Contract, Mechanism Design, and Technological Detail

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Abstract

This paper develops a theoretical framework for studying contract and enforcement in settings of complete but unverifiable information and nondurable trading opportunities. The main point of the paper is that the consideration of renegotiation *necessitates* formal examination of other technological constraints, especially those having to do with the timing and nature of inalienable productive actions. The sets of implementable state-contingent payoffs, under various assumptions about renegotiation opportunities, are characterized and compared. The analysis refutes the validity of the "mechanism design with ex post renegotiation" program in a wide range of contracting environments and it highlights the need for a more structured game-theoretic framework. *JEL Classification: C70, D74, K10.*

Economic models of contract have yielded important insights regarding the nature and impact of contractual imperfections and optimal contractual form. Many of the insights derive from mechanism-design analysis—a methodology whose elegance relies on stripping away institutional detail and focusing on a few fundamental strategic ingredients. To the extent that institutional constraints plays a critical role in the formation and performance of contracts, however, it is important to develop ways of incorporating these constraints into models.

One issue that has received a great deal of attention is the possibility that parties can *renegotiate* in the midst of a contractual relationship. Hart and Moore's (1988) seminal article shows how renegotiation following specific investments can inhibit the parties' ability to induce optimal investment. To incorporate renegotiation into a

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standard mechanism-design model, Maskin and Moore (1999) developed the *mechanism design with ex post renegotiation* (MDER) program, which assumes that parties can renegotiate the contractually-specified outcome after sending messages to an external enforcer. Maskin and Moore's methodology and characterization results have been widely accepted and employed.[1]

In this paper, I study how renegotiation opportunities interact with the technology of trade in contractual relationships. I show that, in order to adequately address renegotiation, our models must accurately represent the nature of the parties' trade actions (such as "the number of units delivered by the seller" or "whether the buyer accepts delivery"). The key issue is whether these actions are modeled as *individual* or as *public* actions. An individual action is one assumed to be taken directly by one of the contracting parties, whereas a public action is one taken directly by an external enforcement authority. In most real settings, trade actions are individual and inalienably so, whereas the MDER and other mechanism-design programs typically treat them as public. I demonstrate that treating as public an action that is actually individual can artificially distort the scope for contracting in the presence of renegotiation.

I present a structured, game-theoretic model that explicitly accounts for the following essential elements: (a) the timing and nature of individual, inalienable actions; (b) the manner in which an external enforcer compels behavior; and (c) at what times the parties have the opportunity to renegotiate their contract.[2] I focus on settings with complete but unverifiable information, verifiable trade actions, and nondurable trading opportunities (where there is a fixed date at which irreversible trade actions must be made). I characterize the sets of implementable outcomes under a variety of assumptions about when renegotiation can take place, and I compare these to the set identified by the standard MDER program. I find that the MDER program fails to accurately describe the implementable set in a wide range of environments.

To see why the modeling of trade actions matters, assume that at Date 6 (as in the model presented here) an irreversible trade action must be taken. Also suppose that the parties can renegotiate their contract at Date 5. If the trade action is modeled as public then it is assumed to be chosen by some external enforcer who simply executes the terms of a contract in force at Date 6. In this setting, the contracting parties direct the trade action through their contract and through messages they send to the

[1]The MDER program builds from Maskin's (1999) work on Nash implementation (without renegotiation). The literature contains numerous, high-profile papers that adopt the MDER program, including the recent work of Che and Hausch (1999), Edlin and Reichelstein (1996), Segal (1999), and Segal and Whinston (2002).

[2]My approach is allied with that of Hart and Moore (1988), MacLeod and Malcomson (1993), and Nöldeke and Schmidt (1995), who model individual trade actions. Also relevant is the work of Myerson (1982,1991), whose mechanism design analysis nicely distinguishes between inalienable private and public actions.

external enforcer prior to Date 5. Then, at Date 5, the parties will know whether the action to be taken by the enforcer is efficient in the current state of the world. If it is not efficient, the parties will renegotiate the contract to achieve an efficient outcome. Importantly, an efficient outcome is realized regardless of the parties' behavior at earlier dates (in or out of equilibrium).

Alternatively, suppose that the trade action is modeled as an individual action taken by one of the parties. In this case, the contract specifies monetary transfers between the parties as a function of the trade action and the messages sent earlier. By using "forcing contracts," it is possible to duplicate the results of treating the trade action as public, because the contract can specify transfers that induce any particular action regardless of the state of the world. However, other contracts may implement outcomes that would not be implementable in the model with the public trade action. The reason is that renegotiation at Date 5 concerns only the *equilibrium* trade action at Date 6; there is no requirement that every selection that could be made at Date 6 must result in an efficient outcome (because there is no "time left" to renegotiate an inefficient trade action chosen at Date 6).

Option contracts provide a practical illustration. Observe that there are two ways of designing an option contract. In one form, the option entails a message that one of the parties sends at, say, Date 4; this message triggers a response by the external enforcer that forces the players to choose a particular trade action. In the other form, the trade action *itself* serves as an option, with the external enforcer simply compelling transfers as a function of the individual trade action.[3] The latter option form is not available when trade actions are treated as public and this makes a difference when renegotiation is possible at Date 5. Thus, in settings with renegotiation, treating individual trade actions as public entails an artificial restriction on the set of contracts. Hence, the MDER approach may fail to describe what can be achieved through contracting in the presence of ex-post renegotiation.

The general modeling framework is described in the next section. Section 2 contains definitions and analysis that are useful for representing the parties' contracting problem as a mechanism-design problem. Section 3 defines and partially characterizes implementation for various settings (differentiated by if and when renegotiation can take place). In Section 4, I present the analysis of the specific example introduced in Section 1. This example illustrates, and supplies intuition for, my general results.

In Section 5, I prove a theorem that ranks by inclusion the sets of implementable state-contingent payoffs under various assumptions about renegotiation. I also pro-

[3] Most mechanism-design models study options of the first type; more structured models, such as Nöldeke and Schmidt's (1995), focus on the second type. The law treats option generally, as a limit on a parties "power to revoke an offer" (Section 25, *Restatement (Second) of Contracts*; see Barnett 1999). In addition to more conventional forms, options are implicitly created by liquidated damage provisions and standard breach remedies. (A party has the option of breaching and then paying the damage amount.)

vide theorems that give conditions under which the inclusion relations are strict; under these conditions, the MDER program fails to identify the implementable set in the presence of ex post renegotiation. Furthermore, I prove that the conditions are satisfied in the wide range of settings delineated by a specific group of assumptions on the technology of trade. Section 6 comprises two examples that further clarify the shortcomings of the MDER program while confirming, with qualifications, that hold-up is indeed a problem in some contractual relationships. Section 7 contains concluding remarks. Proofs of the lemmas and Theorem 4 are contained in the appendices.

My results do not challenge the legitimacy of mechanism design theory for the study of contract. However, the modeling exercise reveals that the *application* of mechanism design theory can be seriously flawed if one does not incorporate the proper technological constraints. Overall, the research clarifies the meaning and implication of "ex post renegotiation."

Numerous authors have argued for the kind of research reported herein. Hurwicz (1994) speaks of the importance of incorporating institutional constraints into design problems—a step that, for the most part, has yet to be taken in any general, compelling way. He suggests that institutional constraints should be represented as limiting design to a class of game forms, whereby the "'desired' game form [is embedded in what he calls] the 'natural' game form" (p.12). My framework may be interpreted as this natural game form. Anderlini, Felli, and Postlewaite (2001), Segal and Whinston (2002), and others recognize the need to study technological and institutional constraints in contracting environments. Furthermore, the contract theory literature has seen several debates regarding renegotiation and its relation to messages and productive actions.[4] Against this backdrop, my message should be clearly understood: To make sense of modeling choices and to have an instructive debate, we must use a theoretical framework that explicitly accounts for the technology of trade and enforcement.

[4]For example, Nöldeke and Schmidt (1995) point out that Hart and Moore's (1988) underinvestment problem disappears if the parties' individual trade actions are verifiable, rather than just partially verifiable. Edlin and Hermalin (2000) argue that Nöldeke and Schmidt (1998) and Bernheim and Whinston (1998) incorrectly model the timing of options, investment, and renegotiation—that, because of ex post renegotiation following opportunities to exercise an option, hold-up is a severe problem. I comment on this in the Conclusion. In work that criticizes Hart and Moore's analysis of incomplete contracts, Lyon and Rasmusen (2001) argue that parties should, in reality, be able to rescind and change option orders after an opportunity for renegotiation expires. See also MacLeod (2001) on renegotiation and the timing of the resolution of uncertainty.

1 The Theoretical Framework

Two contracting parties, whom I call "players 1 and 2," engage in a contractual relationship with external enforcement. Their relationship has the following payoff-relevant components, occurring in the order shown:

The *state of the relationship* θ. The state represents unverifiable events that are assumed to happen early in the relationship. The state may be determined by individual investment decisions and/or by random occurrences, depending on the setting. When the state is realized, it becomes commonly known by the players; however, it cannot be verified to the external enforcer. Let Θ denote the set of possible states.

The *trade actions (or "decisions")* $a = (a_1, a_2)$. This is a profile of individual, inalienable actions that the players choose, determining whether and how the relationship is consummated. The trade actions are commonly observed by the players and are verifiable to the external enforcer. I assume that a is an element of the product set $A \equiv A_1 \times A_2$, where A_1 is the feasible set of actions for player 1 and A_2 is the feasible set of actions for player 2. I assume that the players select their trade actions simultaneously and independently.

The *monetary transfers* $t = (t_1, t_2)$. Here t_i denotes the amount given to player i, for $i = 1, 2$, where a negative value represents an amount taken from this player. These transfers are compelled by the external enforcer, who is not a strategic player but, rather, who behaves as directed by the contract of players 1 and 2. I limit attention to "balanced transfers" by assuming that $t \in \mathbf{R}^2_0$, where

$$\mathbf{R}^2_0 \equiv \{t \in \mathbf{R}^2 \mid t_1 + t_2 = 0\}.$$

This rules out transfers to third parties or money burning by the external enforcer.[5]

I assume that the players' payoffs are additive in money and are thus defined by a function $u : A \times \Theta \to \mathbf{R}^2$. In state θ, with trade action a and transfer t, the payoff vector is $u(a, \theta) + t$. I assume that u is bounded and that the maximal joint payoff, $\max_{a \in A}[u_1(a, \theta) + u_2(a, \theta)]$, exists for every θ. It will not be necessary to put any restrictions on the sets A and Θ.

In addition to the payoff-relevant components of their relationship, I assume that the players can communicate with the external enforcer using public, verifiable messages. Let $m = (m_1, m_2)$ denote the profile of messages that the players send and let

[5]This assumption, which is common in the related literature, is justified by a renegotiation opportunity (the contracting parties would rewrite their contractual instructions just before the court acts).

