

A Theory of the Firm based on Haggling, Coordination and Rent-Seeking*

Sandeep Baliga Tomas Sjöström
Northwestern University Rutgers University

August 21, 2017

Abstract

Two agents want to coordinate their decisions but may also try to extract rents from each other. Decisions are negotiated at the interim stage, when the agents have private information. The agent who owns an asset has the right to make a unilateral decision regarding this asset, so negotiations must satisfy a participation constraint. We compare nonintegration, where each agent owns one asset, with integration, where one agent owns both assets. We derive simple necessary and sufficient conditions for the first best to be implemented with integration or nonintegration. These conditions can never be satisfied simultaneously, so integration sometimes dominates nonintegration and vice versa.

1 Introduction

When preferences depend on the state of the world, a complete contract specifies, *ex ante*, which decision to take in every possible state. As Williamson ([31], [34]) emphasized, if complete contracts are possible then outcomes would be first-best, regardless of ownership and organizational form. In this paper, we study how the allocation of decision rights influences the attainable surplus in a model of coordination and ex post hold-up when complete ex ante contracts are ruled out by assumption.¹ Our work is motivated by the seminal work of Williamson [31] who argued that haggling between independent (nonintegrated) agents may cause surplus loss. He suggested disputes within an integrated firm can be settled more efficiently, because the boss can make unilateral decisions. As Hart [9] and others pointed out, this theory implies that all production should take place within integrated firms so trade across nonintegrated firms

*We thank Robert Gibbons, Massimo Morelli, Andrew Newman and Steve Tadelis for valuable comments. We also thank Philip Marx for research assistance.

¹In the conclusion we discuss possible justifications for this assumption. For now, it may be useful to keep in mind a scenario where the agents do not even meet until after the state has been realized, making ex ante contracting impossible.

cannot be explained. However, haggling can also take place within a firm, for example, if the boss needs information from the subordinate to make an efficient decision. Similarly, efficient decision-making by nonintegrated firms may also rely on the truthful elicitation of private information. We provide a theory of integration based on bargaining under asymmetric information in both integrated and non-integrated firms. We identify environments where one allocation of decision-rights achieves surplus maximization despite haggling and the other does not.

For a concrete example, suppose two companies want to cooperate to make products that work well together, say a smartphone and an operating system. The decisions to be made might concern how much to tailor each product to the other.² The maker of the operating system can make a transaction-specific standard that works well on the other’s phone but not on those made by any other manufacturer. Or it can make an operating system that works on the phones of many manufacturers. Similarly, the smartphone can be customized to work with the other firm’s operating system, or to work on multiple platforms. Microsoft’s Windows Phone, Google’s Android or Apple’s iOS are examples of mobile operating systems that correspond to the standards on one side of a relationship. Nokia, Motorola and Apple smartphones are examples of the products on the other side.

Our formal model is a coordination problem with private information and interim (type-dependent) participation constraints. There are two agents A and B , and two decisions to be made, q_A and q_B . To be specific, we will assume q_i is the decision to either make agent i ’s product “exclusive”, $q_i = E$, or “inclusive”, $q_i = I$. An exclusive product of agent i works mainly if not solely with agent j ’s product. An inclusive product of agent i is designed to work with agent j ’s product but also works well on other agents’ products. Google’s Android is an inclusive standard while Apple’s iOS is exclusive. HTC and Samsung make phones that work on Android and Windows Phone but, even before acquisition, Nokia produced Windows-based products exclusively (of course the iPhone only works with iOS).

Each agent $i \in \{A, B\}$ has a privately known idiosyncratic preference over q_i . This preference is his *type* and is denoted θ_i . In the example, each company could have private information about its relative cost of making a customized product. There are also gains from coordination. If $q_A = q_B = E$, they can ensure their products well together without worrying whether they work well with others. If $q_A = q_B = I$, they can work on design compromises so their products work not only together but on multiple platforms. Formally, we assume each agent enjoys a coordination benefit $\gamma > 0$ if $q_A = q_B$. However, if $q_i = I$ and $q_j = E$ then agent j has tied his fortune to agent i by making a relationship-specific decision while agent i has chosen to sell his product on all platforms. This hurts agent j ’s sales but helps agent i ’s. By choosing to be inclusive while

²Our model also fits more traditional vertical integration examples, such as a car manufacturer and a body supplier who make relationship-specific investments – if we assume the investments are contractible.

agent j is exclusive, agent i “holds up” agent j . We capture this by assuming that if $q_i = I$ and $q_j = E$, then rents $\rho > 0$ are transferred from agent j to agent i .

The decisions q_A and q_B are negotiated at the interim stage, when the agents privately know their preferences (types). At this stage, decisions are contractible and any incentive-compatible agreement that satisfies interim participation constraints is allowed. The participation constraints depend on control rights, i.e., on who has the right to make a decision if negotiations fail. Our main objective is obtain a theory of the optimal allocation of control rights based on private information and interim participation constraints.

Following Grossman-Hart-Moore ([7] and [13]), the control rights (and hence the participation constraints) may be derived from asset ownership. That is, there are two assets A and B , and the owner of asset j has the right to choose q_j if negotiations break down.³ The owner’s incentives depend on the gains from coordination (γ) and rent seeking (ρ), and of course on whether he owns both assets or just one. With *nonintegration*, there is separate ownership of the assets so agent A has the right to choose q_A and agent B has the right to choose q_B . With *integration*, one agent (“the boss”) owns both assets and has the right to choose both q_A and q_B .

To find out if the first best (surplus-maximizing) decisions can be attained at the interim stage, we apply the revelation principle. What matters for implementability of the first best is the sum of the reservation payoffs for the types of each player that have the most to gain from opting out of the mechanism. The higher these are, the higher must be the payoff these types achieve within the mechanism. But then the IC constraints imply that *all* types must get a high payoff if they opt in because they can always pretend to be the type that gets a high payoff. Depending on the parameters, the first best surplus may be enough to compensate all types at one asset allocation but not at another. Thus, the first-best can sometimes be implemented with integration but not with non-integration, and vice versa. For parameter regions where coordination benefits are larger than the incentives to seek rent, the first best can be implemented with nonintegration but not with integration. Conversely, there are parameter regions where the incentives to seek rent are larger than coordination benefits and the first best can be implemented with integration but not with nonintegration.

With nonintegration, types who would try to extract rents ρ (by choosing inclusivity I) if negotiations were to fail have a high reservation payoff when ρ is high. Hence, they must be given a high payoff to sign the interim contract. But even those types who would choose E can *pretend* they would choose I . To avoid this preference falsification (which we refer to as “haggling”), *all* types must get a high expected payoff when ρ is high. Since the expected payoffs are

³Unlike the traditional incomplete contracts literature, there are no *non-contractible* investment decisions in our model. The decisions can be interpreted as *contractible* relationship-specific investments, or as any other (contractible) decisions concerning the two assets.

bounded by the available surplus, with nonintegration the first best is feasible if and only if ρ is low, i.e., if rent-seeking is not a big problem.

To understand if integration can do better, two cases are considered separately. In the *cooperative case*, the rent ρ is small compared to the coordination benefit γ ; in the *rent-seeking case*, ρ is instead large compared to γ . Which of these two cases applies depends on the relationship-specificity of the transaction and the benefits of customization. Returning to our technology example, a hardware manufacturer may make “exclusive” design choices that maximize the utility of a specific operating system; similarly, the operating system can be designed to exploit patents held by this particular hardware manufacturer. These choices release coordination benefits whose magnitude depends on the differentiation between products on each side of the trade. However, if different hardware manufacturers use the same chips and different operating systems run on the same chips, then each agent can use the inclusive strategy to capture revenue from multiple platforms. Thus, if the transaction is not very relationship-specific, the rent-seeking case is likely to occur. Now consider automobile manufacture where the car body is made by one agent and the rest of the car by a car manufacturer. Since the design of a car body is quite unique, a dedicated die is involved in production. The car body manufacturer may not find much to gain by working for multiple car manufacturers who each want their own design and the car manufacturer may find it hard to find multiple suppliers for the same body. In this case, the transaction is highly relationship-specific, and the cooperative case is likely to occur.

Consider first the rent-seeking case; the argument above shows why the first best cannot be implemented with nonintegration. With integration, a more positive result is obtained. On the one hand, the boss’s reservation payoff is high if ρ is high; if negotiations break down he will use his control rights to “hold up” (extract rents from) his subordinate and achieve a payoff of at least ρ . But on the other hand, the subordinate’s reservation payoff is low if ρ is high, because he expects to be held up. Moreover, rent-extraction is inefficient since it involves giving up coordination benefits, so as ρ increases the subordinate’s reservation payoff decreases faster than the boss’s reservation payoff increases. Therefore, the sum of reservation payoffs is low if ρ is big (compared to γ); as a result, the integrated firm can implement the first best in the rent-seeking case.⁴

Next consider the cooperative case. Under integration the boss’s reservation payoff is high, as he gets to choose his preferred action profile if negotiations break down. But the subordinate agent’s payoff is also high, as the boss never extracts rent from him in the cooperative case. In fact, the subordinate has a type that only cares about coordination, and so would get first-best payoffs even if negotiations break down. But any type can claim to be that type, and this threat of haggling implies that the subordinate must get large information rents. Intuitively, even those types of subordinate who in principle would be

⁴This is reminiscent of Williamson’s [?] idea that an exchange of hostages yields a threat point that can enforce efficient trade, and that in a repeated game, the one-shot equilibrium is a threat point which allows cooperation to take place in equilibrium (Halongen [8]).

willing to “bribe” the boss to implement the first best will not have to do so, because they can always pretend to be indifferent. But then the first best does not add enough surplus, compared to the disagreement point, to satisfy all IC and IR constraints. As a result, the integrated firm cannot implement the first best if ρ is small (compared to γ).

The rest of the paper is organized as follows. Section 2 discusses some relevant previous literature. Section 3 contains the basic model and Section 4 describes the first best. In Section 5 we introduce incentive compatibility and individual rationality constraints and prove certain preliminary results. Section 6 contains our main result for nonintegration. In Sections 7.1 and 7.2 we consider integration in the cooperative and rent-seeking cases, respectively. Section 7.3 summarizes and interprets the results, and Section 8 contains some concluding remarks.

