal exam threshold pair that creates a marketplace with only dedicated agents, the firm may maximize its profits by using exam threshold pairs which lead to a marketplace with flexible agents. We show that the flexible agents prefer to serve the same customer class when the firm uses such an optimal exam threshold pair. Therefore, even though there are flexible agents in the marketplaces, the firm ensures that they behave as if they are dedicated agents to maximize its profits. Furthermore, the firm allocates enough capacity to serve all customers from class-A when $\rho_A < 1-F(\mu)$ and class-B when $\rho_B < F(\mu)$ under any optimal pair of exam thresholds allowing for flexible agents. We formally present the above results in Theorem 3.

Theorem 3. *When $S_A + S_B = 1$ and S_A is random with continuous distribution $F(n)$ on support $[0,1]$, we have that*

1. $\alpha_A(\omega_A^*, \omega_B^*) + \alpha_F(\omega_A^*, \omega_B^*) + \alpha_B(\omega_A^*, \omega_B^*) = 1$ *for any threshold levels (ω_A^*, ω_B^*) that maximize the total revenue in the marketplace.*
2. *The threshold levels $(\omega^*, 1-\omega^*)$ maximize the total revenue in the marketplace, where*

$$\omega^* = \begin{cases} F^{-1}(1-\rho_A) & \text{if } F^{-1}(1-\rho_A) > 1/2 \\ 1/2 & \text{if } F^{-1}(1-\rho_A) \le 1/2 \le F^{-1}(\rho_B) \\ F^{-1}(\rho_B) & \text{if } F^{-1}(\rho_B) < 1/2. \end{cases}$$

3. *If $\alpha_F(\omega_A^*, \omega_B^*) > 0$ for a pair of threshold levels (ω_A^*, ω_B^*) that maximize the total revenue in the marketplace, then all of flexible agents serve the same customer class in the equilibrium. Furthermore, the firm favors class-A when $\rho_A < 1 - F(1/2)$ and class-B when $\rho_B < F(1/2)$.*

Up to here, we have considered the firm's skill-mix and capacity decisions when the skills of each agent are perfectly correlated. We, now, turn to discuss marketplaces where an agent's skills are independent.

6.3. Independent Skills

In the previous subsections, we focus on marketplaces where an agent's skills are perfectly correlated. Now, we study the moderating firm's problem in a setting where the skills of an agent are independent and identically distributed. Particularly, we assume that both S_A and S_B are random with continuous distribution $F(\cdot)$ on the support $[0,1]$. We denote the mean of S_A and S_B by μ. As in the previous cases, we can derive the capacity of each agent pool and the reward that a customer expects from a certain pool using (1)-(3) for any given exam threshold pair (ω_A, ω_B).

Recall from Sections 6.1 and 6.2 that at the optimal skill-mix structure, the moderating firm utilizes all of its available capacity when agent skills are perfectly correlated in a seller's market. This is somewhat expected given the fact that total demand exceeds the available service capacity. The main driver of this result is that the moderating firm has no incentive to reduce the service capacity allocated to one customer class while keeping the capacity allocated for the other one unchanged. However, we show that the firm may benefit from limiting the capacity allocated to one class while keeping the capacity for the other class constant in a marketplace where an agent's skills are independent. This can be graphically illustrated by using Curve A and B in Figure 3.

For any threshold pair (ω_A, ω_B) on Curve A, we have that (i) the skill-mix structure is an M-Network with dedicated agents for each class and flexible agents, (ii) the capacity of dedicated agents for class-A is constant and equal to the demand rate of class-A, ρ_A, (iii) all flexible agents choose to serve class-B in equilibrium, and (iv) all agents charge the highest price they can charge. As the firm moves on Curve A in the direction pointed by arrows (which could be achieved by increasing both of the exam thresholds), it faces a trade-off: The profit from class-A increases

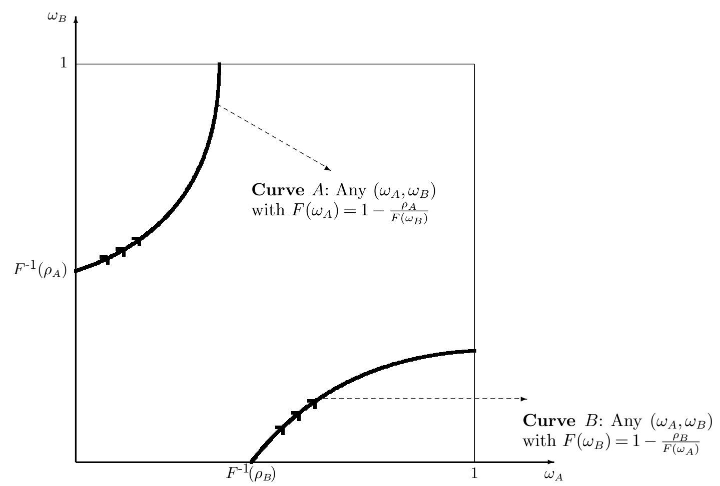


Figure 3 **Curve A (B): Threshold pairs (ω_A, ω_B) where the capacity of dedicated agents for class-A (class-B) is equal to the demand rate of class-A (class-B), ρ_A (ρ_B).**

because the average skill of a dedicated agent serving class-A increases whereas the profit from class-B decreases because the service capacity allocated to this class decreases. When the demand from class-A is less than the critical demand level of $F(\mu)$, we show that the gains from class-A outweighs the losses from class-B on Curve A for exam threshold pairs with zero or low thresholds in Exam-A. Thus, the firm is better off increasing the threshold levels rather than letting all agents pass Exam-A, which implies that there is an interior exam threshold pair that maximizes firm's profit on Curve A. We also show that this interior threshold pair generates more profit than any exam threshold pairs allowing all candidate agents to pass Exam-A, i.e., exam threshold pairs with $\omega_A = 0$. Similarly, the firm does not allow all candidate agents to pass Exam-B when the demand from class-B is less than $F(\mu)$. Hence, when both demand rates are less than $F(\mu)$, it is not optimal to allow all candidate agents in the marketplace to serve customers. In other words, the firm has to fail some of the candidate agents in both exams in order to maximize the revenue generated in a marketplace where agent skills are independent when both demand rates are lower than the critical demand level of $F(\mu)$.

Having both demand rates less than $F(\mu)$ is possible only if the mean of the skill distribution is greater than its median, i.e. $F(\mu) \geq 1/2$. If $F(\mu)$ is less than $1/2$, the higher of the two demand rates is greater than the threshold $F(\mu)$ since we consider a seller's market with $\rho_A + \rho_B > 1$. We assume the higher demand rate corresponds to ρ_A for ease of explanation. As $\rho_A > F(\mu)$, the firm's gains fall short of its losses when the firm increases the exam thresholds on Curve A, and thus the threshold pair $(0, F^{-1}(\rho_A))$ maximizes firm's profit on Curve A. On the other hand, the demand rate of class-B can still be less than $F(\mu)$. If the demand rate of class-B is less than $F(\mu)$, the firm will be better off choosing an interior point on Curve B rather than allowing all agents to pass Exam-B. Thus, the firm compares its profits under two exam threshold pairs: one lets all candidate agents pass Exam-A whereas the other one fails some of the candidate agents in both exams. We show that if the skill distribution function $F(\cdot)$ is convex[3], the firm's profits from the threshold pair failing agents outweigh its profits from the one letting all agents serve customers[4]. Hence, even if the mean of the skill distribution is less than its median, the optimal exam threshold pair prescribes to limit service capacity in a seller's market. We formally present these results in the following theorem.

