The Strategy and Technology of Conflict*

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Abstract

Using a simple bargaining game, we investigate how strategic interactions are shaped by preferences, technology and endowments. We study if changes in relative military capabilities make conflicts more likely, and find a non-monotonic relationship between the cost of conflict and the probability of conflict. The game has strategic complements if the cost of conflict is small and first-mover advantage is large, and strategic substitutes otherwise. This characterization generates predictions regarding strategic investments, e.g., in defense systems. An extension shows how expanding one's territory today may increase the risk of conflict tomorrow.

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1 Introduction

Russia's annexation of Crimea, and China's island-building in the South China Sea, are recent examples of what Schelling (1960) called "strategic moves". A strategic move presents the other side with the stark choice of either conceding and losing the contested territory for sure, or resisting and risking a major confrontation. Such strategic moves have a rich history. After World War II, the Soviet Union gained the first-mover advantage in Eastern Europe by occupying it in violation of the Yalta agreement.[1] If the West had not conceded, for example in Czechoslovakia or Hungary, a military confrontation would have been quite likely, because the Soviets could not have retreated from these countries without a massive loss of reputation. Conversely, US soldiers stationed in Western Europe represented "the pride, the honor, and the reputation of the United States government and its armed forces" (Schelling (1966), p. 47). Therefore, the Soviet Union had every reason to believe that the United States would resist an attack on Western Europe. An East-West confrontation was avoided because the Soviets conceded Western Europe just as the West had conceded Eastern Europe.

A confrontation may be unavoidable if two opposing sides move into the same territory. For example, China and India contested vast border territories, and "[t]he military forces of both sides began pushing into remote and previously mostly unoccupied mountainous frontier regions in 1958 and 1959" (Garver (2006), p. 105). These moves have been attributed to domestic political concerns.[2] Neither side wanted a war, but having made the commitment, neither side was able to back down. War resulted from "the movement and stationing of Chinese and Indian security personnel. They acted in a competitive fashion, and incidents were bound to occur, particularly because

[1] At the Yalta conference in February 1945, it was agreed that the Soviet Union would recover the territory it had lost after 1941. Elsewhere there were supposed to be free elections and democratic governments.

[2] Mao was struggling with the consequences of agricultural collectivization, and his confrontational international policies have often been attributed to his domestic problems (Garver (2006), p. 123). In India, China's repression of Tibet led to intense criticism of Nehru's policy of befriending China. Nehru's "forward policy" established military outposts in the contested border areas. According to Garver (2006), if China had given Nehru "a few face-saving sops for him regarding Tibetan 'autonomy' that Nehru could use in fending off his domestic critics, Nehru might not have felt compelled to prove his toughness on the border issue. Instead of adopting the forward policy, he might have stood by a still-not-discredited friendship policy"(Garver (2006), p. 103).

jurisdictions and border markings had never been jointly defined. The Indians hardened their stance on the borders after a major incident occurred not in the Assam Himalaya but near the Aksai Chin. The conflict spiral possessed a momentum of its own and culminated in the Indian-Chinese border war of October-November 1962" (Hoffmann (2006) p. 183).

We study a simple two-player bargaining game that captures the idea of a strategic move as a "voluntary but irreversible sacrifice of freedom of choice" (Schelling (1960)). Each player may challenge the status quo by demanding an increased share of a contested territory. A costly conflict occurs if both players challenge (since this implies mutually incompatible commitments), or if only one player challenges but the other refuses to concede. In the latter case, the challenger may have a first-mover advantage.

As the optimal challenge is the largest demand the opponent would concede to, the game can be represented as a two-by-two coordination game with actions Hawk (optimal challenge) and Dove (no challenge). The cost of choosing Hawk is a player's privately known *type*. We show how the payoffs in this game can be derived from primitives (such as costs of conflict, relative military strength and endowments, first-mover advantages and risk-aversion), and study how these primitives determine the outcome of the strategic interaction. We also characterize conditions under which actions are strategic complements or strategic substitutes. This characterization is important, as the two scenarios generate dramatically different predictions and policy recommendations.[3] It provides a rigorous foundation for Schelling's ((1960), (1966)) informal discussion of coordination failure amplified by uncertainty as a source of conflict, and Jervis's (1978) discussion of stag hunt (strategic complements) and chicken (strategic substitutes) as canonical conflict games.

An increase in the first-mover advantage increases the probability of conflict in our model. More surprisingly, an increase in the cost of conflict can have the same effect. It encourages the players to challenge the status quo, knowing that the opponent will be willing to make larger concessions to avoid a costly conflict. This logic may explain why many provocations, challenges and proxy wars (Korea, Vietnam...) occurred during the Cold War. For

[3] For example, Baliga and Sjöström (2012) found that with strategic complements, hawkish third parties ("extremists") can trigger conflicts by sending messages that create "fear spirals". With strategic substitutes, hawkish extremists cannot do this – instead dovish third parties ("pacifists") can prevent conflicts by persuading one side to back down. The current model provides a micro foundation which makes it possible to characterize the two scenarios in terms of deeper parameters.

example, Khrushchev assisted the Cuban revolution in 1960, in defiance of the "Truman doctrine". Apparently, he was convinced that the U.S., if faced with a *fait accompli*, would back down rather than risk a major war.[4]

Allison (2015) warned that the rise of China may lead to a "Thucydides trap", with an increased risk of conflict. Even the Chinese leaders are concerned.[5] In Section 5, we assume a "rising power" is militarily weaker and controls a smaller share of the disputed territory than the opponent (the "status quo power"). As the rising power becomes stronger it becomes more hawkish, while the status quo power becomes more dovish. The net effect is ambiguous in general, but if the status quo power is much stronger initially, or if the share of territory it controls corresponds to its greater strength, then the risk of conflict increases when the rising power's military strength increases slightly. However, when the rising power become sufficiently strong then the status quo power becomes sufficiently dovish that the risk of conflict decreases.

When actions are strategic complements, aggression is triggered by fear of the opponent, as in Schelling's (1960) analysis of the reciprocal fear of surprise attack. With strategic substitutes, aggression is instead due to a lack of fear. Our game has strategic substitutes if the cost of conflict is high.[6] This is consistent with the notion that the Cold War was a sequence of chicken races. But if the cost of conflict is low, then the game has strategic complements if utility functions are strictly concave and there is a significant first-mover advantage. This result is not obvious, because there are two opposing effects: when the first-mover advantage increases, the cost of choosing Dove when the opponent chooses Hawk increases, but so does the benefit from choosing

[4] During the Berlin crisis, Khrushchev told an American visitor that Berlin was not worth a war to the US. Khrushchev was then asked whether it was worth a war to the Soviet Union. "No", he replied, "but you are the ones that have to cross a frontier" (Schelling (1966), p. 46) – implying that it would be the West's decision to risk a war by entering East Germany after the Soviets had already occupied it. More recently, Pakistan has employed militants and terrorists to attack the Indian Parliament, Jammu and Mumbai, despite the high potential cost of a conflict between nuclear powers.

[5] "We all need to work together to avoid the Thucydides trap – destructive tensions between an emerging power and established powers" (Presidient Xi Jinping as reported by Valencia (2014)).

[6] This result depends on the assumption that, as in a game of chicken, a dovish player never risks a conflict. If the players were afraid of surprise attacks that automatically lead to conflict, rather than of strategic moves in Schelling's sense, then a high cost of conflict and large first-mover advantages could produce strategic complements.

Hawk when the opponent chooses Dove. The first effect tends to generate strategic complements, while the second effect does the opposite. As long as the marginal utility of land is decreasing, the cost of losing territory exceeds the benefit of acquiring the same amount, so the first effect dominates and the game has strategic complements. If instead the marginal utility of land had been increasing (perhaps due to the contested territory being considered largely indivisible), then the game would have had strategic substitutes. This shows how the primitives determine whether actions are strategic substitutes or complements. The primitives in turn depend on the properties of the contested resource and on the technology of war, which has changed throughout history (see the concluding section).

A controversial Cold War issue was whether investments in defensive systems that reduce the cost of conflict (by protecting US cities against a missile attack) would be destabilizing (Rothstein (1968)). Eventually, President Reagan went ahead with his "Star Wars" program. In Section 7 we show that such investments have two effects familiar from the work of Schelling (1960). First, the US will look tougher (less restrained by fear of conflict), making the Soviets more fearful of US strategic moves. But second, Soviet strategic moves become less profitable as the US becomes less willing to make concessions. If actions are strategic substitutes, as suggested by the interpretation of the Cold War as a sequence of chicken races, the two effects reinforce each other, and the model predicts over-investment (in the sense of Fudenberg and Tirole (1984)) in the defensive technology.

The model also predicts that a player with a low cost of conflict will over-invest in defensive systems, regardless of whether actions are strategic substitutes or complements. For example, if the Palestinians do not have access to sophisticated weaponry, Israel will over-invest in defensive walls. A more subtle strategic investment is to raise the value of one's territory by, for example, building settlements on it. Intuition may suggest that this investment is just another way to "look tough", but in our model this intuition is quite misleading. Since a conflict may lead to the loss of *all* the (divisible) territory, the settlements make the investor *more* willing to give up *some part* of it to avoid a conflict. This encourages the opponent to challenge the status quo. Building settlements may therefore be strategically disadvantageous, even if the game has strategic substitutes.

Our basic bargaining game is one-shot, with no "shadow of the future". However, we would also like to answer questions such as: "When will force create a spiral of hostility? When will concessions lead to reciprocations and when will they lead the other side to expect further retreats?" (Jervis (1976), p. 96). To see how such questions can be addressed in our framework, in Section 8 we consider a dynamic extension. Actions have long-run consequences because today's outcome becomes the new status quo, and an opponent who loses territory today may try to recapture it tomorrow. Farsighted players must therefore trade off the short-run gain from a strategic move against the long-run cost of facing a more aggressive opponent.

2 Related Literature

The game-theoretic literature on commitment in bargaining traces its origins to Nash's (1953) demand game and Schelling's (1960) seminal discussion. The well-known ultimatum game is the simplest model of *one-sided* commitment. With complete information it has an efficient equilibrium, but Fearon (1995) showed how asymmetric information and various forms of non-transferable utility can lead to inefficiencies. His seminal discussion inspired a large literature (see Baliga and Sjöström (2013) for a survey). This literature does not study the coordination problem that arises when *both* sides may try to commit. The two-sided commitment problem was discussed by Schelling (1960), using a metaphor of haggling over the price of a house.[7] Another type of coordination problem, involving a reciprocal fear of surprise attack, was illustrated by Schelling's (1960) metaphor of a night-time burglary.[8] Jervis (1978) suggested that chicken and stag-hunt games are useful

[7] Schelling (1960) argued that either side could benefit by delivering an ultimatum: "If each party knows the other's true reservation price, the object is to be first with a firm offer. Complete responsibility rests with the other, who can take it or leave it as he chooses (and who chooses to take it). Bargaining is all over; the commitment (that is, the first offer) wins" (Schelling (1960), p. 26). He then described the coordination problem: "Interpose some communication difficulty. They must bargain by letter; the invocation becomes effective when signed but cannot be known to the other until its arrival. Now when one person writes such a letter the other may already have signed his own or may yet do so before the letter of the first arrives. There is then no sale; both are bound to incompatible positions" (Schelling (1960), p. 26).