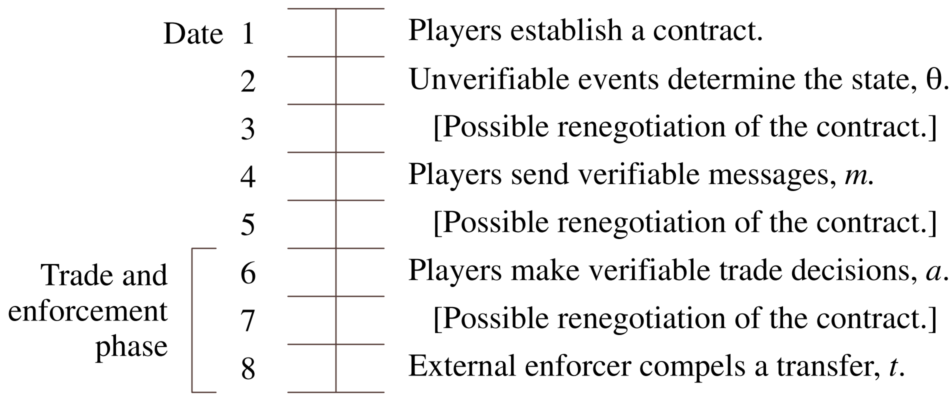

Figure 1: The contractual relationship.

M_1 and M_2 be the sets of feasible messages. The sets M_1 and M_2 will be endogenous in the sense that they are specified by the players in their contract.

I focus on nondurable trading opportunities, meaning that there is a fixed date at which the trade decisions are made. This date is designated as "Date 6" in Figure 1, which shows the time line of the contractual relationship. At even-numbered dates through Date 6, the players make joint observations and they make individual decisions—jointly observing the state at Date 2, sending verifiable messages at Date 4, and selecting the trade actions at Date 6. At Date 8, the external enforcer compels transfers.

At odd-numbered dates, the players make joint contracting decisions—establishing a contract at Date 1 and possibly renegotiating it later. The contract has an externally-enforced component consisting of (i) feasible message spaces M_1 and M_2 and (ii) a function $y : M \times A \to \mathbf{R}^2$ specifying the transfer t as a function of the verifiable items m and a. That is, having seen m and a, the external enforcer compels transfer $t = y(m, a)$. I call y the *transfer function*. The contract also has a self-enforced component, which specifies how the players coordinate their behavior for the times at which they make individual decisions. Renegotiation of the contract amounts to replacing the original transfer function y with some new function y', in which case y' is the one submitted to the external enforcer at Date 8.

I model rational behavior in the contractual relationship as follows. The players' individual decisions at Dates 4 and 6 are assumed to be consistent with sequential rationality; that is, each player maximizes his expected payoff, conditional on what occurred earlier and on what the other player does, and anticipating rational behavior in the future. The joint decisions (initial contracting and renegotiation at odd-numbered periods) are assumed to be consistent with a "black-box" cooperative bargaining solution in which the players divide surplus according to fixed bargaining weights π_1 and π_2 for players 1 and 2, respectively. The bargaining weights are nonnegative and sum to one. I write $\pi = (\pi_1, \pi_2)$. Surplus is defined relative to a disagreement point. More details are given later in this section.[6]

I shall, from this point, ignore the possibility of renegotiation at Date 7. The justification for this is that it has already been incorporated into the analysis by limiting attention to balanced transfers (recall footnote 5).

A *(state-contingent) value function* is a function $v : \Theta \to \mathbf{R}^2$ that gives the players' expected payoff vector from the start of Date 3, as a function of the state. An *implementable* value function is that which results from rational behavior for some contract selected at Date 1. (Formal definitions are in Section 3.) The main theoretical exercise is to determine the set of implementable value functions; this set depends on whether renegotiation is possible at Dates 3 or 5. Calculating the set of value functions is important because the players and society have preferences over them. In some settings, this relates to whether players are given the incentives to make ex ante investments, as is the case in the following example.

Example 1

Here is a simple numerical example that illustrates the model's components. The example is analyzed in detail in Section 4. Player 1 is the buyer of an intermediate good, player 2 is the seller, and the external enforcer is the court. To be concrete, imagine that the buyer is a masonry supply company that hopes to gain new customers at a regional trade show. The seller is an advertisement agency. The buyer wishes to hire the seller to develop an advertisement package for the trade show.

The set of states is $\{\mathrm{H}, \mathrm{L}\}$, where H indicates the "high" state in which the advertisement package will be successful and L denotes the "low" state in which the advertisement will not be successful. The state is determined by an investment that one of the players makes at Date 2. I will consider two versions of the example. In the first version, it is the buyer who makes this investment; think of it as effort that player 1 exerts to evaluate his downstream market and to provide information

[6] Fixed bargaining weights capture the idea that renegotiation activity is non-contractible, so that the parties can exercise bargaining power and hold each other up during the relationship. This assumption is realistic for many applied settings and it is a key ingredient of most recent contract models in the literature. It is standard in the literature to model renegotiation using a cooperative game solution, although in some papers, such as Hart and Moore (1988) and MacLeod and Malcomson (1993), theorists analyze a non-cooperative model of bargaining. In my model, because the players are risk neutral in money, the cooperative solution yields the same expected payoffs as does the following non-cooperative specification of negotiation: Nature selects one of the players to make an ultimatum offer to the other, who either accepts or rejects it; Nature selects player i with probability π_i; and we assume that, if the offer is rejected, then the equilibrium in the continuation of the game does not depend on the identity of the offerer or on the nature of the offer. Technically, the behavioral assumptions made here define a "negotiation equilibrium" in the terminology of Watson (2002a). For more on the relation between non-cooperative bargaining models and cooperative solutions, see Binmore, Rubinstein, and Wolinsky (1986).

to player 2. In the second version of the example, the seller makes the investment; imagine it as player 2's effort to learn about player 1's business and downstream market. I assume that the effort decision is binary (either "exert" effort or "not"), that "exert" entails an immediate cost of c in monetary units, and that the high state is realized if and only if "exert" is chosen.

Suppose that the trade action is the buyer's choice of whether to adopt the advertisement package. Specifically, let $A_1 = \{1, 0\}$ and $A_2 = \emptyset$, where $a = 1$ indicates that the buyer adopts the advertisement and $a = 0$ indicates that he does not adopt it. The buyer's decision to adopt the advertisement can also be described as "the buyer consummates the trade" or "the buyer accepts delivery." The trading opportunity is nondurable; in other words, the buyer's decision of whether to adopt the advertisement cannot be reversed or delayed.[7]

Above any effort costs, the payoffs are defined as follows. In state H, if the buyer adopts the advertisement package and is forced to make a monetary transfer p to the seller, then the buyer gets $5 - p$ and the seller gets $3 + p$. The buyer's value of 5 is the profit generated by a successful advertisement. The seller's value of 3 reflects the extra profit the advertising agency will receive from future clients due to its public success with the masonry firm. In state H, if the buyer decides not to adopt the advertisement package yet transfers p to the seller, then the buyer gets $-p$ and the seller gets p. In state L, the advertisement package is worthless to both the buyer and the seller; in this case, regardless of whether the buyer adopts the advertisement, the payoffs are simply the players' monetary transfers. In the notation of the general model, we thus have $u(1, \mathrm{H}) = (5, 3)$ and $u(0, \mathrm{H}) = u(1, \mathrm{L}) = u(0, \mathrm{L}) = (0, 0)$. Note that the effort cost c is not included in these expressions.

For this example, I assume that the bargaining weights are $\pi_1 = \pi_2 = 1/2$, so the players share equally any gains from renegotiation. Assume that $c \in (0, 8)$, which implies that "exert" is the efficient effort decision at Date 2 (leading to the high state in which the players obtain a joint value of 8 when the buyer accepts delivery). To have a successful relationship, the parties must design a contract at Date 1 that will align their incentives to invest at Date 2. This critically depends on the set of implementable value functions. In the case in which the buyer makes the effort decision at Date 2, he will exert effort only if the value function satisfies $v_1(\mathrm{H}) - c \geq v_1(\mathrm{L})$. This requires $v_1(\mathrm{H}) - v_1(\mathrm{L})$ to be as large as 8, depending on c. In the other case, where the seller makes the effort decision, the seller has the incentive to exert effort only if $v_2(\mathrm{H}) - c \geq v_2(\mathrm{L})$, requiring $v_2(\mathrm{H}) - v_2(\mathrm{L})$ to be as large as 8.

[7] The buyer must choose his trade action just before the trade show begins, at Date 6. After the trade show, there is no use for the advertisement and there is no way to undo the advertisement if it was adopted.

2 Preliminaries for Mechanism-Design Analysis

This section describes how the contractual relationship can be represented in terms of a standard mechanism-design problem. The form of the mechanism-design problem depends on when renegotiation can occur and on whether one treats trade actions as public actions. I first express outcomes of rational behavior from Date 6 as state-contingent payoffs and I use these to write the players' contracting problem. I then discuss the notion of a "forcing contract" and its relation to the common treatment of trade actions as alienable. Finally, I define some notation for describing renegotiation.

Outcomes of the Trade and Enforcement Phase

It is useful to consider the state-contingent payoff vectors that can be achieved from the beginning of Date 6 (the "trade and enforcement phase" shown in Figure 1), for a fixed message profile m. The set of achievable state-contingent payoff vectors is clearly independent of m, because the message is not payoff-relevant. Thus, for the sake of calculating feasible state-contingent payoffs from Date 6, I can ignore m and write the externally enforced transfer function as $\hat{y}\colon A \to \mathbf{R}^2$, where $\hat{y} \equiv y(m, \cdot)$.

Given the state θ, $\hat{y}$ defines a *trading game*, where the space of action profiles is A and the payoffs are given by $u(\cdot, \theta) + \hat{y}(\cdot)$. I focus on pure-strategy Nash equilibria of the trading game. Let $\hat{a}(\theta)$ denote the equilibrium action profile that is chosen by the players in state θ. The state-contingent payoff vector from Date 6 is thus given by the function $w\colon \Theta \to \mathbf{R}^2$ defined by

$$w(\theta) \equiv u(\hat{a}(\theta), \theta) + \hat{y}(\hat{a}(\theta)). \tag{1}$$

I use the term *outcome* for any such function from Θ to $\mathbf{R}^2$. Think of an outcome, therefore, as a state-contingent payoff that results from interaction in the trade and enforcement phase.[8] The set of outcomes is:

$$W \equiv \left\{ w\colon \Theta \to \mathbf{R}^2 \;\middle|\; \text{functions } \hat{y} \text{ and } \hat{a} \text{ exist such that, for every } \theta \in \Theta, \text{ Equation 1} \right.$$
$$\left. \text{holds and } \hat{a}(\theta) \text{ is an equilibrium of the game } \langle A, u(\cdot, \theta) + \hat{y}(\cdot) \rangle \right\}. \tag{2}$$

[8]This should be differentiated from the "trade outcome," which describes the physical trade action and monetary transfer.

Forcing Contracts and the Alienability Issue

Because trade actions are verifiable, the external enforcer can effectively force the players to choose any particular trade action independent of the state. This can be done using a *forcing contract*, which specifies (i) a large transfer from player 1 to player 2 in the event that player 1 does not take his contractually-specified action, and (ii) a large transfer in the other direction if player 2 does not take her contractually-specified action. For instance, in Example 1, the buyer can be forced to adopt the advertisement by a contract that specifies the transfer vector $(-p, p)$ if the buyer adopts and $(-p-6, p+6)$ if the buyer does not adopt, for any given number p. Regardless of the state, the buyer then has a strict incentive to adopt the advertisement. With this contract for a given message profile, the physical outcome from Date 6 will be adoption of the advertisement and a transfer of p from the buyer to the seller.