THEOREM 4. *When S_I is random with continuous distribution $F(\cdot)$ on support $[0,1]$ for all $I \in \{A,B\}$ and $\rho_A \geq \rho_B$, we obtain the following:*

1. *If $\rho_A < F(\mu)$, then we have that $\alpha_A(\omega_A^*, \omega_B^*) + \alpha_F(\omega_A^*, \omega_B^*) + \alpha_B(\omega_A^*, \omega_B^*) < 1$ for any threshold levels (ω_A^*, ω_B^*) that maximize the total revenue in the marketplace. Furthermore, the threshold level $\left(F^{-1}\left(1 - \frac{\rho_A}{F(\hat{\omega}(\rho_A))}\right), \hat{\omega}(\rho_A)\right)$ maximizes the total revenue in the marketplace.*

2. *If $\rho_B < F(\mu)$ and $F(\cdot)$ is convex, then we have that $\alpha_A(\omega_A^*, \omega_B^*) + \alpha_F(\omega_A^*, \omega_B^*) + \alpha_B(\omega_A^*, \omega_B^*) < 1$ for any threshold levels (ω_A^*, ω_B^*) that maximize the total revenue in the marketplace. Furthermore, the threshold level $\left(\omega^*, F^{-1}\left(1 - \frac{\rho_B}{F(\omega^*)}\right)\right)$ maximizes the total revenue in the marketplace, where $\omega^* = \min\{\underline{\omega}, \hat{\omega}(\rho_B)\}$.*

Here, $\hat{\omega}(\rho)$ is the unique solution to $\int_{F^{-1}\left(1-\frac{\rho}{F(\omega)}\right)}^{1} [1 - F(s)]ds = \omega$ for any $\rho < F(\mu)$, $\underline{\omega} = \min\{\omega : F(\omega)[1 - F(\omega)] = \rho_B\}$ when $\rho_B < 1/4$, and $\underline{\omega} = 1$ when $\rho_B \geq 1/4$.

Besides the conditions under which the firm fails some of the candidate agents in a seller's market, the above theorem also describes the optimal skill-mix decision under these conditions. The first claim of Theorem 4 shows that the exam threshold pair that maximizes firm's profit on Curve A is, in fact, the optimal skill-mix decision among all exam threshold pairs. At this optimal exam threshold pair, the marginal gain from increasing the expected reward for class-A (while keeping the capacity of their dedicated agents at ρ_A) is equal to the marginal loss from reducing the available capacity for class-B. Since the optimal exam threshold pair in on Curve A, the optimal skill-mix structure is an M-Network with dedicated agents for each class and flexible agents. The optimal capacity of dedicated agents serving class-A is equal to the demand from this class. In the *Fluid Market Equilibrium* emerging at the optimal skill-mix structure, all agents charge the highest price that they can charge, and all flexible agents serve class-B.

[3] Note that convex $F(\cdot)$ implies that $F(\mu) \leq 1/2$ (See Proposition 3 in Caplin and Nalebuff (1991)).

[4] To prove this result, we actually need to have that $f(F^{-1}(s)) \leq f(F^{-1}(1-s))$ for any $s \leq 1/2$, which is a weaker condition than convex $F(\cdot)$. It is shown that this condition holds for any Beta distribution with shape parameters α and β (i.e., $f(s) = \frac{s^{\alpha-1}(1-s)^{\beta-1}}{B(\alpha,\beta)}$) as long as $\alpha \geq \beta \geq 1$ (See Runnenburg (1978) and Zwet (1979)). For simplicity of our expressions, we do not state our results using this condition.

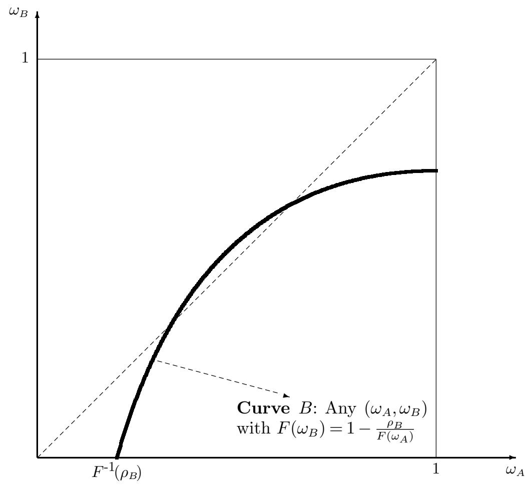


Figure 4 **Threshold pairs (ω_A, ω_B) where the capacity of dedicated agents for class-B is equal to the demand rate of class-B, ρ_B.**

The second claim of the above theorem, similar to the first one, establishes that the firm maximizes its profits by choosing an exam threshold on Curve B when $F(\cdot)$ is convex and $\rho_B < F(\mu)$. The optimal skill-mix structure is also an M-Network, and the optimal capacity of dedicated agents serving class-B is ρ_B. Moreover, all agents charge the highest price that they can charge. However, in contrast to the first claim of Theorem 4, it is possible that the marginal gain from increasing the expected reward for class-B exceeds the marginal loss from reducing the capacity of flexible agents. This means that the firm would benefit from an increase in both exam thresholds. However, the firm cannot improve its profits by failing more agents because such a change would cause a significant shift in the equilibrium behavior of the agents, namely, some of the agents would not charge their highest price. This can happen when the demand rate of class-B is less than 1/4. If $\rho_B < 1/4$, Curve B intersects with the 45^o-line as in Figure 4. At any exam threshold pair on Curve B and above the 45^o-line, dedicated agents serving class-B charge strictly lower than the highest price they can charge.

Note that unlike the first part of Theorem 4, where it is required to have both demand rates lower than a critical level, the condition in the second claim of Theorem 4 requires only the demand rate of class-B to be less than a critical level. Therefore, it may be optimal for the firm to fail candidate agents even when the demand from class-A is very high.

We performed a numerical study to gain further insight about the firm's optimal decisions. We present the optimal skill-mix decision in terms of the percentage of pass and fail in each exam in Table 1. In our numerical study, we observe that the firm may utilize 93% of the total service capacity even though the demand from class-A solely exceeds the total available capacity. Furthermore, the fraction of candidate agents passing a particular exam can be as low as 50%

according to the optimal exam threshold pair. Considering the exams which test fairly independent skills in oDesk.com, these results are consistent with the percentage of agents passing[5] a test in oDesk.com as presented in Table 2.

	$\rho_B = 0.1$	$\rho_B = 0.2$	$\rho_B = 0.3$	$\rho_B = 0.4$
Pass Exam-A	88.73 %	72.86 %	64.46 %	56.94 %
Pass Exam-B	88.73 %	73.70 %	84.40 %	92.90 %
Fail in both	1.27 %	7.14 %	5.55 %	3.06 %

(a)

	$\rho_B = 0.1$	$\rho_B = 0.2$	$\rho_B = 0.3$	$\rho_B = 0.4$
Pass Exam-A	88.73 %	77.35 %	68.38 %	59.52 %
Pass Exam-B	88.73 %	88.32 %	94.87 %	98.81 %
Fail in both	1.27 %	2.65 %	1.62 %	0.48 %

(b)

