

The Strategy of Manipulating Conflict

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Two players choose hawkish or dovish actions in a conflict game with incomplete information. An “extremist”, who can either be a hawk or a dove, attempts to manipulate decision making. If actions are strategic complements, a hawkish extremist increases the likelihood of conflict, and reduces welfare, by sending a public message which triggers hawkish behavior from both players. If actions are strategic substitutes, a dovish extremist instead sends a public message which causes one player to become more dovish and the other more hawkish. A hawkish (dovish) extremist is unable to manipulate decision making if actions are strategic substitutes (complements).

Agents with extreme agendas sometimes take provocative actions that inflame conflicts. For example, Ariel Sharon’s symbolic visit to the Temple Mount in September 2000 helped spark the Second Intifada and derailed the Israeli-Palestinian peace process (Hefetz and Bloom (2006)). How can extremists manipulate conflicts and when is it rational to respond aggressively to provocations?

Provocations play a key role in the conflict between the two nuclear powers India and Pakistan.¹ After 9/11 2001, Pakistani President Musharraf sent troops to the Afghanistan border, and tried to suppress militant groups within Pakistan. In December 2001, militants sponsored by the Pakistani intelligence agency ISI attacked the Indian Parliament. India mobilized for war, and Musharraf shifted his troops from the Afghanistan border to the Indian border. Similarly, in November 2008 a terrorist attack in Mumbai raised tensions at a time when Pakistani President Zardari wanted improved relations with India. ISI-sponsored militants seem to deliberately inflame the conflict between Pakistan and India, partly because India is seen as an implacable foe, but also because the conflict relieves the pressure on extremists supported by the ISI. For Pakistani and Indian leaders, a hawkish stance may be the best response, given the (correct) belief that their opponent will become more aggressive.²

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¹For details on this conflict, see Aneja (2008), Coll (2006), Fair (2010), Rabasa et al. (2009), New York Times (2008), Riedel (2008), Haqqani (2005).

²Of course, provocations are a well-known phenomenon, not just in interstate conflicts. In the early part of the 20th century, African-Americans and Irish-Americans in Chicago viewed each other with suspicion. The former believed “white men have great boxes of guns and ammunition in the cellars of their homes and that white men are forming shooting clubs for the purpose of shooting Negroes in the event of another riot” (Chicago Commission on Race Relations (1919), p. 21-22). The African-Americans newspaper *The Whip* warned: “We are not pacifists, therefore we believe in war, but only when all orderly civil procedure has been exhausted and the points in question are justifiable” (Tuttle (1970), p. 282). In 1919, the tinderbox was deliberately ignited by extremist Irish-American “athletic clubs”, whose provocations

Our model is based on the conflict game of Baliga and Sjöström (2004). There are two countries, A and B . In country $i \in \{A, B\}$, a decision maker, player i , chooses a dovish action D or a hawkish action H . Player i may be interpreted as the median voter, a political leader, or some other pivotal decision maker in country i . The hawkish action might represent accumulation of weapons, sending soldiers to a contested territory, or an act of war. Alternatively, it could represent aggressive bargaining tactics. (For example, in 2000, Ehud Barak and Yasser Arafat had to decide whether to adopt a tough stance H or a conciliatory stance D in peace negotiations.) Finally, H might represent choosing a hawkish agent who will take aggressive actions on the decision maker's behalf. (For example, the median voters in Israel and Palestine decide whether to support Likud or Kadima, or Hamas or Fatah, respectively.)

Each decision maker can be a dominant strategy dove, a dominant strategy hawk, or a “moderate” whose best response depends on his beliefs about the opponent's action. Neither decision maker knows the other's true type. In Baliga and Sjöström (2004), we studied how fear of dominant strategy hawks makes moderates choose H when actions are strategic complements. Now our main purpose is to understand how a third party can manipulate the conflict. In addition, we generalize the conflict game by allowing actions to be strategic substitutes as well as complements.³ Whether actions are strategic complements or substitutes, under fairly mild assumptions on the distribution of types, the conflict game without cheap-talk has a unique communication-free equilibrium.

To study how decision makers can be manipulated by third parties, such as Sharon or the ISI, we add a third player called “the extremist” (player E). The extremist may be at the center of politics in country A , or the leader of an extremist movement located in, or with influence in, country A . We assume his true preferences are commonly known. We consider two cases: a hawkish extremist (“provocateur”) who wants player A to choose H , and a dovish extremist (“pacifist”) who wants player A to choose D . Both kinds of extremists prefer that the opposing player B chooses D . Political insiders, like Ariel Sharon or the ISI, have privileged information about pivotal decision makers in their home countries. But even extremists who are outsiders, moving about the population, may discover the preferences of the country's pivotal decision maker, e.g., the degree of religious fervor of the average citizen. We simplify by assuming the extremist has *perfect* information about the true preferences of the pivotal decision maker in country A .

To isolate the pure logic of manipulation of conflict, we assume the extremist can do nothing except communicate. Before players A and B make their decisions, player E sends a publicly observed cheap-talk message. A visit to the Temple Mount might be a real-world example.⁴ Our main interest is in *communication equilibria*, defined

caused wide-spread rioting (Chicago Commission on Race Relations (1919), p. 11-17, Tuttle (1970)).

³Baliga and Sjöström (2011) show how actions can be either strategic complements or substitutes in a bargaining game with limited commitment to costly conflict. Several empirical articles have tried to establish whether actions are strategic complements or substitutes in the Israel-Palestine conflict. Berrebi and Klor ((2006), (2008)) find that terrorism increases support for Israel's right-wing Likud party, and that there is more terrorism when the left-wing Labor party is in power. Jaeger and Paserman ((2008), (2009)) find that Palestinian violence leads to increased Israeli violence, but Israeli violence either has no effect or possibly a deterrent effect.

⁴In some situations, only *costly* messages (e.g., acts of violence) might be noticed above the background noise and daily concerns of media and politicians. We will show that our results are robust to messages being costly to send and

as equilibria where the extremist's cheap-talk influences the decisions of players A and B . It may be surprising that such equilibria exist. Models of signaling and cheap-talk usually assume the sender's preferences depend directly on his private information. In contrast, we assume it is commonly known exactly what player E wants players A and B to do. Player A knows what player E knows, but player A will pay attention to player E 's message if he thinks it might influence player B , as it will in equilibrium. We show that a communication equilibrium always exists, and find assumptions under which it is unique. Importantly, even if multiple communication equilibria exist, they always have the same structure and the same welfare implications.

In communication equilibrium, some message m_1 will make player B more likely to choose H . A provocateur is willing to send m_1 only if player A also becomes more likely to choose H . Such co-varying actions must be strategic complements. On the other hand, a pacifist is willing to send m_1 only if player A becomes more likely to choose D . Such negative correlation occurs when actions are strategic substitutes. This argument implies that if the underlying game has strategic complements, then only a provocateur can communicate effectively. By sending m_1 , the provocateur triggers an unwanted (by players A and B) cascade of fear and hostility, making both players A and B more likely to choose H . Conversely, if the underlying game has strategic substitutes, then only a pacifist can communicate effectively. By sending m_1 , the pacifist causes player A to back down and choose D .

With strategic complements, message m_1 can be interpreted as a provocation which increases the tension between players A and B . In equilibrium, the provocateur sends m_1 only when player A is a "weak moderate", i.e. a type who would have chosen D in the communication-free equilibrium, but who will choose H out of fear if a provocation makes it more likely that player B chooses H . In response to m_1 , player B indeed chooses H with a very high probability. The *absence* of a provocation reveals that player A is *not* a weak moderate. Eliminating these types makes player B more inclined to choose H than in the communication-free equilibrium, which makes player A more inclined to choose H as well. Thus, with strategic complements, communication increases the probability players A and B choose H , *whether or not a provocation actually occurs*. Because each decision maker always wants the other to choose D , eliminating the provocateur would make all types of players A and B strictly better off. This includes player A 's most hawkish types - even though their preferences are aligned with the provocateur. In view of this, one may ask why players A and B do not jointly agree to ignore the provocation and behave more peacefully.⁵ One answer may be that they do not trust each

receive.

⁵Fromkin (1975) and others have made similar arguments about terrorism:

"Terrorism wins only if you respond to it in the way that the terrorists want you to; which means that its fate is in your hands and not in theirs. If you choose not to respond at all, or else to respond in a way different from that which they desire, they will fail to achieve their objectives. The important point is that the choice is yours. That is the ultimate weakness of terrorism as a strategy. It means that, though terrorism cannot always be prevented, it can always be defeated. You can always refuse to do what they want you to do" (Fromkin (1975), p. 697).

In our model, a *unilateral* deviation along the lines suggested by Fromkin can never be profitable (by definition of equilibrium), but *renegotiating* an equilibrium at some point of the game tree might make both decision makers better off.

other to follow through. In Section II.B, we offer another answer: a player might appear weak if he does not react aggressively to a provocation, and appearing weak is costly.

