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Buying from the Babbling Retailer? The Impact of Availability Information on Customer Behavior

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Provision of real-time information by a firm to its customers has become prevalent in recent years in both the service and retail sectors. In this paper, we study a retail operations model where customers are strategic in both their actions and in the way they interpret information, whereas the retailer is strategic in the way it provides information. This paper focuses on the ability (or the lack thereof) to communicate unverifiable information and influence customers' actions. We develop a game-theoretic framework to study this type of communication and discuss the equilibrium language emerging between the retailer and its customers. We show that for a single retailer and homogeneous customer population setting, the equilibrium language that emerges carries no information. In this sense, a single retailer providing information on its own cannot create any credibility with the customers. We study how the results are impacted due to the heterogeneity of the customers. We provide conditions under which the firm may be able to influence the customer behavior. In particular, we show that the customers' willingness to pay and willingness to wait cannot be ranked in an opposite manner. However, even when the firm can influence each customer class separately, the effective demand is not impacted.

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1. Introduction

Provision of real-time information by a firm to its customers has become prevalent in recent years in both the service and retail sectors. Service providers use delay announcements to inform customers about anticipated service delays, whereas retailers provide the customers with information about the inventory level and the likelihood of being out of stock. Often, this information cannot be credibly verified by the customer. The question of how the information the firm shares with its customers influences their buying behavior is a complex one, and its answer depends both on the dynamics of the underlying operations and the customer behavior.

For example, the Web retailer Sierra Trading Post (http://www.sierratradingpost.com) uses the tag "almost gone!" for some of the products, and in its Frequently Asked Questions section explains this tag as follows:

> If an "almost gone!" label appears next to the item, the sell out risk is very high. We recommend that you place your order immediately.

Several other Web-based retailers, such as Barnes and Noble (http://www.barnesandnoble.com) and Circuit City (http://www.circuitcity.com), allow customers to search for the availability of specific products for in-store pick up. Along the same lines, Web-based travel agencies such as Expedia (http://www.expedia.com) allow customers to view the availability of airline tickets on specific flights, prior to making the purchasing decision. The apparel retailer for men Dickies (http://www.dickies.com) and the online bike store blackoutbmx (http://www.blackoutbmx.com) use the term "low inventory" for certain items, where they claim to carry low inventory.[1] Similarly, brick-and-mortar stores use different display modes to inform customers about availability, which range from showing ample stock per item to showing only a single available item per product. In all of these examples, the information shared cannot be fully verified by the customers. A customer in a brick and mortar does not know if there is more than a single item available even if only one is displayed and cannot verify whether the stock is indeed low, even if a tag "almost gone!" or a sign "limited availability" is attached to an item.

In this paper, we address these issues by proposing a model in which customers treat information provided by the retailer as unverifiable and nonbinding. The model thus treats customers as strategic both in the way they make decisions and process information, and the firm as strategic in the way it provides this

[1] The authors of this paper refrain from claiming that these announcements are indeed made only when the inventory is low.

information. Prior work in operations management analyzes systems assuming that the firm's information sharing strategy is a priori fixed and given. These typically lead to contrasting systems where the firm provides either full information or no information. Usually these papers assume that customers treat information as credible and verifiable, and implicitly assume that the firm restricts itself to truthtelling. The main issues with these assumptions are (a) customers may not blindly trust the information provided to them by firms; and (b) in practice, the information provided rarely translates directly into the inventory level. In this sense, the information may not be quantifiable. For example, the tag "almost gone!" does not reveal the accurate stock level, but it still carries *some* information. These issues are also interrelated and stem from the assumption that customers do not further process the information provided by the firm. In other words, the customers are not assumed to be strategic with respect to the information.

We develop a model in which these issues are addressed by considering a retailer that can provide various kinds of announcements (without restricting it to providing full information, no information, or making quantifiable announcements) and customers who are able to process the announcements and make strategic decisions. Specifically, we study a game played between the firm and its customers. In our model, the firm starts with a fixed inventory that it tries to sell at a fixed price during the regular season. Once the regular season is over, the price of the product drops. This marks the commencement of the sales season. The firm needs to decide what information to reveal to the customers during the regular season. The customers receive this information and decide whether to purchase the product immediately or wait for the sales season. The advantage of waiting is that the price of the product will be lower; however, there are two sources of disutility from buying in the sales period: (a) the customer needs to wait for the product and thus incurs a waiting cost, and (b) there is a chance that he might not be able to purchase the product because of its limited availability during the sales period.

In characterizing the emerging equilibrium language for this game, we account both for the strategic nature of the interested parties—the customers and the firm—as well as the dynamics prevalent in retail operations. We shall begin by showing that an influential language (in which the firm can influence customers using the information that they provide) is not possible between a single retailer and its customers when they are homogeneous in terms of their valuation of the product and the cost of waiting. This result demonstrates that a single monopolistic retailer cannot credibly communicate unverifiable availability information to its customers using cheap talk. When the customers are homogeneous, one can show that although the price may contain information regarding inventory availability, it cannot improve the credibility of the cheap-talk announcements.

We also study a setting in which the firm faces customers of multiple classes with different valuations of the product and time. Even when the customers are heterogeneous, the firm cannot impact the effective demand using availability information. That is, the overall demand realized for the product in both the regular and sales season is independent of the availability information provided by the firm. In this manner, the above result is robust. We also show that even though the firm cannot influence the effective demand, it may be able to influence its composition. That is, the firm may be able to influence each customer segment differently using availability information. In particular, we show that the firm cannot influence customer behavior unless customers with higher valuation of the product have a lower cost of waiting. We then show that the crucial variable that customers infer from every message is the extent by which the availability drops between the regular season and the sales season. For the firm to be able to influence its customers, it must be able to associate different availability drops with each message. Furthermore, we show that the firm must signal this drop in availability to be above and below a specific threshold to induce the influential behavior. In all other cases, the firm is not capable of influencing customer behavior using announcements of the availability risk.

The model formulated in this paper treats the information disclosure as "cheap talk," i.e., a preplay communication that carries no cost. Cheap talk, as described in the literature, consists of costless, nonbinding, nonverifiable messages that may affect the customers' beliefs. It is important to note that although providing the information does not *directly* affect the payoffs, it does have an indirect implication through the customers' reaction and the equilibrium outcomes. However, the information on its own has no impact on the payoffs of the different players per se; i.e., the payoffs of both sides depend only on the actions taken by the customer and inventory dynamics. The firm can neither reward nor penalize a customer based on whether or not he follows the firm's recommendation. In §1.2 we discuss in detail the different modeling assumptions, as well as the differences in the results between our model and the classical cheap-talk literature.

The key contributions of the paper can be summarized as follows:

1. We develop a model that studies the strategic nature of the information transmission in retail operations, where unverifiable and noncommittal

information is provided by a self-interested retailer to selfish customers.

2. The analysis of this model provides what appears to be the first theoretical result that shows that in any equilibrium that emerges in the single-retailer game with homogeneous customers, the availability announcements are *noninfluential*. In other words, the firm can obtain no credibility regarding the information provided about the inventory on hand and thus cannot influence a rational customer in terms of his purchasing decision. (See Proposition 4.1.)

3. We show that when customers have diverse valuations of both their willingness to wait and willingness to pay, the firm may be able to influence customer behavior, albeit in a limited manner. We first show that, in all equilibria, the effective demand (across all customer classes) is independent of the initial inventory and the messages of the firm. We then show that the firm may be able to influence each customer class individually. This can only happen if customers with higher valuation for the product have a higher willingness to wait. We also characterize the relationship between (a) the availability-drop associated with messages that influence the customers, and (b) the customer parameters.

4. We also prove that as the size of the market increases, the necessary conditions for an equilibrium to have influential cheap talk become more stringent in regards to the feasible parameters for the customer classes. In this sense, we expect that it becomes increasingly more difficult for firms to influence customers as they grow, assuming their demand and capacity grow at the same rate. We also discuss settings where the demand and quantity grow disproportionately.