In general, suppose the players want to force themselves to play action profile a^* and have transfer t^*, regardless of the state.[9] This can be accomplished by specifying $\hat{y}$ as follows. Let L be such that

$$L > \sup_{a,\theta} u_i(a,\theta) - \inf_{a,\theta} u_i(a,\theta)$$

for $i = 1, 2$. To induce a^*, one can define $\hat{y}$ so that: (i) For every $a = (a_i, a_j^*)$ for which $a_i \neq a_i^*$, set $\hat{y}_i(a) \equiv t_i^* - L$ and $\hat{y}_j(a) \equiv t_j^* + L$; and (ii) for every other profile a, set $\hat{y}(a) \equiv t^*$. Then a^* is the only Nash equilibrium of the trading game in every state.

Definition 1: *The transfer function* $\hat{y}$ *is called* **forcing** *if there is a unique Nash equilibrium of the trading game* $\langle A, u(\cdot,\theta) + \hat{y}(\cdot)\rangle$ *and this equilibrium is independent of the state.*

Let W^{F} be the subset of outcomes that can be supported using forcing contracts; that is, W^{F} is defined by Expression 2 with the qualifier that $\hat{y}$ is a forcing contract.

Forcing contracts lie implicitly behind the treatment of verifiable actions as public in much of the related literature. The traditional view is that, because the trade actions are verifiable and can therefore be forced, we might as well assume—for modeling simplicity and elegance—that these actions can be taken out of the players' hands (they are *alienable*). In models that take this approach, both a and t are chosen directly by the external enforcer and, thus, the contracted (physical) outcomes

[9] To achieve public randomization over trade actions using forcing contracts, a public randomization device must be included in the model. This is done in the working paper Watson (2002b). In fact, allowing such randomization does not expand the set of implementable value functions here, except in the case of no renegotiation. We could also assume A is a mixture space, but this implies that the external enforcer can observe how the players randomize.

simply are elements of $A \times \mathbf{R}_0^2$. Researchers then perform standard mechanism-design analysis, presuming that the full scope of implementable values can be achieved by conditioning the outcome on the Date 4 messages.[10] Treating trade actions as public is, in the notation of my model, equivalent to restricting attention to the outcome set W^{F} rather than looking at the actual outcome set W.

The following lemma identifies a useful property of the sets W and W^{F}.

Lemma 1: *W and W^{F} are closed under constant transfers. For example, if $w \in W$ and $\bar{t} \in \mathbf{R}_0^2$, then $w + \bar{t} \in W$ as well.*

Contracted Mechanisms

Holding aside the issue of renegotiation for now, the players' contracting problem can be easily stated as a standard mechanism-design problem. The players' contract specifies a mechanism, which maps messages sent at Date 4 to outcomes induced in the trade and enforcement phase. The revelation principle applies in the following sense. We can restrict attention to direct-revelation mechanisms, each of which is defined by a message space $M \equiv \Theta^2$ and a function $f : M \to W$. With such a mechanism, at Date 4 the parties simultaneously and independently report the state. For any report profile m, the mechanism specifies an element $f(m) \in W$, which then determines the payoffs conditional on the state. We can concentrate on equilibria of the mechanism in which the parties report truthfully. If we wish to treat trade actions as public, and so focus on forcing contracts, we constrain attention to the subset of mechanisms in which f maps M to W^{F}.

Any mechanism (Θ^2, f) can be translated back into the notation of contract in the basic model, with y specified appropriately. For each message profile m, we define $y(m, \cdot) \equiv \hat{y}(\cdot)$, where $\hat{y}$ is a transfer function that supports $w = f(m)$ in Expression 2.

Renegotiation

Contract renegotiation at Dates 3 and 5 can be viewed as an opportunity for the players to discard their originally specified f mapping and replace it with another mapping f'. I assume the players divide the renegotiation surplus according to fixed bargaining weights π_1 and π_2. The generalized Nash bargaining solution and several other common bargaining solutions have this representation.

To state the bargaining solution more precisely, I let $\gamma(\theta)$ denote the maximal joint payoff that can be obtained in state θ:

$$\gamma(\theta) \equiv \max_{a \in A} [u_1(a, \theta) + u_2(a, \theta)]. \tag{3}$$

[10] Most models in the mechanism design and contract theory literature implicitly associate verifiability with forcing contracts. Some game theory models, such as that of Bernheim and Whinston (1998), also take this view.

Clearly, we have

$$\gamma(\theta) = \max_{w \in W^{\mathrm{F}}} [w_1(\theta) + w_2(\theta)] \tag{4}$$

because the trade action that solves the maximization problem in Equation 3 can be specified in a forcing contract to yield the outcome that solves the problem in Equation 4. An outcome w is called *efficient in state* θ if $w_1(\theta) + w_2(\theta) = \gamma(\theta)$.

Suppose the original mechanism (M, f) would lead to outcome w in state θ. If w is inefficient in state θ, then the players have a joint incentive to renegotiate the mechanism. The *renegotiation surplus* is

$$r(w, \theta) \equiv \gamma(\theta) - w_1(\theta) - w_2(\theta).$$

The players will select a new mapping f' that induces an efficient outcome. Further, the surplus will be divided according to the players' bargaining weights, so that player i obtains $w_i(\theta) + \pi_i r(w, \theta)$. In practical terms, when the players renegotiate in state θ, they replace the transfer function with one that achieves an outcome w' satisfying $w'(\theta) = w(\theta) + \pi r(w, \theta)$. Equation 4 and Lemma 1 imply that such an outcome w' exists and is supported by a forcing contract.

3 Implementation Conditions

In this section, I define and characterize the set of implementable value functions.[11] I group the analysis into three categories, distinguished by whether the players have the opportunity to renegotiate at Dates 3 and 5. The characterization lemmas in this section are all straightforward variations of well-known theorems from the contract theory literature—in particular, due to Maskin (1999), Maskin and Moore (1999), and Moore and Repullo (1988).

No Renegotiation

First consider the setting in which the players cannot renegotiate. A mechanism (M, f) implies, for each state θ, a *message game* in which the players engage at Date 4. The message game has action profiles given by M and payoffs specified by $f(\cdot)(\theta)$. For this setting, implementability is defined as follows.

Definition 2: *A mechanism (M, f) is said to* **implement** *value function v if $f : M \to W$ and, for each state θ, there is an equilibrium of the message game that leads*

[11] I focus on "implementation" in the weak sense of not requiring uniqueness of equilibrium in each state. I find this a reasonable notion for contractual settings. Regardless of your view about this, however, much of my analysis concerns settings with "ex post renegotiation," in which multiplicity is not a problem. Furthermore the multiplicity issue should be tackled with a theory of the self-enforced component of contract.

to the payoff vector $v(\theta)$. Value function v is said to be **implementable** *if there is a mechanism that implements it.*

Let V^{N} be the set of implementable value functions for the setting in which the players cannot renegotiate.

To characterize the set V^{N}, we invoke the revelation principle to focus on truthful reporting in direct-revelation mechanisms so that, in states θ and θ', the players will send message profiles (θ, θ) and (θ', θ'), respectively, in equilibrium. It is essential that the outcome specified for message profile (θ', θ) be sufficient to simultaneously (i) dissuade player 1 from declaring the state to be θ' when the state is actually θ and (ii) discourage player 2 from declaring "θ" in state θ'. Thus, letting w and w' denote the outcomes specified for messages (θ, θ) and (θ', θ'), respectively, implementation relies on the existence of an outcome $\hat{w}$ satisfying $w_1(\theta) \geq \hat{w}_1(\theta)$ and $w_2(\theta') \geq \hat{w}_2(\theta')$. Combining this with Lemma 1 yields the following characterization.

Lemma 2: *Value function v is an element of V^{N} if and only if (i) for every $\theta \in \Theta$, there is an outcome $w \in W$ such that $w(\theta) = v(\theta)$; and (ii) for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $\hat{w} \in W$ such that $v_1(\theta) + v_2(\theta') \geq \hat{w}_1(\theta) + \hat{w}_2(\theta')$. Also, the set V^{N} is closed under constant transfers.*

When players cannot renegotiate, there is no loss of generality in modeling trade actions as public, as the next lemma confirms.

Lemma 3: *If value function v is implementable, then there is a mechanism (M, f) that implements v and has the property that $f(m) \in W^{\mathrm{F}}$ for every $m \in M$.*

The intuition behind this lemma is standard. Any strategic elements in the actual trading game can be mimicked through the use of messages. The mechanism can be designed so that the players announce what trade actions they want to select and then the external enforcer forces them to take these actions.

Interim Renegotiation

Next consider the setting in which renegotiation is possible at Date 3 but not at Date 5. In other words, the players can renegotiate between the time that they jointly learn the state and when the message game is played. I call this the *interim renegotiation* setting. The players will renegotiate if, in the realized state, their anticipated equilibrium of the message game would yield an inefficient outcome. Thus, if the players' original contract would implement v' without renegotiation, then it leads to payoff vector $v'(\theta) + \pi r(v', \theta)$ in state θ with interim renegotiation.

Definition 3: *Value function v is* **implementable with interim renegotiation** *if there is a value function $v' \in V^{\mathrm{N}}$ such that $v(\theta) = v'(\theta) + \pi r(v', \theta)$ for every state θ.*

Let V^{I} denote the set of implementable value functions when there is interim renegotiation.

Lemma 4: *$v \in V^{\mathrm{I}}$ if and only if $v \in V^{\mathrm{N}}$ and $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ for every $\theta \in \Theta$. Also, V^{I} is closed under constant transfers.*

The "mechanism design with interim renegotiation" (MDIR) program studies the set V^{I}. In this program, as with the case of no renegotiation, there is no loss of generality in modeling trade actions as public because Lemma 3 applies to V^{I}.

Ex Post Renegotiation

Finally, consider the case in which renegotiation is possible at Date 5—between the time the players send messages and the beginning of the trade and enforcement phase. The idea is that the players interact in the contracted mechanism, which leads to an outcome w. But then, just before the outcome would be induced, the players can renegotiate to obtain a different outcome. This is the setting of *ex post renegotiation.* Here, renegotiation implies efficient outcomes in every state and after every message profile in the mechanism. Incidentally, with renegotiation possible at Date 5, implementability is not affected by whether renegotiation can also occur at Date 3.[12]

To characterize implementability for this setting, we must incorporate renegotiation into the definition of an outcome. The set of *ex post renegotiation outcomes* is defined as

$$Z \equiv \Big\{ z : \Theta \to \mathbf{R}^2 \;\Big|\; \text{There is an outcome } w \in W \text{ such that } z(\theta) = w(\theta) + \pi r(w,\theta) \text{ for every } \theta \in \Theta \Big\}.$$

An ex post renegotiation outcome is a state-contingent payoff vector that results when, in every state, the players renegotiate from an outcome in W. Note that all elements of Z are efficient in every state. One can analyze mechanism design in the setting of ex post renegotiation by simply replacing W with Z in definition 2.