[8] "If I go downstairs to investigate a noise at night, with a gun in my hand, and find myself face to face with a burglar who has a gun in his hand, there is a danger of an outcome that neither of us desires. Even if he prefers to leave quietly, and I wish him to, there is a danger that he may *think* I want to shoot, and shoot first. Worse, there is danger that he may think that *I* think *he* wants to shoot. Or he may think that *I* think

lenses through which different kinds of conflicts can be viewed. Intuition suggests that Schelling's haggling and burglar scenarios correspond to chicken (strategic substitutes) and stag-hunt (strategic complements), respectively. We will pinpoint the distinction between the two scenarios in a bargaining model with two-sided commitment, and show how intuition can be enhanced by a formal analysis.

Crawford (1982) provided a formal model of two-sided commitment. In his model, there is symmetric information when demands are made; if the demands are incompatible then the players learn their cost of backing down in a second stage. Crawford's model will typically have multiple equilibria, but Ellingsen and Miettinen (2008) showed that if it is costly to make a commitment then the number of equilibria can be reduced. Specifically, if commitment costs are small enough, and if commitment attempts only succeed with probability $q < 1$, then there is a unique equilibrium where both parties attempt to make the maximum commitment with probability 1. Their model has complete information, yet if the parties have access to efficient commitment technologies, so that q is close to 1, then the probability of an inefficient conflict is also close to 1 (an agreement is almost never reached). In our model, a player's decision to make a commitment depends on his private information. This leads to a unique equilibrium where an inefficient conflict occurs with some probability. But, depending on the primitives, this probability can be small even though the parties have efficient commitment technologies (commitments are irrevocable). More recently, Ellingsen and Miettinen (2014) studied an infinite horizon complete-information model with a unique Markov perfect equilibrium. The equilibrium takes the form of a war of attrition: an agreement is always reached, but only after a commitment has decayed (implying an inefficient delay).

Several papers have studied incomplete-information coordination games with a reduced-form payoff matrix, assuming strategic complements (see Ramsay (2017) for a recent survey). Chassang and Padro i Miquel (2010) showed that increasing weapons stocks can increase the probability of conflict by increasing preemption rather than deterrence. Acharya and Ramsay (2013) showed that communication might not defuse tensions in a global game model. Bueno de Mesquita (2010) studied a coordination model of revolution, where a vanguard group might try to mobilize citizens. In contrast to these papers, our model is founded on bargaining over divisible ter-

he thinks *I* want to shoot. And so on"(Schelling (1960), p. 207).

ritory as in Fearon (1995), which allows us to study a host of different issues. Meirowitz, Morelli, Ramsay and Squintani (2019) adopted a rather different approach, where a private arming decision is followed by communication and bargaining. Finally, Abreu and Gul (2000) and Kambe (1999) found a unique equilibrium in an infinite horizon war of attrition with a positive probability that a player might be a "commitment type". By considering conflict to be a war of attrition, this line of research could perhaps be unified with ours.

3 The Bargaining Game

There are two players, A and B. In the status quo, player $i \in \{A, B\}$ controls a share $\omega_i \in (0, 1)$ of a disputed territory, his *endowment*, where $\omega_A + \omega_B = 1$. Player i's utility of controlling a share σ_i is $u_i(\sigma_i)$, where u_i is an increasing, strictly concave and differentiable function on $[0, 1]$. Without loss of generality we normalize so that $u_i(1) = 1$ and $u_i(0) = 0$. If a conflict occurs, then player i suffers a cost $\phi_i > 0$.

The bargaining game has two stages. In stage 1, player $i \in \{A, B\}$ can either make a claim σ_i, where $\omega_i < \sigma_i \leq 1$, or make no claim. A claim is a *challenge* (to the status quo) which incurs a cost c_i for the challenger. We interpret player i's challenge as a non-revokable instruction to player i's military to cross the status quo demarcation and occupy a fraction σ_i of the contested territory. Aside from the direct material cost (manpower, physical resources), the incursion may be condemned by the international community, leading to a loss of reputation and goodwill, possible sanctions and embargoes, etc.[9] A challenge may also generate benefits that reduce c_i. Such benefits may come from "looking tough" to a domestic audience, or simply from diverting attention from difficult domestic issues (c.f. footnote 2). In addition, recent work on fairness and reciprocity suggests that players who consider the status quo unjust may derive an intrinsic benefit from attempting to correct the perceived injustice (e.g., de Quervain et al. (2004)). These economic, political (international or domestic) and psychological costs/benefits jointly determine c_i.[10]

[9] We assume c_i is independent of whether player j also makes a challenge. In reality, censures may be more severe if only one player challenges the status quo. We show in Appendix A how the model can be generalized in this and other directions without changing the main insights.

[10] Note that $c_i < 0$ if the benefits exceed the costs.

Let λ_i be a parameter that represents player i's relative military strength, where $\lambda_A + \lambda_B = 1$. Let θ be a parameter that measures the first-mover advantage or disadvantage (to be explained below).

The game ends after stage 1 if either no player makes a claim, or both make claims. Stage 2 is reached if only one player makes a claim, in which case the other player (having observed the claim) chooses to concede or not to concede. The final outcome is determined by three rules.

Rule 1. If nobody challenges in stage 1, then the status quo remains in place.

Rule 2. If only player i challenges, and claims $\sigma_i > \omega_i$ in stage 1, then we move to stage 2. In stage 2, if player $j \neq i$ concedes to player i's claim then player i gets σ_i and player j gets $1 - \sigma_i$. If player j does not concede, there is a conflict: with probability $\lambda_i + \theta$, player i (the challenger) wins and takes all of the territory; with probability $\lambda_j - \theta = 1 - (\lambda_i + \theta)$ player j wins and takes all of the territory.

Rule 3. If both players challenge the status quo in stage 1 then there is a conflict. Player i wins, and takes all of the territory, with probability λ_i.

We interpret these rules as follows. If neither player challenges the status quo, then there is no reason why either player should retreat from his initial position, so the status quo remains in place. If only player i makes a challenge then he becomes the *first-mover* and player j the *second-mover*. The challenge is a strategic move in the sense of Schelling (1960): an irrevocable commitment to start a conflict unless player j concedes and lets player i increase his share of the territory to σ_i. If player j does not concede then there is a conflict which player i wins with probability $\lambda_i + \theta$; player i cannot back down and avoid a conflict at this point.[11] Finally, if *both* players challenge the status quo, a conflict occurs because they have made mutually incompatible commitments. Neither player has a first-mover advantage in this case, so the conflict is won by player i with probability λ_i.[12]

[11] As discussed in the Introduction, a player who backs down after making a strategic move would incur an intolerable loss of pride, honor and reputation.

[12] A more general formulation would be that if both decide to challenge, there is some probability $\alpha > 0$ that player $i \in \{A, B\}$ can commit first, in which case player j must decide whether or not to concede. Thus, each player would have a probability α of getting the first-mover advantage. With probability $1 - 2\alpha$, both become committed and there is a conflict where neither has the first-mover advantage. Similarly, like Ellingsen and Miettinen (2008) we could assume that a challenge only leads to a successful commitment

The parameter θ represents the increase in the probability of player i winning when he is a first-mover, compared to the "baseline" of λ_i. We allow θ to be positive (a first-mover advantage) or negative (a second-mover advantage). Since $\lambda_i + \theta$ is a probability, we assume $0 < \lambda_i + \theta < 1$ for $i \in \{A, B\}$. Note that this implies $\theta < 1/2$.

Suppose stage 2 is reached. If player i is the second-mover and concedes to the claim σ_j he gets $u_i(1 - \sigma_j)$. If he doesn't concede, he gets expected payoff

$$(\lambda_j + \theta)u_i(0) + (1 - (\lambda_j + \theta))u_i(1) - \phi_i = \lambda_i - \theta - \phi_i, \tag{1}$$

since $\lambda_i = 1 - \lambda_j$, $u_i(1) = 1$ and $u_i(0) = 0$. Thus, player i prefers to concede if

$$u_i(1 - \sigma_j) \geq \lambda_i - \theta - \phi_i. \tag{2}$$

This is satisfied for $\sigma_j = 1$ if

$$\phi_i \geq \lambda_i - \theta. \tag{3}$$

If (3) holds then when player i is the second-mover he would rather concede the whole territory than have a conflict. If (3) is violated, i.e., if

$$\phi_i < \lambda_i - \theta, \tag{4}$$

then the maximum claim σ_j player i will concede to satisfies (2) with equality, that is,

$$1 - \sigma_j = u_i^{-1}\left[\lambda_i - \theta - \phi_i\right]. \tag{5}$$

Thus, in general, the maximum claim player i would concede to in stage 2 is the claim $\sigma_j = 1 - \eta_i$, where

$$\eta_i \equiv \begin{cases} u_i^{-1}\left[\lambda_i - \theta - \phi_i\right] & \text{if} \quad \phi_i + \theta < \lambda_i, \\ 0 & \text{if} \quad \phi_i + \theta \geq \lambda_i. \end{cases} \tag{6}$$

Notice that if (4) holds then η_i is defined implicitly by

$$u_i(\eta_i) = \lambda_i - \theta - \phi_i \tag{7}$$

and satisfies $\eta_i > 0$. Equation (7) says that player i is indifferent between the share η_i and a conflict when he is the second-mover (c.f. equation (1)). Notice that η_i is decreasing in ϕ_i. The more costly a conflict would be, the more territory the second-mover is willing to concede.

To make the problem interesting, we will assume:

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with probability $q < 1$. Since these generalizations do not add any significant insights, we set $\alpha = 0$ and $q = 1$ for simplicity.

Assumption 1 $\eta_i < \omega_i$ for $i \in \{A, B\}$.

Assumption 1 implies that if the first-mover's claim is sufficiently "modest", i.e., close to the status quo, then the second-mover prefers to concede. Assumption 1 rules out the less interesting case where the second-mover would never concede (even to an arbitrarily small change in the status quo). Assumption 1 is equivalent to the inequality

$$u_i(\omega_i) > \lambda_i - \theta - \phi_i. \tag{8}$$

The left-hand side of (8) is player i's payoff from the status quo, and the right hand side is his expected payoff from Rule 2 when he is the second-mover and does not concede. Assumption 1 can also be re-written as

$$\lambda_j + \theta > 1 - u_i(\omega_i) - \phi_i. \tag{9}$$

This reveals that if the cost of conflict is high enough, specifically if $\phi_i > 1 - u_i(\omega_i)$, then Assumption 1 is automatically satisfied. Note also that strict concavity implies $u_i\left(\frac{1}{2}\right) > \frac{1}{2}u_i(1) + \frac{1}{2}u_i(0) = \frac{1}{2}$. Therefore, in the symmetric case where $\omega_i = 1/2$ the right-hand side of (9) is less than $1/2$, so Assumption 1 is satisfied whenever $\lambda_j + \theta \geq 1/2$, i.e., as long as the first-mover has at least an even chance of winning.