2 Previous Literature

We have already explained our debt to Williamson [31]. Williamson [33] argued that having a destructive outside option might help to ensure efficient trade. Our theory of optimal allocation of decision rights resembles this idea. Also, from Grossman and Hart [7] we take the idea that asset ownership confers residual rights of control. But in our model, there are no ex ante investments and we focus on coordinating verifiable decisions ex post. Hart and Holmström [10] study ex post conflict as do we but they study behavioral phenomena that might destroy surplus. Our approach is classical and private information is at its center. Control rights determine outside options which in turn determine whether efficient coordination is possible in the presence of private information. Coordination problems are also studied by Alonso, Dessein and Matouschek [1] and Rantakari [24], but these are models of centralized versus decentralized decision making when agents use cheap-talk, and hence quite different from our model.

Milgrom and Roberts [19] provided an informal theory based on a comparison of the “bargaining costs” of nonintegration with the “influence costs” of integration. Influence costs occur when the subordinate expends time and effort on “influence activities”, i.e. rent-seeking. Milgrom and Roberts [19] emphasized the distinction between these two kinds of costs. In contrast, we treat nonintegration and integration symmetrically: in each case, decision making requires the revelation of private information, but strategic manipulation may prevent the first best from being implemented.⁵ Baker, Gibbons and Murphy [2] argue that decision rights within an organization are not contractible: the boss cannot formally delegate decision making to an employee, because the boss always has the right to overturn the subordinate’s decision. More generally, they

⁵The resulting surplus loss could of course be labelled a bargaining cost in one case and influence cost in the other.

argue that formal authority can only be allocated via asset ownership, although informal authority can be allocated in a repeated game. In equilibrium the agent who owns the asset must be better off than he would be by making a unilateral decision, since the latter option cannot be contracted away. This is similar to our participation constraint. But our underlying model is different from both Milgrom and Roberts [19] and Baker, Gibbons and Murphy [2], and unlike them we use a mechanism design approach.

Laffont and Maskin [15] and Myerson and Satterthwaite [20] showed that IC and interim IR constraints may preclude surplus maximization in public good and bilateral trading problems (where the decision is how much or whether to produce or trade and with what transfers). In a Myerson-Satterthwaite style trading problem, it is known that the attainable surplus can depend on property rights (Cramton, Gibbons and Klemperer [4]). Our underlying model is different: a decision has to be made for each asset, which raises issues of coordination and rent-extraction that do not exist in the Myerson-Satterthwaite framework, and leads to a richer analysis of the optimal allocation of control rights. Formally, our analysis closely follows Myerson and Satterthwaite [20].⁶

Schmitz [26] allows a seller to invest ex ante to learn his disagreement payoff as well as to increase the value of the object being traded. The buyer has no investment opportunity so the usual Grossman-Hart intuition implies the seller should be allowed to own his assets to minimize hold-up problems. But when there is asymmetric information as the seller learns his outside option, the canonical result is reversed. It can be optimal for the buyer to own the assets to reduce the seller's incentives to collect information. Again, we have no ex ante investments so our model is quite different.

Matouschek [18] allows disagreement payoffs to reflect asset ownership in a Myerson-Satterthwaite style bilateral trade problem and then studies optimal asset ownership. However, he does not allow disagreement payoffs to depend on the realized values and costs of the buyer and seller. But it is entirely natural that there should be some connection between payoffs from trade and no-trade - for example if a seller is efficient and has a low cost of production from trading within the bilateral relationship, his disagreement payoff should also be high as he still has a relatively low cost of producing for a non-ideal partner. In our model, the payoffs at the outside option do depend on both the asset ownership and the realized types. In fact, the way the reservation payoffs vary with the coordination benefits γ and rents ρ drive our results on optimal asset allocation.

Incomplete contracts might arise out of complexity considerations and the fact that details are not filled in might lead to ex post haggling. This is obviously quite different from our approach where incomplete information leads to haggling. For an idea of what an approach based on complexity might look like, see Tadelis [27].

⁶An alternative approach would rely on VCG mechanisms along the lines of Krishna and Perry [14], Makowski and Mezzetti [16], Neeman [21] and Williams [29].

3 Model

3.1 Decisions and Payoff Functions

Consider a relationship involving two agents, A and B . Two decisions have to be made, q_A and q_B . For simplicity, there are only two options, *exclusivity*, E , or *inclusivity*, I , for each decision. For $i \in \{A, B\}$, let θ_i denote agent i 's *type*. Agent i 's type is his private information. The *state of the world* is (θ_A, θ_B) .

Agent i 's payoff function is $v_i(q, \theta_i) + t_i$, where $v_i(q, \theta_i)$ is the benefit type θ_i derives from decision profile $q = (q_A, q_B)$ and t_i is a monetary transfer. An *outcome* of the game consists of a decision profile $q = (q_A, q_B)$ and a transfer profile $t = (t_A, t_B)$. Budget balance requires that the transfers always sum to zero: $t_A + t_B = 0$. There are no individual limited liability constraints.

We represent $v_i(q, \theta_i)$ in the matrix (1). The row indicates q_i and the column indicates q_j .

$$\begin{array}{cc} & \begin{array}{cc} I & E \end{array} \\ \begin{array}{c} I \\ E \end{array} & \begin{array}{cc} \theta_i + \gamma & \theta_i + \rho \\ -\rho & \gamma \end{array} \end{array} \quad (1)$$

The payoff matrix is highly stylized to simplify calculations, but it captures the basic idea that the relationship has cooperative, opportunistic (rent-seeking) and idiosyncratic elements.⁷ If the decisions are coordinated (either both I or both E) then $\gamma > 0$ is added to each agent's payoff. If $q_i = I$ and $q_j = E$ then agent i gains $\rho > 0$ at the expense of agent j . Finally, θ_i (which can be positive or negative) represents an idiosyncratic benefit agent i gets from $q_i = I$.

There are two salient cases. In the *cooperative case* we have $\gamma > \rho$. In this case, the benefit of coordination dominates the possible gains from rent seeking, and the biggest number in payoff matrix (1) is always on the main diagonal. In the *rent-seeking case* we have $\rho > \gamma$. In this case, the gain from rent seeking dominates the benefit of coordination. The incentives to cooperate or rent-peek will determine the optimal allocation of control rights.

Types are independently drawn from a distribution with a differentiable c.d.f. F with support $[\underline{\theta}, \bar{\theta}]$. To simplify the exposition, we assume $\bar{\theta} < \infty$. We make the following two substantive assumptions on F .

Assumption 1. $F'(\theta_i) < 1/(2\gamma)$ for all $\theta_i \in [\underline{\theta}, \bar{\theta}]$ and $\underline{\theta} < -\rho - \gamma$ and $\bar{\theta} > 2\gamma$.

Assumption 2. $\underline{\theta} = -\bar{\theta}$ and $F(\theta_i) = 1 - F(-\theta_i)$ for all $\theta_i \in [\underline{\theta}, \bar{\theta}]$.

Assumption 1 says that there is “sufficient uncertainty”, that is, the density F' is rather flat and not concentrated around one point. This will be used to show uniqueness of equilibrium when each agent i chooses q_i non-cooperatively. Assumption 2 says that F is symmetric around 0. Thus, the idiosyncratic element θ_i is equally likely to favor I or E .

⁷A more general payoff structure, where the gains and losses from opportunism do not cancel out and the gains from coordination depend on which action they coordinate on, would complicate the calculations without adding any new insights.

3.2 Mechanism Design with Incomplete Contracts

Is it possible to achieve the first-best (surplus-maximizing) outcome without complete contracts? What is the role of asset ownership? To answer these questions, we rule out ex-ante comprehensive contracting, but allow the agents to negotiate efficiently once they have learned their own types. To formalize the efficient bargaining, we follow the mechanism design approach and introduce an impartial mediator. At the ex ante stage, before the agents know their own types, he is restricted to allocating control rights (asset ownership).⁸ If he allocates both assets to the same agent (integration), we assume without loss of generality (since the agents are symmetric ex ante) that this is agent A . The mediator’s inability to use a more complex mechanism ex ante formalizes the impossibility of complete contracts. Our main objective is to analyze how the initial allocation of control rights influences the attainable surplus when contracts are incomplete.

To derive the attainable surplus, we assume *interim* contracting is frictionless: after each agent has learned his own type, incentive and participation constraints must be respected, but on top of that there are no *additional* Williamsonian “haggling” inefficiencies. Formally, we assume the mediator is unrestricted at the interim stage and can propose any mechanism; by the revelation principle, we can assume he proposes an incentive compatible revelation mechanism. We then study if surplus losses are a *necessary* consequence of the inability to sign complex (state-contingent) contracts ex ante.⁹

We rule out *ex post* renegotiation of the outcome. Thus, we establish a frictionless *full-committment* benchmark for studying the optimal allocation of control rights when the agents have private information and ex ante contracts are incomplete. (See the concluding section for some remarks on renegotiation.)

The time-line is the following.

Stage 0. Control rights are allocated (the mediator chooses either “integration” or “non-integration”).

Stage 1. Each agent $i \in \{A, B\}$ privately observes his own type θ_i .

Stage 2. The mediator proposes an incentive-compatible revelation mechanism Γ . Then each agent (simultaneously) either accepts or rejects Γ . If both accept, then move to stage 3a, otherwise move to stage 3b.

⁸An alternative model would allow the agents to trade their endowments of assets for cash; the conclusions would be similar to our current model.

⁹It would be possible (and interesting) to model the interim contracting stage in a less abstract way. With integration, agent A might propose a mechanism, while with non-integration, some more symmetric procedure could be used. However, surplus losses might then be caused by “information leakage” (e.g., B might try to infer A ’s type from the mechanism he proposes). This would depend on the exact procedure which is used. The mechanism design approach abstracts from these issues and identifies whether, in principle, the first best is implementable by *some* procedure.

Stage 3. (a) If at stage 2 both agents agreed to participate in Γ , then each agent reveals his type and an outcome is implemented as specified by Γ .
(b) If at least one agent rejected Γ at stage 2, then under *integration* agent A chooses both q_A and q_B , while under *nonintegration* agent A chooses q_A and agent B chooses q_B .