Definition 4: *Value function v is* **implementable with ex post renegotiation** *if it is implemented by a mechanism (M, f) with $f : M \to Z$.*

Letting V^{EP} denote the set of implementable value functions when there is ex post renegotiation, we have:

[12] Renegotiation at Date 5 implies ex post efficiency in both states, which means there is no surplus to be obtained from earlier renegotiation.

Lemma 5: $v \in V^{\mathrm{EP}}$ *if and only if (i)* $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ *for every* $\theta \in \Theta$*; and (ii) for every pair of states* $\theta, \theta' \in \Theta$*, there is an outcome* $\hat{z} \in Z$ *such that* $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$*. Also,* V^{EP} *is closed under constant transfers.*

In the popular "mechanism design with ex post renegotiation" (MDER) program, trade actions are treated as public. Thus, the MDER program limits attention to forcing contracts, where the set of ex post renegotiation outcomes is calculated as

$$Z^{\mathrm{F}} \equiv \left\{ z : \Theta \to \mathbf{R}^2 \;\middle|\; \text{There is an outcome } w \in W^{\mathrm{F}} \text{ such that } z(\theta) = w(\theta) + \pi r(w, \theta) \text{ for every } \theta \in \Theta \right\}.$$

Definition 5: *Value function* v *is* **implementable with ex post renegotiation and a forcing contract** *if it is implemented by a mechanism* (M, f) *with* $f : M \to Z^{\mathrm{F}}$*.*

Let V^{EPF} be the value functions that are implementable with ex post renegotiation and forcing contracts. The MDER program studies precisely the set V^{EPF}.

Lemma 6: $v \in V^{\mathrm{EPF}}$ *if and only if (i)* $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ *for every* $\theta \in \Theta$*; and (ii) for every pair of states* $\theta, \theta' \in \Theta$*, there is an outcome* $\hat{z} \in Z^{\mathrm{F}}$ *such that* $v_1(\theta) + v_2(\theta') \geq \hat{z}_1(\theta) + \hat{z}_2(\theta')$*. Also,* V^{EPF} *is closed under constant transfers.*

4 The Main Point Via Example 1

Example 1 demonstrates the importance of accounting for the technology of trade. In this section, I calculate the sets V^{I}, V^{EP}, and V^{EPF} for the example. I show that the MDER program does not accurately describe the scope of contracting with ex post renegotiation.

MDIR Program

I first perform the analysis of the example for the setting of interim renegotiation. Without loss, I constrain attention to direct-revelation mechanisms and truthful reporting in equilibrium (by the revelation principle) and forcing contracts (by Lemma 3). The calculations are a simple application of Lemmas 2 and 4. Note that, because renegotiation only occurs before the message phase, the contracted mechanism may lead to an ex post inefficient outcome in some state, for some message profiles. However, an ex post efficient outcome occurs in equilibrium. Thus, to incorporate renegotiation, we must specify "adoption of the advertisement package" when (H, H) is the message profile. We can further limit attention to mechanisms that specify no adoption when message profiles (H, L) and (L, H) are sent, because this makes for the most relaxed incentive constraints. Let $p^{\theta\theta'}$ denote the monetary transfer from player 1 to player 2 that is specified by the mechanism for message profile (θ, θ'), for $\theta, \theta' \in \Theta$.

The game form implies a message game for each state, as pictured below.

B \ S	H	L
H	$-p^{HH},\ p^{HH}$	$-p^{HL},\ p^{HL}$
L	$-p^{LH},\ p^{LH}$	$-p^{LL},\ p^{LL}$

Message game in state L

B \ S	H	L
H	$5-p^{HH},\ 3+p^{HH}$	$-p^{HL},\ p^{HL}$
L	$-p^{LH},\ p^{LH}$	$5-p^{LL},\ 3+p^{LL}$

Message game in state H

We look for equilibria with truthful reporting. For truthful reporting to be a Nash equilibrium in each state, it must be that $p^{LH} \leq p^{LL} \leq p^{HL}$, $5 - p^{HH} \geq -p^{LH}$, and $3 + p^{HH} \geq p^{HL}$. Combining these inequalities yields

$$p^{LL} + 5 \geq p^{HH} \geq p^{LL} - 3,$$

which implies that the set of implementable value functions in the MDIR setting is:

$$V^{\mathrm{I}} = \left\{ v\colon \{\mathrm{H},\mathrm{L}\} \to \mathbf{R}^2 \;\middle|\; v(\mathrm{L}) = (\alpha, -\alpha),\ v(\mathrm{H}) = (5 + \alpha - \beta, 3 - \alpha + \beta), \right.$$
$$\left. \text{for any } \alpha \in \mathbf{R} \text{ and } \beta \in [-3, 5] \right\}. \tag{5}$$

Recall that, in this example, the effort cost c is not included in the value function. The maximal difference $v_1(\mathrm{H}) - v_1(\mathrm{L})$ is 8, which is sufficient to give the buyer the incentive to exert effort in the version of the example in which he makes the effort decision. Likewise, the maximal difference $v_2(\mathrm{H}) - v_2(\mathrm{L})$ is also 8, which means the seller can be motivated to exert effort in the version in which she makes the effort decision.

MDER Program

I next turn to the setting of ex post renegotiation and forcing contracts. This is the standard program of mechanism design with ex post renegotiation. We can assume that the mechanism specifies "adoption of the advertisement package" when the report profile is (H, H) and when it is (L, L).[13] Furthermore, it is easy to verify

[13]Any incentive-compatible mechanism that specifies "no adoption" when the report profile is (H, H) will be renegotiated in the H state. One can alter the mechanism so that the renegotiated outcome is specified for (H, H), without affecting the incentive conditions. This is the "renegotiation-proofness principle" (Dewatripont 1989, Hart and Tirole 1988, Laffont and Tirole 1990, Brennan and Watson 2001) in action.

that the incentive constraints are most relaxed if "no adoption" is specified for report profile (L,H) and "adoption" is specified for profile (H,L). Note that the mechanism would be renegotiated in state H in the off-equilibrium case in which the buyer reports L while the seller reports H. Incorporating the renegotiation activity, a game form implies the following message games in the two states.

B \ S	H	L
H	$-p^{HH},\ p^{HH}$	$-p^{HL},\ p^{HL}$
L	$-p^{LH},\ p^{LH}$	$-p^{LL},\ p^{LL}$

Message game in state L

B \ S	H	L
H	$5-p^{HH},\ 3+p^{HH}$	$5-p^{HL},\ 3+p^{HL}$
L	$4-p^{LH},\ 4+p^{LH}$	$5-p^{LL},\ 3+p^{LL}$

Message game in state H

As in the previous subsection, $p^{\theta\theta'}$ denotes the transfer from the buyer to the seller that is specified for message profile (θ,θ').

For truthful reporting to be a Nash equilibrium in each state, it must be that $p^{LH} \leq p^{LL} \leq p^{HL}$, $5 - p^{HH} \geq 4 - p^{LH}$, and $3 + p^{HH} \geq 3 + p^{HL}$. Combining these inequalities yields

$$p^{LL} + 1 \geq p^{HH} \geq p^{LL}.$$

The set of implementable value functions for the MDER program is thus:

$$V^{\mathrm{EPF}} = \left\{ v\colon\{\mathrm{H},\mathrm{L}\} \to \mathbf{R}^2 \;\middle|\; v(\mathrm{L}) = (\alpha,-\alpha),\ v(\mathrm{H}) = (5+\alpha-\beta, 3-\alpha+\beta), \text{ for any } \alpha \in \mathbf{R} \text{ and } \beta \in [0,1] \right\}. \tag{6}$$

Note that the opportunity to renegotiate at Date 5, specifically following out-of-equilibrium message profiles, causes a refinement in the set of implementable values relative to the case of interim renegotiation. In both the seller-effort and buyer-effort versions of the example, there are values of c for which efficiency requires that effort be exerted yet there is no mechanism that induces this goal.

Trade Actions as Options

I next show that, with ex post renegotiation, the set of implementable value functions significantly expands when parties depart from forcing contracts and, instead, use trade actions as options. Suppose that at Date 1 the parties write the following contract: If the buyer adopts the advertisement, then he must pay $p' + \beta$ to the seller; if the buyer does not adopt, then he pays p'; further, the external enforcer is instructed to ignore messages sent at Date 4. For $\beta \in (0,5)$, this is not a forcing contract—that is, it neither compels the buyer to adopt the advertisement in both states, nor compels the buyer to *not* adopt the advertisement in both states. Instead, this is an option contract, but one that uses the buyer's *trade decision*, rather than the buyer's message, as the way to exercise the option. With $\beta \in [0,5]$, the buyer has the incentive to adopt the advertisement in state H and not to adopt in state L.

From Date 6, this contract yields a payoff vector of $(5-p'-\beta, 3+p'+\beta)$ in state H and $(-p', p')$ in state L. Because the contract leads to the efficient trade action in each state, it would not be renegotiated at either Date 5 or Date 3. The contract thus implements value $(5-p'-\beta, 3+p'+\beta)$ in state H and $(-p', p')$ in state L.

Clearly, by using the trade decision as an option, the parties are able to reduce the detrimental effect of renegotiation at Date 5. Because the trading opportunity is nondurable, there is no way for the parties to reverse it through renegotiation after Date 6. The parties could use a more complicated contract that involves transfers contingent on both trade actions and messages. However, in this example, more complicated contracts cannot improve on the scope of the simple option scheme described in the preceding paragraph.[14] Thus, the set of implementable value functions in the case of ex post renegotiation is:

$$V^{\mathrm{EP}} = \Big\{ v : \{\mathrm{H},\mathrm{L}\} \to \mathbf{R}^2 \;\Big|\; v(\mathrm{L}) = (\alpha, -\alpha),\ v(\mathrm{H}) = (5+\alpha-\beta, 3-\alpha+\beta), \text{ for any } \alpha \in \mathbf{R} \text{ and } \beta \in [0,5] \Big\}. \tag{7}$$

With ex post renegotiation, the supported range of β is sufficient to give the seller the incentive to exert effort when it is efficient to do so. To see this, note that $v_2(\mathrm{H}) - v_2(\mathrm{L}) = 8$ when one selects $\beta = 5$. On the other hand, in the version of the model in which the buyer makes the investment, there are still values of c under which the buyer cannot be given the incentive to exert effort.

Insights from the Example

Note that, in the case of ex post renegotiation, there is a discrepancy between the set of implementable value functions and the *strictly smaller* set identified by the MDER program. The MDER program misses how trade decisions can be used as options, precisely because the MDER program treats trade actions as part of the abstract, public "outcome." For this reason, we should reject the MDER program as incorporating implicit assumptions about contractual incompleteness. We should instead focus on structured models that incorporate the technology of trade and enforcement, and, where appropriate, on the MDIR program.