We conclude this section with a review of the relevant literature. Section 2 describes the model for a single retailer. Section 3 analyzes the game played among the customers when the firm provides no information. Section 4 studies the cheap-talk game played between the retailer and the customers. Section 5 studies the impact of heterogeneity among the consumers. Section 6 provides discussion and conclusions. Proofs of results stated in §§3 and 4 are in the main body of the paper, and the proofs of the remaining results are relegated to the appendix.

1.1. Literature Review

Recent literature in operations management analyzes and models the impact of strategic customers. This literature can be broadly divided into two categories: (a) models where no availability information is provided to the customer, and (b) models where customers are provided with complete information regarding availability.

Our single-retailer model is related to the model introduced in Aviv and Pazgal (2008), which falls in the first category. The authors study pricing strategies for a retailer facing a stochastic arrival stream of customers. When customers arrive, they have no information about the current state of the inventory. A related model is studied in Dana (2001), where the firm signals availability using prices. Cachon and Swinney (2009) consider a model of a retailer that sells a product with uncertain demand over a finite selling season. The authors characterize the rational expectation equilibrium (REE) between the firm, who sets its initial quantity level, and the strategic customers, who choose whether to buy during the selling season or during the clearance season. The authors also study the impact of quick response and the interplay between the existence of strategic customers and this option. Su and Zhang (2008) show that the presence of strategic customers can impact the performance of a centralized supply chain when the customers form rational expectations regarding quantities and prices. They show that, whereas firms cannot commit to specific levels of inventory, decentralized supply chains can use contractual arrangements as indirect commitment devices to attain the desired outcomes with commitment. Liu and van Ryzin (2008) investigate whether it is optimal for a firm to create rationing risk by deliberately understocking products. The authors develop a model that determines the optimal amount of rationing risk to maximize their profits. In a related paper, Debo and van Ryzin (2009) study how customers infer the quality of the product from stockouts and availability (see Veeraraghavan and Debo 2009 for a service setting).

Yin et al. (2009) and Su and Zhang (2009) belong to the second category. Yin et al. (2009) consider a retailer that announces the regular price and the sales-season clearance price (or a contingent price) at the beginning of the selling season, as in our model. In the presence of either myopic customers or strategic customers, the authors compare two display modes: one where the retailer displays all the available units (and corresponds to providing full information to the customers) and one where it shows only one unit. Customers treat this one unit as verifiable proof that the firm has at least one unit in stock. The authors show that the retailers will earn higher expected profits under the "display one unit" format when the customers are strategic. Su and Zhang (2009) study the role of availability and its impact on consumer demand by analyzing a newsvendor model with strategic customers who incur some search costs in order to visit the retailer. They contrast the REE in a game where the availability information is not provided to the customer with the scenario where such information is provided. It is shown that the retailer can improve its profits in the latter. To deal with the lack of credibility of the above information, the authors study availability guarantees, in which the seller compensates the consumers in the event of stockouts.

Our paper contributes to the above papers by proving that, indeed, the firm cannot influence its customers using availability information when customers are homogeneous, thus showing that indeed "display one" (which is equivalent to a babbling equilibrium in our setting) is the only viable option for a firm that would like to announce information. Our paper also contributes to the above by showing that a firm may be able to influence customers using more refined information when customers are heterogeneous.

A closely related paper in terms of the underlying framework is Allon et al. (2011), which appears to be the first paper in the operations management literature to consider a model in which a firm communicates *unverifiable* information to its customers. Both papers focus on analyzing the problem of information communication in an operational setting by considering a model in which the firm and the customers act strategically: the firm in choosing its announcements, and the customers in interpreting this information and in making the decision. The settings considered, however, are very different, and the results are driven by the characterizing features of service systems and inventory systems. The paper concludes with §6.

1.2. Classical Cheap-Talk Game

In this section, we provide an overview of the cheap-talk game introduced in Crawford and Sobel (1982) and compare the model to the one studied in this paper. The classical cheap-talk game is played between a *sender*, who has some private information, and a *receiver*, who takes the payoffs-relevant actions. The game proceeds as follows: The sender observes the state of the world, which we shall denote by Q. The sender then sends a signal (or a message) denoted by $m \in \mathcal{M}$. (Here, $\mathcal{M}$ denotes the set of all signals that can be used by the sender.) The receiver, who cannot observe the state of the world Q, but does know its distribution, processes the signal (using Bayes rule) and chooses an action y that determines the players payoff. Both the sender and the receiver obtain utilities that depend on (a) the action taken by the receiver y, and (b) the state of the world Q. A distinctive feature of their model is that the distribution of the state of the world is exogenous and independent of the actions of the players.[2]

[2] A variety of papers study mixed-motive economic interaction involving private information and the impact of cheap talk on the outcomes. Farrell and Gibbons (1989) study cheap talk in bargaining; in political context cheap talk has been studied in multiple papers, including Austen-Smith (1990) and Matthews (1989).

Driven by the applications in operations management, our model has two novel features: first, the game is played with multiple receivers (customers) whose actions have externalities on other receivers; second, the stochasticity of the state of the world (i.e., the state of the system) is not exogenously given but is determined endogenously. In particular, the private information in this model (i.e., the availability of inventory both in the regular and the sales season) is driven by the equilibrium strategies of both the firm and the customers. In particular, the customers' actions are payoff relevant as well as system-dynamic relevant. As we shall see, the multiplicity of receivers with externalities as well as the endogenization of the uncertainty impact both the nature of the communication, when one exists, as well as the outcome for the various players. Hence, although the framework used in this paper echoes the cheap-talk model described in the literature, the above mentioned distinguishing features lead to different results.[3]

2. Model

We study a two-period model in which a firm aims to maximize its revenue. We will refer to the first period as the regular period and the second period as the sales period. In our model, the firm starts with an initial inventory Q_0. The customers do not know the initial inventory; however, they have beliefs regarding the actual inventory. The customer believes that the initial inventory has a cumulative distribution function F_{Q_0}. Further, we assume that $F(0) = 0$.

During the regular season, the *potential* customer demand is realized. The potential number of customers is Poisson with mean λ; we denote the realized potential demand by D_1. Each customer obtains a value v from the product. The potential demand captures the number of customers who are interested in buying the product but who will time their purchase to maximize their utility. The firm provides an announcement that signals the inventory level at the beginning of the regular period. The price of the product during the regular is set to p.[4] The customers (who form the potential demand) decide whether to buy[5] in the regular period or wait for the next period.

[3] One should note that the results in Crawford and Sobel (1982) and most of the cheap-talk literature are stated, based on the *bias* between the sender's and the receiver's preferred actions, which are exogenously given. In our model, the extent of the misalignment depends endogenously on the preferred action of the customers as it arises in equilibrium in the game they play. Thus, even the most basic results cannot be directly borrowed from this literature.

[4] We also provide a discussion of the setting where the pricing is a decision of the firm and contingent on the initial inventory in §6.

[5] Note that when we say that a customer decides to buy, we merely mean that the customer attempts to buy, but they might not be able to purchase because of limited availability.

The customers who decide to buy during the regular season form the *effective* demand for the regular period. The firm allocates/satisfies (as much as possible) the effective demand. We assume that, if the effective demand is higher than the quantity, then the product would be rationed uniformly among the customers who form the effective demand.

After the regular season, the firm is left with Q_s units of inventory. Note that the actual quantity on hand at the beginning of the sales period Q_s is determined by both the potential demand and the customers' buying decisions. The latter depends on the information they have, including among other things, the information provided by the firm.

The price in the sales season may be contingent on the quantity left and is denoted by $s(Q_s)$. During the sales season, new customers, which we refer to as *bargain hunters*, are also interested in buying the product. We will denote by D_2 the number of such customers, where the distribution of D_2 is denoted by F_{D_2}.[6] Thus, the effective demand during the sales season is formed by both the customers who decided to wait for the sales season and the bargain hunters. The customers who arrived during the regular season but decided to wait for the sales season will incur a cost of c for waiting. As before, the firm satisfies as much of this overall demand in the sales period as possible. Thus, the firm's revenue is $p(Q_0 - Q_s) + S(Q_s)\min(Q_s, D_1 + D_2 - Q_0 + Q_s)$.