At stage 3a, truthtelling is an equilibrium of the mechanism Γ . For each revealed type profile $\theta = (\theta_A, \theta_B)$, Γ implements an outcome, i.e., a probability of implementing each decision profile and a transfer profile. Decisions and transfers are contractible in the sense that if both agent agree to participate in Γ then the outcome at stage 3a is final and not subject to moral hazard or renegotiation. An incentive compatible revelation mechanism which always implements the first best decision profile is said to be a *first best mechanism*.

At stage 2, each agent has an outside option, namely, to refuse to participate in Γ and move to stage 3b. For all types to participate, Γ must satisfy interim individual rationality (IR) constraints: each type must expect to get at least his reservation payoff, i.e., what he expects to get if stage 3b is reached. This reservation payoff depends on the allocation of control rights: under integration, agent A chooses both q_A and q_B at stage 3b, while under nonintegration, agent A chooses q_A and agent B chooses q_B simultaneously and independently. (No monetary transfers are ever made at stage 3b.) Again, the outcome is final and not subject to renegotiation. If a mechanism Γ satisfies all interim IR constraints, then Γ is said to be *individually rational*. (In this paper, individual rationality is always taken to mean interim IR.)

The mediator's objective is to maximize the social surplus. We will characterize the conditions under which a first best mechanism can satisfy the IR constraints with integration or non-integration. If the IR constraints can be satisfied with integration (resp. non-integration), then the social surplus is maximized by choosing integration (resp. non-integration) at stage 0.¹⁰

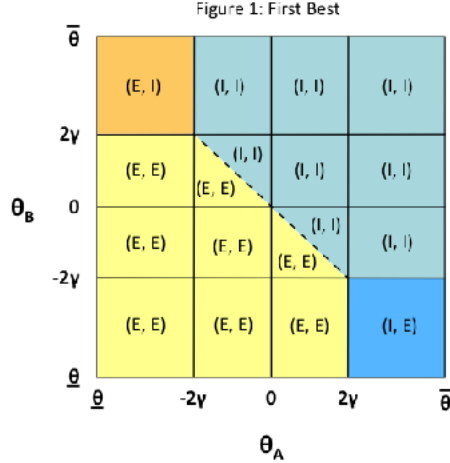
4 The First Best

The *social surplus* is the sum of agent A 's and agent B 's payoff. We represent the social surplus in the following matrix. The row indicates q_A and the column indicates q_B .

$$\begin{array}{cc}
 & \begin{array}{cc} I & E \end{array} \\
 \begin{array}{c} I \\ E \end{array} & \begin{array}{cc} \theta_A + \theta_B + 2\gamma & \theta_A \\ \theta_B & 2\gamma \end{array}
 \end{array} \tag{2}$$

This matrix highlights the fact that neither ρ nor (t_A, t_B) matter for social surplus.

¹⁰If the first-best is unattainable, then the mediator will choose a second-best allocation of control rights, but in this paper we will focus on conditions for implementability of the first-best.



For any type profile $\theta = (\theta_A, \theta_B)$, the *first best* (socially optimal) decision profile is denoted $q^*(\theta) = (q_A^*(\theta), q_B^*(\theta))$. By definition, $q^*(\theta)$ maximizes the social surplus, i.e., it selects the biggest number in the matrix (2).¹¹ Figure 1 illustrates the first best. It is easy to see that if $\theta_i \leq -2\gamma$ (**Case 1**) or $\theta_i \geq 2\gamma$ (**Case 3**) it is optimal to set $q_i^*(\theta) = E$ and $q_i^*(\theta) = I$ respectively. In the intermediate case (**Case 2**), where $-2\gamma < \theta_i < 2\gamma$, the surplus maximizing decision is always on the main diagonal of matrix (2). Specifically, $q^*(\theta) = (E, E)$ if $\theta_A + \theta_B < 0$, and $q^*(\theta) = (I, I)$ if $\theta_A + \theta_B > 0$.

The following matrix indicates the *social surplus generated by agent A*. The row indicates q_A and the column indicates q_B . It differs from matrix (1) because ρ , which is irrelevant for social surplus calculations, has been dropped.

$$\begin{array}{cc}
 & \begin{array}{cc} I & E \end{array} \\
 \begin{array}{c} I \\ E \end{array} & \begin{array}{cc} \theta_A + \gamma & \theta_A \\ 0 & \gamma \end{array}
 \end{array}$$

We will calculate the *ex ante* expected social surplus S generated under the first best decision rule. With probability $F(-2\gamma)$, Case 1 applies to θ_A . In this case, $q^*(\theta) = (Down, Down)$ if $\theta_B < 2\gamma$, and then the social surplus generated by agent A is γ ; but $q^*(\theta) = (Down, Up)$ if $\theta_B > 2\gamma$ and then agent A generates no social surplus. Now $\theta_B < 2\gamma$ with probability $F(2\gamma)$, so corresponding to Case 1, S_A contains the term $F(-2\gamma)F(2\gamma)\gamma$. Considering the other two cases

¹¹If two numbers are the same, either decision profile can be chosen, but this will happen with probability zero.

in a similar way, we obtain

$$\begin{aligned}
S_A &= F(-2\gamma)F(2\gamma)\gamma + \int_{-2\gamma}^{2\gamma} (\gamma + \theta_A F(\theta_A)) dF(\theta_A) + \int_{2\gamma}^{\bar{\theta}} (\theta_A + \gamma F(2\gamma)) dF(\theta_A) \\
&= 2\gamma \left[(1 - F(2\gamma))F(2\gamma) + F(2\gamma) - \frac{1}{2} \right] + \int_{-2\gamma}^{2\gamma} \theta_A F(\theta_A) dF(\theta_A) \\
&\quad + \int_{2\gamma}^{\bar{\theta}} \theta_A dF(\theta_A)
\end{aligned}$$

This calculation used Assumption 2. Integration by parts yields

$$\begin{aligned}
\int_{-2\gamma}^{2\gamma} sF(s)dF(s) &= \frac{1}{2} \int_{-2\gamma}^{2\gamma} sd(F(s))^2 \\
&= \gamma [(F(2\gamma))^2 + (1 - F(2\gamma))^2] - \frac{1}{2} \int_{-2\gamma}^{2\gamma} (F(s))^2 ds.
\end{aligned}$$

Using this, we obtain a simplified expression:

$$S_A = 2F(2\gamma)\gamma - \frac{1}{2} \int_{-2\gamma}^{2\gamma} (F(s))^2 ds + \int_{2\gamma}^{\bar{\theta}} s dF(s). \quad (3)$$

The two agents are perfectly symmetric, so ex ante the expected social surplus under the first best is $S = 2S_A$. Explicitly, we have

$$S = 4\gamma F(2\gamma) - \int_{-2\gamma}^{2\gamma} (F(s))^2 ds + 2 \int_{2\gamma}^{\bar{\theta}} s dF(s). \quad (4)$$

5 Incentive Compatible First Best Mechanisms

From now on, we will consider a first best mechanism Γ and study the constraints imposed by incentive compatibility. The results from Section 4 will be used to compute any type's expected payoff from Γ , assuming all types participate in it (later, we will check if interim IR constraints hold). The agents are symmetric, so it suffices to consider agent A .

Let

$$t(\theta_A) \equiv \int_{\underline{\theta}}^{\bar{\theta}} t_A(\theta_A, \tilde{\theta}_B) dF(\tilde{\theta}_B)$$

denote agent A 's expected transfer when his type is θ_A , and let

$$u_A(\theta_A) \equiv \int_{\underline{\theta}}^{\bar{\theta}} v_A(q^*(\theta_A, \tilde{\theta}_B), \theta_A) dF(\tilde{\theta}_B) + t(\theta_A)$$

denote type θ_A 's expected payoff. The *ex ante* expected payoff for agent A is

$$V_A \equiv \int_{\underline{\theta}}^{\bar{\theta}} u_A(\theta_A) dF(\theta_A).$$

As explained in Section 4, there are three cases.

Case 1: $\theta_A \leq -2\gamma$. Then $q_A = E$. Agent A gets γ with probability $F(2\gamma)$ and $-\rho$ with probability $1 - F(2\gamma)$. Thus, agent A 's expected payoff is

$$u_A(\theta_A) = t(\theta_A) + \gamma F(2\gamma) - \rho(1 - F(2\gamma)) = t(\theta_A) - \rho + (\gamma + \rho)F(2\gamma). \quad (5)$$

Case 2: $-2\gamma < \theta_A < 2\gamma$. Then the decision is (E, E) if $\theta_B < -\theta_A$, and (I, I) otherwise, so agent A expects

$$u_A(\theta_A) = t(\theta_A) + \gamma + \theta_A(1 - F(-\theta_A)) = t(\theta_A) + \gamma + \theta_A F(\theta_A). \quad (6)$$

using Assumption 2.

Case 3: $\theta_A \geq 2\gamma$. Then $q_A = I$. Agent A gets γ with probability $1 - F(-2\gamma) = F(2\gamma)$, and ρ with probability $1 - F(2\gamma)$. Thus, agent A expects

$$u_A(\theta_A) = t(\theta_A) + \theta_A + \gamma F(2\gamma) + \rho(1 - F(2\gamma)) = t(\theta_A) + \rho + \theta_A + (\gamma - \rho)F(2\gamma). \quad (7)$$

Incentive compatibility imposes restrictions on the transfer function. In Case 1, the decision profile is independent of θ_A , so the expected transfer must equal some constant t_A for all such types, and then from (5) the expected payoff is constant as well. Thus, for all $\theta_A < -2\gamma$ we have $t(\theta_A) = t_A$ and

$$u_A(\theta_A) = \underline{u}_A \equiv t_A - \rho + (\gamma + \rho)F(2\gamma). \quad (8)$$

In Case 2, the arguments of Myerson and Satterthwaite [20] imply that u_A is differentiable almost everywhere, and $u'_A(\theta_A) = F(\theta_A)$ for θ_A such that $-2\gamma < \theta_A < 2\gamma$. This implies that

$$u_A(\theta_A) = \underline{u}_A + \int_{-2\gamma}^{\theta_A} F(s) ds$$

if $-2\gamma < \theta_A < 2\gamma$.