In response to my point, a mechanism design theorist might be inclined to conclude that I am mis-applying the MDER program. The theorist would argue that, if we

[14] One can easily verify this by considering message games as in the previous subsections and examining non-forcing transfer functions for message profiles (H,L) and/or (L,H).

think the trading opportunity is nondurable (and so trade decisions can be used as options without being reversed by renegotiation), then the set of implementable values is actually characterized by the *MDIR* program. Segal and Whinston (2002) and others take this position. However, the argument is flawed because, clearly, the MDIR program actually does *not* characterize the set of implementable value functions in the setting of ex post renegotiation (compare Expressions 5 and 7).

The key issue is that, in designing option contracts, the fixed technology of trade is not as flexible as are messages. Thus, neither the MDER nor the MDIR programs accurately model Example 1 in the setting of ex post renegotiation. The renegotiation opportunity, in a sense, occurs "in the middle of the mechanism." To analyze this renegotiation opportunity, one must examine the structured model that explicitly accounts for the technology of trade and external enforcement. In fact, mechanism design methodology is applicable, but it relies on precise modeling of the technology of trade. In particular, one cannot view the "outcome" as merely a specification of the transfer and the trade action. Rather, an "outcome" must indicate the equilibrium trade actions and transfers that result in the different states, given the technology of trade and the instructions for the external enforcer.

Some researchers may claim that the MDER program accurately describes the scope of contracting in settings with renegotiation and *durable* trading opportunities, where parties always have the opportunity to reverse a trade decision. The analysis in Watson (2003) shows that this, too, is an erroneous line of thinking. Watson (2003) studies an infinite-period version of the model herein, where trade actions chosen in one period can be reversed in future periods. With durability and reversibility, stationary contracts are shown to be optimal. An implication is that, when forcing contracts are used, the problem of contracting in an infinite-period relationship is equivalent to the problem studied here. To justify use of the MDER program, one must assume that long-term contracts cannot be written.

5 General Inclusion Results

The following result generalizes the weak inclusion relations that Example 1 exhibits.

Theorem 1: $V^{\mathrm{EPF}} \subset V^{\mathrm{EP}} \subset V^{\mathrm{I}} \subset V^{\mathrm{N}}$.

Proof: The relation $V^{\mathrm{EPF}} \subset V^{\mathrm{EP}}$ follows from Lemmas 5 and 6. By the definition of Z, we see that $Z \subset W$, so condition (ii) in Lemma 5 implies condition (ii) in Lemma 2. Further, condition (i) in Lemma 5 implies condition (i) in Lemma 2, because the maximum joint value exists in every state. Thus, the conditions of Lemma 5 imply those of Lemma 4 and, as a result, $V^{\mathrm{EP}} \subset V^{\mathrm{I}}$. Finally, $V^{\mathrm{I}} \subset V^{\mathrm{N}}$ is clear from Lemma 4. *Q.E.D.*

I next address the question of under what conditions the inclusion relations are strict. First consider the issue of whether it is appropriate to focus on forcing contracts in settings with ex post renegotiation. There is no loss in limiting attention to forcing contracts if and only if $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$, in which case I call the MDER program *justified.*

Theorem 2: $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ *if and only if, for every pair of states* $\theta, \theta' \in \Theta$ *and every* $\hat{z} \in Z$*, there is an ex post renegotiation outcome* $\tilde{z} \in Z^{\mathrm{F}}$ *such that* $\tilde{z}_1(\theta) + \tilde{z}_2(\theta') \leq \hat{z}_1(\theta) + \hat{z}_2(\theta')$*.*

Proof: Under the hypothesis of the theorem, condition (ii) of Lemma 6 implies condition (ii) of Lemma 5, proving $V^{\mathrm{EP}} \subset V^{\mathrm{EPF}}$. This and Theorem 1 yield the result. *Q.E.D.*

The intuition behind this result concerns the punishment outcome $\hat{z}$ that is specified for a particular message profile (θ, θ') with $\theta \neq \theta'$. This outcome must deter player 1 from declaring "θ" in state θ' and it must deter player 2 from declaring "θ'" in state θ. The total *punishment level* is $\hat{z}_1(\theta) + \hat{z}_2(\theta')$. Lower punishment levels support a greater range of value functions.

The sets V^{EP} and V^{I} can be compared in a similar way.

Theorem 3: $V^{\mathrm{EP}} = V^{\mathrm{I}}$ *if and only if, for all* $\theta, \theta' \in \Theta$ *and every* $\hat{w} \in W^{\mathrm{F}}$*, there is an ex post renegotiation outcome* $\hat{z} \in Z$ *such that* $\hat{z}_1(\theta) + \hat{z}_2(\theta') \leq \hat{w}_1(\theta) + \hat{w}_2(\theta')$*.*

Proof: By Lemmas 3 and 4, we can assume that $\hat{w} \in W^{\mathrm{F}}$ in the interim-renegotiation implementation condition for any given message profile (θ, θ') with $\theta \neq \theta'$. Then, under the hypothesis of the theorem, the conditions for implementation with ex post renegotiation (Lemma 5) imply the conditions for implementation with interim renegotiation (Lemma 4), proving $V^{\mathrm{I}} \subset V^{\mathrm{EP}}$. This and Theorem 1 yield the result. *Q.E.D.*

Results For a Class of Trade Technologies

I elaborate on Theorems 2 and 3 by considering the general class of trading environments in which player 1 alone makes the trade decision (player 2's trade action is trivial). I make several assumptions that put some structure on the technology.

Assumption 1: $A_2 = \emptyset$, so $A = A_1$. Also, A and Θ are compact subsets of $\mathbf{R}$, and $u_1(\cdot, \theta)$ and $u_2(\cdot, \theta)$ are continuous functions of a for every $\theta \in \Theta$.

Define $U(a, \theta) \equiv u_1(a, \theta) + u_2(a, \theta)$. Define $\underline{a} \equiv \min A$, $\overline{a} \equiv \max A$, $\underline{\theta} \equiv \min \Theta$, $\overline{\theta} \equiv \max \Theta$. Assumption 1 guarantees that these exist and that $\max_{a \in A} U(a, \theta)$ exists.

Assumption 2: $U(\cdot, \theta)$ is quasiconcave and has a unique maximizer, for every $\theta \in \Theta$.

Define $a^*(\theta) \equiv \text{argmax}_{a \in A} U(a, \theta)$.

Assumption 3: u_1 is strictly supermodular, meaning that $u_1(a, \theta) - u_1(a', \theta) > u_1(a, \theta') - u_1(a', \theta')$ whenever $a > a'$ and $\theta > \theta'$.

Assumption 4: There is a state $\theta^* > \underline{\theta}$ such that $a^*(\theta^*) > \underline{a}$.

Assumption 5: Player 1's bargaining weight is positive: $\pi_1 > 0$.

Assumption 2 means that U is increasing below, and decreasing above, its maximum. Assumption 3 means that, without considering transfers, player 1's marginal value of increasing his trade action rises with the state; that is, higher trade actions are more attractive to him as the state increases. In words, Assumption 4 says that there is some non-minimal state in which the efficient trade action is greater than the minimum action. A variation on Assumption 4 is:

Assumption 4′: There is a state $\theta^* < \overline{\theta}$ such that $a^*(\theta^*) > \underline{a}$.

I consider these technical assumptions to be quite weak, in that many interesting examples satisfy them. Example 1 satisfies all of these assumptions, as does Example 2 in the next section. More generally, a wide range of buyer/seller relationships fit in. Specifically, suppose a is the number of units of an intermediate good to be transferred from the seller to the buyer. The buyer's benefit of obtaining a units in state θ is $B(a, \theta)$; the seller's cost of production and delivery is $C(a, \theta)$. Suppose that B is increasing and concave in a and that C is decreasing and concave in a. If choosing a is the buyer's decision (he selects how many units to install, for example), then $u_1 \equiv B$ and $u_2 \equiv C$. If the seller chooses a (she decides how many units to deliver, say), then $u_1 \equiv C$ and $u_2 \equiv B$. In either case, Assumptions 1 and 2 are satisfied. Assumption 3 adds the weak monotonicity requirement on the payoff of the player who selects a. Assumptions 4 and 4′ require that it is efficient to trade a positive amount in some non-minimal state and in some non-maximal state.

Theorem 4: *Under Assumptions 1-5, $V^{\text{EPF}} \neq V^{\text{EP}}$ and so the MDER program is not justified. Furthermore, under Assumptions 1-3 and 4′, $V^{\text{EP}} \neq V^{\text{I}}$.*

The proof, which is rather involved yet instructive, is in Appendix B. In the first part of the proof, I show how the punishment level $\hat{z}_1(\theta) + \hat{z}_2(\theta')$ can be written as

$$u_1(a, \theta) + \pi_1 R_1 + u_2(a', \theta') + \pi_2 R_2 + \sigma,$$

where (i) a and a' are the trade actions that player 1 is induced to select in states θ and θ', respectively; (ii) R_1 and R_2 are the renegotiation surpluses in these states and given these actions; and (iii) σ is a "slack" term on the difference in transfers between the states that the equilibrium conditions imply (so σ is a function of the states and actions). The slack term is zero in a forcing contract, where $a = a'$. I show that, under the assumptions, one can depart from $a = a'$ in a particular way so that the slack term will decrease more than the sum of the other terms increases, proving the first claim of the theorem via Theorem 2. The second claim is proved using the same expression for the punishment level and invokes Theorem 3.[15]

6 Two More Examples

To further demonstrate how the proper accounting of the technology of trade improves our understanding of contractual imperfections, I present two more examples.

Example 2: Efficient Cooperative Investments

Consider a contractual setting with what Che and Hausch (1999) call "cooperative investments" (more descriptively, *cross investments*). A buyer (player 1) and a seller (player 2) contract to trade one unit of an intermediate good. At Date 2, the seller makes an investment $\theta \in [0, \overline{\theta}]$, at immediate cost θ, which enhances the buyer's value of trade, denoted B. The trade action at Date 6 is either "trade" ($a = 1$) or "not trade" ($a = 0$). There are two versions of the model; in one version, the buyer makes the trade decision, whereas, in the other version, the seller makes this decision. Payoffs are defined by $u(0, \theta) = (0, 0)$ and $u(1, \theta) = (B(\theta), -\varepsilon[\overline{\theta} - \theta])$ for every θ, where ε is a small positive number. The term $\varepsilon[\overline{\theta} - \theta]$ is the seller's delivery cost and is decreasing in θ. Note that I do not include the seller's investment cost in the specification of u. I assume that B is differentiable and concave, with $B'(\theta) > \varepsilon[\overline{\theta} - \theta]$, $B'(0) > 1$, and $B(\overline{\theta}) < 1$. In this example, trade is always ex post efficient. The efficient investment θ^* solves $\max_{\theta \in [0, \overline{\theta}]} B(\theta) - (1 - \varepsilon)\theta$ and is interior. I consider the setting of ex post renegotiation and assume that the players have equal bargaining weights.