Each customer arriving during the regular season faces a decision whether to buy immediately or wait for the sales season. The main trade-off customers face is whether to buy now at a given (high) price with relative high availability or wait and buy, facing much greater availability risk. This is driven by the customers' parameters (v, c). If he decides to buy during the regular season, then he obtains a value $(v-p)$ $A(Q_0, D_E)$, where D_E is the effective demand and $A(x, y)$ is the availability function. We assume that, if the demand is higher than the quantity on hand, then the likelihood of obtaining the product is identical among the customers who decide to buy; i.e., if the demand for the product is x and the quantity on hand is y then the likelihood is $A(x, y) = \min\{x/y, 1\}$. Similarly, if he decides to wait for the sales season, then his value is given by $(v-s)A(Q_s, D_2 + D_1 - D_E) - c$, where c is the cost of waiting between the regular and the sales season. Here, c is associated with the inconvenience of not obtaining the product immediately (we thus refer to $1/c$ as the willingness to wait, because it is the amount of time the customer is willing to wait for a dollar). Note that the quantity on hand in the sales period is given by Q_s, and the overall demand during the sales period is given by $D_1 + D_2 - D_E$. We shall refer to $A(Q_s, D_2 + D_1 - D_E)$ as the *availability* of the product during the sales season.

We can immediately make the following observation about the customer strategies: The customer has the option to leave the market, and obtain zero utility, but it can be easily seen because $v > p$ that the option of leaving the market is strictly dominated by the buying option during the regular season because $Q > 0$ with probability one. Thus, the customer decision can be reduced to whether he buys immediately or he waits for the sales season.

3. Providing No Information

The main focus of this paper is to characterize the ability or inability of a retailer to communicate unverifiable information to strategic customers. To be able to discuss the specific model of communication, we will initially discuss the customers behavior when the firm provides no information about inventory availability. In §4, we discuss whether this strategy of not providing information emerges in equilibrium.

In this setting, we assume that the firm is not providing any information with regards to the inventory position. Because the customers cannot observe the state of the system, they have to rely on their belief about the inventory level. Further, because all agents are a priori identical, we will be focusing on symmetric strategies for the customers. We will represent their strategy by $y \in [0, 1]$, which is the probability that a customer tries to buy the product in the regular season. We next define the notion of Bayesian Nash equilibrium (BNE) under no information (see Chaps. 6 and 13 of Fudenberg and Tirole 1991 for a definition of Bayesian Nash Equilibrium).

Definition 3.1. We say that the pair $y^* \in [0, 1]$ forms a BNE under no information in the retail cheap-talk game if and only if it satisfies the following condition:

$$y \in \arg\max_{y \in [0,1]} y\mathbb{E}[(v-p)A(Q_0, Z_y + 1) - (v - S(Q_0 - Z_y)) \cdot A((Q_0 - Z_y)^+, D_2 + D_1 - Z_y) + c \mid D_1 \geq 1], \tag{1}$$

where Z_y is a binomial random variable with $(D_1 - 1)^+$ trials each with probability y of success.

The above definition requires that the customers do not have any unilateral profitable deviation from the strategy profile that defines the equilibrium. Specifically, the condition requires that when fixing the strategy of the rest of the customers and assuming the firm

[6] One can view a more detailed description of the bargain hunter. For instance, D_2 can emerge as an aggregate number of arrivals during the regular season of customers whose valuation is below the regular price p.

provides no information, a customer should not have any profitable deviation. In the condition, the random variable Z_y is the effective demand (excluding the customer who is making the decision). Because all customers randomize with probability y and the demand (excluding the deciding customer) is given by $(D_1 - 1)^+$, we obtain that the effective demand is the number of successes in $(D_1 - 1)^+$ binomial trials, each with a success probability y. Thus, the objective in (1) is the difference between the utility if the customer decides to buy during the regular season and the utility if the customer decides to buy during the sales season.

To study whether such an equilibrium always exists, let $U^R(y)$ and $U^S(y)$ be the utility of a customer when he decides to buy during the regular season and sales season, respectively, when other customers are buying with probability y during the regular season. It is easy to verify that these functions are continuous decreasing functions of y. Thus, we obtain the following two cases:

Case 1. If $U^R(0) > U^S(0)$ and $U^R(1) < U^S(1)$, then there exists $U^R(y) = U^S(y)$. Let us denote the solution to $U^R(y) = U^S(y)$ by y^*. Then $y = y^*$ forms a BNE. (In this case, there is a mixed strategy BNE.)

Case 2. If Case 1 is not satisfied, it must be the case that either $U^R(0) < U^S(0)$ or $U^R(1) > U^S(1)$ are true. Thus, either $y = 0$ or $y = 1$ (or both) form an equilibrium, respectively. In this case, there is a pure strategy BNE. This completes the proof.

These are summarized by the following proposition, whose proof follows directly from the above discussion.

Proposition 3.1. *There exists a BNE for the game under no information.*

The above theorem shows that there exists an equilibrium among the customers when the firm does not provide any information.[7] One can view this equilibrium as self-organization of the customers among themselves in the absence of any information.

4. Cheap-Talk Equilibrium

In this section, we explore the game played between the firm and its customers, where the firm is allowed to use any information provision strategy. To define the single-retailer game formally, we shall start by defining the strategy of the customer followed by the strategy of the firm.

Let $\mathcal{M}$ be the Borel set, which is comprised of feasible signals that the firm can use, and let Ω denote its σ-algebra. Let y: $\mathcal{M} \mapsto [0, 1]$ represent the strategy of a customer. Here, $y(m)$ is the probability that a customer arriving during the regular season and receiving a signal $m \in \mathcal{M}$, buys the product during the regular season. Thus, this customer waits for the sales period with probability $1 - y(m)$. Let the space of feasible strategies for the customers be denoted by $\mathcal{Y}$.

Next, we describe the strategy of the firm. In doing so, we allow the firm to randomize over the set of messages in the set $\mathcal{M}$, i.e., given a specific quantity on hand q, the firm may randomize among different messages. To capture this, let ν: $\mathbb{Z} \times \Omega \mapsto [0, 1]$ represent the strategy of the firm. Here, we require that $\nu(q, \cdot)$ induces a probability measure on $\mathcal{M}$ from which the firm announces a realization, if the quantity on hand is q. Thus, if the quantity on hand is q at the beginning of the regular period, the probability that the firm signals a message from a measurable Borel-subset $S \subseteq \mathcal{M}$ is given by $\nu(q, S) = \int_{m \in S} d\nu(q, m)$. (For example, if the firm announces messages m_1 and m_2 with probability half when the quantity on hand is 5, then the measure $\nu(5, \cdot)$ is defined as follows: $\nu(5, \{m_1\}) = \nu(5, \{m_2\}) = 0.5$, and $\nu(5, S) = 0$ for all $S \subset \Omega$ and $S \cap \{m_1, m_2\} = 0$.) Let the space of feasible strategies for the firm be denoted by $\mathcal{G}$. Note that the quantity on hand at the beginning of the sales period Q_s is determined by the customer's strategy as well as the firm's strategy ν. Let $\mu_{\nu, y}$ represent the distribution of the signal transmitted if the firm follows strategy ν and the customers follow strategy y. A random variable with measure μ shall be represented by X_μ. Further, let the firm's profit under the strategy pair (ν, y) be written as $\Pi(\nu, y)$. Also, let $Q_{\nu, y, s}$ be the inventory on hand at the beginning of the sales period under the strategy pair (ν, y).

Definition 4.1. We say that the pair $(\nu^*, y^*) \in \mathcal{G} \times \mathcal{Y}$ forms a BNE in the retail cheap-talk game if and only if it satisfies the following two conditions:

1. For all $m \in \mathcal{M}$ $y_i^*(m) \in \arg\max_{y \in [0, 1]}$,

$$y\mathbb{E}[(v-p)A(Q_0, Z_y+1) - (v - S(Q_0 - Z_y)) \cdot A((Q_0 - Z_y)^+, D_2 + D_1 - Z_y) + c \mid D_1 \geq 1, X_{\mu_{\nu^*, y}} = m],$$

where Z_y is a binomial random variable with $(D_1 - 1)^+$ trials each with probability y of success.