In Case 3, the arguments of Myerson and Satterthwaite [20] imply $u'_A(\theta_A) = 1$. Using integration by parts and Assumption 2, this implies that

$$\begin{aligned} u_A(\theta_A) &= u_A(2\gamma) + \theta_A - 2\gamma = \underline{u}_A + \int_{-2\gamma}^{2\gamma} F(s) ds + \theta_A - 2\gamma \\ &= \underline{u}_A + \theta_A \end{aligned} \quad (9)$$

if $\theta_A > 2\gamma$.

Now we combine the three cases to compute the *ex ante* expected payoff for agent A , denoted V_A . Using integration by parts, we get¹²

$$\begin{aligned} V_A &= t_A - \rho + (\gamma + \rho)F(2\gamma) + \int_{-2\gamma}^{2\gamma} \left[\int_{-2\gamma}^{\theta_A} F(s)ds \right] dF(\theta_A) + \int_{2\gamma}^{\bar{\theta}} \theta_A dF_A(\theta_A) \\ &= t_A - \rho + (3\gamma + \rho)F(2\gamma) - \int_{-2\gamma}^{2\gamma} (F(\theta_A))^2 d\theta_A + \int_{2\gamma}^{\bar{\theta}} \theta_A dF_A(\theta_A). \end{aligned}$$

There is a similar expression for agent B 's *ex ante* expected payoff so the sum of the *ex ante* expected payoffs is

$$\begin{aligned} &V_A + V_B \\ &= t_A + t_B - 2\rho + (6\gamma + 2\rho)F(2\gamma) \\ &\quad - 2 \int_{-2\gamma}^{2\gamma} (F(s))^2 ds + 2 \int_{2\gamma}^{\bar{\theta}} s dF(s). \end{aligned} \tag{10}$$

Budget balance requires that the sum of the expected payoffs equals the expected social surplus:

$$V_A + V_B = S. \tag{11}$$

Above, we showed that all types $\theta_i < -2\gamma$ get the same expected transfer $t(\theta_i) = t_i$, and their expected payoff from participating in Γ is

$$\underline{u}_i \equiv t_i - \rho + (\gamma + \rho)F(2\gamma).$$

Substituting from (10) and (4) in (11), we obtain:

$$\underline{u}_A + \underline{u}_B \equiv t_A + t_B - 2\rho + 2(\gamma + \rho)F(2\gamma) = \int_{-2\gamma}^{2\gamma} (F(s))^2 ds. \tag{12}$$

The well-known d'Aspremont and Gérard-Varet [3] (AGV) mechanism is a first best mechanism. Given the revealed types (θ_A, θ_B) , the first best decision $q^*(\theta_A, \theta_B)$ is chosen, and agent A gets the AGV transfer

$$\begin{aligned} t_A(\theta_A, \theta_B) &\equiv \int_{\underline{\theta}}^{\bar{\theta}} v_B(q^*(\theta_A, \tilde{\theta}_B), \tilde{\theta}_B) dF(\tilde{\theta}_B) \\ &\quad - \int_{\underline{\theta}}^{\bar{\theta}} v_A(q^*(\tilde{\theta}_A, \theta_B), \tilde{\theta}_A) dF(\tilde{\theta}_A) + k_A \end{aligned} \tag{13}$$

¹²This calculation uses the fact that

$$\begin{aligned} \int_{-2\gamma}^{2\gamma} \left[\int_{-2\gamma}^{\theta_A} F(s)ds \right] dF(\theta_A) &= F(2\gamma) \int_{-2\gamma}^{2\gamma} F(s)ds - \int_{-2\gamma}^{2\gamma} (F(\theta_A))^2 d\theta_A \\ &= F(2\gamma)2\gamma - \int_{-2\gamma}^{2\gamma} (F(\theta_A))^2 d\theta_A. \end{aligned}$$

where k_A is a constant. Player B 's transfer is given by the analogous expression, with a constant k_B . Budget balance requires $k_A + k_B = 0$.

Suppose our model allowed comprehensive ex ante contracting: at stage 0, the mediator would first allocate control rights, and then propose a mechanism Γ which the agents must accept or reject before learning their own types. If someone rejects Γ then at the interim stage the agents make decisions non-cooperatively, based on their assigned control rights. But if both accept, they are committed to playing Γ at the interim stage. This commitment removes the interim participation constraints: they are *forced* to play Γ interim. Then for any initial allocation of control rights, the mediator could implement the first best by offering an AGV mechanism at stage 0 which both would accept, i.e., which would satisfy *ex ante* participation constraints.¹³ This would correspond to a complete state-contingent contract; the outcome for each state would be specified ex ante by the AGV mechanism. The Coase theorem would hold so the issue of integration versus nonintegration would be moot.¹⁴ It is precisely because we are interested in this issue, and more generally the implications of incomplete contracts, that our model rules out such comprehensive ex ante contracting.

6 Implementing the First Best with Nonintegration

Whether or not a first best mechanism satisfies individual rationality constraints depends on asset ownership.¹⁵ We begin by studying nonintegration where each agent owns his own asset and identify the type whose individual rationality constraint is the most difficult to satisfy at the first best. Recall that if stage 3b is reached, each agent $i \in \{A, B\}$ chooses q_i knowing his own type θ_i but not the other agent's type θ_j .

Consider a noncooperative (Bayesian-Nash) equilibrium of this stage 3b game.¹⁶ Since higher types are more inclined to choose I , each agent must

¹³The ex ante participation constraints can be satisfied by a judicious choice of k_A . Agent i 's ex ante reservation payoff r_i^e would depend on how decisions are expected to be made if someone rejects the mechanism. However, since the AGV mechanism is first best, its total expected surplus can be no smaller than $r_A^e + r_B^e$. Setting k_A appropriately ensures that each agent i gets at least r_i^e and so is willing, ex ante, to accept the mechanism.

¹⁴The Coase theorem would also hold for contracting under *complete information*, where each agent knows $\theta = (\theta_A, \theta_B)$. Consider the following mechanism. Agent A makes a take-it-or-leave-it offer consisting of a proposed decision profile and a transfer to agent B . If agent B rejects the offer, then he gets some reservation payoff (which may depend on control rights). In subgame perfect equilibrium, agent A proposes the first-best decision profile and a transfer such that agent B gets exactly his reservation payoff.

¹⁵Recall that in our model the IR constraints are *interim* constraints.

¹⁶To simplify, we make two assumptions about the noncooperative game played at stage 3b: (i) *passive beliefs*: each agent maintains his prior beliefs about the other agent's type, and (ii) *no cheap talk*. Either assumption could be changed, and the results would be qualitatively the same. Quantitatively the results would change, because the reservation payoffs would change. But the disagreement point would still not be first best, and control rights would still

use a cutoff strategy. Suppose agent B chooses $q_B = E$ if and only if $\theta_B \leq x$, which happens with probability $F(x)$. Then, agent A prefers $q_A = E$ if

$$F(x)\gamma - (1 - F(x))\rho \geq \theta_A + F(x)\rho + (1 - F(x))\gamma$$

which is equivalent to

$$\theta_A \leq (2F(x) - 1)\gamma - \rho.$$

Thus, if agent B uses cutoff x , agent A 's best response is to use the cutoff y defined by

$$y = (2F(x) - 1)\gamma - \rho. \quad (14)$$

By Assumption 1, the best response function has slope less than one:

$$\frac{dy}{dx} = 2F'(x)\gamma < 1.$$

Therefore, there is a unique noncooperative equilibrium. By the symmetry of the game, this equilibrium must be symmetric, and it can be found by setting $x = y$ in (14). Thus, in the unique equilibrium, each agent uses the cutoff θ^* defined by

$$\theta^* + \gamma + \rho = 2F(\theta^*)\gamma. \quad (15)$$

It can be checked that $-\gamma - \rho < \theta^* < -\rho$. Figure 2 illustrates the noncooperative equilibrium under nonintegration.

Agent A 's noncooperative equilibrium payoff is

$$\gamma F(\theta^*) - \rho(1 - F(\theta^*)) = -\rho + (\gamma + \rho)F(\theta^*) \quad (16)$$

if $\theta_A < \theta^*$ and

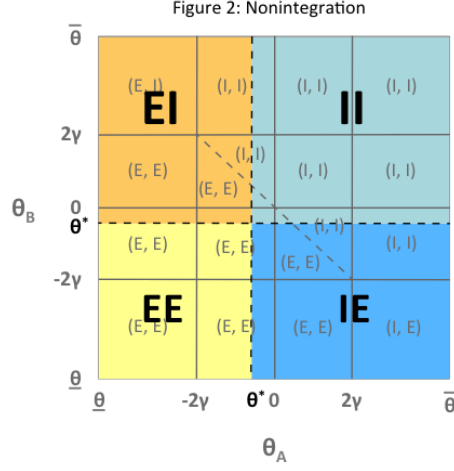
$$\theta_A + \gamma + (\rho - \gamma)F(\theta^*) \quad (17)$$

if $\theta_A > \theta^*$.

Since the noncooperative equilibrium is played at stage 3b if Γ is rejected at stage 2, agent A 's reservation payoff at stage 2 is given by (16) if $\theta_A < \theta^*$ and (17) if $\theta_A > \theta^*$.

Let Γ be a first best mechanism. Which type of agent A is most reluctant to accept Γ at stage 2? If stage 3b is reached, there is no check on opportunism, and type $\theta_A > \theta^*$ benefits by playing I , thus gaining ρ when $\theta_B < \theta^*$. Since ρ does not count as a gain for social surplus computations, this opportunistic benefit is reduced by Γ . Indeed, consider type θ_A such that $\theta^* < \theta_A \leq 2\gamma$.

matter for the attainable surplus. Passive beliefs were introduced by Rubinstein [25] and is a common assumption. The model with cheap talk would be reminiscent of Alonso, Dessein and Matouschek [1] and Rantakari [24], but with a comparison of integration and nonintegration, rather than centralization versus decentralization.