Games such as chicken and stag-hunt, which are often used as metaphors for conflicts, have multiple equilibria when the true payoffs are common knowledge. This raises the difficult issue of equilibrium selection. For example, who backs down in a chicken race? But common knowledge of payoffs is an unrealistic assumption. For example, Mao and Nehru could probably not fully understand each other's domestic concerns prior to the India-China war (see footnote 2). If there is significant uncertainty about payoffs, the indeterminacies disappear. The following assumption leads to a particularly simple analysis. We assume all parameters of the game are commonly known, with one exception: player i's cost of making a challenge is not known to player j. Formally, for each $i \in \{A, B\}$, the cost c_i is independently drawn from a distribution F with support $[\underline{c}, \overline{c}]$ and density $f(c) = F'(c)$. Player $i \in \{A, B\}$ knows c_i but not c_j. We refer to c_i as player i's type.

If the support of F is small, or if the support is large but the density is highly concentrated around one point, then the players are fairly certain about each others' types, and there can be multiple equilibria. We rule this out by assuming that (i) the support of F is quite large, and (ii) the density is quite "flat". To simplify the expressions, define

$$\Omega_i \equiv \lambda_i - \phi_i - u_i(\eta_i) - u_i(1-\eta_j) + u_i(\omega_i). \tag{10}$$

Assumption 2 (Sufficient uncertainty about types) (i)

$$\underline{c} < \min\{u_i(1-\eta_j) - u_i(\omega_i),\ \lambda_i - \phi_i - u_i(\eta_i)\}$$

and

$$\bar{c} > \max\{u_i(1-\eta_j) - u_i(\omega_i),\ \lambda_i - \phi_i - u_i(\eta_i)\}$$

for $i \in \{A, B\}$. (ii)

$$f(c) < \frac{1}{|\Omega_i|}$$

for all $c \in [\underline{c}, \bar{c}]$ and $i \in \{A, B\}$.

If F is uniform, then (ii) is redundant because (i) implies (ii). Indeed, the uniform distribution is maximally "flat". However, we do not restrict attention to the uniform distribution. In the non-uniform case, (ii) guarantees that the density is not highly concentrated at one point.[13]

4 Equilibrium, First-Mover Advantage and the Costs of War

If player i challenges and player j doesn't, then player j will concede if and only if player i's claim σ_i satisfies $\sigma_i \leq 1-\eta_j$.[14] Let p_j denote the equilibrium probability that player j challenges.

Lemma 1 *Consider a perfect Bayesian equilibrium such that $p_j < 1$. If player i challenges in equilibrium, then he will claim $\sigma_i = 1 - \eta_j$.*

[13] The significant uncertainty distinguishes our approach from Harsanyi (1973), who introduced uncertainty concentrated on a very small interval $[-\epsilon, \epsilon]$ in order to provide an interpretation of mixed-strategy equilibria. His approach does not eliminate the indeterminacies of multiple equilibria.

[14] Sequential rationality forces player j to concede whenever $\sigma_i < 1 - \eta_j$. It is without loss of generality to assume that player j concedes when equality holds (see footnote 15).

Proof. The size of player i's claim only matters if player j does not challenge. Therefore, to find player i's optimal claim we may restrict attention to this event, which by hypothesis happens with probability $1 - p_j > 0$.

Thus, suppose only player i makes a challenge. If player i claims $\sigma_i > 1 - \eta_j$ then player j will reject the claim and there is conflict. By Rule 2, this conflict gives player i expected payoff

$$(\lambda_i + \theta)u_i(1) + (1 - (\lambda_i + \theta))\, u_i(0) - \phi_i = \lambda_i + \theta - \phi_i \tag{11}$$

using our normalizations. If instead player i claims $\sigma_i \leq 1 - \eta_j$ then player j will concede. Clearly, claiming strictly less than $1 - \eta_j$ is strictly worse than claiming $1 - \eta_j$. If player i claims $1 - \eta_j$ then his payoff is $u_i(1 - \eta_j)$ which is strictly greater than (11). To see this, note that it certainly holds if $\eta_j = 0$. If instead $\eta_j > 0$ and player i claims $1 - \eta_j$ then player j's payoff is $u_j(\eta_j)$ whether there is a conflict or not (see (7)). But conflicts are inefficient (since the players are risk-averse and $\phi_i > 0$), so player i strictly prefers to not have a conflict and get $1 - \eta_j$ for sure. Thus, (11) is strictly smaller than $u_i(1 - \eta_j)$, so claiming $\sigma_i = 1 - \eta_j$ is strictly better than claiming $\sigma_i > 1 - \eta_j$. Hence, if player i makes a challenge in equilibrium, he must claim $\sigma_i = 1 - \eta_j$.[15] ■

We now show that the probability that player i challenges must be strictly less than one, because he will not challenge if c_i is too high.

Lemma 2 *In perfect Bayesian equilibrium, $p_i < 1$ must hold for each $i \in \{A, B\}$.*

Proof. By challenging, player i can get at most

$$-c_i + p_j\,(\lambda_i - \phi_i) + (1 - p_j)\, u_i(1 - \eta_j), \tag{12}$$

using the fact that if $p_j < 1$ then his optimal challenge is $1 - \eta_j$. His expected payoff from not challenging is at least

$$p_j u_i(\eta_i) + (1 - p_j) u_i(\omega_i), \tag{13}$$

using the fact that if player j challenges then player i gets at least $u_i(\eta_i)$. Assumption 2(i) implies that (13) is strictly greater than (12) when $c_i = \bar{c}$. Thus, if c_i is sufficiently close to $\bar{c}$ then player i will not challenge. ■

[15] If player j does not concede when indifferent, i.e., when $\sigma_i = 1 - \eta_j$, then player i would like to claim strictly less than, but arbitrarily close to $1 - \eta_j$ (as such claims will necessarily be accepted). This would be incompatible with equilibrium. This standard argument implies that player j must concede when indifferent.

Combining Lemmas 1 and 2, we find that in any perfect Bayesian equilibrium, each player i will either not challenge, or choose the *optimal challenge* $\sigma_i = 1 - \eta_j$ (which the opponent will concede to, if he did not himself make a challenge). We label the optimal challenge *Hawk* (or H). To not make any challenge is to choose *Dove* (or D). Thus, the two-stage bargaining game can be reduced to the following 2×2 payoff matrix. Player i chooses a row, player j a column, and only player i's payoff is indicated.

$$\begin{array}{lcc} & \text{Hawk (claim } \sigma_j = 1-\eta_i) & \text{Dove (no challenge)} \\ \text{Hawk (claim } \sigma_i = 1-\eta_j) & \lambda_i - \phi_i - c_i & u_i(1-\eta_j) - c_i \\ \text{Dove (no challenge)} & u_i(\eta_i) & u_i(\omega_i) \end{array} \tag{14}$$

The payoff matrix reveals that, under the reasonable assumption that $\theta \geq 0$ (so there is no *second*-mover advantage), player i wants player j to choose Dove, whatever action player i himself chooses.[16]

Lemmas 1 and 2 imply that the set of perfect Bayesian equilibria of the two-stage game of Section 3 is isomorphic to the set of Bayesian Nash equilibria (henceforth "equilibria") of the simultaneous-move Hawk-Dove game (where player i is privately informed about c_i). Hence, we turn to an analysis of the Hawk-Dove game. A *strategy* for player i is a function $g_i : [\underline{c}, \overline{c}] \rightarrow \{H, D\}$ which specifies an action $g_i(c_i) \in \{H, D\}$ for each type $c_i \in [\underline{c}, \overline{c}]$. In equilibrium, all of player i's types maximize their expected payoff. Type c_i is a *dominant strategy hawk* if Hawk (H) dominates Dove (D) for this type.[17] Type c_i is a *dominant strategy dove* if D dominates H.[18] Assumption 2(i) implies that the support of F includes dominant strategy types of both kinds.

Suppose player i thinks player j will choose H with probability p_j. Player i's expected payoff from playing H is

$$-c_i + p_j \left(\lambda_i - \phi_i\right) + \left(1 - p_j\right) u_i(1 - \eta_j),$$

while his expected payoff from D is

$$p_j u_i(\eta_i) + (1 - p_j) u_i(\omega_i).$$

[16] To see this, first note that $\omega_i > \eta_i$ implies $u_i(\omega_i) > u_i(\eta_i)$. Second, we showed above that $u_i(1-\eta_j) > \lambda_i + \theta - \phi_i$ so if $\theta \geq 0$ then $u_i(1-\eta_j) - c_i > \lambda_i - \phi_i - c_i$. Even if $\theta < 0$, it can be checked that player i prefers player j to choose Dove if $\omega_i \geq 1/2$ and $\lambda_i \leq 1/2$.

[17] Formally, H (interim) dominates D if $c_i \leq u_i(1-\eta_j) - u_i(\omega_i)$ and $c_i \leq \lambda_i - \phi_i - u_i(\eta_i)$ with at least one strict inequality. For *strict* domination, both inequalities must be strict.

[18] Formally, D (interim) dominates H if $c_i \geq u_i(1-\eta_j) - u_i(\omega_i)$ and $c_i \geq \lambda_i - \phi_i - u_i(\eta_i)$ with at least one strict inequality. For *strict* domination, both inequalities must be strict.

Thus, if he chooses H instead of D, his *net* gain is

$$-c_i + p_j\left(\lambda_i - \phi_i - u_i(\eta_i)\right) + (1-p_j)\left(u_i(1-\eta_j) - u_i(\omega_i)\right). \qquad (15)$$

Therefore, $g_i(c_i) = H$ if (15) is positive, and $g_i(c_i) = D$ if (15) is negative. If (15) is zero then type c_i is indifferent, and for convenience we assume he chooses H in this case.

Player i uses a *cutoff strategy* if there is a *cutoff point* $x \in [\underline{c}, \bar{c}]$ such that $g_i(c_i) = H$ if and only if $c_i \leq x$. Because the expression in (15) is monotone in c_i, all equilibria must be in cutoff strategies. Therefore, we can without loss of generality restrict attention to cutoff strategies. Any such strategy is identified with its cutoff point $x \in [\underline{c}, \bar{c}]$. If player j uses cutoff point x_j, the probability he plays H is $p_j = F(x_j)$. Therefore, using (15), player i's best response to player j's cutoff x_j is the cutoff $x_i = \Gamma_i(x_j)$, where

$$\Gamma_i(x) \equiv F(x)\left(\lambda_i - \phi_i - u_i(\eta_i)\right) + (1-F(x))\left(u_i(1-\eta_j) - u_i(\omega_i)\right). \qquad (16)$$

The function Γ_i is the best-response function for cutoff strategies.