2. Fixing y^*, ν^* solves

$$\nu^* \in \arg\max_{\tilde{\nu} \in \mathcal{G}} \Pi(\tilde{\nu}, y).$$

The above definition requires that both the firm and the customers do not have any unilateral profitable deviation from the strategy profile that defines the equilibrium. Specifically, the first condition in the definition requires that when fixing the strategy of the rest of the customers and the firm, a customer should not have any profitable deviation. Thus, given that all other customers interpret the messages and

[7] It is worth noting that the equilibrium may not be unique.

purchase the product as prescribed by the equilibrium, and thus given the availability that is driven by such behavior, a customer has no incentive to deviate from this prescription. Similarly, the second condition requires that given the customer's action rule y^* as fixed, the firm maximizes its profit by using strategy ν^*. That is, given that the firm knows how customers interpret its messages, the firm should not have any incentives to use this message on a different realization of capacity than the one prescribed by the equilibrium.[8]

4.1. Main Result

Next, we characterize the emerging equilibria in the homogeneous customer's cheap-talk game. We prove that it is impossible for the firm to influence the customer behavior using availability information if all customers are homogeneous in their valuation of the product and their cost of waiting and, in that sense, for the firm to credibly communicate any meaningful information to its customers. In particular, we show that any equilibrium that can emerge in the game played between the firm and its customers is such that the customers are not influenced by what the firm announces. We shall refer to such equilibrium as noninfluential (formal definition is defined below). Such equilibria can manifest themselves in several ways: the firm may either provide no information (in which case, naturally, the customers base their decisions only on their expectations of the inventory levels), or provide any type of information that is uncorrelated with the state of the system, yet the customers disregard it and, again, base their decisions on their expectations of the inventory levels. The key feature of a *noninfluential* equilibrium is that the action taken by the customers in equilibrium is independent of what the firm announces, due to the lack of credibility of such an announcement. Note that the definition focuses on the ability to induce different actions and these actions directly affect the profit of the firm and the utility of the customers. We shall next formally define the class of noninfluential equilibria.

Definition 4.2. We say that the pair $(y, \nu) \in \mathcal{Y} \times \mathcal{G}$ forms a noninfluential equilibrium if there exists a constant p such that for all $q \in \{0, 1, 2, \ldots, Q(0)\}$, we have that $\nu(q, \{m\colon y(m) = p\}) = 1$; i.e., on the equilibrium path, irrespective of the firm's announcement, the probability that a customer buys during the regular season is p.

In the previous section, we showed that a noninfluential equilibrium always exists in the sense that an equilibrium exists in the game with no information, but we have not yet excluded other more influential types of equilibria. We next show that all equilibria that can arise in the single-retailer cheap-talk game are noninfluential.

Proposition 4.1 (The Noninfluential Result). *Under any BNE of the single-retailer cheap-talk game, the customer's realized buying behavior satisfies the following*:

$$y(X_{\mu_{\nu,y}}) = y^* \quad a.s.,$$

where there exists an equilibrium with noninfluential cheap talk under which the customers purchase with probability y^* during the regular season. Thus, any BNE is noninfluential.

Proof. The proof is based on the following observation: because the firm prefers customers to buy in the regular season, it will provide a message that maximizes the probability of buying. Thus, under any strategy profile in which customers react differently to different messages, the firm will always have an incentive to deviate and use the one that maximizes the buying probability. Hence, in equilibrium, the customers behave as if the buying probability is fixed and is not impacted by the message provided by the firm.

We next make this rigorous. Consider any BNE of the above cheap-talk game, $(y, \nu) \in \mathcal{Y} \times \mathcal{G}$. When the firm signals m, the effective demand observed during the regular season is Poisson with mean $\lambda y(m)$. Thus, the profit of the firm if it provides message m is given by

$$\mathbb{E}\Bigg[p \min\Bigg(Q_0, \sum_{i=1}^{D_1} \mathbb{I}_{\{\omega_i^L \le y(m)\}}\Bigg) + S\Bigg(\Bigg(Q_0 - \sum_{i=1}^{D_1} \mathbb{I}_{\{\omega_i^L \le y(m)\}}\Bigg)^+\Bigg) \cdot \min\Bigg(\Bigg(Q_0 - \sum_{i=1}^{D_1} \mathbb{I}_{\{\omega_i^L \le y(m)\}} v\Bigg)^+, \sum_{i=1}^{D_1} \mathbb{I}_{\{\omega_i^L > y(m)\}} + D_2\Bigg)\Bigg].$$

Let $\mathcal{M}^* = \arg\max_m y(m)$. Thus, it is easy to see that under any equilibrium the firm must use signals from the set $\mathcal{M}^*$ only; i.e., any strategy where the firm signals a message that is not in $\mathcal{M}^*$ cannot form an equilibrium. Based on this, we obtain that the customer can only buy with probability $y^* = \max_m y(m)$ during the regular season in equilibrium and thus any equilibrium is noninfluential. This completes the proof. □

The above proposition shows that, in equilibrium, no matter what signaling rule the firm uses, the customers would simply ignore all the signals and make their buying decisions irrespective of any information provided. Thus, in this cheap-talk game no credibility whatsoever can be created. We next study how this result extends when the customers are heterogeneous.

[8] We assume that, off the equilibrium path, a message that was not supposed to be used will result in a "wait" decision by the customer.

5. Multiple Customer Classes

Whereas until now we have assumed that all the customers (those who actively shop in the regular season) were homogeneous, we next consider a model where they are heterogeneous both in terms of the value they derive from the product and the cost of waiting they incur when they decide to postpone purchase to the sales season. Specifically, we assume that there are two customer classes; we shall denote these classes by H and L and class specific attributes by subscript $i \in \{H, L\}$. Thus, the value of the product is v_i and the waiting cost per unit of time is c_i for a class i customer. We assume that the potential demand rate of each class is λ_i.

The key result of the previous section is that the firm cannot influence the customer's purchasing behavior by providing availability information. We next answer the question of whether or not the heterogeneity of customers' preferences allows the firm to influence customer behavior. To answer this question, consider two settings: (a) one in which the customer's class is observable to the firm and the firm can tailor the information provided to each class separately; and (b) one where the classes are unobservable, and thus the firm cannot tailor its message. The former may be applied by online retailers or catalog-based retailers, whereas the latter is more applicable for brick-and-mortar retailers. For the first setting, it is easy to show that our noninfluential result from the previous setting extends. We will thus focus on the latter case, in which the firm cannot observe the classes and thus need to use a unified message.

Our description is reminiscent of Farrell and Gibbons (1989), who compare settings in which the sender of information may choose between providing the information that is class dependent (they refer to such a setting as private messaging) to the setting where the information is provided without the knowledge of the customer class (they refer to such settings as public messaging). They show that if the sender can create credibility when providing information that is class dependent, it can also do so in the setting where the class is unobserved. However, if the sender cannot create credibility in the observable case, Farrell and Gibbons (1989) do not rule out the possibility of establishing credibility in the unobservable case.

We begin by demonstrating that even when customers are heterogeneous, the firm cannot impact the effective demand using messages on the availability of inventory. To study the setting where the customer class is unobservable, we define the tuple $\{\nu, y_L, y_H\}$ that represents the strategy of the various customer classes and the signaling rule for the firm. Here, $y_i(m)$ is the probability that a customer of class i attempts to purchase the product in the regular season when the firm signals m. As before, $\nu(q, \cdot)$ denotes the measure induced on the messages from which the firm signals during the regular season when the quantity on hand is q. We can define the BNE for the cheap-talk game in an analogous manner to Defintion 4.1.

As with the single-class setting, our work is centered around the question of whether or not an equilibrium is influential. To define the noninfluential equilibrium, we need the following two definitions: one focuses on the overall purchasing probability and the other one is customer class specific.