If he accepts Γ , the first best decision rule will, with some probability, make him forego the opportunistic gain ρ by implementing $q_A = E$. As the benefit from $q_A = E$ is decreasing in the type, in the range $[\theta^*, 2\gamma]$ the disadvantage of the first best is the greatest for type 2γ . On the other hand, if $\theta_A > 2\gamma$ then $q_A = I$ both in the noncooperative equilibrium and in the first best, so the disadvantage is not any greater for types above 2γ than it is for type 2γ . This intuitive argument suggests that the most difficult interim IR constraint to satisfy is for type 2γ . The following lemma verifies that the argument is correct.

Lemma 1 *Let Γ be a first best mechanism. With nonintegration, agent i 's interim IR constraints are satisfied for all his types if the interim IR constraint is satisfied for type $\theta_i = 2\gamma$, which is true if and only if*

$$\underline{u}_i \geq \gamma + (\rho - \gamma)F(\theta^*). \quad (18)$$

The proof of this and subsequent lemmas can be found in the appendix. Lemma 1 has the following implication:

Theorem 2 *With nonintegration, there exists an individually rational first best mechanism if and only if*

$$\int_{-2\gamma}^{2\gamma} (F(s))^2 ds \geq 2\rho F(\theta^*) + 2\gamma(1 - F(\theta^*)). \quad (19)$$

Proof. The necessity of (19) follows from (12) and Lemma 1.

To prove sufficiency of (19), consider the AGV mechanism with transfers given by (13). Since the mechanism is incentive compatible and the outcome is

first best, the results of Section 5 apply. If $k_A = k_B = 0$ then the mechanism is *symmetric*, so $\underline{u}_A = \underline{u}_B$. From (12) we obtain

$$\underline{u}_A = \frac{1}{2} \int_{-2\gamma}^{2\gamma} (F(s))^2 ds. \quad (20)$$

Inequality (19) says that (20) exceeds $\gamma + (\rho - \gamma)F(\theta^*)$. Therefore, by Lemma 18, all IR constraints are satisfied. ■

Theorem 2 is hard to interpret because θ^* is an endogenous variable which depends on γ and ρ . The following result helps us express Theorem 2 in terms of γ and ρ .

Lemma 3 *For any γ , there is $\rho^*(\gamma) \in (0, \gamma)$ such that (19) holds if and only if $\rho \leq \rho^*(\gamma)$.*

Combining Theorem 2 and Lemma 3, we get the following result.

Theorem 4 *With nonintegration, there exists an individually rational first best mechanism if and only if $\rho \leq \rho^*(\gamma)$, where $0 < \rho^*(\gamma) < \gamma$.*

To understand this result intuitively, note that Lemma 1 shows that the most difficult IR constraint to satisfy is for type 2γ . The noncooperative non-integrated equilibrium favors type 2γ inasmuch as he can benefit from the opportunistic benefit ρ . Still, when ρ is low, type 2γ does not need a big transfer to accept Γ . Then, other types would not gain much by pretending to be type 2γ , i.e., the benefits of “haggling” are low. Transfers to all types can be correspondingly low without violating incentive compatibility. Then there is enough surplus at the first best to satisfy all incentive and participation constraints without violating budget balance. But when ρ is high, it takes a big expected transfer to make type 2γ accept Γ . The incentive constraints imply the expected transfer must be high to all types. In terms of the AGV mechanism, we need $k_A > 0$ and $k_B > 0$ which violates budget balance, so the first best cannot be implemented.

7 Implementing the First Best with Integration

We will determine when individual rationality constraints are satisfied at a first best mechanism under integration. Recall that if stage 3b is reached, agent A unilaterally chooses the decision profile (q_A, q_B) which is best for himself, given θ_A . That is, he picks the greatest number in the payoff matrix (1), for $i = A$. Notice that agent A 's decision doesn't depend on his beliefs about θ_B , since θ_B doesn't influence his payoff.

Since agent A has right to choose the decision profile unilaterally if stage 3b is reached, his reservation payoff at stage 2 is high. If agent B 's reservation

payoff were correspondingly low, agent B would be willing to pay a high transfer to agent A at stage 3a in order to implement the first best. In this case, an AGV mechanism with a high k_A would be individually rational. But agent B 's reservation payoff depends on agent A 's incentive to cooperate or rent-seek. In the cooperative case where $\gamma > \rho$, at stage 3b agent A will never behave opportunistically (in the sense of transferring ρ from agent B to himself). In the *rent-seeking case* where $\rho > \gamma$, at stage 3b agent A will sometimes choose $(q_A, q_B) = (I, E)$, a rent-seeking policy which transfers ρ from agent B to himself. Since the IR constraints differ in the two cases, we treat them separately. We emphasize the key fact that agent B 's reservation payoff at stage 2 is quite different in the two cases: high in the cooperative case (making it hard to satisfy the IR constraints) but low in the rent-seeking case (making it easy to satisfy the IR constraints).

7.1 The Cooperative Case

Suppose $\rho < \gamma$ and Γ is rejected at stage 2. At stage 3b agent A chooses (E, E) if $\theta_A < 0$ and (I, I) otherwise. Therefore, if agent A rejects Γ at stage 2 then his payoff at stage 3b will be

$$\gamma + \max\{0, \theta_A\}. \quad (21)$$

This is agent A 's reservation payoff at stage 2. Of course, since agent A does not take θ_B into account, his decision is unlikely to be first best. At stage 3b agent B gets γ if $\theta_A < 0$ and $\theta_B + \gamma$ otherwise, so if agent B rejects Γ at stage 2 his expected payoff is

$$\gamma + (1 - F(0))\theta_B = \gamma + \theta_B/2. \quad (22)$$

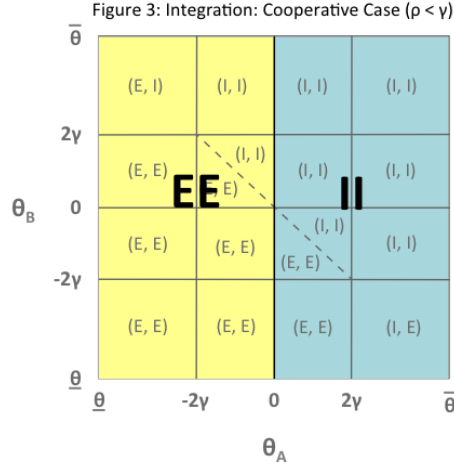
This is agent B 's reservation payoff at stage 2. Figure 3 illustrates what happens at stage 3b.

In the cooperative case, agent A will never hold up agent B and take ρ away at stage 3b. In fact, although agent A 's choice is not *always* first best, it *is* first best when $\theta_B = 0$, since then only θ_A matters for social efficiency. Type $\theta_B = 0$ knows that the decision profile will be the same whether they reach stage 3a or 3b, so he is not willing to pay anything to participate in Γ .¹⁷ Clearly, the IR constraint for type $\theta_B = 0$ will be difficult to satisfy, and this will be used in Section 7.1 to show that Γ cannot be individually rational. Here, we prove a preliminary result. Recall that in the first best mechanism Γ the expected payoff for any type $\theta_i < -2\gamma$ is given by

$$\underline{u}_i \equiv t_i - \rho + (\gamma + \rho)F(2\gamma).$$

Lemma 5 *Let Γ be a first best mechanism and suppose $\rho < \gamma$. With integration the following is true. (a) Agent A 's interim IR constraints are satisfied for all*

¹⁷Type $\theta_B \neq 0$ does care about whether they coordinate on $(Down, Down)$ or (Up, Up) . So the most difficult IR constraint to satisfy is for $\theta_B = 0$.



his types if the interim IR constraint is satisfied for type $\theta_A = 2\gamma$, which is true if and only if

$$\underline{u}_A \geq \gamma. \quad (23)$$

(b) Agent B's interim IR constraints are satisfied for all his types if the interim IR constraint is satisfied for type $\theta_B = 0$, which is true if and only if

$$\underline{u}_B \geq \gamma - \int_{-2\gamma}^0 F(s)ds. \quad (24)$$

Then, from Lemma 5 and (12), if the first best is implementable under integration then

$$\underline{u}_A + \underline{u}_B = \int_{-2\gamma}^{2\gamma} (F(s))^2 ds \geq 2\gamma - \int_{-2\gamma}^0 F(s)ds. \quad (25)$$

But this never holds, because the middle term and the third term are equal at $\gamma = 0$ and the derivative of the middle term with respect to γ is always strictly less than the derivative of the third term. Therefore, the inequality in (25) is violated for all $\gamma > 0$. Thus, we have the following result:

Theorem 6 *If $\rho < \gamma$, then with integration no first best mechanism is individually rational.*

Thus, it is impossible to implement the first best with integration in the cooperative case. To see this intuitively, recall that since agent A has control

rights if stage 3b is reached, his reservation payoff at stage 2 is high. If agent B 's reservation payoff were correspondingly low, then an AGV mechanism with a large positive k_A and correspondingly large negative k_B would be individually rational. However, as argued above, type $\theta_B = 0$ knows that the decision profile will be first best whether they reach stage 3a or stage 3b, so he will certainly reject an AGV mechanism with a large negative k_B . But agent A will insist that k_A should be large and positive on account of his advantageous bargaining position (he has the right to choose both q_A and q_B). Since budget balance forces $k_B = -k_A$ the first best cannot be implemented.

Notice that if $\theta_B \neq 0$ then at stage 3b agent B may suffer relative to the first best outcome, as agent A 's unilateral decision may cause agent B to lose surplus if $\theta_B < 0$ or not capture enough if $\theta_B > 0$. So if agent B has strong idiosyncratic preferences, he would in principle be willing to pay a large transfer to reach the first best. But, because his type is private information, agent B can haggle, i.e., pretend to be type $\theta_B = 0$ in order to avoid the transfer. Types $\theta_B \neq 0$ must receive information rents to preserve incentive compatibility. These rents limit the ability to extract transfers from agent B 's types and prevent implementation of the first best.