With strategic substitutes, message m_1 can be interpreted as a “peace rally” in country A , organized by a pacifist who wants his key audience to renounce violence. For example, the Campaign for Nuclear Disarmament, formed by Bertrand Russell during the Cold War, proposed unilateral disarmament even at the cost of giving in to communism.⁶ In our model, a peace rally occurs only when player A is a “tough moderate” who would have chosen H in the communication-free equilibrium, but who is deterred from doing so if he fears a hawkish opponent. Following a peace rally, player B indeed becomes more hawkish, and the tough moderate type of player A backs down and chooses D . Since peace protests in country A make player B more hawkish, player A would like to ban them if he could. On the other hand, because player A becomes more dovish, the peace rally makes player B better off.

We consider several extensions of the basic model. The structure of the communication equilibrium carries over to the case of provocateurs in both countries, although, surprisingly, the probability of peace may increase when the second provocateur is added. The basic results also go through with a small amount of uncertainty about whether actions are strategic substitutes or complements (in which case provocation can result in a player backing off, and a peace rally can result in mutual de-escalation), and when player E may not know player A 's true type (in which case provocation can backfire: player A might stick to D while player B switches to H).

In related work, Levy and Razin (2004) also consider cheap-talk with multiple audiences: a democratic leader sends a message to his own citizens and to another country. The citizens have the same state-contingent preferences as their leader, and the leader would prefer to send them a private message but this is assumed to be impossible. In our model, the preferences of the sender (the extremist) differ from both receivers (the decision makers), and private messages would not be useful, because the extremist seeks to *indirectly* influence player A by *publicly* provoking player B .

In Baliga and Sjöström (2004), we show that communication between players A and B can be good for peace when actions are strategic complements. Although neither player wants to provoke the other to choose H , some types are more conflict-averse than others. This allows the construction of a “peaceful” cheap-talk equilibrium where moderate types who exchange “peaceful messages” coordinate on D . This construction relies on the fact that both players send messages and their preferences depend directly on their privately known types. In our current model, the provocateur's preferences are commonly known, and his messages are bad for peace. The logic behind his manipulation of the conflict is quite different from the role played by communication in our earlier work.

Jung (2007) shows how communication by a hawkish Ministry of Propaganda can refine the set of equilibria in a version of the Baliga and Sjöström (2004) model. For this purpose it is crucial that messages are not cheap-talk. In contrast, we study cheap-talk

⁶“If no alternative remains except Communist domination or the extinction of the human race, the former alternative is the lesser of two evils” (Russell, quoted by Rees (2002)).

equilibria which do not replicate the outcome of any communication-free equilibrium. Edmond (2008) considers a global game where citizens can overthrow a dictator by coordinating on a revolution, but the dictator increases his chances of survival by jamming the citizens' signals about how likely it is that a revolution will succeed. Bueno de Mesquita (2010) studies a related model where the level of violence inflicted by uninformed extremists generates information for the population. In contrast to the global games literature, we do not assume highly correlated types (in fact types are uncorrelated).

I. The Model

A. The Conflict Game without Cheap-Talk

The conflict game without cheap-talk is similar to the game studied in Baliga and Sjöström (2004). Two decision makers, players A and B , simultaneously choose either a hawkish (aggressive) action H or a dovish (peaceful) action D . The payoff for player $i \in \{A, B\}$ is given by the following payoff matrix, where the row represents his own choice, and the column represents the choice of player $j \neq i$.

$$(1) \quad \begin{array}{cc} & \begin{array}{cc} H & D \end{array} \\ \begin{array}{c} H \\ D \end{array} & \begin{array}{cc} -c_i & \mu - c_i \\ -d & 0 \end{array} \end{array}$$

We assume $d > 0$ and $\mu > 0$, so player j 's aggression reduces player i 's payoff. Notice that d captures the cost of being caught out when the opponent is aggressive, while μ represents a benefit from being more aggressive than the opponent. If $d > \mu$, player i 's incentive to choose H over D increases with the probability that player j chooses H , so the game has *strategic complements*. If $d < \mu$, player i 's incentive to choose H decreases with the probability player j chooses H and the game has *strategic substitutes*.

Player i has a privately known cost c_i of taking the hawkish action, referred to as his "type". Types are independently drawn from the same distribution. Let F denote the continuous cumulative distribution function, with support $[\underline{c}, \bar{c}]$, and where $F'(c) > 0$ for all $c \in (\underline{c}, \bar{c})$. When taking an action, player i knows c_i but not c_j , $j \neq i$.

Player i is a *dominant strategy hawk* if H is a dominant strategy ($\mu \geq c_i$ and $d \geq c_i$ with at least one strict inequality). Player i is a *dominant strategy dove* if D is a dominant strategy ($\mu \leq c_i$ and $d \leq c_i$ with at least one strict inequality). Player i is a *coordination type* if H is a best response to H and D a best response to D ($\mu \leq c_i \leq d$). Player i is an *opportunistic type* if D is a best response to H and H a best response to D ($d \leq c_i \leq \mu$). Coordination types exist only in games with strategic complements, and opportunistic types exist only in games with strategic substitutes. Assumption 1 states that the support of F is big enough to include dominant strategy types of both kinds.

Assumption 1 If the game has strategic complements then $\underline{c} < \mu < d < \bar{c}$. If the game has strategic substitutes then $\underline{c} < d < \mu < \bar{c}$.

Suppose player j chooses H with probability p_j . Player i 's expected payoff from playing H is $-c_i + \mu(1 - p_j)$, while his expected payoff from D is $-p_j d$. Thus, if player i chooses H instead of D , his *net* gain is

$$(2) \quad \mu - c_i + (d - \mu)p_j.$$

A *strategy* for player i is a function $\sigma_i : [\underline{c}, \bar{c}] \rightarrow \{H, D\}$ which specifies an action $\sigma_i(c_i) \in \{H, D\}$ for each cost type $c_i \in [\underline{c}, \bar{c}]$. In Bayesian Nash equilibrium (BNE), all types maximize their expected payoff. Therefore, $\sigma_i(c_i) = H$ if the expression in (2) is positive, and $\sigma_i(c_i) = D$ if it is negative. If expression (2) is zero then type c_i is indifferent, but for convenience we will assume he chooses H in this case.

Player i uses a *cutoff strategy* if there is a *cutoff point* $x \in [\underline{c}, \bar{c}]$ such that $\sigma_i(c_i) = H$ if and only if $c_i \leq x$. Because (2) is monotone in c_i , all BNE must be in cutoff strategies. Any such strategy can be identified with its cutoff point $x \in [\underline{c}, \bar{c}]$. By Assumption 1, dominant strategy doves and hawks have positive probability, so all BNE must be interior: each player chooses H with probability strictly between 0 and 1.

If player j uses cutoff point x_j , the probability he plays H is $p_j = F(x_j)$. Therefore, using (2), player i 's best response to player j 's cutoff x_j is the cutoff $x_i = \Gamma(x_j)$, where

$$(3) \quad \Gamma(x) \equiv \mu + (d - \mu)F(x).$$

The function Γ is the best response function for cutoff strategies. Notice that $\Gamma'(x) = (d - \mu)F'(x)$, so the best response function is upward (downward) sloping if actions are strategic complements (substitutes). Moreover, $\Gamma(\underline{c}) = \mu > \underline{c}$ and $\Gamma(\bar{c}) = d < \bar{c}$. Since Γ is continuous, a fixed-point $\hat{x} \in (\underline{c}, \bar{c})$ exists. Thus, a BNE exists (where by the symmetry of the game each player uses cutoff \hat{x}).

Assumption 2 states that the density of F is not too large anywhere, i.e., that there is significant uncertainty about types.⁷

Assumption 2 $F'(c) < |\frac{1}{d-\mu}|$ for all $c \in (\underline{c}, \bar{c})$.

Assumption 2 implies that $0 < \Gamma'(x) < 1$ if $d > \mu$ and $-1 < \Gamma'(x) < 0$ if $d < \mu$. Hence, in both cases a well-known sufficient condition for uniqueness is satisfied: the best response functions have slope strictly less than one in absolute value (see Vives (2001)). Thus, we have:

THEOREM 1: *The conflict game without cheap-talk has a unique Bayesian Nash equilibrium.*

Theorem 1 says that without cheap-talk there is a unique BNE, which we refer to as the *communication-free equilibrium*, whether actions are strategic substitutes or complements. In equilibrium, player i chooses H if $c_i \leq \hat{x}$, where \hat{x} is the *unique* fixed point of

⁷As long as Assumption 1 is satisfied, the uniform distribution on $[\underline{c}, \bar{c}]$ satisfies Assumption 2. But Assumption 2 is much weaker than uniformity. What it rules out is having probability mass highly concentrated around one particular type. This guarantees that the BNE is unique. See Morris and Shin (2005) for a detailed discussion of uniqueness in this type of game.