Che and Hausch (1999) study the forcing contract set V^{EPF} and they show that, in settings with cross investments, the hold-up problem severely restricts implementability and leads to inefficiently low investment. In fact, for the model here with ε sufficiently close to zero, they prove that the "null contract"—forcing no trade, regardless

[15] The assumptions can be relaxed to some extent at the cost of added complexity and logical steps. For example, one can easily extend the result to more a general setting in which A and Θ are unbounded. Also, we do not have to assume that Θ is a subset of $\mathbf{R}$. The arguments in the proof go through without modification if there is a subset of Θ that is isomorphic to a compact subset of $\mathbf{R}$, such that the assumptions are satisfied using the mapping from $\mathbf{R}$ to Θ. We can also dispense with the assumption that U has a unique maximizer.

of the messages—is best. Through renegotiation, it gives the seller the incentive to invest at the inefficient level that solves $\max_{\theta \in [0,\overline{\theta}]} B(\theta)/2 - \varepsilon[\overline{\theta} - \theta]/2 - \theta$.

Explicitly accounting for the technology of trade, however, reveals that Che and Hausch's conclusions are very sensitive to the trade technology. Consider the version of the example in which the *buyer* makes the trade decision and let value function v^* be defined by $v_2^*(\theta) = B(\theta^*) - \varepsilon[\overline{\theta} - \theta^*]$ for all $\theta \geq \theta^*$ and $v_2^*(\theta) = B(\theta)/2 - \varepsilon[\overline{\theta} - \theta]/2$ for $\theta < \theta^*$. The following mechanism implements v^* with ex post renegotiation. The parties direct the external enforcer to ignore messages and to compel a transfer of $B(\theta^*)$ from the buyer to the seller if and only if the buyer accepts delivery; otherwise, there is no transfer. Clearly, with this contract, the buyer will accept delivery if and only if $\theta \geq \theta^*$. If the seller selects $\theta \geq \theta^*$ at Date 2, then the parties would not renegotiate; on the other hand, the parties would renegotiate and split the surplus of $B(\theta) - \varepsilon[\overline{\theta} - \theta]$ if the seller chooses $\theta < \theta^*$. Thus, v^* is implemented. At Date 2 the seller solves $\max_{\theta \in [0,\overline{\theta}]} v_2^*(\theta) - \theta$, and the solution is the efficient investment θ^*.

In the version of the example in which the *seller* makes the trade decision, Che and Hausch's conclusions are valid. To see this, note that player 2's incentive to invest is heightened by making $v_2(\theta) - v_2(\theta')$ large for $\theta > \theta'$, which means $v_1(\theta) + v_2(\theta')$ is low. With reference to the condition of Lemma 5, we thus want the punishment level $\hat{z}_1(\theta) + \hat{z}_2(\theta')$ to be small. In a non-forcing contract, the seller can be given the incentive to trade in state θ and not trade in state θ' if and only if $\theta \geq \theta'$. However, it is not difficult to verify that such a contract produces a higher level of $\hat{z}_1(\theta) + \hat{z}_2(\theta')$ than does the contract that forces no trade.

This example satisfies Assumptions 1-5 and $4'$ in the previous section, which implies that $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}} \neq V^{\mathrm{I}}$. That Che and Hausch's conclusions hold in one version of the example is due to the nature of investment (that the seller makes it). In other words, the added implementability afforded by V^{EP} relative to V^{EPF} does not help increase the seller's incentive to invest in the particular case in which the seller makes the trade decision.

Example 3: Complexity and Hold-up

Next consider an example along the lines of Segal (1999) and Hart and Moore (1999). This example will reiterate these authors' main point—that hold-up problems can exist in cases of "complex, pure self investment"—and show that the insight is still valid when one properly accounts for the technology of trade. A buyer (player 1) and a seller (player 2) contract to trade one unit of an intermediate good. As with the previous example, the state represents the seller's Date 2 investment. The seller can either invest "high," which yields state H, or invest "low," yielding state L. The high investment entails a cost c, which is immediately paid by the seller and is not included in the u specification below. Low investment is costless.

The buyer makes the trade decision at Date 6, which is either a^h or a^l. Function

u is defined by: $u(a^h, \mathrm{H}) = (10, 0)$, $u(a^l, \mathrm{H}) = (22, -22)$, $u(a^h, \mathrm{L}) = (0, 0)$, and $u(a^l, \mathrm{L}) = (10, -8)$. This example exhibits pure self investment in that, if the optimal trade action is chosen (a^h in state H, a^l in state L) then the seller's investment would affect only his own cost (0 in state H, 8 in state L). However, the environment is complex because the seller's investment affects the buyer's value when the suboptimal trade decision is made. Assume that $c \in (0, 8)$, which means that high investment is efficient. Also assume that the players can renegotiate ex post and that the buyer has all of the bargaining power during renegotiation.

The analysis of forcing contracts runs as follows. By the revelation principle and the renegotiation-proofness principle, one can focus on contracts that force a^h at price p^H when the message profile is (H,H), and force a^l at price p^L when the message profile is (L,L). The contract specifies either a^h or a^l when the message profile is (L,H)—that is, when the buyer reports L and the seller reports H. Consider these cases separately.

First, suppose that the contract forces a^h at price $\hat{p}$ when the message profile is (L,H). For truthful reporting to be an equilibrium of the message game in both states, it must be that player 1 has no incentive to send message L in state H and player 2 has no incentive to send message H in state L. If player 1 deviates in state H, then there would be no renegotiation (because a^h is still specified). Player 1 thus reports truthfully in state H if and only if

$$10 - p^H \geq 10 - \hat{p}.$$

If player 2 deviates in state L, then the players would renegotiate the contractually-specified trade action, but player 1 would get all of the surplus. Player 2 thus reports truthfully in state L if and only if

$$p^L - 8 \geq \hat{p}.$$

Combining the two inequalities yields the constraint $p^L \geq p^H + 8$.

Second, suppose that the contract forces a^l at price $\hat{p}$ when the message profile is (L,H). In this case, renegotiation would occur if player 1 reports L in state H, but renegotiation would not occur if player 2 reports H in state L. Equilibrium conditions for the message game are

$$10 - p^H \geq 22 - \hat{p} + 10$$

and

$$p^L - 8 \geq \hat{p} - 8,$$

which simplify to $p^L \geq p^H + 22$.

Clearly, in both cases player 2's payoff in state L must be at least as high as is his payoff in state H, which means no forcing contract induces the seller to invest high. In fact, with ex post renegotiation, *no* contract (forcing or non-forcing) can induce high investment.

To see why non-forcing contracts cannot improve on forcing contracts in this example, consider the scope of non-forcing contracts. Suppose that, given a particular message profile, the contract specifies a price of $\overline{p}^h$ if the buyer chooses trade action a^h and a price of $\overline{p}^l$ if the buyer chooses trade action a^l. This is necessarily a forcing contract either if $\overline{p}^l - \overline{p}^h > 12$ (in which case player 1 has the incentive to choose a^h in both states) or if $\overline{p}^l - \overline{p}^h < 10$ (in which case player 1 has the incentive to choose a^l in both states). If $\overline{p}^l - \overline{p}^h \in [10, 12]$ then player 1 has the incentive to select a^h in state L and to select a^l in state H. There are no values of $\overline{p}^l$ and $\overline{p}^h$ that give player 1 the incentive to select a^h in state H and a^l in state L.

Thus, there is only one type of non-forcing contractual provision: that which has $\overline{p}^l - \overline{p}^h \in [10, 12]$. This leads to outcome w defined by $w(\mathrm{H}) = (22 - \overline{p}^l, \overline{p}^l - 22)$ and $w(\mathrm{L}) = (-\overline{p}^h, \overline{p}^h)$. The implied ex post renegotiation outcome is z, where $z(\mathrm{H}) = (22 - \overline{p}^l + 10, \overline{p}^l - 22)$ and $z(\mathrm{L}) = (2 - \overline{p}^h, \overline{p}^h)$. If this contractual provision is specified for the message profile (L,H) then equilibrium conditions for the message game are

$$10 - p^H \geq 22 - \overline{p}^l + 10$$

and

$$p^L - 8 \geq \overline{p}^h.$$

Combining these inequalities with $\overline{p}^l - \overline{p}^h \in [10, 12]$, we obtain $p^L \geq p^H + 18$. Again, player 2's payoff in state L must be higher than it is in state H, implying that he does not have the incentive to invest high.

By ordering a^h and a^l so that a^h is "lower" than a^l, this example satisfies Assumptions 1-5, but it does not satisfy Assumption 4′. We have $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}}$, but the difference between these sets has to do with the lower bound of $p^H - p^L$, whereas it is the upper bound of $p^H - p^L$ that is critical for the seller's investment incentive. The upper bound is -8 when there is ex post renegotiation, with both forcing and non-forcing contracts.

7 Conclusion

I have demonstrated that, to appropriately study institutional constraints, the analysis of contract must start with an understanding of the technology of trade. When parties can renegotiate just before making trade decisions, this technology greatly affects implementability. Thus, researchers should take a structured, game-theoretic approach to studying contract and enforcement. One must be clear about exactly what is verifiable, the nature and timing of inalienable actions, and how external enforcement occurs.

My analysis here has implications for the applicability of popular mechanism design models. Some theorists, including Segal and Whinston (2002), have stated that future work in applied contract theory will be geared toward discovering whether it is the MDER or MDIR program that is the "right" model of any particular setting. I find that this objective is in error. The MDER program under-represents the true scope of implementability. Contracting parties can often overcome contractual imperfections—to motivate efficient cross investment, for example—even when they have the opportunity to renegotiate just before making trade decisions. Finally, I emphasize that the mechanism design methodology is still applicable and useful, as long as one correctly defines the outcome set. Unfortunately, differential methods may be less applicable than is currently thought, because the trading game does not generally exhibit the constant-sum condition that underlies the analysis of Segal and Whinston (2002).

I conclude that the MDER program makes hidden assumptions of contractual incompleteness. It can be justified on the basis of a restriction to forcing contracts or, as noted in Watson (2003) in the context of durable trading opportunities, a restriction to short-term contracts. I can think of no ready defense of these incompleteness assumptions, just as I see no reason to believe that parties would limit themselves to forcing or short-term contracts.[16]

My analysis also has practical relevance. In the least, it should remind us that, to some extent, messages are a theoretical construct. While we sometimes do observe contracts that require parties to send verifiable messages (in real estate transactions, for example), we also often see option contracts that merely specify transfers on the basis of productive actions.

There are myriad promising opportunities for fruitful research on contracting with institutional constraints. Given the importance of the technology of trade, further analysis of specific technologies and their relation to unverifiable activity are in order. This will provide a theoretical base for empirical analysis of contractual form

[16] My message can be further illustrated in the context of Edlin and Hermalin's (2000) debate with Nöldeke, G. and K. Schmidt (1998). In their discussion of whether a party could let an option expire and then renegotiate from scratch, Edlin and Hermalin appeal to the "outside option principle," whereby the outside option implies an inequality constraint on the outcome of negotiation rather than serving as the disagreement point. While there are bargaining models that justify treating outside options in this way, these models blur the distinction between verifiable trade actions and noncontractible renegotiation opportunities. If parties can exercise trade-based options in the process of renegotiating, then either trade actions are really not fully verifiable or the opportunity to renegotiate can be partially controlled by the external enforcer (because he can observe a party's actions whenever an option can be exercised—which, for example, would be in every round of an alternating-offer bargaining game). Edlin and Hermalin may have one of these justifications in mind, or they may be thinking of an institutional constraint that limits the time in which an option may be exercised. In any case, I assert that one must model the timing of renegotiation and trade in order to understand exactly what is being assumed. Note that, in my framework, the non-contractible renegotiation opportunity is *separated* in time from the verifiable trade actions, so that a party cannot delay the trading opportunity by refusing to make an agreement at the time of renegotiation. Other modeling approaches may be useful for comparison.

and the implications of contractual imperfections. It will also be instructive to examine settings with partial verifiability, expanding on the analysis of Hart and Moore (1988), and to look at how legal institutions constrain the writing and enforcement of contracts.