Combining Theorems 6 and 4 we get:

Theorem 7 Cooperative Case. *Suppose $\gamma > \rho$. Then, with integration no first best individually rational mechanism exists; with nonintegration such a mechanism exists if and only if $\rho \leq \rho^*(\gamma)$, where $0 < \rho^*(\gamma) < \gamma$.*

7.2 The Rent-Seeking Case

Suppose $\rho > \gamma$ and Γ is rejected at stage 2. At stage 3b, agent A chooses (E, E) if $\theta_A < \gamma - \rho$ and (I, E) otherwise. Thus, in this case agent A 's reservation payoff at stage 2 is

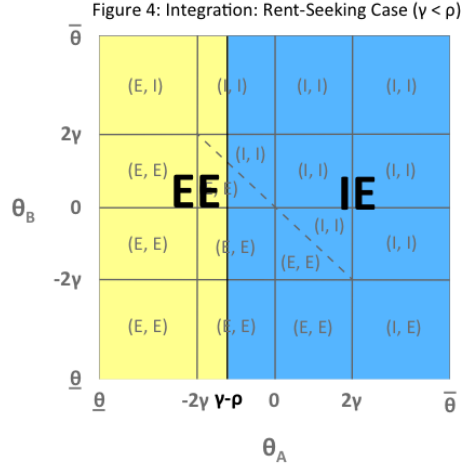
$$\max \{ \gamma, \theta_A + \rho \}.$$

At stage 3b agent B receives γ if $\theta_A < \gamma - \rho$ and $-\rho$ otherwise, so agent B 's reservation payoff at stage 2 is

$$F(\gamma - \rho)\gamma - \rho(1 - F(\gamma - \rho)) = -\rho + F(\gamma - \rho)(\gamma + \rho).$$

Notice that $\gamma - \rho < 0$ so $F(\gamma - \rho) < 1/2$. Figure 4 illustrates the stage 3b rent-seeking policy.

At stage 3b, agent A will always choose $q_B = E$ in the rent-seeking case. Since the high types of agent B would derive a large benefit from $q_B = I$, they are eager to accept Γ . In contrast, the low types of B do not mind if $q_B = E$. Indeed, if $\theta_B \leq -2\gamma$ then $q_B = E$ in the first best for sure, i.e., q_B will be the same at stages 3a and 3b when $\theta_B \leq -2\gamma$. These types all have the same IR constraint, and it is intuitively clear that this is most difficult IR constraint among agent B 's types.



Lemma 8 Let Γ be a first best mechanism and suppose $\rho > \gamma$. With integration the following is true. (a) Agent A's interim IR constraints are satisfied for all his types if the interim IR constraint is satisfied for type $\theta_A = 2\gamma$, which is true if and only if

$$\underline{u}_A \geq \rho. \quad (26)$$

(b) Agent B's interim IR constraints are satisfied for all his types if the interim IR constraint is satisfied for type $\theta_B = -2\gamma$, which is true if and only if

$$\underline{u}_B \geq -\rho + F(\gamma - \rho)(\gamma + \rho). \quad (27)$$

From Theorem 4, as $\rho^*(\gamma) < \gamma$, we already know that the first best cannot be implemented under nonintegration. High rents imply that the benefits of haggling are high under nonintegration and this creates inefficiency. In contrast, there are circumstances where the first best can be implemented under integration:

Theorem 9 If $\rho > \gamma$, then with integration a first best individually rational mechanism exists if and only if

$$\int_{-2\gamma}^{2\gamma} (F(s))^2 ds \geq F(\gamma - \rho)(\gamma + \rho). \quad (28)$$

Proof. The necessity of (28) follows from Lemma 8 and (12). To prove sufficiency of (28), consider the AGV mechanism with transfers given by (13). Since the mechanism is incentive compatible and the outcome is first best, the results of Section 5 apply. Choose k_A such that $\underline{u}_A = \rho$. By Lemma 8, all of agent A 's IR constraints are satisfied. From (12) and (28),

$$\underline{u}_B = \int_{-2\gamma}^{2\gamma} (F(s))^2 ds - \rho \geq F(\gamma - \rho)(\gamma + \rho) - \rho.$$

Again by Lemma 8, all of agent B 's IR constraints are satisfied as well. ■

To interpret Theorem 9, notice that (28) holds in the following three scenarios: (i) when the idiosyncratic preference is relatively unimportant; (ii) when ρ is only slightly bigger than γ ; (iii) when ρ is much bigger than γ .

To prove (i), define $z \equiv \rho - \gamma > 0$. We make θ_i less important, without changing the relative size of ρ and γ , by simultaneously increasing ρ and γ , while keeping $z \equiv \rho - \gamma$ fixed. Now (28) can be written as

$$\int_{-2\gamma}^{2\gamma} (F(s))^2 ds \geq F(-z)(2\gamma + z).$$

Raising γ increases the left hand side at the rate

$$2(F(2\gamma))^2 + 2(F(-2\gamma))^2 = 2(F(2\gamma))^2 + 2(1 - F(2\gamma))^2$$

which always exceeds 1 and, in fact, will be close to 2 for γ large. But the right hand side goes up only at the rate $2F(-z) < 1$ because $-z < 0$ and $F(0) = 1/2$. Therefore, (28) is bound to hold for γ large enough, for any given z .

To prove (ii), note that (28) holds with strict inequality when $\rho = \gamma$, because the left side lies strictly between γ and 2γ while the right side equals γ when $\rho = \gamma$. Thus (28) also holds for ρ slightly above γ .

To prove (iii), note that $F(\gamma - \rho) = 0$ whenever $\gamma - \rho < \underline{\theta}$. Therefore, (28) holds if $\rho > \gamma - \underline{\theta}$.

To visualize inequality (28), suppose F is the normal distribution with mean 0 and variance 1. In Figure 5, the light blue shaded (or lightly shaded) area is the set of (γ, ρ) such that $\rho > \gamma$ and the inequality (28) holds. Thus, if (γ, ρ) belongs to the light blue shaded area, the first best can be implemented under integration. The dark (blue) curve is the set of (γ, ρ) such that there is equality in (28). If both γ and ρ increase simultaneously then we move into the light blue area (property (i)); the light blue area includes all (γ, ρ) slightly above the 45 degree line (property (ii)); and if ρ is increased while γ is fixed, we move into the light blue area (property (iii)).

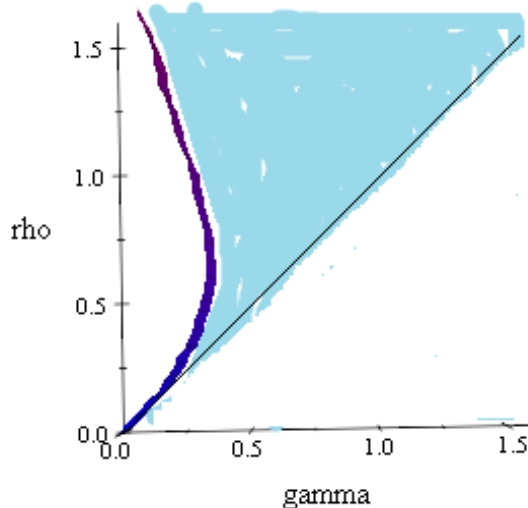


Figure 5

To understand why it is possible to implement the first best with integration in the rent-seeking case, notice that if agent B rejects Γ at stage 2, there is a chance that he will be held up at stage 3b (the outcome will be (I, E) and agent B loses ρ). This imposes an expected cost on all of player B 's types; in terms of the AGV mechanism, he is willing to agree to set $k_A = -k_B > 0$. Haggling (misrepresenting information) is not a serious problem: because all of agent B 's types will suffer from hold up if negotiations break down, there is no type which has to be given a large expected payoff to participate, hence agent B does not have much to gain from misrepresenting his type. Surplus is destroyed if stage 3b is reached because actions are not efficiently coordinated, and the extra surplus generated by the first best can be sufficient to satisfy both agents' individual rationality constraints.

Combining Theorem 4 and Theorem 9, we have the following result:

Theorem 10 *Rent-Seeking Case.* Suppose $\gamma < \rho$. Then, with nonintegration no first best individually rational mechanism exists; with integration such a mechanism exists if and only if

$$\int_{-2\gamma}^{2\gamma} (F(s))^2 ds \geq F(\gamma - \rho)(\gamma + \rho).$$

7.3 Summary of the Results

Our results show how the attainable surplus depends on the allocation of control rights. The revelation mechanism is a model of efficient bargaining with asymmetric information. With nonintegration, each agent has a fairly good bargaining position, because he has some decision rights if negotiations fail. But the reservation payoff, which determines his bargaining strength, is type-dependent. Each agent has some type who could gain a lot from rent-seeking, so he has a high reservation payoff. But the strong bargaining position of this type spills over to other types, since they can always *pretend* to be this type. The threat of such preference falsification, or “haggling”, by nonintegrated agents may make the first-best unattainable. In particular, if ρ is large compared to γ (so the rent-seeking motive is strong), then to both induce the type who benefits the most from rent-seeking to participate *and* prevent other types from haggling would require more surplus than is available.

Williamson [31] suggested that integration might eliminate haggling between nonintegrated opportunistic agents, and therefore increase the attainable surplus. But to make good decisions with asymmetric information, both integration and nonintegration require information transmission, and whether the threat of haggling is more serious in one case than in the other depends on the parameters. Under integration, agent A has a high reservation payoff, since he has all the decision rights if negotiations break down. Agent B 's reservation payoff depends on agent A 's relative incentives to cooperate or rent-seek. In the rent-seeking case (where ρ is large compared to γ), agent B 's reservation payoff is low because he expects to be held up. Accordingly, agent B is willing to make a large monetary transfer to agent A in order to get the first best decision. This large transfer persuades agent A to refrain from rent-seeking, so the first best is implementable with integration. Intuitively, in the rent-seeking case, if negotiations break down the subordinate's losses will exceed the boss's gains, and this makes it possible to agree on the first best. In the cooperative case, however, we showed earlier that agent B 's type 0 has a high reservation payoff. The possibility of haggling means agent B 's other types can pretend to be type 0, giving them significant bargaining power as well. Thus, in this case it is not possible to make agent B pay a large transfer to agent A . But without such a transfer, agent A prefers to exercise his control rights unilaterally. Therefore, if ρ is small compared to γ , the first best cannot be implemented with integration.

For the case where F is the normal distribution with mean 0 and variance 1, Figure 6 summarizes the results for both integration and nonintegration.