Assumption 2(i) rules out corner solutions, where all types do the same thing. Indeed, Assumption 2(i) implies that $\Gamma_i(x) \in (\underline{c}, \bar{c})$ for all $x \in [\underline{c}, \bar{c}]$ so the equilibrium cutoff point will lie strictly between $\underline{c}$ and $\bar{c}$. Since the function $(\Gamma_A(x_B), (\Gamma_B(x_A)) : [\underline{c}, \bar{c}]^2 \to (\underline{c}, \bar{c})^2$ is continuous, a fixed-point $(\hat{x}_A, \hat{x}_B) \in (\underline{c}, \bar{c})^2$ exists. This is an equilibrium (where player i uses cutoff $\hat{x}_i$). Thus, an equilibrium $(\hat{x}_A, \hat{x}_B)$ exists, where $\hat{x}_i = \Gamma_i(\hat{x}_j)$ for $i, j \in \{A, B\}$, $i \neq j$. The slope of the best response function is $\Gamma_i'(x) = \Omega_i f(x)$, where Ω_i is defined by (10). A standard sufficient condition for the existence of a *unique* equilibrium is that the absolute value of the slope of each player's best response function is less than 1. Assumption 2(ii) implies this. Thus, while Assumption 2(i) guarantees that every equilibrium is interior, Assumption 2(ii) guarantees that there is a unique equilibrium:

Proposition 1 *The Hawk-Dove game has a unique Bayesian Nash equilibrium.*

The unique equilibrium can be reached via iterated deletion of strictly dominated strategies. The precise argument depends on whether actions are strategic complements or substitutes. Strategic complements (substitutes) means that the incentive to choose Hawk is greater (smaller), the more likely it is that the opponent chooses Hawk. If player j chooses Hawk, then if player i switches from Dove to Hawk player i's net gain, from the payoff matrix (14), is

$$\lambda_i - \phi_i - c_i - u_i(\eta_i). \tag{17}$$

If instead player j chooses Dove, then if player i switches from Dove to Hawk player i's net gain is

$$u_i(1-\eta_j) - c_i - u(\omega_i). \tag{18}$$

Actions are *strategic complements for player i* if (17) is strictly greater than (18), which is equivalent to $\Omega_i > 0$, where Ω_i is defined by (10). They are *strategic substitutes for player i* if $\Omega_i < 0$. The game has strategic substitutes (resp. complements) if the actions are strategic substitutes (resp. complements) for both players.

For the rest of this section, assume the players are (ex ante) *symmetric* in the sense that they have the same utility function, $u_A = u_B = u$, the same cost of conflict, $\phi_A = \phi_B = \phi$, the same military strength, $\lambda_A = \lambda_B = \frac{1}{2}$, and the same initial endowment of territory, $\omega_A = \omega_B = \frac{1}{2}$. Then $\eta_A = \eta_B = \eta$. The unique equilibrium must be symmetric, and the equilibrium cutoff $\hat{x}$ is the same for both players and implicitly defined by the equation

$$\hat{x} - \Omega F(\hat{x}) = u(1-\eta) - u\left(\frac{1}{2}\right) \tag{19}$$

where

$$\Omega \equiv \frac{1}{2} - \phi - u(\eta) - u(1-\eta) + u\left(\frac{1}{2}\right). \tag{20}$$

Now consider the iterated deletion of strictly dominated strategies, starting with the case of strategic substitutes, $\Omega < 0$. If $c_i > u(1-\eta) - u(\frac{1}{2})$ then H is strictly dominated and can be deleted for type c_i. Similarly, if $c_i < \frac{1}{2} - \phi - u(\eta)$ then D is deleted. This completes the first round of deletion; it makes no use of player i's beliefs about player j. Next, note that type $c_i = \frac{1}{2} - \phi - u(\eta)$ is willing to choose D if and only if he thinks player j chooses H with probability 1. However, under Assumption 2(i), the first round of deletion eliminated H for a set of types that has strictly positive probability. This makes D strictly dominated for type $c_i = \frac{1}{2} - \phi - u(\eta)$. The same must be true for "almost dominant strategy hawks" with c_i slightly above $\frac{1}{2} - \phi - u(\eta)$, so D is eliminated for such types as well. Similarly, H is eliminated for "almost dominant strategy doves". This completes the second round; it utilizes player i's beliefs about player j's dominant strategy types.

The third round utilizes player i's beliefs about player j's beliefs about player i's dominant strategy types to delete strategies for "almost-almost dominant strategy types", etc. Eventually, under Assumption 2, this iterative process will completely resolve the uncertainty about what any type will do.[19]

With strategic complements, $\Omega > 0$, the fear of dominant strategy hawks causes "almost dominant strategy hawks" (c_i slightly above $u(1-\eta) - u(\frac{1}{2})$) to play H, which in turn causes "almost-almost dominant strategy hawks" to play H, etc. This "hawkish cascade" causes higher and higher cost-types to choose H. Meanwhile, "almost dominant strategy doves"(c_i slightly below $\frac{1}{2} - \phi - u(\eta)$) will play D, knowing that the opponent may be a dominant strategy dove. The "dovish cascade" causes lower and lower types to choose D. Again, the iterative process will completely resolve the uncertainty about what any type will do. In asymmetric scenarios with strategic complements for one player and strategic substitutes for the other, the iterative process uses a combination of the arguments in this and the previous paragraph.

Turning to comparative statics,[20] consider first how η depends on ϕ and θ. If $\phi + \theta > 1/2$ then $\eta = 0$ and $d\eta/d\theta = d\eta/d\phi = 0$. But if $\phi + \theta < 1/2$ then (7) holds, and the second-mover concedes more if θ or ϕ increases:

$$\frac{d\eta}{d\theta} = \frac{d\eta}{d\phi} = -\frac{1}{u'(\eta)} < 0. \tag{21}$$

An increase in the first-mover advantage θ allows the first-mover to extract a larger concession ($d\eta/d\theta \leq 0$), which makes it more tempting to choose H. Thus, a conflict becomes more likely. A perhaps more surprising result concerns the cost of conflict ϕ. When ϕ increases there are two

[19] The mathematics requires that a non-negligible fraction of types is removed in each round; Assumption 2(ii) guarantees this. Note the importance of *two-sided* incomplete information. For suppose there is significant uncertainty only about player B's type, while player A is commonly known to be an "opportunist" such that $\frac{1}{2} - \phi - u(\eta) < c_A < u(1-\eta) - u(\frac{1}{2})$. Thus, player A prefers to play H against D and D against H. Since player A has no dominant strategy types, there is nothing to eliminate for him in the first round. Even after player B's first-round elimination, there may be nothing to eliminate for player A. In this case, there will be multiple equilibria: one where player A surely plays D and player B plays H unless he is a dominant strategy dove, another where player A surely plays H and player B plays D unless he is a dominant strategy hawk, and a third where player A plays H with some probability.

[20] If Assumption 2 is relaxed then uniqueness is lost, but we can still obtain comparative statics of the "highest" and "lowest" equilibria (Milgrom and Roberts (1990)). However, for the results to be interesting, at least one of these equilibria must be interior, which requires that at least one dominant strategy type is in the support of F.

opposing effects. On the one hand, it becomes worse to choose H when the opponent chooses H. But on the other hand, when one player chooses H and the other D, the former may now extract a larger concession from the latter ($d\eta/d\phi \leq 0$). When ϕ is low, the second effect dominates, so the incentive to choose H increases with ϕ. When ϕ becomes high enough, however, $\eta = 0$ and the second effect vanishes. Any further increases in ϕ will reduce the incentive to choose H. This non-monotonicity is consistent with the "stability-instability" paradox (Hart (1960), Snyder (1965)): increasing ϕ from an initially low level first causes "instability" as each player tries to exploit the first-mover advantage, but then "stability" as ϕ becomes sufficiently large to make the players more cautious. We state this result formally.[21]

Proposition 2 *Suppose the players are ex ante symmetric. An increase in ϕ increases the probability of conflict if $\phi + \theta < 1/2$, but reduces the probability of conflict if $\phi + \theta > 1/2$.*

Proof. If $\phi + \theta < 1/2$ then totally differentiate (19) with respect to ϕ and use (21) to obtain

$$\frac{d\hat{x}}{d\phi} = \frac{1}{1 - \Omega f(\hat{x})} \frac{u'(1-\eta)}{u'(\eta)} (1 - F(\hat{x})) > 0.$$

Thus, when ϕ increases $\hat{x}$ increases, making conflicts more likely.

If $\phi + \theta > 1/2$ then $\eta = 0$ and $d\eta/d\phi = 0$. We get

$$\frac{d\hat{x}}{d\phi} = -\frac{1}{1 - \Omega f(\hat{x})} F(\hat{x}) < 0.$$

■

5 Power Imbalances and Conflict

We now consider the implications of (ex ante) asymmetries. For simplicity, suppose $u_A = u_B = u$ and $\phi_A = \phi_B = \phi$, so the asymmetries only concern endowments of territory and military strength. Suppose player B is a *rising power* who controls a smaller part of the territory ($\omega_B < \omega_A$), but his

[21] Like the other substantive results in this paper, Proposition 2 holds also in the generalized model of Appendix A.

military position is improving relative to the opponent (λ_B increases, with a corresponding decrease in $\lambda_A \equiv 1 - \lambda_B$). Using (6), player i's best response function (16) can be written

$$\Gamma_i(x) = F(x) \min\{\lambda_i - \phi, \theta\} + (1 - F(x)) \left(u(1 - \eta_j) - u(\omega_i)\right). \qquad (22)$$

Since $1 - \eta_A$ is non-decreasing in λ_B, the increase in λ_B shifts Γ_B up: player B becomes "tougher" (more inclined to choose H). On the other hand, Γ_A shifts down: player A (the *status quo power*) becomes "softer". Whether a conflict becomes more or less likely depends on which of these two effects dominates. In turn, this depends on the nature of the initial equilibrium $(\hat{x}_A, \hat{x}_B)$. The equilibrium probability of conflict is $W = F(\hat{x}_A)F(\hat{x}_B)$, so a change in $\hat{x}_i$ has a bigger impact on W if $F(\hat{x}_j)$ is big, i.e., if player j is likely to choose H.

Part (i) of our next proposition shows that if the status quo power is militarily strong (i.e., λ_A is large) and more likely than the opponent to choose H (i.e., $\hat{x}_A > \hat{x}_B$), then an increase in λ_B makes a conflict more likely. Part (ii) proves that this is also true at a *balanced* equilibrium, where each player is equally likely to choose H. But part (iii) shows that if λ_B increases sufficiently, then the rising power will become more likely to choose H, and further increases in λ_B make a conflict less likely.