A Appendix: Proofs of the Lemmas

Proof of Lemma 1: The result follows from the fact that one can add a constant transfer $\bar{t} \in \mathbf{R}_0^2$ to any given function $\hat{y}$ without altering the players' incentives in the trading game in any state. *Q.E.D.*

Proof of Lemma 2: For any direct-revelation mechanism (Θ^2, f), define $w^{(\theta_1,\theta_2)} \equiv f(\theta_1, \theta_2)$ for all $\theta_1, \theta_2 \in \Theta$. Observe that truthful reporting is a Nash equilibrium if and only if $w_1^{(\theta,\theta)}(\theta) \geq w_1^{(\theta_1,\theta)}(\theta)$ and $w_2^{(\theta,\theta)}(\theta) \geq w_2^{(\theta,\theta_2)}(\theta)$, for every $\theta \in \Theta$ and all $\theta_1, \theta_2 \in \Theta$. Combining this fact with the definition of implementability implies that v is implementable if and only if condition (i) of the lemma holds and, for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $\tilde{w} \in W$ such that $v_1(\theta) \geq \tilde{w}_1(\theta)$ and $v_2(\theta') \geq \tilde{w}_2(\theta')$. Lemma 1 implies that $v_1(\theta) \geq \tilde{w}_1(\theta)$ and $v_2(\theta') \geq \tilde{w}_2(\theta')$ hold for some $\tilde{w} \in W$ if and only if condition (ii) of the lemma is true. That V^{N} is closed under constant transfers also follows from Lemma 1. *Q.E.D.*

Proof of Lemma 3: Suppose that (Θ^2, g) implements v. We can assume that $M = \Theta^2$ by the revelation principle. For every message profile m, write $\hat{a}^m$ and $\hat{y}^m$ as the functions that support $g(m)$ as described in Equation 1. That is, $\hat{y}^m(a)$ is the transfer specified for message m and trade action a; $\hat{a}^m(\theta)$ is the equilibrium action profile in state θ following message m.

Define mechanism (Θ^2, f) as follows. For every message profile (θ_1, θ_2), the external enforcer is directed to *force* the action profile $(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2))$ and the transfer $\hat{y}^{(\theta_1,\theta_2)}(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2))$. Players are given the incentive to select the assigned trade action by the threat of severe punishment for any deviation; recall the construction discussed in the text. This forcing contract yields outcome $w^{(\theta_1,\theta_2)}$ that, written in terms of the functions supporting (Θ^2, g), has the following payoff vector in state θ:

$$w^{(\theta_1,\theta_2)}(\theta) = u(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2), \theta) + \hat{y}^{(\theta_1,\theta_2)}(\hat{a}_1^{(\theta_1,\theta_2)}(\theta_1), \hat{a}_2^{(\theta_1,\theta_2)}(\theta_2)).$$

Define $f(m) \equiv w^m$ for every $m \in \Theta^2$.

Note that $f(\theta, \theta)(\theta)$, which we can write as $w^{(\theta,\theta)}(\theta)$, is equal to $g(\theta, \theta)(\theta)$. To complete the proof of the lemma, we must show that truthful reporting is a Nash equilibrium of the message game in every state. Suppose that, in state θ, player 1 deviates by reporting θ_1 while player 2 reports θ. Then player 1 gets a payoff of

$$w_1^{(\theta_1,\theta)}(\theta) = u_1(\hat{a}_1^{(\theta_1,\theta)}(\theta_1), \hat{a}_2^{(\theta_1,\theta)}(\theta), \theta) + \hat{y}^{(\theta_1,\theta)}(\hat{a}_1^{(\theta_1,\theta)}(\theta_1), \hat{a}_2^{(\theta_1,\theta)}(\theta)),$$

which is weakly less than

$$u_1(\hat{a}_1^{(\theta_1,\theta)}(\theta), \hat{a}_2^{(\theta_1,\theta)}(\theta), \theta) + \hat{y}^{(\theta_1,\theta)}(\hat{a}_1^{(\theta_1,\theta)}(\theta), \hat{a}_2^{(\theta_1,\theta)}(\theta))$$

because $\hat{a}^{(\theta_1,\theta)}(\theta)$ is a Nash equilibrium of the trading game $\langle A, u(\cdot,\theta) + \hat{y}^{(\theta_1,\theta)}(\cdot)\rangle$. But this last value is exactly player 1's expected payoff conditional on message profile (θ_1, θ) in state θ, under mechanism (Θ^2, g). Write this payoff as $g(\theta_1,\theta)_1(\theta)$. It, in turn, is weakly less than $g(\theta,\theta)_1(\theta)$, the payoff for player 1 when both players report truthfully in state θ. Thus, for mechanism (Θ^2, f), we have $f(\theta_1,\theta)_1(\theta) \le f(\theta,\theta)_1(\theta)$. The analogous calculation holds for player 2, which means truthful reporting is an equilibrium in every state. *Q.E.D.*

Proof of Lemma 4: Suppose v is implementable with interim renegotiation and let $v' \in V^{\mathrm{N}}$ be a value function that satisfies the expressions in Definition 3. Obviously, v is efficient in every state, so $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ for every $\theta \in \Theta$. By Equation 4, v satisfies condition (i) of Lemma 2. Furthermore, $v(\theta) \ge v'(\theta)$ in the vector sense, for every θ; thus, condition (ii) of Lemma 2 is also satisfied, implying $v \in V^{\mathrm{N}}$. To prove the "sufficiency" direction of the lemma, simply note that $v_1(\theta) + v_2(\theta) = \gamma(\theta)$ implies $r(v,\theta) = 0$, so, by Definition 3, $V^{\mathrm{I}} \subset V^{\mathrm{N}}$. The last claim of the lemma follows from Lemma 1. *Q.E.D.*

Proof of Lemma 5: Lemma 1 implies that Z is closed under constant transfers. Since the maximum joint value exists in every state, condition (i) holds if and only if, for every $\theta \in \Theta$, there is an outcome $z \in Z$ such that $z(\theta) = v(\theta)$ for every $\theta \in \Theta$. The rest of the proof follows the proof of Lemma 2 with Z in place of W. *Q.E.D.*

Proof of Lemma 6: Recognizing that $Z^{\mathrm{F}} \subset Z$ and Z^{F} is closed under constant transfers, this lemma is proved in the same manner as was Lemma 5. *Q.E.D.*

B Appendix: Proof of Theorem 4

I first prove the claim about the relation between V^{EPF} and V^{EP}. The method involves characterizing how non-forcing contracts can be used to induce player 1 to select different actions in different states. This enables the problem of finding a punishment outcome to be written in terms of the trade actions that are selected in two states. I then invoke Theorem 2.

Consider outcomes (state-contingent payoffs from Date 6). For any given transfer function $\hat{y}$, the following are necessary conditions for player 1 to select trade action a in state θ and action a' in state θ':

$$\begin{gathered} u_1(a,\theta) + \hat{y}_1(a) \ge u_1(a',\theta) + \hat{y}_1(a') \quad \text{and} \\ u_1(a',\theta') + \hat{y}_1(a') \ge u_1(a,\theta') + \hat{y}_1(a). \end{gathered} \tag{8}$$

Transfer function $\hat{y}$ can be specified so that player 1 is harshly punished for selecting any trade action other than a or a'. Then, in every state, either a or a' maximizes player 1's payoff from Date 6 and we have an equilibrium. Thus, we have:

Fact 1: *Expression 8 is necessary and sufficient for the existence of functions $\hat{y}: A \to \mathbf{R}^2_0$ and $\hat{a}: \Theta \to A$ such that (i) $\hat{a}(\theta)$ is a Nash equilibrium of $\langle A, u(\cdot, \theta) + \hat{y}(\cdot)\rangle$, for every state θ, and (ii) $\hat{a}(\theta) = a$ and $\hat{a}(\theta') = a'$.*

Expression 8 yields two other useful facts as well. First, by summing the inequalities, we see that there are values $\hat{y}(a), \hat{y}(a') \in \mathbf{R}^2_0$ that satisfy (8) if and only if

$$u_1(a, \theta) - u_1(a', \theta) \geq u_1(a, \theta') - u_1(a', \theta'). \tag{9}$$

Assumption 3 then implies:

Fact 2: *If $\theta > \theta'$ then Inequality 9 holds if and only if $a \geq a'$. If $\theta < \theta'$ then Inequality 9 holds if and only if $a \leq a'$.*

In words, this means that player 1 can only be given the incentive to choose greater trade actions in higher states.

For any two states $\theta, \theta' \in \Theta$, define

$$E(\theta, \theta') \equiv \{(a, a') \in A^2 \mid \text{Inequality 9 is satisfied.}\}.$$

Also, for states $\theta, \theta' \in \Theta$ and trade actions $a, a' \in A$ with $(a, a') \in E(\theta, \theta')$, define

$$Y(a, a', \theta, \theta') \equiv \{\hat{y}: A \to \mathbf{R}^2_0 \mid \text{Inequality 8 is satisfied.}\}.$$

Expression 8, combined with the identity $\hat{y}_1 = -\hat{y}_2$, implies:

Fact 3: *Given $\theta, \theta' \in \Theta$ and $a, a' \in A$, with $(a, a') \in E(\theta, \theta')$, we have*

$$\min_{\hat{y} \in Y(a, a', \theta, \theta')} \hat{y}_1(a) + \hat{y}_2(a') = u_1(a', \theta) - u_1(a, \theta).$$

Using Expression 2, any given $w \in W$ can be written in terms of the trade decisions and transfers that support it. Specifically, we have $w(\theta) = u(\hat{a}(\theta), \theta) + \hat{y}(\hat{a}(\theta))$ and $w(\theta') = u(\hat{a}(\theta'), \theta') + \hat{y}(\hat{a}(\theta'))$, where $\hat{a}$ and $\hat{y}$ are the functions that support w in Expression 2.