Definition 5.1. We say that an equilibrium (ν, y_L, y_H) is aggregate noninfluential (AGNI) if there exists a constant θ

$$\lambda_L y_L(X_{\nu, y_L, y_H}) + \lambda_H y_H(X_{\nu, y_L, y_H}) = \theta \quad \text{a.s.} \qquad (2)$$

on the equilibrium path.

Definition 5.2. We say that an equilibrium (ν, y_L, y_H) is class-wise noninfluential (CWNI) if there exist constants θ_i such

$$\lambda_i y_i(X_{\nu, y_L, y_H}, t) = \theta_i \quad \text{a.s.} \qquad (3)$$

for $i = L, H$ on the equilibrium path.

Of course, based on the above definition, any equilibrium that is CWNI is also AGNI. Also, if an equilibrium is not CWNI, we will refer to it as class-wise influential (CWI).

Note that the above definitions imply that, under an AGNI equilibrium, the probability that a customer attempts to purchase the product in the regular season is independent of the initial inventory level because it is independent of any message the firm provides. Similarly, under CWNI equilibrium, the probability that a customer of class i attempts to purchase the product in the regular season is independent of the inventory for every class i customer where $i \in \{L, H\}$. Note that if an equilibrium is CWNI then it must be AGNI, but the converse is not true.

We will start by showing that any equilibrium must be AGNI, regardless of the customer parameters. (One can show that the result holds even if not restricting attention to symmetric strategies.)

Proposition 5.1. *Every equilibrium for the multiclass cheap-talk game, (ν, y_L, y_H), is AGNI.*

The AGNI property says that the firm cannot impact the aggregate purchasing probability using availability information. Loosely speaking, this property sets a limit on the ability to influence customer behavior. Even if an equilibrium that is not CWNI exists, it is still true that the overall purchasing probability is unaffected by the messages provided by the firm. We thus have the following corollary.

Corollary 5.1. *In the two-class cheap-talk game, for a class-wise influential equilibrium to exist in pure strategies, it must be the case that* $\lambda_L = \lambda_H$.

The above result says that for a CWI to exist in pure strategies, it must be the case that the demand rates of both classes are identical. The result stems immediately from the AGNI results because, for an influential equilibria, it must be the case that there exist two messages such that one class buys during the regular season with one and the other class buys during the regular season with the other. Because the overall buying rate with each message is the same, for an equilibrium to exist it must be the case that both classes have the same demand rate. Otherwise, if restricted to pure strategies, the firm would try to induce only the class with higher demand to buy immediately, which would break the equilibrium.

Proposition 5.1 raises the question, however, whether a firm is capable of influencing customer classes separately, within the limitation of AGNI. We next provide necessary conditions for the existence of a class-wise influential equilibrium.

Proposition 5.2. *In the two-class cheap-talk game, for a class-wise influential equilibrium to exist it must be the case that*

$$c_H - c_L \leq \Delta v_L, \tag{4}$$

where Δ *is a constant that depends only on* λ_L, λ_H *and the distributions of* D_1, D_2, *and* Q.

Before providing an intuition for the condition, we state an immediate corollary for similar conditions when the market becomes large (i.e., the number of potential buyers and items grow).

Corollary 5.2. *For the two-class cheap-talk game with large market,*[9] *for a class-wise influential equilibrium to exist it must be the case that*

$$(c_H - c_L)(v_H - v_L) \leq 0. \tag{5}$$

Thus, the above corollary shows that it is possible for an equilibrium to be CWI while being AGNI when the customers are heterogeneous. Recall first that the effective demand cannot be influenced by the retailer. Thus, the only way the firm can influence each class differently is if the customers have different preferences for time and value and thus may take different actions given the same information. Moreover, we cannot influence customers if $v_H > v_L$ and $c_H > c_L$; i.e., it must be the case that customers with higher willingness to pay are those that are also willing to postpone their purchase. The key factor contributing to the existence of an influential equilibrium is the fact that, in the regular season, the customers need to trade off the availability of the product and the price drop in the sales period. Furthermore, the conditions state that for an influential equilibrium to be possible, the different classes must trade off the benefits and the costs of waiting differently.

One may observe that conditions in Proposition 5.2 are easier to satisfy than those in Corollary 5.2. One of the main difficulties in sustaining an influential equilibrium is the fact that when providing a message on availability, customers of different classes may have the same beliefs regarding the inventory availability. Because the effective demand is unaffected by the messages (due to AGNI), for an influential equilibrium to exist, the same message must elicit different actions from different classes. It turns out that when customers are nonatomistic, they may have different beliefs even if provided with the same message. The key intuition is that the customers in our model are nonatomistic. Thus, the players from one customer class will have a different belief about the availability even when they are given the same message. This discrepancy in beliefs helps sustain an influential (class-wise) equilibrium. However, as the market size grows, their views get closer and coincide. Thus, it is difficult to influence the customers differently. This suggests that it is harder to influence large markets than small. In the larger market, it is necessary to have more markedly different valuations of the product and time in order to influence customer behavior.

To provide a better characterization of influential equilibria, one can rewrite the problem a customer is facing when deciding whether to purchase the product in the regular season or wait. In particular, a customer of class i will purchase the product in the regular season when provided with message m if

$$(A^m_{R,i} - A^m_{S,i})v + c \geq pA^m_{R,i} - sA^m_{S,i}, \tag{6}$$

where A^m_R is the availability during the regular season when the customer receives the message m, and $A^m_{S,i}$ is the availability during the regular season when the customer receives the message m. Note that $A^m_{R,i}$ and $A^m_{S,i}$ depend not only on the customer behavior and the message but also on the customer's own class. We will denote the $\beta^m_i := A^m_{R,i} - A^m_{S,i}$, i.e., the perceived availability drop in equilibrium associated with message m from the perspective of a class i customer. As we described earlier, for an equilibrium to be influential the customer must trade off the value of the product and the cost of waiting. To convert this trade-off into an influential action, the firm must use signals

[9] If the demand and the quantity do not grow proportionally, it is easy to see that the results are quite trivial. For example, assume that $p - s > c$. If Q grows faster than the demand, then everybody waits. If the demand grows faster, everybody purchases immediately. In both cases the messages play no role.

that have certain restrictions on the availability drop. We next provide this characterization of an influential equilibrium.

Theorem 5.1. *For an equilibrium to be influential, there must be two messages (that the firm uses with positive probability) m_1 and m_2 such that for $i = L, H$*

$$(A_{R,i}^{m_1} - A_{S,i}^{m_1}) \le \frac{c_L - c_H}{v_H - v_L} + \frac{\Delta v_H}{v_H - v_L}, \tag{7}$$

$$\frac{c_L - c_H}{v_H - v_L} - \frac{\Delta v_H}{v_H - v_L} \le (A_{R,i}^{m_2} - A_{S,i}^{m_2}). \tag{8}$$

Before we provide an intuition we state the result for large markets.

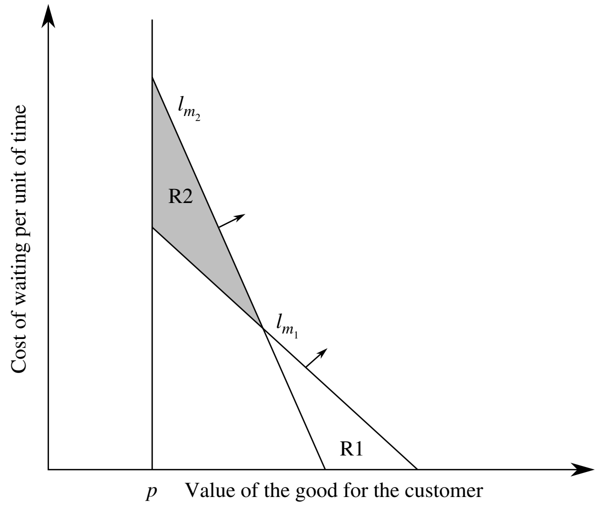
Corollary 5.3. *For the two-class cheap-talk game with a large market, for an equilibrium to be influential, there must be two messages (that the firm uses with positive probability) m_1 and m_2 such that for $i = L, H$*

$$(A_{R,i}^{m_1} - A_{S,i}^{m_1}) \le \frac{|c_L - c_H|}{|v_L - v_H|} \le (A_{R,i}^{m_2} - A_{S,i}^{m_2}). \tag{9}$$

Recall that for an equilibrium to be influential, there must be at least two messages that result in two different actions. In equilibrium, each of these messages will be associated with availability in the regular and the sales season. The above theorem shows that for an equilibrium to be influential, it must be the case that there exists a threshold such that the *availability drop* associated with one message has to be below this threshold and the availability drop of a second message has to be above this threshold. The threshold depends on the value of the product and cost of waiting for the two customer classes.