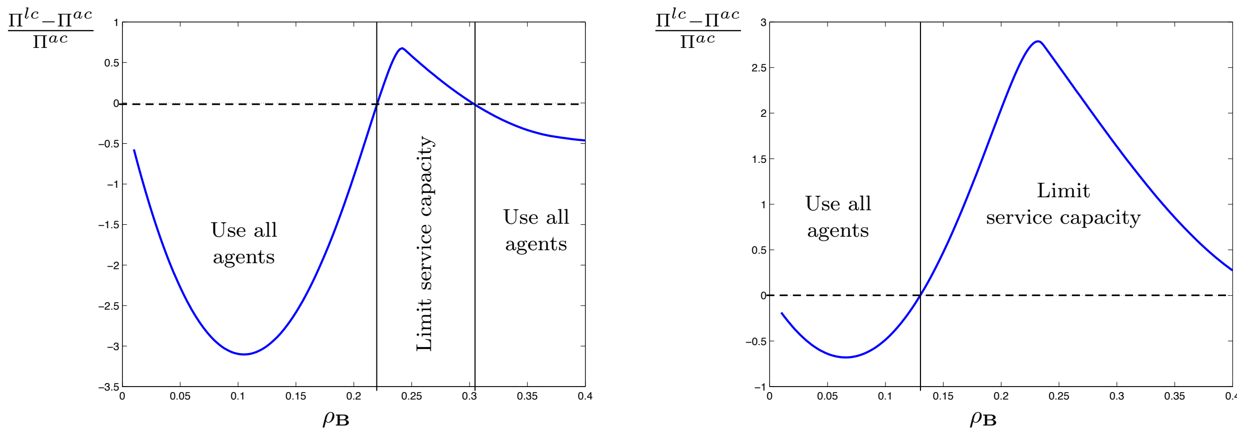
Table 1 The pass and fail percentages according to the optimal skill-mix decision when $\rho_A = 1$ and (a)$F(s) = s$, (b) $F(s) = s^{3/2}$.

Test ID	Test Title	Category	# of Provider Taken the Test	Passing Rate
584	Windows XP Test	Computer Skills	92530	72.11 %
681	Book keeping Test	Finance and Accounting	9651	64.22 %
518	HTML 4.01 Test	Web Designing	38746	63.61 %
692	English To Spanish Translation Skills Test	Translation Skills	4244	89.80 %
474	Adobe Photoshop 6.0	Graphics Designing	10998	72.00 %
561	Programming with C++ Test	General Programming	10143	37.96 %
571	Telephone Etiquette Certification	Office Skills	35703	92.81 %

Table 2 The passing rates in a set of exams that test independent skills in oDesk.com.

For the cases, which are not discussed in Theorem 4, we can show that the optimal skill-mix and capacity decisions of the firm fall into two cases: 1) the firm utilizes all candidate agents, and 2) the optimal decisions depend on the comparison between two exam threshold pairs. The second case arises when the boundary threshold pair $(0, F^{-1}(\rho_A))$ maximizes firm's profit on Curve A from Figure 3 and the firm's profit is maximized on Curve-B from Figure 3 at an interior point. The description of this interior point is given in Theorem 4.2. Then, the firm has to compare two exam threshold pairs one of which utilizes all candidate agents and the other does not. Note that Theorem 4.2 shows that the one that maximizes the firm's profit among the two is the interior optimal on Curve-B when the skill distribution $F(\cdot)$ is convex. However, if $F(\cdot)$ is not convex, comparing these two exam threshold pairs becomes analytically intractable. For instance, when $\rho_B < \min\{F(\mu), 1 - F(\mu)\}$ and $F(\cdot)$ is not convex, we need to compare the threshold pair $(0, F^{-1}(1 - \rho_B))$, which lets all agents pass Exam-A, and the optimal threshold pair described in Theorem 4.2, which does not utilize all candidate agents. As we numerically illustrate in Figure 5, either of these two pairs of exam thresholds can be optimal. We denote the firm's profit under the threshold pair utilizing all candidate agents by Π^{ac} and the profit under threshold pair limiting the service capacity by Π^{lc}. Figure 5 depicts Π^{lc} relative to Π^{ac} displaying the values as percentages. The details are omitted due to the space constraints.

[5] In oDesk.com, service providers must obtain a score of 2.5 out of 5.0 to “pass” a test for most of the tests.



(a) (b)

Figure 5 The firm's profit under the threshold pair described in Theorem 4.2 relative to its profit under the threshold pair $(0, F^{-1}(1-\rho_B))$ when $\rho_A = 1$, (a) $S_A \sim \mathbf{Beta}(0.5, 0.8)$ and (b) $S_A \sim \mathbf{Beta}(0.5, 0.6)$.

7. Conclusion

In this paper, we study a marketplace in which many small service providers, who are distinguished with respect to their service skills, compete with each other in providing service to two groups of self-interested customers, each of which has different needs. An important aspect of these marketplaces that our model captures is that customers cannot learn the skills of a provider before the completion of the service. However, the moderating firm, which sets up the marketplace, may help customers by providing them with further information about the ability of candidate agents through a skill screening mechanism. Such a screening mechanism consists of skill tests determining whether a candidate agent is eligible to serve customers and helps the firm to create different skill-mix structures in the marketplace. The main focus of this paper is on how the moderating firm uses the skill tests while making skill and capacity management decisions. To this end, we explore the optimal skill-mix and capacity decisions of the firm by considering three different structures for the correlation between the skills of an agent.

The firm faces a natural trade-off while setting up the skill tests: As the tests become more comprehensive, agents who pass the exams generate more value whereas the marketplace has less eligible agents. It turns out the firm is better off increasing the eligible service capacity when it focuses on the profits from only one customer class. This implies that the firm has an incentive to utilize all candidate agents if it can isolate its business with one class from the other one. We show that such an isolation is possible when an agent's skills are perfectly correlated, and thus the optimal skill-mix and capacity decision of the firm utilizes all candidate agents. On the other hand, the firm's profit from one class depends highly on the capacity allocated to the other class when agent skills are independent. As a result of this interdependence, we show that the firm benefits from limiting the capacity allocated to one class while keeping the capacity for the other one unchanged if the demand rate from one class is low enough. Therefore, the optimal skill-mix decision of the firm may not utilize all candidate agents even in a seller's market, where service capacity is scarce.

The decision of limiting the potential service capacity might also be optimal in other (more general) correlation structures. We identify some correlation structures where the firms optimal skill-mix decision is narrowed down to selecting between two candidate exam threshold pairs one of which utilizes all candidate agents and the other does not. Despite the fact that comparing these thresholds becomes analytically intractable, we numerically observe that the firm's profits under the exam threshold pair which limits the available capacity might outweigh its profits under the exam threshold utilizing all candidate agents.

In this paper, we study the moderating firm's skill and capacity management problem in a marketplace where the service capacity is scarce but our key findings would continue to hold in a buyer's market, where the service capacity is ample. We first note that in a buyer's market, the firm would always choose to limit the service capacity up to a level where the capacity of eligible agent is equal to the total demand rate because the ample capacity would make service providers charge very low prices due to the intensified competition. However, the moderating firm would not fail more agents than that when an agent's skills are correlated. Along the same lines with our findings in Theorems 2 and 3, the firm's profit from one class would be independent from its decisions regarding the other class. On the other hand, in a marketplace with independent skills, the optimal skill-mix might limit the potential service capacity even further by making the eligible service capacity strictly less than the total demand similar to our results in Theorem 4.

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We provide the formal presentations and the proofs of the supplementary lemmas used in our proofs in the Supporting Document. The Supporting Document also presents the extended proofs of our results.

Appendix A: Proofs in Section 5

A.1. Proof of Proposition 1

Below, we state and prove an extended version of Proposition 1. Proposition 1 is a direct implication of Proposition 3. When $\rho_I > \rho_\ell^0 + y_{I_\ell}$, the result holds since $\rho D_{I_\ell}^{FCE} > y_{I_\ell}$ by parts 2 and 3. When, $\rho_I \in [\rho_\ell^0, \rho_\ell^0 + y_{I_\ell}]$, this time the result holds by part 1. Finally, when $\rho_I < \rho_\ell^0$, the result holds again by part 1.