To understand the balanced case, note that in general we have

$$\frac{dW}{d\lambda_B} = F(\hat{x}_B)F'(\hat{x}_A)\frac{d\hat{x}_A}{d\lambda_B} + F(\hat{x}_A)F'(\hat{x}_B)\frac{d\hat{x}_B}{d\lambda_B}. \qquad (23)$$

This expression simplifies in a balanced equilibrium where $\hat{x}_A = \hat{x}_B = \hat{x}$, in which case

$$\frac{dW}{d\lambda_B} = F'(\hat{x})F(\hat{x}) \left(\frac{d\hat{x}_A}{d\lambda_B} + \frac{d\hat{x}_B}{d\lambda_B}\right). \qquad (24)$$

Player i's incentive to choose H depends on both λ_i and ω_i. In a balanced equilibrium, player A's greater endowment, $\omega_A > \omega_B$, perfectly reflects his greater military strength, $\lambda_A > \lambda_B$, so that neither side has more reason to challenge the status quo than the other. If the rising power gains military strength, then the status quo is no longer legitimate in this sense. The rising power becomes more hawkish, and the status quo power becomes more dovish. Part (ii) shows that the former effect dominates at the balanced equilibrium, so (24) is positive and the probability of conflict increases.[22]

[22] Herrera, Morelli and Nunnari (2019) also find, in a model that is quite different from

Concavity of utility functions plays a key role in the proof of part (ii). Intuitively, the rising power is weak initially and cannot extract much territory by a strategic move ($1-\eta_A$ is small). But concavity means that the marginal value of territory to the status quo power is small, so an increase in λ_B has a big effect on $1-\eta_A$. Again by concavity, extra territory is very valuable to the rising power, so his incentive to choose H increases significantly, making $d\hat{x}_B/d\lambda_B$ large. But there is a much smaller effect on $1-\eta_B$, and territory is not so valuable to the status quo power, so the change in his incentive to choose H is rather small. Accordingly, $d\hat{x}_A/d\lambda_B$ is smaller in absolute value than $d\hat{x}_B/d\lambda_B$, so that (24) is positive.

Proposition 3 *Suppose $u_A = u_B = u$ and $\phi_A = \phi_B = \phi$. (i) If $\lambda_A > \max\{\phi+\theta, 1-\phi-\theta\}$ and $\hat{x}_A > \hat{x}_B$ then a small increase in λ_B makes conflicts more likely. (ii) If $\lambda_A > \lambda_B$, $\omega_A > \omega_B$ and $\hat{x}_A = \hat{x}_B$, then a small increase in λ_B makes conflicts more likely. (iii) If $\lambda_B > \max\{\phi+\theta, 1-\phi-\theta\}$ and $\omega_A > \omega_B$ then a small increase in λ_B makes conflicts less likely.*

The proof is in Appendix B. Parts (i) and (ii) suggest the existence of a "Thucydides trap". But when λ_B increases, the trap itself implies $F(\hat{x}_B)$ will become bigger than $F(\hat{x}_A)$, so increases in $\hat{x}_B$ start to have less impact on W than reductions in $\hat{x}_A$. Eventually, conflicts become less likely (part (iii)). Thus, the Thucydides trap contains the seeds of its own destruction, and the probability of conflict is a *non-monotonic* function of λ_B. Intuitively, once player B is strong enough, player A has little incentive to challenge the status quo or to fight over territory. If player B makes a challenge, the likely outcome is (D, H), resulting in a peaceful transfer of territory from A to B. Even if the Thucydides trap no longer operates, the status quo power is made worse off as the rising power gets increasingly aggressive and the status quo power backs down.

6 Escalation and Deterrence: Strategic Complements and Substitutes

We begin by showing that actions are strategic substitutes if there is no military advantage to being the first-mover.

ours, that conflicts may be due to a mismatch between military capabilities and the allocation of resources.

Proposition 4 *The game has strategic substitutes if $\theta \leq 0$.*

Proof. Recall that actions are strategic substitutes for player i if $\Omega_i < 0$. If $\eta_i > 0$ then (7) holds so

$$\Omega_i = \theta - u_i(1 - \eta_j) + u_i(\omega_i) < \theta$$

since $1 - \eta_j > \omega_i$. If $\eta_i = 0$ then (3) holds so

$$\Omega_i = \lambda_i - \phi_i - u_i(1 - \eta_j) + u_i(\omega_i) < \lambda_i - \phi_i \leq \theta$$

where the first inequality is due to $1 - \eta_j > \omega_i$. ■

To simplify the exposition, for the remainder of this section we again assume the two players are symmetric ex ante. We therefore drop the subscripts on u_i, ϕ_i and η_i and observe that $\omega_A = \omega_B = 1/2$ and $\lambda_A = \lambda_B = 1/2$. We have $\Omega_A = \Omega_B = \Omega$ as defined by (20). The game has strategic substitutes if $\Omega < 0$ and strategic complements if $\Omega > 0$. Totally differentiating Ω yields

$$\frac{d\Omega}{d\theta} = -\left(u'(\eta) - u'(1-\eta)\right)\frac{d\eta}{d\theta} \geq 0 \tag{25}$$

with strict inequality when $\phi + \theta < 1/2$, in view of (21) and strict concavity (note that $\eta < 1 - \eta$). Also,

$$\frac{d\Omega}{d\phi} = -\left(u'(\eta) - u'(1-\eta)\right)\frac{d\eta}{d\phi} - 1 < 0. \tag{26}$$

If $\phi + \theta > 1/2$ then (26) follows from $d\eta/d\phi = 0$. If $\phi + \theta < 1/2$ then (21) implies

$$\frac{d\Omega}{d\phi} = \left(u'(\eta) - u'(1-\eta)\right)\frac{1}{u'(\eta)} - 1 = -\frac{u'(1-\eta)}{u'(\eta)} < 0.$$

Thus, actions are more likely to be strategic complements the bigger is θ and the smaller is ϕ.

It is intuitive that if ϕ is large, then the most important consideration is to avoid a conflict – just as in the classic chicken race, where a collision would be disastrous. Thus, we have the following result.

Proposition 5 *Suppose the players are symmetric ex ante. If $\phi > u(1/2) - 1/2$ then the game has strategic substitutes.*

Proof. By concavity,

$$u(\eta) + u(1-\eta) \geq u(0) + u(1) = 1.$$

Therefore, $\phi > u(1/2) - 1/2$ implies

$$\Omega = \frac{1}{2} - u(\eta) - u(1-\eta) + u(1/2) - \phi < 0.$$

■

If ϕ is small, however, then the players will be more concerned about territorial gains and losses than about avoiding a conflict, and actions become strategic complements if θ is large. Intuitively, a large θ has two effects: it will be very costly to be caught out and play Dove against Hawk, but it will be very rewarding to play Hawk against Dove. The first effect tends to make actions strategic complements, while the second effect does the opposite. It can be seen in (25) that the first effect dominates: intuitively, concavity means that it is more important to preserve your own territory than to acquire the opponent's territory. Thus, we have:

Proposition 6 *Suppose the players are symmetric ex ante and $\phi < u(1/2) - 1/2$. There exists $\theta^* \in (0, 1/2)$ such that the game has strategic substitutes if $\theta < \theta^*$ and strategic complements if $\theta > \theta^*$.*

Proof. Fix ϕ such that $\phi < u(1/2) - 1/2$. Then $\phi < 1/2$ since $u(1/2) < u(1) \equiv 1$. From Proposition 4, we have $\Omega < 0$ if $\theta \leq 0$. Now define

$$\bar{\theta} \equiv \frac{1}{2} - \phi \in (0, 1/2).$$

From (6) it follows that $\eta = 0$ if and only if $\theta \geq \bar{\theta}$. When $\eta = 0$ we have

$$\Omega = u\left(\frac{1}{2}\right) - \frac{1}{2} - \phi > 0$$

so that $\Omega > 0$ when $\theta \geq \bar{\theta}$. Thus, there exists $\theta^* \in (0, \bar{\theta})$ such that if $\theta = \theta^*$ then $\Omega = 0$ and $\eta > 0$. It follows from (25) that $\Omega < 0$ if $\theta < \theta^*$ and $\Omega > 0$ if $\theta > \theta^*$. ■

Note that θ^* from Proposition 6 depends on ϕ, so we can write $\theta^* = \theta^*(\phi)$. It is easy to check that the function $\theta^*(\phi)$ is decreasing in ϕ.[23] Hence, the parameter range where the game has strategic complements is decreasing in the cost of conflict.

Note also the key role played by (25) in the proof of Proposition 6. This inequality comes from the maintained assumption of strict concavity (decreasing marginal utility of land). The concavity assumption would be violated if the contested territory were considered (more or less) indivisible, perhaps because controlling it is a matter of national prestige, so that players are willing to gamble on a territorial expansion. With convex utility functions, the game would surely have strategic substitutes. To see this, note first that the proof of Proposition 4 does not use concavity, so the game has strategic substitutes if $\theta \leq 0$. If θ increases, the marginal gain from choosing H against D (which is $u(1-\eta) - u(1/2)$) increases faster than the marginal gain against H (which is $\frac{1}{2} - \phi - u(\eta)$). This is true because $d\eta/d\theta \leq 0$, and $u'(1-\eta) > u'(\eta)$ under convexity. This means Ω cannot increase (the inequality in (25) is reversed), and actions remain strategic substitutes also for $\theta > 0$.[24] Intuitively, the inequality in (25) is reversed because with increasing marginal utility of land, it is more important to acquire the opponent's territory than to preserve your own territory. Thus, strategic complementarity is impossible when utility functions are convex. This confirms the intuition that risk-seeking leads to chicken races, while stag hunts are for risk-averse players only.

[23] Since η depends on ϕ and θ, we can write $\eta = \eta(\phi, \theta)$. The function $\theta^* = \theta^*(\phi)$ identified in Proposition 6 is such that $\Omega = 0$ when $\eta = \eta(\phi, \theta^*)$. Substitute $\eta = \eta(\phi, \theta^*(\phi))$ in (10) to get

$$\frac{1}{2} - u(\eta(\phi, \theta^*(\phi)))) - u(1 - \eta(\phi, \theta^*(\phi))) + u\left(\frac{1}{2}\right) - \phi \equiv 0$$

for all $\phi < u(1/2) - 1/2$. The proof of Proposition 6 implies that $\theta^* < 1 - \phi$, so $\eta(\phi, \theta^*(\phi)) > 0$ satisfies (7). Using this fact, totally differentiating the identity above yields

$$\frac{d\theta^*(\phi)}{d\phi} = \frac{-u'(1-\eta)}{u'(\eta) - u'(1-\eta)} < 0.$$

[24] We assume Lemma 1 obtains even with convex utility functions. This is true if the cost of conflict is high enough.

7 Strategic Investments

Suppose player A can make an *ex ante* move (before types are drawn) that changes the parameters of his payoff function in the bargaining game. For example, player A might invest in defensive technology, such as a defensive wall, an anti-missile system or cybersecurity software, to reduce the cost of conflict. Allowing businesses or settlements to develop on contested land might make the land more worth fighting for. Alternatively, player A might announce that his endowment is sacred (losing territory then becomes more costly, as it implies loss of face), or delegate decision-making to an agent who is less conflict-averse than he is, or who values territory more. These tactics influence the resolve to fight for territory and shift the best response curves.