$\Gamma(x)$ in $[\underline{c}, \bar{c}]$. See Figure 1 for the case of strategic complements (the equilibrium is the intersection of the best response curves $x_B = \Gamma(x_A)$ and $x_A = \Gamma(x_B)$).⁸

The unique communication-free equilibrium can be reached via iterated deletion of dominated strategies. With strategic complements, the fear of dominant strategy hawks causes coordination types who are “almost dominant strategy hawks” (i.e., types slightly above μ) to play H , which in turn causes “almost-almost dominant strategy hawks” to play H , etc. The “hawkish cascade” causes higher and higher types to choose H . Meanwhile, since dominant strategy doves play D , “almost dominant strategy doves” (i.e., types slightly below d) also play D , knowing that the opponent may be a dominant strategy dove. The “dovish cascade” causes lower and lower types to choose D . With sufficient uncertainty about types, these two cascades completely resolve the ambiguity about what coordination types will do.⁹

B. Cheap-Talk

We now introduce a third player, player E , the extremist. His payoff function is similar to player A 's, with one exception: player E 's cost type c_E differs from player A 's cost type c_A . Thus, player E 's payoff is obtained by setting $c_i = c_E$ in the payoff matrix (1), and letting the row represent player A 's choice and the column player B 's choice. There is no uncertainty about c_E . Formally, c_E is common knowledge among the three players. Player E knows c_A but not c_B .

We consider two possibilities. First, if player E is a hawkish extremist (a “provocateur”), then $c_E < 0$. Thus, the provocateur enjoys a *benefit* ($-c_E > 0$) if player A is aggressive. The provocateur is guaranteed a strictly positive payoff if player A chooses H , but he gets at most zero when player A chooses D , so he certainly wants player A to choose H . Second, if player E is a dovish extremist (a “pacifist”), then $c_E > \mu + d$. The most the pacifist can get if player A chooses H is $\mu - c_E$, while the worst he can get when player A chooses D is $-d > \mu - c_E$, so he certainly wants player A to choose D . Notice that, holding player A 's action fixed, the extremist (whether hawkish or dovish) is better off if player B chooses D .

Before players A and B play the conflict game described in Section I.A, player E sends a publicly observed cheap-talk message $m \in M$, where M is his message space. The time line is as follows.

1. The cost type c_i is determined for each player $i \in \{A, B\}$. Players A and E learn c_A . Player B learns c_B .
2. Player E sends a (publicly observed) cheap-talk message $m \in M$.
3. Players A and B simultaneously choose H or D .

In a “babbling” equilibrium, messages are disregarded and at time 3 players A and B behave just as in the unique communication-free equilibrium of Section I.A. Cheap-talk

⁸It is obvious from Figure 1 that the equilibrium is also unique if Assumption 2 is replaced by the assumption that F is concave. Sometimes concavity of F is convenient to work with (c.f. Section II.B) but it is hard to justify intuitively. In contrast, Assumption 2 formalizes the intuitive notion of sufficient uncertainty about types.

⁹Strategic substitutes generates a different kind of spiral. Fearing dominant strategy hawks, “almost dominant strategy doves” back down and play D . This emboldens “almost dominant strategy hawks” to play H , and so on.

is *effective* if there is a positive measure of types that choose *different* actions at time 3 than they would have done in the communication-free equilibrium. For cheap-talk to be effective, player E 's message must reveal some information about player A 's type. A Perfect Bayesian Equilibrium (PBE) with effective cheap-talk is a *communication equilibrium*. We will show that communication equilibria have a very specific structure, allowing us to unambiguously compare communication equilibrium payoffs with the payoffs in the babbling (communication-free) equilibrium.

A strategy for player E is a function $m : [\underline{c}, \bar{c}] \rightarrow M$, where $m(c_A)$ is the message sent by player E when player A 's type is c_A . Without loss of generality, each player $j \in \{A, B\}$ uses a "conditional" cutoff strategy: for any message $m \in M$, there is a cutoff $c_j(m)$ such that if player j hears message m , he chooses H if and only if $c_j \leq c_j(m)$. The next lemma shows that any communication equilibrium can be taken to involve just two messages, say m_0 and m_1 . One message, say m_1 , must make player B behave more hawkishly than the other message, m_0 .

LEMMA 1: *In communication equilibrium, it is without loss of generality to assume that M contains only two messages, $M = \{m_0, m_1\}$. The probability that player B plays H is higher after m_1 than after m_0 . That is, $c_B(m_1) > c_B(m_0)$.*

All omitted proofs are in the Appendix. Lemma 1 applies for both strategic substitutes and strategic complements, and for both pacifists and provocateurs. The proof of the lemma does not use Assumption 2.

II. Cheap-Talk with Strategic Complements

In this section, we consider the case of strategic complements, $d > \mu > 0$.

A. Main Results

From Lemma 1, we can assume only two messages, m_0 and m_1 , are sent in equilibrium. Player B is more likely to choose H after m_1 than after m_0 . If player A 's action does not depend on the message, then the extremist certainly prefers to send m_0 . If player A 's action depends on the message, then player A must be a coordination type who (by strategic complements) plays H in response to m_1 and D in response to m_0 .

If player E is a pacifist, then he wants both players A and B to choose D , so he must always send m_0 in equilibrium. But a constant message is not informative, and the outcome must be equivalent to the unique communication-free equilibrium of Section I.A. Thus, we have the following result.

THEOREM 2: *If player E is a pacifist and the game has strategic complements, then cheap-talk cannot be effective.*

Now suppose player E is a provocateur. We will show there exists a communication equilibrium where the provocateur uses cheap-talk to increase the risk of conflict above the level of the communication-free equilibrium. The communication equilibrium has

the following structure. If c_A is either very high or very low, then player A 's action will not depend on the message, and sending m_0 is optimal as it reduces the probability that player B will choose H . The provocateur can only benefit from message m_1 if it causes player A to switch from D to H . Thus, the provocateur's strategy must be non-monotonic: he sends message m_1 if and only if player A belongs to an intermediate range of coordination types who play D following m_0 but H following m_1 .

By this logic, if message m_1 is sent then player B knows that player A will play H . Therefore, player B plays H unless he is a dominant strategy dove. That is, his optimal cutoff point is $c_B(m_1) = d$, and the probability that he plays H is $F(d)$. Accordingly, player A 's best response is to choose H if and only if $c_A \leq \Gamma(d)$, where Γ is defined by equation (3). That is, $c_A(m_1) = \Gamma(d)$. Thus, conditional on message m_1 , players A and B must use cutoffs $c_A(m_1) = \Gamma(d)$ and $c_B(m_1) = d$, respectively.

Since player B is less likely to play H after m_0 than after m_1 , by strategic complements, so is player A . Thus, $c_A(m_0) < c_A(m_1) = \Gamma(d)$. If player A is of type $c_A \leq c_A(m_0)$ then he plays H following any message; if his type is $c_A > c_A(m_1) = \Gamma(d)$ then he plays D following any message. But if $c_A \in (c_A(m_0), \Gamma(d)]$, then player A chooses D after m_0 and H after m_1 . As the provocateur wants player A to be hawkish, he sends m_1 if and only if $c_A \in (c_A(m_0), \Gamma(d)]$.

It remains to determine the cutoffs used by players A and B conditional on message m_0 , denoted $y^* = c_A(m_0)$ and $x^* = c_B(m_0)$. These cutoffs, and the associated strategy profiles, are indicated in Figure 2. As always, optimal cutoffs are determined by the probability that the opponent plays H . Player B uses cutoff x^* after m_0 so he plays H with probability $F(x^*)$. Therefore, player A 's optimal cutoff is $y^* = \Gamma(x^*)$, where Γ is defined by equation (3). Now, the message m_0 is sent when c_A is either below y^* or above $\Gamma(d)$, and player A chooses H in the former case and D in the latter case. Therefore, conditional on m_0 , player A chooses H with probability

$$(4) \quad \frac{F(y^*)}{1 - F(\Gamma(d)) + F(y^*)}.$$

Player B 's optimal cutoff x^* is the best response to the belief that player A chooses H with probability given by (4). Since $y^* = \Gamma(x^*)$, to prove existence of communication equilibrium we use a fixed-point argument to show that x^* and y^* exist. This is given in the proof of part (i) of Theorem 3 (in the Appendix).

THEOREM 3: *Suppose player E is a provocateur and the game has strategic complements. (i) A communication equilibrium exists. (ii) All types of players A and B prefer the communication-free equilibrium to any communication equilibrium. Player E is better off in communication equilibrium if and only if $\hat{x} < c_A \leq \Gamma(d)$ (where \hat{x} is the cutoff in the communication-free equilibrium). (iii) If*

$$(5) \quad \frac{F'(y)}{1 - F(\Gamma(d)) + F(y)} < \frac{1}{d - \mu}$$

for all $y \in (\underline{c}, \bar{c})$ then the communication equilibrium is unique.

Proving part (ii) of Theorem 3 involves showing that players A and B behave more hawkishly than in the communication-free equilibrium, no matter which message is sent. Intuitively, we interpret m_1 as a “provocation” which occurs when player A is a “weak” coordination type $c_A \in (y^*, \Gamma(d)]$. Following a provocation, player B chooses H (except if he is a dominant strategy dove) and this causes player A to toughen up and play H . It is as if the provocation makes players A and B coordinate on a “bad” equilibrium of a stag-hunt game: they behave aggressively because they believe (correctly) that the other will be aggressive.

The cutoffs conditional on m_0 are lower than the cutoffs conditional on m_1 , so the decision makers behave less aggressively following m_0 than following m_1 , which justifies interpreting m_0 as the *absence* of a provocation. This absence is informative, just as Sherlock Holmes, in the story *Silver Blaze*, found it informative that a dog did not bark (Conan Doyle (1894)). Specifically, message m_0 reveals that player A is *not* a weak coordination type ($c_A \notin (y^*, \Gamma(d)]$). The weak coordination types would have chosen D in communication-free equilibrium, so eliminating these types is bad for peace. Thus, message m_0 actually triggers more aggression than the communication-free equilibrium (although not as much as m_1 does). Formally, the proof of Theorem 3 shows that the cutoffs after message m_0 are higher than the cutoffs in the communication-free equilibrium: $x^* > \hat{x}$ and $y^* > \hat{y}$.