PROPOSITION 3. *Given any* $(\mathbf{r_A}, \mathbf{y_A})$, *let* $D^{FCE}_{A_n}$ *be the fraction of customers choosing sub-pool-*A_n *in a customer equilibrium, we obtain the following:*

1. *If* $\rho_A \in [\rho^0_\ell, \rho^0_\ell + y_{A_\ell}]$ *for some* $\ell \in \{1,\dots,N_A\}$, *then we have that* $D^{FCE}_{A_n} = \big((r_{A_n} + c_A m_a)y_{A_n}\big)/\big((r_{A_\ell} + c_A m_a)\rho_A\big)$ *for any* $n < \ell$, *and* $\sum_{n\geq \ell} D^{FCE}_{A_n} = (\rho_A - \rho^0_\ell)/(\rho_A)$.
2. *If* $\rho_A \in (\rho^0_\ell + y_{A_\ell}, \rho^0_{\ell+1})$ *for some* $\ell \in \{1,\dots,N_A - 1\}$, *then we have that* $D^{FCE}_{A_n} = \big((r_{A_n} + c_A m_a)y_{A_n}\big)/\big(\sum_{m=1}^{\ell}(r_{A_m} + c_A m_a)y_{A_m}\big)$ *for any* $n \leq \ell$, *and* $D^{FCE}_{A_n} = 0$ *for any* $n > \ell$.
3. *If* $\rho_A \in (\rho^0_{N_A} + y_{A_{N_A}}, \infty)$, *then we have that* $D^{FCE}_{A_n} = (r_{A_n} + c_A m_a)y_{A_n}/\big(\max\big\{\sum_{m=1}^{N_A}(r_{A_m} + c_A m_a)y_{A_m}, \rho_A c_A m_a\big\}\big)$ *for any* $n \leq N_A$.

Proof: 1. We first show that the expected utility of customers in the equilibrium, say CU^{FCE}_A, is r_{A_ℓ} in this case. To do this, we first suppose $CU^{FCE}_A < r_{A_\ell}$. Then, by Lemma 1.2, we have that $\sum_{n=1}^{N_A} D^{FCE}_{A_n} > 1$. However, this is clearly a contradiction since $\sum_{n=1}^{N_A} D^{FCE}_{A_n}$ cannot exceed 1. Similarly, $CU^{FCE}_A > r_{A_\ell}$ leads to a contradiction because it implies that $\sum_{n=1}^{N_A} D^{FCE}_{A_n} < 1$ despite customer utility is strictly positive. As a direct implication of $CU^{FCE}_A = r_{A_\ell}$, we have $\rho_A D^{FCE}_{A_1} = (r_{A_1} + c_A m_a)y_{A_1}/(r_{A_\ell} + c_A m_a)$. Furthermore, sub-pool-1 serves only its customers. Therefore, customers picking sub-pool-2 can only get their service from sub-pool-2, and because of that we should $\rho_A D^{FCE}_{A_2} = (r_{A_2} + c_A m_a)y_{A_1}/(r_{A_\ell} + c_A m_a)$. We can apply this argument to any sub-pool-n for $n < \ell$, and conclude that $\rho_A D^{FCE}_{A_n} = (r_{A_n} + c_A m_a)y_{A_n}/(r_{A_\ell} + c_A m_a)$ and sub-pool-n serves only its own customers for any $n < \ell$. Finally, since $\rho_A - \rho_A \sum_{n=1}^{\ell-1} D^{FCE}_{A_n} = \rho_A - \rho^0_\ell \leq y_{A_\ell}$, we should have that $\sum_{n=1}^{N_A} D^{FCE}_{A_n} = 1$. Thus, we have that $\sum_{n=\ell}^{N_A} D^{FCE}_{A_n} = (\rho_A - \rho^0_{A_\ell})/\rho$.

2. Similar to the proof in Part 1, we can argue that $CU^{FCE}_A \in (r_{A_{\ell+1}}, r_{A_\ell})$ in this case. Let $u = CU^{FCE}_A$. Then, we have that that $\rho_A D^{FCE}_{A_n} = (r_{A_n} + c_A m_a)y_{A_n}/(u + c_A m_a)$ and sub-pool-n serves only its own customers for any $n \leq \ell$. Therefore, customers choosing sub-pool-n for any $n > \ell$ cannot earn more than $r_{A_{\ell+1}}$, and we should have that $D^{FCE}_{A_n} = 0$ for any $n > \ell$. Finally, since $u > r_{A_{\ell+1}} \geq 0$, we have that $\sum_{n=1}^{\ell} D^{FCE}_{A_n} = 1$, and this implies that $\rho_A(u + c_A m_a) = \sum_{n=1}^{\ell}(r_{A_n} + c_A m_a)y_{A_n}$. Thus, we have that $D^{FCE}_{A_n} = (r_{A_n} + c_A m_a)y_{A_n}/\big(\sum_{m=1}^{\ell}(r_{A_m} + c_A m_a)y_{A_m}\big)$. The proof of Part 3 is very similar to Part 2 and can be seen in Appendix S.1.2 in the Supporting Document.

A.2. Proof of Proposition 2

WLOG we assume $r_{I_n} > r_{I_{n+1}}$ for all $n < N_I$. for notational convenience. Since $(\mathbf{r_I}, \mathbf{y_I})$ is an *FME*, there exists a sequence of (ϵ^k, δ^k)-*FME*, say $(\mathbf{r_I}^k, \mathbf{y_I}^k)$, such that $(\mathbf{r_I}, \mathbf{y_I}) = \lim_{k\to\infty}(\mathbf{r_I}^k, \mathbf{y_I}^k)$ where $\lim_{k\to\infty}(\epsilon^k, \delta^k) = (0,0)$.

1. In this proof, we focus only on class-A and dedicated agents. Let $\mathcal{N}_{AD} = \big\{n : \exists$ *dedicated agents in sub-pool-n*, $n \leq N_A\big\}$ and $\underline{n} = \min\{n : n \in \mathcal{N}_{AD}\}$. We prove our claim by contradiction. Therefore, we suppose $\overline{V}_{AD_{\hat{n}}} > 0$ for some $\hat{n} \in \mathcal{N}_{AD}$ and find a contradiction for $\rho_A \geq \rho^0_{\underline{n}+1}$ and $\rho_A < \rho^0_{\underline{n}+1}$, where ρ^0_ℓ is defined as in Proposition 1 for all $\ell \in \{1,\dots,N_A\}$. When $\rho_A \geq \rho^0_{\underline{n}+1}$, a small fraction of dedicated agents from sub-pool-$A_{\underline{n}}$ can improve their revenues after increasing their prices by an arbitrarily small ζ This contradicts with the definition of (ϵ,δ)-*FME*. On the other hand, when $\rho_A < \rho^0_{\underline{n}+1}$, we should have that $\overline{V}_{AD_n} = 0$ for all $n \in \{\underline{n}+1,\dots,N_A\}$. Then, a small fraction of dedicated agents from sub-pool-A_n can improve their revenues after charging a strictly positive price $p' = (R_A - r_{A_{\underline{n}}})/2$ for some $n \in \mathcal{N}_{AD}$ with $n \neq \underline{n}$. This also contradicts with the definition of (ϵ,δ)-*FME*.

2. Let $\overline{V}_{IF}(\omega_A,\omega_B)$ be the equilibrium revenue of a flexible agent serving class-I. We prove our claim by contradiction. Thus, we suppose $\overline{V}_{AF} \neq \overline{V}_{BF}$. When $\overline{V}_{AF} > \overline{V}_{BF} \geq 0$, a small fraction of flexible agents serving class-B can improve their revenues by charging $p' = (\overline{V}_{AF} + \overline{V}_{BF})/2$. This contradicts with the definition of (ϵ,δ)-*FME*. Similarly, when $\overline{V}_{BF} > \overline{V}_{AF} \geq 0$, a small group of flexible agents serving class-A can improve their revenues.