Borrowing terminology from Fudenberg and Tirole (1984), we say that player A *over-invests* if he derives a strategic advantage from an ex ante investment, i.e., if player B becomes more likely to choose D. (If the investment makes player B more likely to choose H, then player A under-invests). The familiar logic of Fudenberg and Tirole (1984) (see also Tirole (2016)) suggests that if actions are strategic substitutes, then player A benefits by becoming tougher (shifting Γ_A up); with strategic complements, he benefits by becoming softer (shifting Γ_A down).[25] There is, however, a complicating factor: player A's investment may change the minimum amount η_A he would be willing to accept if he is challenged, which shifts Γ_B. Thus, player A's investment can shift both best-response functions. For example, an investment which reduces ϕ_A tends to make player A tougher and player B softer (as player A becomes less inclined to make concessions). Both effects must be taken into account when determining if the investment is strategically advantageous.

To simplify the exposition, assume $\lambda_A = \lambda_B = 1/2$ and $\omega_A = \omega_B = 1/2$. Moreover, assume utility functions are piecewise linear. There are constants $v_i > 0$ and $g_i > 0$ such that each unit of player i's own endowment is worth v_i to him, but each unit of player j's endowment is worth only $g_i < v_i$ to player i. Normalizing the status quo utility to zero,[26] we obtain the utility

[25] This is true in their model when entry is accommodated, which is the relevant case for our comparison.

[26] Thus, to simplify formulas, here we do not normalize $u_i(1) = 1$ and $u_i(0) = 0$.

function

$$u_i(\sigma_i) = \begin{cases} v_i\,(\sigma_i - 1/2) & \text{if} \quad \sigma_i - 1/2 \leq 0 \\ \\ g_i(\sigma_i - 1/2) & \text{if} \quad \sigma_i - 1/2 \geq 0 \end{cases}$$

This utility function is concave but not strictly concave. However, strict concavity is only important in as much as it guarantees that each unit of player i's own endowment is strictly more valuable to him than each unit of player j's endowment.[27] This property holds here, because $g_i < v_i$. The payoff matrix of the row player, player i, is now

$$\begin{array}{ccc} & H & D \\ H & g_i/4 - v_i/4 - \phi_i - c_i & g_i(1/2 - \eta_j) - c_i \\ D & v_i(\eta_i - 1/2) & 0 \end{array} \tag{27}$$

Actions are strategic complements for player i if

$$\frac{v_i - g_i}{4} - v_i\eta_i + g_i\eta_j - \phi_i > 0 \tag{28}$$

and strategic substitutes if the opposite inequality holds. Player i's best-response function is

$$\Gamma_i(x) = F(x)\left(\frac{v_i + g_i}{4} - \phi_i - v_i\eta_i\right) + (1 - F(x))\, g_i\left(\frac{1}{2} - \eta_j\right) \tag{29}$$

where

$$\eta_i = \begin{cases} (1-2\theta)\,\frac{v_i+g_i}{4v_i} - \frac{\phi_i}{v_i} & \text{if} \quad \phi_i < (1-2\theta)\,\frac{v_i+g_i}{4} \\ 0 & \text{if} \quad \phi_i \geq (1-2\theta)\,\frac{v_i+g_i}{4} \end{cases} \tag{30}$$

We consider the strategic effects of ex ante moves (investments) that either reduce ϕ_A, the cost of conflict, or increase v_A, player A's valuation of his own endowment (in each case by a small amount).[28]

Suppose player A's investment reduces ϕ_A by a small amount (say, by installing defense systems). First, suppose actions are strategic substitutes

[27] That is, all the results of the paper go through if strict concavity of u_i is replaced by the weaker assumption: if $0 < x < \omega_i < y < 1$ then $u_i'(x) > u_i'(y)$.

[28] To simplify, we disregard what Fudenberg and Tirole (1984) call *direct effects* of the investment (including any ex ante costs of investing), as these are irrelevant to the analysis of under- versus over-investment.

for player B. There are two cases; both lead to the conclusion that A will over-invest, but for quite different reasons. The first case occurs when $\phi_A > (1-2\theta)\frac{v_A+g_A}{4}$ so $\eta_A = 0$ and $\frac{d\eta_A}{d\phi_A} = 0$. From (29), player A's investment causes Γ_A to shift up: player A becomes tougher. It has no effect on Γ_B. Since actions are strategic substitutes for player B, player A over-invests to persuade player B to choose D. This is the traditional interpretation of Schelling's commitment tactic – become tough in order to deter aggression.

The second case occurs when $\phi_A < (1-2\theta)\frac{v_A+g_A}{4}$, so $\eta_A > 0$. Substituting from (30) into $\Gamma_A(x)$, we find

$$\Gamma_A(x) = F(x)\theta\frac{v_A+g_A}{2} + (1-F(x))\,g_A\left(\frac{1}{2}-\eta_B\right). \tag{31}$$

Thus, reducing ϕ_A has no effect on Γ_A. However, Γ_B shifts down because η_A increases, as player A is less willing to concede territory without a fight. There is an unambiguous strategic advantage for player A in making player B less likely to choose H. Hence, player A again over-invests in cost-reduction – not to make himself tough, but to make the opponent soft.

Next, suppose actions are strategic complements for player B. Now the two cases lead to different conclusions. The first case occurs when $\phi_A > (1-2\theta)\frac{v_A+g_A}{4}$ so $\eta_A = 0$. Again, the investment shifts Γ_A up. By strategic complements, player B becomes more likely to choose H, which is disadvantageous for player A. Therefore, player A will under-invest, to make B feel less threatened and more likely to choose D. The second case occurs when $\phi_A < (1-2\theta)\frac{v_A+g_A}{4}$. As $\eta_A > 0$ in this case, as discussed above player A's investment has no effect of Γ_A but shifts Γ_B down. Player A benefits from making player B soft, and hence will over-invest.

We summarize the discussion so far:

Proposition 7 *Suppose investment by player A reduces ϕ_A. If actions are strategic complements for player B and $\phi_A > (1-2\theta)\frac{v_A+g_A}{4}$, then player A under-invests. Otherwise, he over-invests.*

Thus, if ϕ_A is low then player A will surely over-invest in defensive technology. But if ϕ_A is high, then player A will over-invest or under-invest depending on whether actions are strategic substitutes or complements for player B, following the usual logic of Fudenberg and Tirole (1984).

Now consider an investment that increases v_A by a small amount (say, building settlements on the disputed territory, making it more valuable).

Suppose first that $\phi_A > (1-2\theta)\,(v_A + g_A)/4$ so $\eta_A = 0$. In this case the increase in v_A has no effect on Γ_B. But it makes it more costly for player A to lose all of the territory, as he will do when he is the second-mover. By choosing H, he may avoid this bad outcome, so Γ_A shifts up. The usual logic of "looking tough" means player A over-invests in settlements when actions are strategic substitutes for player B, but under-invests when they are strategic complements.

The logic is more subtle when $\phi_A < (1-2\theta)(v_A + g_A)/4$. Again, the investment which increases v_A causes Γ_A to shift up. Whether Γ_B shifts down or up depends on whether η_A increases or decreases. Now, from (30),

$$\frac{\partial \eta_A}{\partial v_A} = -\frac{(1-2\theta)\frac{g_A}{4} - \phi_A}{v_A^2} \tag{32}$$

so Γ_B shifts down if $\phi_A > (1-2\theta)g_A/4$, but shifts up if $\phi_A < (1-2\theta)g_A/4$.

To understand this, note that a small change $dv_A > 0$ has two effects on player A's incentive to concede. On the one hand, if player A concedes to player B's demand $1-\eta_A$ then player A must give $\frac{1}{2} - \eta_A$ units of his endowment to player B. Thus, increasing v_A by dv_A increases player A's cost of conceding by $\left(\frac{1}{2} - \eta_A\right) dv_A$. On the other hand, if player A does *not* concede, he will lose *all* of his endowment (which is of size $1/2$) with probability $\theta + 1/2$. Thus, increasing v_A by dv_A increases player A's expected cost of *not* conceding by $(1/2)(\theta + 1/2)dv_A$. The first effect dominates, making player A less willing to concede ($\partial\eta_A/\partial v_A > 0$), when $\eta_A < (1-2\theta)/4$. From (32) this inequality is equivalent to $\phi_A > (1-2\theta)g_A/4$. But if $\phi_A < (1-2\theta)g_A/4$ then the second effect dominates and therefore $\partial\eta_A/\partial v_A < 0$.

Intuitively, since a conflict may lead to the loss of *all* the territory, settlements can make player A more willing to give up some part of the territory in order to avoid a conflict. This makes player B tougher (shifts Γ_B up), which tends to make settlements a strategic disadvantage for player A. On the other hand, Γ_A also shifts up, which tends to make settlements a strategic advantage for player A if actions are strategic substitutes. Thus, if $\phi_A < (1-2\theta)g_A/4$ then the net effect of an increase in v_A is ambiguous; it is possible that the shift of Γ_B dominates, so that player A under-invests in settlements even if the game has strategic substitutes. Similarly, the net effect is ambiguous if $\phi_A > (1-2\theta)g_A/4$ and the game has strategic complements. The remaining cases are unambiguous. The results can be summarized as follows:

Proposition 8 *Suppose investment by player A increases v_A. (i) Suppose $\phi_A > (1-2\theta)(v_A+g_A)/4$. Player A over-invests if actions are strategic substitutes for player B, and under-invests otherwise. (ii) Suppose $\phi_A < (1-2\theta)(v_A+g_A)/4$. Player over-invests if $\phi_A > (1-2\theta)g_A/4$ and actions are strategic substitutes for player B, but under-invests if $\phi_A < (1-2\theta)g_A/4$ and actions are strategic complements for player B.*

8 The Shadow of the Future

Suppose bargaining takes place both today (period 1) and in the future (period 2), in each period following the rules of Section 3. The two periods are linked because today's outcome may change the status quo, which will influence tomorrow's negotiations. A player who loses territory today will in the future become more aggressive as he tries to recapture what he lost. This leads to a strong tendency for actions to be *dynamic* strategic complements, which tempers the incentive to make challenges today.

Suppose the cost of making a challenge is independently drawn at the beginning of each period. At the beginning of period 1, the status quo allocation is $(\omega_A, \omega_B) = (1/2, 1/2)$. Period 1 bargaining will either lead to a conflict, in which case the game ends, or it will generate some new division of the territory, say $(\hat{\omega}_A, \hat{\omega}_B)$. Then $(\hat{\omega}_A, \hat{\omega}_B)$ becomes the new status quo, the starting point for the period 2 bargaining.