It follows from these arguments that if a type would have chosen H in the communication-free equilibrium, then he necessarily chooses H in communication equilibrium. Moreover, after any message, there are types (of each player) who choose H , but who would have chosen D in the communication-free equilibrium. Since all types of players A and B want their opponent to choose D , they are all harmed by the third party’s cheap-talk.

For the provocateur, the benefits of cheap-talk are ambiguous. If either $c_A \leq \hat{x}$ or $c_A > \Gamma(d)$, then player A ’s action is the same in the communication equilibrium and in the communication-free equilibrium, but player B is more likely to choose H in the former, making player E worse off. On the other hand, if $\hat{x} < c_A \leq \Gamma(d)$, then player A would have chosen D in the communication-free equilibrium, but in the communication equilibrium he plays H , making player E better off.

Part (iii) of Theorem 3 shows that the communication equilibrium is unique if a “conditional” version of Assumption 2 holds.¹⁰ Intuitively, after m_0 is sent player B knows that player A ’s type is either below y^* or above $\Gamma(d)$. Thus, the continuation equilibrium must be the equilibrium of a “conditional” game where player A ’s type distribution G has support $[\underline{c}, y^*] \cup (\Gamma(d), \bar{c}]$ and density

$$G'(c) = \frac{F'(c)}{1 - F(\Gamma(d)) + F(y^*)}.$$

Furthermore, following m_0 , player A ’s cutoff type $y^* = c_A(m_0)$ is indifferent between H and D . Therefore, in the “conditional” game, the only possible cutoff type is y^* . Theo-

¹⁰That is, except for trivial re-labeling of messages, there is only one PBE *with effective cheap-talk*. Of course, the “babbling” PBE always exists as well.

rem 1 showed that equilibrium in the communication-free game is unique if Assumption 2 holds, i.e., if the distribution is sufficiently diffuse. The analogous “conditional” diffuseness condition for communication equilibrium turns out to be $G'(y^*) < 1/(d - \mu)$ for all y^* .¹¹ Note that even if this condition is violated, the only possible non-uniqueness comes from the possibility of multiple fixed points (x^*, y^*) , but the structure of the communication equilibrium is always the same (i.e., provocations occurring for weak coordination types, welfare effects given by part (ii) of Theorem 3, etc.).

The current model assumes a third party extremist communicates while players A and B are silent. In Baliga and Sjöström (2004), we found that (in the absence of an extremist) the two decision makers could reduce conflict by sending their own messages. These messages separated out “tough” coordination types who would have played H in the communication-free equilibrium, which cut the “hawkish cascade” and allowed the intermediate types to coexist peacefully. In the current model, a provocation separates out “weak” coordination types, who would have played D in the communication-free equilibrium but now switch to H . This brings conflict when peace could have prevailed. Even when no provocation occurs, the situation is still worse than the communication-free equilibrium, because the absence of weak coordination types leads to a less favorable type-distribution (the “dovish cascade” is cut off).

B. Extensions

PROVOCATEURS IN BOTH COUNTRIES

Extremists may not be confined to just one country. Suppose each country $i \in \{A, B\}$ has its own provocateur, player E^i , who knows the type of player i (the decision maker in country i). The two provocateurs simultaneously send (publicly observed) messages. We obtain the following *symmetric* version of the communication equilibrium of Section II.A. There are two cutoffs \tilde{x} and \tilde{y} , with $\mu < \tilde{x} < \tilde{y} < d$. In each country $i \in \{A, B\}$, player E^i sends m_1 (a “provocation”) if $c_i \in (\tilde{x}, \tilde{y}]$, and m_0 otherwise. Player $i \in \{A, B\}$ behaves as follows. If player E^j sends m_1 , where $i \neq j$, then player i chooses H if and only if $c_i \leq d$. If player E^j sends m_0 and player E^i sends m_1 then player i chooses H if and only if $c_i \leq \tilde{y}$. Finally, if both extremists send m_0 , then player i chooses H if and only if $c_i \leq \tilde{x}$. The existence proof (in the Appendix) uses a fixed-point argument to find \tilde{x} and \tilde{y} .

THEOREM 4: *With a provocateur in each country and strategic complements, a symmetric communication equilibrium exists.*

The logic of this equilibrium is just as in Section II.A. Extreme cost-types with $c_i \leq \tilde{x}$ or $c_i > \tilde{y}$ are not responsive to provocation so player E^i sends m_0 to minimize the probability that player j chooses H . If instead $c_i \in (\tilde{x}, \tilde{y}]$, the message sent by player E^i is pivotal if and only if player E^j sends m_0 . Then, if player E^i sends m_1 instead of m_0

¹¹For example, suppose F is uniform on $[0, \bar{c}]$. Then inequality (5) holds if \bar{c} is big enough, more precisely if $(\bar{c} - d)\bar{c} > (d - \mu)d$.

he changes player i 's action from D to H , which he prefers. Therefore, each extremist is provocative only in the intermediate range. When player E^j sends m_1 , player $i \neq j$ knows player j will play H , so player i chooses H unless he is a dominant strategy dove. When player E^j sends m_0 , player i 's incentive to choose H depends on the message sent by player E^i . Player j is more hawkish when player E^i sends m_1 rather than m_0 and hence by strategic complementarities so is player i (that is, $\tilde{x} < \tilde{y}$).

It might seem as if two provocateurs will create more conflict than one, but this is not necessarily the case. If no information is revealed about player j , then player i 's type $\Gamma(d)$ is the highest type that could conceivably be convinced to play H (because player j 's types above d play D for sure). In the communication equilibrium of Section II.A, the provocateur in country A actually achieves this upper bound: for $c_A = \Gamma(d)$, as well as for lower types, a provocation occurs which causes player A to choose H and player B to choose H with probability $F(d)$. Thus, a single provocateur has a remarkable ability to provoke aggression. But now player E^B reveals information about player B . If player E^B sends m_0 , then player A knows that $c_B \notin (\tilde{x}, \tilde{y}]$. Since the removed types in $(\tilde{x}, \tilde{y}]$ are not dominant strategy doves, player A knows that the probability that player B will choose H must be strictly less than $F(d)$, so if $c_A = \Gamma(d)$ then player A strictly prefers D , and the same is true for types slightly below $\Gamma(d)$. Thus, the information revealed about player B actually makes it harder to convince player A to choose H . Formally, in the communication equilibrium of Section II.A the peaceful outcome DD occurred when $c_A > \Gamma(d)$ and $c_B > x^*$. Here, with two extremists, the outcome DD occurs when $c_A > \tilde{y}$ and $c_B > \tilde{y}$. It can be shown that $x^* < \tilde{y} < \Gamma(d)$, so it is not possible to say if the peaceful outcome is more or less likely.

In what follows, we assume there is an extremist only in country A .

COSTLY MESSAGES

What happens if it is costly for the provocateur to ensure that his message is heard? In our model, the provocateur is willing to incur a cost to manipulate the conflict game, so such costs do not change the nature of our arguments. Suppose the "provocative" message m_1 imposes a cost $\tau_j > 0$ on player $j \in \{A, B, E\}$. The other message, m_0 , involves no costs. The extremist does not internalize τ_A and τ_B , and as these costs are already incurred when players A and B move, they do not affect strategic behavior. We now argue that if τ_E is not prohibitively big, then the communication equilibrium exists as before. Player E 's expected payoff from m_1 when $c_A \in (y^*, \Gamma(d)]$ is $-c_E + (1 - F(d))\mu - \tau_E$, as player A plays H and player B plays H unless he is a dominant strategy dove. If player E instead chooses m_0 , then player A plays D and player E 's expected payoff is $-dF(x^*)$. Player E prefers m_1 if

$$dF(x^*) - c_E + (1 - F(d))\mu > \tau_E.$$

The left hand side is strictly positive, so if τ_E is not too big, the communication equilibrium of Section II.A still exists. In what follows, we return to the case of pure cheap-talk.

CREDIBILITY, RENEGOTIATION AND DOMESTIC POLITICS

The provocateur's messages create conflict, which is bad for players A and B . Given that the messages are publicly observed, the two decision makers cannot simply agree to disregard the messages and behave as in the communication-free equilibrium, because the messages convey information about player A 's type. Neither can they convince the provocateur to voluntarily refrain from provoking conflict, because he benefits from it. The question is whether, *conditional on the information revealed by the extremist's message*, players A and B can "renegotiate" their strategies. In the communication equilibrium, message m_1 triggers a hawkish continuation equilibrium. But since the message in fact reveals that player A is a weak coordination type, there also exists a dovish continuation equilibrium, where player A chooses D and player B chooses D unless he is a dominant strategy hawk. However, renegotiation would face several problems. The first is information leakage: if renegotiation is not anticipated, but player B wants to renegotiate, player A might fear that player B is a dominant strategy hawk out to trick him. Second, even if there is no information leakage, there is a credibility problem. Each player, regardless of type, has an incentive to try to convince the opponent to become more dovish, even if he doubts that this will work so that he himself plans to stick to the original hawkish equilibrium. Therefore, an appeal to renegotiate and behave more peacefully is not informative of the player's own intentions, and may therefore not convince the opponent to deviate from the original equilibrium (c.f. Aumann (1990)).