To better understand the intuition behind the characterization, we next provide a graphical representation of the incentive-compatibility conditions of each class with respect to each message for large markets. We shall represent the customer classes based on their two characteristics: value of the product, v, and the cost of waiting, c. (In Figure 1 the horizontal axis is the value, v, and the vertical axis is the cost of waiting, c.) Note that the IC condition divides the value-cost space into two sets via a line (labeled as l_{m_i} for message m_i) whose slope is the availability drop of the corresponding message. The set "above" the IC condition denotes the parameters for the customer classes that will buy during the regular season when they obtain a message m_1, the other set, i.e., the region below the IC condition denotes the parameters for the customer class that will postpone their purchase to the sales period. Thus, for the CWI equilibrium, it must be the case that the IC constraints for two messages align as shown in the figure. Specifically, we need that the two sets be formed by the intersection of the "buy" from one message and "not buy" from the other message, i.e., regions R1 and R2, are nonempty. Combining this with the fact that the slope of IC constraint is the drop in the availability, we obtain the condition provided in Theorem 5.1. As shown in Corollary 5.2, for an influential equilibrium to exist, the two classes' parameters (v_H, c_H) and (v_L, c_L) cannot be on a line with a positive slope.

Figure 1 **The (v, c) Parameters**



Note. If class 1 parameters lie in region R1 and class 2 parameters lie in region R2, then the firm can sustain a CWI equilibrium.

5.1. Example of Class-Wise Influential Equilibrium

We next provide an example where the customers do react to the announcements made by the firm and hence the equilibrium is class-wise influential. We assume that $Q(0) = 20, 40,$ and 60 with probability 1/3, 1/2, and 1/6, respectively. There are two customer classes, and both have a demand that has Poisson distribution with mean 30. We will next outline the equilibrium and the implied availability functions. When the firm has quantity 20 or 60, it announces m_1, otherwise the firm announces m_2. Based on the message m_i for $i = 1, 2$, class i customers purchase in the regular season whereas the other class' customers wait for the sales period. The price in the regular season is \$10, whereas the sales season price is \$5.

We next characterize the incentive compatibility conditions on the value (v_1, v_2) and the cost of waiting (c_1, c_2). To this end, note that when the message m_1 is provided, the customers' beliefs are that the quantity at the beginning of the regular season is 20 and 60 with probability 2/3 and 1/3, respectively. Thus, following the equilibrium behavior, we have that the availability during the regular season and sales season is $A_R^{m_1} = 0.7912$ and $A_S^{m_1} = 0.3045$, respectively. Similarly, for message m_2 we have $A_R^{m_2} = 0.9979$ and $A_S^{m_2} = 0.3482$. These numbers were computed numerically by approximating the Poisson demand by a truncated Poisson random variable. Thus, we obtain the following four incentive compatibility conditions:

$$v_1(0.4871) + c_1 \geq 6.3942, \quad (10)$$

$$v_2(0.4871) + c_2 \leq 6.3942, \quad (11)$$

$$v_1(0.6497) + c_1 \leq 8.2380, \quad (12)$$

$$v_2(0.6497) + c_2 \geq 8.2380. \quad (13)$$

One can observe that these reduce to the condition of (v_1, c_1) and (v_2, c_2) being on opposite sides of the following two lines:

$$l_{m_1}\colon v(0.4871) + c = 6.3942, \quad (14)$$

$$l_{m_2}\colon v(0.6497) + c = 8.2380. \quad (15)$$

It is easy to see the existence of such (v_1, v_2, c_1, c_2). For instance, $v_1 = 11$, $v_2 = 13$, $c_1 = 1.05$, and $c_2 = 0$ is one such instance. This is in line with Corollary 5.2, where $v_2 > v_1$ and $c_1 < c_2$.

It is important to note that the paper provides necessary conditions for the existence of CWI equilibrium. When these conditions are violated, any arising equilibrium is CWNI. However, even if these conditions are satisfied, it is not guaranteed that a CWI equilibrium exists. Note that for an equilibrium with CWI cheap talk to exist in pure strategies, it must be the case that there are two messages corresponding to the two classes such that, when the firm announces these message, only the customers of the corresponding class attempt to buy during the regular season. Thus, one can compute the availability drops that correspond to the different realizations of initial quantity on hand. Using these, one can construct a linear program; the solution of which provides the signaling rule that the firm can use to influence customers. Further, if this set of linear equations do not have a solution then there is no CWI.

6. Concluding Remarks and Future Directions

In this paper, we study a retail operations model where customers are strategic not only in their actions but also in the way they interpret information, and the retailer is strategic in the way it provides information. This paper focuses on the ability to influence consumer behavior when providing unverifiable information on inventory availability. We develop a game-theoretic framework to study this class of communication and discuss the equilibrium language emerging between the retailer and its customers. In this setting, we observe that the firm is limited in its capacity to influence customers. In fact, in most circumstances, the firm cannot influence the purchasing behavior using messages that signal the quantity on hand. This is consistent with a recent *New York Times* article (Rosenbloom 2009), which quotes Mr. Tansky, president and chief executive of Neiman Marcus, saying, "We've told our customers that the availability is less than they're used to seeing in the stores. We've suggested that it would be prudent to shop early." The article then adds that "Some surveys have found that, so far, the prospect of lean inventories is not prompting consumers to hasten their holiday shopping," demonstrating that the proclamation that the firm is using lean inventories is noninfluential.

We do show, however, that the firm is capable of influencing customer behavior if it targets customer populations with diverse preferences (that is, their waiting-time cost and product valuations do not "line up"). We provide necessary conditions for an equilibrium with influential cheap talk to exist, and demonstrate how to construct one using a simple example. One may take one of two views of this ability to influence customer behavior—seeing the glass as half full or empty. The firm may indeed be able, under certain conditions, to influence customer behavior using availability information. Yet, a careful look at the conditions on the emerging equilibrium and the customer attributes shows that these require the firms' signaling rule to be very sophisticated, and the customer attributes to be ordered in a specific manner; moreover, the effective demand cannot be impacted using these messages. It is important to note that in the cases in which the firm is unable to influence customer behavior, no matter what information the firm provides, or what technique it employs to lure customers to buy the product, a rational customer will ignore it. In these settings, if a customer reacts in any way to the availability information provided or to the actions taken by the firm (such as displaying the item in a specific format), it will continuously use this method to maximize the number of customers buying in the regular season and, in turn, maximize its profits. Thus, the only equilibrium language is one where customers disregard any information provided by the firm.

6.1. Endogenous Pricing

Our paper focuses on the ability to influence customer behavior using *costless* unverifiable information provided to the customers. Customers may also be impacted by the firm's pricing decisions. In our model we can show that even when customers react to price changes, the firm cannot improve the credibility of the costless signal in aggregate. That is, the cheap talk remains noninfluential even when complemented with pricing decisions. In that sense, our results that the firm cannot influence the effective demand using messages about inventory availability continue to hold even if the pricing is endogenized.

This is in line with a recent paper, Kartik (2007), that proves that cheap talk can be influential with money burning if and only if it can be influential without money burning. The optimal pricing problem itself is very complex and is thus outside the scope of the current paper.