A.3. Proof of Theorem 1

Let $(p_D,p_F;\alpha_D,\alpha_F)$ be a symmetric *FME* in a marketplace with one customer class and two agent pools. Since $(p_D,p_F;\alpha_D,\alpha_F)$ is an equilibrium, there exists a sequence of (ϵ^k,δ^k)-*FME*, say $(p_D^k,p_F^k;\alpha_D,\alpha_F)$, such that $(p_D,p_F) = \lim_{k\to\infty}(p_D^k,p_F^k)$ where $\lim_{k\to\infty}(\epsilon^k,\delta^k) = (0,0)$. We let $\overline{V}_D^{FME}(k)$ and $\overline{V}_F^{FME}(k)$ be the revenue of a dedicated and a flexible agent, respectively, according to $(p_D^k,p_F^k;\alpha_D,\alpha_F)$. Then, we have that $\overline{V}_i^{FME} = \lim_{k\to\infty}\overline{V}_i^{FME}(k)$ for all $i \in \{D,F\}$. We suppose $R_D \geq R_L$ in this proof without loss of generality.

1 We show that $\overline{V}_D^{FME} = \overline{V}_F^{FME} = 0$ by contradiction. Thus, we suppose that either $\overline{V}_D^{FME} > 0$ or $\overline{V}_F^{FME} > 0$ is true on the contrary and find a contradiction for any possible price pair (p_D,p_F) satisfying either of these conditions. To this end, we follow a case-by-case analysis:

i. $(\mathbf{R_D - p_D = R_F - p_F})$: Notice to that dedicated agents are under-utilized since $\rho < \alpha_D$, and thus their revenue is at most $(\rho p_D^k)/\alpha_D$. When a small fraction of dedicated agents deviate and cut their prices by an arbitrarily small ζ, the revenue of deviating agents will be $p_D^k - \zeta$ for large k by Proposition 1. As this deviation improves the revenue of deviating agents, any (p_D,p_F) satisfying $R_D - p_D = R_F - p_F$ cannot emerge as an equilibrium price pair.

ii. $(\mathbf{R_F - p_F > R_D - p_D})$: Let $\rho_D^0 = \frac{R_F - p_F + cm_a}{R_D - p_D + cm_a}\alpha_F$. In this case, we have two sub-cases: **a)** $(\rho \leq \boldsymbol{\rho_D^0})$**:** By Corollary 1, we have that $\lim_{k\to\infty}\overline{V}_D^{FME}(k) = 0$ since $\rho \leq \rho_D^0$. We also should have that $\overline{V}_F^{FME} > 0$, which implies that $p_F > 0$. Then, when a small fraction of dedicated agents deviate and charge p' with $0 < p' < p_F$, the revenue of deviating agents will be p' for large k by Proposition 1. **b)** $(\rho > \boldsymbol{\rho_D^0})$**:** By Corollary 1, we have that $\lim_{k\to\infty}\overline{V}_D^{FME}(k) < p$. Then, as in Part 1.i, a small group of dedicated agents can improve their revenues by cutting their price.

iii. $(\mathbf{R_D - p_D > R_F - p_F})$: By Corollary 1, we have that $\lim_{k\to\infty}\overline{V}_D^{FME}(k) = \rho p_D$ and $\lim_{k\to\infty}\overline{V}_F^{FME}(k) = 0$. We should have that $p_D > 0$ to make sure $\overline{V}_D^{FME} > 0$. Then, as in **Part 1.i**, a small group of dedicated agents can improve their revenues by cutting their price.

2. We suppose that either $\overline{V}_D^{FME} > R_D - R_F$ or $\overline{V}_F^{FME} > 0$ on the contrary and follow a case-by-case analysis:

i. $(\mathbf{R_D - p_D = R_F - p_F})$: Notice that the utilization of at least one group of agents, say group-i where $i \in \{D,F\}$, should be less than the demand rate over total capacity, i.e. $\rho/(\alpha_D + \alpha_F)$. Then, when a small fraction of group-i agents deviate and cut their prices by an arbitrarily small ζ, the revenue of deviating agents will be $p_i^k - \zeta$ for large k by Proposition 1.

ii. $(\mathbf{R_F - p_F > R_D - p_D})$: The proof is the same as in **Part 1**.

iii. $(\mathbf{R_D - p_D > R_F - p_F})$: By Corollary 1, we have that $\lim_{k\to\infty}\overline{V}_D^{FME}(k) = p_D$ and $\lim_{k\to\infty}\overline{V}_F^{FME}(k) = 0$. We should also have that $p_D > R_D - R_F$ to make sure that $\overline{V}_D^{FME} > R_D - R_F$. Then, when a small fraction of flexible agents deviate and charge p' with $0 < p' < (R_F - R_D + p_D)$, the revenue of deviating agents will be p' for large k by Proposition 1.

3. We suppose that either $\overline{V}_D^{FME} \neq R_D - R_F$ or $\overline{V}_F^{FME} > 0$ on the contrary and follow a case-by-case analysis:

i. ($\mathbf{R_D - p_D = R_F - p_F}$): If $p_F > 0$, the proof is very similar to the proof of **Part 2.i**. If $p_F = 0$, we have that $\overline{V}_F^{FME} = 0$ and $p_D = R_D - R_F$. We should also have that $\overline{V}_D^{FME} < R_D - R_F$ and $R_D > R_F$ because otherwise we could not satisfy $\overline{V}_D^{FME} \neq R_D - R_F$ condition. Then, a small fraction of dedicated agents deviate can improve their revenues after cuting their prices by an arbitrarily small ζ.

ii. ($\mathbf{R_F - p_F > R_D - p_D}$): The proof is the same as in **Part 1**.

iii. ($\mathbf{R_D - p_D > R_F - p_F}$): When a small fraction of dedicated agents increase their price by $\zeta = \min\left\{(R_D - p_D) - (R_F - p_F), \left(1 - \frac{\alpha_D}{\rho}\right)(R_D - p_D + cm_a)\right\}/2$, their revenue will be $p_D^k + \zeta$ for large k by Proposition 1 by the choice of ζ. The proofs for Parts 4 and 5 are similar to **Part 3**.

Asymmetric equilibria: The possibility of asymmetric equilibrium does not affect our results for the revenue of dedicated agents in parts 1 and 2 and for the revenue of flexible agents in parts 1-4 because we do not exclude the possibility of zero revenue in these cases. In the remaining cases, we can show that there is not any asymmetric equilibrium as follows: Suppose there is an asymmetric equilibrium, where dedicated agents charge different prices, when $\rho > \alpha_D$ and $R_D > R_F$. By Proposition 2, we should have that all of the dedicated agents earn zero in the equilibrium. However, a small group of dedicated agents can guarantee a strictly positive revenue by charging a very low price ζ since $\rho > \alpha_D$. Similarly, we can rule out any asymmetric equilibrium where flexible agents charge different prices if $\rho > 1$.

Appendix B: Proofs in Section 6

B.1. Proof of Theorem 2

1. Note that we have $\alpha_A(\omega_A, \omega_B) + \alpha_F(\omega_A, \omega_B) + \alpha_B(\omega_A, \omega_B) < 1$ only if $\omega_I > 0$ for all $I \in \{A, B\}$. Therefore, it is sufficient to show that $\Pi(\omega_A, \omega_B) < \Pi(0,0)$ for any (ω_A, ω_B) with $\omega_I > 0$ for all $I \in \{A, B\}$. We consider two different regions: 1) $\omega_A \geq \omega_B$, and 2) $\omega_B > \omega_A$.