For any state θ'' and trade action a'', define $R(a'', \theta'')$ to be the renegotiation surplus if, without renegotiation, player 1 would select a''. That is, $R(a'', \theta'') = U(a^*(\theta''), \theta'') - U(a'', \theta'')$. Combining the expressions for w in the previous paragraph with Fact 1 and the definition of ex post renegotiation outcomes, we obtain:

Fact 4: *Consider any two states $\theta, \theta' \in \Theta$ and let α be any number. There is an ex post renegotiation outcome $z \in Z$ that satisfies $z_1(\theta) + z_2(\theta') = \alpha$ if and only if there are trade actions $a, a' \in A$ and a transfer function $\hat{y}$ such that $(a, a') \in E(\theta, \theta')$, $\hat{y} \in Y(a, a', \theta, \theta')$, and*

$$\alpha = u_1(a, \theta) + \hat{y}_1(a) + \pi_1 R(a, \theta) + u_2(a', \theta') + \hat{y}_2(a') + \pi_2 R(a', \theta'). \tag{10}$$

In the last line, the first three terms are $w_1(\theta)$ plus player 1's share of the renegotiation surplus in state θ, totaling $z_1(\theta)$. The last three terms are $w_2(\theta')$ plus player 2's share of the renegotiation surplus in state θ', totaling $z_2(\theta')$.

Our objective is to find the best (minimum) punishment level for states θ and θ', which means minimizing $\hat{z}_1(\theta) + \hat{z}_2(\theta')$ by choice of $\hat{z} \in Z$. To this end, we can use Fact 3 to substitute for $\hat{y}_1(a) + \hat{y}_2(a')$ in Expression 10. This yields the best punishment level for trade decisions a and a' in states θ and θ', respectively, written

$$\nu(a, a', \theta, \theta') \equiv u_1(a', \theta) + \pi_1 R(a, \theta) + u_2(a', \theta') + \pi_2 R(a', \theta'). \tag{11}$$

Assumption 1 guarantees that $\nu(a, a', \theta, \theta')$ has a minimum by choice of $(a, a') \in E(\theta, \theta')$. We therefore have:

Fact 5: *The minimum punishment level in the setting of ex post renegotiation,* $\min_{\hat{z} \in Z} \hat{z}_1(\theta) + \hat{z}_2(\theta')$*, exists and is equal to* $P^{\mathrm{EP}}(\theta, \theta') \equiv \min_{(a,a') \in E(\theta,\theta')} \nu(a, a', \theta, \theta')$.

We obtain a similar characterization of the minimal punishment level for the setting in which attention is restricted to forcing contracts. The characterization is exactly as in Fact 5 except with the additional requirement that $a = a'$.

Fact 6: *The minimum punishment level for the MDER program,* $\min_{\hat{z} \in Z^{\mathrm{F}}} \hat{z}_1(\theta) + \hat{z}_2(\theta')$*, exists and is equal to* $P^{\mathrm{EPF}}(\theta, \theta') \equiv \min_{a \in A} \nu(a, a, \theta, \theta')$.

Observe that, by Theorem 2, $V^{\mathrm{EP}} = V^{\mathrm{EPF}}$ if and only if $P^{\mathrm{EPF}}(\theta, \theta') = P^{\mathrm{EP}}(\theta, \theta')$ for all $\theta, \theta' \in \Theta$. We can compare the minimization problems to determine if this is the case. I proceed by focusing on states θ^* and $\underline{\theta}$, where θ^* is a state that satisfies Assumption 4. Let b^1 denote a solution to the MDER problem $\min_{a \in A} \nu(a, a, \theta^*, \underline{\theta})$ and let b^2 denote a solution to the MDER problem $\min_{a \in A} \nu(a, a, \underline{\theta}, \theta^*)$. I shall establish a series of additional claims which, ultimately, demonstrate that $V^{\mathrm{EP}} \neq V^{\mathrm{EPF}}$.

Consider the implications of $b^1 < a^*(\theta^*)$. In this case, we know that $(a^*(\theta^*), b^1) \in E(\theta^*, \underline{\theta})$ because $\theta^* > \underline{\theta}$. Note that if the MDER program is justified, it must be that $\nu(b^1, b^1, \theta^*, \underline{\theta}) \leq \nu(a^*(\theta^*), b^1, \theta^*, \underline{\theta})$. Some algebraic manipulation reveals this inequality to be equivalent to $U(a^*(\theta^*), \theta^*) \leq U(b^1, \theta^*)$, which contradicts Assumption 2. [The algebra uses Assumption 5 to divide by π_1; it is also simplified by recalling the definition of R and by noting that $u_2(a, \theta) - \pi_2 U(a, \theta) = \pi_1 U(a, \theta) - u_1(a, \theta)$.] A similar contradiction is produced in the case of $b^2 > a^*(\underline{\theta})$. We thus obtain:

Fact 7: *If $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ then $b^1 \geq a^*(\theta^*)$ and $b^2 \leq a^*(\underline{\theta})$.*

Next compare, for states θ^* and $\underline{\theta}$, the minimum punishment level of the MDER problem with the value of choosing $a = b^1$ and $a' = \underline{a}$ in the minimization problem that defines P^{EP}. Note that $(b^1, \underline{a}) \in E(\theta^*, \underline{\theta})$ because $\theta^* > \underline{\theta}$ and $b^1 \geq \underline{a}$. If the MDER program is justified then it must be that $\nu(b^1, b^1, \theta^*, \underline{\theta}) \leq \nu(b^1, \underline{a}, \theta^*, \underline{\theta})$. Algebraic manipulation reveals that this inequality is equivalent to

$$u_1(b^1, \theta^*) - u_1(\underline{a}, \theta^*) - [u_1(b^1, \underline{\theta}) - u_1(\underline{a}, \underline{\theta})] \leq \pi_1[U(\underline{a}, \underline{\theta}) - U(b^1, \underline{\theta})].$$

Presuming that the MDER program is justified, Fact 7 and Assumption 4 imply that $b^1 > \underline{a}$. Assumption 3 then implies that the left side of this inequality is strictly positive, which means $U(\underline{a}, \underline{\theta}) > U(b^1, \underline{\theta})$. Using Assumption 2, we obtain:

Fact 8: *If $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ then $U(\underline{a}, \underline{\theta}) > U(\overline{a}, \underline{\theta})$*

We can perform a similar analysis by reversing the order of states θ^* and $\underline{\theta}$ and comparing the minimum punishment level of the MDER problem with the value of choosing $a = b^2$ and $a' = \overline{a}$ in the minimization problem that defines P^{EP}. Clearly, $(b^2, \overline{a}) \in E(\underline{\theta}, \theta^*)$, so if the MDER program is justified then it must be that $\nu(b^2, b^2, \underline{\theta}, \theta^*) \leq \nu(b^2, \overline{a}, \underline{\theta}, \theta^*)$. Algebra reveals that this inequality is equivalent to

$$u_1(\overline{a}, \theta^*) - u_1(b^2, \theta^*) - [u_1(\overline{a}, \underline{\theta}) - u_1(b^2, \underline{\theta})] \leq \pi_1[U(\overline{a}, \theta^*) - U(b^2, \theta^*)]. \tag{12}$$

By the definition of b^2 (solving the minimization problem that defines P^{EPF}), we have that $\nu(b^2, b^2, \underline{\theta}, \theta^*) \leq \nu(\overline{a}, \overline{a}, \underline{\theta}, \theta^*)$. This inequality is equivalent to

$$\begin{aligned} &\pi_1[U(\overline{a}, \theta^*) - U(b^2, \theta^*) - U(\overline{a}, \underline{\theta}) + U(b^2, \underline{\theta})] \\ &\quad \leq u_1(\overline{a}, \theta^*) - u_1(b^2, \theta^*) - [u_1(\overline{a}, \underline{\theta}) - u_1(b^2, \underline{\theta})]. \end{aligned} \tag{13}$$

Combining Inequalities 12 and 13, we get

$$U(\overline{a}, \theta^*) - U(b^2, \theta^*) - U(\overline{a}, \underline{\theta}) + U(b^2, \underline{\theta}) \leq U(\overline{a}, \theta^*) - U(b^2, \theta^*),$$

which simplifies to $U(b^2, \underline{\theta}) \leq U(\overline{a}, \underline{\theta})$. Recalling that $b^2 \leq a^*(\underline{\theta})$ (from Fact 7) and using Assumption 2, we obtain:

Fact 9: *If $V^{\mathrm{EPF}} = V^{\mathrm{EP}}$ then $U(\underline{a}, \underline{\theta}) \leq U(\overline{a}, \underline{\theta})$*

Facts 8 and 9 provide the contradiction that proves $V^{\mathrm{EPF}} \neq V^{\mathrm{EP}}$.

I next prove the claim about the relation between V^{I} and V^{EP}. Using the characterization of Lemma 4 to justify limiting attention to forcing contracts (via Lemma 3), we have:

Fact 10: *The minimum punishment level in the setting of interim renegotiation,* $\min_{w \in W^{\mathrm{F}}} w_1(\theta) + w_2(\theta')$*, exists and is equal to* $P^{\mathrm{I}}(\theta, \theta') \equiv \min_{a'' \in A} u_1(a'', \theta) + u_2(a'', \theta')$.

Observe that, by Theorem 3, $V^{\mathrm{I}} = V^{\mathrm{EP}}$ if and only if $P^{\mathrm{EP}}(\theta, \theta') = P^{\mathrm{I}}(\theta, \theta')$ for all $\theta, \theta' \in \Theta$. A sufficient condition for $V^{\mathrm{I}} \neq V^{\mathrm{EP}}$ is that, for some θ', $a^*(\theta')$ does *not* solve $\min_{a'' \in A} u_1(a'', \theta) + u_2(a'', \theta')$. To see why this is the case, take any θ and consider any solution to the minimization problem that defines $P^{\mathrm{EP}}(\theta, \theta')$; let (b, b') denote this solution. Because $R \geq 0$, the only way to get $P^{\mathrm{EP}}(\theta, \theta') = P^{\mathrm{I}}(\theta, \theta')$ is if b' solves $\min_{a'' \in A} u_1(a'', \theta) + u_2(a'', \theta')$ and if $R(b, \theta) = R(b'\theta') = 0$. However, if b' solves this minimization problem then $b' \neq a^*(\theta')$ and so $R(b', \theta') > 0$.

Specifically, consider states $\overline{\theta}$ and θ^*, where θ^* satisfies Assumption 4′. I will show that $a^*(\theta^*)$ does not solve $\min_{a'' \in A} u_1(a'', \overline{\theta}) + u_2(a'', \theta^*)$. Compare the punishment level of $a^*(\theta^*)$ to that of $\underline{a}$, noting that $\underline{a} < a^*(\theta^*)$. Trade action $\underline{a}$ produces a strictly lower value if

$$u_1(\underline{a}, \overline{\theta}) + u_2(\underline{a}, \theta^*) < u_1(a^*(\theta^*), \overline{\theta}) + u_2(a^*(\theta^*), \theta^*).$$

Using the identity $u_2 = U - u_1$ and rearranging terms, we see that this inequality is equivalent to

$$U(\underline{a}, \theta^*) - U(a^*(\theta^*), \theta^*) < u_1(a^*(\theta^*), \overline{\theta}) - u_1(\underline{a}, \overline{\theta}) - [u_1(a^*(\theta^*), \theta^*) - u_1(\underline{a}, \theta^*)].$$

The left side of this inequality is strictly negative (by Assumption 4′) and the right side is strictly positive (by Assumption 3). Thus, $a^*(\theta^*)$ does not solve $\min_{a'' \in A} u_1(a'', \overline{\theta}) + u_2(a'', \theta^*)$.

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