6.2. Multiple Channels for Information Provision

Given that we observe that the firm cannot influence the customers in aggregate and in some cases even class-wise, we next summarize a setting in which this inability can be alleviated. There are numerous cases in practice where multiple channels sell items from the same pool of inventory and independently provide availability information (this inventory may either be physically colocated or virtually pooled). For example, the Web retailers Dick's Sporting Goods (http://www.dickssportinggoods.com) and Modell's Sporting Goods (http://www.modells.com)—whose operations are both run by GSI commerce—compete over the same potential customers yet provide information on the same pool of inventory for the same items. Also, many businesses have multiple retail channels catering to the same pool of customers and sharing the same pool of inventory. In these settings, one can observe that as the customers can obtain information from two sources about the same inventory, richer (and specifically more influential) language is possible in equilibrium. Allon and Bassamboo (2009) study such a model and show that in such scenarios, the firm can sustain a fully revealing equilibrium. This result stems from the fact that the customer can compare the two messages from the different channels and possibly "punish" the one channel that he believes tries to induce him to purchase early.

6.3. Comparison with Allon et al. (2011)

The above results contrast with those on information sharing in services, explored in Allon et al. (2011).[10] There it was shown that a single service provider can "create" some credibility with respect to sharing real-time system information. The difference in the nature of equilibrium emerges due to the following distinguishing features of the service and retail operations: In retail operations, the incentives of the customers and the firm are aligned for low inventory levels, i.e., both "agree" that the customer should purchase in these states, and misaligned for high inventory levels, i.e., the firm would like the customers to purchase; however, given that the inventory level is high the customers can improve their utility by postponing the purchase to the sales season. In service operations, the service provider's and its customers' incentives are aligned when the number of customers waiting in the system is either "high" or "low." The only misalignment is when the number of customers is moderate. Because misalignment is limited in services, it helps the provider create some credibility. Thus, the one-sided-only agreement in retail operations games prevents the firm from creating any credibility when it is providing the information on its own. This logic does not fully extend to the multiclass setting, allowing the firm to partially influence the customers. Another difference between the results of the two models pertains to the ability of the firm to use the announcements to extract information from customers regarding their class identity: In both the retail and the queueing games, the firm may be able to extract information from customers regarding their class identity. In the queueing game, the firm may use it to achieve its first best profit. In the retail game, due to AGNI property, this information does not help the firm improve its profit. Furthermore, in the queueing game, if customers have private information regarding their class, the ability of the firm to influence its customers is (weakly) diminished compared to the case in which the firm has perfect information. This result is reversed in the retail game, where, as shown above, the firm can do weakly better when its customers have private information.

[10] Although the two systems are very different from an operational point of view, in both cases the game played between the firm and a single customer is best described as a single-stage game in which the customer has expectations regarding the evolution of the system and needs to make a decision: in the service setting, whether to join or balk, and in the retail setting, whether to buy immediately or wait for the sales period. Allon et al. (2011) study a system in steady state and the current paper studies a transient system; however, in both models customers make decisions upon their arrivals, based on the belief about the state of the system, which may be based on messages provided by the firm.

Appendix. Proofs

Proof of Proposition 5.1. The proof follows along the same line as the proof of Proposition 4.1. As before the firm's incentive compatibility condition requires that the firm signals from the set of messages that maximizes

$$\lambda_L y_L(m) + \lambda_H y_H(m).$$

Thus, given the equilibrium (ν, y_L, y_H), we have that there exists a constant θ for all realization of quantity on hand that satisfies the definition of AGNI. This completes the proof. □

Proof of Proposition 5.2. Let $A_{R,i}^{m_1}$ be the implied availability in equilibrium for class i customer during the regular season when the firm signals m_1. Let $A_{S,i}^{m_1}$ be the implied availability in equilibrium for class i customer during the sales season when the firm signals m_1. We know from Lemma A.1 that

$$|A_{R,1}^{m_1} - A_{R,2}^{m_1}| \le \Delta^R, \quad |A_{S,1}^{m_1} - A_{S,2}^{m_1}| \le \Delta^S. \tag{16}$$

For the equilibrium to be CWI, there must be a message such that

$$\begin{aligned}(A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L+c_L &\geq pA^{m_1}_{R,L}-sA^{m_1}_{S,L},\\ (A^{m_1}_{R,H}-A^{m_1}_{S,H})v_H+c_H &\leq pA^{m_1}_{R,H}-sA^{m_1}_{S,H}.\end{aligned}\tag{17}$$

Using (16), we have that

$$\begin{aligned}&|pA^{m_1}_{R,L}-sA^{m_1}_{S,L}-(pA^{m_1}_{R,H}-sA^{m_1}_{S,H})|\\ &\quad\leq p\Delta^R+s\Delta^S\leq v_L(\Delta^R+\Delta^S).\end{aligned}\tag{18}$$

Thus, (17) implies that

$$\begin{aligned}&(A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L+c_L\\ &\quad\geq (A^{m_1}_{R,H}-A^{m_1}_{S,H})v_H+c_H-v_L(\Delta^R+\Delta^S).\end{aligned}\tag{19}$$

Rearranging, we obtain

$$\begin{aligned}c_H-c_L &\leq (A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L-(A^{m_1}_{R,H}-A^{m_1}_{S,H})v_H+v_L(\Delta^R+\Delta^S)\\ &\leq (A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L-(A^{m_1}_{R,H}-A^{m_1}_{S,H})v_L\\ &\quad+(A^{m_1}_{R,H}-A^{m_1}_{S,H})(v_L-v_H)+v_L(\Delta^R+\Delta^S)\\ &\overset{(a)}{\leq} (A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L-(A^{m_1}_{R,H}-A^{m_1}_{S,H})v_L+v_L(\Delta^R+\Delta^S)\\ &\overset{(b)}{\leq} 2v_L(\Delta^R+\Delta^S),\end{aligned}$$

where (a) follows by noting that $v_H>v_L$ and $A^{m_1}_{R,H}\geq A^{m_1}_{S,H}$, and (b) follows from (16). This completes the proof. □

Lemma A.1. *For any equilibrium, we have that for any message m*

$$|A^{m_1}_{R,L}-A^{m_1}_{R,H}|\leq\Delta^R:=2e^{-\lambda_L}+2e^{-\lambda_H}+\mathbb{E}\left[\frac{1}{Q_0}\right],\tag{20}$$

$$|A^{m_1}_{S,L}-A^{m_1}_{S,H}|\leq\Delta^s:=2e^{-\lambda_L}+2e^{-\lambda_H}+\mathbb{E}\left[\frac{1}{D_2}\right].\tag{21}$$

Proof of Lemma A.1. Let y be the equilibrium behavior for the customers. Thus, for the message m_1 customer of class L and class H purchase during the regular season with probability $y_L(m_1)$ and $y_H(m_1)$. Thus, we can represent the

$$A^{m_1}_{R,L}=\mathbb{E}\left[\frac{Q_0}{\sum_{i=2}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}+1+\sum_{j=1}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}}\wedge 1\;\Big|\;N_L\geq 1\right],\tag{22}$$

$$A^{m_1}_{R,H}=\mathbb{E}\left[\frac{Q_0}{\sum_{i=1}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}+1+\sum_{j=2}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}}\wedge 1\;\Big|\;N_H\geq 1\right],\tag{23}$$

where ω_j^H ω_j^L are an i.i.d. sequence of uniform random variables on $[0,1]$. Let us define

$$X_L=\left[\frac{Q_0}{\sum_{i=2}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}+1+\sum_{j=1}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}}\wedge 1\right],$$

$$X_H=\left[\frac{Q_0}{\sum_{i=1}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}+1+\sum_{j=2}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}}\wedge 1\right].$$