Region-1 ($\boldsymbol{\omega_A \geq \omega_B}$): Note that for any (ω_A, ω_B) in this region, we have that $\alpha_A = 0$. Furthermore, the equilibrium revenue of flexible agents and dedicated agents for class-B cannot exceed R_F and R_B, respectively. Hence, for any (ω_A, ω_B) in this region with $\omega_I > 0$ for all $I \in \{A, B\}$, we have that $\Pi(\omega_A, \omega_B) \leq R_B\alpha_B + R_F\alpha_F = \int_{\omega_B}^1 sf(s)ds < \int_0^1 sf(s)ds = \Pi(0,0)$, where the inequality holds since $\Pi(\omega_A, \omega_B)$ decreasing in ω_B, and the last equality holds because $\alpha_F(0,0) = 1$ and the equilibrium revenue of agents is $R_F(0,0)$ by Theorem 1 and the fact that $\rho_A + \rho_B > 1$ (Otherwise, we would have that $\gamma_A \geq \rho_A$ and $1 - \gamma_A \geq \rho_B$, where γ_A is the fraction of agents serving class-A. This would imply $\rho_A + \rho_B \leq 1$.)

Region-2 ($\boldsymbol{\omega_B > \omega_A}$): As in Region-1, for any (ω_A, ω_B) in this region with $\omega_I > 0$ for all $I \in \{A, B\}$, we have that $\Pi(\omega_A, \omega_B) \leq R_A\alpha_A + R_F\alpha_F = \int_{\omega_A}^1 sf(s)ds < \int_0^1 sf(s)ds = \Pi(0,0)$.

2. By part 1, any (ω_A, ω_B) with $\omega_I > 0$ for all $I \in \{A, B\}$ is dominated by $(0,0)$. Furthermore, we have that $\Pi(\omega_A, \omega_B) \leq \int_{\omega_I}^1 sf(s)ds = \int_0^1 sf(s)ds$ for any (ω_A, ω_B) with $\omega_I = 0$ for some $I \in \{A, B\}$.

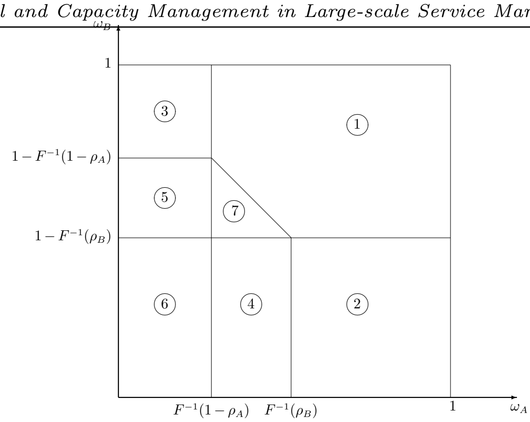

Figure 6 Different regions that a given (ω_A, ω_B) falls when the skills of an agent are negatively correlated.

B.2. Proof of Theorem 3

1. Let $\pi^{NC}(\omega) = \int_\omega^1 sf(s)ds + \int_0^\omega (1-s)f(s)ds$. We will show that $\Pi(\omega_A, \omega_B) < \Pi(\omega^*, 1-\omega^*)$ for any (ω_A, ω_B) with $\omega_A + \omega_B > 1$. We follow a case-by-case analysis based on the regions described in Figure 6.

Region-1: In this region, we have that the equilibrium revenue of agents serving class-I is R_I for $I \in \{A, B\}$ by Theorem 1. Thus, for any (ω_A, ω_B) in this region with $\omega_A + \omega_B > 1$, we have that $\Pi(\omega_A, \omega_B) = \int_{\omega_A}^1 sf(s)ds + \int_0^{1-\omega_B}(1-s)f(s)ds < \max\{\pi^{NC}(\omega_A) : F^{-1}(1-\rho_A) \le \omega_A \le F^{-1}(\rho_B)\} = \pi^{NC}(\omega^*) = \Pi(\omega^*, 1-\omega^*)$.

Region-2: In this region, we have that the equilibrium revenue of agents serving class-A is R_A, and the equilibrium revenue of agents serving class-B is 0 by Theorem 1. Thus, for any (ω_A, ω_B) in this region with $\omega_A + \omega_B > 1$, we have that $\Pi(\omega_A, \omega_B) = \int_{\omega_A}^1 sf(s)ds \le \int_{F^{-1}(\rho_B)}^1 sf(s)ds < \pi^{NC}\big(F^{-1}(\rho_B)\big) \le \pi^{NC}(\omega^*)$.
Region-3: As in Region-2, we have that $\Pi(\omega_A, \omega_B) < \pi^{NC}\big(F^{-1}(1-\rho_A)\big) \le \pi^{NC}(\omega^*)$ for any (ω_A, ω_B) in this region with $\omega_A + \omega_B > 1$.

2. We will show that $\Pi(\omega_A, \omega_B) \le \pi^{NC}(\omega^*)$ for any (ω_A, ω_B) with $\omega_A + \omega_B \le 1$. Then, our claim holds since $\Pi(\omega^*, 1-\omega^*) = \pi^{NC}(\omega^*)$. As in part 1, we follow a case-by-case analysis focusing on different regions
Region-6: We first note that the equilibrium revenue of flexible agents cannot exceed $\min\{R_{AF}, R_{BF}\}$. Furthermore, the equilibrium revenue of dedicated agents serving class-I cannot exceed R_I by definition. Then, for any (ω_A, ω_B) in this region, we have that $\Pi(\omega_A, \omega_B) \le R_A\alpha_A + R_{AF}\alpha_F + R_B\alpha_B = \pi^{NC}(\omega_A) \le \max_{\omega \le F^{-1}(1-\rho_A)} \pi^{NC}(\omega)$ and $\Pi(\omega_A, \omega_B) \le R_A\alpha_A + R_{BF}\alpha_F + R_B\alpha_B = \pi^{NC}(1-\omega_B) \le \max_{\omega \ge F^{-1}(\rho_B)} \pi^{NC}(\omega)$. Thus, $\Pi(\omega_A, \omega_B) \le \min\{\pi^{NC}\big(F^{-1}(1-\rho_A)\big), \pi^{NC}\big(F^{-1}(\rho_B)\big)\} < \pi^{NC}(\omega^*)$ for any (ω_A, ω_B) in Region-6.

Region-4: We first note that the equilibrium revenue of flexible agents cannot exceed R_{AF} in this region. Hence, for any (ω_A, ω_B) in this region, we have that $\Pi(\omega_A, \omega_B) \le R_A\alpha_A + R_{AF}\alpha_F + R_B\alpha_B = \pi^{NC}(\omega_A) \le \max\{\pi^{NC}(\omega) : F^{-1}(1-\rho_A) \le \omega \le F^{-1}(\rho_B)\} = \pi^{NC}(\omega^*)$.

Region-5: As in Region-4, we have that the equilibrium revenue of flexible agents cannot exceed R_{BF} in this region since $\alpha_A + \alpha_F > \rho_A$. Hence, for any (ω_A, ω_B) in this region $\Pi(\omega_A, \omega_B) \le R_A\alpha_A + R_{BF}\alpha_F + R_B\alpha_B = \pi^{NC}(1-\omega_B) \le \max\{\pi^{NC}(\omega) : F^{-1}(1-\rho_A) \le \omega \le F^{-1}(\rho_B)\} = \pi^{NC}(\omega^*)$.

Region-7: For any (ω_A, ω_B) in this region, we have that $\alpha_I + \alpha_F \le \rho_I$ for all $I \in \{A, B\}$. Let $\hat{I} = \arg\max_{I \in \{A,B\}} R_{IF}$. As $\alpha_{\hat{I}} + \alpha_F \le \rho_{\hat{I}}$, the equilibrium revenue of flexible agents serving class-$\hat{I}$ is $R_{\hat{I}F}$ by Theorem 1 even when all of the flexible agents choose to serve class-$\hat{I}$. Thus, all of the flexible agents choose to serve class-$\hat{I}$ in equilibrium because they cannot improve their revenue by serving the other class. Furthermore, the equilibrium revenue of dedicated agents serving class-I is R_I by Theorem 1. Therefore, for any (ω_A, ω_B) in this region $\Pi(\omega_A, \omega_B) = \max\{\pi^{NC}(\omega_A), \pi^{NC}(1-\omega_B)\} \le \pi^{NC}(\omega^*)$.