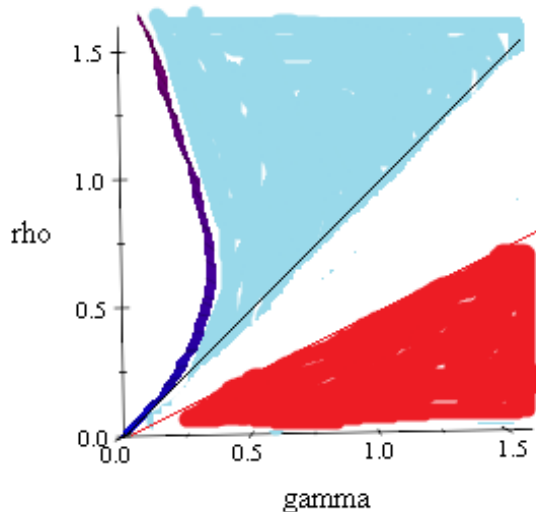


Figure 6

The light blue or lightly shaded area is the set of (γ, ρ) such that with integration a first best individually rational mechanism exists. The red or darkly shaded area is the set of (γ, ρ) such that with nonintegration a first best individually rational mechanism exists (that is, in this area we have $\rho < \rho^*(\gamma)$).

8 Concluding Remarks

We have derived conditions for implementability of the first best when agents are unable to sign complete state-contingent contracts before learning their private information. The incomplete contracting assumption has previously been defended based on the argument that, *ex ante*, the number of possible states and actions is extremely large and impossible to fully describe (Grossman and Hart [7], Hart and Moore [13]). At the interim stage, the set of possible types is known to lie in a much smaller set and the set of feasible actions is verifiable. Maskin and Tirole [17] argued that *ex ante* contracts could be written even if states and actions are indescribable *ex ante*. However, it is not clear that a court would enforce a complex long-run contract if the parties, unable to describe states and actions, could not have forecast scenarios for breach or the circumstances under which an investment would fail or they would be unable to perform. In such situations, the *penalty doctrine* in contract law specifies that courts should not (excessively) penalize an agent who walks away from the contract (see Eisenberg [5]). An interim agreement, made with better knowledge of

the current conditions, would be more likely to be enforced. In this case, asset ownership would play the same role as in our model.

A good illustration of the logic behind our model is the Japanese automobile industry (see, e.g., Holmström and Roberts [11]). Nonintegration (subcontracting) is the norm, and car manufacturers and their suppliers have exclusive relationships. The die used to make car parts is owned by the supplier, so if the relationship should break down, neither the supplier nor the automobile manufacturer is in a good position to capture rents. There are no alternate suppliers for the car part and, as the die is owned by the supplier, it is costly for the manufacturer to find one. Similarly, the die is useless for making car parts for other manufacturers so the supplier is not a good position to work with someone else either. Thus, both the manufacturer and the supplier parties have poor outside options which, by the logic of our model, helps to make surplus maximization both incentive-compatible and consistent with voluntary participation.

Private information plays a key role in our model. In reality, information asymmetries seem pervasive. For example, Pisano [23] observes that in the biotechnology industry:¹⁸

The firm in charge of R&D will accumulate asymmetric information on the technology; likewise, the partner in charge of marketing will gain asymmetric information on the technology's commercial potential. Strategic misrepresentation of new information by either party is a possibility (Pisano [23]).

Coordination problems also seem pervasive. In our idealized model, they are solved by efficient negotiations at the interim stage. In reality, however, coordination failure is not uncommon. Microsoft and Nokia initially tried to cooperate under nonintegration, although most Windows Phone devices sold were made by Nokia, and Nokia exclusively produced Windows Phone devices (O'Brien [22]). The nonintegrated companies each controlled assets such as patents that gave them good outside options. In terms of our model, this should have made it difficult to maximize surplus, and perhaps they would have to agree on a second-best outcome. Warren [28] describes how in fact they failed to coordinate their decisions:

“There are real-world examples of situations where Nokia was building a phone and keeping information about it secret from us,” said Joe Belfiore, a corporate vice president at Microsoft who's in charge of the company's Windows Phone project. “We would make changes in the software, or prioritize things in the software, unaware of the work that they're doing. And then late in the cycle we'd find

¹⁸Pisano [23] also describes the different ownership structures in the biotech industry: one firm may own a controlling share in a separate jointly controlled venture or become a large but minority shareholder in its trading partner; or two firms may have fifty-fifty ownership and control specific parts of the joint venture, say with one controlling decisions related to R&D and the other decisions related to marketing.

out and say, ‘If we had known that we would have done this other thing differently and it would have turned out better!’ ” (Warren [28])

This coordination failure suggests that decision making did not quite meet our assumption of efficient interim contracting. Eventually, Microsoft purchased Nokia’s Devices and Services unit. Integration consolidated patents in the hands of Microsoft, removed Nokia’s outside option, and made coordination easier (Wingfield [35]).

Our model assumes efficient interim contracting in order to establish a benchmark and derive intuitions about key variables. There are several ways in which this assumption could be relaxed. In particular, the agents may be unable to commit not to renegotiate an inefficient outcome. As is well known, this will make it more difficult to reach an efficient agreement. For example, suppose after an agent has rejected the proposed mechanism, renegotiation may occur; perhaps they can engage a new mediator to help them coordinate. This will make the IR constraints more difficult to satisfy for the “original” mediator, and the attainable surplus will fall (under both integration and nonintegration). Formalizing this requires facing a possible infinite regress: if the renegotiation under the “new mediator” breaks down, a disagreement point must be specified, which in turn may be renegotiated under a “new new mediator”, etc. Gains from coordination and rents from hold up may interact in different ways under renegotiation which is an interesting topic for future study.

Finally, we have not studied the second-best. For instance, in the uncolored area in Figure 6, neither integration nor nonintegration can implement the first best, so the mediator has to choose a second-best allocation of control rights at stage 0. This poses no philosophical problem for our approach as either integration or nonintegration will be optimal so there will still be a theory of optimal allocation of decision-rights based on haggling. However, the calculation of the second best is complex, as it involves optimal constrained allocations with type-dependent participation constraints. If F is uniform, it can be shown computationally that integration should always be chosen in the rent-seeking case. In the cooperative case, nonintegration is the second-best for a range of parameters that borders the region where it is the first-best. The qualitative principle that underlies the analysis of the first-best are valid also for the second-best: the optimal allocation of control rights minimizes the total value of outside options for the types that are most likely to walk away from negotiations, which makes it easier to pay the information rents required for incentive-compatibility. But the precise conditions under which each institution is optimal and the nature of distortions away from efficiency must be the topic for future work.

References

- [1] Ricardo Alonso, Wouter Dessein and Niko Matouschek (2008): “When Does Coordination Require Centralization?,” *American Economic Review* 98 (1), 145-179.

- [2] George Baker, Robert Gibbons and Kevin Murphy (1999) “Informal Authority in Organizations” *Journal of Law, Economics and Organization* 15 (1): 56-73.
- [3] Claude d’Aspremont and L. A. Gérard-Varet (1979): “Incentives and Incomplete Information,” *Journal of Public Economics*, 11, 25-45.
- [4] Peter Cramton, Robert Gibbons, and Paul Klemperer (1987): “Dissolving a Partnership Efficiently,” *Econometrica*, 55, 615–632.
- [5] Melvin Aron Eisenberg (1995): “The Limits of Cognition and the Limits of Contract,” *Stanford Law Review*, 47, 211-259.
- [6] Robert Gibbons (2005): “Four Formal(izable) Theories of the Firm,” *Journal of Economic Behavior and Organization*, 58, 202-47.
- [7] Sanford J. Grossman and Oliver D. Hart (1986): “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration,” *Journal of Political Economy*, 94, 691–719.
- [8] Maija Halonen (2002): “Reputation and The Allocation of Authority,” *Economic Journal*, 112, 539–558.
- [9] Oliver Hart (1995): *Firms, Contracts and Financial Structure*, Oxford University Press, Oxford.
- [10] Oliver Hart and Bengt Holmström (2010): “A Theory of Firm Scope”, *Quarterly Journal of Economics*, May, 125: 483-513.
- [11] Bengt Holmström and John Roberts (1998): “The Boundaries of the Firm Revisited,” *Journal of Economic Perspectives*, 12 (4), 73-94.
- [12] Benjamin Klein, Robert Crawford, Armen Alchian (1978): “Vertical Integration, Appropriable Rents and the Competitive Contracting Process,” *Journal of Law and Economics*, 21, 297-326.
- [13] Oliver D. Hart and John Moore (1990), “Property Rights and the Nature of the Firm,” *Journal of Political Economy*, 98, 1119–1158.
- [14] Vijay Krishna and Motty Perry (2000): “Efficient Mechanism Design,” mimeo, Penn State.
- [15] Jean-Jacques Laffont and Eric Maskin (1979): “A differentiable approach to expected utility-maximizing mechanisms”, in J.-J. Laffont (ed.), *Aggregation and Revelation of Preferences*. North-Holland, Amsterdam.
- [16] Louis Makowski and Claudio Mezzetti (1994), “Bayesian and Weakly Robust First Best Mechanisms: Characterizations,” *Journal of Economic Theory*, 64, 500–519.