Because

$$\begin{aligned}&\left|\left[\sum_{i=2}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}+1+\sum_{j=1}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}\right]-\left[\sum_{i=1}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}+1+\sum_{j=2}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}\right]\right|\\ &\quad=\left|\mathbb{1}_{\{\omega_1^H\leq y_H\}}-\mathbb{1}_{\{\omega_1^L\leq y_L\}}\right|\leq 1,\end{aligned}$$

we obtain that

$$|X_L-X_H|\leq\frac{1}{Q_0}.\tag{24}$$

Using the definitions of X_L and X_H, we have

$$\begin{aligned}&|A^{m_1}_{R,L}-A^{m_1}_{R,H}|\\ &=|\mathbb{E}[X_L\mid N_L\geq 1]-\mathbb{E}[X_H|N_H\geq 1]|\\ &=\left|\frac{\mathbb{E}[X_L\mathbb{1}_{\{N_L\geq 1\}}]}{\mathbb{P}(N_L\geq 1)}-\frac{\mathbb{E}[X_H\mathbb{1}_{\{N_H\geq 1\}}]}{\mathbb{P}(N_H\geq 1)}\right|\\ &\leq\left|\frac{\mathbb{E}[X_L\mathbb{1}_{\{N_L\geq 1,\,N_H\geq 1\}}]}{\mathbb{P}(N_L\geq 1)}-\frac{\mathbb{E}[X_H\mathbb{1}_{\{N_H\geq 1,\,N_L\geq 1\}}]}{\mathbb{P}(N_H\geq 1)}\right|\\ &\quad+\left|\frac{\mathbb{E}[X_L\mathbb{1}_{\{N_L\geq 1,\,N_H=0\}}]}{\mathbb{P}(N_L\geq 1)}\right|+\left|\frac{\mathbb{E}[X_H\mathbb{1}_{\{N_H\geq 1,\,N_L=0\}}]}{\mathbb{P}(N_H\geq 1)}\right|\\ &\overset{(a)}{\leq}\left|\frac{\mathbb{E}[X_L\mathbb{1}_{\{N_L\geq 1,\,N_H\geq 1\}}]}{\mathbb{P}(N_L\geq 1)}-\frac{\mathbb{E}[X_H\mathbb{1}_{\{N_H\geq 1,\,N_L\geq 1\}}]}{\mathbb{P}(N_H\geq 1)}\right|+e^{-\lambda_L}+e^{-\lambda_H}\\ &=\left|\frac{\mathbb{E}[X_L\mathbb{P}(N_H\geq 1)\mathbb{1}_{\{N_L\geq 1,\,N_H\geq 1\}}]-\mathbb{E}[X_H\mathbb{P}(N_L\geq 1)\mathbb{1}_{\{N_H\geq 1,\,N_L\geq 1\}}]}{\mathbb{P}(N_L\geq 1)\mathbb{P}(N_H\geq 1)}\right|\\ &\quad+e^{-\lambda_L}+e^{-\lambda_H}\\ &=\left|\frac{\mathbb{E}[(X_L\mathbb{P}(N_H\geq 1)-X_H\mathbb{P}(N_L\geq 1))\mathbb{1}_{\{N_L\geq 1,\,N_H\geq 1\}}]}{\mathbb{P}(N_L\geq 1)\mathbb{P}(N_H\geq 1)}\right|+e^{-\lambda_L}+e^{-\lambda_H}\\ &=|\mathbb{E}[(X_L\mathbb{P}(N_H\geq 1)-X_H\mathbb{P}(N_L\geq 1))\mid N_L\geq 1,N_H\geq 1]|\\ &\quad+e^{-\lambda_L}+e^{-\lambda_H}\\ &\overset{(b)}{\leq}|\mathbb{E}[(X_L-X_H)\mid N_L\geq 1,N_H\geq 1]|+2(e^{-\lambda_L}+e^{-\lambda_H})\\ &\overset{(c)}{\leq}\mathbb{E}\left[\frac{1}{Q_0}\right]+2(e^{-\lambda_L}+e^{-\lambda_H}).\end{aligned}$$

Here, (a) and (b) follows by noting that X_L and X_H are bounded by 1, $\mathbb{P}(N_L=0)=e^{-\lambda_L}$ and $\mathbb{P}(N_H=0)=e^{-\lambda_H}$. The inequality in (c) follows by (24) and mutual independence of random variables Q_0, N_L, and N_H. This completes the proof for (20). The proof of (21) follows along the same line and using the definitions

$$A^{m_1}_{S,L}=\mathbb{E}\left[\frac{Q_0-\sum_{i=2}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}-\sum_{j=1}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}}{N_B+1+\sum_{i=2}^{N_L}\mathbb{1}_{\{\omega_i^L>y_L\}}+\sum_{j=1}^{N_H}\mathbb{1}_{\{\omega_j^H>y_H\}}}\wedge 1\,|\,N_L\geq 1\right],\tag{25}$$

$$A^{m_1}_{S,H}=\mathbb{E}\left[\frac{Q_0-\sum_{i=1}^{N_L}\mathbb{1}_{\{\omega_i^L\leq y_L\}}-\sum_{j=2}^{N_H}\mathbb{1}_{\{\omega_j^H\leq y_H\}}}{N_B+1+\sum_{i=1}^{N_L}\mathbb{1}_{\{\omega_i^L>y_L\}}+\sum_{j=2}^{N_H}\mathbb{1}_{\{\omega_j^H>y_H\}}}\wedge 1\,|\,N_H\geq 1\right].\quad\square\tag{26}$$

Proof of Theorem 5.1. As in the proof of Proposition 5.2, we let $A^{m_1}_{R,i}$ be the implied availability in equilibrium for class i customer during the regular season when the firm signals m_1, and $A^{m_1}_{S,i}$ be the implied availability in equilibrium for class i customer during the regular season when the firm signals m_1. We know from Lemma A.1 that

$$|A^{m_1}_{R,1}-A^{m_1}_{R,2}|\leq\Delta^R,\quad |A^{m_1}_{S,1}-A^{m_1}_{S,2}|\leq\Delta^S.\tag{27}$$

For the equilibrium to be CWI, there must be a message m_1 and m_2 such that

$$\begin{aligned}(A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L+c_L &\ge pA^{m_1}_{R,L}-sA^{m_1}_{S,L},\\ (A^{m_1}_{R,H}-A^{m_1}_{S,H})v_H+c_H &\le pA^{m_1}_{R,H}-sA^{m_1}_{S,H};\\ (A^{m_2}_{R,L}-A^{m_2}_{S,L})v_L+c_L &\le pA^{m_2}_{R,L}-sA^{m_2}_{S,L},\\ (A^{m_2}_{R,H}-A^{m_2}_{S,H})v_H+c_H &\ge pA^{m_2}_{R,H}-sA^{m_2}_{S,H}.\end{aligned}\tag{28}$$

Further, using (18), we have

$$\begin{aligned}c_H-c_L &\le (A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L-(A^{m_1}_{R,H}-A^{m_1}_{S,H})v_H+v_L(\Delta^R+\Delta^S)\\ &\le (A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L-(A^{m_1}_{R,H}-A^{m_1}_{S,H})v_L\\ &\quad+(A^{m_1}_{R,H}-A^{m_1}_{S,H})(v_L-v_H)+v_L(\Delta^R+\Delta^S)\\ &\le (A^{m_1}_{R,L}-A^{m_1}_{S,L})v_L-(A^{m_1}_{R,H}-A^{m_1}_{S,H})v_L\\ &\quad+v_L(\Delta^R+\Delta^S)+(A^{m_1}_{R,H}-A^{m_1}_{S,H})(v_L-v_H)\\ &\le 2v_L(\Delta^R+\Delta^S)+(A^{m_1}_{R,H}-A^{m_1}_{S,H})(v_L-v_H).\end{aligned}$$

We thus obtain

$$\frac{c_L-c_H}{v_H-v_L}\ge -2(\Delta^R+\Delta^S)\frac{v_L}{v_H-v_L}+(A^{m_1}_{R,H}-A^{m_1}_{S,H}),$$

$$\frac{c_L-c_H}{v_H-v_L}+2(\Delta^R+\Delta^S)\frac{v_L}{v_H-v_L}\ge (A^{m_1}_{R,H}-A^{m_1}_{S,H}).$$

The other inequalities follows in the same manner. This completes the proof. □

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