3. In Parts 1 and 2, we show that $\Pi(\omega_A, \omega_B) < \pi^{NC}(\omega^*)$ for any (ω_A, ω_B) in Regions 2, 3, and 6. Therefore, we have that $\omega_A^* \le F^{-1}(\rho_B)$ and $\omega_B^* \le 1 - F^{-1}(1-\rho_B)$. We also have that $\omega_A^* + \omega_B^* < 1$ as $\alpha_F(\omega_A^*, \omega_B^*) > 0$.

Felxible agents serve the same class: We prove our first claim by contradiction. Thus, we suppose there are flexible agents serving each customer class for some (ω_A^*, ω_B^*). Then, by Proposition 2.2, the equilibrium revenue of all flexible agents is less than or equal to $\min\{R_{AF}, R_{BF}\}$. This implies that $\Pi(\omega_A^*, \omega_B^*) \le \min\{\pi^{NC}(1-\omega_B^*), \pi^{NC}(\omega_A^*)\}$. Consider the case where $\omega^* = 1/2$. We should have either $\omega_A^* < 1/2$ or $\omega_B^* < 1/2$ for any (ω_A^*, ω_B^*). This implies that $\Pi(\omega_A^*, \omega_B^*) < \pi^{NC}(\omega^*)$, which clearly contradicts with the optimality of (ω_A^*, ω_B^*). Now, we consider the case where $\omega^* = F^{-1}(1-\rho_A)$. For any (ω_A^*, ω_B^*) with $\omega_B^* < 1 - F^{-1}(1-\rho_A)$, we have that $\Pi(\omega_A^*, \omega_B^*) \le \pi^{NC}(1-\omega_B^*) < \pi^{NC}(\omega^*)$. For any (ω_A^*, ω_B^*) with $\omega_B^* = 1 - F^{-1}(1-\rho_A)$, we have that the equilibrium revenue of all flexible agents is zero, which implies that $\Pi(\omega_A^*, \omega_B^*) \le \int_{1-\omega_B^*}^1 sf(s)ds + \int_0^{\omega_A^*} sf(s)ds < \pi^{NC}(\omega^*)$. These two observations clearly contradict with the optimality of (ω_A^*, ω_B^*). Similarly, our assumption that flexible agents serve both classes leads to $\Pi(\omega_A^*, \omega_B^*) < \pi^{NC}(\omega^*)$ when $\omega^* = F^{-1}(\rho_B)$.

The firm favors Class-A if $\rho_{\mathbf{A}} < \mathbf{1} - \mathbf{F}(\mathbf{1/2})$: Note that (ω_A^*, ω_B^*) is in either of the Regions 4, 5, and 7. For any (ω_A^*, ω_B^*) that is in Region-5, we should have that $\omega_B^* = 1 - F^{-1}(1-\rho_A)$, which implies that $\alpha_A(\omega_A^*, \omega_B^*) = \rho_A$. For any (ω_A^*, ω_B^*) that is in Region 4 or 7, we should have that $\omega_A^* = F^{-1}(1-\rho_A)$, which implies that $\alpha_A(\omega_A^*, \omega_B^*) + \alpha_F(\omega_A^*, \omega_B^*) = \rho_A$. Moreover, all flexible agents serve class-A since $R_{AF} > R_{BF}$. Similarly, we can show that the firm favors class-B when $\rho_B < F(1/2)$

B.3. Proof of Theorem 4

1. Let $\hat{\pi}(\omega, \rho) = \int_\omega^1 sf(s)ds + F(\omega)\left(\int_{w^c(\omega,\rho)}^1 sf(s)ds\right)$ where $w^c(\omega, \rho) = F^{-1}\left(1 - \frac{\rho}{\max\{\rho, F(\omega)\}}\right)$. We will show that $\Pi(\omega_A, \omega_B) \le \hat{\pi}(\hat{\omega}(\rho_A), \rho_A)$ for any (ω_A, ω_B) and $\Pi(\omega_A, \omega_B) < \hat{\pi}(\hat{\omega}(\rho_A), \rho_A)$ for any (ω_A, ω_B) with $\omega_I = 0$ for some $I \in \{A, B\}$. Note that $\alpha_A(\omega_A, \omega_B) + \alpha_F(\omega_A, \omega_B) + \alpha_B(\omega_A, \omega_B) = 1$ only if $\omega_I = 0$ for some $I \in \{A, B\}$. We follow a case-by-case analysis based on the regions described in Figure 7.a.

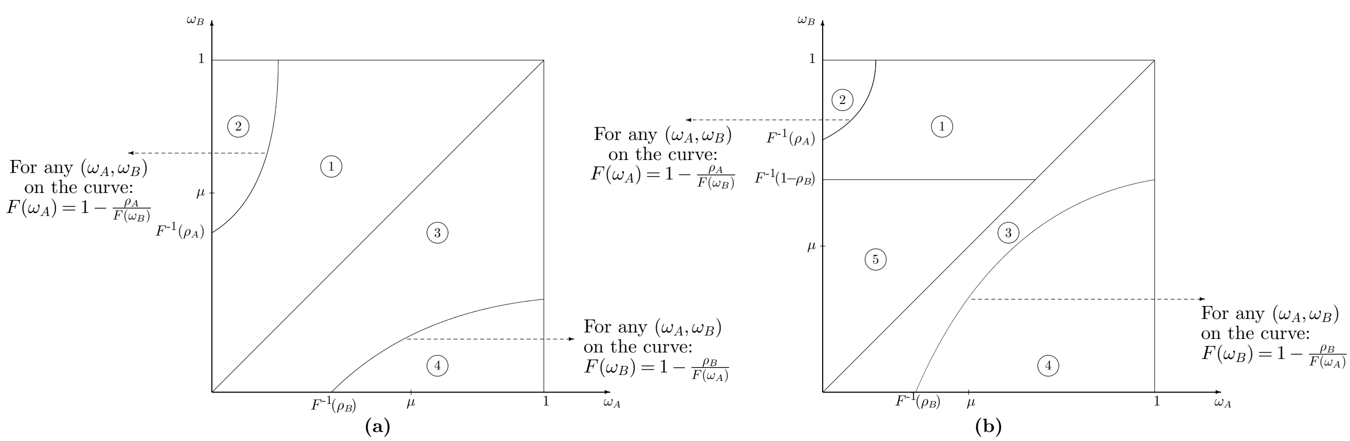

Figure 7 **Different regions that a given thresholds decision (ω_A, ω_B) falls when the skills of an agent are independent and (a) $\rho_A < F(\mu)$, (b) $\rho_B < F(\mu)$ and $F(\cdot)$ is convex.**

Region-1: For any (ω_A, ω_B) in this region, we have that $R_A \leq R_B$ since $\omega_A \leq \omega_B$. Therefore, the equilibrium revenue of flexible agents cannot exceed R_B. Furthermore, the equilibrium revenue of dedicated agents serving class-I cannot exceed R_I for all $I \in \{A, B\}$. Thus, for any (ω_A, ω_B) in this region, we have that $\Pi(\omega_A, \omega_B) \leq R_B[\alpha_B + \alpha_F] + R_A\alpha_A = \int_{\omega_B}^1 sf(s)ds + F(\omega_B)\int_{\omega_A}^1 sf(s)ds \leq \hat{\pi}(\omega_B, \rho_A) \leq \hat{\pi}\big(\hat{\omega}(\rho_A), \rho_A\big)$, where the second inequality holds since $\int_{\omega_A}^1 sf(s)ds$ is decreasing in ω_A. Furthermore, we have that $\Pi(0, \omega_B) < \hat{\pi}\big(\hat{\omega}(\rho_A), \rho_A\big)$ for any $\omega_B \leq F^{-1}(\rho_A)$ because $\hat{\pi}(\omega_B, \rho_A)$ is increasing in ω_B by Lemma 2 in Appendix S.2.2.