- [17] Eric Maskin and Jean Tirole (1999), “Unforeseen Contingencies and Incomplete Contracts,” *Review of Economic Studies*, 66:83-114.
- [18] Niko Matouschek (2004): “Ex Post Inefficiencies in a Property Rights Theory of the Firm,” *Journal of Law, Economics, & Organization*, 20 (1), 125-147.
- [19] Paul Milgrom and John Roberts (1990): “Bargaining costs, influence costs, and the organization of economic activity,” in J. E. Alt and K. A. Shepsle (eds.) *Perspectives on positive political economy*, Cambridge University press, 57-89.
- [20] Roger B. Myerson and Mark A. Satterthwaite (1983), “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory*, 29, 265–281.
- [21] Zvika Neeman (1999): “Property Rights and Efficiency of Voluntary Bargaining Under Asymmetric Information,” *Review of Economic Studies*, 66, 679–691.
- [22] Kevin O’Brien (2011): “Together, Nokia and Microsoft Renew a Push in Smartphones,” *New York Times*, Feb 12.
- [23] Gary Pisano (1989): “Using Equity Participation to Support Exchange: Evidence from the Biotechnology Industry,” *Journal of Law, Economics and Organization*, 5, 109-126.
- [24] Heikki Rantakari (2008): “Governing Adaptation,” *Review of Economic Studies*, 75(4), 1257-1285.
- [25] Ariel Rubinstein (1985): “Choices of Conjectures in a Bargaining Game with Incomplete Information,” in A. Roth (ed.), *Game Theoretic Models of Bargaining*, 99-114, Cambridge University Press.
- [26] Patrick W. Schmitz (2006): “Information Gathering, Transaction Costs, and the Property Rights Approach,” *American Economic Review*, Vol. 96, 422-434.
- [27] Steven Tadelis (2002): “Complexity, Flexibility and the Make or Buy Decision,” *American Economic Review: Papers and Proceedings*, 92(2):433-437.
- [28] Tom Warren (2013): “Nokia kept secrets from Microsoft over its Windows Phone plans,” *CNET*, September 7.
- [29] Steven R. Williams (1999): “A Characterization of Efficient, Bayesian Incentive Compatible Mechanisms,” *Economic Theory*, 14, 155–180.
- [30] Oliver E. Williamson (1971): “The Vertical Integration of Production: Market Failure Considerations,” *American Economic Review*, 61, 112-123.
- [31] Oliver E. Williamson (1975): *Markets and Hierarchies: Analysis and Antitrust Implications*, Free Press, New York, N.Y.

- [32] Oliver E. Williamson (1979) “Transaction-Cost Economics: The Governance of Contractual Relations” *Journal of Law and Economics* Vol. 22, No. 2, 233-261
- [33] Oliver E. Williamson (1983): “Credible Commitments: Using Hostages To Support Exchange,” *American Economic Review*, 73(4), 519-540.
- [34] Oliver E. Williamson (1985): *The Economic Institutions of Capitalism*, Free Press, New York, N.Y.
- [35] Nick Wingfield (2013): “Microsoft Gets Nokia Units, and Leader,” *New York Times*, Sept 3.

9 Appendix

Proof of Lemma 1. By symmetry, it suffices to consider agent A . The IR constraints require that agent A 's payoff under Γ , which is either (5), (6) or (7) depending on θ_A , exceeds the reservation payoff, which is either (16) or (17), again depending on θ_A . Thus, we consider the possible cases that can occur.

If $\theta_A \leq \min\{\theta^*, -2\gamma\}$, then the IR constraint is that (5) should be no less than (16), that is,

$$u_A(\theta_A) = \underline{u}_A \geq -\rho + (\gamma + \rho)F(\theta^*).$$

If $\min\{\theta^*, -2\gamma\} \leq \theta_A \leq \max\{\theta^*, -2\gamma\}$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \int_{-2\gamma}^{\theta_A} F(s)ds \geq -\rho + (\gamma + \rho)F(\theta^*).$$

If $\max\{\theta^*, -2\gamma\} \leq \theta_i \leq 2\gamma$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \int_{-2\gamma}^{\theta_A} F(s)ds \geq \theta_A + \gamma + (\rho - \gamma)F(\theta^*).$$

If $\theta_i \geq 2\gamma$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \theta_A \geq \theta_A + \gamma + (\rho - \gamma)F(\theta^*). \quad (29)$$

Notice that

$$\gamma + (\rho - \gamma)F(\theta^*) > -\rho + (\gamma + \rho)F(\theta^*)$$

using (15) and the fact that $\theta^* < 0$. Therefore, the individual rationality constraint for $\theta_A = 2\gamma$, which can be written as

$$\underline{u}_A \geq \gamma + (\rho - \gamma)F(\theta^*),$$

implies all the others. ■

Proof of Lemma 5. The proof is analogous to the proof of Lemma 1. The IR constraints require that the payoff under the mechanism, which is either (5), (6) or (7) depending on the agent's type, exceeds the reservation payoff, which is (21) for agent A and (22) for agent B . Thus, we consider the possible cases that can occur. First, we consider agent A .

If $\theta_A \leq -2\gamma$ then the IR constraint is that (5) should be no less than γ , that is,

$$u_A(\theta_A) = \underline{u}_A \geq \gamma.$$

If $-2\gamma \leq \theta_A \leq 0$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \int_{-2\gamma}^{\theta_A} F(s)ds \geq \gamma.$$

If $0 \leq \theta_A < 2\gamma$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \int_{-2\gamma}^{\theta_A} F(s)ds \geq \gamma + \theta_A.$$

If $\theta_A \geq 2\gamma$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \theta_A \geq \gamma + \theta_A.$$

It is easy to see that all of agent A 's IR constraints are satisfied if the IR constraint holds for type $\theta_A = 2\gamma$, which is (23).

For agent B , the individual rationality constraints are as follows.

If $\theta_B \leq -2\gamma$, then the IR constraint is

$$u_B(\theta_B) = \underline{u}_B \geq \gamma + \frac{1}{2}\theta_B.$$

If $-2\gamma \leq \theta_B \leq 2\gamma$, then the IR constraint is

$$u_B(\theta_B) = \underline{u}_B + \int_{-2\gamma}^{\theta_B} F(s)ds \geq \gamma + \frac{1}{2}\theta_B.$$

If $\theta_B \geq 2\gamma$, then the IR constraint is

$$u_B(\theta_B) = \underline{u}_B + \theta_B \geq \gamma + \frac{1}{2}\theta_B.$$

It is easy to check that all of agent B 's IR constraints are satisfied if the IR constraint holds for type $\theta_B = 0$, which is (24). ■

Proof of Lemma 8. The proof is analogous to the proofs of Lemmas 1 and 5. Suppose first that $\gamma - \rho < -2\gamma$. Then, the individual rationality constraints for agent A are as follows.

If $\theta_A \leq \gamma - \rho$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A \geq \gamma. \tag{30}$$

If $\gamma - \rho \leq \theta_A \leq -2\gamma$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A \geq \theta_A + \rho. \quad (31)$$

If $-2\gamma \leq \theta_A \leq 2\gamma$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \int_{-2\gamma}^{\theta_A} F(s) ds \geq \theta_A + \rho. \quad (32)$$

If $\theta_A \geq 2\gamma$, then the IR constraint is

$$u_A(\theta_A) = \underline{u}_A + \theta_A \geq \theta_A + \rho.$$

If instead $\gamma - \rho > -2\gamma$ then (30) applies when $\theta_A \leq -2\gamma$, (32) applies when $\gamma - \rho \leq \theta_A \leq 2\gamma$, and (31) is replaced by the following IR constraint for the case $-2\gamma \leq \theta_A \leq \gamma - \rho$:

$$u_A(\theta_A) = \underline{u}_A + \int_{-2\gamma}^{\theta_A} F(s) ds \geq \gamma.$$

In either case, it can be checked that all of agent A 's individual rationality constraints hold if and only if the individual rationality constraint holds for type $\theta_A = 2\gamma$, which is (26).

For agent B , the individual rationality constraints are as follows.

If $\theta_B \leq -2\gamma$ then the IR constraint is

$$\underline{u}_B \geq -\rho + F(\gamma - \rho)(\gamma + \rho).$$

If $-2\gamma \leq \theta_B \leq 2\gamma$, then the IR constraint is

$$u_B(\theta_B) = \underline{u}_B + \int_{-2\gamma}^{\theta_B} F(s) ds \geq -\rho + F(\gamma - \rho)(\gamma + \rho).$$

If $\theta_B \geq 2\gamma$, then the IR constraint is

$$u_B(\theta_B) = \underline{u}_B + \theta_B \geq -\rho + F(\gamma - \rho)(\gamma + \rho).$$

It can be checked that all of agent B 's individual rationality constraints hold if and only if the individual rationality constraint holds for $\theta_B = -2\gamma$, which is (27). ■

Proof of Lemma 3. The left side of (19) can be written as

$$\begin{aligned} \int_{-2\gamma}^{2\gamma} (F(s))^2 ds &= \int_{-2\gamma}^0 (1 - F(-s))^2 ds + \int_0^{2\gamma} (F(s))^2 ds \\ &= \int_0^{2\gamma} \left[(F(s))^2 + (1 - F(s))^2 \right] ds. \end{aligned}$$

Since $1/2 < (F(s))^2 + (1 - F(s))^2 < 1$, this expression lies strictly between γ and 2γ . Therefore, if $\rho \geq \gamma$, it is impossible to satisfy (19). If ρ is close to 0,

then θ^* is close to 0 and the right hand side of (19) is close to γ . If ρ is close to γ , then the right hand side of (19) is close to 2γ .

We now claim that the right hand side of (19) strictly increases from γ to 2γ as ρ increases from 0 to γ . As the left hand side lies strictly between γ and 2γ and is independent of ρ , this claim will prove the lemma.

From (15), we get

$$\frac{d\theta^*}{d\rho} = \frac{1}{2F'(\theta^*)\gamma - 1} < 0.$$

The derivative of the right side of (19) with respect to ρ is

$$2F(\theta^*) + 2(\rho - \gamma)F'(\theta^*)\frac{d\theta^*}{d\rho} = 2F(\theta^*) + 2(\rho - \gamma)F'(\theta^*)\frac{1}{2F'(\theta^*)\gamma - 1}.$$

We claim this is positive. This is equivalent to showing

$$F(\theta^*) > \frac{(\rho - \gamma)F'(\theta^*)}{1 - 2F'(\theta^*)\gamma}$$

which is the same as

$$F(\theta^*) > (\rho - \gamma + 2\gamma F(\theta^*))F'(\theta^*) = (2\rho + \theta^*)F'(\theta^*) \quad (33)$$

where the equality uses (15). Because $2F'(\theta^*)\gamma < 1$,

$$(2\rho + \theta^*)F'(\theta^*) < (2\rho + \theta^*)/2\gamma.$$

Therefore, to show (33) holds we need to show that $2F(\theta^*)\gamma > 2\rho + \theta^*$. This is true because of (15) and $\rho < \gamma$. ■