A third problem is that a leader who does not react hawkishly to a provocation may look weak, and less likely to stay in power. For example, Jimmy Carter lost the presidential election in 1980 in part because he failed to deal effectively with the Iranian hostage crisis. To capture this, suppose player B gets an extra payoff $R > 0$ if he plays H after m_1 , interpreted as rents from increased popularity. Assume for convenience $\bar{c} > R + d$ to rule out corner solutions. The communication equilibrium of Section II.A is modified as follows to take R into account. Player A 's cutoff points are $c_A(m_0) = y^{**}$ and $c_A(m_1) = \Gamma(R + d)$. Player B 's cutoff points are $c_B(m_0) = x^{**}$ and $c_B(m_1) = R + d$. Player E sets $m(c_A) = m_1$ if and only if $c_A \in (y^{**}, \Gamma(R + d)]$. As before, a fixed-point argument is used to find x^{**} and y^{**} . But now messages are not cheap-talk, and we can obtain a stronger result than before. Specifically, if $R + \mu > d$, F is concave, and a condition analogous to (5) holds, namely

$$(6) \quad \frac{F'(y)}{1 - F(\Gamma(R + d)) + F(y)} < \frac{1}{d - \mu},$$

then the unique (modified) communication equilibrium is renegotiation-proof in the following strong sense: following any message there is a unique continuation equilibrium. Thus, even abstracting from the information leakage and credibility problems, there is no self-enforcing agreement where players A and B behave more dovishly following m_1 . Intuitively, player B is sufficiently aggressive following m_1 that the iterated deletion of dominated strategies (the hawkish cascade) generates a unique continuation equilibrium.

Moreover, there can be no "babbling" PBE. To see this, notice that if $c_B \leq R + \mu$,

then following m_1 , H dominates D for player B . Thus, in any PBE, $c_B(m_1) \geq R + \mu$. If $c_B \geq d$, then following m_0 , D dominates H for player B . Thus, in any PBE, $c_B(m_0) \leq d$. If $R + \mu > d$ then $c_B(m_1) > c_B(m_0)$, and $c_A(m_1) > c_A(m_0)$ by strategic complements. The provocateur therefore prefers to send m_1 if $c_A(m_0) < c_A \leq c_A(m_1)$ (since this makes player A choose H) but m_0 otherwise (since this minimizes the probability that player B chooses H). Thus, a provocation necessarily occurs if and only if player A is an intermediate type.

THEOREM 5: *If $R > d - \mu$, F is concave and inequality (6) holds for all $y \in (\underline{c}, \bar{c})$, then the (modified) communication equilibrium is the unique PBE, and it is renegotiation-proof.*

PARTIALLY UNINFORMED CHEAP-TALK

If c_A is either very high or very low, then the fact that the provocateur knows c_A makes him worse off because of the “dog that did not bark” effect (part (ii) of Theorem 3). He cannot escape this logic by staying silent, because it will simply be equated with sending m_0 (and hence informative). However, suppose the provocateur is known to be informed only with probability p , where $0 < p < 1$. His “silence” is less informative and players A and B are more peaceful. But this means there will be more scope for provocation to create conflict for intermediate c_A .

First, we informally discuss the provocateur’s incentive to be provocative when he does *not* know c_A . That is, he does not know how player A will react to his message. If each player $i \in \{A, B\}$ plays H with probability p_i , then player E ’s expected payoff is

$$p_A [-c_E + (1 - p_B) \mu] - (1 - p_A) p_B d.$$

Suppose a provocation increases each decision maker i ’s probability of playing H from p_i to $p'_i = p_i + \delta_i > p_i$. After some manipulations, the change in player E ’s expected payoff can be expressed as the following weighted sum of δ_A and δ_B :

$$(-c_E + (1 - p'_B) \mu + p'_B d) \delta_A - (p_A \mu + (1 - p_A) d) \delta_B.$$

This expression confirms that the increase in p_A makes the provocateur better off (the first term is positive), but the increase in p_B makes him worse off (the second term is negative). Depending on the relative sizes of δ_A and δ_B , either term might dominate, so in general, we cannot say whether provocations would pay for the uninformed extremist. However, the weight on δ_A is bigger, the bigger is p'_B (as $d > \mu$). Intuitively, if tensions are high, so player B is likely to choose H , increasing p_A is very valuable to the provocateur, because he reduces the chance of incurring the cost d . On the other hand, the weight on δ_B is smaller (in absolute value) the bigger is p_A . Intuitively, if player A is likely to choose H , increasing p_B is not so costly to the provocateur, because he is unlikely to incur the cost d . Thus, provocations are more likely to benefit an uninformed extremist in situations where tensions are high and hawkish behavior not unlikely. In contrast, a provocation where tension is low may backfire by causing the outcome DH .

Suppose, in fact, the uninformed provocateur prefers to send m_0 to reduce the risk of the outcome DH .

The informed provocateur will, following the logic of Section II.A (where in effect $p = 1$), send m_1 to provoke conflict when c_A is in some intermediate range. But the “dog that did not bark” effect is diluted since message m_0 may come from someone who has no information about c_A . Therefore, player B is more likely to play D after message m_0 if $p < 1$ than if $p = 1$. By strategic complements, so is player A . This causes the informed provocateur to send m_1 even when c_A is fairly low, to prevent player A from choosing D . Because the absence of a provocation may simply mean that the extremist is uninformed, there is less conflict in this case, and so the informed extremist resorts to provocations more frequently to prevent peace.

III. Cheap-Talk with Strategic Substitutes

In this section, we consider the case of strategic substitutes, $0 < d < \mu$.¹² Lemma 1 still applies, but now the message m_1 which makes player B more likely to play H must make player A more likely to play D . Since $\mu > 0$ and $d > 0$, player E always prefers player B to play D . Also, a hawkish extremist (provocateur) wants player A to choose H , so he clearly would always send m_0 . This gives us the following result.

THEOREM 6: *If player E is a provocateur and the game has strategic substitutes, then cheap-talk cannot be effective.*

If player E is a pacifist, however, a communication equilibrium exists. Since m_1 makes player B more hawkish ($c_B(m_1) > c_B(m_0)$), by strategic substitutes it makes player A more dovish ($c_A(m_1) < c_A(m_0)$). The pacifist will send m_1 if and only if player A is an opportunistic type who is induced by m_1 to switch from H to D (i.e., when $c_A(m_1) < c_A \leq c_A(m_0)$). Intuitively, we can interpret message m_1 as a “peace rally” which signals that player A will back down and choose D for sure. This causes player B to choose H , unless he is a dominant strategy dove ($c_B(m_1) = \mu$). Player A ’s optimal cutoff point is $c_A(m_1) = \Gamma(\mu)$. The cutoff points following m_0 , denoted $y^* = c_A(m_0)$ and $x^* = c_B(m_0)$, are constructed in the Appendix. The same argument as in Section II.A implies that for uniqueness, we must impose a “conditional” version of Assumption 2, specifically,

$$(7) \quad \frac{F'(y)}{1 - F(y) + F(\Gamma(\mu))} < \frac{1}{\mu - d}.$$

¹²If we had assumed $0 > \mu > d$, then player E would prefer that player B plays H in the strategic substitutes case. In this case, a relabeling of player B ’s strategies, $H \rightarrow d$ and $D \rightarrow h$, would restore strategic complementarity; again, only hawkish extremists would be able to communicate effectively. However, we in fact assume that the provocateur always wants player A to choose H and player B to choose D , while the pacifist always wants both to choose D . Maintaining $\mu > 0$ and $d > 0$, the strategic substitutes and complements cases are not isomorphic; a relabeling of strategies cannot turn one case into the other.

THEOREM 7: *Suppose player E is a pacifist and the game has strategic substitutes. (i) A communication equilibrium exists. (ii) All of player A's types prefer the communication-free equilibrium to any communication equilibrium. All of player B's types have the opposite preference. Player E is better off in the communication equilibrium if and only if $\Gamma(\mu) < c_A \leq \hat{x}$ (where \hat{x} is the unique fixed point of $\Gamma(x)$ in $[\underline{c}, \bar{c}]$). (iii) If condition (7) holds for all $y \in (\underline{c}, \bar{c})$ then the communication equilibrium is unique.*

The communication equilibrium has a “better red than dead” flavour, in the sense that the pacifist sends m_1 to make player A back down, even at the cost of making player B more hawkish. Evidently, player B benefits from message m_1 . In fact player B benefits from message m_0 as well, as it eliminates types of player A who would have played H in communication-free equilibrium. This makes player B more likely to choose H , and hence player A more likely to choose D , than in the communication-free equilibrium. In summary, whichever message is sent, player B is more hawkish and player A more dovish - hence player B is better off and player A worse off - than in communication-free equilibrium. (Formally, player B's cutoffs x^* and μ are both strictly greater than \hat{x} , while player A's cutoffs y^* and $\Gamma(\mu)$ are both strictly smaller than \hat{x} .) It is not possible to unambiguously say if the pacifist is good for peace, since he makes one player more dovish but the other more hawkish.