If there is no conflict in period 1, then in period 2 the players must behave exactly as in the one-shot game, given endowments $(\hat{\omega}_A, \hat{\omega}_B)$, because period 2 is the last period. Therefore, before nature has determined the cost of challenging in period 2, player A's expected period 2 payoff is

$$\begin{aligned}
&F(\hat{x}_A)\,F(\hat{x}_B)\left[\frac{1}{2}u_A(1)+\frac{1}{2}u_A(0)-\phi_A-E\{c_A : c_A \leq \hat{x}_A\}\right]\\
&+F(\hat{x}_A)\,(1-F(\hat{x}_B))\left[u_A(1-\eta_B)-E\{c_A : c_A \leq \hat{x}_A\}\right]\\
&+(1-F(\hat{x}_A))\,F(\hat{x}_B)\,u_A(\eta_A)\\
&+(1-F(\hat{x}_A))\,(1-F(\hat{x}_B))\,u_A(\hat{\omega}_A). \qquad (33)
\end{aligned}$$

Here $\hat{x}_i$ and $1-\eta_j$ are player i's continuation equilibrium cutoff and optimal challenge in period 2, calculated as in a one-shot game with endowments $(\hat{\omega}_A, \hat{\omega}_B)$. $E\{c_A : c_A \leq \hat{x}_A\}$ is player A's expected cost of making a challenge in period 2, conditional on a challenge being made. Note that with probability $(1 - F(\hat{x}_A))(1 - F(\hat{x}_B))$ neither player will challenge in period 2, so player A will hold on to the share $\hat{\omega}_A$ and get payoff $u_A(\hat{\omega}_A)$.

In period 1, the "shadow of the future" must be taken into account. If by challenging the initial status quo, player A manages to increase his share from $\omega_A = 1/2$ to $\hat{\omega}_A > 1/2$ in period 1, then player B will be a tougher opponent in period 2. Indeed, as player B's territory shrinks, he will have more to gain, and less to lose, from challenging the new status quo $(\hat{\omega}_A, \hat{\omega}_B)$ in period 2. This is verified by the following proposition.

Proposition 9 *Suppose $u_A = u_B = u$, $\phi_A = \phi_B = \phi$, and $\lambda_A = \lambda_B = 1/2$. Consider equilibrium of a one-shot game with initial endowments $(\hat{\omega}_A, \hat{\omega}_B)$ such that $\hat{\omega}_A > \hat{\omega}_B$. Player B is more likely to choose H, the bigger is $\hat{\omega}_A$. That is, $d\hat{x}_B/d\hat{\omega}_A > 0$.*

The proof is in Appendix B. The increase in player B's period 2 cutoff $\hat{x}_B$ reduces player A's expected future payoff, given by (33). Thus, when contemplating a period 1 challenge, player A must trade off the short-term benefit of additional territory against the long-run cost of a more aggressive opponent.

Since space constraints prevent a comprehensive analysis, we will illustrate in a simple example. There is no discounting: each player wants to maximize the sum of his payoffs in the two periods. Also, $\theta = 0$, the distribution F is uniform on $[0, 1]$, and $\phi_A = \phi_B = \frac{1}{2}$. Finally, each $u_i(x_i)$ is piecewise linear as in Section 7, with $g_i = 1/2$ and $v_i = 3/2$. Thus, $u_i(x_i) = \frac{3}{2}\left(x_i - \frac{1}{2}\right)$ if $x_i \leq 1/2$, $u_i(x_i) = \frac{1}{2}(x_i - \frac{1}{2})$ if $x_i \geq 1/2$.

The optimal challenge in period 2 turns out to be to demand all of the territory ($\eta_A = \eta_B = 0$). We obtain the period 2 cutoffs ($\hat{x}_A$, $\hat{x}_B$) by solving $\hat{x}_A = \Gamma_A(\hat{x}_B)$ and $\hat{x}_B = \Gamma_B(\hat{x}_A)$ as in the one-shot game; the best response functions are given by (16) with initial endowments $(\hat{\omega}_A, \hat{\omega}_B)$. Player A's expected total payoff if he is the first-mover in period 1, and player B concedes to $\hat{\omega}_A$, is the period 1 payoff $(u_A(\hat{\omega}_A) - c_A)$ plus the expected period 2 payoff in (33). It is easy to show numerically that this expected total payoff is strictly decreasing in $\hat{\omega}_A$ on the interval $[\frac{1}{2}, 1]$.[29] Therefore, claiming more than half of the disputed territory cannot be optimal in period 1, since if player A makes such a claim, he will strictly gain by reducing it. The same goes for player B. Thus, in equilibrium no challenges are made in period 1. Knowing that a successful strategic move today will make a conflict more likely tomorrow, the farsighted players become cautious, preferring to leave the status quo intact rather than going for temporary advantages.

If no challenges are made, nobody will wish to challenge: it is better to leave the initial status quo in place, since as we know, taking more than a half will reduce one's total expected payoff.[30] Thus, the unique equilibrium path is as follows. In period 1, there are no challenges, so the probability of a conflict is zero. In period 2, each player i uses the cutoff $\hat{x}_i = 0.2$, just as in a one-shot game with equal endowments. The path would be the same if future payoffs were discounted by $\delta < 1$ sufficiently close to 1. As δ falls the shadow of the future becomes less important. If $\delta = 0$ then the future casts no shadow and each myopic player challenges with 20 percent probability in period 1 (as in the one-shot game), implying a four percent chance of conflict in period 1.

The situation is more complicated when the initial status quo is asymmetric. Even a very farsighted player may challenge the status quo in period 1 if he is militarily strong but his initial endowment is very small, making the status quo highly unbalanced. Moreover, it would clearly become more tempting to challenge the status quo today if, following Fearon (1996), we modify the model so that gaining territory today increases one's military capabilities tomorrow. The message from the dynamic model is not that strategic moves are irrational when the future casts a shadow over the present. It is rather that farsighted players must take into account the long-run as well as the short-run implications of a strategic move.[31]

[29] In this example, (33) becomes:

$$-\hat{x}_A \hat{x}_B \left(\frac{3}{4} + \frac{\hat{x}_A}{2}\right) + \hat{x}_A (1 - \hat{x}_B) \left(\frac{1}{4} - \frac{\hat{x}_A}{2}\right) - (1 - \hat{x}_A)\, \hat{x}_B \frac{3}{4}$$
$$+ (1 - \hat{x}_A)(1 - \hat{x}_B) \frac{1}{2}(\hat{\omega}_A - \frac{1}{2}).$$

[30] This is true even without considering the cost c_i of making a challenge. After taking this cost into account, it becomes even less profitable to challenge.

[31] Herrera, Morelli and Nunnari (2019) study a dynamic model where there is no bar-

9 Concluding Comments

A key distinction in the theory of conflict is whether actions are strategic substitutes or complements. In this paper we have tried to answer Jervis's ((1976), p. 96) question: "What are the conditions under which one model rather than the other is appropriate?" Our simple bargaining game has strategic complements when first-mover advantages are big and conflicts are not too costly, and strategic substitutes otherwise. Mapping the technology of war onto the parameters of our model helps us understand which scenario is at play in a given environment. For example, McNeil (1982) discusses how first-mover advantages changed over time. During the era when forts could withstand a siege for many years, the offensive advantage was not large. Then, mobile and powerful siege cannon, developed in France in the late 1400s, gave the advantage to the attacker. This was later neutralized by the *trace italienne*, a fort design that combined wide ditches reinforced with soft earth and a star-shaped structure that facilitated counter-attack (Duffy (1997)). By the end of the nineteenth century, Napoleon's use of trained mass armies and Prussia's rapid, well-planned attacks with breech-loading, long range guns led to short wars and a significant offense dominance (Bueno de Mesquita (2013)). By World War I, trench warfare instead made defense dominant, and wars were long and costly. But in Germany and Great Britain, it was believed that offense was still dominant (Jervis (1978)). The ensuing conflict shows how policies can backfire, if based on an incorrect identification of the scenario at play. In the current era, nuclear warfare with its destructiveness and second-strike capability may be closer to trench warfare than to the Napoleonic mass army.[32]

Our analysis focussed on "small" changes in parameters, but large changes can also be analyzed using this framework. Consider the introduction of a powerful missile defense. If the main effect is a large reduction in the cost of conflict, actions may go from being strategic substitutes to strategic comple-

gaining over resources, so going to war is the only way the status quo can be changed, but a war also changes future military capabilities. They find that the dynamic incentives to start a war may amplify the static incentives.

[32] For the analogy between nuclear war and World War I, and the lack of foresight of policymakers, see Clark (2014): "In the 1950s and 1960s, decision-makers and the general public alike grasped in a visceral way the meaning of nuclear war - images of mushroom clouds over Hiroshima and Nagasaki entered the nightmares of ordinary citizens....[T]he protagonists of 1914 were sleepwalkers, watchful but unseeing, haunted by dreams, yet blind of the reality of the horror they were about to bring to the world."

ments. This could have dramatic implications: with strategic complements, confidence-building diplomacy can enhance cooperation; with strategic substitutes, deterrence is key and looking weak is dangerous. On the other hand, if the main effect is a large reduction in first-mover advantages, actions may go from being strategic complements to strategic substitutes, with equally dramatic (but very different) implications. Note also that if the cost of conflict is very high to begin with, a small reduction in this cost will make challenges more likely, but if the cost falls sufficiently then further reductions will make challenges less likely. Again, our model helps to elucidate the possible equilibrium effects and the risks and rewards of new technologies of conflict.

Gurantz and Hirsch (2017) studied deterrence with one-sided challenges and incomplete information about the challenger's type. Dovish types are deterred from challenging if challenges are resisted, so a challenge signals that the challenger is a Hawkish type who, most likely, will start a war in the future. This renders it credible to resist challenges, even if the contested territory is not very valuable to the defender, as a war is anyway expected to occur sooner or later. Thus, incomplete information can facilitate credible deterrence. It would be interesting to explore similar issues in our model with two-sided challenges. If we modify the dynamic model of Section 8 by assuming types are positively correlated over time, then beliefs about future types (and hence about future actions) may depend on past actions, as in Gurantz and Hirsch (2017). A player who chooses H today signals that he is likely to also choose H tomorrow. This will influence both the incentive to choose H today and the incentive to concede if stage 2 is reached.

Our model predicts a peaceful transfer of territory if only one player challenges the status quo. Following Fearon (1995), the model could be modified to make unilateral challenges riskier. There could be uncertainty about what concession the opponent is willing to make, say because his cost of conflict is his private information. A player with a high cost of conflict would get information rents, as the opponent modulates his demand to reduce the risk of rejection. In a dynamic model, a concession might lead to an increase in the opponent's military strength ("shifting power"), making appeasement costlier. These forces would reduce the expected payoff from a unilateral challenge, influencing comparative statics, whether the game has strategic complements or substitutes, etc. Future work may show how different theories of conflict can be distinguished along these dimensions.