Now we show the optimality of $(\tilde{\omega}_A^*, \tilde{\omega}_B^*)$ with $\tilde{\omega}_A^* = F^{-1}\left(1 - \frac{\rho_A}{F(\hat{\omega}(\rho_A))}\right)$ and $\tilde{\omega}_B^* = \hat{\omega}(\rho_A)$. Notice that $\tilde{\alpha}_B^* + \tilde{\alpha}_F^* > \rho_B$, $\tilde{\alpha}_A^* = \rho_A$. As $\tilde{\alpha}_B^* + \tilde{\alpha}_F^* > \rho_B$ and $R_A^* \leq R_B^*$, all of the flexible agents choose to serve class-B in equilibrium. Furthermore, all agents charge the highest price they can charge in equilibrium. Thus, we have that $\Pi(\tilde{\omega}_A^*, \tilde{\omega}_B^*) = \tilde{R}_B^*[\tilde{\alpha}_B^* + \tilde{\alpha}_F^*] + \tilde{R}_A^*\tilde{\alpha}_A^* = \int_{\tilde{\omega}_B^*}^1 sf(s)ds + F(\tilde{\omega}_B^*)\int_{\tilde{\omega}_A^*}^1 sf(s)ds = \hat{\pi}\big(\hat{\omega}(\rho_A), \rho_A\big)$.

Region-2: For any (ω_A, ω_B) in this region, we have that $\alpha_A > \rho_A$, and thus, the equilibrium revenue of dedicated agents serving class-A is zero by Theorem 1. Hence, for any (ω_A, ω_B) in this region, we have that $\Pi(\omega_A, \omega_B) \leq R_B[\alpha_B + \alpha_F] = \int_{\omega_B}^1 sf(s)ds \leq \int_{F^{-1}(\rho_A)}^1 sf(s)ds < \hat{\pi}\big(F^{-1}(\rho_A), \rho_A\big)$.

Region-3-4: The proof is very similar to the proofs for Region 1 and 2, respectively.

2. We will show that $\Pi(\omega_A, \omega_B) \leq \hat{\pi}(\omega_A^*, \rho_B)$ for any (ω_A, ω_B) and $\Pi(\omega_A, \omega_B) < \hat{\pi}(\omega_A^*, \rho_B)$ for any (ω_A, ω_B) with $\omega_I = 0$ for some $I \in \{A, B\}$. We follow a case-by-case analysis based on regions described in Figure 7.b.

Region-1: As in Part 1, we have that $\Pi(\omega_A, \omega_B) \leq \hat{\pi}(\omega_B, \rho_A)$ for any (ω_A, ω_B) in this region. Furthermore, we have that $\hat{\pi}(\omega_B, \rho_A) \leq \hat{\pi}(F^{-1}(1-\rho_B), \rho_A)$ since $\hat{\pi}(\omega_B, \rho_A)$ is decreasing in ω_B for any $\omega_B > \mu$ by Lemma 2.2 in Appendix S.2.2 and the fact that $\rho_A \geq F(\mu)$. Finally, we have that $\hat{\pi}(F^{-1}(1-\rho_B), \rho_A) \leq \hat{\pi}(F^{-1}(\rho_B), \rho_B) < \hat{\pi}(\omega_A^*, \rho_B)$, where the inequalities hold by Lemma 2.4 in Appendix S.2.2.

Region-2: The proof is very similar to the proof for Region-2 in Part 1.

Region-3: As in Region-1 in Part 1, we have that $\Pi(\omega_A, \omega_B) \leq \hat{\pi}(\omega_A, \rho_B)$ for any (ω_A, ω_B) in this region. Let $\overline{\omega} = \max\{\omega : F(\omega)[1 - F(\omega)] = \rho_B\}$ for any $\rho_B < 1/4$, and $\overline{\omega} = 1$ for $\rho \geq 1/4$. Notice that (ω_A, ω_B) with $\omega_A \in (\underline{\omega}, \overline{\omega})$ is not Region-3 even if $\omega_A > F^{-1}(\rho_B)$ (which can only be possible when $\rho_B < 1/4$). Using this observation, for any (ω_A, ω_B) in this region, we have that $\Pi(\omega_A, \omega_B) \leq \max_{\omega \notin (\underline{\omega}, \overline{\omega})} \hat{\pi}(\omega, \rho_B)$. We also have that $\hat{\pi}(\omega, \rho_B)$ is increasing in ω for any $\omega < \min\{\underline{\omega}, \hat{\omega}(\rho_B)\}$ by Lemma 2.1. Combining these two observation, we have that $\max_{\omega \notin (\underline{\omega}, \overline{\omega})} \hat{\pi}(\omega, \rho_B) = \hat{\pi}(\omega_A^*, \rho_B)$ and $\Pi(\omega_A, 0) < \hat{\pi}(\omega_A^*, \rho_B)$ for any $\omega_A \leq F^{-1}(\rho_B)$. Finally, note that we have $\alpha_A^* + \alpha_F^* > \rho_A$, $\alpha_B^* = \rho_B$. Then, as we show in Region-1 in Part 1, $\Pi(\omega_A^*, \omega_B^*) = \hat{\pi}\big(\omega_A^*, \rho_A\big)$.

Region-4: Similar to Region-2, we have that $\Pi(\omega_A, \omega_B) \leq \hat{\pi}(F^{-1}(\rho_B), \rho_B)$ for any (ω_A, ω_B) in this region. Furthermore, we have that $\hat{\pi}(F^{-1}(\rho_B), \rho_B) < \hat{\pi}\big(\omega_A^*, \rho_A\big)$ as we show in Region-3.

Region-5: For any (ω_A, ω_B) in this region, we have that $\alpha_B + \alpha_F > \rho_B$ and $R_B \geq R_A$. Therefore, the equilibrium revenue of flexible agents cannot exceed R_A. This implies that the revenue of dedicated agents serving class-B is also R_A by Theorem 1. Hence, for any (ω_A, ω_B) in this region, we have that $\Pi(\omega_A, \omega_B) \leq R_A[\alpha_A + \alpha_B + \alpha_F] = \frac{1 - F(\omega_A)F(\omega_B)}{1 - F(\omega_A)} \int_{\omega_A}^1 sf(s)ds$. Notice that for any (ω_A, ω_B) with $\omega_A \notin (\underline{\omega}, \overline{\omega})$, we have that $\Pi(\omega_A, \omega_B) \leq \big[1 + F(\omega_A)\big] \int_{\omega_A}^1 sf(s)ds \leq \hat{\pi}(\omega_A, \rho_B)$. For any (ω_A, ω_B) with $\omega_A \in (\underline{\omega}, \overline{\omega})$, we have that $\Pi(\omega_A, \omega_B) \leq \big[1 + \frac{\rho_B}{1 - F(\omega_A)}\big] \int_{\omega_A}^1 sf(s)ds \leq \hat{\pi}(\underline{\omega}, \rho_B)$, where the last inequality holds by Lemma 3 in Appendix S.2.2. Finally, we have that $\Pi(0, \omega_B) \leq \mu = \hat{\pi}(0, \rho_B) < \hat{\pi}(\omega_A^*, \rho_B)$, where the strict inequality holds since $\hat{\pi}(\omega, \rho_B)$ is increasing in ω.