An interesting generalization is that the slope of a best response function may be uncertain. We will argue that the communication equilibria of Theorems 3 and 7 are robust to a small amount of uncertainty of this kind, but they fail to exist if there is too much uncertainty. Specifically, suppose the parameter μ in the payoff matrix (1) is μ_A for player A and μ_B for player B. Player i 's best response function is $\Gamma_i(x) \equiv \mu_i + (d - \mu_i)F(x)$. For simplicity, μ_A is fixed, but μ_B can take two values, $\mu_B \in \{\mu, \mu'\}$, where $\mu < d < \mu'$. The probability that $\mu_B = \mu'$ is η , where $0 < \eta < 1$. Only player B knows the true μ_B . Notice that with probability η , player B's best response function slopes down ($\Gamma'_B(x) < 0$), as with strategic substitutes, but with probability $1 - \eta$ it slopes up ($\Gamma'_B(x) > 0$), as with strategic complements.

Suppose that following message m , player j chooses H with probability $p_j(m)$. From (2), player B's optimal cutoff following m is $\mu_B + (d - \mu_B)p_A(m)$. Thus,

$$(8) \quad p_B(m) = (1 - \eta)F(\mu + (d - \mu)p_A(m)) + \eta F(\mu' + (d - \mu')p_A(m)).$$

Suppose m_0 minimizes $p_B(m)$. If player E is a provocateur and $\mu_A > d$ (so player A's best response function slopes down), or if player E is a pacifist and $\mu_A < d$ (so player A's best response function slopes up), then player E would always send m_0 , so communication is ineffective in these two cases (mimicking our earlier results).

Now suppose player E is a provocateur and $\mu_A < d$, so player A's best response function slopes up. If $\eta > 0$ is small enough, there exists a communication equilibrium similar to the one described in Theorem 3. Player E will send m_0 if player A's action is not responsive to the message, but he will send $m_1 \neq m_0$ (a provocation) if it changes player A's action from D to H . Therefore, in equilibrium, following m_1 player A must choose H for sure: $p_A(m_1) = 1$. From (8), we get $p_B(m_1) = F(d)$. In contrast,

$p_A(m_0) < 1$. If η is small, then $p_B(m_0) < F(d) = p_B(m_1)$, because $d > \mu$. Therefore, since he considers actions to be strategic complements, there will indeed be a set of types of player A who want to play D following m_0 but H following m_1 . This allows the equilibrium construction to go through as in the proof of Theorem 3. Thus, the communication equilibrium is robust to a small amount of uncertainty about whether player B 's best response function has positive or negative slope. Indeed, if there is a (small) chance that a provocation causes "the enemy" (player B) to back down, this actually strengthens the extremist's incentive to be provocative. However, if η is sufficiently big then equation (8) implies $p_B(m_0) > F(d) = p_B(m_1)$. In this case, if the provocation causes player A to become more hawkish, then the probability that player B becomes more dovish is so large that player A would also want to be more dovish (since his best response function slopes up), a contradiction. Thus, when η is too big, the communication equilibrium construction fails. Intuitively, the provocateur cannot create a hawkish cascade if player B is very likely to react to aggression by backing down.

A similar reasoning reveals that if player E is a pacifist and $\mu_A > d$, a communication equilibrium similar to the one described in Theorem 7 exists if η is sufficiently big, so it is likely that players A and B agree that actions are strategic substitutes. With $\eta < 1$ there is even a chance that a peace rally will make player B more peaceful, which strengthens the pacifist's incentive to stage the rally: it might bring about the outcome DD . However, if η is too small, then if the peace rally causes player A to become more dovish, the probability that player B also becomes more dovish is so large that player A would actually want to be more hawkish (as his best response function slopes down), a contradiction. Thus, in this case the communication equilibrium construction fails when η is too small.

IV. Conclusion

The International Relations literature distinguishes fear-spirals, like the one preceding World War I, from conflicts like World War II where lack of deterrence emboldened Hitler (Nye (2007), p. 111). Games with strategic complements or substitutes are stylized representations of these two kinds of strategic interactions. We have studied how a hawkish extremist can trigger conflicts when actions are strategic complements. When actions are strategic substitutes, the hawkish extremist is powerless, but a dovish extremist can convince one side to back down.

Provocateurs gain extra power if, unlike in our model, their actions cannot be clearly distinguished from those of the country's highest leaders. For example, Ellsberg (2002) describes how elements within the U.S. government wanted to provoke North Vietnam. On January 28, 1965, U.S. naval patrols "with the mission of provoking an attack, were ordered back into the Tonkin Gulf" (Ellsberg (2002), p. 66). The mission succeeded, and paved the way for heavy American involvement in Vietnam. It was probably unclear to the Vietnamese whether these provocative patrols had been approved at the highest levels of the U.S. government. In contrast, our model illuminates how a provocative act can trigger conflict even if it is commonly known to be the act of a third party. For example, after the 2008 Mumbai terrorist attack, Indian government officials clearly distinguished

between Pakistan's civilian government, which India believed was not involved in the attacks, and the ISI, which is believed to be outside the control of Pakistan's political leaders (Walsh (2010)).

It is sometimes argued that the ISI wants to force India to relinquish Kashmir by making India's presence in Kashmir costly. However, our model suggests that the ISI's optimal strategy may depend on the preferences of Pakistan's highest military and political leadership, because without their cooperation, the ISI will find it very difficult to drive India out of Kashmir. If Pakistan's leaders are sufficiently hawkish, the ISI's best option might be to develop a network of insurgents and lay the groundwork for a surprise attack by the Pakistani military, corresponding to the outcome HD (as in the 1999 Kargil war, for example). Since the ISI would not be aiming to provoke India, it would correspond to message m_0 . Of course, if India understands this strategy, the absence of provocations will not be very reassuring. In contrast, if the ISI thinks Pakistan's leaders are indecisive, the ISI's best option might be to use provocations to raise tensions between the two countries (corresponding to message m_1). Recent provocations by ISI-sponsored militants occurred when Pakistan's leaders were preoccupied with the "war on terror" rather than the struggle over Kashmir. According to our theory, these provocations were actually (moderately) good news, in the sense that they indicated the ISI believed Pakistan's political leaders were not dominant strategy hawks on Kashmir.¹³ However, if the ISI thinks Pakistan's current leaders are too weak to ever turn hawkish, the ISI's best option may again be to lay the groundwork for a future conflict, anticipating the arrival a more hawkish Pakistani leader. Since provoking India would not be the objective, it would again correspond to m_0 . In this way, a non-monotonic strategy could come about naturally, perhaps without being explicitly formulated in advance.

In Section III we showed that the communication equilibrium is robust to a small amount of uncertainty about whether actions are truly strategic substitutes or complements. In reality there may be *significant* uncertainty on this point. For example, the Cold War was characterized by disagreements about whether toughness would make the Soviet Union back down or become more aggressive. The model of Baliga and Sjöström (2008) emphasized this kind of uncertainty, but there was no third party who manipulated the conflict. Third party manipulation in such environments is an interesting topic for future research.

V. Appendix

Proof of Lemma 1. Suppose strategy μ is part of a BNE. Because unused messages can simply be dropped, we may assume that for any $m \in M$, there is c_A such that $m(c_A) = m$. Now consider any two messages m and m' . If $c_B(m) = c_B(m')$, then the probability player B plays H is the same after m and m' , and this means each type of player A also behaves the same after m as after m' , so having two separate messages m

¹³Many other examples of this logic could be given. For example, the provocative takeover of the American embassy by Iranian radicals would signal that Iranian leaders were not dominant strategy hawks (i.e., not necessarily implacable foes of the U.S.). Hamas's attacks during the Oslo peace accords and before Israeli elections would signal that the leaders of the Palestinian Authority were moderates who, unlike Hamas, wanted peace.

and m' is redundant. Hence, without loss of generality, we can assume $c_B(m) \neq c_B(m')$ whenever $m \neq m'$. Whenever player A is a dominant strategy type, player E will send whatever message minimizes the probability that player B plays H . Call this message m_0 . Thus,

$$(9) \quad m_0 = \arg \min_{m \in M} c_B(m).$$

Message m_0 is the *unique* minimizer of $c_B(m)$, since $c_B(m) \neq c_B(m_0)$ whenever $m \neq m_0$.

Player E cannot always send m_0 , because then messages would not be informative and cheap-talk would be ineffective (contradicting the definition of communication equilibrium). But, since message m_0 uniquely maximizes the probability that player B chooses D , player E must have some other reason for choosing $m(c_A) \neq m_0$. Specifically, if player E is a hawkish extremist (who wants player A to choose H) then it must be that type c_A would choose D following m_0 but H following $m(c_A)$; if player E is a dovish extremist (who wants player A to choose D) then it must be that type c_A would choose H following m_0 but D following $m(c_A)$. This is the only way player E can justify sending any other message than m_0 .