10 Appendix A

Our model can be generalized in various directions without changing the essential insights. For example, it is plausible that the cost of choosing H depends on what happens subsequently. In particular, a player who chooses H may suffer harsher sanctions and greater loss of reputation if the opponent chooses D than if the opponent chooses H. To account for this, let each player's *expected* cost of choosing H be his private information. To be specific, suppose if stage 2 is reached because player j is the only one to challenge, then player j suffers an additional cost $a_j \geq 0$ while player i derives a benefit $b_i \geq 0$ (perhaps player i receives some international support in this case). Assume a_j and b_i are commonly known. The maximum claim player i would concede to in stage 2 is again $x_j = 1 - \eta_i$, where η_i is defined by (6). The new payoff matrix is

	Hawk	Dove
Hawk	$\lambda_i - \phi_i - c_i$	$u_i(1-\eta_j) - c_i - a_i$
Dove	$u_i(\eta_i) + b_i$	$u_i(\omega_i)$

Assumption 1 remains unchanged ($\eta_i < \omega_i$), but to ensure that there is a unique (interior) equilibrium, Assumption 2 must be modified as follows:

Assumption 2 (i)

$$\underline{c} < \min\{u_i(1-\eta_j) - a_i - u_i(\omega_i),\ \lambda_i - \phi_i - u_i(\eta_i) - b_i\}$$

and

$$\bar{c} > \max\{u_i(1-\eta_j) - a_i - u_i(\omega_i),\ \lambda_i - \phi_i - u_i(\eta_i) - b_i\}$$

for $i \in \{A, B\}$. (ii)

$$f(c) < \frac{1}{|\Omega_i|}$$

for all $c \in [\underline{c}, \bar{c}]$ and $i \in \{A, B\}$.

Whether we have strategic substitutes or complements depends on the sign of

$$\Omega_i \equiv \lambda_i - \phi_i - u_i(\eta_i) - b_i - u_i(1-\eta_j) + a_i + u_i(\omega_i)\,. \tag{34}$$

It is straightforward to check that all the substantive results of the paper remain qualitatively the same in this more general model. The model can be generalized further so that the additional cost of making a unilateral challenge depends on whether or not the opponent concedes. It could also depend on the challenger's privately known type. After appropriate modifications to Assumption 2, the basic insights again go through.

11 Appendix B

11.1 Proof of Proposition 3

(i) Suppose $\lambda_A > \max\{\phi+\theta, 1-\phi-\theta\}$. Then $\lambda_B = 1-\lambda_A < \phi+\theta$, so the two best response functions can be written

$$\Gamma_A(x) = F(x)\theta + (1-F(x))\,\Delta_A \tag{35}$$

and

$$\Gamma_B(x) = F(x)(\lambda_B - \phi) + (1-F(x))\,\Delta_B \tag{36}$$

where $\Delta_i \equiv u(1-\eta_j) - u(\omega_i)$. Differentiating the equilibrium conditions $\hat{x}_i = \Gamma_i(\hat{x}_j)$, using (35), (36) and the fact that $d\Delta_A/d\lambda_B = 0$ when $\lambda_B < \phi+\theta$, yields

$$\frac{d\hat{x}_A}{d\lambda_B} = F'(\hat{x}_B)\,(\theta - \Delta_A)\,\frac{d\hat{x}_B}{d\lambda_B} \tag{37}$$

and

$$\frac{d\hat{x}_B}{d\lambda_B} = \frac{F(\hat{x}_A) + (1-F(\hat{x}_A))\frac{d\Delta_B}{d\lambda_B}}{1 - F'(\hat{x}_A)F'(\hat{x}_B)\,(\theta-\Delta_A)\,(\lambda_B - \phi - \Delta_B)} > 0. \tag{38}$$

The inequality is due to $d\Delta_B/d\lambda_B = u'(1-\eta_A)/u'(\eta_A) > 0$ by Equation (7), and the denominator is positive by Assumption 2(ii). Plugging (37) into (23) yields

$$\frac{dW}{d\lambda_B} = F'(\hat{x}_B)\left[F(\hat{x}_A) + F(\hat{x}_B)F'(\hat{x}_A)\,(\theta - \Delta_A)\right]\frac{d\hat{x}_B}{d\lambda_B}. \tag{39}$$

Since $F(\hat{x}_B) < F(\hat{x}_A)$ by hypothesis, Assumption 2(ii) implies that the expression in square brackets is positive (note that $\theta - \Delta_A = \Omega_A$). Since $d\hat{x}_B/d\lambda_B > 0$, we have $dW/d\lambda_B > 0$.

(ii) It suffices to show that the expression in parenthesis in (24) is positive. Suppose $\lambda_B > \phi+\theta$. Then also $\lambda_A > \phi+\theta$ so (7) holds for both players, and we can write each player i's best-response function as

$$\Gamma_i(x) = F(x)\theta + (1-F(x))\,\Delta_i. \tag{40}$$

Differentiating the equilibrium conditions yields, using (40),

$$\frac{d\hat{x}_A}{d\lambda_B} = F'(\hat{x}_B)\frac{d\hat{x}_B}{d\lambda_B}(\theta - \Delta_A) + (1 - F(\hat{x}_B))\frac{d\Delta_A}{d\lambda_B} \tag{41}$$

and

$$\frac{d\hat{x}_B}{d\lambda_B} = F'(\hat{x}_A)\frac{d\hat{x}_A}{d\lambda_B}(\theta - \Delta_B) + (1 - F(\hat{x}_A))\frac{d\Delta_B}{d\lambda_B}. \tag{42}$$

Equation (40) implies that $\Delta_A = \Delta_B = \Delta$ at a balanced equilibrium. Adding equations (41) and (42) yields

$$\frac{d\hat{x}_A}{d\lambda_B} + \frac{d\hat{x}_B}{d\lambda_B} = (1 - F(\hat{x}))\frac{\frac{d\Delta_A}{d\lambda_B} + \frac{d\Delta_B}{d\lambda_B}}{1 - F'(\hat{x})(\theta - \Delta)}. \tag{43}$$

Assumption 2(ii) implies the denominator on the right-hand side of (43) is strictly positive. The numerator is also strictly positive. To see this, note that Equation (7) implies

$$\frac{d\Delta_A}{d\lambda_B} = -\frac{u'(1-\eta_B)}{u'(\eta_B)} < 0 \tag{44}$$

and

$$\frac{d\Delta_B}{d\lambda_B} = \frac{u'(1-\eta_A)}{u'(\eta_A)} > 0. \tag{45}$$

As $\lambda_B < \lambda_A$ we have $\eta_A > \eta_B$, and since u is strictly concave, (45) is greater in absolute value than (44). Thus, (43) is strictly positive, and so is (24). This proves part (ii) for the case $\lambda_B > \phi + \theta$. The proof for the case $\lambda_B \leq \phi + \theta$ is similar, and is omitted.

(iii) Suppose $\lambda_B > \max\{\phi + \theta, 1 - \phi - \theta\}$ so $\lambda_A = 1 - \lambda_B < \phi + \theta$. Since $\lambda_A < \lambda_B$ and $\omega_A > \omega_B$ it is obvious that player B is more likely to challenge, $\hat{x}_B > \hat{x}_A$. A similar argument to part (i) implies $dW/d\lambda_B < 0$.

Remark 1 *In the proof of part (ii), the condition for a balanced equilibrium is $\Delta_A = \Delta_B$. Intuitively, player i's incentive to challenge the status quo depends on how much increasing his territory from ω_i to $1 - \eta_j$ would be worth to him. Thus, if player A controls more territory ($\omega_A > \omega_B$), in a balanced equilibrium this perfectly reflects his greater military strength ($1 - \eta_B > 1 - \eta_A$), so the status quo is in this sense legitimate.*

Remark 2 *If the game has strategic complements, then (37) and (38) imply that both* $\hat{x}_A$ *and* $\hat{x}_B$ *start to fall when the rising power gets sufficiently strong. With strategic substitutes,* $\hat{x}_A$ *and* $\hat{x}_B$ *move in opposite directions. But even then, the proof of part (ii) uses* $F(\hat{x}_B) > F(\hat{x}_A)$ *to show that the expression in square brackets in (39) is positive. That is, the decrease in* $\hat{x}_A$ *dominates the increase in* $\hat{x}_B$*, reducing the probability of conflict.*

11.2 Proof of Proposition 9

Player i's best-response function is

$$\Gamma_i(x) = \Omega_i F(x) + u(1-\eta) - u(\hat{\omega}_i) \tag{46}$$

where

$$\Omega_i = \frac{1}{2}u(0) + \frac{1}{2}u(1) - u(\eta) - u(1-\eta) + u(\hat{\omega}_i) - \phi. \tag{47}$$

Notice that $\Omega_A > \Omega_B$ as $\hat{\omega}_A > \hat{\omega}_B$. Differentiating the equilibrium conditions $\hat{x}_i = \Gamma_i(\hat{x}_j)$, using the fact $\hat{\omega}_B = 1 - \hat{\omega}_A$, yields

$$\frac{d\hat{x}_A}{d\hat{\omega}_A} = \Omega_A F'(\hat{x}_B)\frac{d\hat{x}_B}{d\hat{\omega}_A} - (1 - F(\hat{x}_B))\, u'(\hat{\omega}_A)$$

and

$$\frac{d\hat{x}_B}{d\hat{\omega}_A} = \Omega_B F'(\hat{x}_A)\frac{d\hat{x}_A}{d\hat{\omega}_A} + (1 - F(\hat{x}_A))\, u'(\hat{\omega}_B).$$

We solve to obtain

$$\frac{d\hat{x}_A}{d\hat{\omega}_A} = \frac{\Omega_A F'(\hat{x}_B)\,(1 - F(\hat{x}_A))\, u'(\hat{\omega}_B) - (1 - F(\hat{x}_B))\, u'(\hat{\omega}_A)}{1 - \Omega_A \Omega_B F'(\hat{x}_A) F'(\hat{x}_B)} \tag{48}$$

and

$$\frac{d\hat{x}_B}{d\hat{\omega}_A} = \frac{(1 - F(\hat{x}_A))\, u'(\hat{\omega}_B) - \Omega_B F'(\hat{x}_A)\,(1 - F(\hat{x}_B))\, u'(\hat{\omega}_A)}{1 - \Omega_A \Omega_B F'(\hat{x}_A) F'(\hat{x}_B)}. \tag{49}$$

We have

$$\begin{aligned} &\frac{d\hat{x}_B}{d\hat{\omega}_A} - \frac{d\hat{x}_A}{d\hat{\omega}_A} \\ &= \frac{(1 - F(\hat{x}_A))\,[1 - \Omega_A F'(\hat{x}_B)]\, u'(\hat{\omega}_B) + (1 - F(\hat{x}_B))\,[1 - \Omega_B F'(\hat{x}_A)]\, u'(\hat{\omega}_A)}{1 - \Omega_A \Omega_B F'(\hat{x}_A) F'(\hat{x}_B)} > 0 \end{aligned} \tag{50}$$

as $|\Omega_A F'(x_A)| < 1$ and $|\Omega_B F'(x_B)| < 1$ by Assumption 2(ii). Because $\hat{x}_A = \hat{x}_B$ when $\hat{\omega}_A = \hat{\omega}_B$, (50) implies than $\hat{x}_B > \hat{x}_A$ when $\hat{\omega}_A > \hat{\omega}_B$. That is, since his endowment is smaller, player B is more likely to choose H (whether actions are strategic complements or substitutes). Moreover, (49) is strictly positive, as $F(\hat{x}_A) < F(\hat{x}_B)$, $|\Omega_B F'(\hat{x}_A)| < 1$ and $u'(\hat{\omega}_B) \geq u'(\hat{\omega}_A)$ by concavity.

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