Thus, if player E is a hawkish extremist, then whenever he sends a message $m_1 \neq m_0$, player A will play H . Player B therefore responds with H whenever $c_B < d$. That is, $c_B(m_1) = d$. But $c_B(m) \neq c_B(m')$ whenever $m \neq m'$, so m_1 is unique. Thus, $M = \{m_0, m_1\}$.

Similarly, if player E is a dovish extremist, then whenever he sends a message $m_1 \neq m_0$, player A will play D . Player B 's cutoff point must therefore be $c_B(m_1) = \mu$. Again, this means $M = \{m_0, m_1\}$ and this completes the proof.

Proof of Theorem 3. The argument in the text showed that any communication equilibrium must have the following form. Player E sends message m_1 if and only if $c_A \in (y^*, \Gamma(d)]$. Player A 's cutoff points are $c_A(m_0) = y^*$ and $c_A(m_1) = \Gamma(d)$. Player B 's cutoff points are $c_B(m_0) = x^*$ and $c_B(m_1) = d$. Moreover, $y^* = \Gamma(x^*)$ and x^* is a best response to player A 's playing H with probability $F(y^*) / [1 - F(\Gamma(d)) + F(y^*)]$. To show part (i) of the theorem, we need to show that such x^* and y^* exist.

Conditional on message m_0 , player A will choose H with probability $F(y^*) / [1 - F(\Gamma(d)) + F(y^*)]$, so player B prefers H if and only if

$$(10) \quad -c_B + \frac{1 - F(\Gamma(d))}{1 - F(\Gamma(d)) + F(y^*)} \mu \geq \frac{F(y^*)}{1 - F(\Gamma(d)) + F(y^*)} (-d).$$

Inequality (10) is equivalent to $c_B \leq \Omega(y^*)$, where

$$\Omega(y) \equiv \frac{[1 - F(\Gamma(d))] \mu + F(y)d}{[1 - F(\Gamma(d))] + F(y)}.$$

Thus, $x^* = \Omega(y^*)$. We now show graphically that we can find x^* and y^* such that $x^* = \Omega(y^*)$ and $y^* = \Gamma(x^*)$.

By Assumption 2, Γ is increasing with a slope less than one. Since $F(\underline{c}) = 0$ and $F(\bar{c}) = 1$, we have $\Gamma(\underline{c}) = \mu > \underline{c}$ and $\Gamma(\bar{c}) = d < \bar{c}$. Furthermore,

$$\Gamma(d) - \mu = F(d)(d - \mu) < d - \mu.$$

Therefore,

$$(11) \quad \Gamma(d) < d.$$

Also,

$$\Gamma(\mu) = \mu(1 - F(\mu)) + dF(\mu) > \mu$$

as $d > \mu$. Let \hat{x} be the unique fixed point of $\Gamma(x)$ in $[\underline{c}, \bar{c}]$. Clearly, $\mu < \hat{x} < \Gamma(d)$ (see Figure 1).

Figure 3 shows three curves: $x = \Omega(y)$, $y = \Gamma(x)$ and $x = \Gamma(y)$. The curves $x = \Gamma(y)$ and $y = \Gamma(x)$ intersect on the 45 degree line at the unique fixed point $\hat{x} = \Gamma(\hat{x})$. Notice that

$$\Omega'(y) = \frac{F'(y)(d - \mu)(1 - F(\Gamma(d)))}{([1 - F(\Gamma(d))] + F(y))^2}$$

so Ω is increasing. It is easy to check that $\Omega(y) > \Gamma(y)$ whenever $y \in (\underline{c}, \Gamma(d))$. Moreover, $\Omega(\underline{c}) = \Gamma(\underline{c}) = \mu$ and

$$\Omega(\Gamma(d)) = \Gamma(\Gamma(d)) < \Gamma(d)$$

where the inequality follows from (11) and the fact that Γ is increasing. These properties are shown in Figure 3. Notice that the curve $x = \Omega(y)$ lies to the right of the curve $x = \Gamma(y)$ for all y such that $\underline{c} < y < \Gamma(d)$ (because $\Omega(y) > \Gamma(y)$ for such y), but the two curves intersect when $y = \underline{c}$ and $y = \Gamma(d)$.

As shown in Figure 3, the two curves $x = \Omega(y)$ and $y = \Gamma(x)$ must intersect at some (x^*, y^*) , and it must be true that

$$(12) \quad \hat{x} < y^* < x^* < \Gamma(d) < d.$$

By construction, $y^* = \Gamma(x^*)$ and $x^* = \Omega(y^*)$. Thus, a communication equilibrium exists. The welfare comparisons in part (ii) follow from the fact that $\hat{x} < y^* < x^*$ and the argument in the text.

Finally, part (iii) is equivalent to showing uniqueness of (x^*, y^*) . It can be verified that (5) implies $0 < \Omega'(y) < 1$. This implies, since $0 < \Gamma'(x) < 1$, that the two curves $x = \Omega(y)$ and $y = \Gamma(x)$ intersect only once, as indicated in Figure 3.

Proof of Theorem 4. Consider the continuous function $F : [\mu, d]^2 \rightarrow [\mu, d]^2$, defined by

$$F(x, y) = \begin{bmatrix} F^x(x, y) \\ F^y(x, y) \end{bmatrix}$$

where

$$F^x(x, y) \equiv \frac{(1 - F(y))\mu + F(x)d}{1 - F(y) + F(x)}$$

and

$$F^y(x, y) \equiv \frac{(1 - F(d))\mu + (F(x) + F(d) - F(y))d}{1 - F(y) + F(x)}.$$

There exists a fixed point $(\tilde{x}, \tilde{y}) = F(\tilde{x}, \tilde{y})$. It is easy to check that $\mu < \tilde{x} < \tilde{y} < d$.

Consider the strategy profile described in the text. Player E^i maximizes his payoff by sending m_1 if and only if $c_i \in (\tilde{x}, \tilde{y}]$. Now consider player A . If player E^B sends m_1 , then player B is expected to choose H . Therefore, player A plays H unless D is his dominant strategy. Suppose instead that player E^B sends m_0 and player E^A sends m_1 . Then either $c_B \leq \tilde{x}$ or $c_B > \tilde{y}$, and player B chooses H if and only if $c_B \leq d$. Therefore, the probability that player B chooses H is

$$\frac{F(\tilde{x}) + F(d) - F(\tilde{y})}{1 - F(\tilde{y}) + F(\tilde{x})}.$$

It can be checked that $\tilde{y} = F^y(\tilde{x}, \tilde{y})$ implies that player A 's type \tilde{y} is indifferent between H and D . Thus, the best response is to choose H when $c_A \leq \tilde{y}$.

Finally, suppose both extremists send m_0 . Again, either $c_B \leq \tilde{x}$ or $c_B > \tilde{y}$. Player B chooses H in the former case and D in the latter case. Thus, the probability that player B chooses H is

$$\frac{F(\tilde{x})}{1 - F(\tilde{y}) + F(\tilde{x})}.$$

It can be checked that $\tilde{x} = F^x(\tilde{x}, \tilde{y})$ implies that player A 's type \tilde{x} is indifferent between H and D . Thus, the best response is to choose H when $c_A \leq \tilde{x}$. Hence, player A maximizes his payoff. The situation for player B is symmetric.

Proof of Theorem 5. The argument in the text proves that there can be no ‘‘babbling’’ (uninformative) PBE. Communication equilibria (with informative messages) must have the familiar form. Arguing as in Section II.A, $y^{**} = \Gamma(x^{**})$ where Γ is defined by equation (3), and x^{**} is a best response to player A playing H with probability

$$\frac{F(y^{**})}{1 - F(\Gamma(R + d)) + F(y^{**})}.$$

The function Ω is modified to take R into account:

$$\hat{\Omega}(y) \equiv \frac{[1 - F(\Gamma(R + d))]\mu + F(y)d}{[1 - F(\Gamma(R + d))] + F(y)}.$$

As before, it can be shown that the two curves $x = \hat{\Omega}(y)$ and $y = \Gamma(x)$ intersect at some point (x^{**}, y^{**}) , where

$$(13) \quad \hat{x} < y^{**} < x^{**} < \Gamma(R + d) < d.$$

There is only one intersection if (6) holds, so a unique communication equilibrium exists as before. Moreover, (6) guarantees that there is a unique continuation equilibrium following m_0 . We need to show that there is also a unique continuation equilibrium following m_1 . Specifically, following m_1 player B must expect that player A will play H and thus player B plays H if $c_B \leq R + d$ (i.e., unless D is his dominant action following m_1).

Any continuation equilibrium must consist of a pair of cutoff points, x for player B and y for player A , that are best responses to each other, conditional on m_1 having revealed to player B that $c_A \in (y^{**}, \Gamma(R + d)]$. If player A uses a cutoff $y \in [y^{**}, \Gamma(R + d)]$, player B prefers H if and only if

$$(14) \quad R - c_B + \frac{\mu (F(\Gamma(R + d)) - F(y))}{F(\Gamma(R + d)) - F(y^{**})} \geq \frac{-d (F(y) - F(y^{**}))}{F(\Gamma(R + d)) - F(y^{**})}.$$

Inequality (14) is equivalent to $c_B \leq \Theta(y)$ where

$$\Theta(y) \equiv \frac{(d - \mu) F(y)}{F(\Gamma(R + d)) - F(y^{**})} + R + \frac{\mu F(\Gamma(R + d))}{F(\Gamma(R + d)) - F(y^{**})} - \frac{d F(y^{**})}{F(\Gamma(R + d)) - F(y^{**})}.$$

Thus, player B 's best response is $x = \Theta(y) \in [R + \mu, R + d]$. (Types below $R + \mu$ or above $R + d$ have dominant actions following m_1 .)

Player A 's best response to x is given by Γ . If $R + \mu > d$ then $\Gamma(R + \mu) > y^{**} = \Gamma(x^{**})$. To see this, notice that $R + \mu > d$ implies

$$(15) \quad R + \mu > x^{**} = \frac{[1 - F(\Gamma(R + d))] \mu + F(y^{**})d}{[1 - F(\Gamma(R + d))] + F(y^{**})}.$$

Thus, $\Gamma(R + \mu) > y^{**}$, and since Γ is increasing, player A 's best response to $x \geq R + \mu$ is $y = \Gamma(x) > y^{**}$.

So far we have shown that the cutoffs conditional on m_1 satisfy $x = \Theta(y) \geq R + \mu$ and $y = \Gamma(x) > y^{**}$. In fact, the curves $y = \Gamma(x)$ and $x = \Theta(y)$ intersect at $(x, y) = (R + d, \Gamma(R + d))$ which yields the strategy played in the unique communication equilibrium: after message m_1 player A is expected to play H (all types $c_A \in (y^{**}, \Gamma(R + d)]$ play H) and player B plays H if $c_B \leq R + d$. The curves can have no other intersection if F is concave, since both Γ and Θ are concave and can intersect at most once in the relevant region where $x \in [R + \mu, R + d]$ and $y \in [y^{**}, \Gamma(R + d)]$. Thus, the continuation equilibrium following m_1 is unique.

Proof of Theorem 7. Arguing as in Section II.A, $y^* = \Gamma(x^*)$, and x^* is a best response to player A playing H with probability $F(\Gamma(\mu)) / [1 - F(y^*) + F(\Gamma(\mu))]$. To show the existence of x^* and y^* is again a fixed-point argument. Let

$$\tilde{\Omega}(y) \equiv \frac{[1 - F(y)] \mu + F(\Gamma(\mu))d}{[1 - F(y)] + F(\Gamma(\mu))}.$$

The cutoffs (x^*, y^*) is an intersection of the two curves $x = \tilde{\Omega}(y)$ and $y = \Gamma(x)$. With strategic substitutes, Assumption 2 implies $-1 < \Gamma'(x) < 0$. Furthermore, $\Gamma(\underline{c}) = \mu < \bar{c}$ and $\Gamma(\bar{c}) = d > \underline{c}$. Also,

$$\Gamma(\mu) - d = (1 - F(\mu))(\mu - d)$$

where

$$0 < (1 - F(\mu))(\mu - d) < \mu - d.$$

Therefore,

$$(16) \quad d < \Gamma(\mu) < \mu.$$

Let \hat{x} be the unique fixed point of $\Gamma(x)$ in $[\underline{c}, \bar{c}]$. It is easy to check that $d < \hat{x} < \mu$.

The curves $x = \Gamma(y)$ and $y = \tilde{\Omega}(x)$ intersect on the 45 degree line at the fixed point $\hat{x} = \Gamma(\hat{x})$. It is easy to check that $\tilde{\Omega}(y) > \Gamma(y)$ whenever $y \in (\Gamma(\mu), \bar{c})$. Moreover, $\tilde{\Omega}(\bar{c}) = \Gamma(\bar{c}) = d$ and

$$\tilde{\Omega}(\Gamma(\mu)) = \Gamma(\Gamma(\mu)) > \Gamma(\mu)$$

where the inequality follows from (16) and the fact that Γ is decreasing. Consider now the intersection of the two curves $x = \tilde{\Omega}(y)$ and $y = \Gamma(x)$. A figure analogous to Figure 3 reveals that there exists $(x^*, y^*) \in [\underline{c}, \bar{c}]^2$ such that $y^* = \Gamma(x^*)$ and $x^* = \tilde{\Omega}(y^*)$, and

$$(17) \quad d < \Gamma(\mu) < y^* < \hat{x} < x^* < \mu.$$

This proves parts (i) and (ii) of Theorem 7. For part (iii), it can be checked that (7) implies $-1 < \tilde{\Omega}'(y) < 0$. Since $-1 < \Gamma'(x) < 0$, the two curves $x = \tilde{\Omega}(y)$ and $y = \Gamma(x)$ intersect only once.

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FIGURE 1. STRATEGIC COMPLEMENTS: COMMUNICATION-FREE EQUILIBRIUM

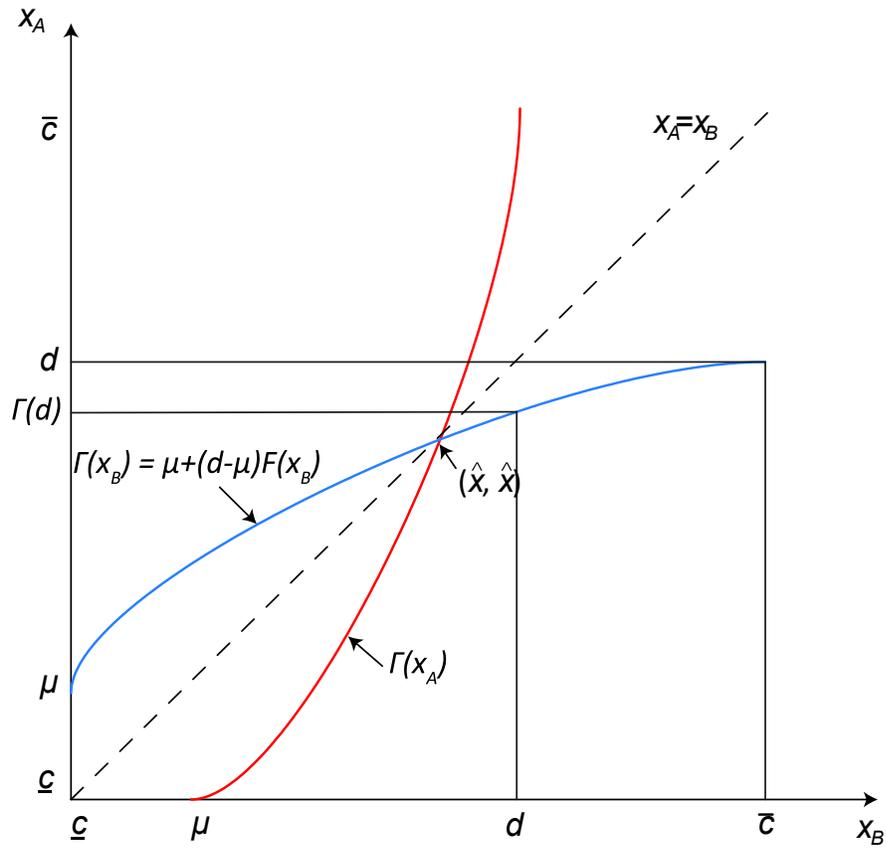


FIGURE 2. STRATEGIC COMPLEMENTS: STRATEGIES IN COMMUNICATION EQUILIBRIUM

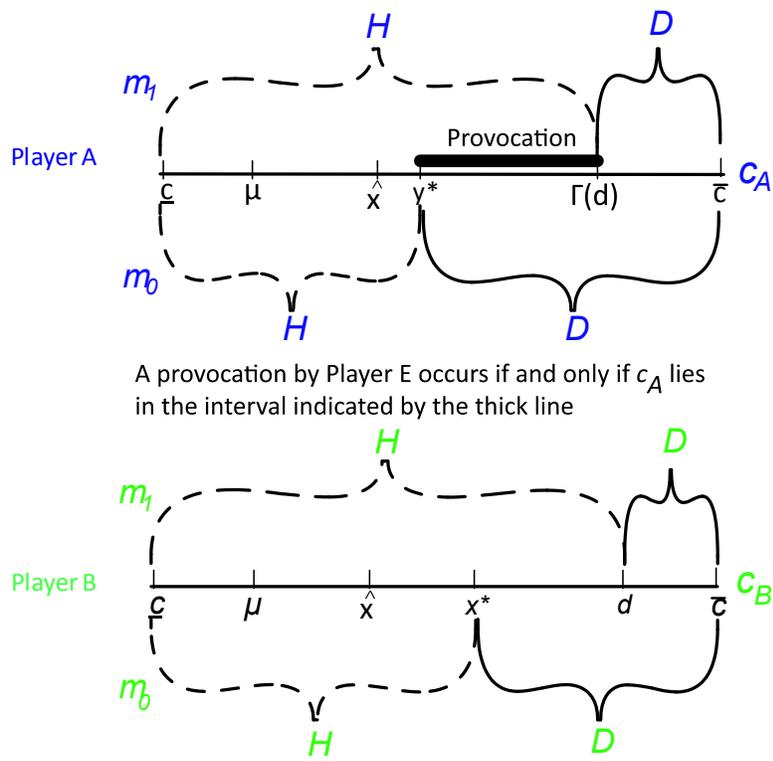


FIGURE 3. STRATEGIC COMPLEMENTS: THEOREM 3

